Exploring chirality structure in nucleon decay

Hiroki Takahashi (2nd year in PhD)

with

Koichi Hamaguchi (UTokyo), Shihwen Hor (TDLI), Natsumi Nagata (UTokyo)

JHEP 01 (2025) 175

The Frontier of Particle Physics 2025@YITP

東京大学 THE UNIVERSITY OF TOKYO

Take-home: Nucleon decay → Chirality structure

Decay channel: $p \to \pi^0 e^+$ and $p \to \eta e^+$

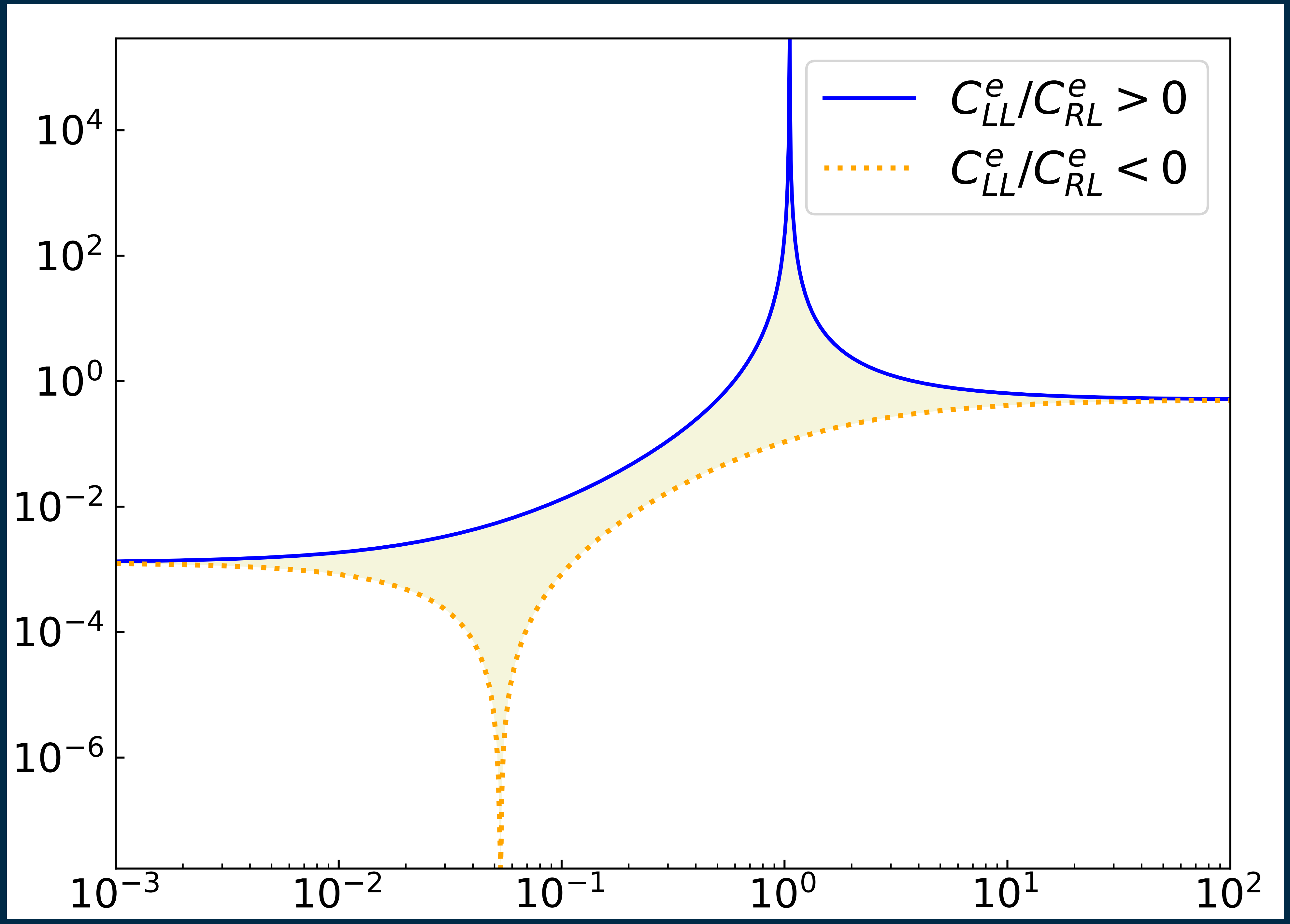
$$\mathcal{L}_{\text{eff}} = C^{\ell}_{RL}\left[\epsilon_{abc}(u^a_R d^b_R)(u^c_L \ell_L)\right] + C^{\ell}_{LR}\left[\epsilon_{abc}(u^a_L d^b_L)(u^c_R \ell_R)\right] + C^{\ell}_{LL}\left[\epsilon_{abc}(u^a_L d^b_L)(u^c_L \ell_L)\right] + C^{\ell}_{RR}\left[\epsilon_{abc}(u^a_R d^b_R)(u^c_R \ell_R)\right]$$

Take-home: Nucleon decay → Chirality structure

Decay channel: $p \to \pi^0 e^+$ and $p \to \eta e^+$

$$\mathcal{L}_{\rm eff} = C^\ell_{RL}\left[\epsilon_{abc}(u^a_R d^b_R)(u^c_L \ell_L)\right] + C^\ell_{LR}\left[\epsilon_{abc}(u^a_L d^b_L)(u^c_R \ell_R)\right] + C^\ell_{LL}\left[\epsilon_{abc}(u^a_L d^b_L)(u^c_L \ell_L)\right] + C^\ell_{RR}\left[\epsilon_{abc}(u^a_R d^b_R)(u^c_R \ell_R)\right]$$

$$\frac{\Gamma(p \to \eta e^+)}{\Gamma(p \to \pi^0 e^+)}$$



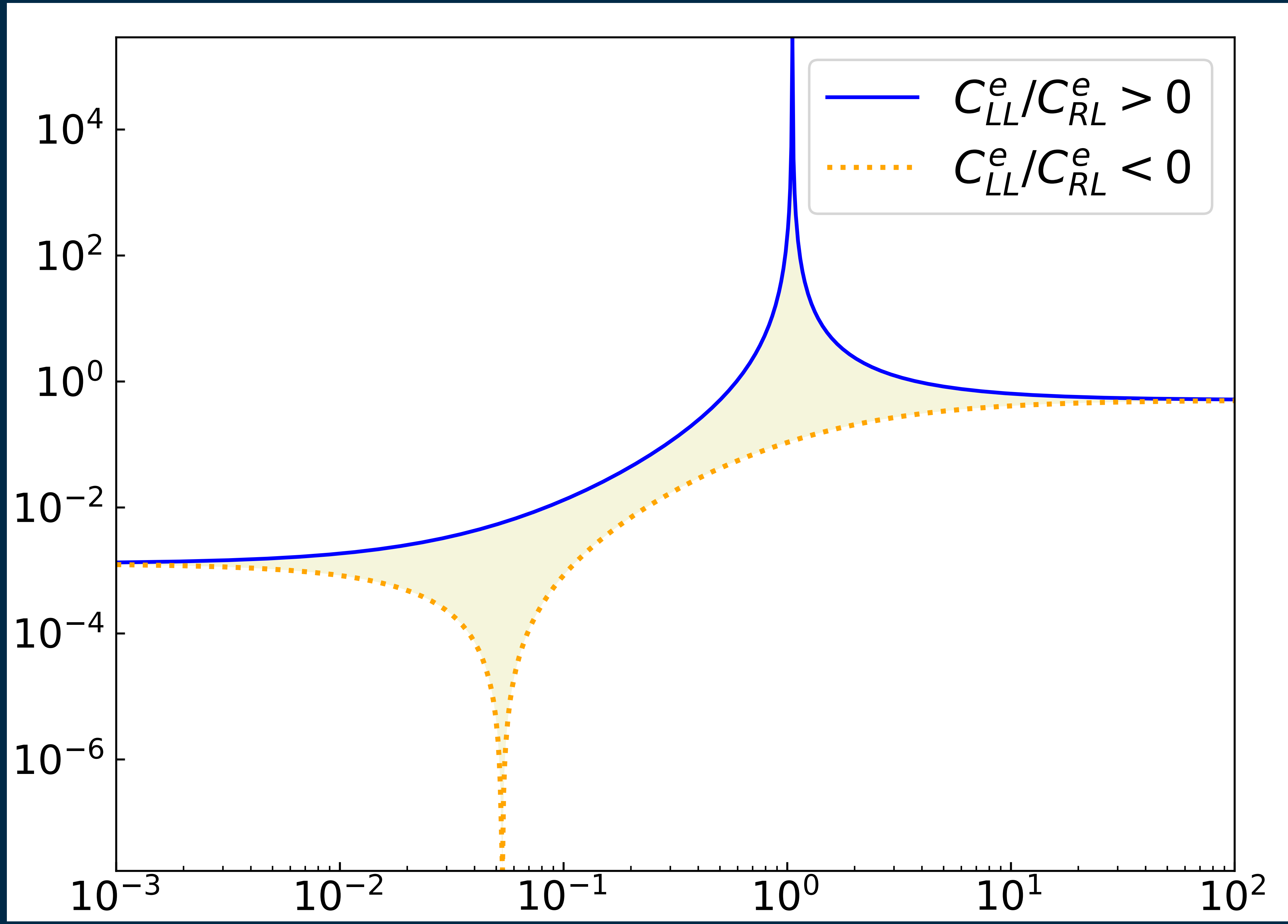
$$\left|\frac{C^e_{LL}}{C^e_{RL}}\right|$$

Take-home: Nucleon decay → Chirality structure

Decay channel: $p \to \pi^0 e^+$ and $p \to \eta e^+$

$$\mathcal{L}_{\text{eff}} = C^{\ell}_{RL}\left[\epsilon_{abc}(u^a_R d^b_R)(u^c_L \ell_L)\right] + C^{\ell}_{LR}\left[\epsilon_{abc}(u^a_L d^b_L)(u^c_R \ell_R)\right] + C^{\ell}_{LL}\left[\epsilon_{abc}(u^a_L d^b_L)(u^c_L \ell_L)\right] + C^{\ell}_{RR}\left[\epsilon_{abc}(u^a_R d^b_R)(u^c_R \ell_R)\right]$$

$$\frac{\Gamma(p \to \eta e^+)}{\Gamma(p \to \pi^0 e^+)}$$



$$\left|\frac{C^e_{LL}}{C^e_{RL}}\right|$$

Plan:

- Introduction
- EFT discussion
- Application to SUSY GUTs

Baryon number violation

Baryon number is expected to be violated in physics beyond the standard model.

- Baryon asymmetry of the universe

  Sakharov's three conditions → B violation A. D. Sakharov (1966)

Baryon number violation

Baryon number is expected to be violated in physics beyond the standard model (SM).

- Baryon asymmetry of the universe

  Sakharov's three conditions → B violation   A. D. Sakharov (1966)

- Gauge coupling unification: $\mathrm{SU(3)}_C \otimes \mathrm{SU(2)}_L \otimes \mathrm{U(1)}_Y \to \mathrm{SU(5)}$

  nicely incorporated in **Grand Unified Theory (GUT)**

  H. Georgi and S. L. Glashow (1974)

  - Quark-lepton unification

$$\mathbf{10}_i = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u^\dagger_{Ri3} & -u^\dagger_{Ri2} & u^1_{Li} & d^1_{Li} \\ -u^\dagger_{Ri3} & 0 & u^\dagger_{Ri1} & u^2_{Li} & d^2_{Li} \\ u^\dagger_{Ri2} & -u^\dagger_{Ri1} & 0 & u^3_{Li} & d^3_{Li} \\ -u^1_{Li} & -u^2_{Li} & -u^3_{Li} & 0 & e^\dagger_{Ri} \\ -d^1_{Li} & -d^2_{Li} & -d^3_{Li} & -e^\dagger_{Ri} & 0 \end{pmatrix}, \quad \bar{\mathbf{5}}_i = \begin{pmatrix} d^\dagger_{Ri1} \\ d^\dagger_{Ri2} \\ d^\dagger_{Ri3} \\ e_{Li} \\ -\nu_{Li} \end{pmatrix}$$

  - B violation

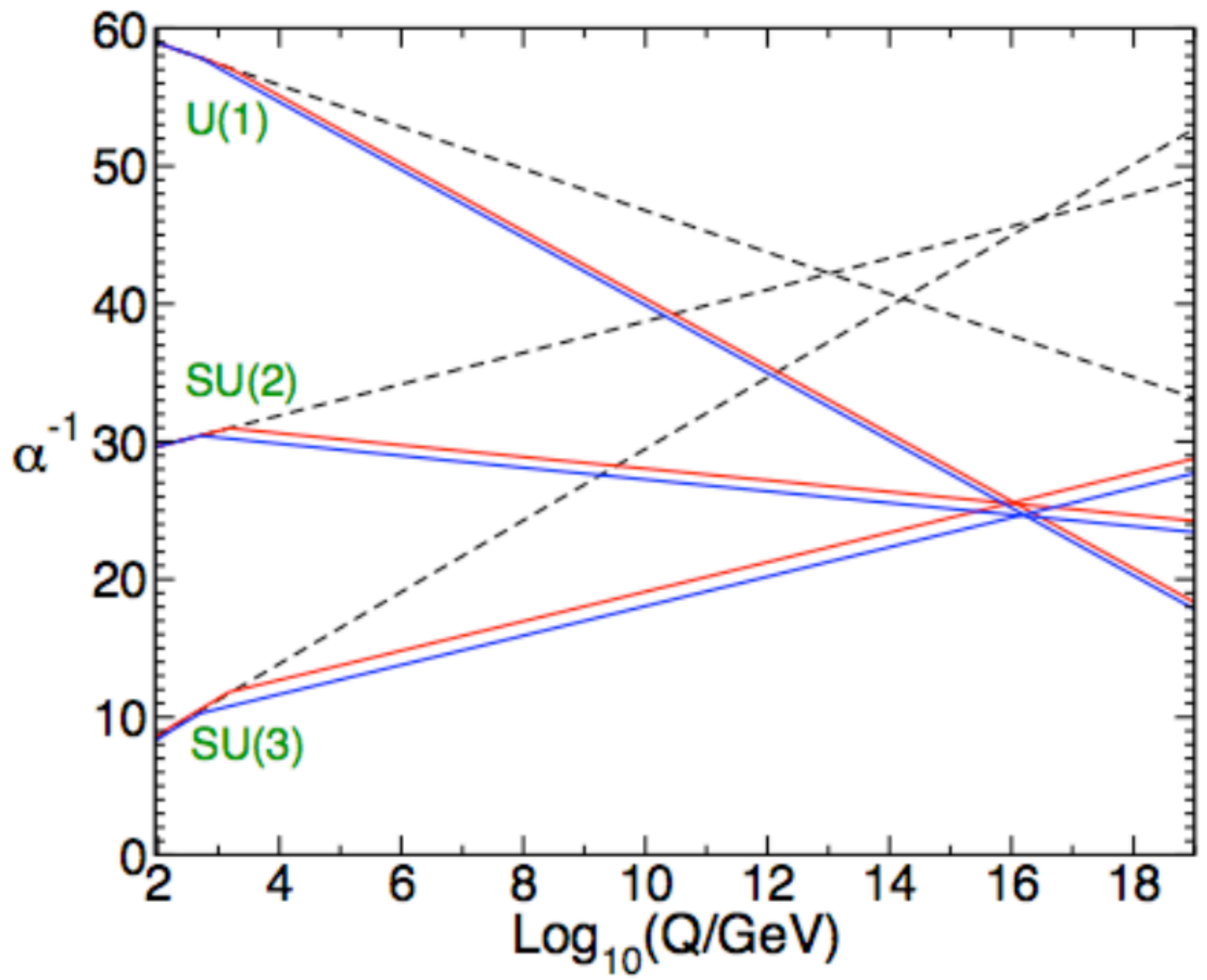


Fig. from P. Langacker (1981)

