Nambu-Goldstone theorem in cosmology?

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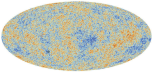
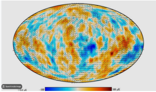
with Takahiro Tanaka (Kyoto)

+ partly with Tadashi Kuramoto (SOKENDAI)

Massless modes in cosmology 1

The unique method to address the model of inflation is…





PLANCK

→ The spectrums of ζ (~ density contrast) and GWs γ_{ij}.

In principle, we need to solve the entire history from inflation to the last scattering surface (of CMB).

But, in most case, not necessary because ζ and GWs are frozen in time in large scale (IR) limit .

WHY? and WHEN?

Massless modes in cosmology 2

$\mu = 0, 1, 2, 3$
$i = 1, 2, 3,$
$a(t)$: scale factor

Massless free scalar field $\Box\phi = 0$

- with Lorentz invariance

$$\phi \propto e^{ik(t \pm x)}$$

$$\Box\phi = \frac{1}{\sqrt{-g}}\partial_\mu(\underline{\sqrt{-g}}g^{\mu\nu}\partial_\nu)\phi$$

physical spatial volume $\propto a^3(t)$

- in cosmology

$$\phi \sim (\text{const}) + \int \frac{dt}{a^3(t)} + \mathcal{O}\left(\frac{\partial_i}{aH}\right)$$

corrections due to spatial variation

Being massless (Shift sym.). $\leftrightarrow$ **$\exists$Time independent mode.**

Weinberg's adiabatic mode

Weinberg (2002)

Puzzles

Why ζ and GWs are massless (or have shift sym)?

Diffeomorphism invariance

$$dl_3^2 = a^2(t)\, e^{2\zeta}\, dx^2$$

+ ???

$$x \to e^{-\lambda} x$$
$$\zeta \to \zeta + \lambda$$

Classically Shift symmetry
(or without hard modes)

∃Examples w/ ~~shift~~ sym. while w/ diff. invariance

- Open and closed universe ($K \neq 0$)
- α vacua (which requires cuffoff)

Especially, w/UV radiative corrections, the presence of the shift symmetry is not very clear.

Physical or unphysical?

Nambu-Goldstone theorem (for global symmetry)

Nambu (1960), Goldstone (1961, 1962), Ezawa and Swieca (1967).

0. The theory is Lorentz invariant. + 1 $\Rightarrow \exists Q$ with $\dot{Q} = 0$.

1. ∃Conserved current $j_\mu(x)$, which satisfies $\partial^\mu j_\mu(x) = 0$.

2. ∃Spacetime translation invariance, described by $U(a)$. unitary
   ∃ A unique vacuum state $|0\rangle$ with $U(a)|0\rangle = 0$.

3. The conserved current $j_\mu(x)$ transforms covariantly as

$$j_\mu(x+a) = U(a) j_\mu(x) U^{-1}(a).$$

4. Micro causality, i.e.,$[j_\mu(x), j_\nu(y)] = 0$ for x-y: spacelike.

Ezawa and Swieca (1967)

Unless there is a massless particle which interacts with the vacuum state via $j_0(x)$, the following should hold

$$\lim_{R\to\infty} \langle 0 | [j_0(f_R, t), A] | 0 \rangle = 0 \qquad \text{for all } A\text{s}$$

i.e., If LHS is non-zero, having SSB, there should be a massless field.

(spatially) smeared current $\qquad j_\mu(f_R, t)$

Using the spectral decomposition, in the limit R → ∞,

$$\lim_{R\to\infty} \langle 0 | [j_0(f_R, t), A] | 0 \rangle \sim \int_0^\infty d\mu^2 \times \lambda\,\delta(\mu^2)$$

the spectral fn. is dominated by $\mu^2=0$ pole.

