Undulators as axion/dark photon factories


https://nanoterasu.jp/

TOKYO METROPOLITAN UNIVERSITY
東京都立大学

Based on WY, J. Yoshida, 2408.17451
WY, in preparation

@2025/6/19, The Frontier of Particle Physics:
Exploring Muons, Quantum Science and the Cosmos

Wen Yin (Tokyo Metropolitan University)

Contents

- 1. Introduction

Axion (Axion-like particle)

$$\mathscr{L}_{\rm int} = -\frac{g_{\phi\gamma\gamma}}{4}\phi\epsilon^{\mu\nu\delta\epsilon}F_{\mu\nu}F_{\delta\epsilon} = -g_{\phi\gamma\gamma}\phi\vec{E}\cdot\vec{B}$$

- explains the dark matter, the inflation, and/or strong CP, etc.

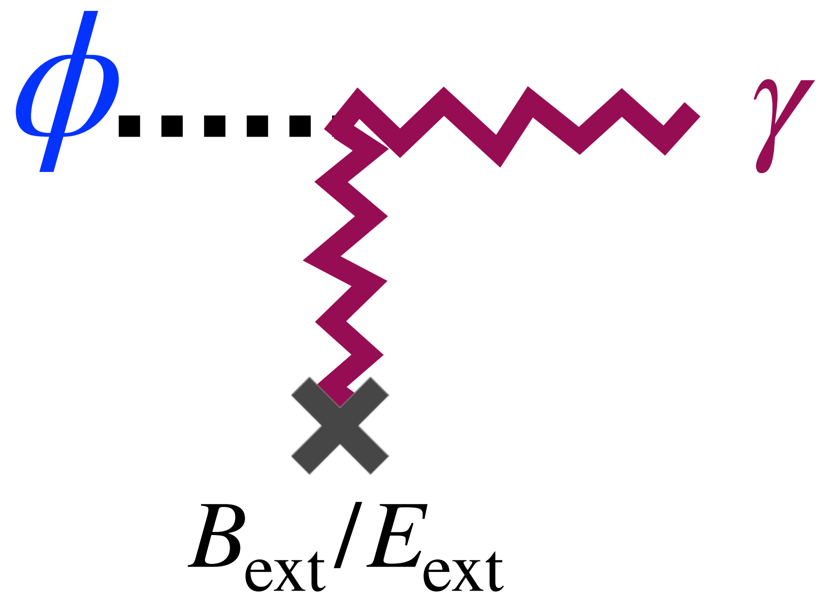
e.g. Daido, Takahashi, WY 1702.03284; Narita, Takahashi, WY, 2308.12154; Takahashi WY 2301.10757 etc for ``and”.

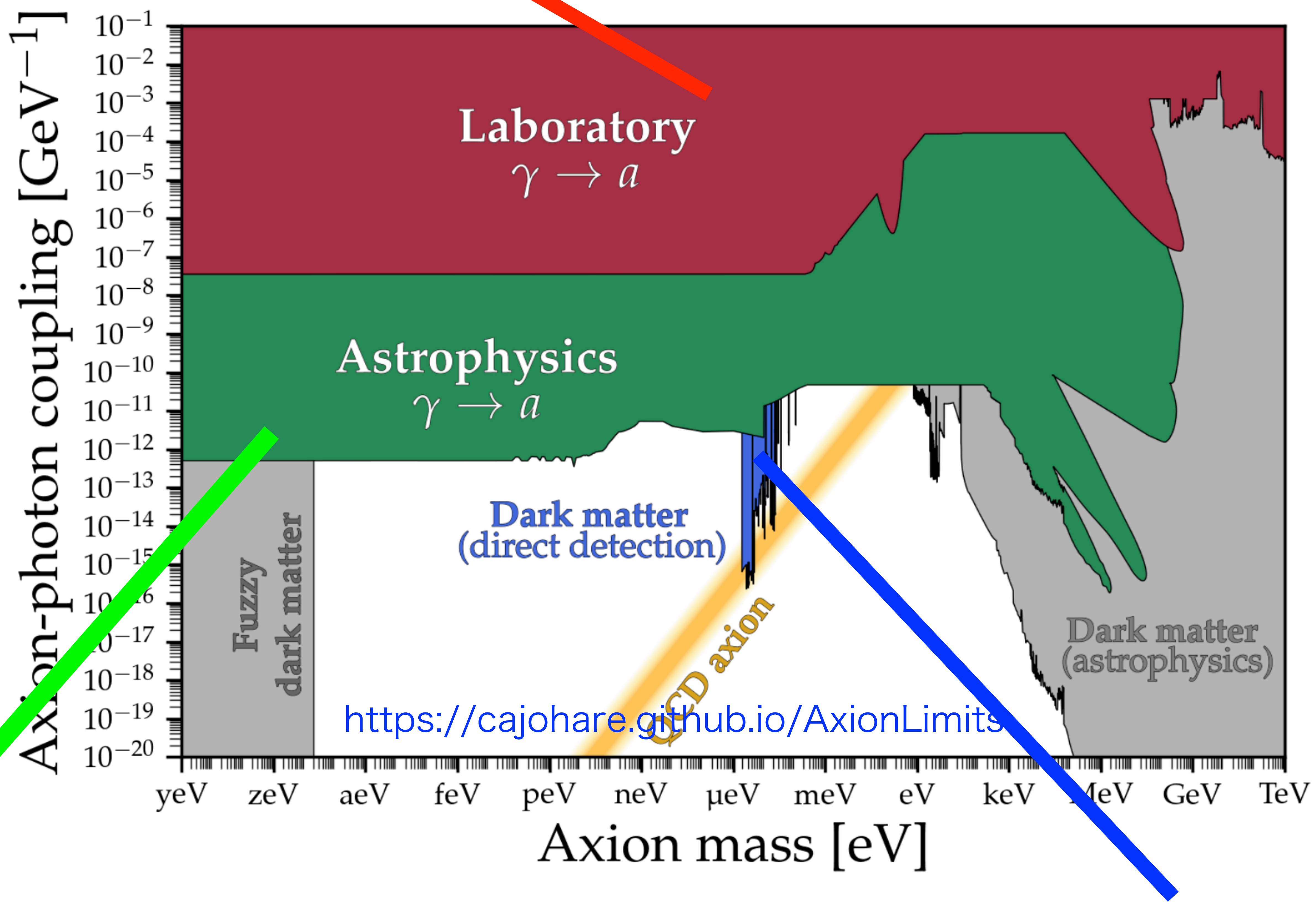
- appears as a remnant of extra dimension, or as a pseudo-Nambu-Goldstone boson.
- mass is generated with some non-perturbative effects and scales exponentially.

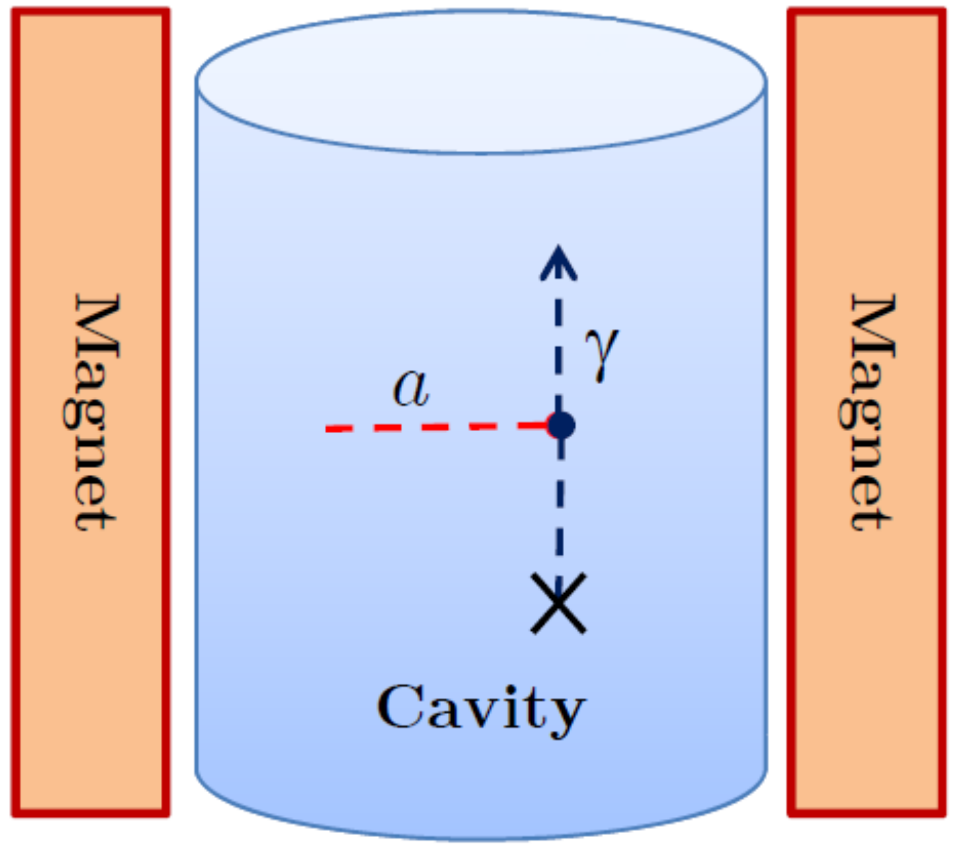
Search of light axions with axion-photon conversion

- Laboratory produced axion

To be discussed







- Astrophysically produced axion

Supernova axion + (extra)Galactic magnetic fields.
Solar axion + magnets by hand etc

- Cosmologically produced axion

DM axion + magnets/cavity
by hand, etc

Search of light axions with axion-photon conversion

- Laboratory produced axion

- Both production and detection are conducted in laboratories.
- Usually weaker limit. c.f. ALPS-II
- **Less systematics** and thus **more robust**.

There is also a series of models that can only be probed in laboratories.

Masso and Redondo, 0504202; Jaeckel et al. 0610203; Brax, 0703243

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- Astrophysically produced axion
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Robustness

Sensitivity

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Robustness

Today's talk

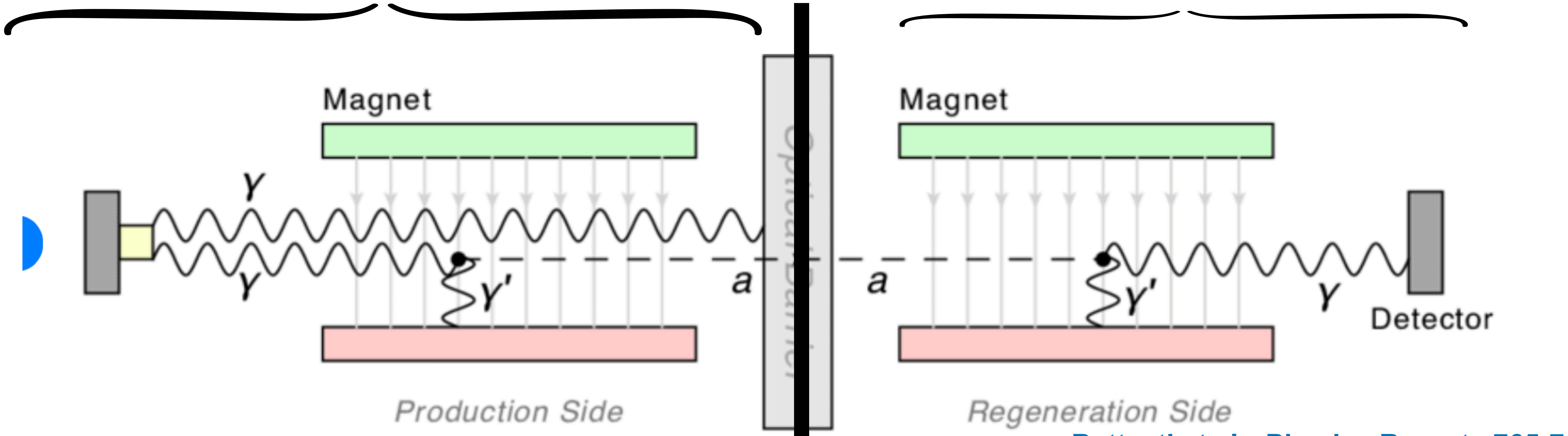
↕

Sensitivity

Light-Shining-Through-a-Wall (LSW) experiment

Sikivie,1983;Anselm,1985;VanBibber,1987;

Axion factory **Wall** **Axion detector**



Battesti et al Physics Reports 765-766(15)

- Light source
- Magnets

- Magnets
- Photon detector

See also experiments of vacuum birefringence, and laser collider experiments: DellaValle:2015xxa, SAPPHIRES:2022bqg

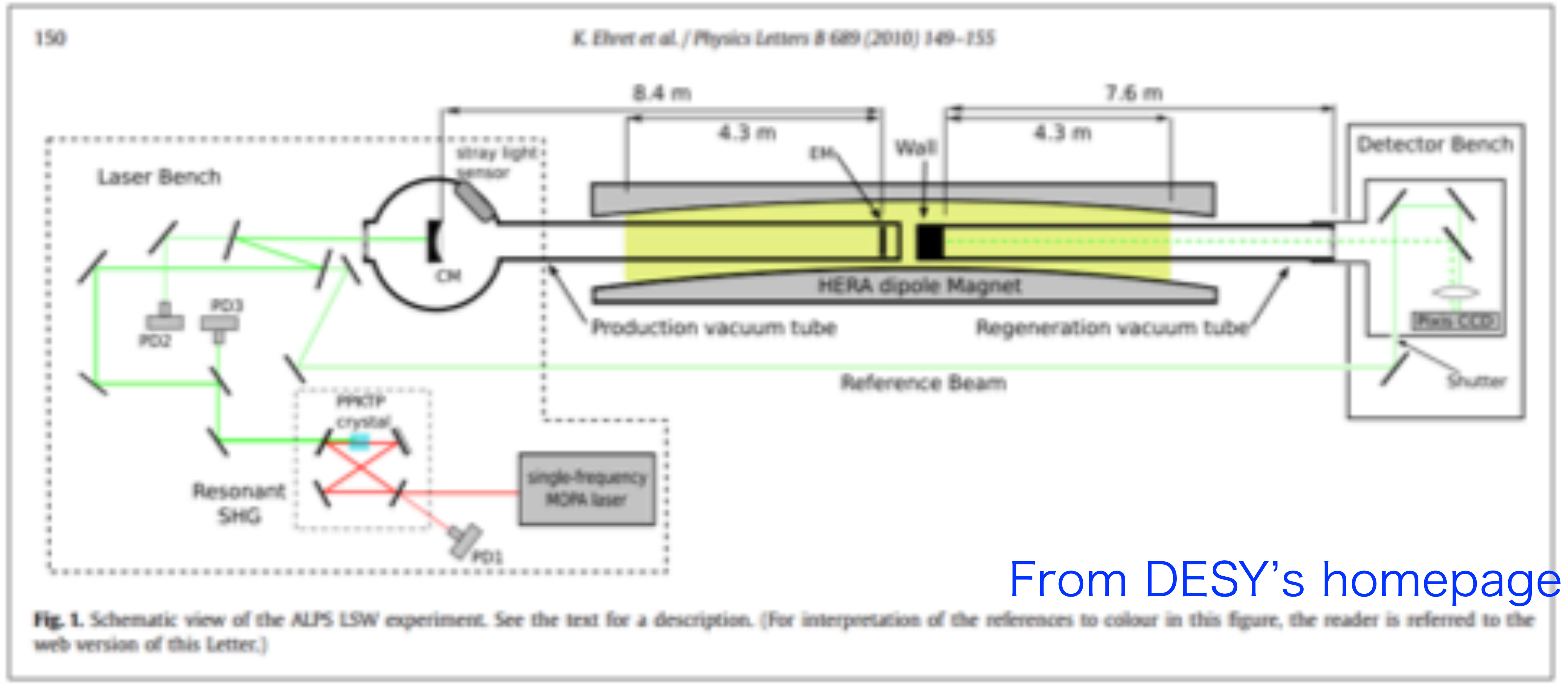
Examples of LSW experiments, $k = 0$.

Optical Laser as light source

Undulator as (X-ray) light source

Ehret et al, 1004.1313

The ALPS Collaboration @ DESY
- Light axion search by optical laser
- 5Tesla * 4.3m * 2sections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K. Ehret et al. / Physics Letters B 689 (2010) 149–155

Fig. 1. Schematic view of the ALPS LSW experiment. See the text for a description. (For interpretation of the references to colour in this figure, the reader is referred to the web version of this Letter.)

From DESY's homepage

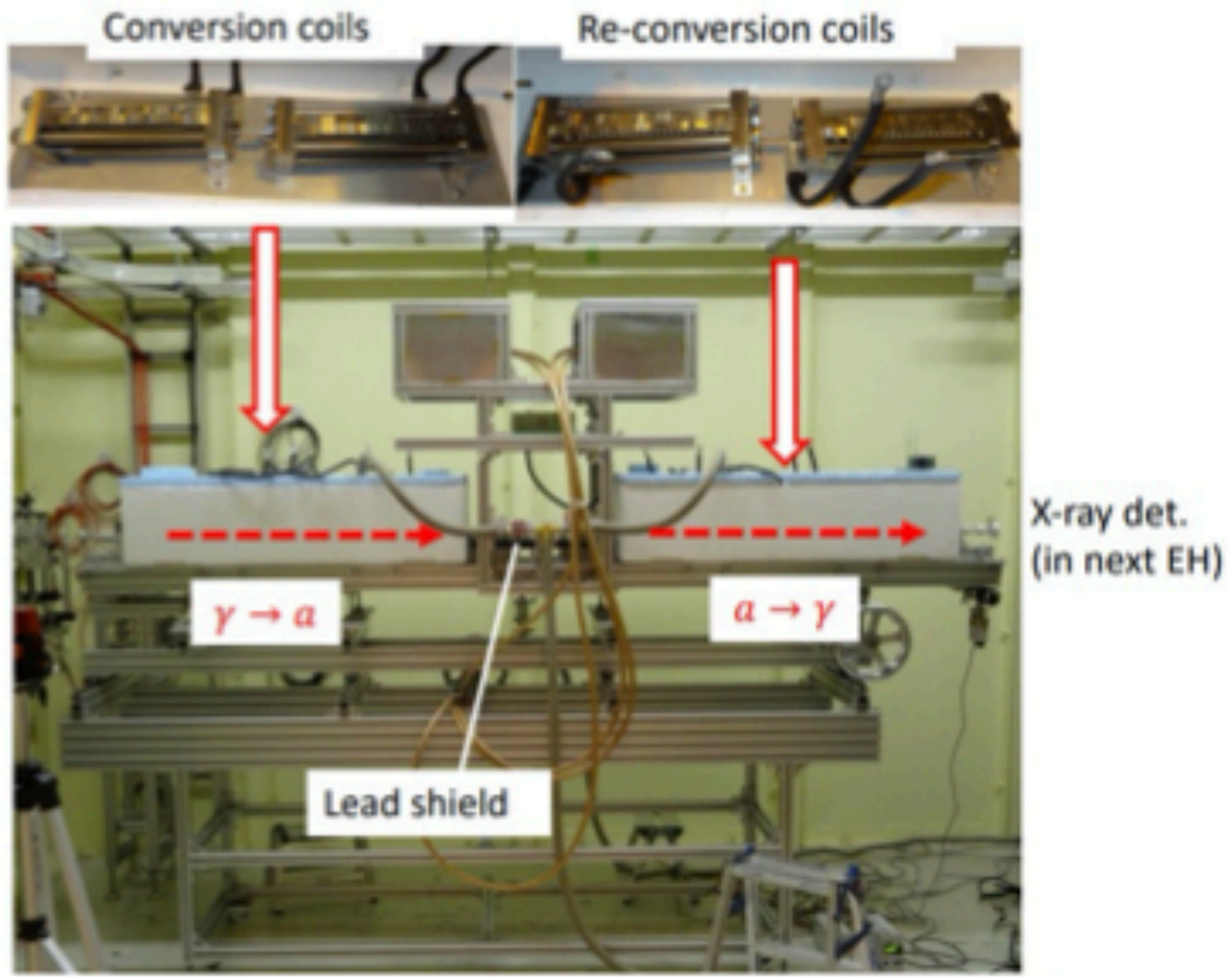
Upgraded ALPS II has 5.3Tesla * 120m * 2sections

ALPS II will run by O(years).

It will obtain a very powerful bound.

Taken (and modified) from J. Yoshida-san's slide

ICEPP, UT group @ SPring-8 BL19LXU
- 9.5 keV X ray for relatively heavy axion
- 14.1Tesla * 20cm * 2coils * 2sections
- Net 2 days operation (28,000 excitations * 1msec)



From slide of T. Namba

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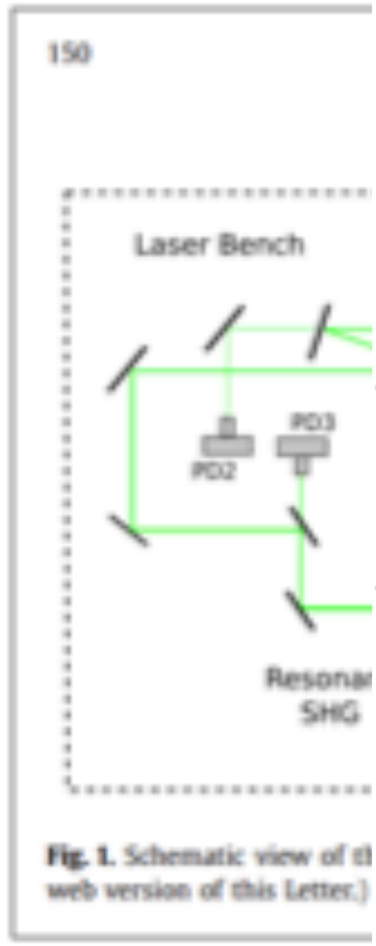
The ALP
- Light
- 5Tesla