Nucleon decay

Nucleon decay has been the main probe of baryon-number violation.

e.g. $\tau(p \to \pi^0 e^+) > 2.4 \times 10^{34}$ years Super-Kamiokande Collaboration (2020)

Nucleon decay

Nucleon decay has been the main probe of baryon-number violation.

e.g. $\tau(p \to \pi^0 e^+) > 2.4 \times 10^{34}$ years Super-Kamiokande Collaboration (2020)

Next-generation nucleon decay experiments:

- Hyper-Kamiokande (HK)
- JUNO
- DUNE

Effective interactions for nucleon decay

S. Weinberg (1979)
F. Wilczek and A. Zee (1979)
L. F. Abbott and M. B. Wise (1980)

$$\mathcal{L}_{\mathrm{SM,eff}} = \sum_{I,ijkl} C^{ijkl}_{(I)} \mathcal{O}^{(I)}_{ijkl} + \mathrm{h.c.}$$

i, j, k, l: flavor

$\boxed{SU(3)_C \otimes SU(2)_L \otimes U(1)_Y}$

$$\mathcal{O}^{(1)}_{ijkl} = \epsilon_{abc}\epsilon^{\alpha\beta}(u^a_{Ri} d^b_{Rj})(Q^c_{Lk\alpha} L_{Ll\beta})$$

$$\mathcal{O}^{(2)}_{ijkl} = \epsilon_{abc}\epsilon^{\alpha\beta}(Q^a_{Li\alpha} Q^b_{Lj\beta})(u^c_{Rk} e_{Rl})$$

$$\mathcal{O}^{(3)}_{ijkl} = \epsilon_{abc}\epsilon^{\alpha\beta}\epsilon^{\gamma\delta}(Q^a_{Li\alpha} Q^b_{Lj\gamma})(Q^c_{Lk\delta} L_{Ll\beta})$$

$$\mathcal{O}^{(4)}_{ijkl} = \epsilon_{abc}(u^a_{Ri} d^b_{Rj})(u^c_{Rk} e_{Rl})$$

Effective interactions for nucleon decay

S. Weinberg (1979)
F. Wilczek and A. Zee (1979)
L. F. Abbott and M. B. Wise (1980)

$$\mathcal{L}_{\mathrm{SM,eff}} = \sum_{I,ijkl} C^{ijkl}_{(I)} \mathcal{O}^{(I)}_{ijkl} + \mathrm{h.c.}$$

i, j, k, l: flavor

$\mathrm{SU(3)}_C \otimes \mathrm{SU(2)}_L \otimes \mathrm{U(1)}_Y$

$$\boxed{\begin{aligned}\mathcal{O}^{(1)}_{ijkl} &= \epsilon_{abc}\epsilon^{\alpha\beta}(u^a_{Ri}d^b_{Rj})(Q^c_{Lk\alpha}L_{Ll\beta})\\ \mathcal{O}^{(2)}_{ijkl} &= \epsilon_{abc}\epsilon^{\alpha\beta}(Q^a_{Li\alpha}Q^b_{Lj\beta})(u^c_{Rk}e_{Rl})\end{aligned}}$$

Mixed-type

$$\mathcal{O}^{(3)}_{ijkl} = \epsilon_{abc}\epsilon^{\alpha\beta}\epsilon^{\gamma\delta}(Q^a_{Li\alpha}Q^b_{Lj\gamma})(Q^c_{Lk\delta}L_{Ll\beta})$$

$$\mathcal{O}^{(4)}_{ijkl} = \epsilon_{abc}(u^a_{Ri}d^b_{Rj})(u^c_{Rk}e_{Rl})$$

Effective interactions for nucleon decay

S. Weinberg (1979)
F. Wilczek and A. Zee (1979)
L. F. Abbott and M. B. Wise (1980)

$$\mathcal{L}_{\mathrm{SM,eff}} = \sum_{I,ijkl} C^{ijkl}_{(I)} \mathcal{O}^{(I)}_{ijkl} + \mathrm{h.c.}$$

i, j, k, l: flavor

$\boxed{\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y}$

Mixed-type

$$\mathcal{O}^{(1)}_{ijkl} = \epsilon_{abc}\epsilon^{\alpha\beta}(u^a_{Ri} d^b_{Rj})(Q^c_{Lk\alpha} L_{Ll\beta})$$

$$\mathcal{O}^{(2)}_{ijkl} = \epsilon_{abc}\epsilon^{\alpha\beta}(Q^a_{Li\alpha} Q^b_{Lj\beta})(u^c_{Rk} e_{Rl})$$

Pure-type

$$\mathcal{O}^{(3)}_{ijkl} = \epsilon_{abc}\epsilon^{\alpha\beta}\epsilon^{\gamma\delta}(Q^a_{Li\alpha} Q^b_{Lj\gamma})(Q^c_{Lk\delta} L_{Ll\beta})$$

$$\mathcal{O}^{(4)}_{ijkl} = \epsilon_{abc}(u^a_{Ri} d^b_{Rj})(u^c_{Rk} e_{Rl})$$

Effective interactions for nucleon decay

S. Weinberg (1979)
F. Wilczek and A. Zee (1979)
L. F. Abbott and M. B. Wise (1980)

$$\mathcal{L}_{\rm SM,eff} = \sum_{I,ijkl} C^{ijkl}_{(I)} \mathcal{O}^{(I)}_{ijkl} + \text{h.c.}$$

i, j, k, l: flavor

$$\begin{aligned}
\mathcal{O}^{(1)}_{ijkl} &= \epsilon_{abc}\epsilon^{\alpha\beta}(u^a_{Ri} d^b_{Rj})(Q^c_{Lk\alpha} L_{Ll\beta}) \\
\mathcal{O}^{(2)}_{ijkl} &= \epsilon_{abc}\epsilon^{\alpha\beta}(Q^a_{Li\alpha} Q^b_{Lj\beta})(u^c_{Rk} e_{Rl})
\end{aligned}$$

Mixed-type

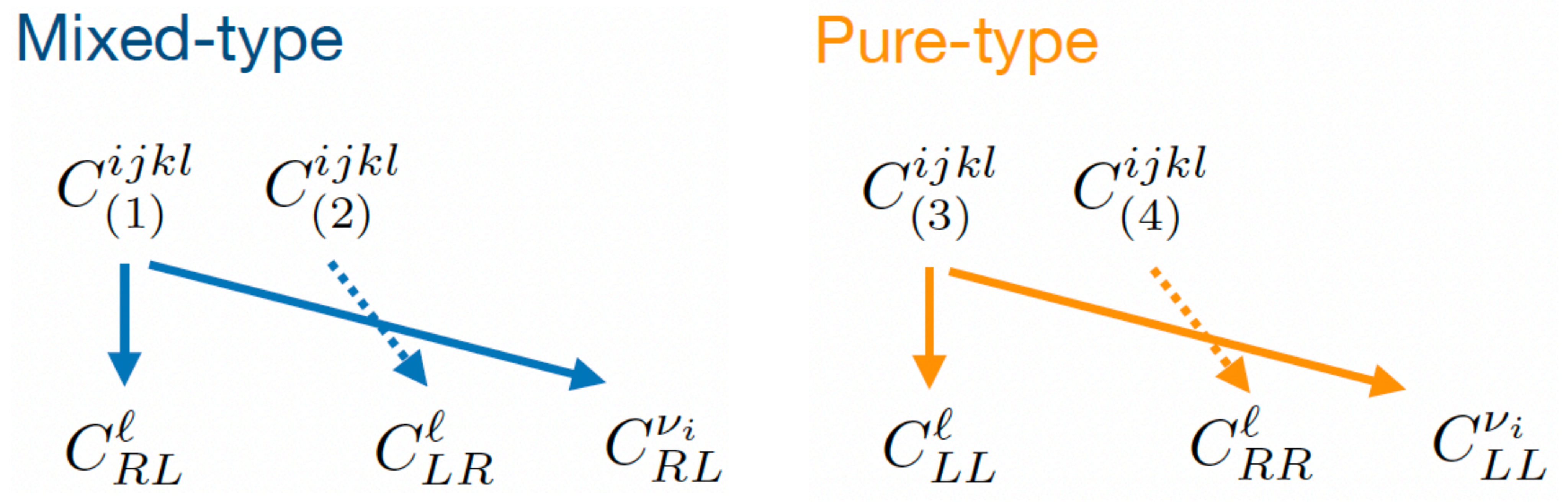
$$\begin{aligned}
\mathcal{O}^{(3)}_{ijkl} &= \epsilon_{abc}\epsilon^{\alpha\beta}\epsilon^{\gamma\delta}(Q^a_{Li\alpha} Q^b_{Lj\gamma})(Q^c_{Lk\delta} L_{Ll\beta}) \\
\mathcal{O}^{(4)}_{ijkl} &= \epsilon_{abc}(u^a_{Ri} d^b_{Rj})(u^c_{Rk} e_{Rl})
\end{aligned}$$

Pure-type

$\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$

$\downarrow$

$\mathrm{SU}(3)_C \otimes \mathrm{U}(1)$



Matching

$$\begin{aligned}
\mathcal{L}_{\rm eff} = {} & C^{\ell}_{RL}\left[\epsilon_{abc}(u^a_R d^b_R)(u^c_L \ell_L)\right] + C^{\ell}_{LR}\left[\epsilon_{abc}(u^a_L d^b_L)(u^c_R \ell_R)\right] \\
& + C^{\ell}_{LL}\left[\epsilon_{abc}(u^a_L d^b_L)(u^c_L \ell_L)\right] + C^{\ell}_{RR}\left[\epsilon_{abc}(u^a_R d^b_R)(u^c_R \ell_R)\right] \\
& + C^{\nu_i}_{RL}\left[\epsilon_{abc}(u^a_R d^b_R)(d^c_L \nu_{Li})\right] + C^{\nu_i}_{LL}\left[\epsilon_{abc}(u^a_L d^b_L)(d^c_L \nu_{Li})\right] + \text{h.c.}
\end{aligned}$$

Effective interactions for nucleon decay

S. Weinberg (1979)
F. Wilczek and A. Zee (1979)
L. F. Abbott and M. B. Wise (1980)

$$\mathcal{L}_{\mathrm{SM,eff}} = \sum_{I,ijkl} C_{(I)}^{ijkl} \mathcal{O}_{ijkl}^{(I)} + \mathrm{h.c.}$$

i, j, k, l: flavor

$$\mathcal{O}_{ijkl}^{(1)} = \epsilon_{abc}\epsilon^{\alpha\beta}(u_{Ri}^a d_{Rj}^b)(Q_{Lk\alpha}^c L_{Ll\beta})$$

$$\mathcal{O}_{ijkl}^{(2)} = \epsilon_{abc}\epsilon^{\alpha\beta}(Q_{Li\alpha}^a Q_{Lj\beta}^b)(u_{Rk}^c e_{Rl})$$

Mixed-type

$$\mathcal{O}_{ijkl}^{(3)} = \epsilon_{abc}\epsilon^{\alpha\beta}\epsilon^{\gamma\delta}(Q_{Li\alpha}^a Q_{Lj\gamma}^b)(Q_{Lk\delta}^c L_{Ll\beta})$$

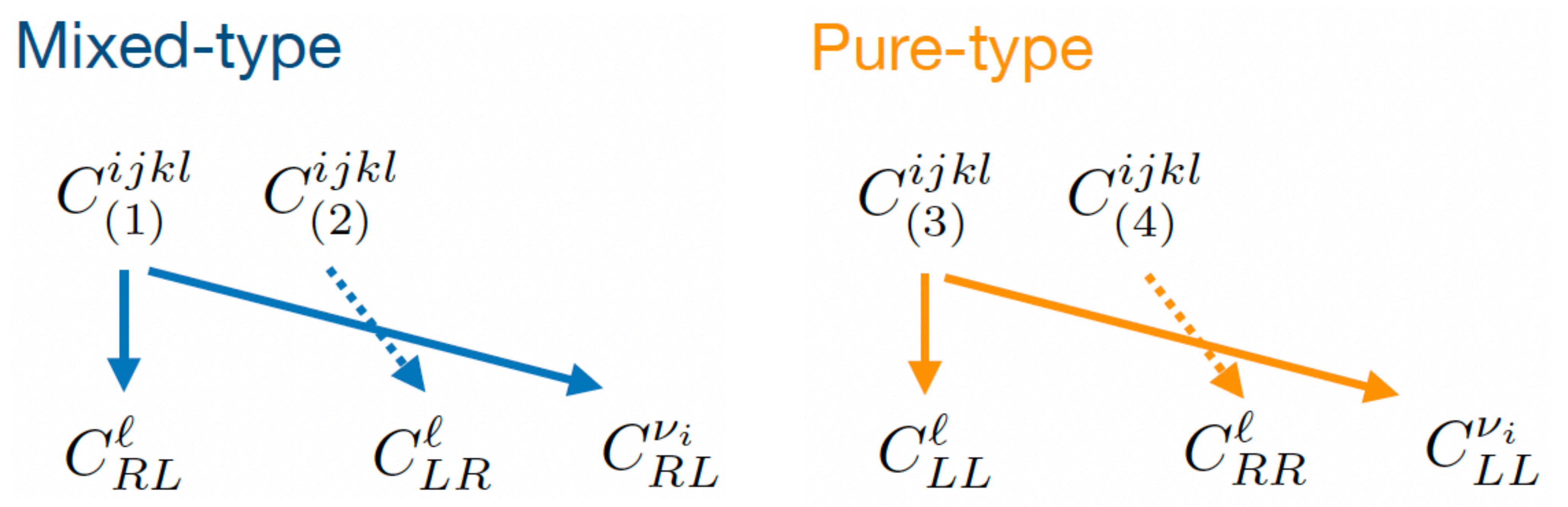
$$\mathcal{O}_{ijkl}^{(4)} = \epsilon_{abc}(u_{Ri}^a d_{Rj}^b)(u_{Rk}^c e_{Rl})$$

Pure-type

Matching

$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$

$SU(3)_C \otimes U(1)$ $\quad \Delta S = 0$



$$\begin{aligned}\mathcal{L}_{\mathrm{eff}} = {} & C_{RL}^{\ell}\left[\epsilon_{abc}(u_R^a d_R^b)(u_L^c \ell_L)\right] + C_{LR}^{\ell}\left[\epsilon_{abc}(u_L^a d_L^b)(u_R^c \ell_R)\right] \\ & + C_{LL}^{\ell}\left[\epsilon_{abc}(u_L^a d_L^b)(u_L^c \ell_L)\right] + C_{RR}^{\ell}\left[\epsilon_{abc}(u_R^a d_R^b)(u_R^c \ell_R)\right] \\ & + C_{RL}^{\nu_i}\left[\epsilon_{abc}(u_R^a d_R^b)(d_L^c \nu_{Li})\right] + C_{LL}^{\nu_i}\left[\epsilon_{abc}(u_L^a d_L^b)(d_L^c \nu_{Li})\right] + \mathrm{h.c.}\end{aligned}$$

$$\boxed{\begin{array}{ll} p \to \pi^0 \ell^+ & \\ & p \to \eta \ell^+ \\ n \to \pi^- \ell^+ & \end{array}}$$