Large gauge transformation (LGT)

Definition of LGT

Small GTs ($G_S \ni g$) $\quad g \to 1$ in $x \to \infty$

Large GTs ($G_L \ni g$) $\quad g \not\to 1$ in $x \to \infty$

Possible LGTs are restricted by the structure in the infinity

An example of LGT in QED

Gauge parameter $\quad \alpha(x) = \ell^\mu x_\mu \not\to 0$ in $x \to \infty$

$$\delta\psi(x) = -ie\,\ell_\mu x^\mu \psi(x), \qquad A_\mu(x) \to A_\mu(x) - \underbrace{\ell_\mu}_{\text{const. shift}}$$

Generator and Noether charge

U(1) generator

$$Q_{\text{gen}}[\alpha] \equiv \int d^3x\, \alpha(x)\, \mathcal{G}(x)$$

with Gauss law const. $\mathcal{G}(x) \equiv \partial_i E^i(x) - e\, \underbrace{j^0(x)}_{\text{charge density}} \approx 0$

Noether charge

$$Q_{\text{Noether}} = \sum_\alpha \delta\Phi_\alpha \cdot \pi_\alpha$$

$$Q_{\text{gen}}[\alpha] = Q_{\text{Noether}}[\alpha] + Q_{\text{bdry}}[\alpha]$$

$$Q_{\text{Noether}}[\alpha] = -\int_V d^3x\, \partial_i \alpha(x) E^i(x) - e\int_V d^3x\, \alpha(x)\, j^0(x)$$

$$Q_{\text{bdry}}[\alpha] = \int_V \underbrace{dS_i}_{\propto r^2}\, \alpha(x)\, \underbrace{E^i(x)}_{\propto \frac{1}{r^2}}$$

For small GTs, $Q_{\text{bdry}} = 0$, while for large GTs, $Q_{\text{bdry}} \neq 0$.

SSB for LGTs

$$Q_{gen}[\alpha] = Q_{Noether}[\alpha] + Q_{bdry}[\alpha]$$

For small GTs, $Q_{bdry} = 0$, while for large GTs, $Q_{bdry} \neq 0$.

What does this imply?

Physical state has to satisfy $Q_{gen}[\alpha]|phys\rangle = 0$

For small GTs, $Q_{Noether}[\alpha]|phys\rangle = 0$ No SSB

* Consistent with Elitzur's theorem

For large GTs, $Q_{Noether}[\alpha]|phys\rangle \neq 0$ "SSB"

… unless E falles off faster than $\propto \frac{1}{r^2}$

NG theorem for LGT in QED

Ferrari and Picasso (1971)

Generalization of Ezawa and Swieca (1967) to LGTs

Key difference

Translation and the LGT are not commutable, so

$J_\mu(x)$: Conserved current for LGT

$$J_\mu(x+a) = U(a) J_\mu(x) U^{-1}(a) + \ldots$$

Does not yield additional poles at $\mu^2=0$

Using the spectral decomposition, in the limit $R \to \infty$,

$$\lim_{R\to\infty} \langle 0 | [J_0(f_R,t), A] | 0 \rangle \sim \int_0^\infty d\mu^2 \times \lambda\,\delta(\mu^2)$$

the spectral fn. is dominated by $\mu^2=0$ pole

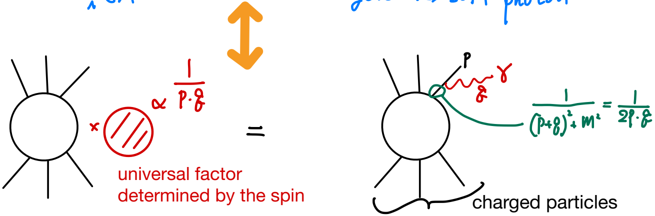
Photon as NGb and soft theorem

Ferrari and Picasso (1971)

The soft photon corresponds to NGb of the LGT and the corresponding WT identity (order parameter) gives

$$lim_{R\to\infty} < 0|\underbrace{[J^0_R, A]}_{= \frac{1}{i}\delta A}|0 > = lim_{R\to\infty} < 0|J^0_R EA - AEJ^0_R|0 >$$

generates soft photon



Soft theorem

Weinberg (1965)

What about cosmology?