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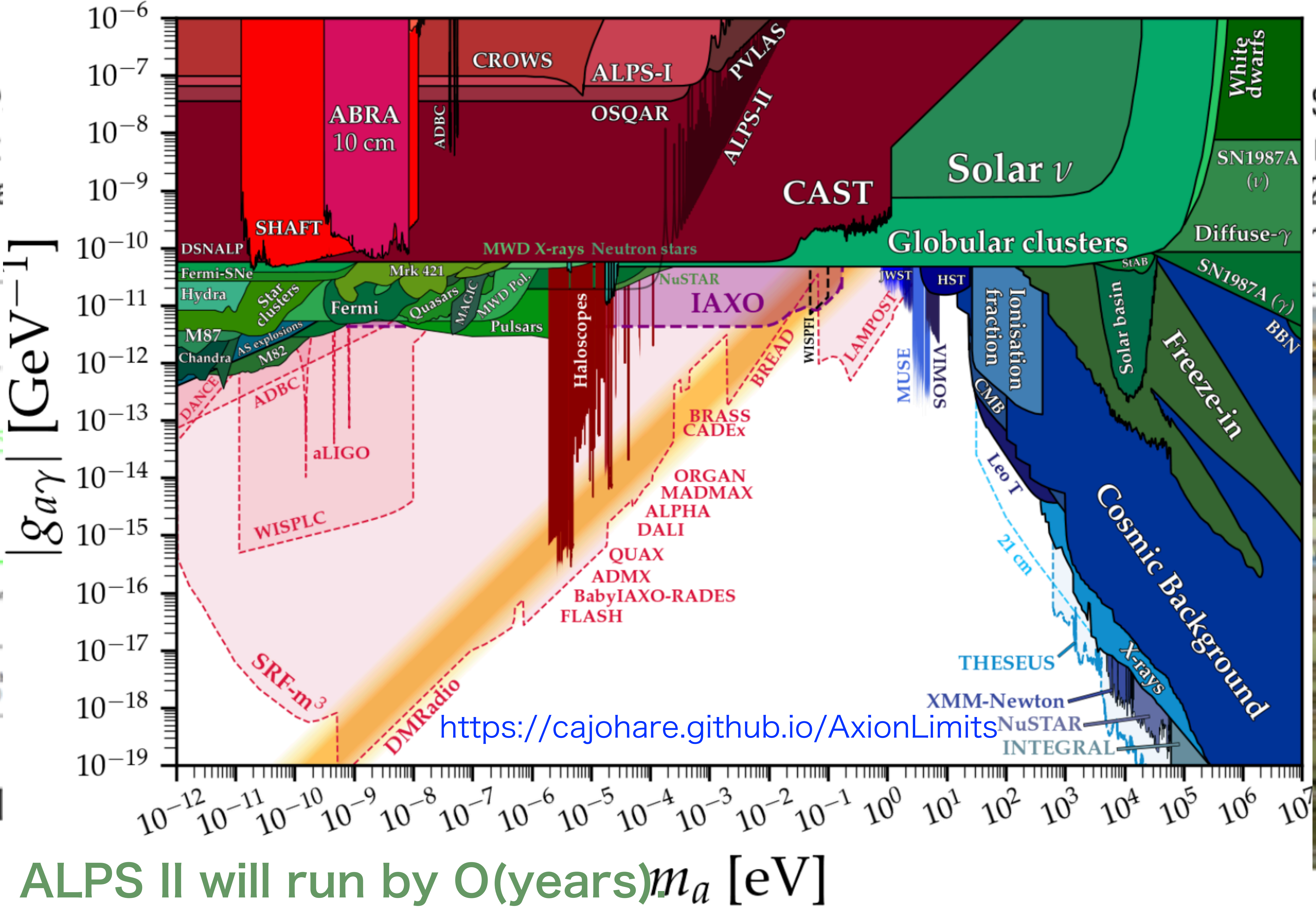
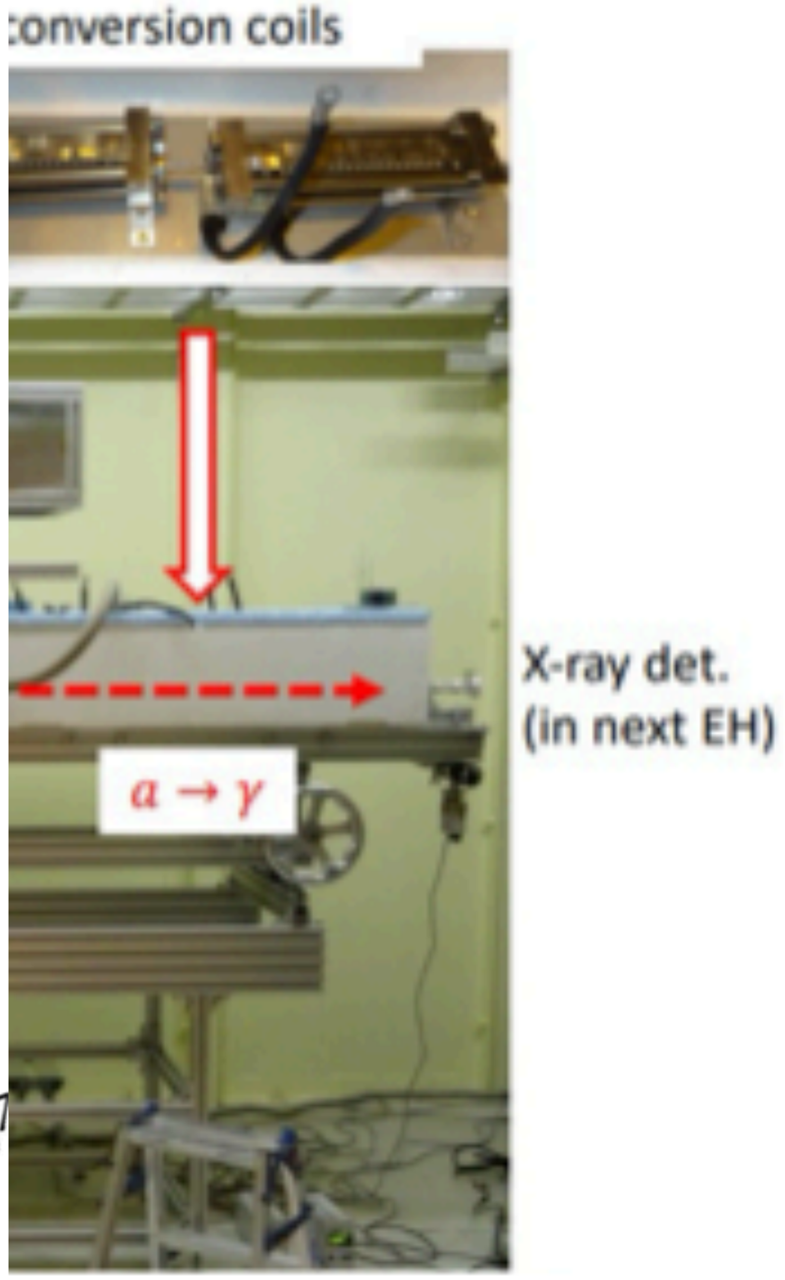


https://cajohare.github.io/AxionLimits

SPring-8 BL19LXU
relatively heavy axion
2coils * 2sections
n (28,000 excitations * 1msec)



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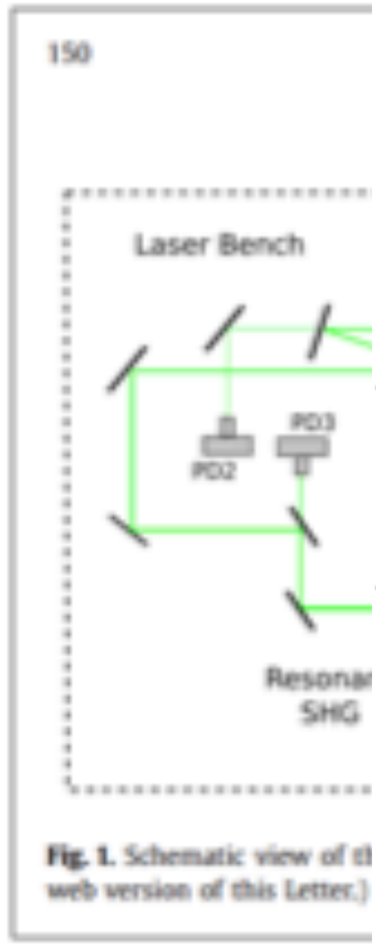

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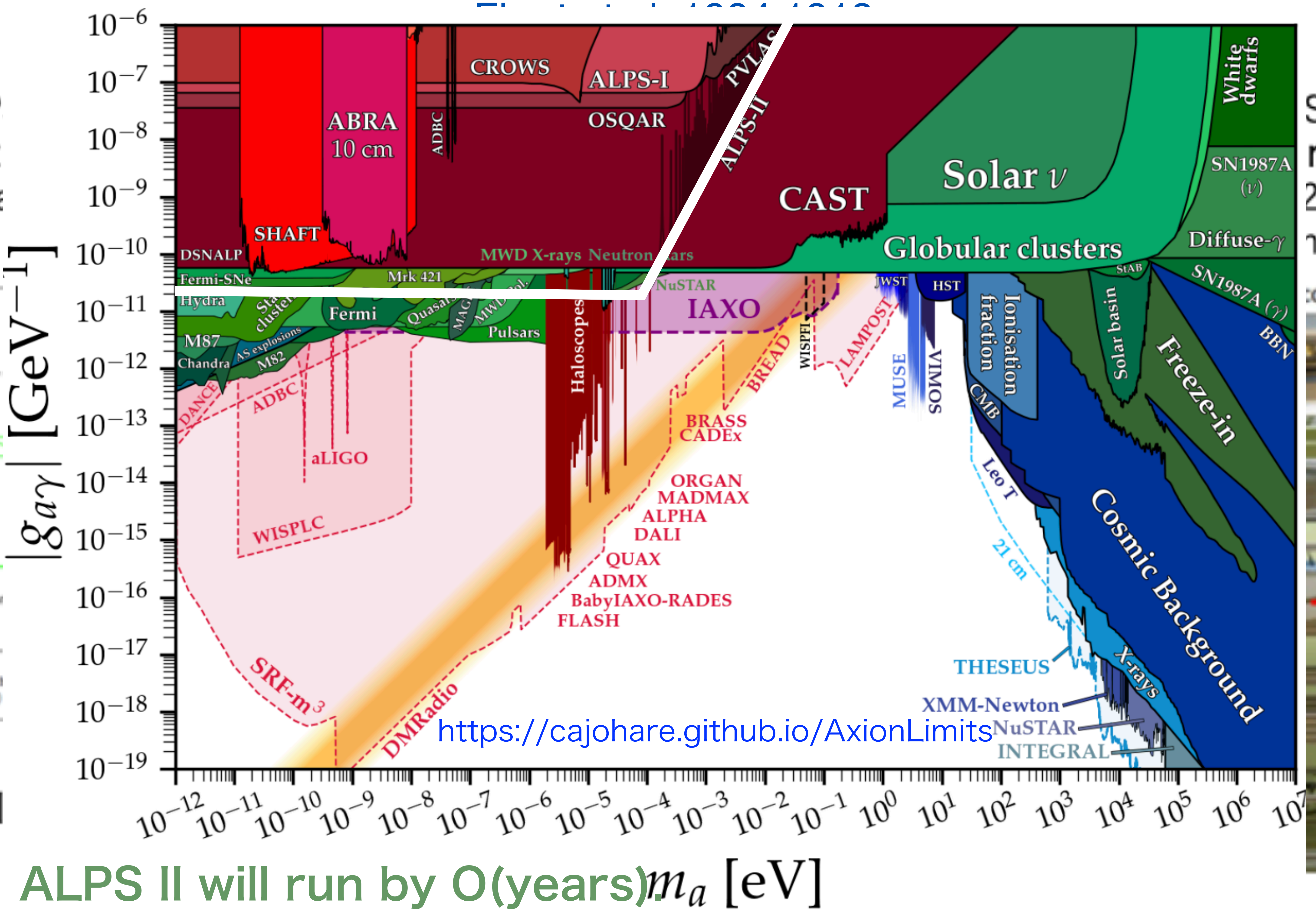

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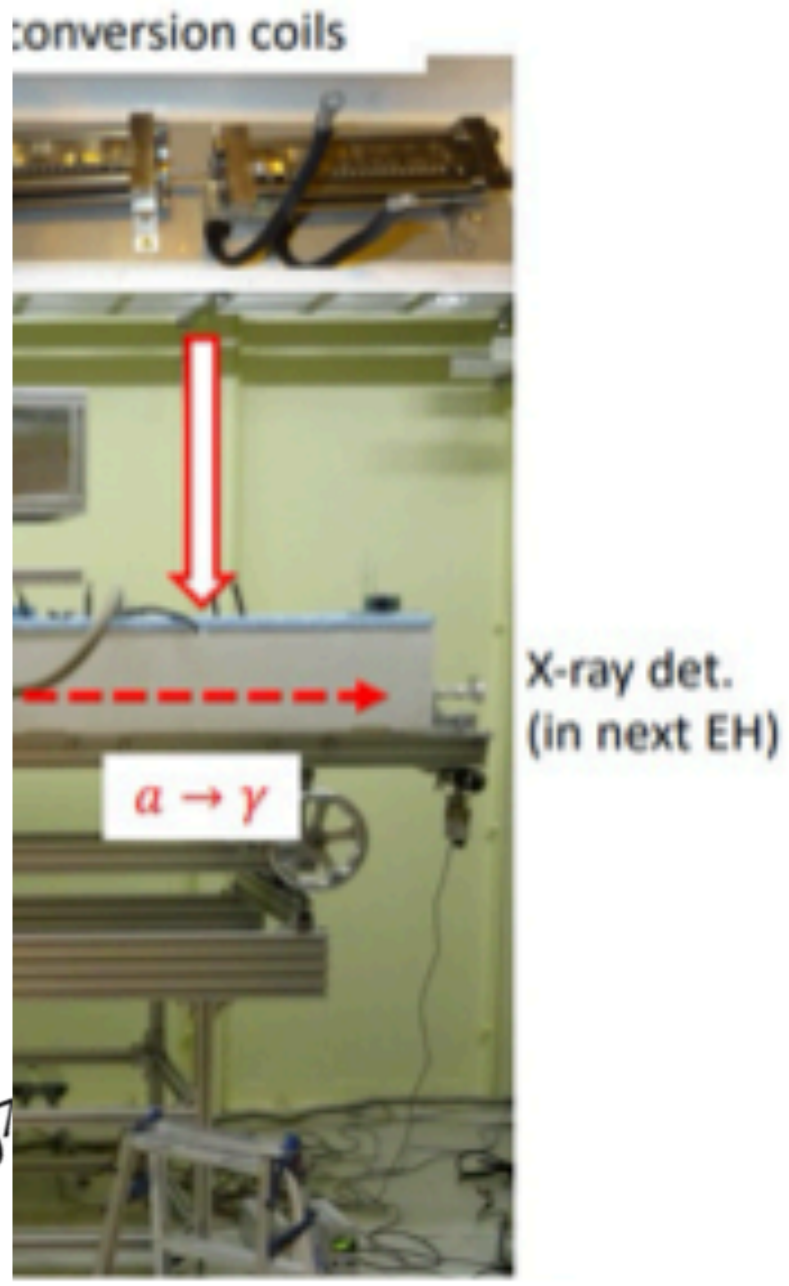

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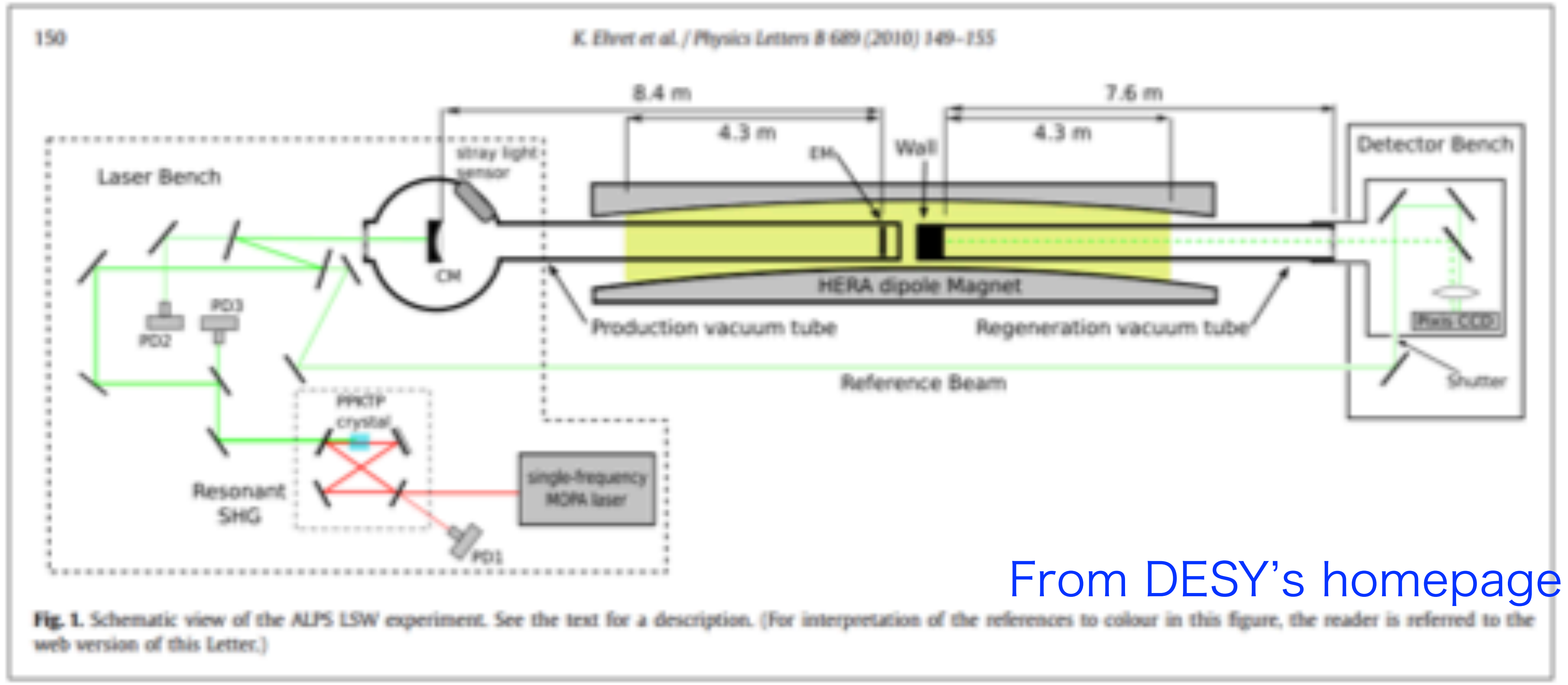

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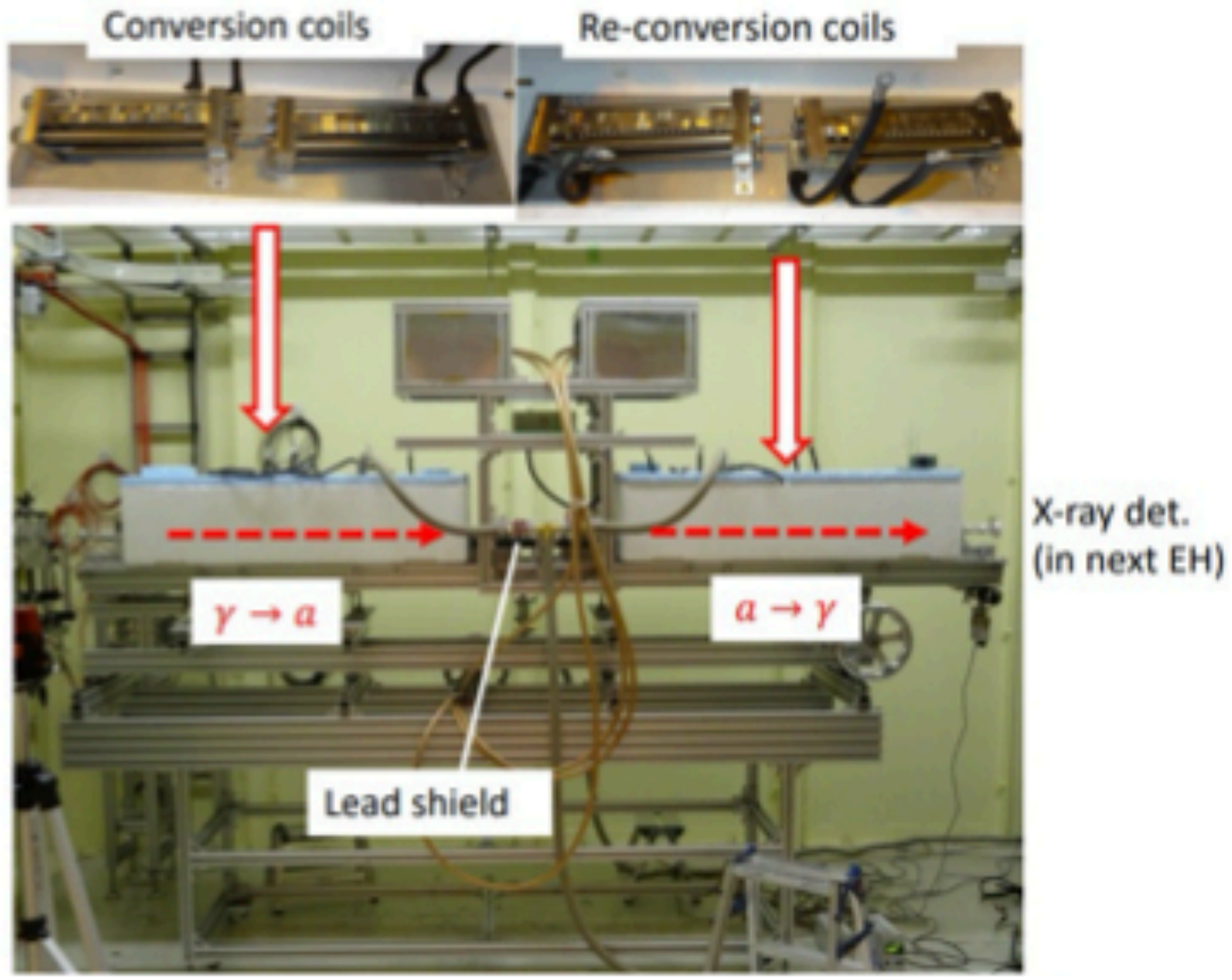

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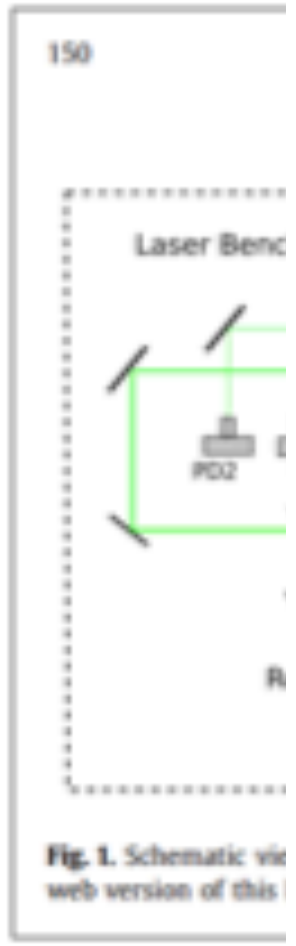

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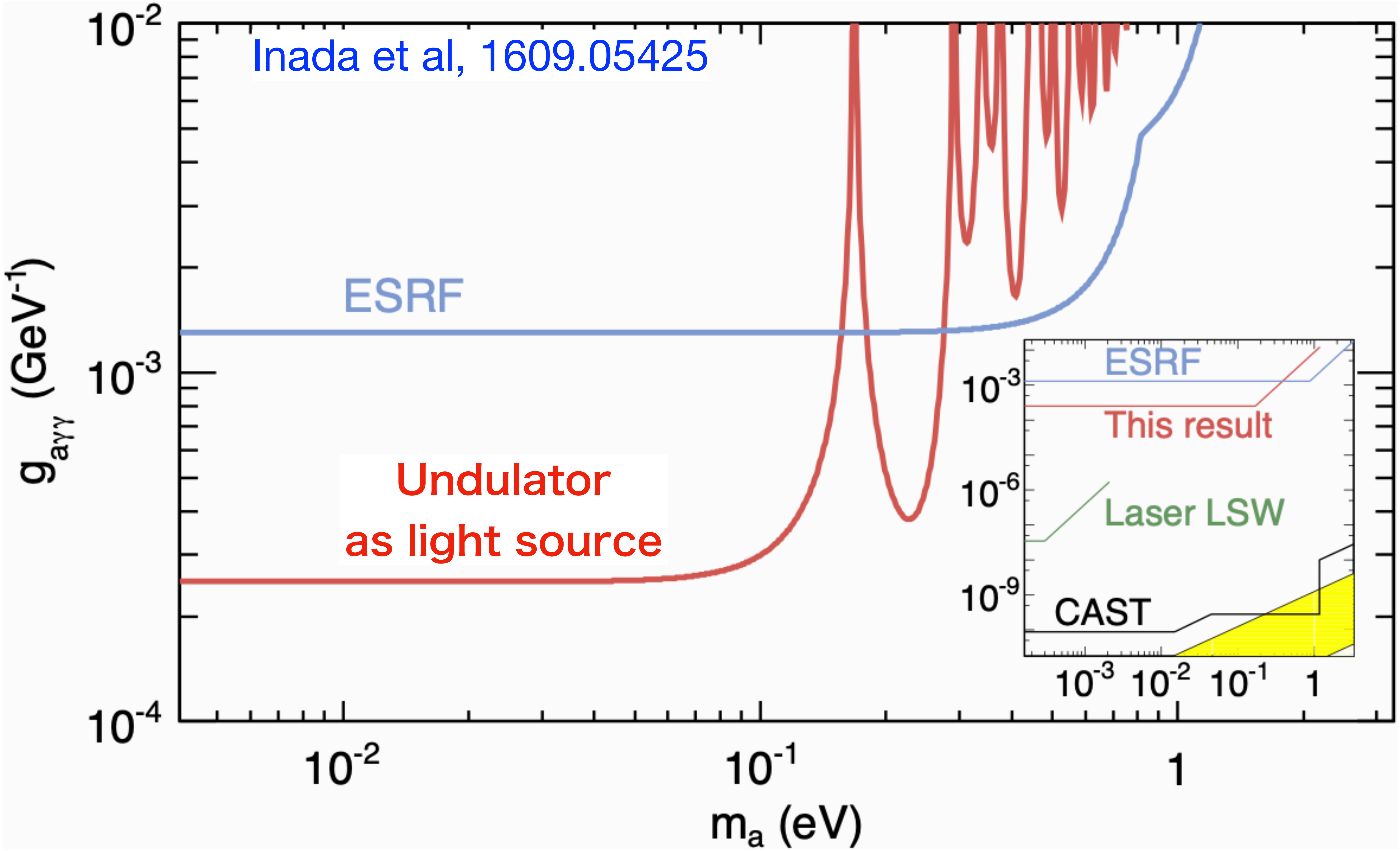

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From slide of T. Namba

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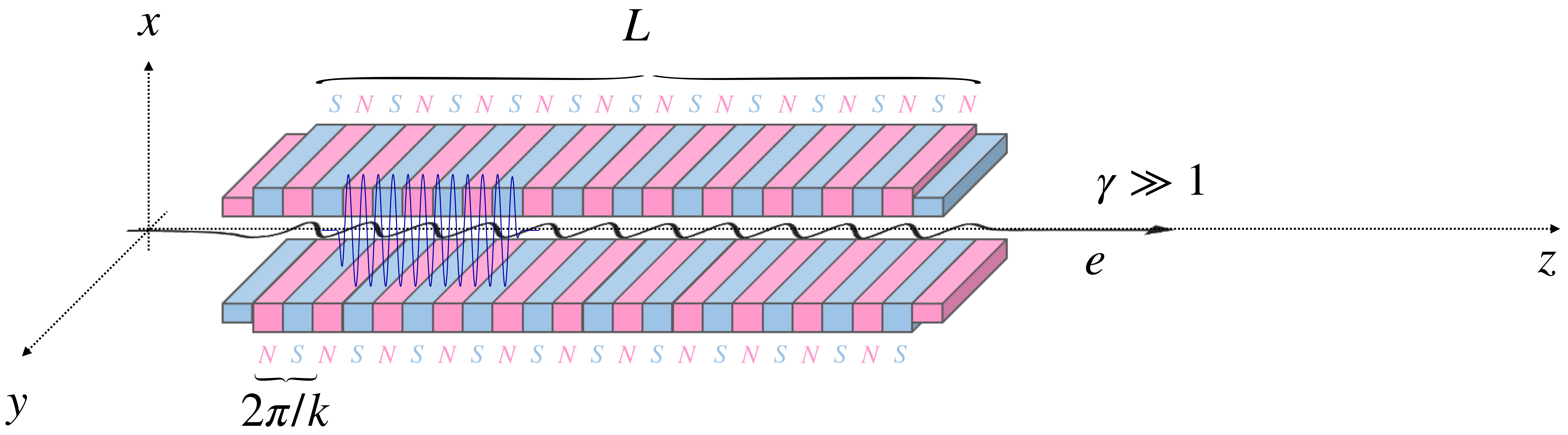
Taken (and modified) from J. Yoshida-san's slide

What is undulator?