$\ell = e, \mu$

Effective interactions for nucleon decay

S. Weinberg (1979)
F. Wilczek and A. Zee (1979)
L. F. Abbott and M. B. Wise (1980)

$$\mathcal{L}_{\mathrm{SM,eff}} = \sum_{I,ijkl} C^{ijkl}_{(I)} \mathcal{O}^{(I)}_{ijkl} + \mathrm{h.c.}$$

i, j, k, l: flavor

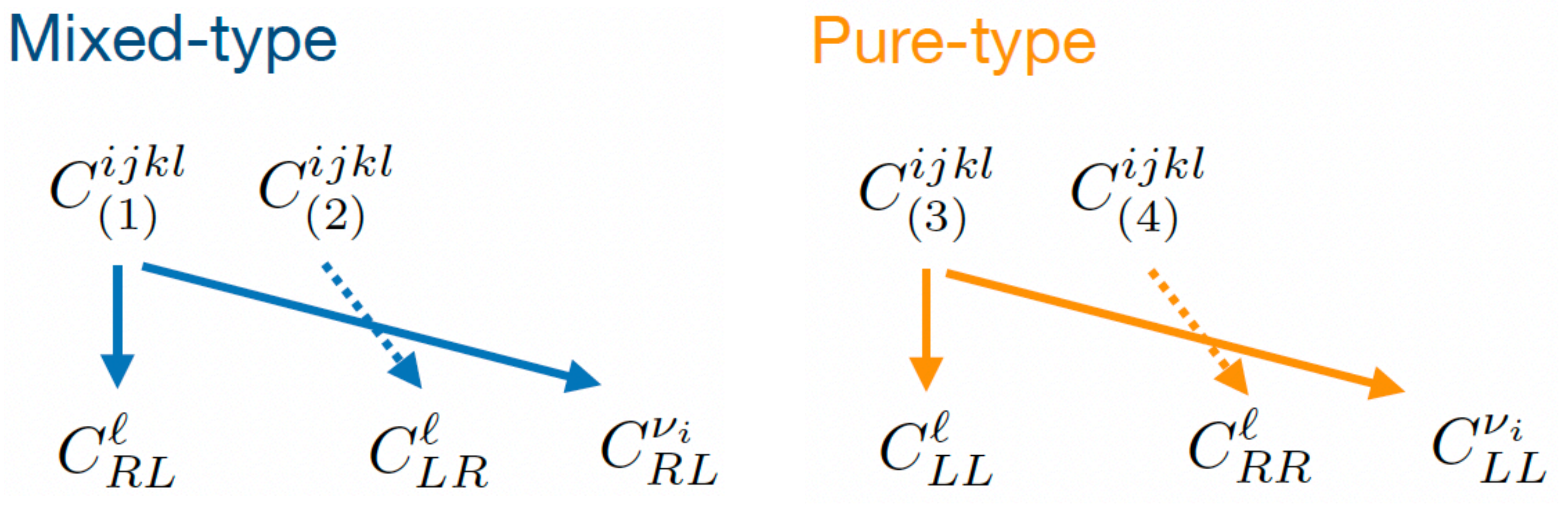
Mixed-type

$$\mathcal{O}^{(1)}_{ijkl} = \epsilon_{abc}\epsilon^{\alpha\beta}(u^a_{Ri}d^b_{Rj})(Q^c_{Lk\alpha}L_{Ll\beta})$$
$$\mathcal{O}^{(2)}_{ijkl} = \epsilon_{abc}\epsilon^{\alpha\beta}(Q^a_{Li\alpha}Q^b_{Lj\beta})(u^c_{Rk}e_{Rl})$$

Pure-type

$$\mathcal{O}^{(3)}_{ijkl} = \epsilon_{abc}\epsilon^{\alpha\beta}\epsilon^{\gamma\delta}(Q^a_{Li\alpha}Q^b_{Lj\gamma})(Q^c_{Lk\delta}L_{Ll\beta})$$
$$\mathcal{O}^{(4)}_{ijkl} = \epsilon_{abc}(u^a_{Ri}d^b_{Rj})(u^c_{Rk}e_{Rl})$$

SU(3)$_C$ ⊗ SU(2)$_L$ ⊗ U(1)$_Y$



Matching

SU(3)$_C$ ⊗ U(1)

$\Delta S = 0$

$$\begin{aligned}\mathcal{L}_{\mathrm{eff}} = {} & C^{\ell}_{RL}\left[\epsilon_{abc}(u^a_R d^b_R)(u^c_L \ell_L)\right] + C^{\ell}_{LR}\left[\epsilon_{abc}(u^a_L d^b_L)(u^c_R \ell_R)\right] \\ & + C^{\ell}_{LL}\left[\epsilon_{abc}(u^a_L d^b_L)(u^c_L \ell_L)\right] + C^{\ell}_{RR}\left[\epsilon_{abc}(u^a_R d^b_R)(u^c_R \ell_R)\right] \\ & + C^{\nu_i}_{RL}\left[\epsilon_{abc}(u^a_R d^b_R)(d^c_L \nu_{Li})\right] + C^{\nu_i}_{LL}\left[\epsilon_{abc}(u^a_L d^b_L)(d^c_L \nu_{Li})\right] + \mathrm{h.c.}\end{aligned}$$

$p \to \pi^0 \ell^+$ $\quad$ $p \to \eta \ell^+$
$n \to \pi^- \ell^+$

$\ell = e, \mu$

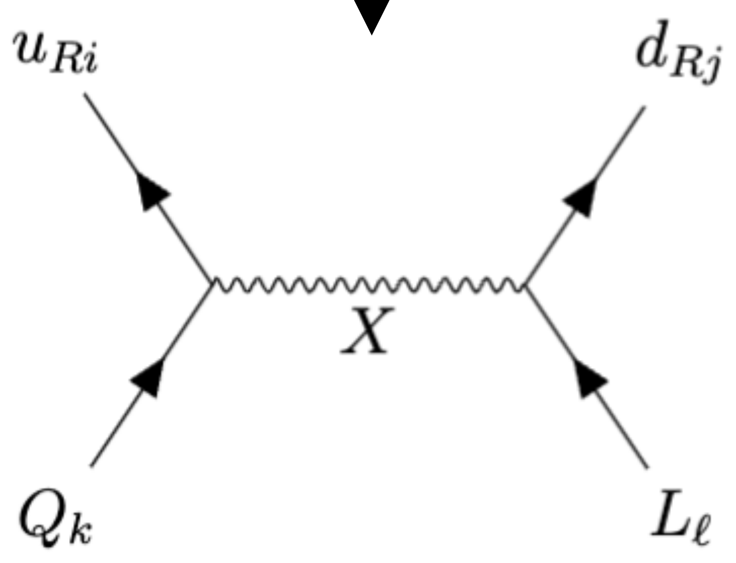
$p \to \pi^+ \bar{\nu}_i$ $\quad$ $n \to \eta \bar{\nu}_i$
$n \to \pi^0 \bar{\nu}_i$

Chirality structure and UV mechanisms

H. Georgi and S. L. Glashow (1974)

Dominant in
non-SUSY GUTs

$X_\mu \psi_{R\dot{\alpha}} \bar{\sigma}^{\mu\dot{\alpha}\alpha} \chi_{L\alpha}$



Gauge boson exchange

↓

Dimension-six Kähler

↓

$C^{ijkl}_{(1)}$ $C^{ijkl}_{(2)}$

Mixed-type

Chirality structure and UV mechanisms

H. Georgi and S. L. Glashow (1974)

S. Weinberg (1982)
N. Sakai and T. Yanagida (1982)

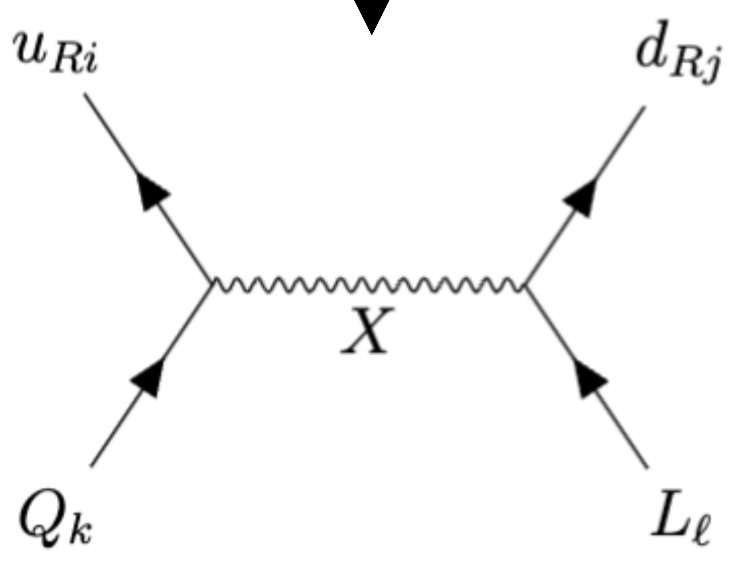
Gauge boson exchange → Dimension-six Kähler → $C_{(1)}^{ijkl}$ $C_{(2)}^{ijkl}$ Mixed-type

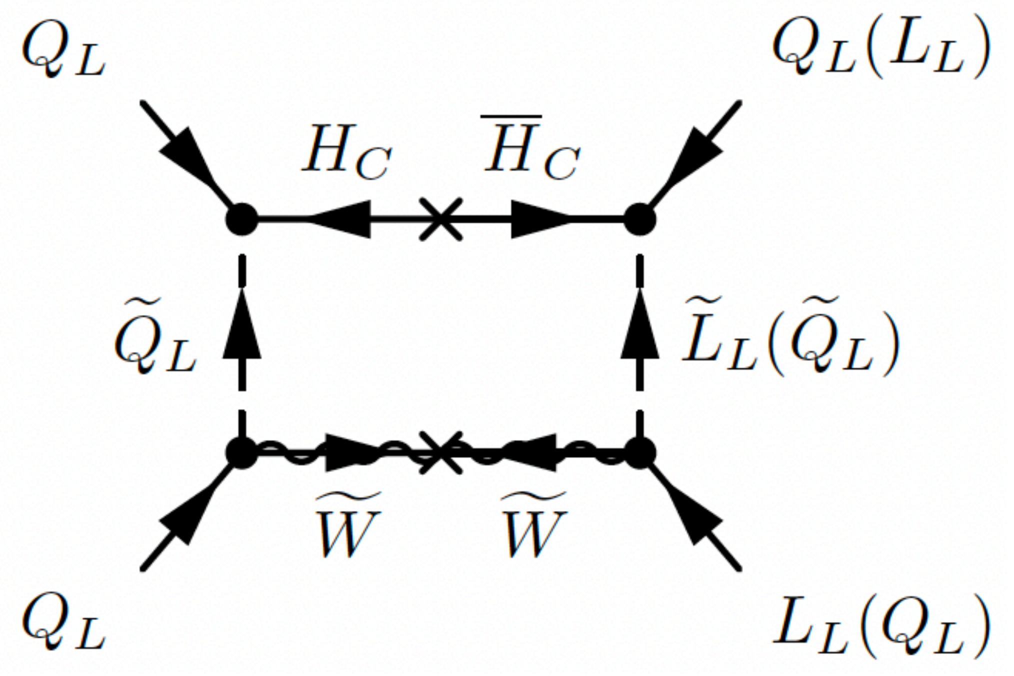
Color-triplet Higgs exchange → Dimension-five superpotential → Wino / Gluino/bino (flavor violation) → $C_{(3)}^{ijkl}$ $C_{(4)}^{ijkl}$ Pure-type

Dominant in
non-SUSY GUTs

Dominant in
SUSY GUTs

$X_\mu \psi_{R\dot{\alpha}} \bar{\sigma}^{\mu\dot{\alpha}\alpha} \chi_{L\alpha}$





Chirality structure and UV mechanisms

H. Georgi and S. L. Glashow (1974)

S. Weinberg (1982)
N. Sakai and T. Yanagida (1982)

Gauge boson exchange

Color-triplet Higgs exchange

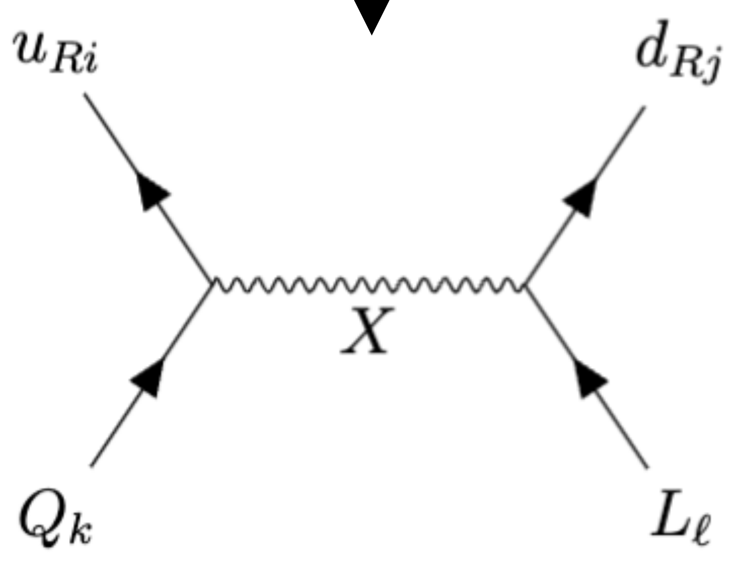
Dominant in
non-SUSY GUTs

Dominant in
SUSY GUTs

Dimension-six Kähler

Dimension-five superpotential

$X_\mu \psi_{R\dot{\alpha}} \bar{\sigma}^{\mu\dot{\alpha}\alpha} \chi_{L\alpha}$



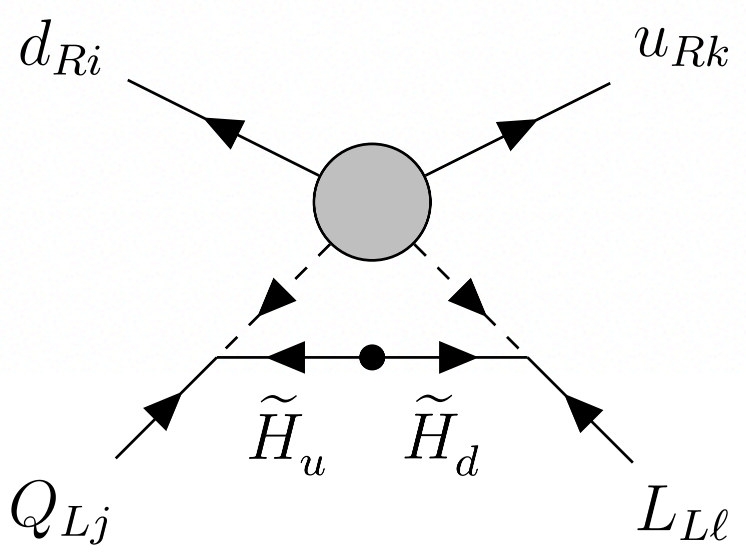
Higgsino

Wino

Gluino/bino
(flavor violation)