Physical spatial distance $dl_3^2 = a^2(t) e^{2\zeta} dx^2$

Lagrangian is invariant under $x^i \to e^{-\lambda} x^i, \quad \zeta \to \zeta + \lambda$ LGT

<u>A hypothesis</u>

ζ and GWs are NG modes for spontaneously broken LGTs

→ They are massless (having shift symmetry).

If so,....

Conditions to apply NG th. ⟷ Conditions for massless ζ/GWs

<u>Difficulties</u>

・Spectral decomposition is not possible.

・In gauge-inv. prescription, the coordinates (x^μ) are operators.

Two aspects

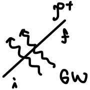
1. Why do we expect NG in cosmology?
    - Infrared universalities
2. Existence of massless modes in cosmology
    - EFT (Shift sym. of effective action)

Some examples of NG argument w/o Lonrenz sym.

Watanabe and Brauner(2011), Watanabe and Murayama (2012), Hidaka (2012)

Infrared triangle

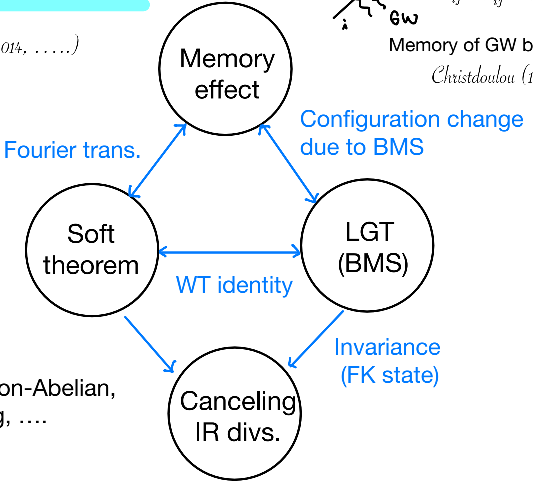
Strominger et al. (2013, 2014, ...)



$$\Delta h_{ij} = h_{ij}^{(f)} - h_{ij}^{(i)}$$

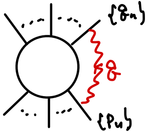
Memory of GW burst

Christdoulou (1991)



Similar examples
e.g., in Abelian, non-Abelian,
gravitonscattering, ...

Cancelling IR divergence

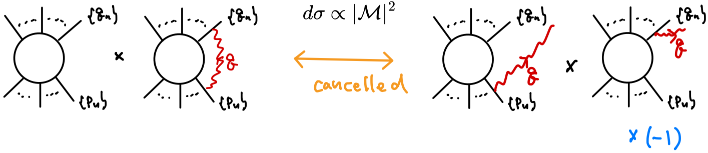


$$\int \frac{d^4q}{(q^2 - i\epsilon)(p_n \cdot q - i\epsilon)(-p_m \cdot q - i\epsilon)} \propto \text{log div.}$$

Bloch & Nordsieck (1937)

We can only resolve photons with E(q) > E_{th}

Inclusive cross section (summing E(q) < E_{th}) is free from IR div.

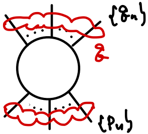


In/Out states are Fock states. S-matrix itself diverges

Cancelling IR divergence 2

Kulish & Faddeev (1970)

In/Out states dress the cloud of infinite IR photons



IR photon clouds

: Sum of infinite IR photons

In/Out states are not Fock states. S-matrix has no IR div.

<u>FK state = coherent state</u>

$$|\Phi_p^{FK}\rangle = e^{R(p)}\ |p\rangle_{Hard} \otimes |0\rangle_{soft}$$

$$R(p) \sim e \int \frac{d^3q}{2\omega_q} \left[\underbrace{\frac{p^\mu \varepsilon_\mu(q)}{p\cdot q}} a^\dagger_{soft}(q) - h.c. \right]$$

IR pole (recall soft th.)