- Intense and concentrated line-like photons.
- Controllable photon energy and polarization.
- Used in synchrotron radiation facilities (**NanoTerasu, Spring-8, and KEK photon factory (PF)** etc.), free-electron laser, and in the design of ILC, etc

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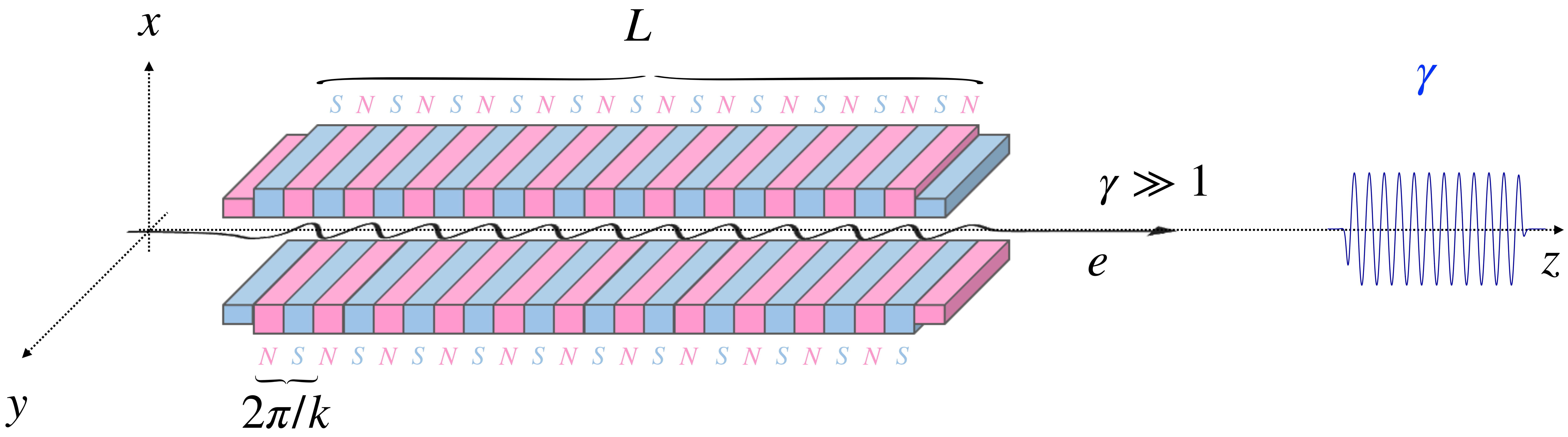

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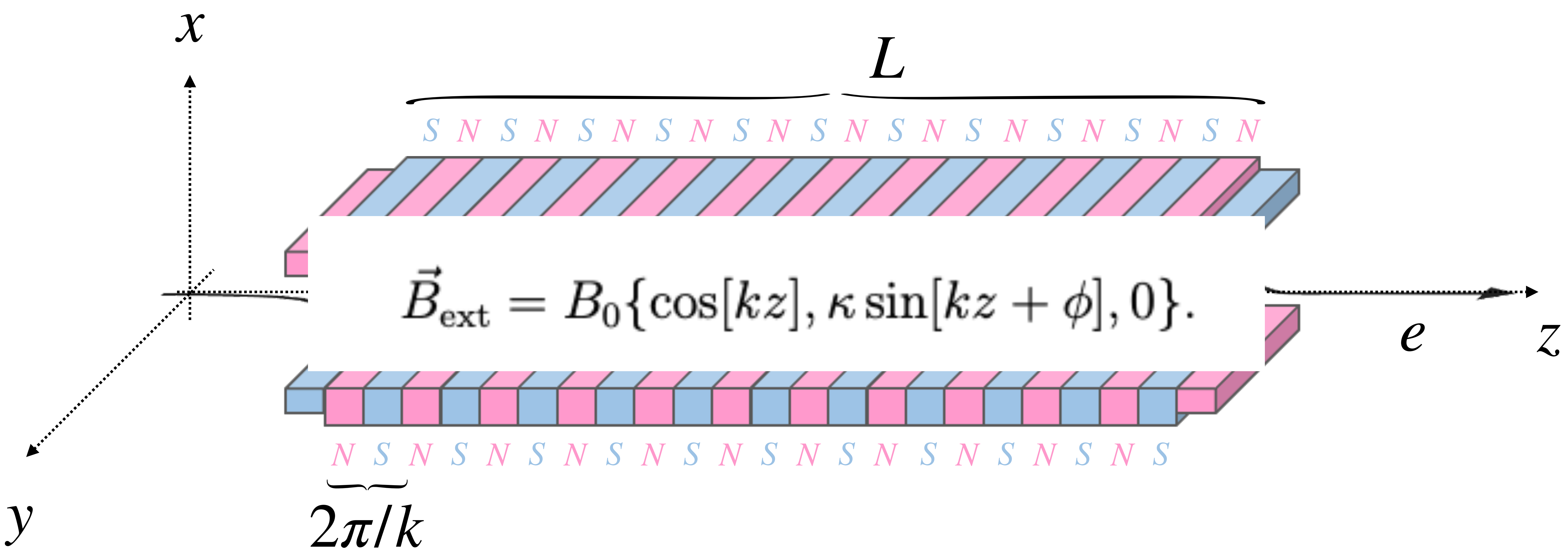
Modeling the undulator and electron motion



$$\vec{B}_{\rm ext} = B_0\{\cos[kz], \kappa\sin[kz+\phi], 0\}.$$

$$\vec{\beta} \simeq \left(-\frac{K\kappa}{\gamma}\cos[k\beta^z t_e + \phi], -\frac{K}{\gamma}\sin[k\beta^z t_e], \beta^z\right), \qquad \beta^z \simeq \sqrt{1-\gamma^{-2}\left(1+\frac{1}{2}(1+\kappa^2)K^2\right)},$$

$$\boxed{K \equiv \frac{B_0 e}{k m_e}.}$$

With $K \lesssim 1$, it is called undulator. ($K \gg 1$ is called Wigler.)

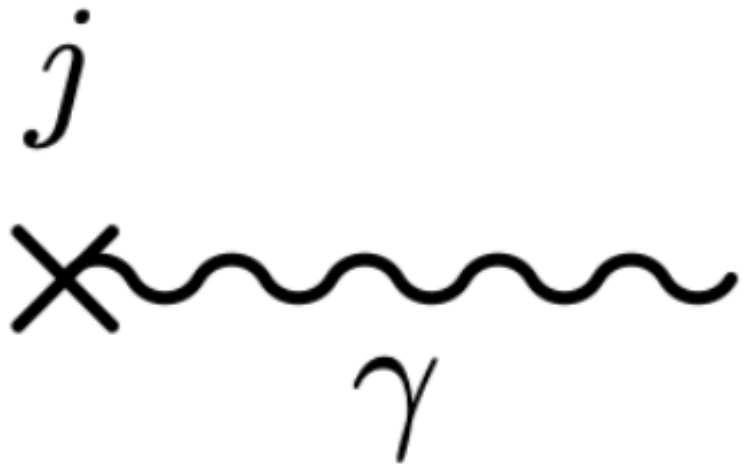
$\kappa = 1, \phi = 0$ Helical undulator

$\kappa = 0$ Linear undulator(Fig)

Synchrotron radiation for particle theorist: QFT Approach, c.f. textbooks

$$\mathscr{L}_{\text{int}} = -A^\mu j_\mu, \quad j_\mu = -ev_\mu(t)\delta^3(\vec{x} - \vec{r}_e[t]), \qquad v^\mu = (1, \vec{\beta}_e)$$

- (1)Draw Feynman Diagram



- (2)Calculate the amplitude

$$\delta_X(q) \equiv \int_0^X dx e^{iqx}$$

By expanding K/γ and seeing the leading term, we get

$$\sqrt{2w_\gamma}\frac{{}_{\text{out}}\langle \gamma, \epsilon, k_\gamma | 0_j \rangle_{\text{in}}}{\pi} \simeq i\frac{eK}{2k\gamma}\left(k\left(\kappa e^{-i\phi}\epsilon^x + i\epsilon^y\right) + \epsilon^z\left(\kappa e^{-i\phi}k_\gamma^x + ik_\gamma^y\right)\beta^z\right)\delta_{\frac{L}{\beta^z}}\left(w_\gamma(1 - \beta^z\cos[\theta]) - k\beta^z\right) + O(K^2)\,.$$

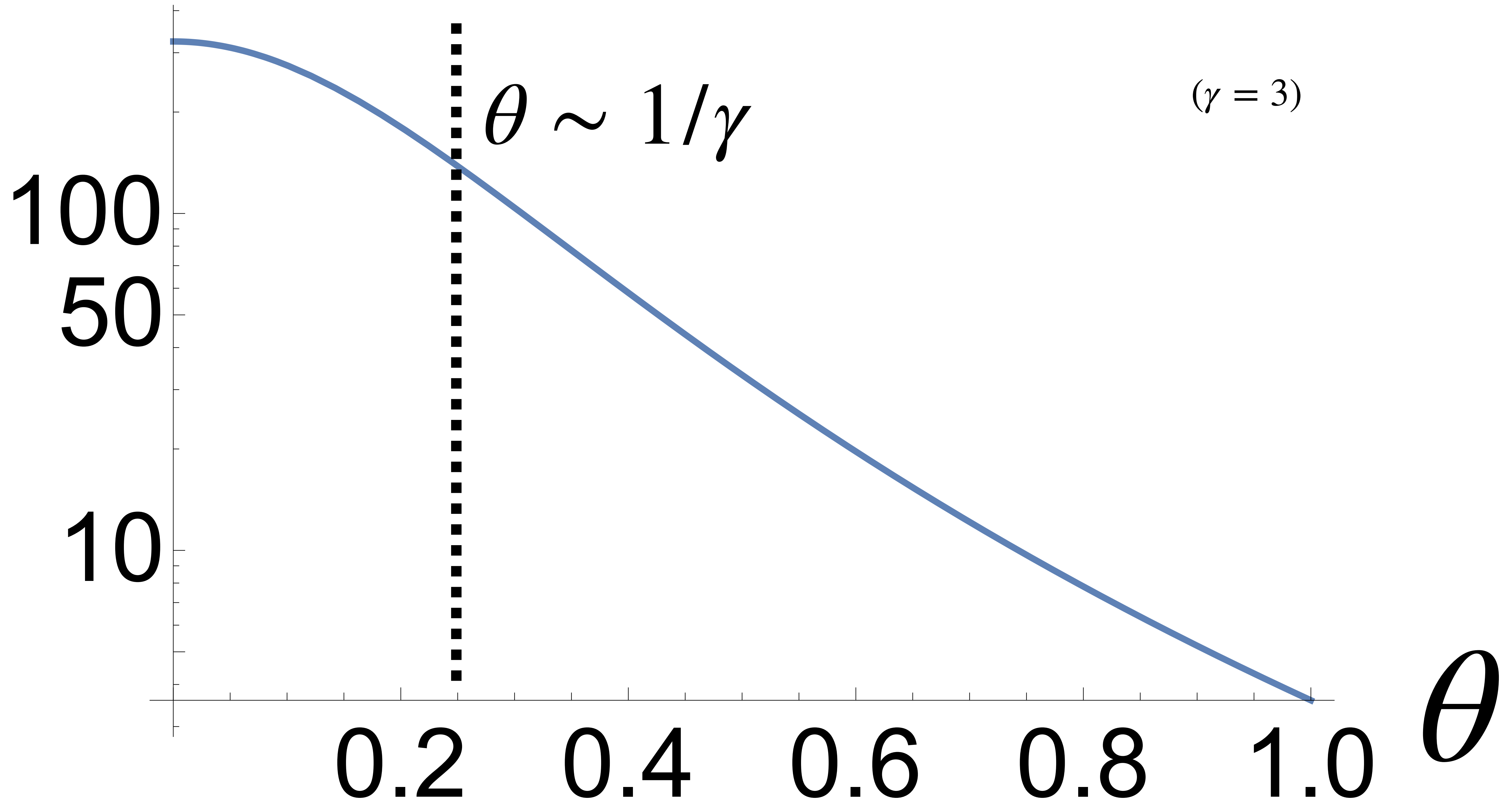
$$\to w_\gamma = \frac{k\beta^z}{1 - \beta^z\cos\theta}$$

- (3)Estimate photon production

$$\frac{dn}{d\Omega} \equiv \int \frac{dk_\gamma k_\gamma^2}{(2\pi)^3}\sum_\epsilon |\langle \gamma, \epsilon, \vec{k}_\gamma | 0_j \rangle|^2 \approx \frac{L}{\beta^z}\frac{e^2K^2k\beta_z}{16\pi^2\gamma^2}\frac{1}{(1 - \cos\theta\beta^z)^2}\left(1 - \frac{\gamma^{-2}\sin^2\theta + \mathcal{O}(\gamma^{-4})}{2(1 - \cos\theta\beta^z)^2}\right)$$

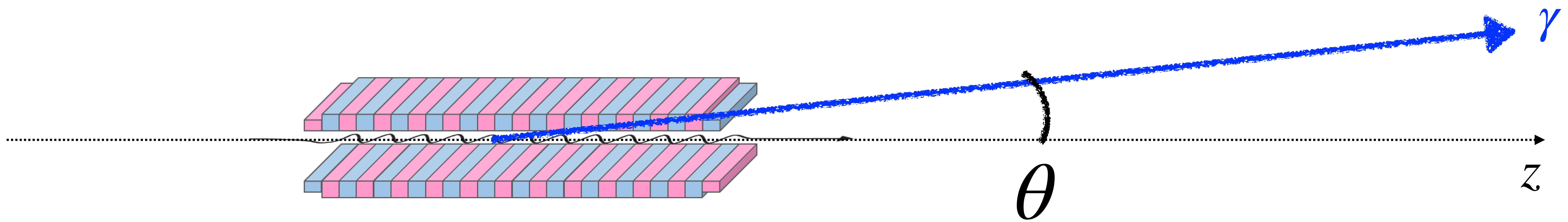
Agreeing with the conventional estimation!

➡ $$\frac{dn_\gamma}{d\Omega} \approx \frac{L}{\beta^z}\frac{e^2K^2k\beta_z}{16\pi^2\gamma^2}\frac{1}{(1-\cos\theta\beta^z)^2}$$



- Forward direction $< 1/\gamma$
- $w_\gamma = \dfrac{k\beta^z}{1-\beta^z\cos\theta} \simeq 2k\gamma^2$

(from delta-like function)

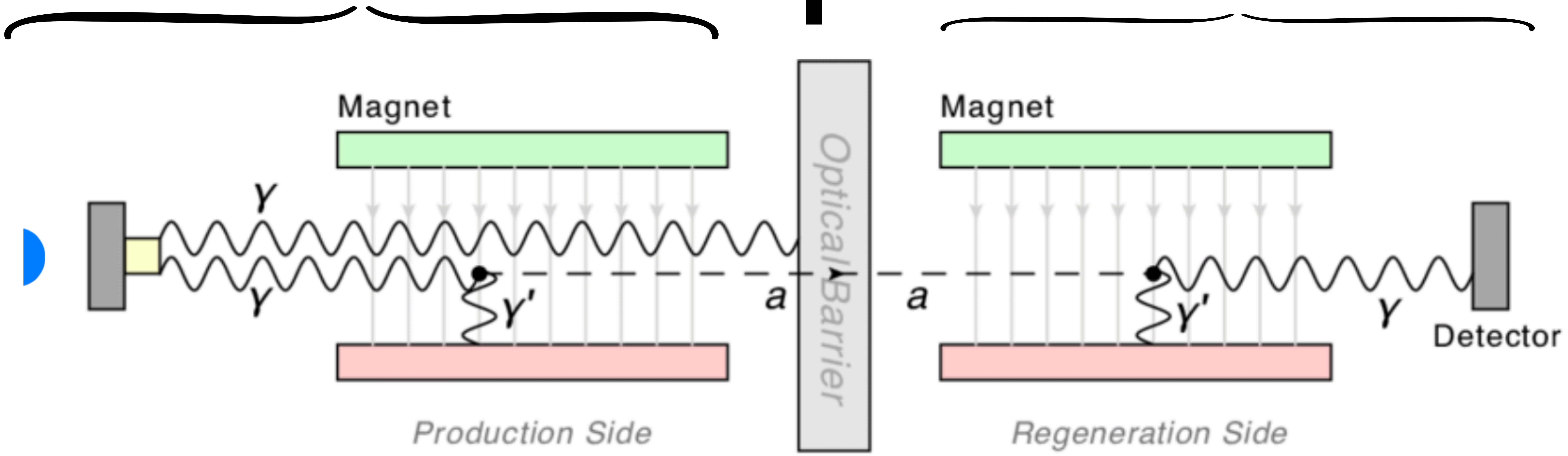


Is an undulator axion factory?

Axion factory

Wall

Axion detector



Battesti et al Physics Reports 765-766(15)

- Light source
- Magnets

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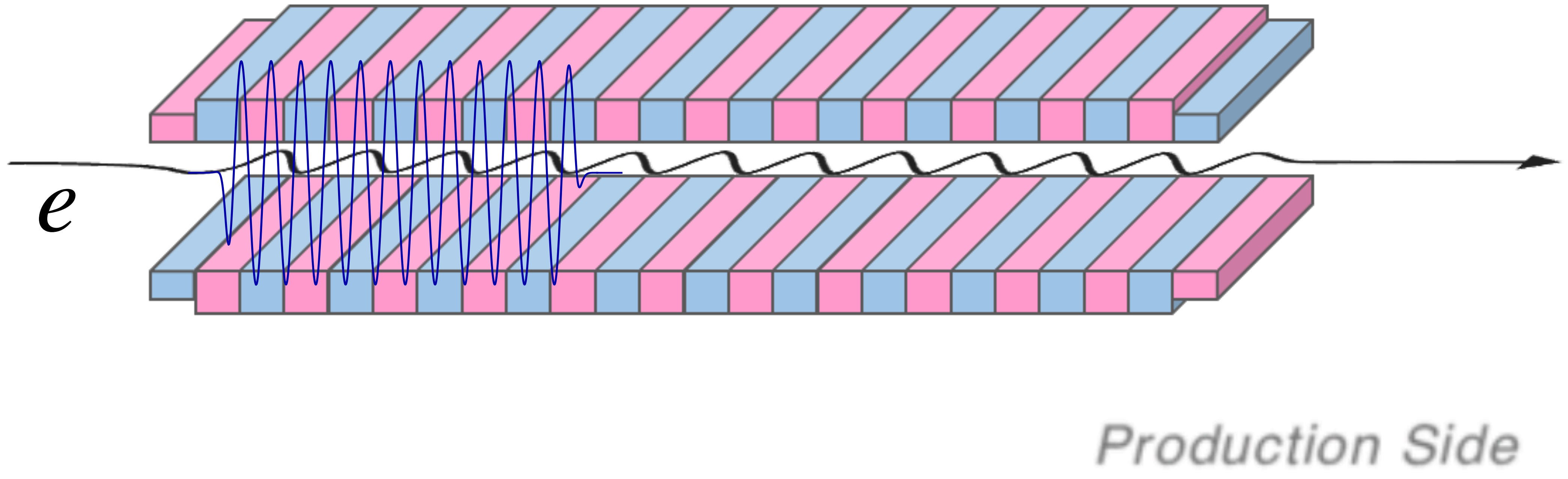
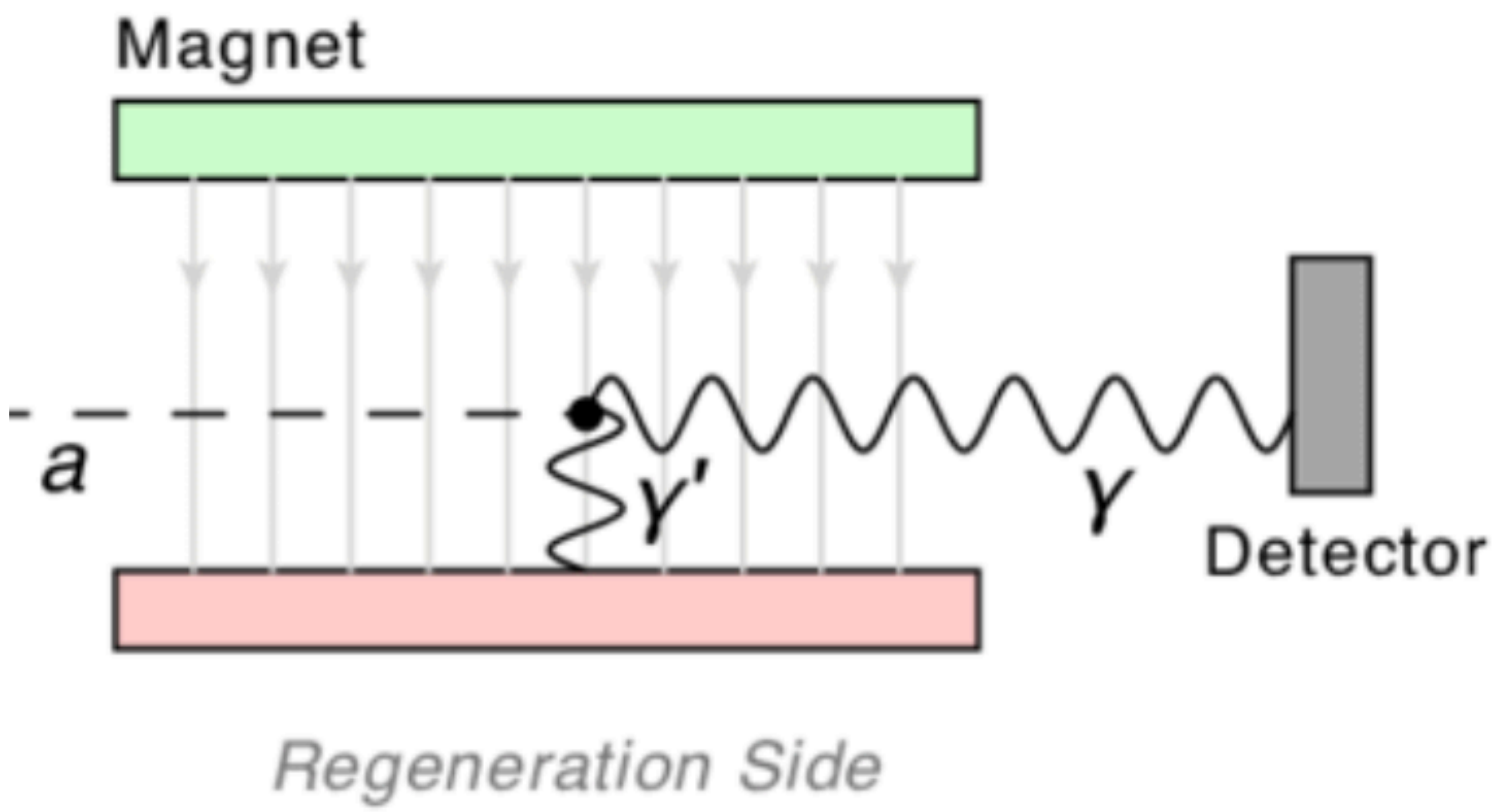
Axion factory | **Wall** | **Axion detector**