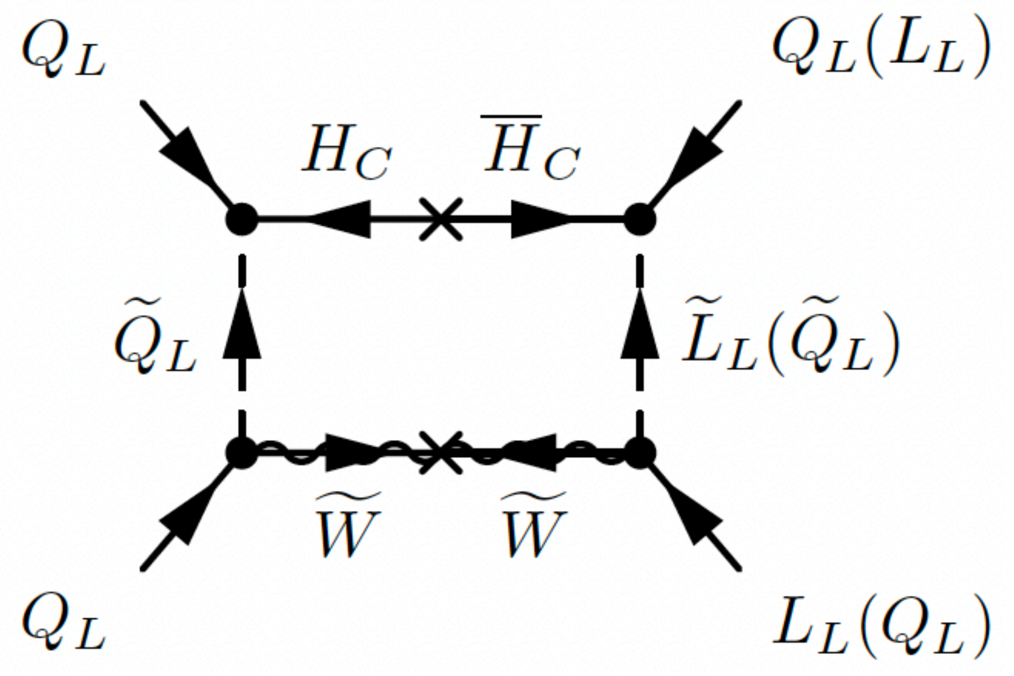
$C^{ijkl}_{(1)}$ $C^{ijkl}_{(2)}$

Mixed-type



$C^{ijkl}_{(3)}$ $C^{ijkl}_{(4)}$

Pure-type



Chirality structure is sensitive to UV physics.

Summary so far

Chirality structure ←-------→ UV physics

Mixed-type Pure-type

Mediator, mass spectrum

Summary so far

Chirality structure ←- - - - - - -→ UV physics

Mixed-type   Pure-type

Mediator, mass spectrum

Q. How to probe chirality structure?

Summary so far

Chirality structure ⟷ UV physics

Mixed-type Pure-type

Mediator, mass spectrum

Q. How to probe chirality structure?

A. By means of **ratios of branching fractions**

Summary so far

Chirality structure

Mixed-type Pure-type

UV physics

Mediator, mass spectrum

Q. How to probe chirality structure?

A. By means of **ratios of branching fractions**

Chirality structure and branching fractions

- **Case 1:** mixed-type only case, $C^{\ell}_{LL} = C^{\ell}_{RR} = 0$, with $m_\ell = 0$.

$$\Gamma(N \to \Pi \ell^+) = \frac{m_N}{32\pi}\left(1 - \frac{m_\Pi^2}{m_N^2}\right)^2 |W^{LR}_{N\Pi\ell,0}|^2 \left[|C^{\ell}_{RL}|^2 + |C^{\ell}_{LR}|^2\right]$$

Chirality structure and branching fractions

- **Case 1:** mixed-type only case, $C^{\ell}_{LL} = C^{\ell}_{RR} = 0$, with $m_\ell = 0$.

$$\Gamma(N \to \Pi \ell^+) = \underbrace{\frac{m_N}{32\pi}\left(1 - \frac{m_\Pi^2}{m_N^2}\right)^2}_{\text{Kinematic factor}} |W^{LR}_{N\Pi\ell,0}|^2 \left[|C^{\ell}_{RL}|^2 + |C^{\ell}_{LR}|^2\right]$$

Chirality structure and branching fractions

- **Case 1:** mixed-type only case, $C^{\ell}_{LL} = C^{\ell}_{RR} = 0$, with $m_\ell = 0$.

$$\Gamma(N \to \Pi \ell^+) = \frac{m_N}{32\pi}\left(1 - \frac{m_\Pi^2}{m_N^2}\right)^2 \underline{|W^{LR}_{N\Pi\ell,0}|^2} \left[|C^{\ell}_{RL}|^2 + |C^{\ell}_{LR}|^2\right]$$

Hadron matrix elements

$$\langle \Pi(\boldsymbol{p}) | \mathcal{O}_{\chi\chi'} | N(\boldsymbol{k}) \rangle = P_{\chi'} \left[\underline{W_0^{\mathcal{O}}(q^2)} + \frac{\not{q}}{m_N} W_1^{\mathcal{O}}(q^2)\right] u_N(\boldsymbol{k}) \qquad \mathcal{O}_{\chi\chi'} = \epsilon_{abc}(q^a_\chi q'^b_\chi) q''^c_{\chi'}$$

$$q^\mu = k^\mu - p^\mu$$

Chirality structure and branching fractions

- **Case 1:** mixed-type only case, $C^{\ell}_{LL} = C^{\ell}_{RR} = 0$, with $m_\ell = 0$.

$$\Gamma(N \to \Pi\ell^+) = \frac{m_N}{32\pi}\left(1 - \frac{m_\Pi^2}{m_N^2}\right)^2 \underline{|W^{LR}_{N\Pi\ell,0}|^2} \left[|C^{\ell}_{RL}|^2 + |C^{\ell}_{LR}|^2\right]$$

Hadron matrix elements

$$\langle \Pi(\boldsymbol{p})|\mathcal{O}_{\chi\chi'}|N(\boldsymbol{k})\rangle = P_{\chi'}\left[\underline{W_0^{\mathcal{O}}(q^2)} + \frac{\not{q}}{m_N} W_1^{\mathcal{O}}(q^2)\right] u_N(\boldsymbol{k}) \qquad \mathcal{O}_{\chi\chi'} = \epsilon_{abc}(q^a_\chi q'^b_\chi) q''^c_{\chi'}$$

We use the form factors obtained by <u>lattice simulations</u>

Y. Aoki et al. (2017)
J. S. Yoo, et al. (2022)

$\longrightarrow$ Uncertainty: $\mathcal{O}(10)\ \%$

Chirality structure and branching fractions

- **Case 1:** mixed-type only case, $C^{\ell}_{LL} = C^{\ell}_{RR} = 0$, with $m_{\ell} = 0$.

$$\Gamma(N \to \Pi \ell^{+}) = \frac{m_N}{32\pi}\left(1 - \frac{m_{\Pi}^2}{m_N^2}\right)^2 |W^{LR}_{N\Pi\ell,0}|^2 \left[\underline{|C^{\ell}_{RL}|^2 + |C^{\ell}_{LR}|^2}\right]$$

  - Wilson coefficients which reflect UV physics.
  - **Does not depend on whether** $p \to \pi^0 e^+$ **or** $p \to \eta e^+$

Chirality structure and branching fractions

- **Case 1:** mixed-type only case, $C^{\ell}_{LL} = C^{\ell}_{RR} = 0$, with $m_{\ell} = 0$.

$$\Gamma(N \to \Pi \ell^+) = \frac{m_N}{32\pi}\left(1 - \frac{m_\Pi^2}{m_N^2}\right)^2 |W^{LR}_{N\Pi\ell,0}|^2 \left[|C^{\ell}_{RL}|^2 + |C^{\ell}_{LR}|^2\right]$$

- The ratio $\Gamma(p \to \eta e^+)/\Gamma(p \to \pi^0 e^+)$ are **solely determined by low-energy quantities.**

$$\frac{\Gamma(p \to \eta e^+)}{\Gamma(p \to \pi^0 e^+)} = \frac{(1 - m_\eta^2/m_p^2)^2}{(1 - m_\pi^2/m_p^2)^2} \cdot \frac{|W^{XX'}_{p\eta\ell}|^2}{|W^{XX'}_{p\pi^0\ell}|^2} \simeq \begin{cases} 0.0013 & \text{(Mixed only)} \\ 0.51 & \text{(Pure only)} \end{cases}$$

Chirality structure and branching fractions

- **Case 1:** mixed-type only case, $C^{\ell}_{LL} = C^{\ell}_{RR} = 0$, with $m_\ell = 0$.

$$\Gamma(N \to \Pi \ell^+) = \frac{m_N}{32\pi}\left(1 - \frac{m_\Pi^2}{m_N^2}\right)^2 |W^{LR}_{N\Pi\ell,0}|^2 \left[|C^{\ell}_{RL}|^2 + |C^{\ell}_{LR}|^2\right]$$

- The ratio $\Gamma(p \to \eta e^+)/\Gamma(p \to \pi^0 e^+)$ are **solely determined by low-energy quantities.**

$$\frac{\Gamma(p \to \eta e^+)}{\Gamma(p \to \pi^0 e^+)} = \frac{(1 - m_\eta^2/m_p^2)^2}{(1 - m_\pi^2/m_p^2)^2} \cdot \frac{|W^{\chi\chi'}_{p\eta\ell}|^2}{|W^{\chi\chi'}_{p\pi^0\ell}|^2} \simeq \begin{cases} 0.0013 & \text{(Mixed only)} \\ 0.51 & \text{(Pure only)} \end{cases}$$

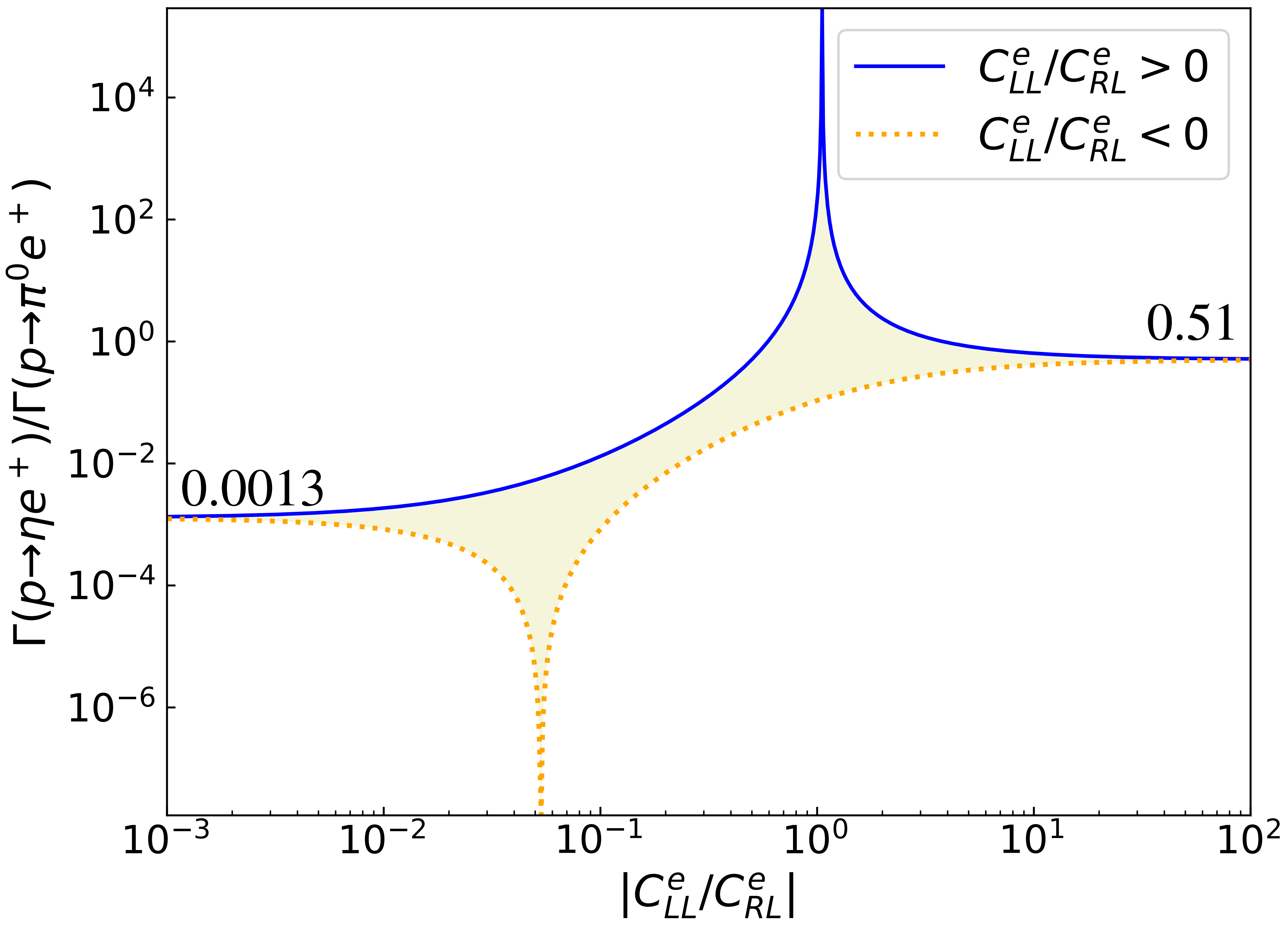
→ Parity invariance of QCD plays the role:

$$\boxed{W^{RL}_{N\Pi\ell,I} = W^{LR}_{N\Pi\ell,I}\,, \qquad W^{LL}_{N\Pi\ell,I} = W^{RR}_{N\Pi\ell,I}}$$

$$\langle \Pi(\boldsymbol{p})|\mathcal{O}_{\chi\chi'}|N(\boldsymbol{k})\rangle = P_{\chi'}\left[W_0^{\mathcal{O}}(q^2) + \frac{\not{q}}{m_N} W_1^{\mathcal{O}}(q^2)\right] u_N(\boldsymbol{k})$$