FK state and SSB of LGT

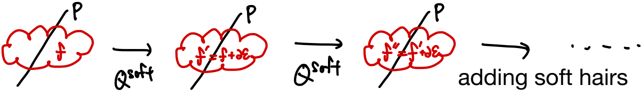
Kapec, Perry, Raclariu, Strominger (2017)

FK state = coherent state whose (soft) eigenvalue is shifted by LGT.

$$|\Psi^{FK}_{p:f}\rangle = e^{R_f(p)} \, |p\rangle_{Hard} \otimes |0\rangle_{soft} \qquad R_f(p) \sim \int_{S_2} f_p a^{+}_{soff} - h.c.$$

Q : charge of LGT which shifts soft mode $\quad f_p \to f_p + \partial_z \varepsilon$

$$Q = Q^{soft} + Q^{hard}$$



Infinite number of degenerate vacua with different soft hairs T

SSB of LGT: Choosing one of the different clouds

FK state and SSB of LGT 2

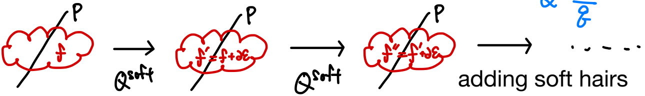
Kapec, Perry, Raclariu, Strominger (2017)

The entire FK state is invariant under the LGT.

$$Q\,|\Psi^{FK}_{p:f}\rangle = 0 \quad \Rightarrow \quad Q^{soft}\,|\Psi^{FK}_{p:f}\rangle = -\,Q^{hard}\,|\Psi^{FK}_{p:f}\rangle$$

IR pole due to soft dressing

$\propto \frac{1}{\omega}$



The shift of the soft mode is exactly cancelled by soft dressing.

graviton scattering *Ware, Saotome, Akhoury (13), Choi, Kol, Akhoury (17)*

Infrared “triangle” in cosmology

Remarks for non-cosmologists

- No Lorentz invariance, IR = Long (spatial) distance
- Target is not $\mathcal{M}$ (in-out), but expectation value (in-in)

Relevant large gauge transformations

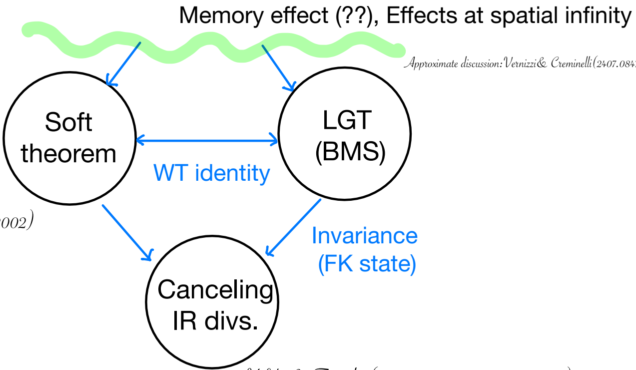
3 dim spatial line element $$dl^2 = a^2(t)e^{2\zeta(t,x)}[e^{\gamma(t,x)}]_{ij}dx^i dx^j$$

$$\gamma_{ii} = \partial_i \gamma_{ij} = 0$$

- Dilatation $x^i \to x^i_\lambda = e^\lambda x^i$ $\zeta^\lambda(t,x^i) \simeq \zeta(t,x^i) - \lambda(1 + x^i\partial_i)$
- (Shear) distortion $x^i \to [e^{\frac{1}{2}\Lambda}]^i{}_j\, x^j$ $\gamma^\Lambda_{ij}(t,x^i) \simeq \gamma_{ij}(t,x^i) - \Lambda \cdots$

Constant shift of ζ/GW

Infrared "triangle" in cosmology 2



Approximate discussion: Vernizzi & Creminelli (2407.08472)

Maldacena (2002)

Y.U. & Tanaka (2009, 2010, 2013, 2014, …)
Gidding & Sloth (2010), Senatore & Zaldarriaga (2012)
…..