Sheild for facility



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An undulator in use satisfies the more than half of the condition of LSW

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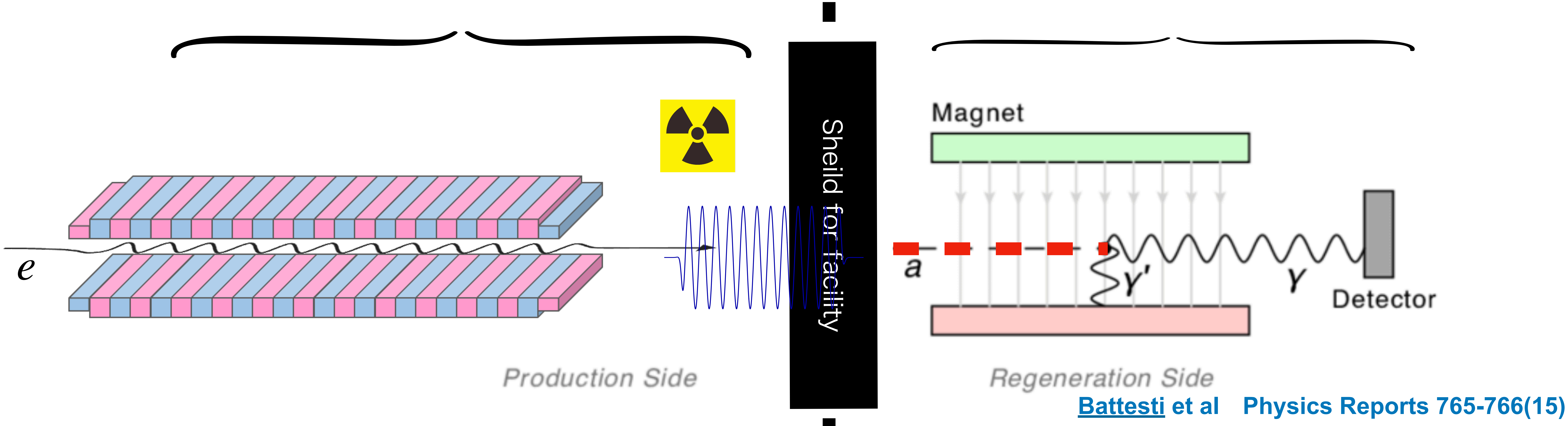
Axion factory **Wall** **Axion detector**



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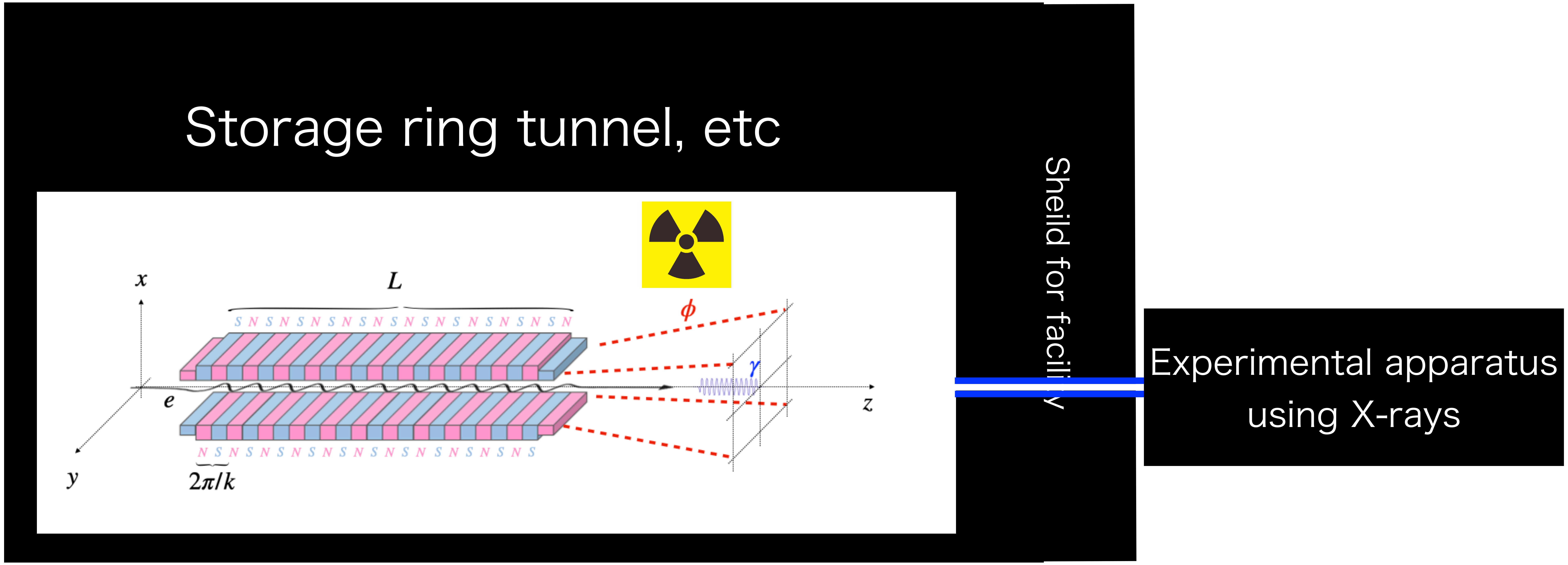
If an undulator is an axion factory,

WY, Junya Yoshida, arXiv:2408.17451



Storage ring tunnel, etc

Sheild for facili y

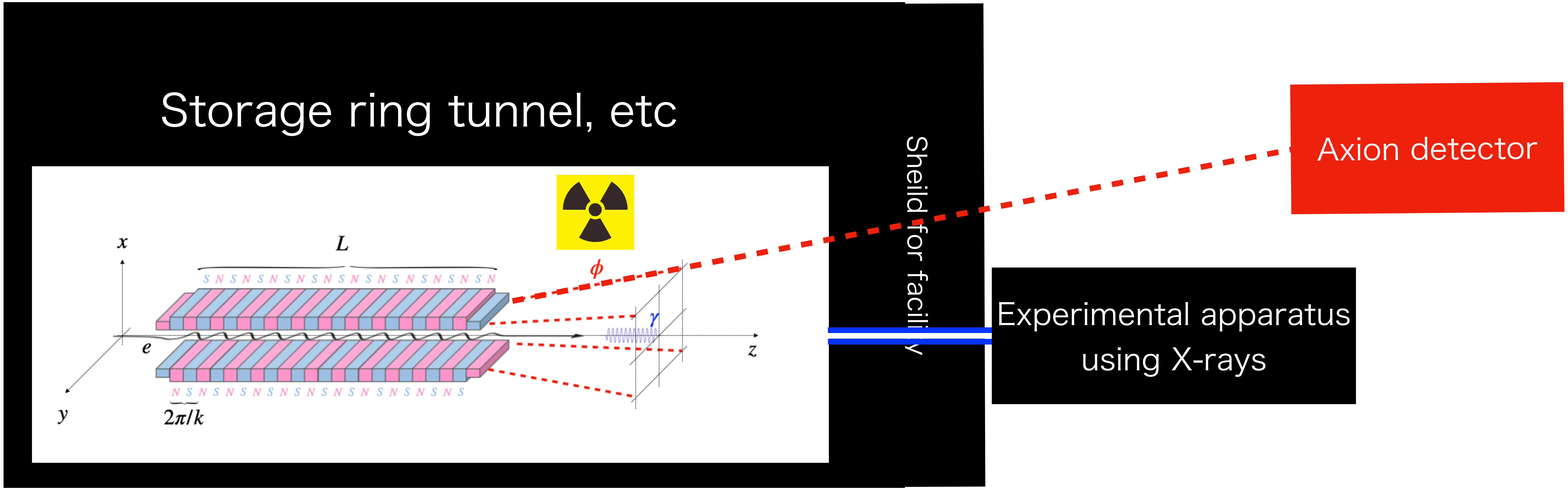
Experimental apparatus using X-rays

If an undulator is an axion factory,

WY, Junya Yoshida, arXiv:2408.17451

we have a super low cost strategy:

Install an ‘axion detector’ and wait!



Storage ring tunnel, etc

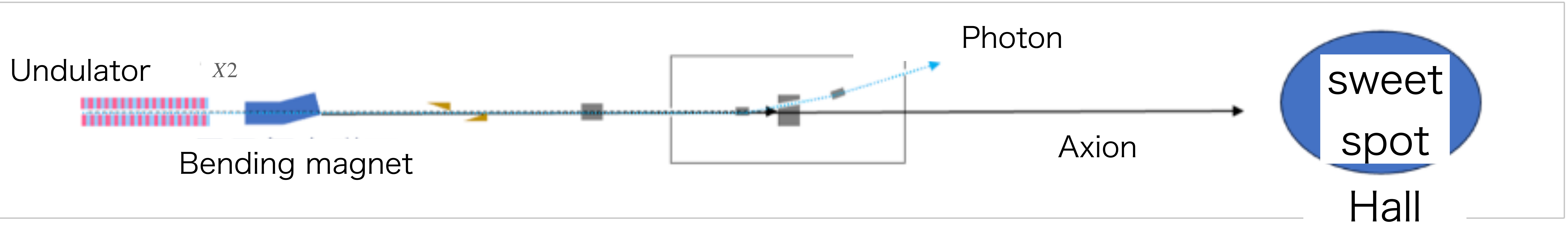
Sheild for facility

Axion detector

Experimental apparatus using X-rays

Case of NanoTerasu,

J. Yoshida, WY, preliminary



BL14U beam line

- Photon beam spread $\Delta\theta \sim 1/10^4$
- Distance to sweet spot $O(10m)$
- Detector with scale $\gg 10m \times 1/10000 \sim 1\text{mm}$

will cover the interesting directions.

Case of NanoTerasu,

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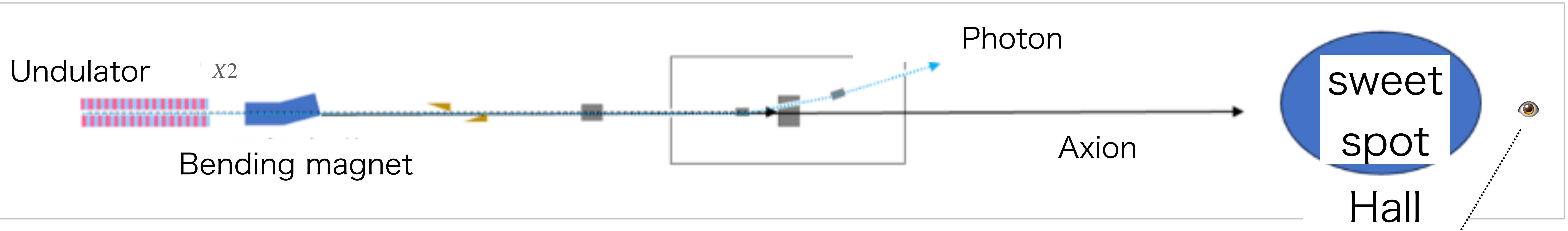

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From Yoshida-san

2. Undulator Axions

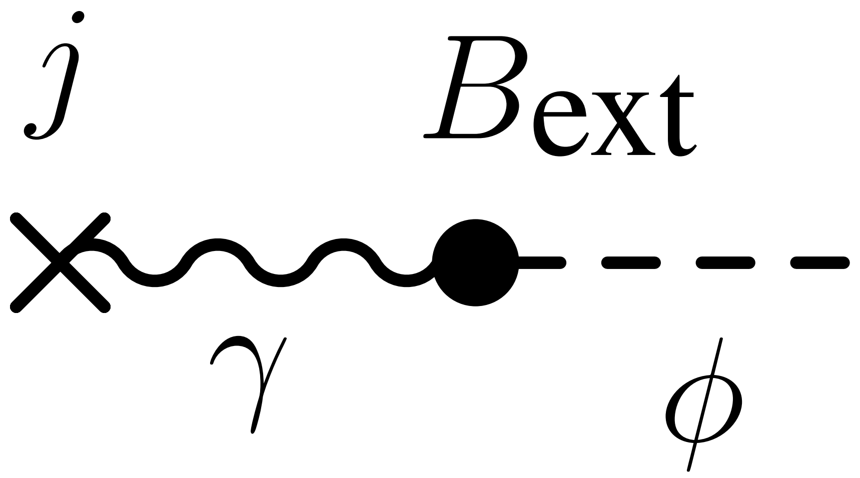
Based on WY, J. Yoshida, 2408.17451

Let us take the QFT approach

$$\mathscr{L}_{\text{int}} = -g_{\phi\gamma\gamma}\phi\overrightarrow{E}\cdot\overrightarrow{B}_{\text{ext}} - J_{\mu}^{\text{background}}A^{\mu}$$

From electron motion

- (1)Draw Feynman Diagram
- (2)Calculate the amplitude



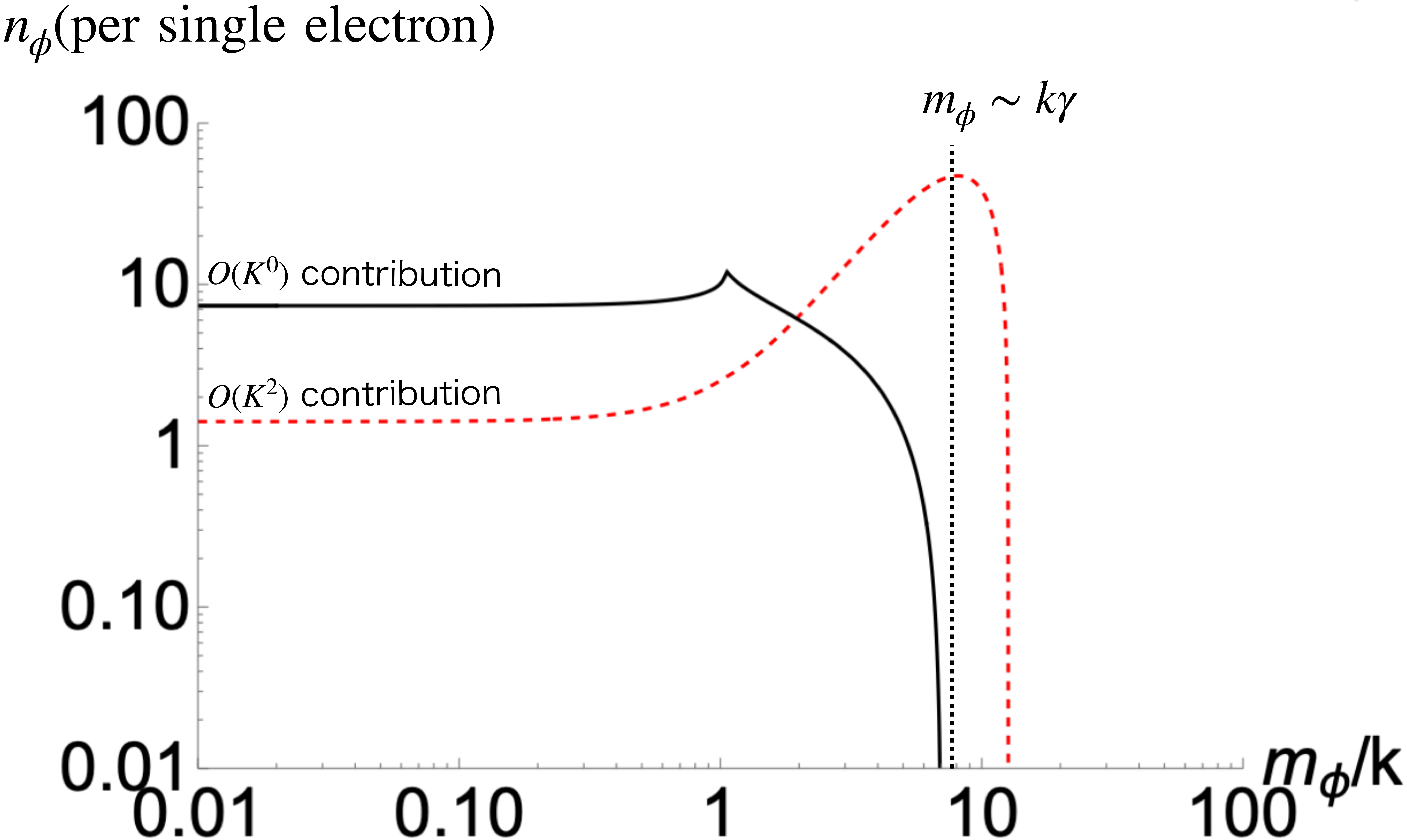
$$\langle\phi, k_\phi|0_{jB}\rangle = \frac{g_{\phi\gamma\gamma}}{\sqrt{2w_\phi}}\int d^4x d^4y e^{i(w_\phi t - \vec{k}_\phi \vec{x})}\sum_{l=x.u.z} B^l_{\text{ext}}(x)\left(\partial_0\Delta_{l\mu}(x,y) - \partial_l\Delta_{0\mu}(x,y)\right)j^\mu(y).$$

Neglecting terms $\delta_X(x)$ *with* $x \gg 2\pi/X$.

$$\frac{\langle\phi, k_\phi|0_{jB}\rangle}{g_{\phi\gamma\gamma}B_0} \simeq \frac{e(k^x_\phi + i\kappa k^y_\phi \exp(-i\phi))}{2\sqrt{2w_\phi}\left(\gamma^{-2}w^2_\phi/\beta^z + k^2_{\phi,x} + k^2_{\phi,y}\right)}\delta_{\frac{L}{\beta^z}}\left(w_\phi - \beta^z(k + k^z_\phi)\right)$$
$$- \frac{eKe^{-2i\phi}\left(\kappa k^x_\phi + ik^y_\phi e^{i\phi}\right)\left(k^x_\phi e^{i\phi} + i\kappa k^y_\phi\right)}{8\gamma k\sqrt{2w_\phi}\left(-w^2_\phi + (w_\phi - k\beta^z)^2\beta_z^{-2} + k^2_{\phi,x} + k^2_{\phi,y}\right)}\delta_{\frac{L}{\beta^z}}\left(w_\phi - \beta^z(2k + k^z_\phi)\right), +O(K^2)$$

(3) estimate axion production (with massive axion)

$$k^2 \lesssim m_\phi^2 \lesssim \frac{-k^2+3k^2\beta^z}{1-\beta^z},$$



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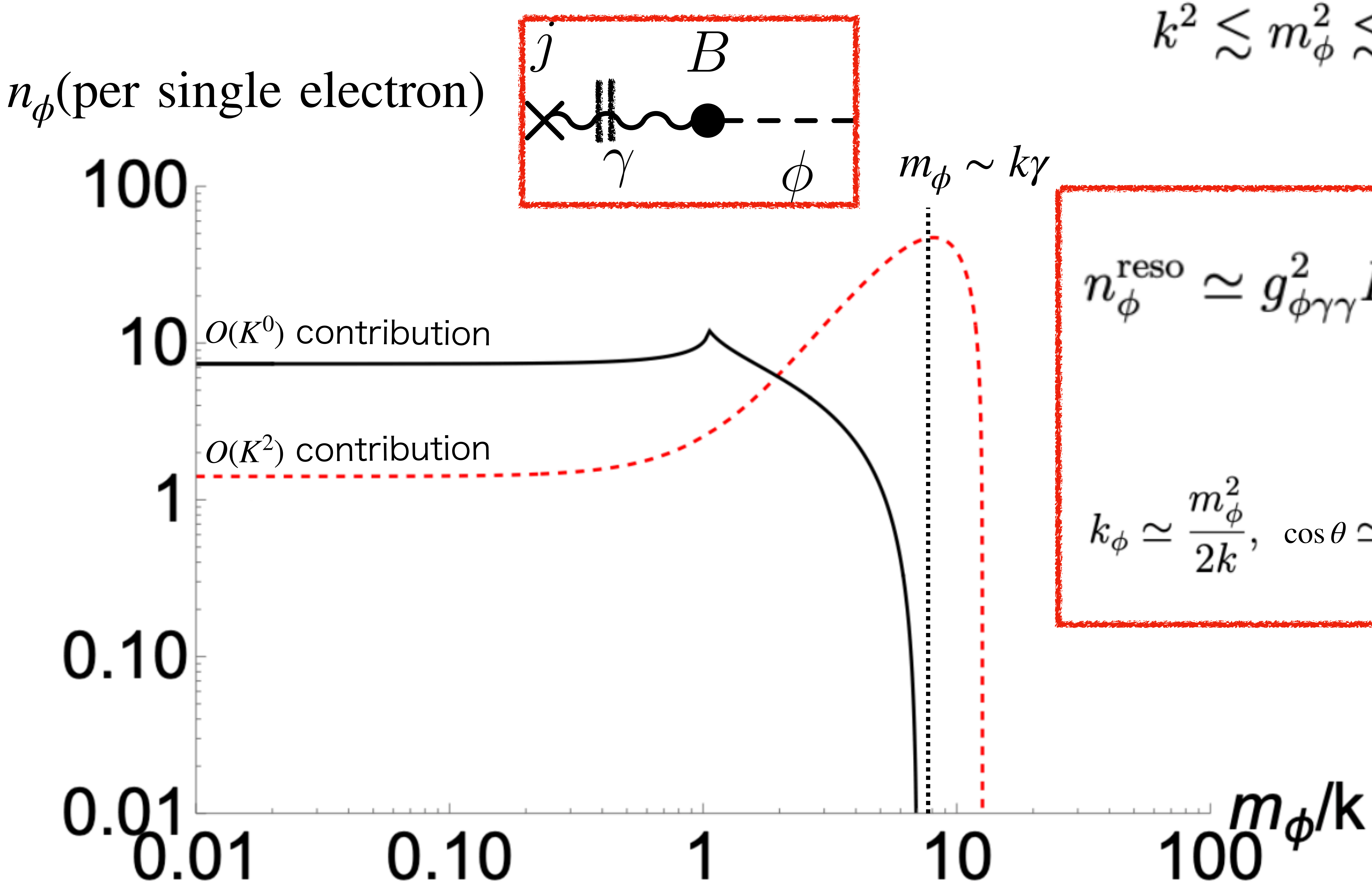

$$n_\phi^{\rm reso} \simeq g_{\phi\gamma\gamma}^2 B_0^2 L^2 e^2 \frac{K^2}{512\pi} \frac{m_\phi^2}{k^2\gamma^2} f_2$$

$$k_\phi \simeq \frac{m_\phi^2}{2k}, \quad \cos\theta \simeq 1 - \frac{2k^2}{m_\phi^2} + \frac{1 + \frac{1+\kappa^2}{2}K^2}{2\gamma^2}.$$