Chirality structure and branching fractions

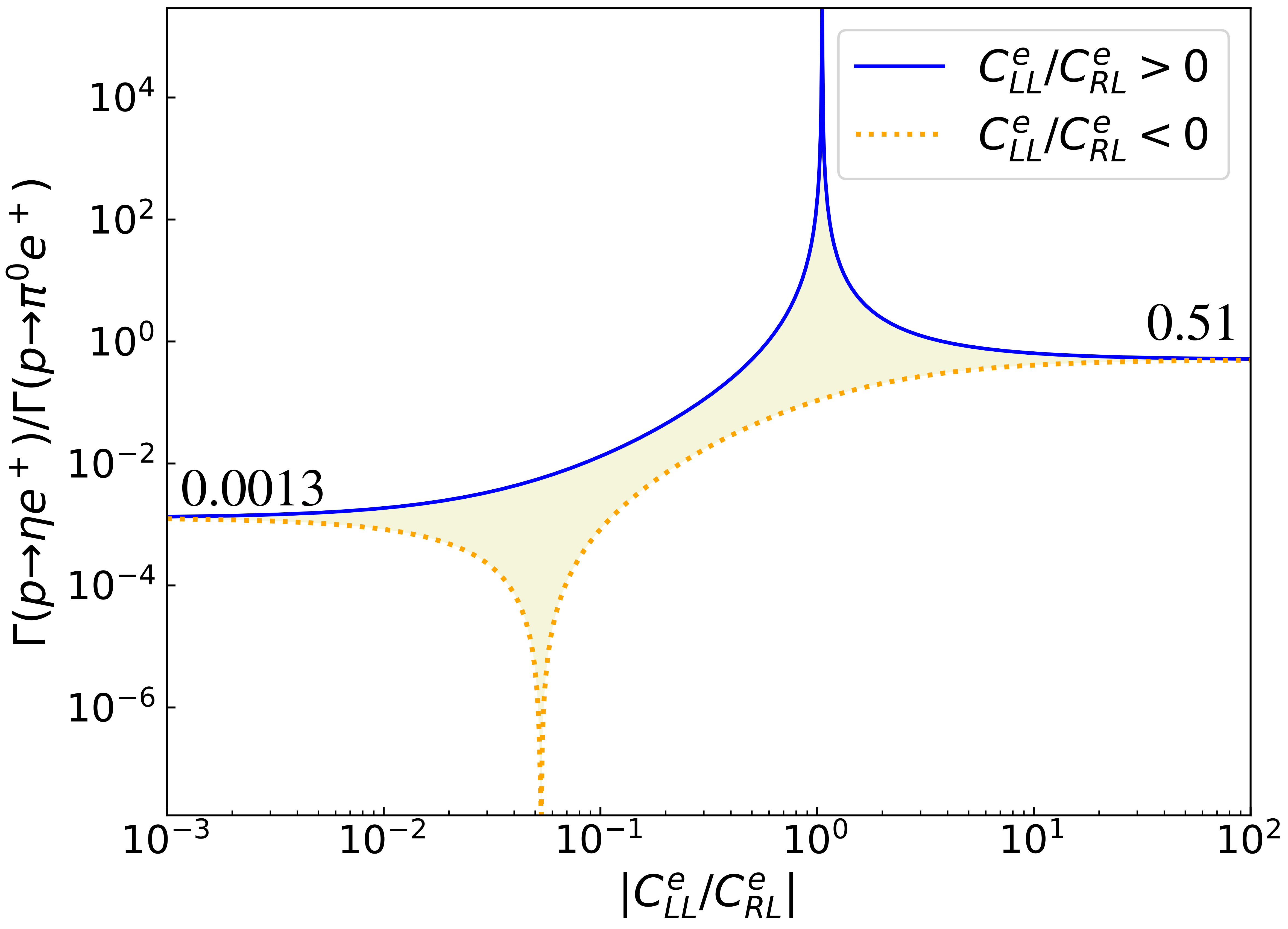
- **Case 2:** both-type case, $C^e_{RR} = C^e_{LR} = 0$ with $m_e = 0$.



- $|C^e_{LL}/C^e_{RL}| \ll 1$ and $\gg 1$ approach **orders of magnitude different values.**

Chirality structure and branching fractions

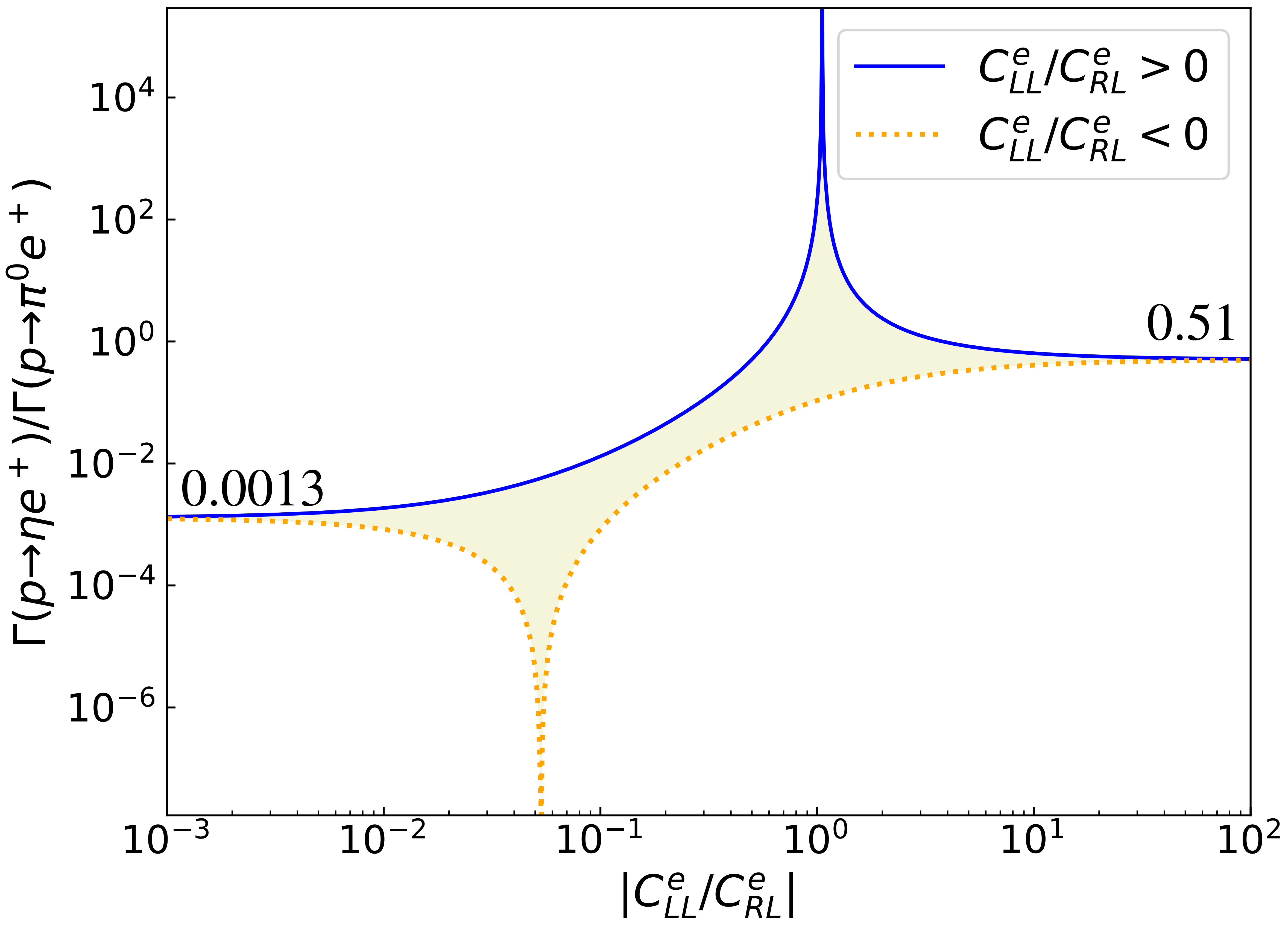
- **Case 2:** both-type case, $C^e_{RR} = C^e_{LR} = 0$ with $m_e = 0$.



- $|C^e_{LL}/C^e_{RL}| \ll 1$ and $\gg 1$ approach **orders of magnitude different values.**
- Peaks at
  - $|C^e_{LL}/C^e_{RL}| \simeq 1$ for $C^e_{LL}/C^e_{RL} > 0$
  - $|C^e_{LL}/C^e_{RL}| \simeq 0.1$ for $C^e_{LL}/C^e_{RL} < 0$

Chirality structure and branching fractions

- **Case 2:** both-type case, $C^e_{RR} = C^e_{LR} = 0$ with $m_e = 0$.



- $|C^e_{LL}/C^e_{RL}| \ll 1$ and $\gg 1$ approach **orders of magnitude different values.**
- Peaks at
  - $|C^e_{LL}/C^e_{RL}| \simeq 1$ for $C^e_{LL}/C^e_{RL} > 0$
  - $|C^e_{LL}/C^e_{RL}| \simeq 0.1$ for $C^e_{LL}/C^e_{RL} < 0$

$\Gamma(p \to \eta e^+)/\Gamma(p \to \pi^0 e^+)$ **is sensitive to chirality structure.**

Summary so far

Chirality structure

Mixed-type Pure-type

UV physics

Mediator, mass spectrum

Based on EFT

The ratios of branching fractions

Summary so far

Chirality structure

Mixed-type Pure-type

UV physics

Mediator, mass spectrum

SUSY SU(5)
example

The ratios of branching fractions

Chirality structure and UV mechanisms

H. Georgi and S. L. Glashow (1974)

S. Weinberg (1982)
N. Sakai and T. Yanagida (1982)

Gauge boson exchange

Color-triplet Higgs exchange

Dominant in non-SUSY GUTs

Dominant in SUSY GUTs

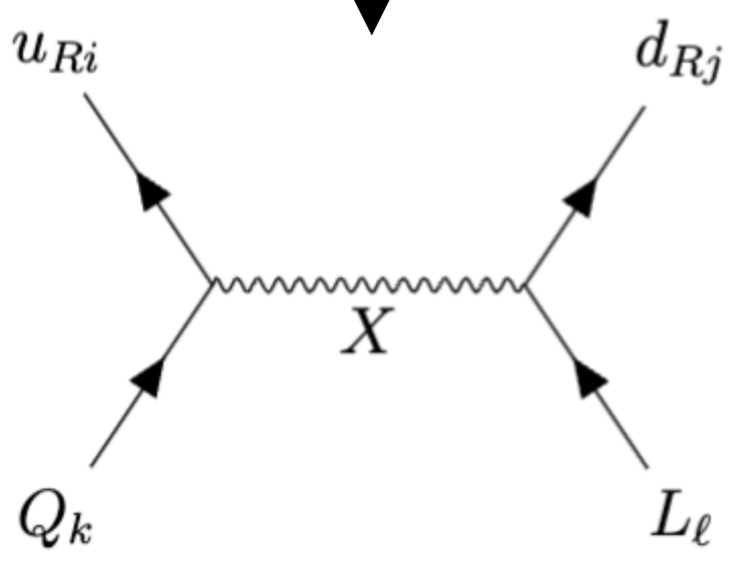
Dimension-six Kähler

Dimension-five superpotential

Higgsino

Wino

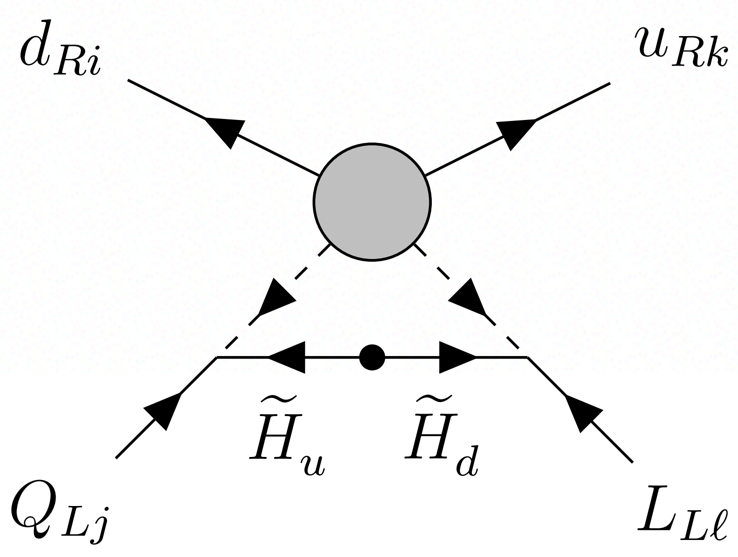
$X_\mu \psi_{R\dot{\alpha}} \bar{\sigma}^{\mu\dot{\alpha}\alpha} \chi_{L\alpha}$