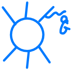
Consistency relation (Soft theorem)

$|vac\rangle$: Euclidean vacuum (Free theory limit is Bunch-Davies)

$$-\sum_{i=1}^{n}\left(\mathbf{k}_i\frac{\partial}{\partial \mathbf{k}_i}+\cdots\right)\langle vac|\mathcal{O}^H(t_1,\mathbf{k}_1)\cdots\mathcal{O}^H(t_n,\mathbf{k}_n)|vac\rangle' = \frac{\langle vac|\zeta_{\mathbf{q}}\,\mathcal{O}^H(t_1,\mathbf{k}_1)\cdots\mathcal{O}^H(t_n,\mathbf{k}_n)|vac\rangle'}{P^{(\zeta)}(q)}$$

spin dep.

w/o delta fn.



for $\mathcal{O}=\zeta$

- Consistency relation with all modes being superhorizon.

Maldacena (2002), Creminelli and Zaldarriaga (2004)

n=2, f_{NL} = -0. 9 ± 5. 1 [PLANCK 18]

- This relation holds even if short modes are sub horizon.

Pimental, Senatore, and Zaldarriga (2012)

for $\mathcal{O}$: general integer spin fields

- This relation holds even if short modes are sub horizon.

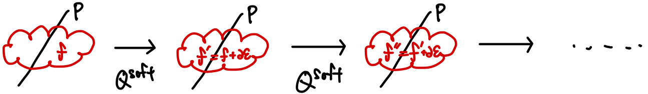
Tanaka & Urakawa (2017)

Euclidean vacuum and FK

Kuramoto, Tanaka & Y.U. (in progress)

FK state $\quad Q\,|\Phi^{FK}_{p:f}\rangle = 0$

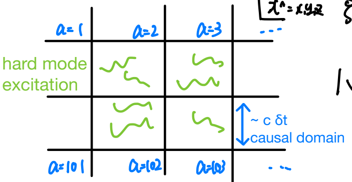
degenerate coherent states



The shift of the soft mode is cancelled by soft dressing.

Euclidean vacuum in cosmology

Different spacing = Different causality connected regions



$x^i = x,y,z$

ξ_a : spatial average of ζ in spacing a

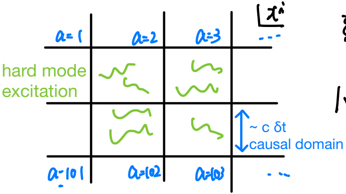
$$|vac\rangle = \prod_{a=1} \int d\bar{\xi}_a \, |\psi(\bar{\xi}_a)| \, |\bar{\xi}_a\rangle \, |hard\rangle_{\bar{\xi}_a}$$

Approx. Gaussian weight

FK state

Euc. vacuum and soft theorem

Tanaka & Y.U. (2017, in progress)
Kuramoto, Tanaka, & Y.U. (in progress)



$\bar{\xi}_a$: spatial average of ζ in spacing a

$$|vac\rangle = \prod_{a=1} \int d\bar{\xi}_a \, |\Psi(\bar{\xi}_a)| \, |\bar{\xi}_a\rangle \, |hard\rangle_{\bar{\xi}_a}$$

Approx. Gaussian weight

FK state

Each coherent state for a given $\bar{\xi}_a$ satisfies. $Q\,|\bar{\xi}_a\rangle |hard\rangle_{\bar{\xi}_a} = 0$

$Q\ (= Q^{soft} + Q^{hard})$: Charge of LGT

Tanaka & Y.U. (2017)

The shift of ζ's soft mode is cancelled by hard modes (of arbitrary sources of gravity) w/soft dressing.

$$\langle vac|\left[iQ^{hard}, \prod_{a=1}^{n} \mathcal{O}^H(t_a, x_a)\right]|vac\rangle = -\langle vac|\left[iQ^{soft}, \prod_{a=1}^{n} \mathcal{O}^H(t_a, x_a)\right]|vac\rangle$$