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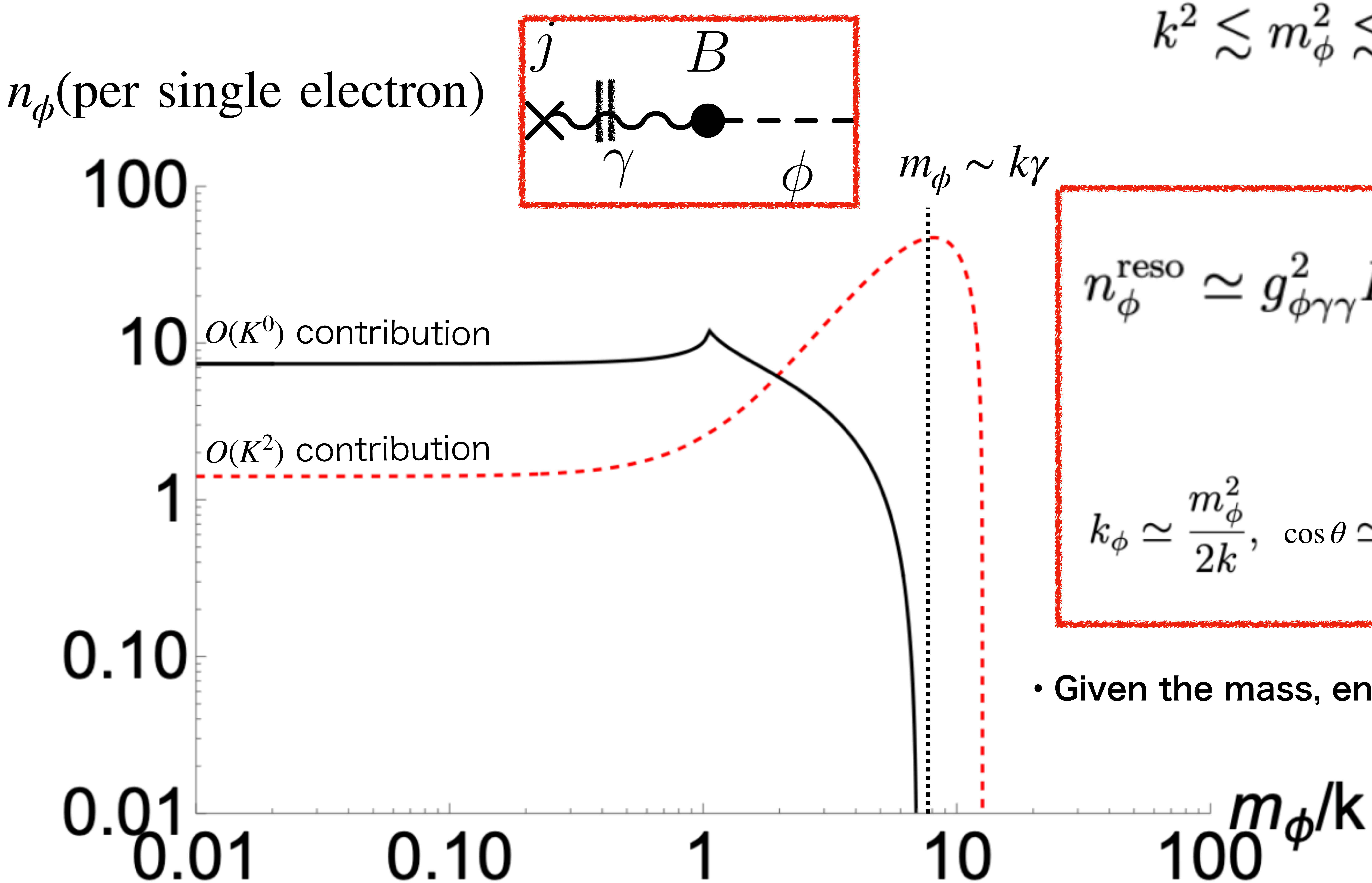

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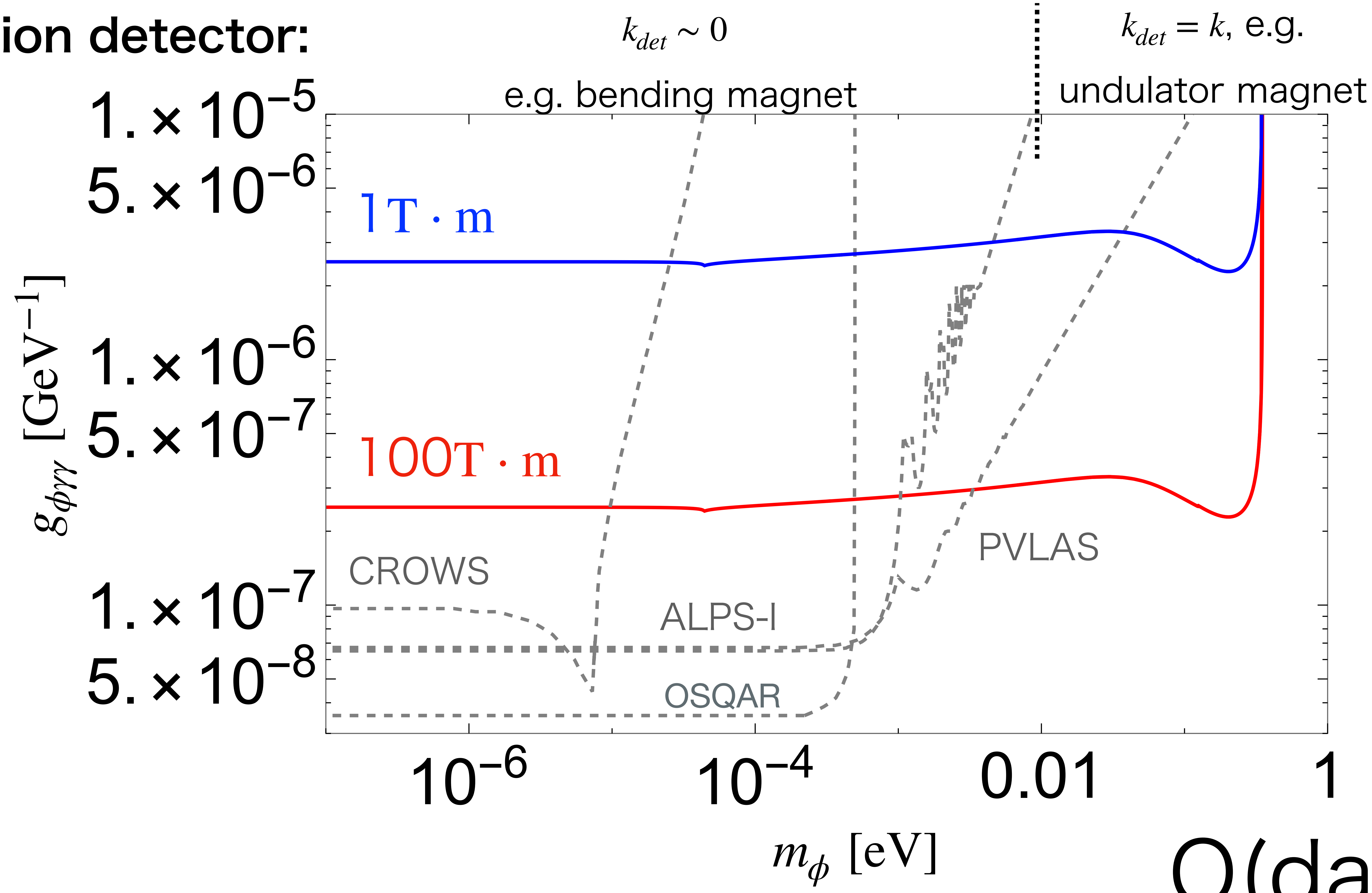
- Given the mass, energy and polar angle are fixed

Heavy mass range with resonance can exceed existing limits from the laboratory.

Axion detector:

WY, J. Yoshida, 2408.17451



NanoTerasu-like undulator assumed for axion factory

3yrs operation is assumed

4events

O(days)vs O(years)

3. Undulator dark photons

WY, Preliminary

Dark Photon Model

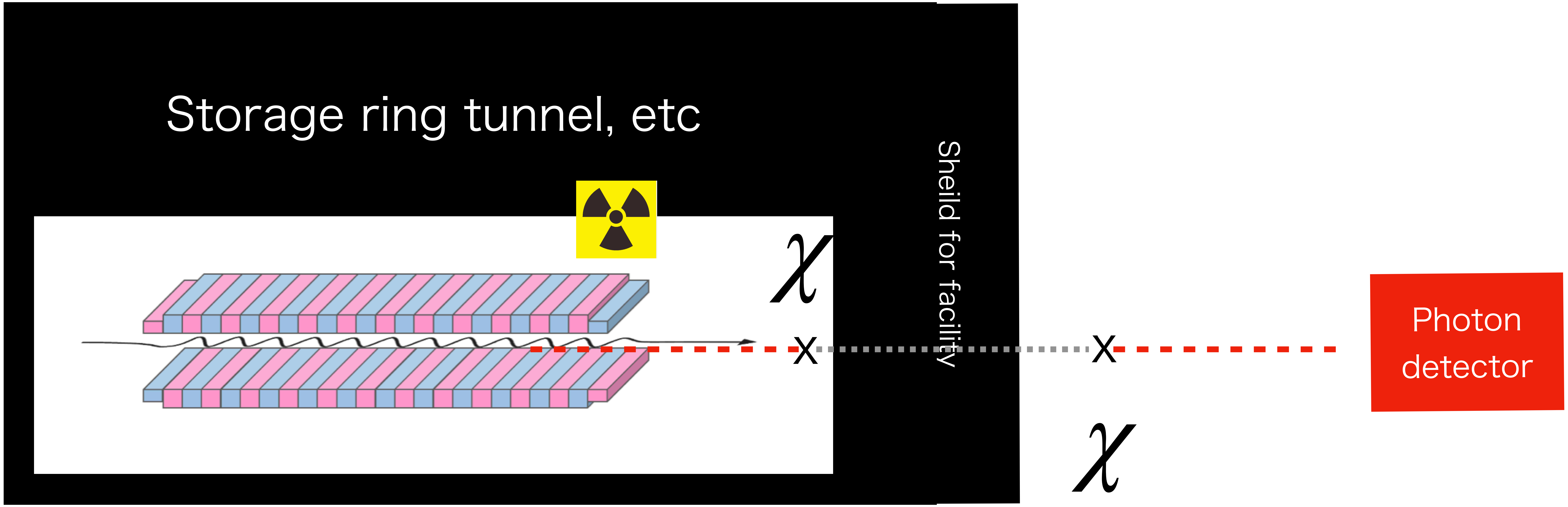
See e.g. Jaeckel, Ringwald, 1002.0329

$$\Delta\mathscr{L} \supset -\underline{\frac{\chi}{2}F^{\rm dark}_{\mu\nu}F^{\mu\nu}} - \frac{1}{4}F^{\mu\nu}_{\rm dark}F^{\rm dark}_{\mu\nu} - \frac{m^2_{\gamma'}}{2}A^{\rm dark}_{\mu}A^{\mu}_{\rm dark}$$

- One of the simplest extensions of Standard Model.
- Dark photon couples to electron via the mixing with ordinary photon: $e_{\rm dark} = \chi e$.
- Photon-dark-photon oscillation occurs similar to neutrino.
- =>LSW without a magnet

we have an extremely low cost strategy:

Install an ‘photon detector’ and wait!

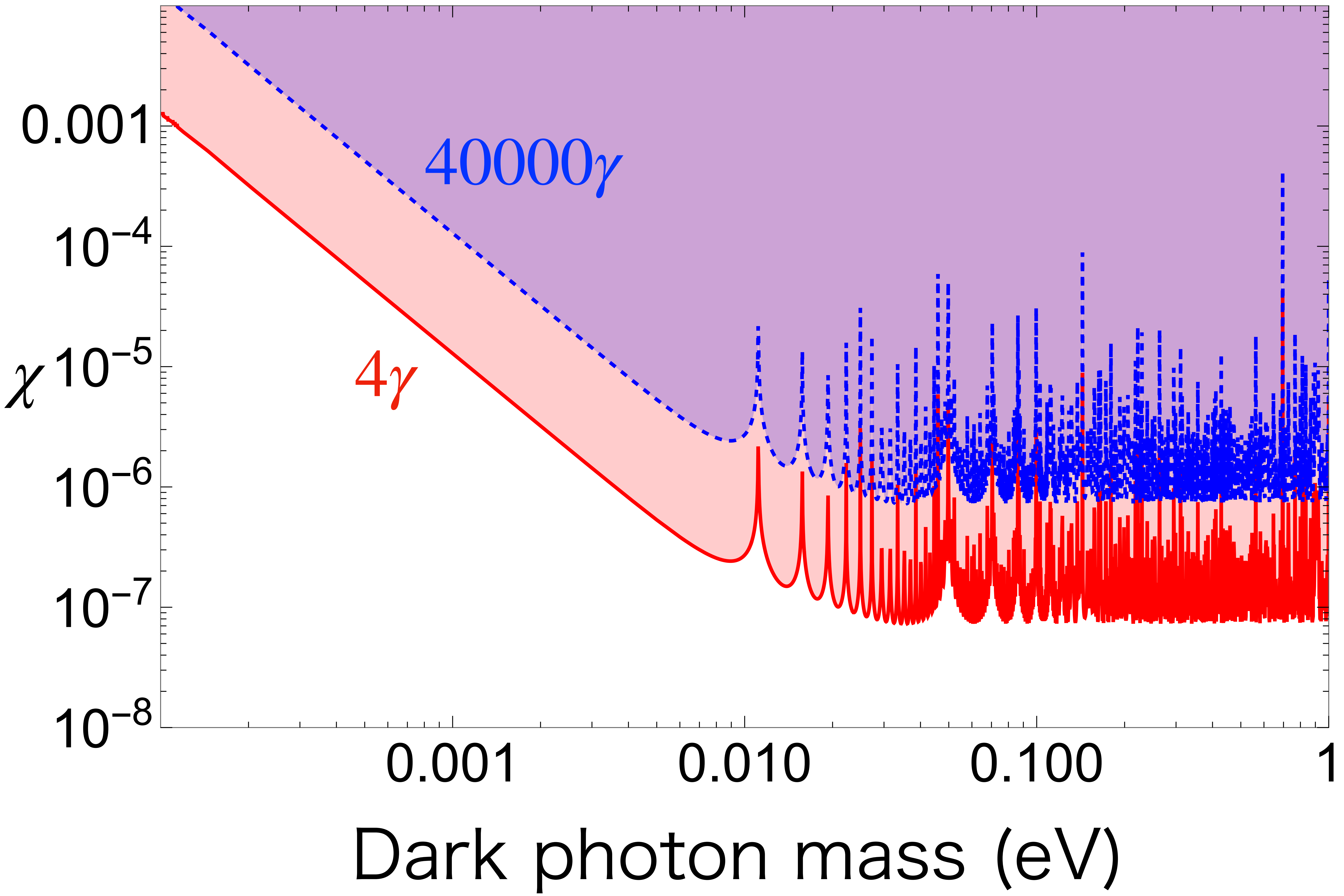


$$P_{\gamma-\gamma'} = 16\chi^4\left(\sin\left(\frac{\Delta k L_1}{2}\right)\sin\left(\frac{\Delta k L_2}{2}\right)\right)^2$$

$$n_{\gamma,signal} = P_{\gamma-\gamma'}n_{\gamma}, \quad \Delta k = E_\gamma - \sqrt{E_\gamma^2 - m_{\gamma'}^2}$$

Undulator prospects (by using known formulas)

$$P_{\gamma-\gamma'} = 16\chi^4 \left(\sin\left(\frac{\Delta k L_1}{2}\right)\sin\left(\frac{\Delta k L_2}{2}\right)\right)^2 \qquad n_{\gamma,signal} = P_{\gamma-\gamma'} n_\gamma, \quad \Delta k = E_\gamma - \sqrt{E_\gamma^2 - m_{\gamma'}^2} \sim m_{\gamma'}^2/2E_\gamma$$



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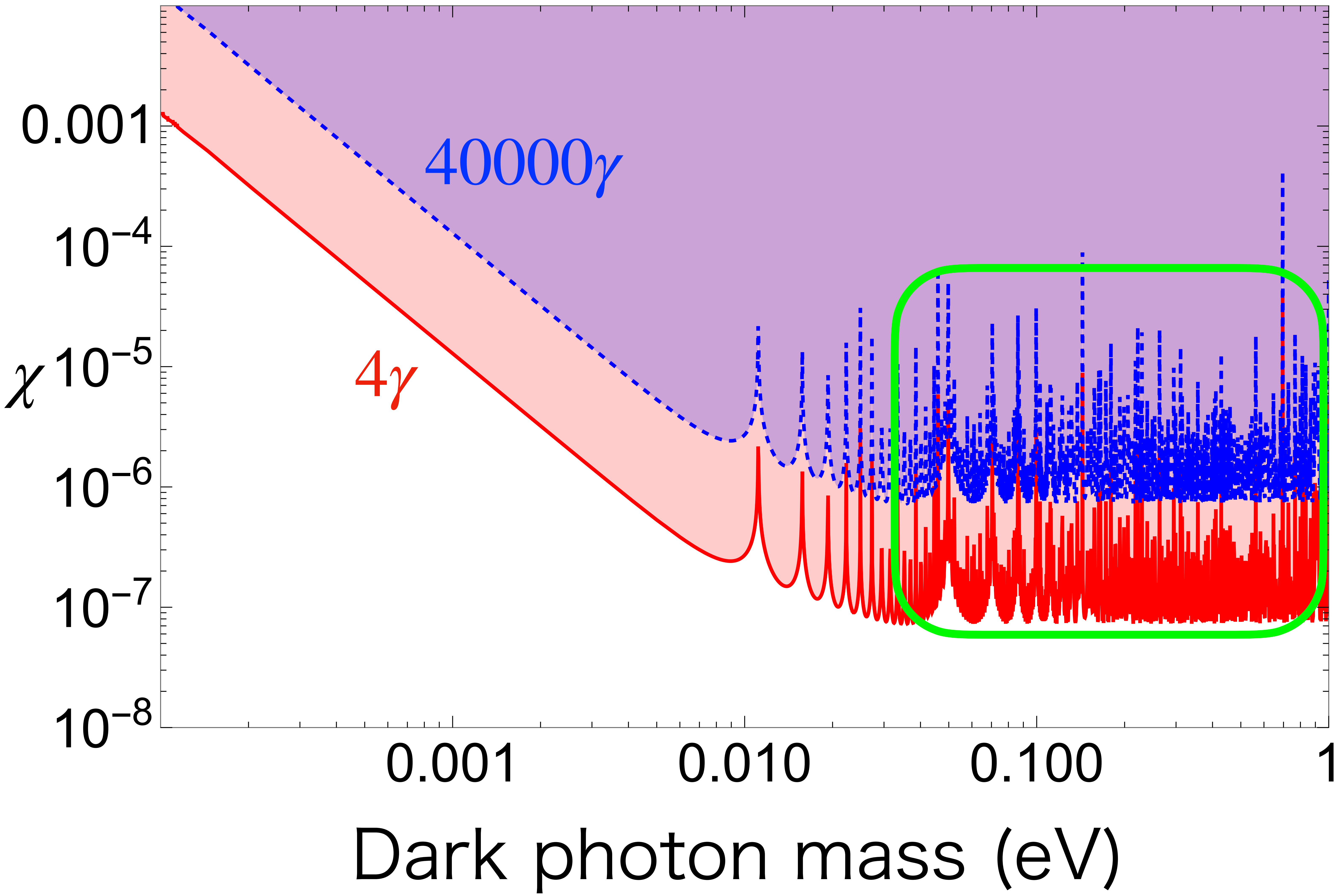

Caveats:

- Asymptotic mass eigenstates should be considered for

$$\Delta E_\gamma \partial_{E_\gamma}\frac{m_{\gamma'}^2 L_i}{2E_\gamma} \sim \frac{m_{\gamma'}^2 L_i}{2E_\gamma}\frac{\Delta E_\gamma}{E_\gamma} \gg 1.$$

- When

$$m_{\gamma'} \gtrsim k\gamma \to m_{\gamma'}^2/E_\gamma \gtrsim k$$

the production of heavy mode is suppressed.

Undulator prospects (by using known formulas)

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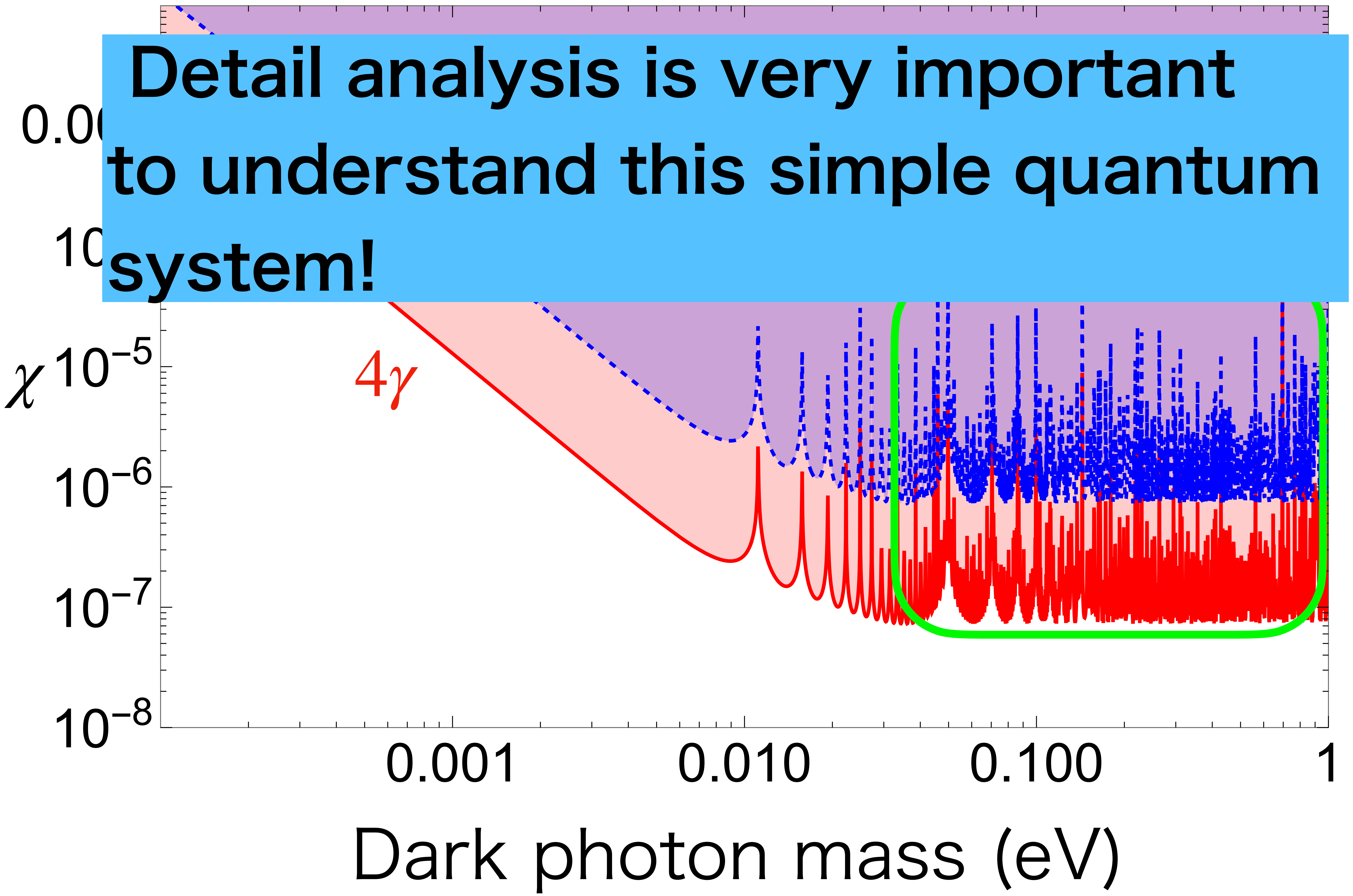

Detail analysis is very important to understand this simple quantum system!