$\propto \dfrac{1}{M_X^2}$

$C^{ijkl}_{(1)}$ $C^{ijkl}_{(2)}$

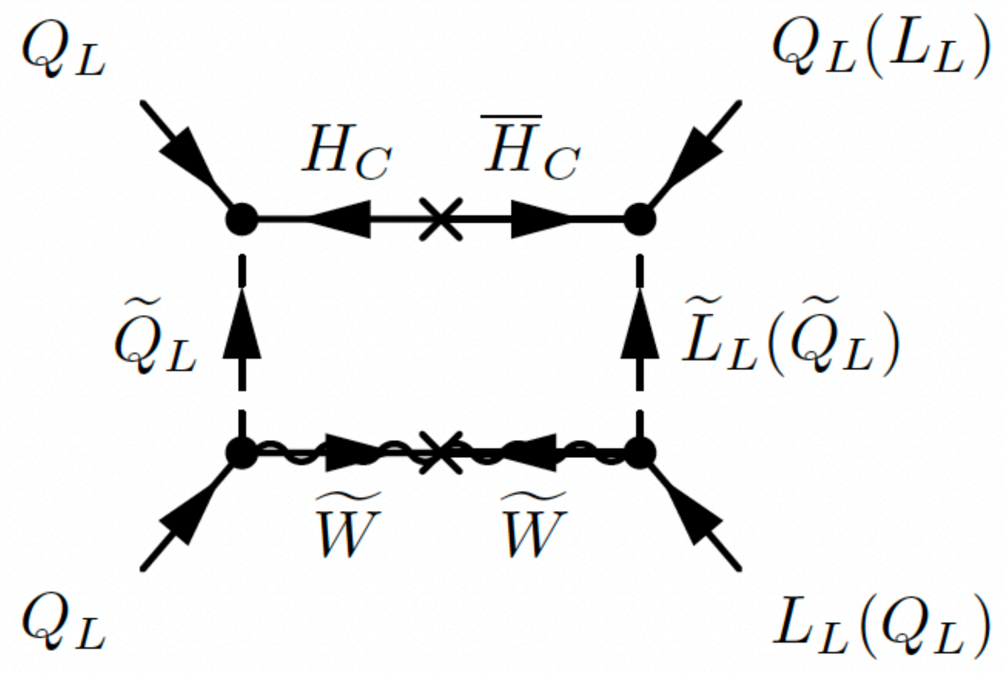
Mixed-type



$\propto \dfrac{\mu_H}{M_{H_C} M_{\mathrm{SUSY}}^2}$

$C^{ijkl}_{(3)}$ $C^{ijkl}_{(4)}$

Pure-type

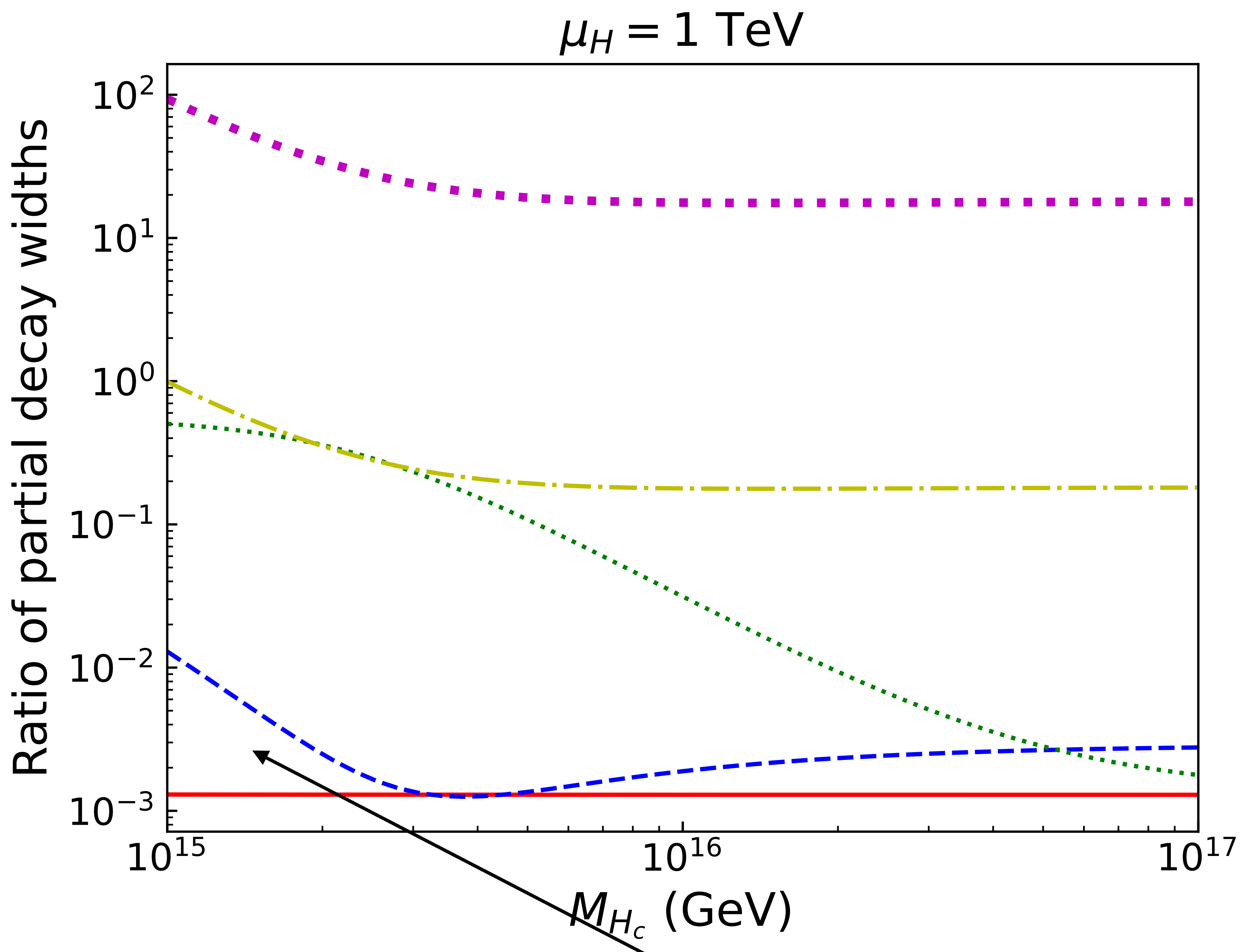


$\propto \dfrac{m_{\tilde{g}}}{M_{H_C} M_{\mathrm{SUSY}}^2}$

- M_{SUSY}: sfermion mass
- $m_{\tilde{g}} \leq M_{\mathrm{SUSY}}$

The minimal SU(5) with high-scale SUSY

No sfermion flavor violation

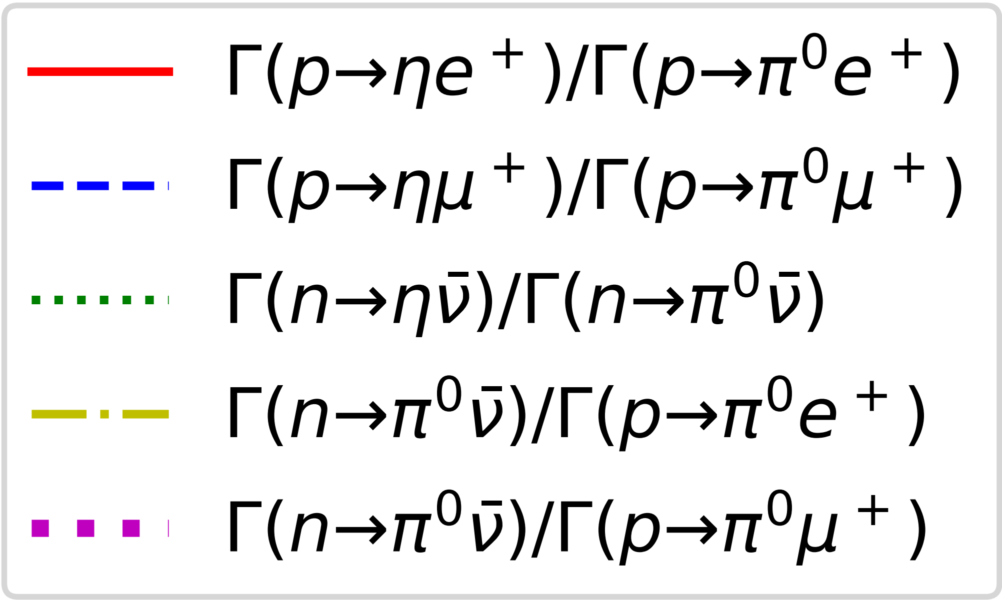


- $M_X = 10^{16}$ GeV
- $M_2 = 1$ TeV
- $M_3 = 10$ TeV
- $M_{\mathrm{SUSY}} = 100$ TeV
- $\tan\beta = 3$

Mini-split type mass spectrum

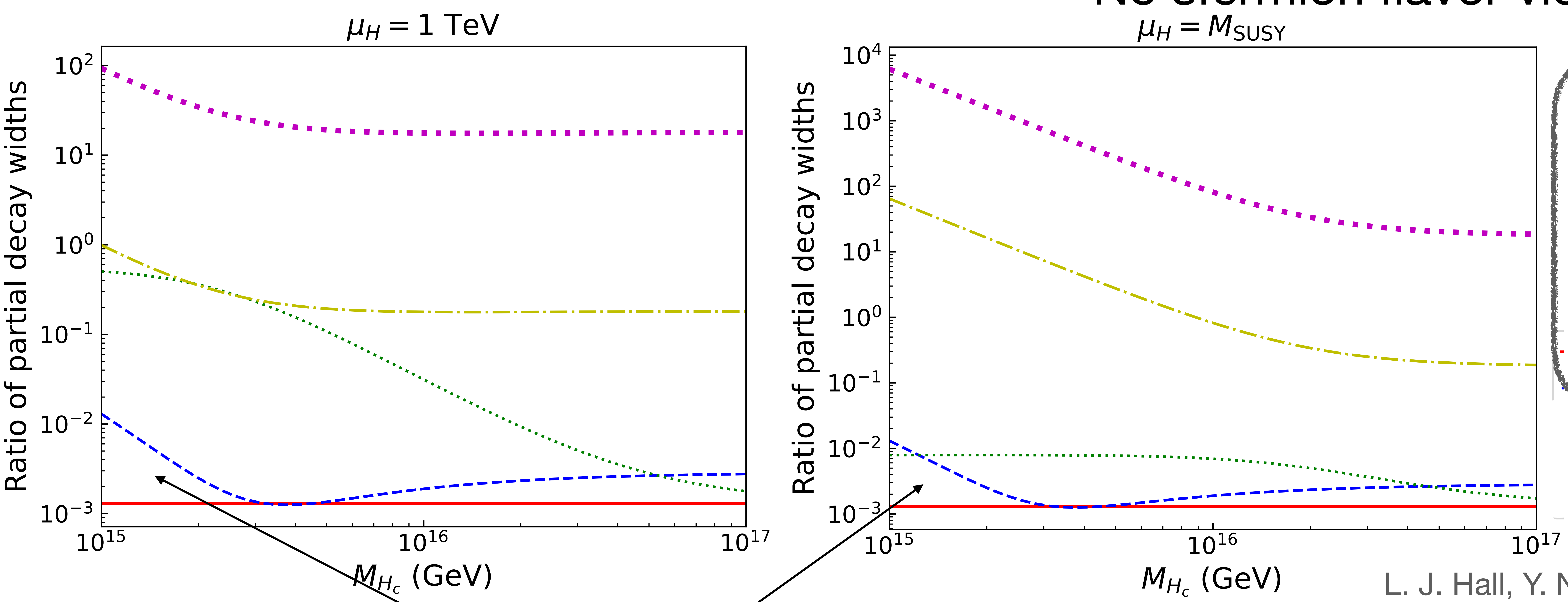
L. J. Hall, Y. Nomura and S. Shirai (2012)
M. Ibe, S. Matsumoto and T. T. Yanagida (2012)

- $\Gamma(p \to \eta\mu^+)/\Gamma(p \to \pi^0\mu^+)$: wino-exchange contributes (pure-type) as M_{H_C} decreases.



The minimal SU(5) with high-scale SUSY

No sfermion flavor violation

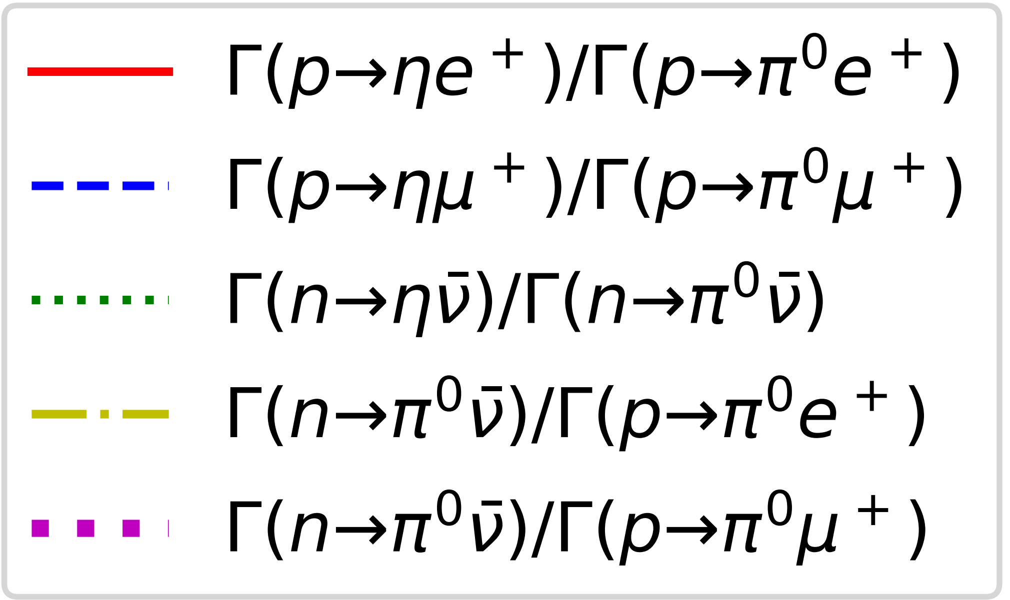


- $M_X = 10^{16}$ GeV
- $M_2 = 1$ TeV
- $M_3 = 10$ TeV
- $M_{\rm SUSY} = 100$ TeV
- $\tan\beta = 3$

Mini-split type mass spectrum

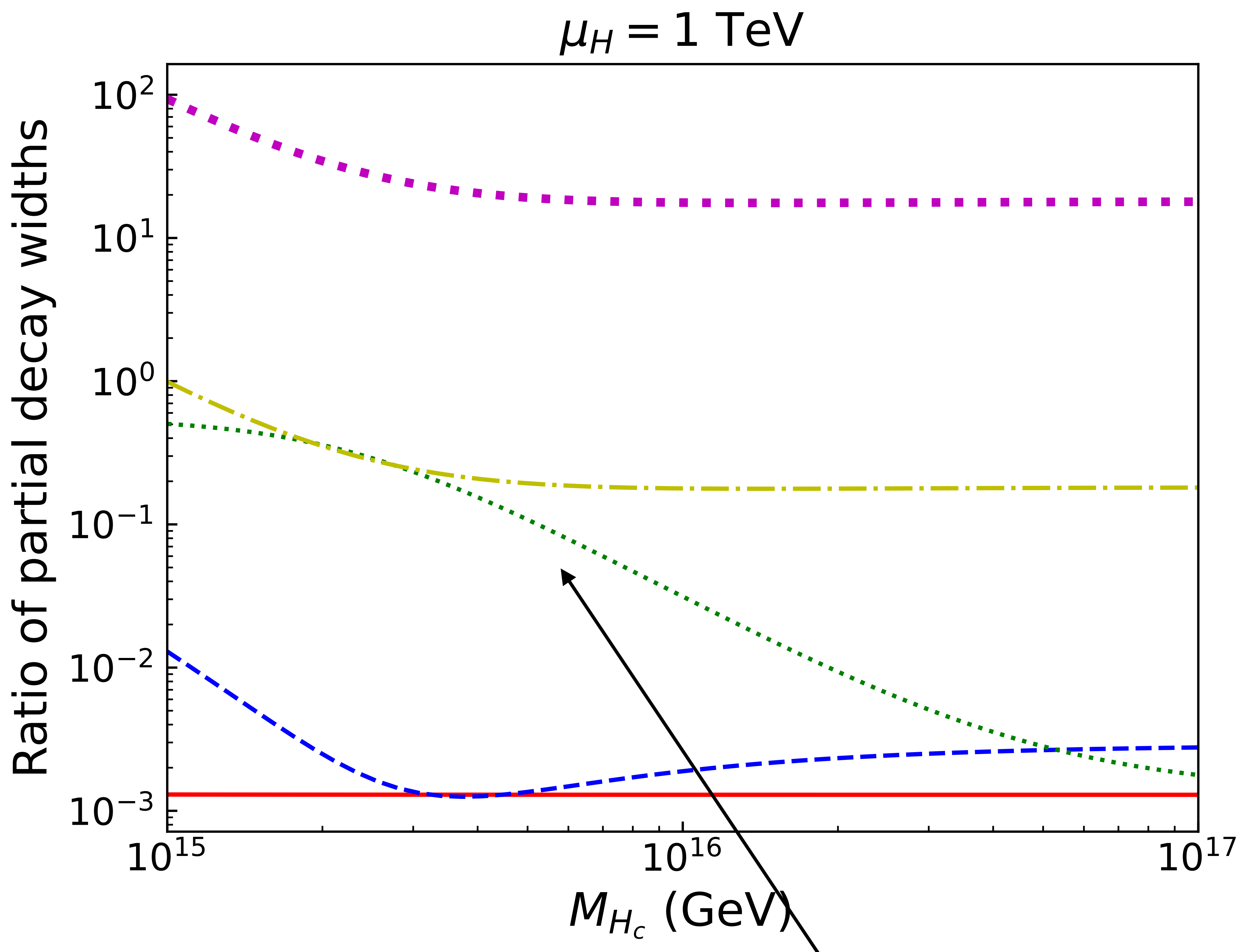
L. J. Hall, Y. Nomura and S. Shirai (2012)
M. Ibe, S. Matsumoto and T. T. Yanagida (2012)