Soft theorem (Consistency relation)

Claims

Tanaka & Y.U. (2017, in progress)

- The theory is local and spatial Diff invariance.
- The quantum stat |Ψ> is LGT(dilatation/shear) invariant,i.e.,

$$Q^{dilatation}\,|\zeta^w\rangle\,|hard\rangle_{\zeta^w}=0\,,\qquad Q^{shear}\,|\gamma^w\rangle\,|hard\rangle_{\gamma^w}=0$$

$\propto\langle\zeta^{(w)}|\Psi\rangle$

- Soft theorem

$$-\sum_{i=1}^{n}\left(\mathbf{k}_i\cdot\frac{\partial}{\partial \mathbf{k}_i}+\cdots\right)\langle\Psi|O^H(t_1,\mathbf{k}_1)\cdots O^H(t_n,\mathbf{k}_n)|\Psi\rangle' = \frac{\langle\Psi|\zeta^w_{\mathbf{q}}\,O^H(t_1,\mathbf{k}_1)\cdots O^H(t_n,\mathbf{k}_n)|\Psi\rangle'}{P^{(\zeta)}(q)}$$

$\zeta=\zeta^w+\cdots$ is subject to LGT.

general!

important if $\zeta\to\zeta^w$ in $t\gg1$

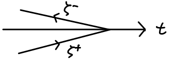
t ↑ $\zeta_w\gg\zeta$, $\zeta_w\ll\zeta$??

- Cancellation of IR divergence
- Shift symmetry of the effective action

Shift symmetry in EFT (~ ∃NG mode)

Feynman - Vernon's influence functional

Integrating out the hard modes



$$S_{eff}[\zeta^+, \zeta^-] = S[\zeta^+] - S[\zeta^-] + S_{IF}[\zeta^+, \zeta^-]$$

Correction due to integrating out hard modes

Tanaka & Y.U. (2015, 2017, in progress)

inflaton + massive fields

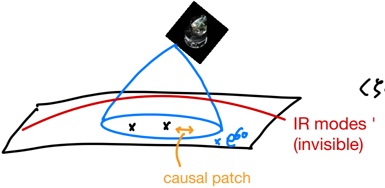
↳ a general theory w/ LGT

Perturbatively,….

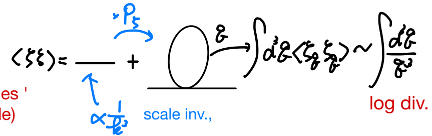
$$S_{eff}[\zeta^+ - s, \zeta^- - s] = S_{eff}[\zeta^+, \zeta^-] + s \cdot (\underbrace{\text{soft th.}}_{\neq 0})$$
$$+ s^2 (\underbrace{\text{soft th}}_{\neq 0}) + \cdots$$

Shift symmetry → Existence of NG mode

Cancellation of IR divergence



A naive computation yields....



Tanaka & Y.U. (2009, 2010, 2013, 2014, 2017)

These diverging IR modes = Shift of ζ due to LGT

→ LGT invariant quantities are free from IR div.

physical distance (LGT inv) $dl_{\rm phys} = a(t)e^{\zeta}dl$

Evaluate correlators using $l_{\rm phys}$ but not l .

Dressing of IR modes $\bar{\zeta}$ (~ FK)

Summary and prospects

The overall IR structure of QED/perturbed gravity in asymptotically flat spacetime and the one in cosmology share the same properties.

LGT inv. $\longrightarrow$ Soft th, Cancellation of IR div

$\exists$Shift symmetry (massless mode)

$\downarrow$ for cosmology

time independent mode
($\neq$ conservation of ζ)

, while there are also differences in a detailed view.

- $|E\,\mathrm{vac}\rangle \sim \int d\xi\,(\mathrm{weight})\,|FK\rangle$?? · explicit op. on hard modes.
- In cosmology one can consider LGT non-inv. state, but what about QED or perturbed gravity (in AF)?