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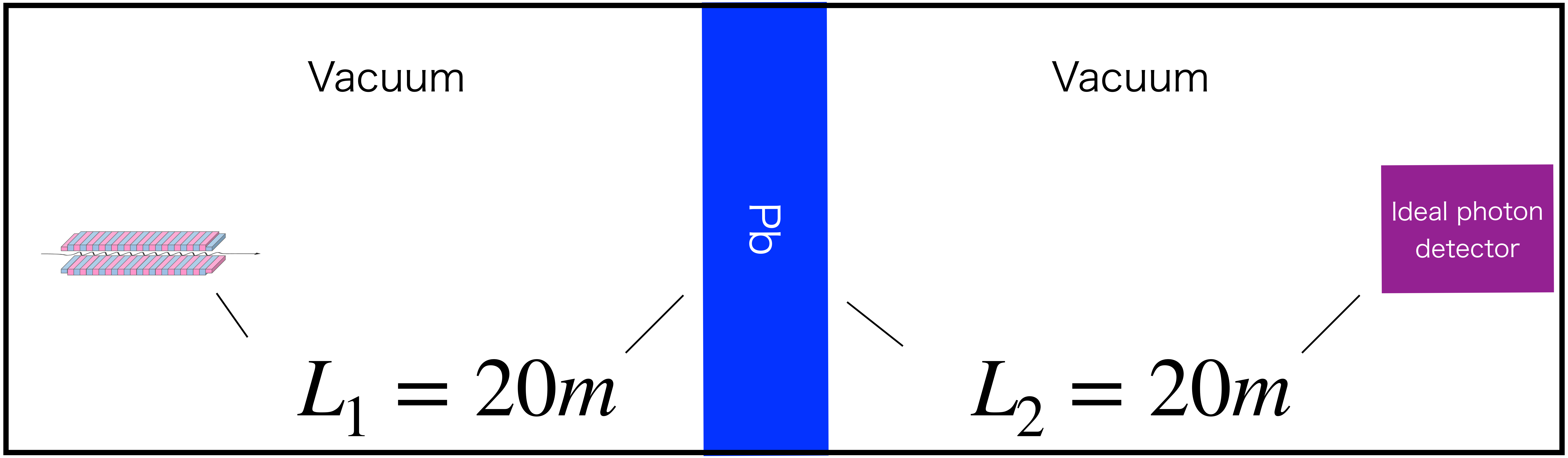
To study in more detail let me assume an ideal setup.

(For more realistic case, the right volume is not vacuum, and ``detector” may lose sensitivity for certain heavy $m_{\gamma'}$.)



Vacuum

Vacuum

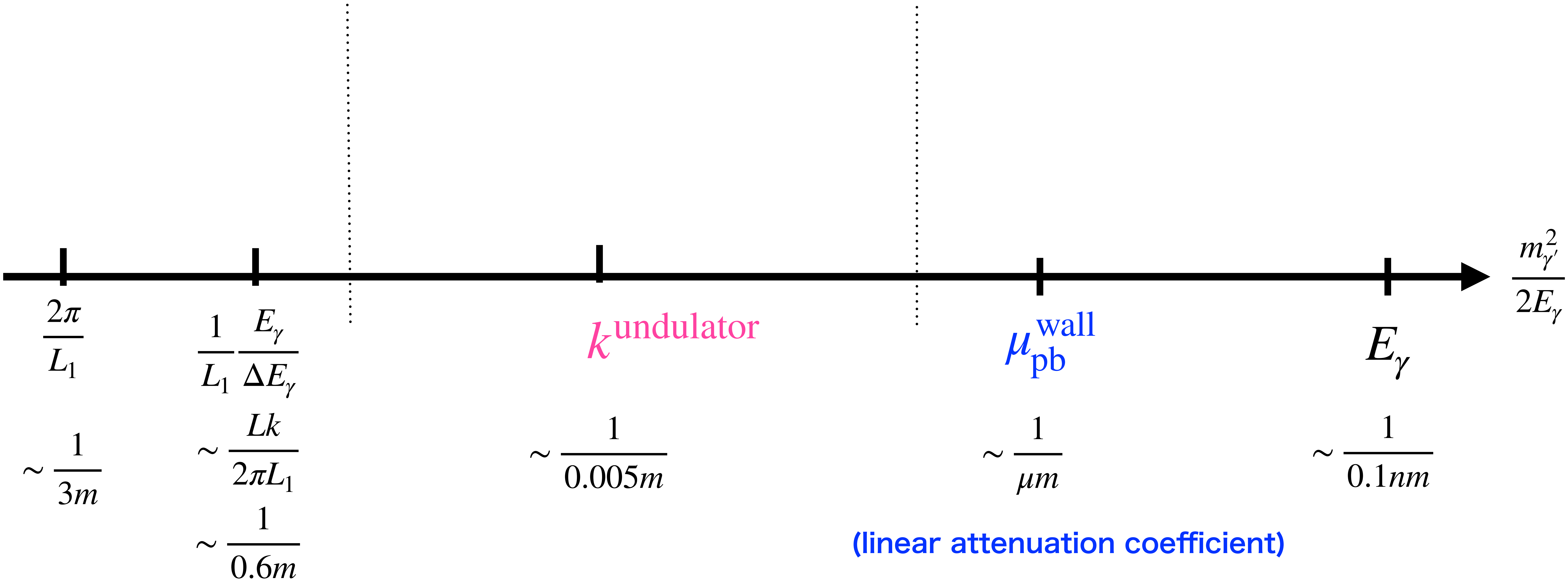
Pb

Ideal photon detector

$L_1 = 20m$

$L_2 = 20m$

Calculation strategy, and some results (For more detail, ask me!)



$\frac{m_{\gamma'}^2}{2E_\gamma}$

$\frac{2\pi}{L_1} \sim \frac{1}{3m}$

$\frac{1}{L_1}\frac{E_\gamma}{\Delta E_\gamma} \sim \frac{Lk}{2\pi L_1} \sim \frac{1}{0.6m}$

$k^{\text{undulator}} \sim \frac{1}{0.005m}$

$\mu_{\text{pb}}^{\text{wall}} \sim \frac{1}{\mu m}$

(linear attenuation coefficient)

$E_\gamma \sim \frac{1}{0.1nm}$

Calculation strategy, and some results (For more detail, ask me!)

Oscillation
with wave packet

$\frac{m_{\gamma'}^2}{2E_\gamma}$

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$\sim \frac{1}{3m}$

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Calculation strategy, and some results (For more detail, ask me!)

Oscillation
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QFT analysis
with undulator background

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Oscillation
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QFT analysis
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Suppressed production
of massive mode from
undulator

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Kinetic equation

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Calculation strategy, and some results (For more detail, ask me!)

Oscillation with wave packet

QFT analysis with undulator background

Kinetic equation

Suppressed production of massive mode from undulator

Absorption of both photon and dark photon

$$\frac{m_{\gamma'}^2}{2E_\gamma}$$

$$\frac{2\pi}{L_1} \sim \frac{1}{3m}$$

$$\frac{1}{L_1}\frac{E_\gamma}{\Delta E_\gamma} \sim \frac{Lk}{2\pi L_1} \sim \frac{1}{0.6m}$$

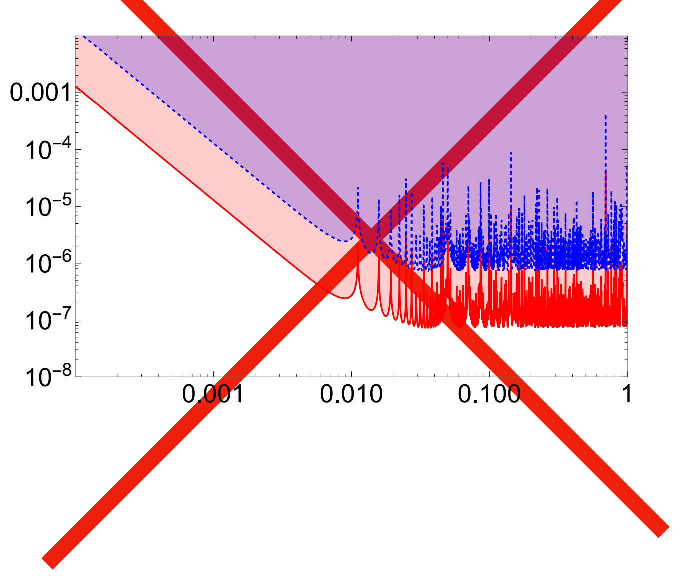
$$k^{\text{undulator}} \sim \frac{1}{0.005m}$$

$$\mu_{\text{pb}}^{\text{wall}} \sim \frac{1}{\mu m}$$

(linear attenuation coefficient)

$$E_\gamma \sim \frac{1}{0.1nm}$$

$L_1 = L_2 = 20m, k_\gamma = 3keV$
$P = 15kW, L = 200/k, k = 2\pi/(0.03m)$



Undulators are dark photon factories

Conclusions:

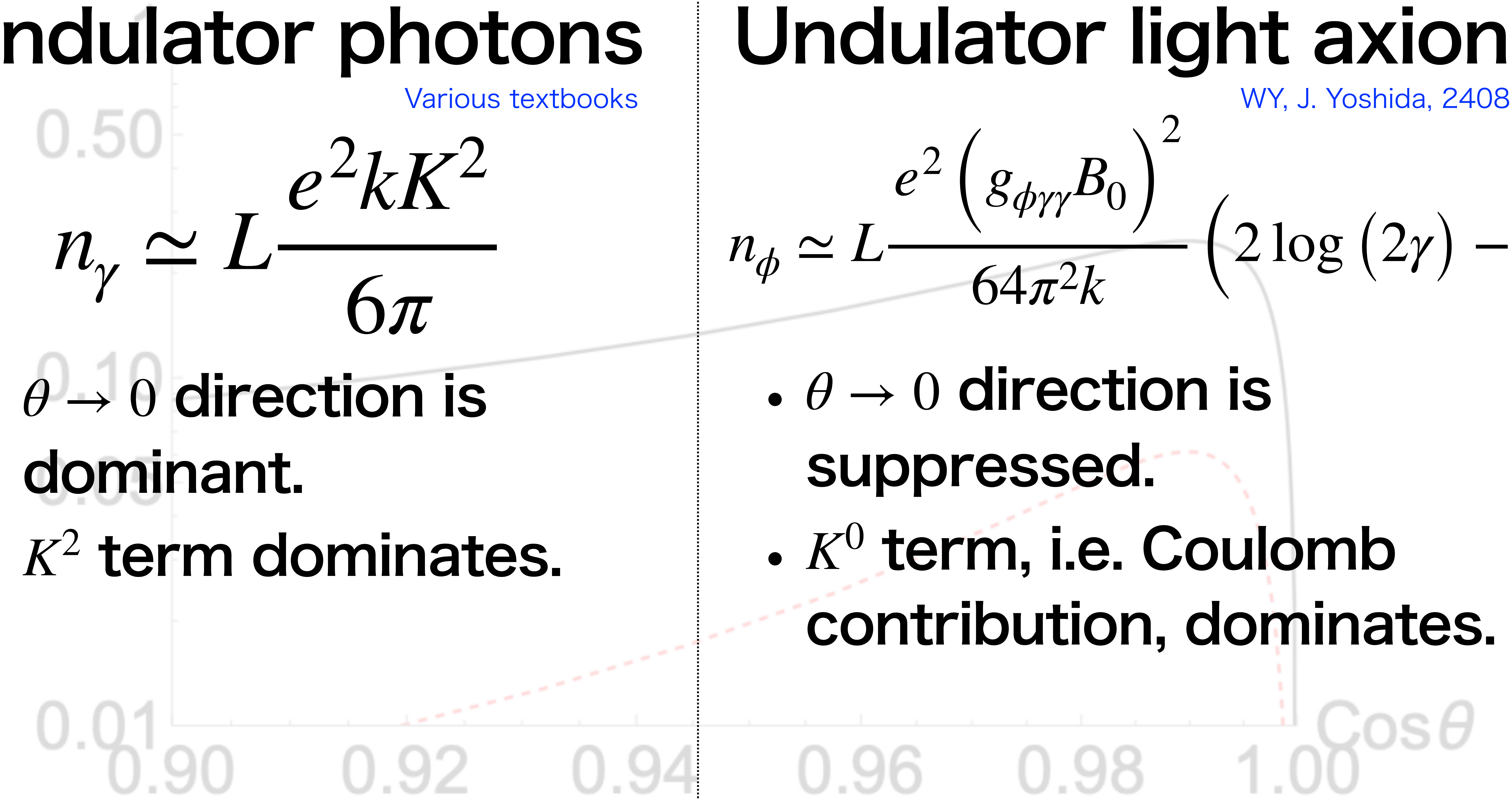
Undulators are axion/dark photon factories!

A sustainable and coexisting strategy:
Install an 'BSM detector' out of the experimental hatch and wait!

- Less time/space restriction.
- More (would-be) photons since we do not filter them.

More theoretical/experimental studies are needed.
If you are interested in, feel free to join us!

Each relativistic electron passing through the undulator, we have



Undulator photons

Various textbooks

$$n_\gamma \simeq L\frac{e^2kK^2}{6\pi}$$

- $\theta \to 0$ direction is dominant.
- K^2 term dominates.

Undulator light axions

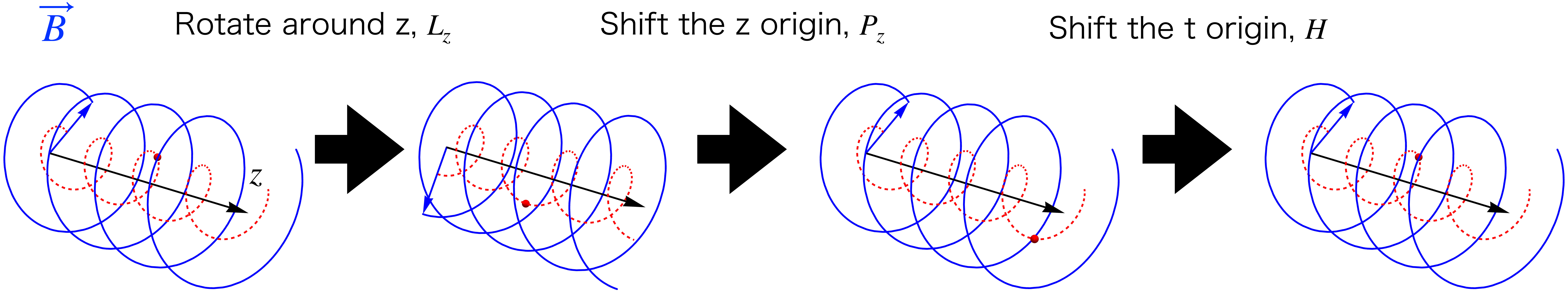
WY, J. Yoshida, 2408.17451

$$n_\phi \simeq L\frac{e^2\left(g_{\phi\gamma\gamma}B_0\right)^2}{64\pi^2k}\left(2\log\left(2\gamma\right)-1\right).$$

- $\theta \to 0$ direction is suppressed.
- K^0 term, i.e. Coulomb contribution, dominates.

Symmetry of (helical) undulator system

Consider the transformation leaving `vacuum' with the background fields invariant.



Rotate around z, L_z

Shift the z origin, P_z

Shift the t origin, H

A conserving charge (with $L \to \infty$):

$$\hat{Q} \equiv -\hat{H} + \beta^z \hat{P}^z + \beta^z k \hat{L}^z$$

Understanding from symmetry

Any particle produced from the helical undulator has

$$\mathbf{-w_i + \beta_z p_i^z + \beta^z k(s_i^z + l_i^z) = 0}$$

Undulator Photon

$$w_\gamma(1 - \cos\theta\beta^z) = \beta^z k(1 + l_\gamma^z)$$

$$\therefore \theta = 0 \text{ allowed}$$

Undulator Axion

$$w_\phi(1 - \cos\theta\beta^z) = \beta^z k(0 + l_\phi^z)$$

$$\therefore \theta \neq 0$$

->Measurement of spin!

Spin difference

Nothing but the arguments of the “delta functions”

$$\mathscr{L}_{\text{int}} = -J_\mu A^\mu$$

$At\ K \to 0$, the electron moves straight, we have additional accidental symmetry.

$$\hat{\tilde{Q}} \equiv -\hat{H} + \beta^z \hat{P}^z$$

which is never satisfied.

=> <u>K^0 contribution is forbidden</u>.

$$\mathscr{L}_{\text{int}} = -g_{\phi\gamma\gamma}\phi\vec{E}\cdot\vec{B} - J_\mu A^\mu$$

$At\ K \to 0$, we do not have accidental symmetry due to $\vec{B}_{\text{ext}} \neq 0$.

<u>No symmetry forbids Coulomb Contribution.</u>

Accidental symmetry

How undulator emits synchrotron radiation

(1)Maxwell equations

$$\partial^\mu F_{\mu\nu} = J_\nu$$

(2)Delayed vector potential

$$A_\mu(\vec{x}, t) = \int d^3x' \frac{1}{|\vec{x}' - \vec{x}|} j_\mu\left(\vec{x}', t - |\vec{x}' - \vec{x}|\right)$$

(3) Electromagnetic field

$$-\vec{E} = \frac{e(1-\beta^2)}{4\pi R^2 b^3}\left(\frac{\vec{R}}{R} - \vec{\beta}\right) + \frac{e}{4\pi R b^3}\frac{\vec{R}}{R} \times \left\{\left(\frac{\vec{R}}{R} - \vec{\beta}\right) \times \dot{\vec{\beta}}\right\},$$

$$\vec{B} = \frac{\vec{R}}{R} \times \vec{E}$$

(4)Poynting vector, $\vec{S} = \vec{E} \times \vec{B}$

At large R the dominant contribution is

$$\vec{S} = e^2 \frac{\vec{a}^2}{16\pi^2 R^2} \frac{b^2 - \gamma^{-2}\cos^2\theta'}{b^6} \frac{\vec{R}}{R}.$$

(5)Power of radiation

$$\frac{dP}{d\Omega} = R^2 \vec{S} \cdot \frac{\vec{R}}{R} \frac{dt}{dt_R} = \frac{e^2 \vec{a}^2}{16\pi^2} \frac{1}{(1-\beta\cos\theta)^3}\left(1 - \frac{\gamma^{-2}\cos^2\theta'}{(1-\beta\cos\theta)^2}\right).$$

(6)Electron motion+approximations:

$$\frac{dP}{d\Omega} = \frac{e^2 K^2 k^2 \beta^2}{16\pi^2 \gamma^2} \frac{1}{(1-\beta\cos\theta)^3}\left(1 - \frac{\gamma^{-2}\sin^2\theta}{2(1-\beta\cos\theta)^2}\right)$$

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(Radiation term)^2

Numerical check with background electric magnetic field

Helical undulator, $g_{\phi\gamma\gamma} = 1GeV^{-1}$, Using E, B estimated from Maxwell eqs as background.

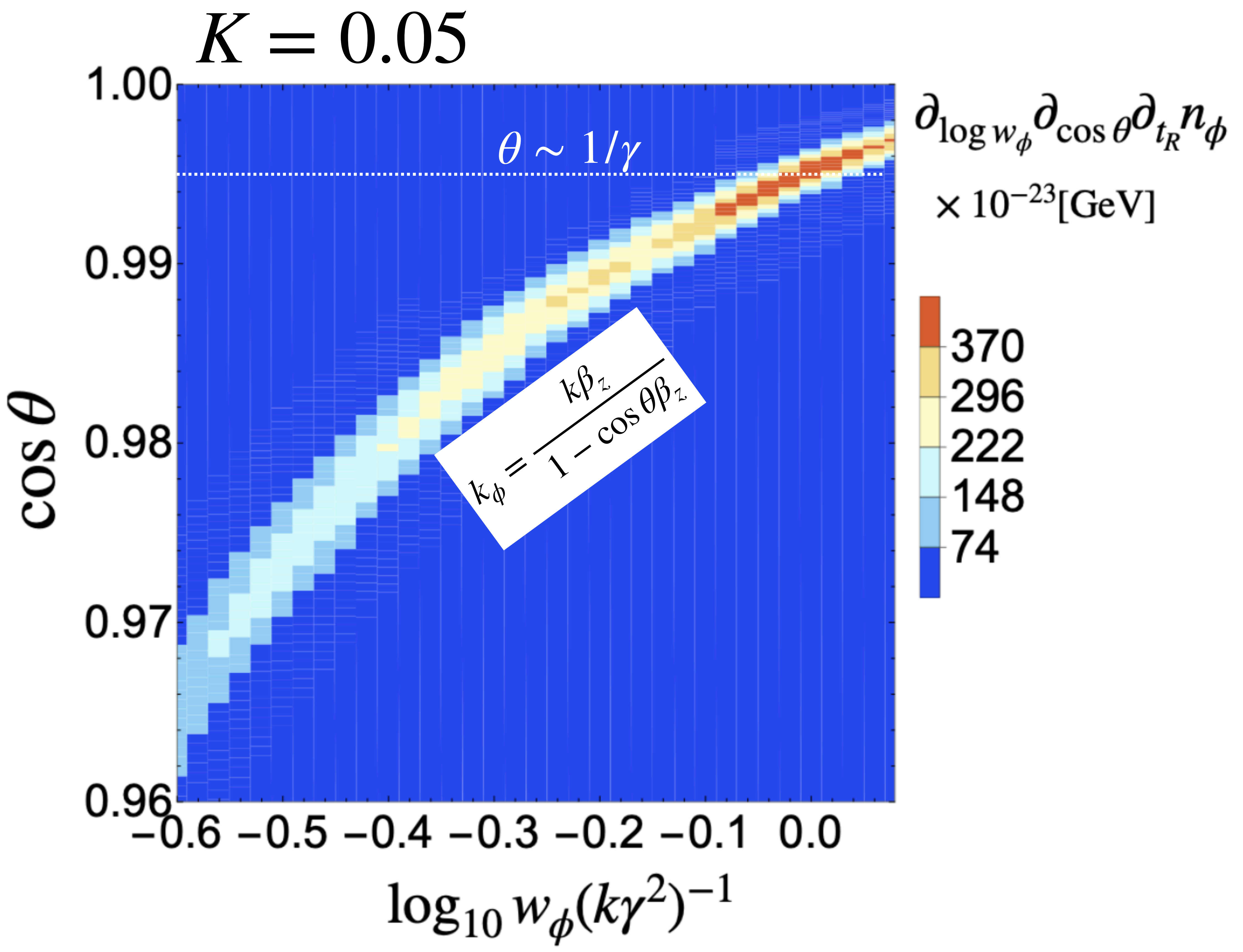
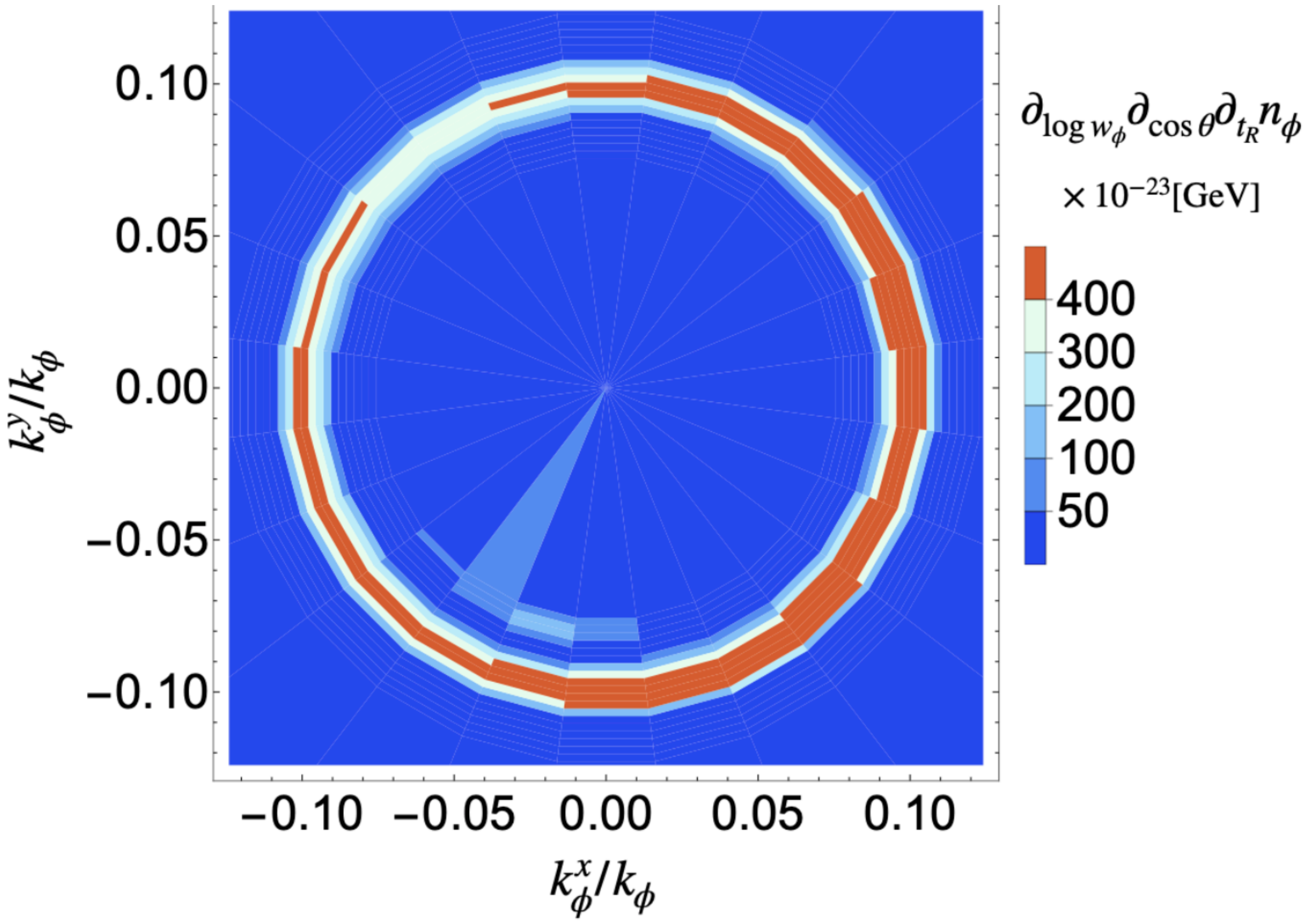