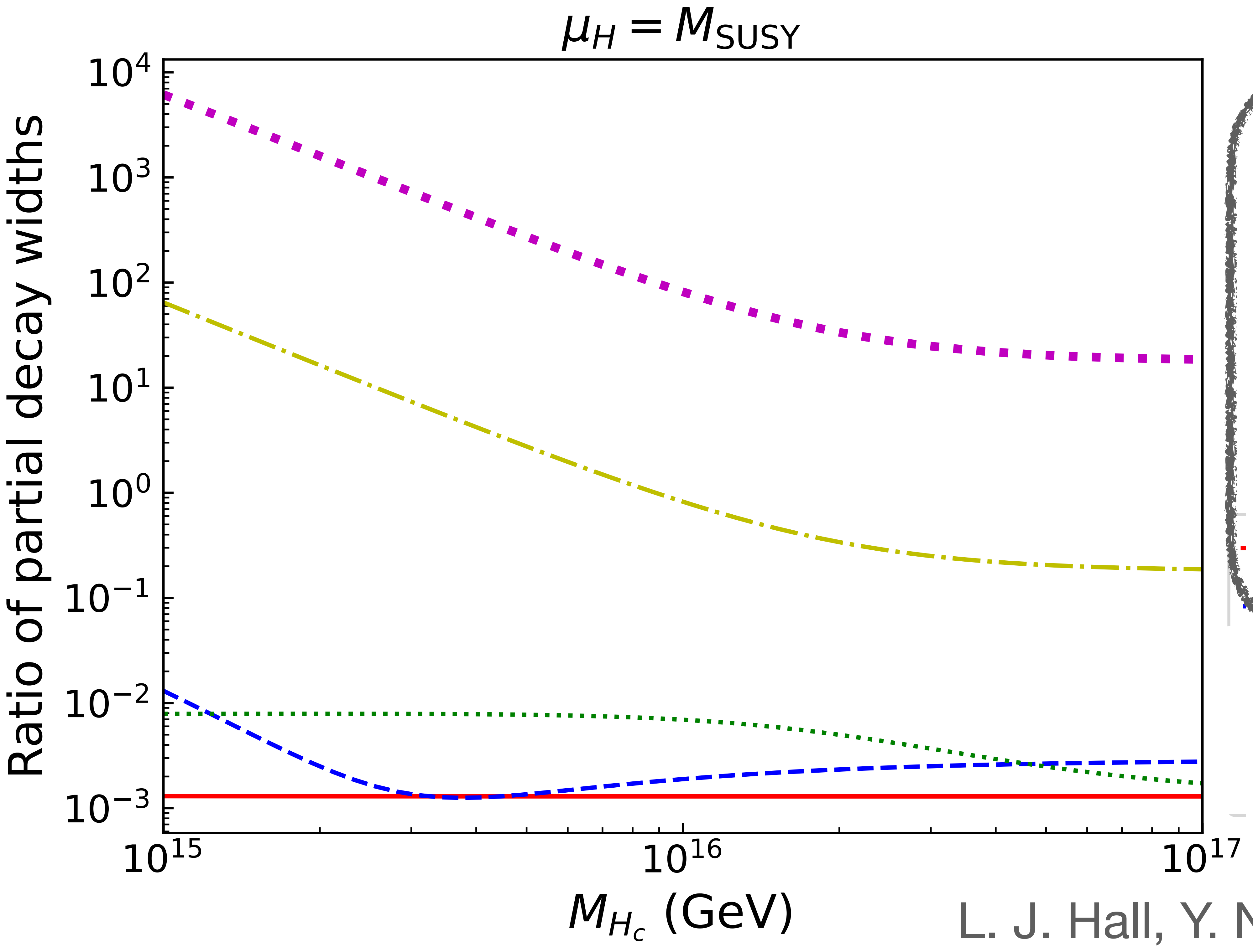
- $\Gamma(p \to \eta\mu^+)/\Gamma(p \to \pi^0\mu^+)$: wino-exchange contributes (pure-type) as M_{H_C} decreases.



The minimal SU(5) with high-scale SUSY

No sfermion flavor violation

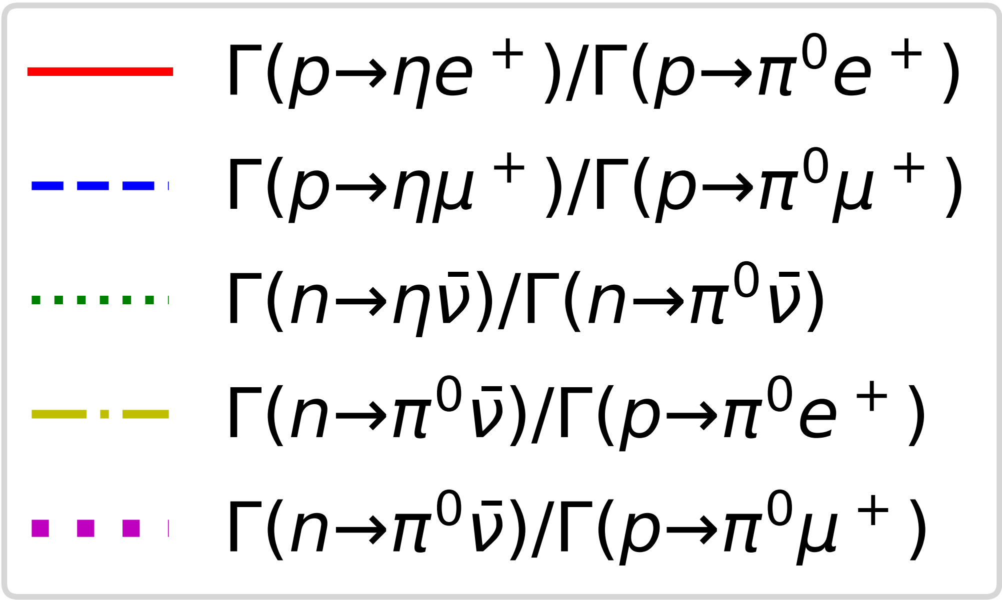




- $M_X = 10^{16}$ GeV
- $M_2 = 1$ TeV
- $M_3 = 10$ TeV
- $M_{\rm SUSY} = 100$ TeV
- $\tan\beta = 3$

Mini-split type mass spectrum

L. J. Hall, Y. Nomura and S. Shirai (2012)
M. Ibe, S. Matsumoto and T. T. Yanagida (2012)

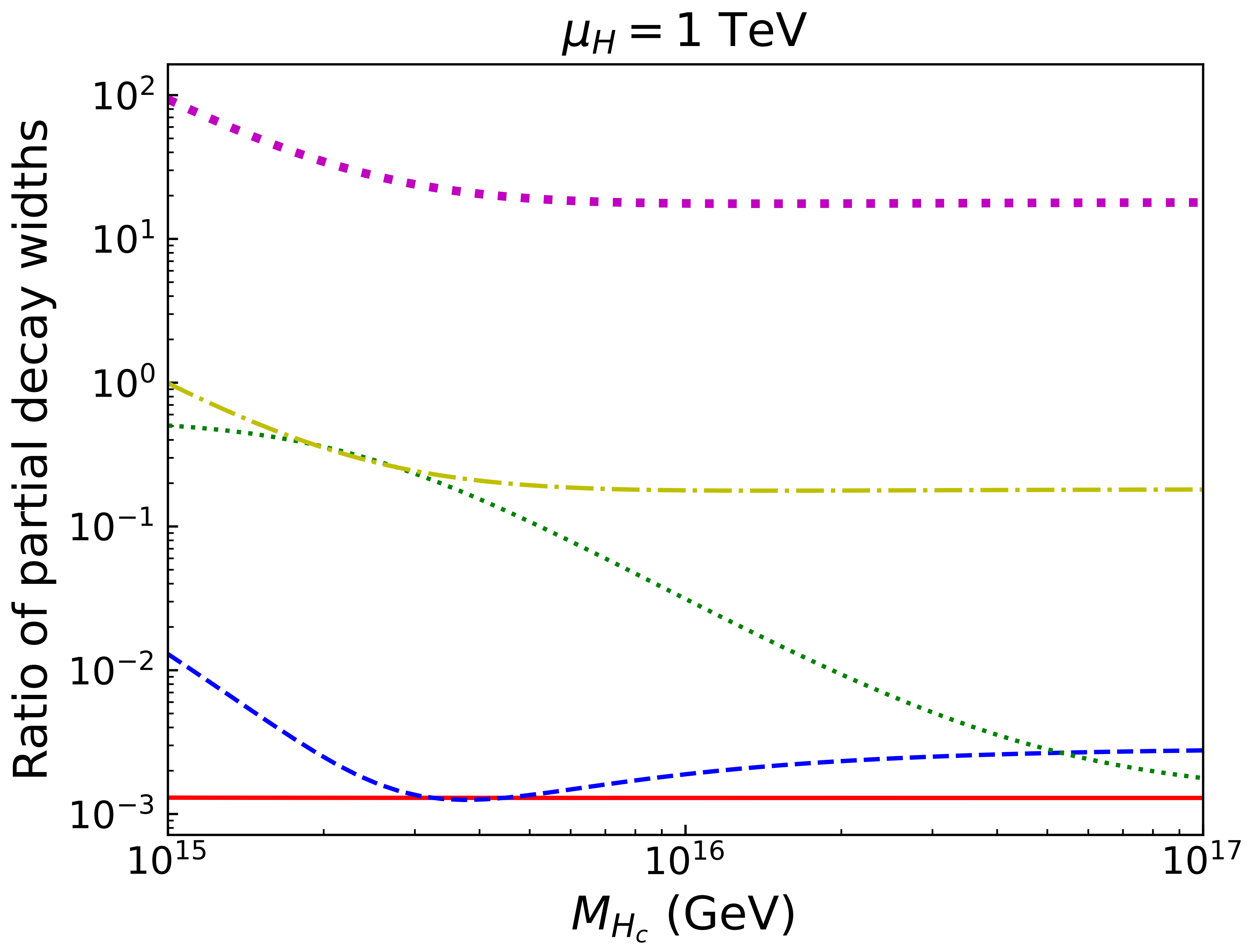


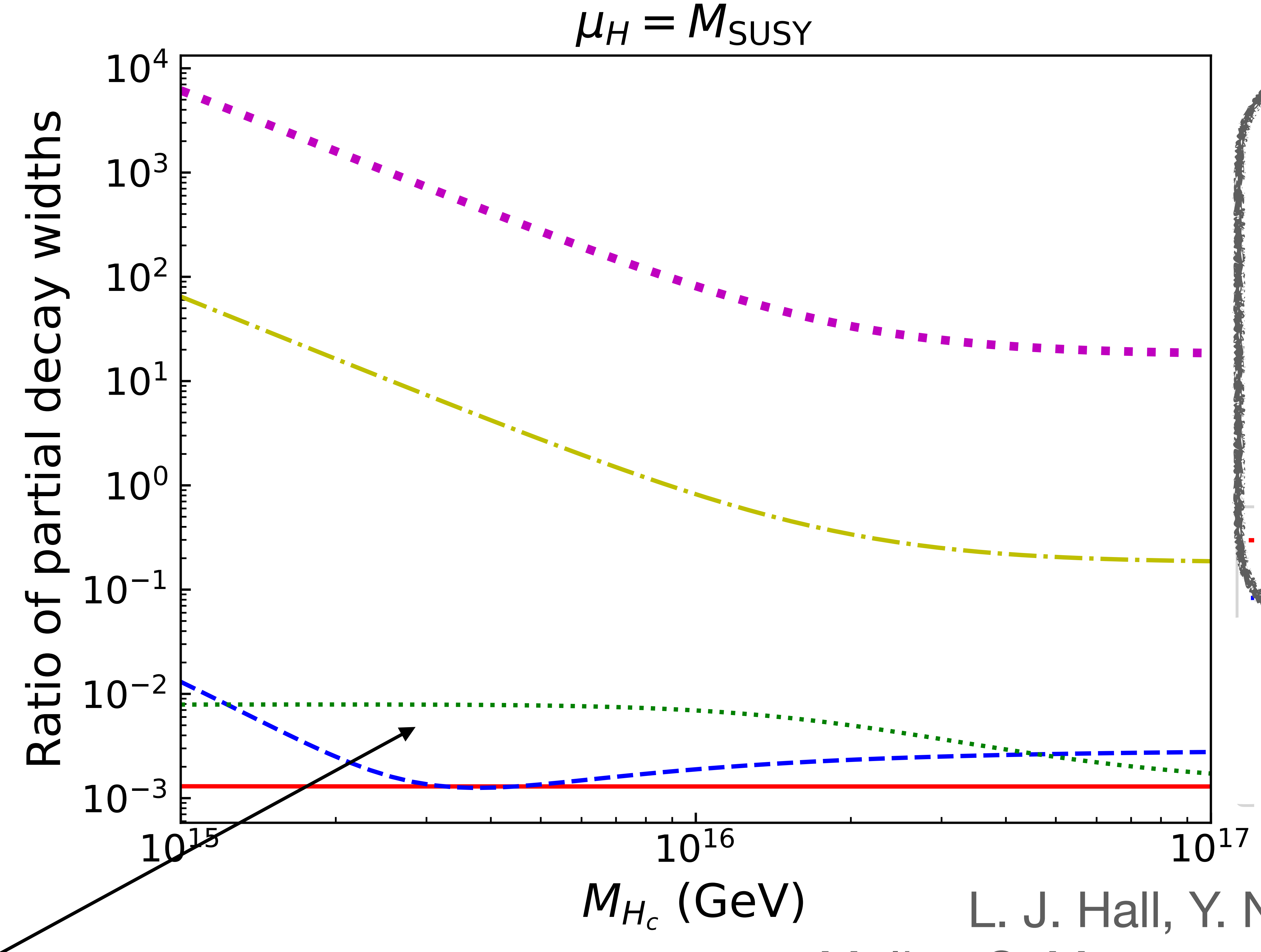
$\Gamma(n \to \eta\bar{\nu})/\Gamma(n \to \pi^0\bar{\nu})$:

- Left: larger contribution of the wino-exchanging process (pure-type).