Fig. 7: The contour plot of $\partial_{w_\phi}\partial_{\cos\theta}\partial_{t_R} n_\phi$ in the $\log_{10} w_\phi (k\gamma^2)^{-1}, \cos\theta$ plane. We set $\gamma = 10$, $2\pi/k = 0.5\text{mm}$, $B_0 = 1\text{Tesla}$, $L = 5 \times 2\pi/k$ and $m_\phi = 0$. m_e and e are taken to be the realistic values. This corresponds to $K \approx 0.05$.

$$\langle \phi, k_\phi | 0_{E,B} \rangle = -i \int d^4x \frac{g_{\phi\gamma\gamma}}{\sqrt{2w_\phi}} e^{i(-\vec{k}_\phi \cdot \vec{x} + w_\phi t)} \vec{E} \cdot \vec{B}_{\text{ext}},$$



c.f. analytic formula for the dependence on azimuth angle, $\tilde{\phi}$,

$$\partial_{\tilde{\phi}} n_\phi |_{f_1} \propto \kappa \sin(\tilde{\phi})(\kappa \sin(\tilde{\phi}) + 2\cos(\tilde{\phi})\sin(\phi)) + \cos^2(\tilde{\phi})$$

$$= 1 \quad (\text{with } \kappa = 1, \phi = 0)$$

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$K = 0.5$

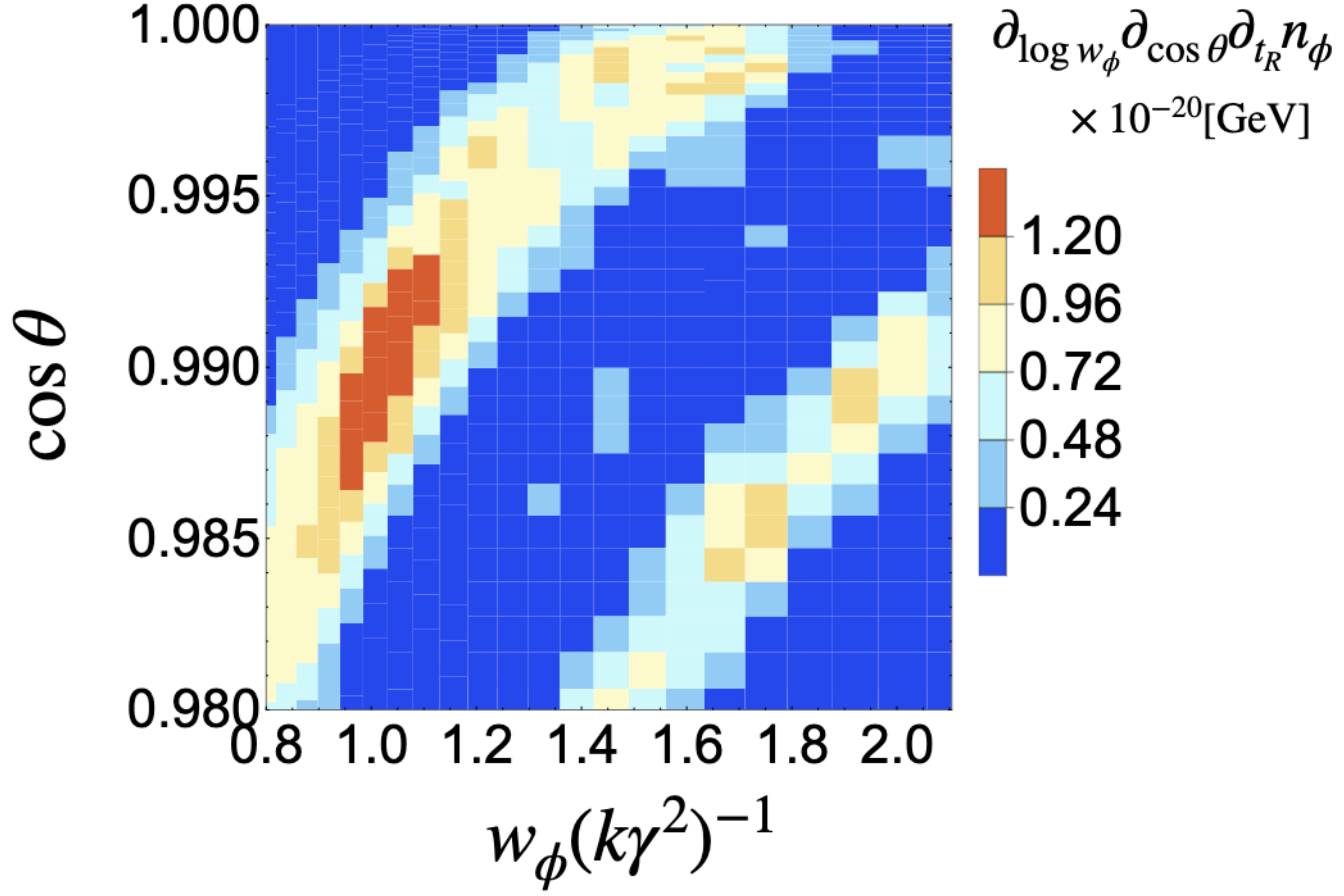

Fig. 8: The contour plot of $\partial_{w_\phi}\partial_{\cos\theta}\partial_{t_R}n_\phi$ in the $w_\phi(k\gamma^2)^{-1}, \cos\theta$ plane. We set $\gamma = 6$, $2\pi/k = 5\text{mm}$, $B_0 = 1\text{Tesla}$, $L = 3 \times 2\pi/k$ and $m_\phi = 0$. m_e and e are taken to be the realistic values. This corresponds to $K \approx 0.5$.

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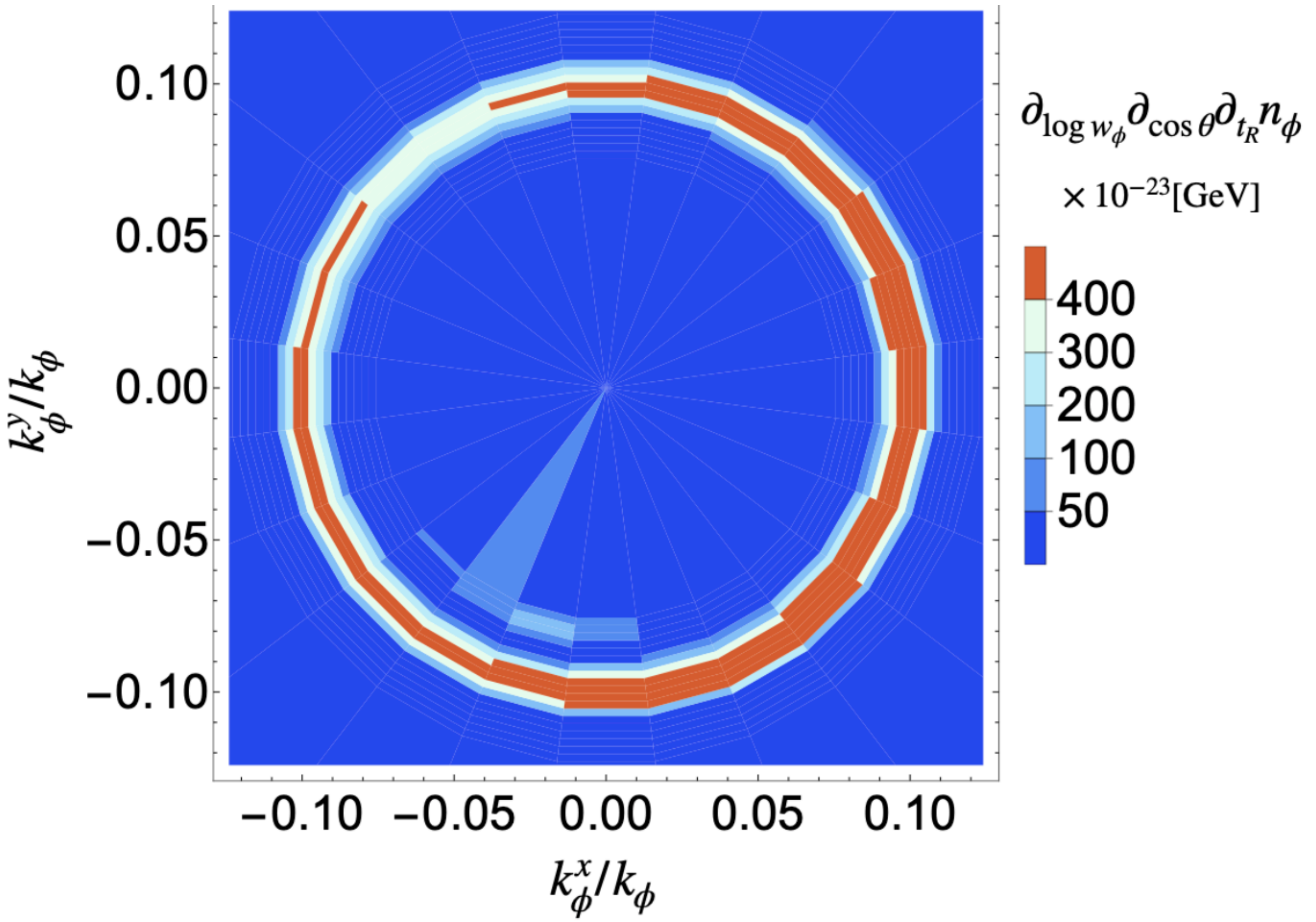

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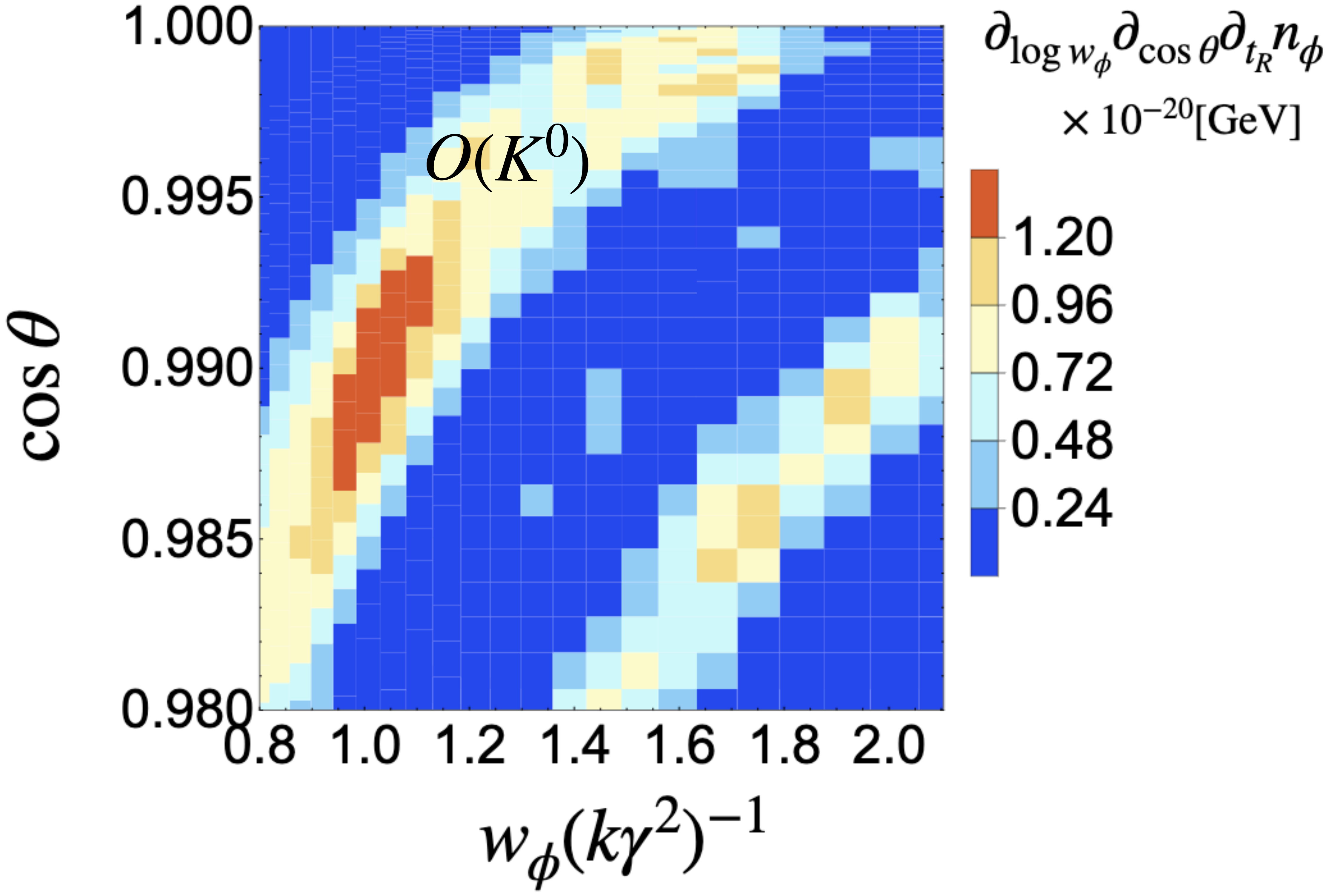

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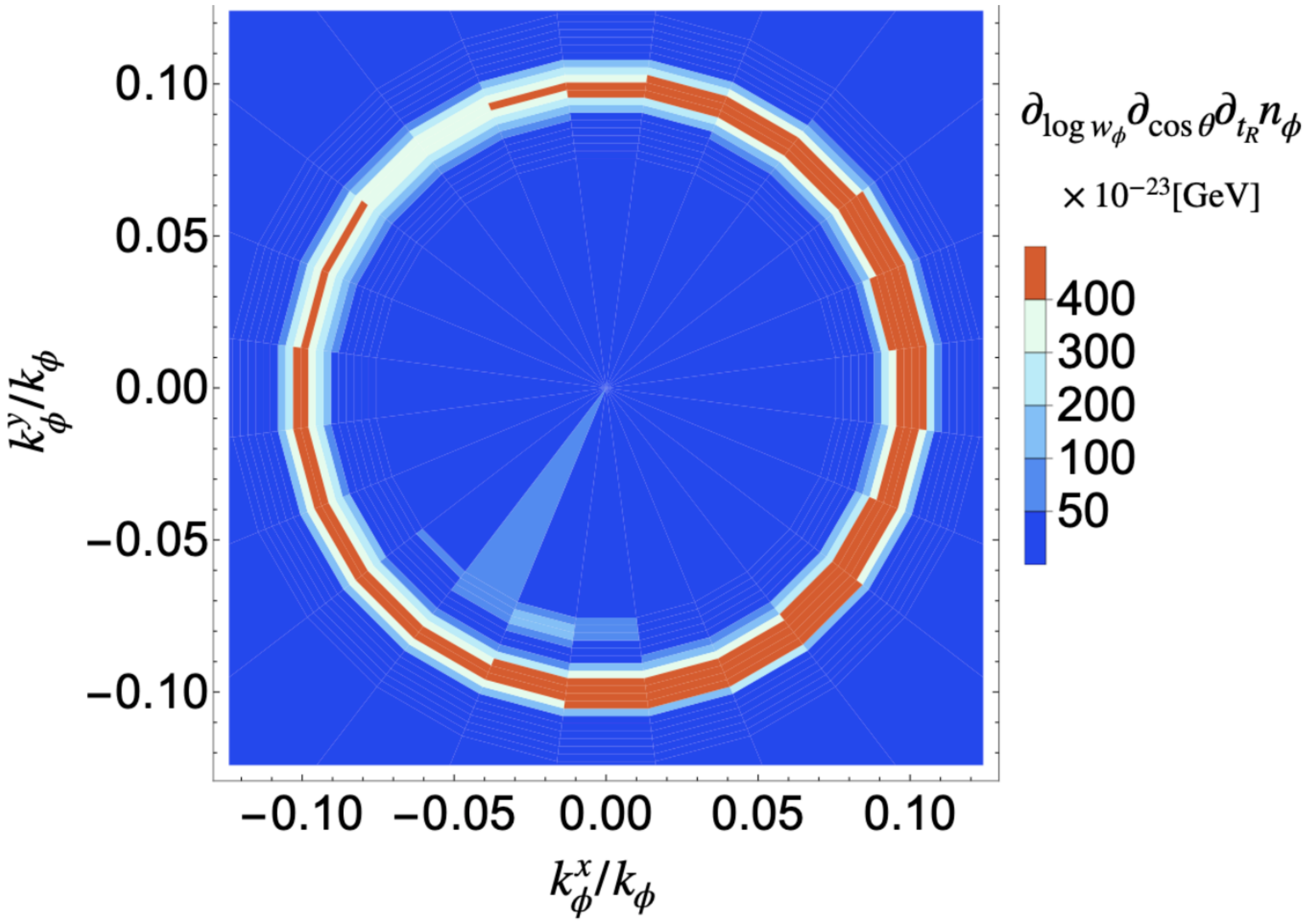


c.f. analytic formula for the dependence on azimuth angle, $\tilde{\phi}$,

$$\partial_{\tilde{\phi}} n_\phi|_{f_1} \propto \kappa\sin(\tilde{\phi})(\kappa\sin(\tilde{\phi}) + 2\cos(\tilde{\phi})\sin(\phi)) + \cos^2(\tilde{\phi})$$

$$= 1 \quad (\text{with } \kappa = 1, \phi = 0)$$

Numerical check with background electric magnetic field

Helical undulator, $g_{\phi\gamma\gamma} = 1GeV^{-1}$, Using E, B estimated from Maxwell eqs as background.

$K = 0.5$

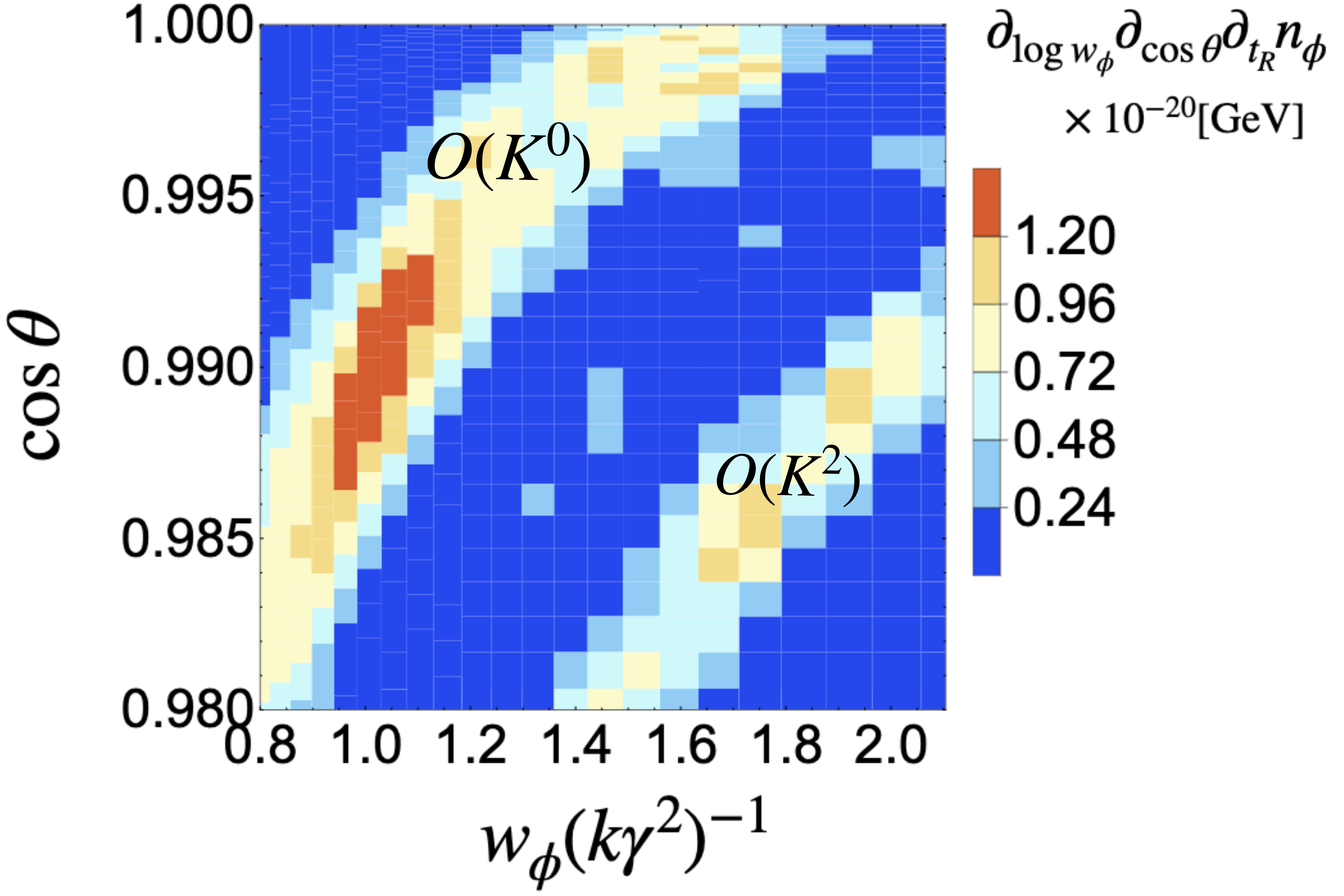
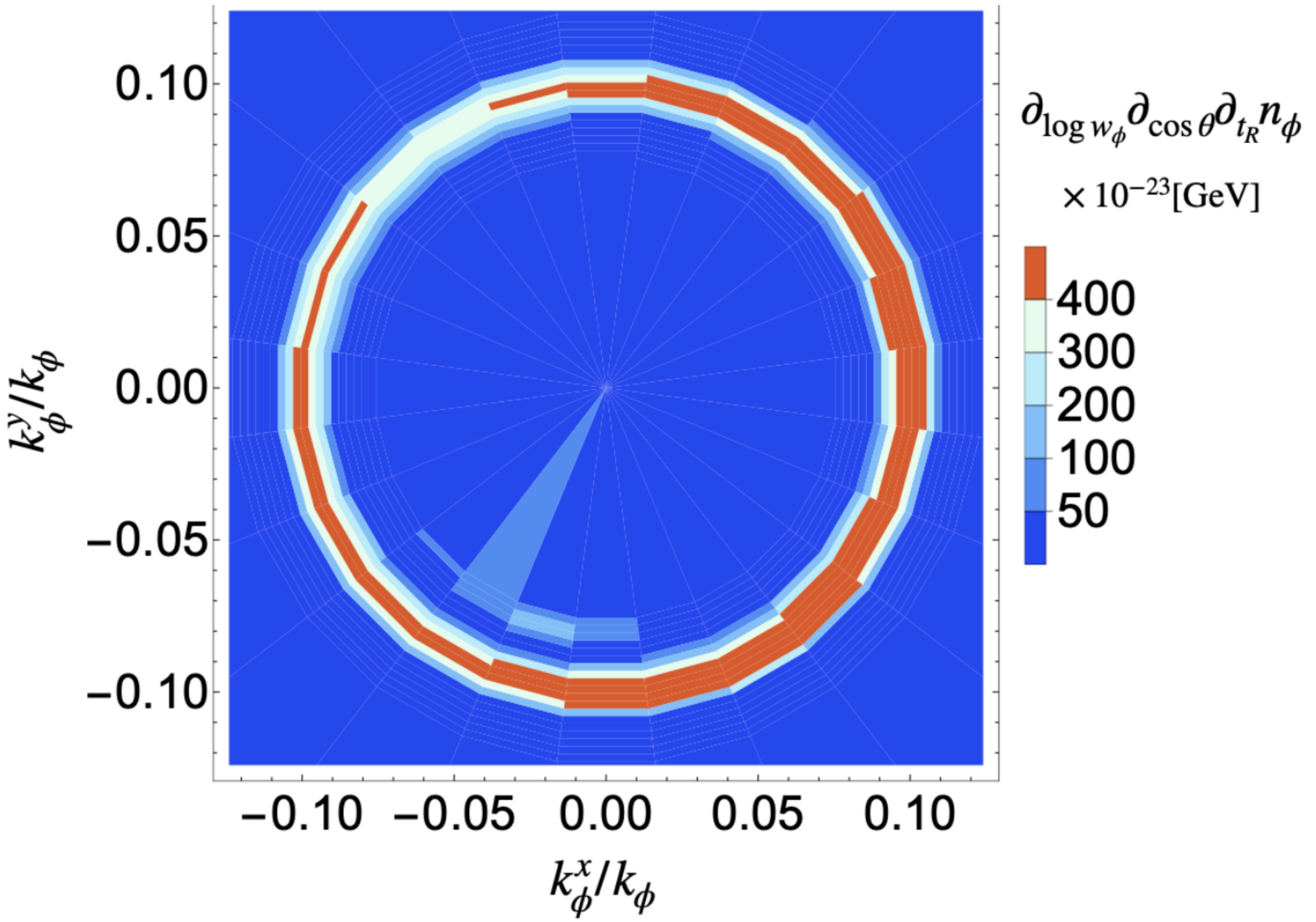


Fig. 8: The contour plot of $\partial_{w_\phi}\partial_{\cos\theta}\partial_{t_R}n_\phi$ in the $w_\phi(k\gamma^2)^{-1}, \cos\theta$ plane. We set $\gamma = 6$, $2\pi/k = 5$mm, $B_0 = 1$Tesla, $L = 3 \times 2\pi/k$ and $m_\phi = 0$. m_e and e are taken to be the realistic values. This corresponds to $K \approx 0.5$.

$$\langle \phi, k_\phi | 0_{E,B} \rangle = -i \int d^4x \frac{g_{\phi\gamma\gamma}}{\sqrt{2w_\phi}} e^{i(-\vec{k}_\phi \cdot \vec{x} + w_\phi t)} \vec{E} \cdot \vec{B}_{\text{ext}},$$



c.f. analytic formula for the dependence on azimuth angle, $\tilde{\phi}$,

$$\partial_{\tilde{\phi}} n_\phi|_{f_1} \propto \kappa \sin(\tilde{\phi})(\kappa \sin(\tilde{\phi}) + 2\cos(\tilde{\phi})\sin(\phi)) + \cos^2(\tilde{\phi})$$

$$= 1 \quad (\text{with } \kappa = 1, \phi = 0)$$

Numerical check

$$k_\phi \simeq \frac{m_\phi^2}{2k}, \cos\theta \simeq 1 - \frac{2k^2}{m_\phi^2} + \frac{1+\frac{1+\kappa^2}{2}K^2}{2\gamma^2}.$$

$$\eta^{\rm reso} = g_{\phi\gamma\gamma}^2 B_0^2 \frac{3\pi L}{128k} f_2 f_1^{-1} \frac{m_\phi^2}{\gamma^2 k^2}$$

Not very small with $m_\phi \sim k\gamma$

$$\eta_{\rm LSW} \sim g_{\phi\gamma\gamma}^2 (B_0 L)^2 \sim \frac{L}{\lambda}\eta_{\rm reso} = \frac{meter}{O(10)cm}\eta^{\rm reso}$$

, for conventional LSW.

$$m_\phi = k\gamma$$

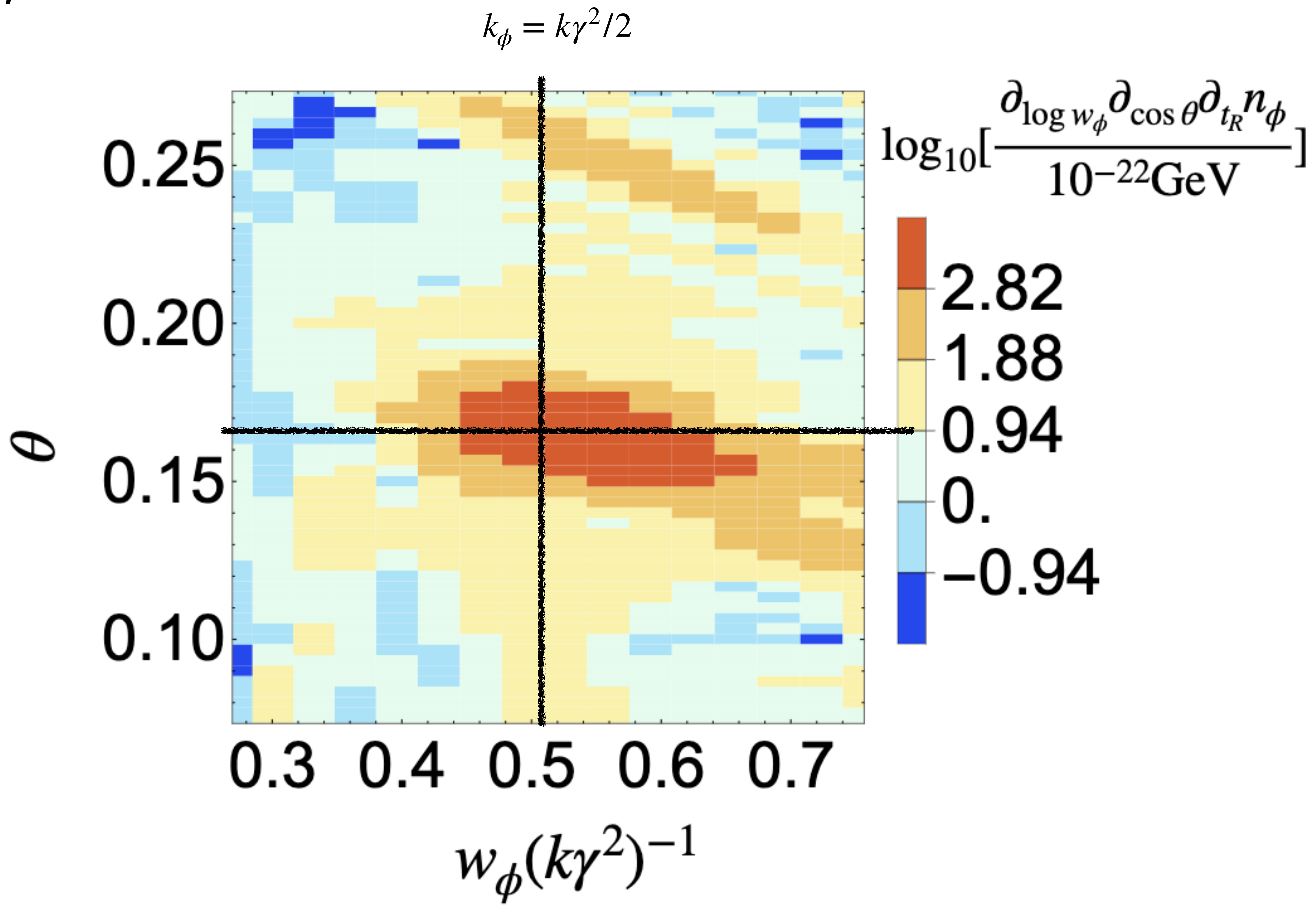
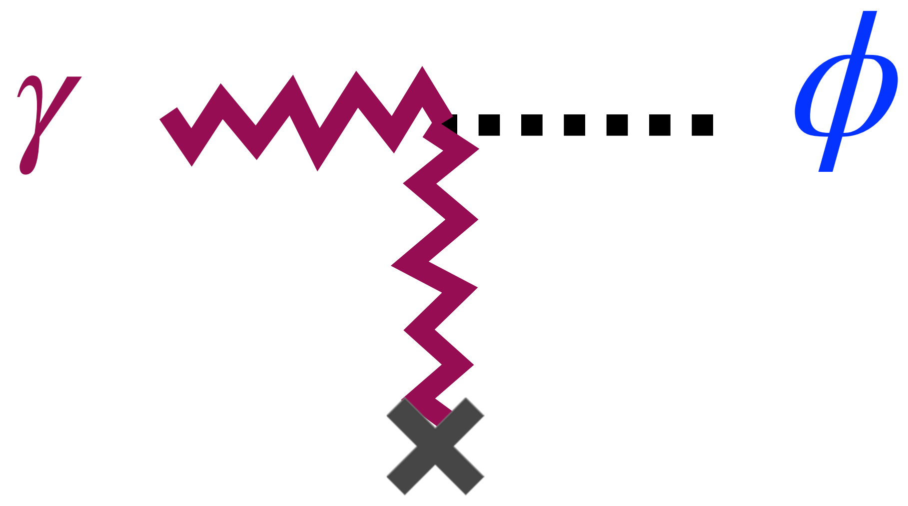


Fig. 9: The contour plot of $\log_{10}[\partial_{w_\phi}\partial_{\cos\theta}\partial_{t_R}n_\phi]$ in $w_\phi(k\gamma^2)^{-1},\theta$ plane, with $m_\phi = k\gamma$. We set $\gamma = 10$, $2\pi/k = 5$mm, $B_0 = 1$Tesla, $L = 6\times 2\pi/k$, $m_\phi = \gamma k$. m_e, e are taken to be the realistic values. This corresponds to $K \approx 0.5$.

Axion-photon conversion with magnets



$$\mathscr{L}_{\rm int} = -g_{\phi\gamma\gamma}\phi\vec{E}\cdot\vec{B}$$

$\vec{B}_{\rm ext} = \vec{B}_0\cos[zk]$ for 0<z<L

$${}_{\rm out}\langle\phi, k_\phi\,|\,\gamma, k_\gamma\rangle_{\rm in} \propto g_{\phi\gamma\gamma}\delta^{(3)}(p_\phi - p_\gamma)\int_0^L dz e^{i(k_z - k_{\phi,z})z} i\vec{\epsilon}\cdot\vec{B}_{\rm ext}$$

$$\propto g_{\phi\gamma\gamma}\delta^{(3)}(p_\phi - p_\gamma)\delta_L(k_{\gamma,z} - k_{\phi,z} \pm k)B_0$$

$\delta_L(x)$: Delta-like function

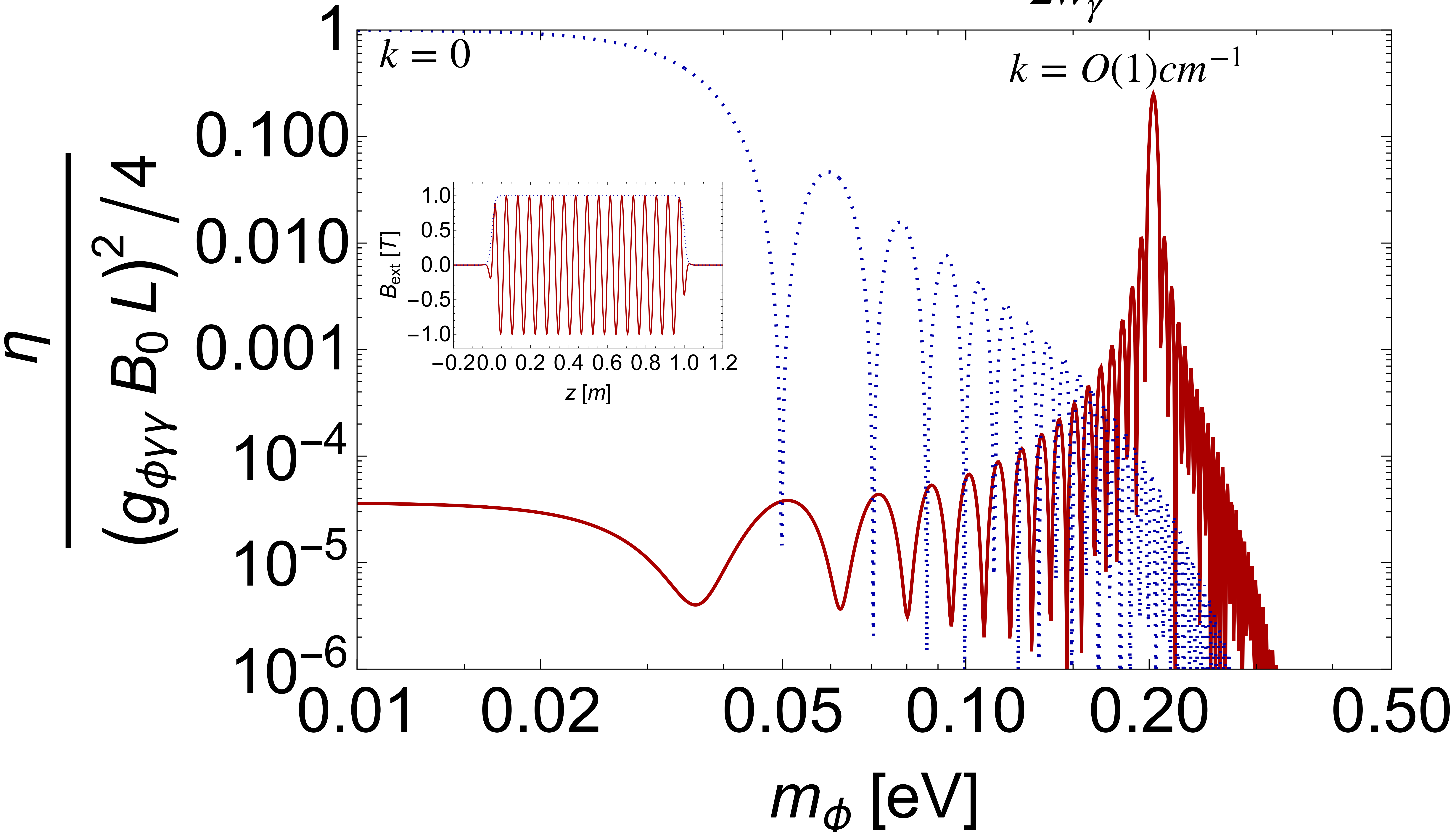
with finite volume, $\delta_L(0) \sim \delta_L(\pi/L) \sim L$.

k: "Momentum" from

Bulk magnetic field

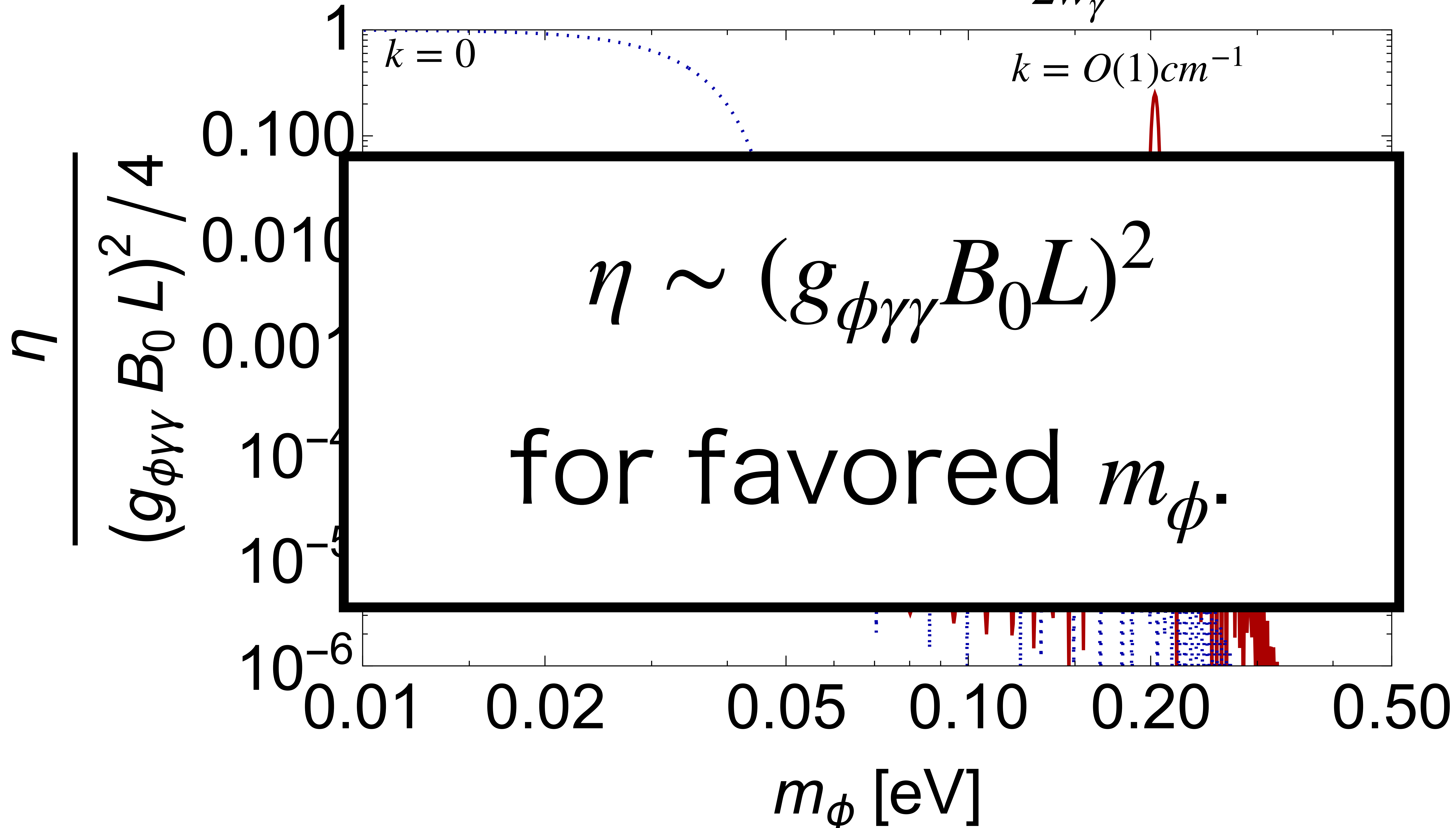
Axion-photon conversion rate $\eta \equiv n_\phi^{\rm out}/n_\gamma^{\rm in}(= n_\gamma^{\rm out}/n_\phi^{\rm in})$:

$$\eta \propto \delta_L^2(k_{\gamma,z} - k_{\phi,z} - k) \simeq \delta_L^2(\frac{m_\phi^2}{2w_\gamma} - k)$$



Axion-photon conversion rate $\eta \equiv n_\phi^{\rm out}/n_\gamma^{\rm in} (= n_\gamma^{\rm out}/n_\phi^{\rm in})$:

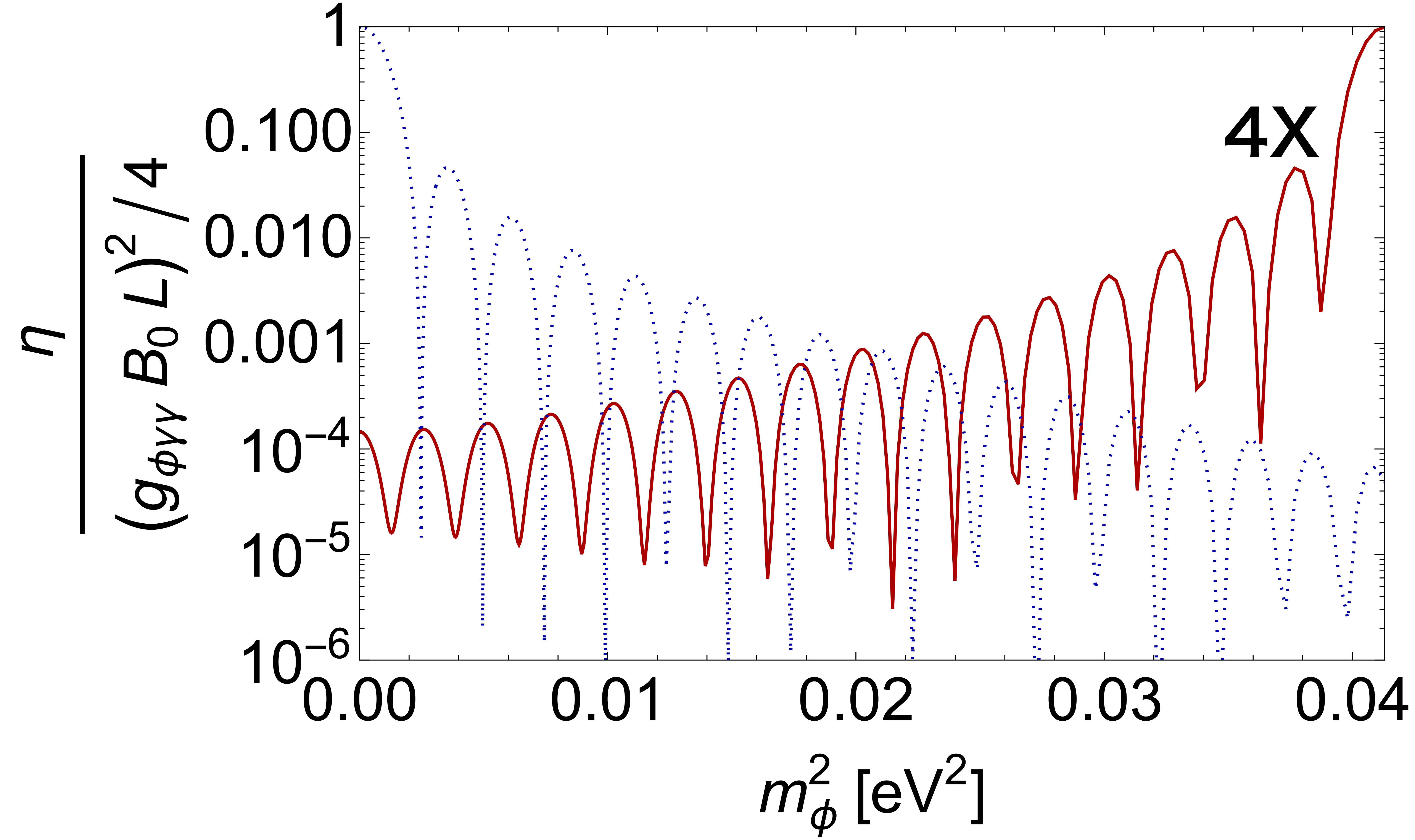
$$\eta \propto \delta_L^2(k_{\gamma,z} - k_{\phi,z} - k) \simeq \delta_L^2(\frac{m_\phi^2}{2w_\gamma} - k)$$



$$\eta \sim (g_{\phi\gamma\gamma}B_0L)^2$$

for favored m_ϕ.

A Resonance?



4X
1
0.100
0.010
0.001
10^{-4}
10^{-5}
10^{-6}
0.00
0.01
0.02
0.03
0.04
$\frac{\eta}{(g_{\phi\gamma\gamma} B_0 L)^2/4}$
m_ϕ^2 [eV^2]