The minimal SU(5) with high-scale SUSY

No sfermion flavor violation

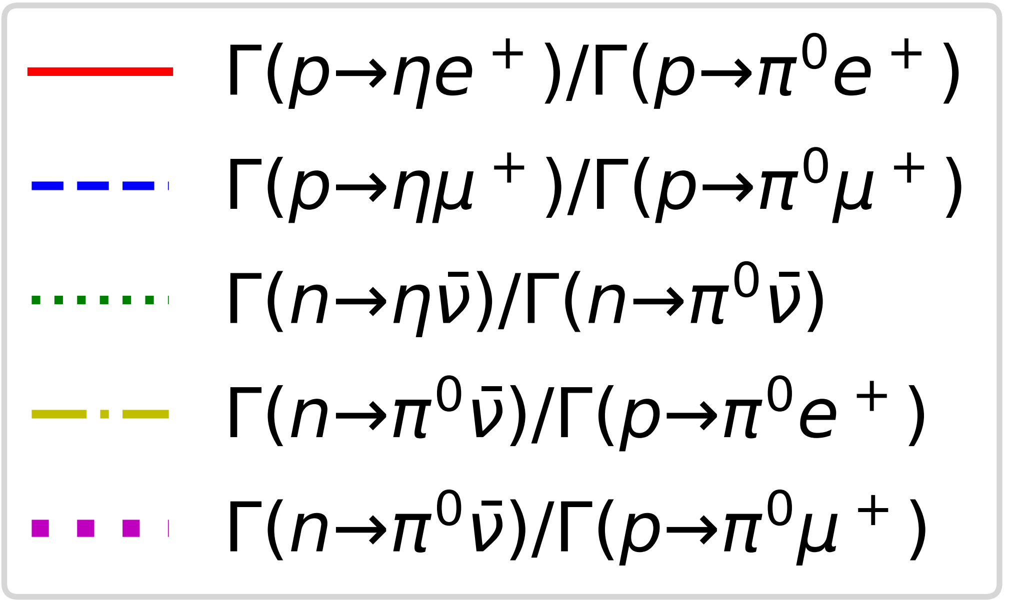




- $M_X = 10^{16}$ GeV
- $M_2 = 1$ TeV
- $M_3 = 10$ TeV
- $M_{\rm SUSY} = 100$ TeV
- $\tan\beta = 3$

Mini-split type mass spectrum

L. J. Hall, Y. Nomura and S. Shirai (2012)
M. Ibe, S. Matsumoto and T. T. Yanagida (2012)



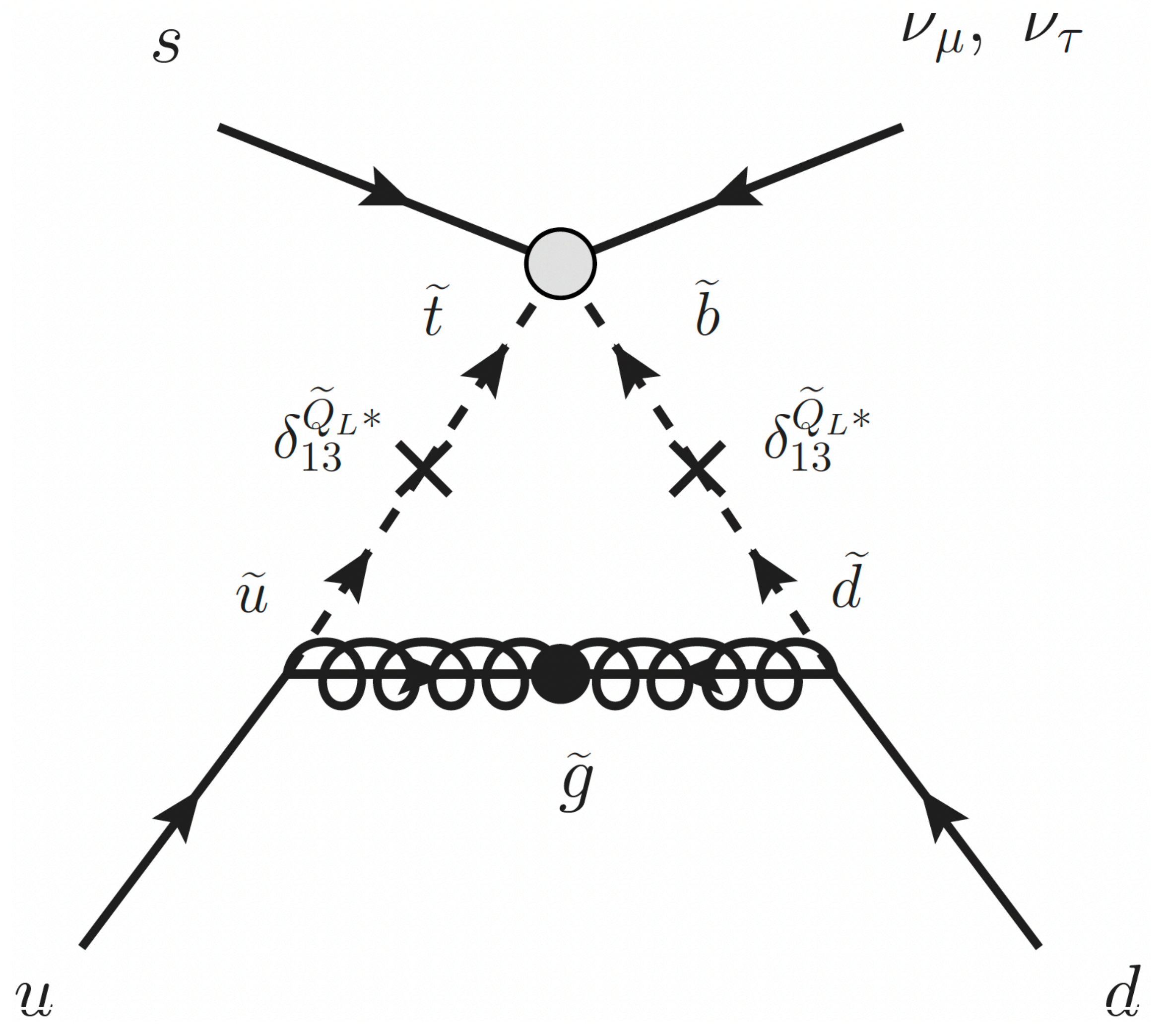
$\Gamma(n \to \eta\bar{\nu})/\Gamma(n \to \pi^0\bar{\nu})$:

- Left: larger contribution of the wino-exchanging process (pure-type).
- Right: higgsino exchange (mixed-type) dominates in smaller M_{H_C}.

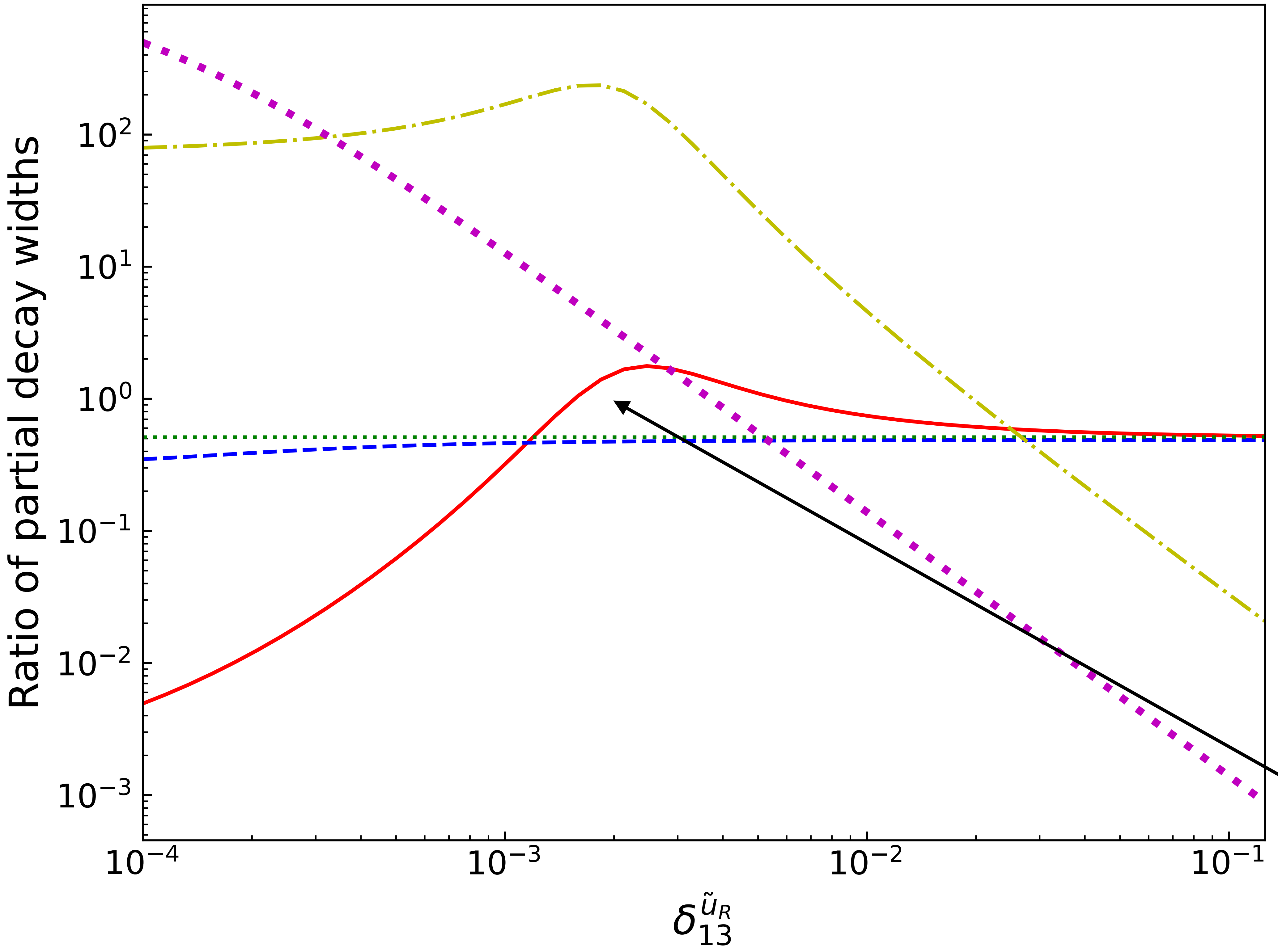
Sfermion flavor violation and ratios

- $M_X = 10^{17}$ GeV
- $M_{H_C} = 10^{16}$ GeV
- $M_1 = 5$ TeV
- $M_2 = 1$ TeV
- $M_3 = 10$ TeV
- $M_{\text{SUSY}} = 100$ TeV
- $\tan\beta = 3$
- $\mu_H = 200$ GeV

$$\widetilde{m}^2_{\tilde{f}} = M^2_{\text{SUSY}} \begin{pmatrix} 1 & \delta^{\tilde{f}}_{12} & \delta^{\tilde{f}}_{13} \\ \delta^{\tilde{f}*}_{12} & 1 & \delta^{\tilde{f}}_{23} \\ \delta^{\tilde{f}*}_{13} & \delta^{\tilde{f}*}_{23} & 1 \end{pmatrix}$$

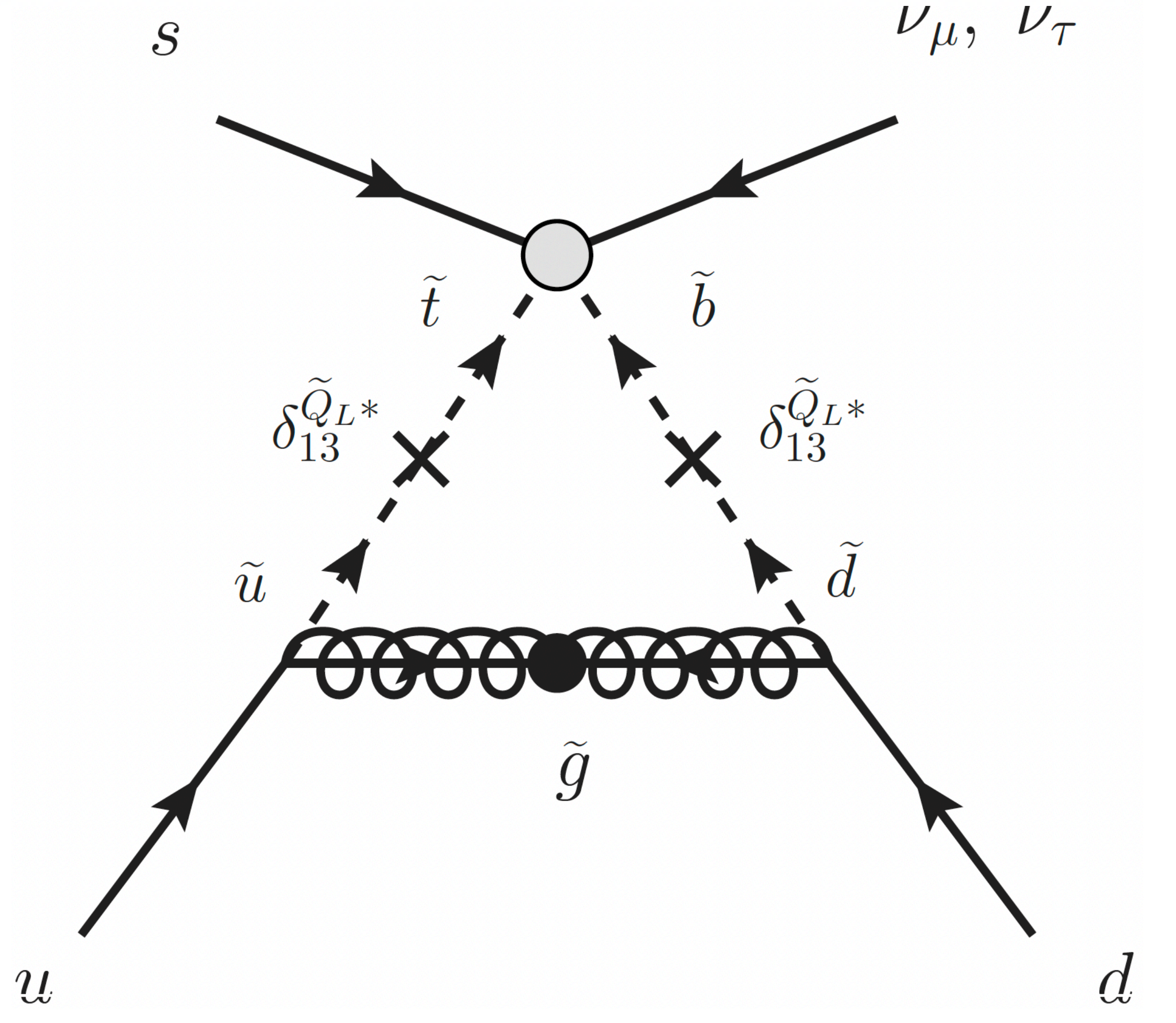


Sfermion flavor violation and ratios

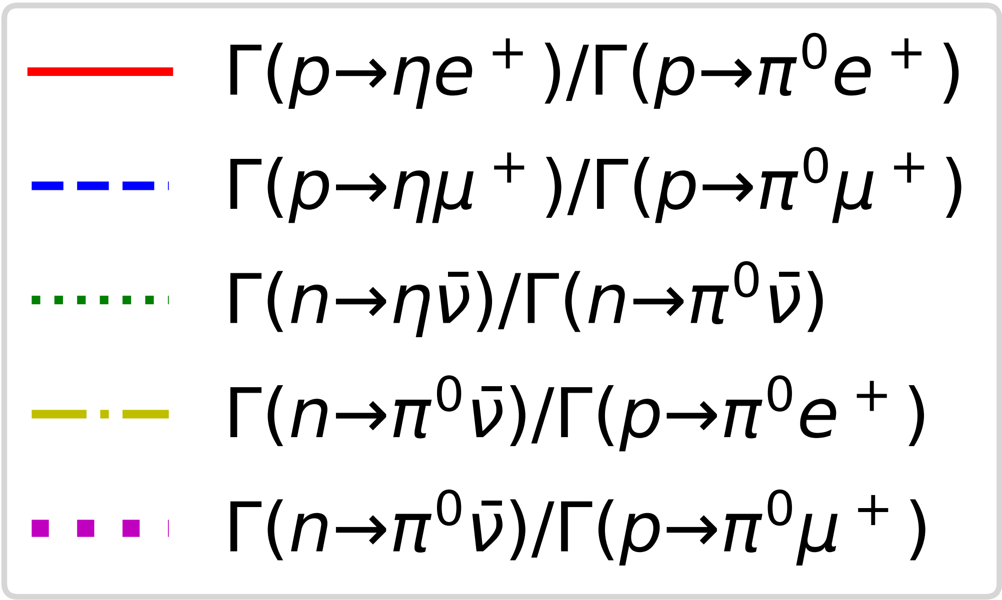


- $M_X = 10^{17}$ GeV
- $M_{H_C} = 10^{16}$ GeV
- $M_1 = 5$ TeV
- $M_2 = 1$ TeV
- $M_3 = 10$ TeV
- $M_{\text{SUSY}} = 100$ TeV
- $\tan\beta = 3$
- $\mu_H = 200$ GeV

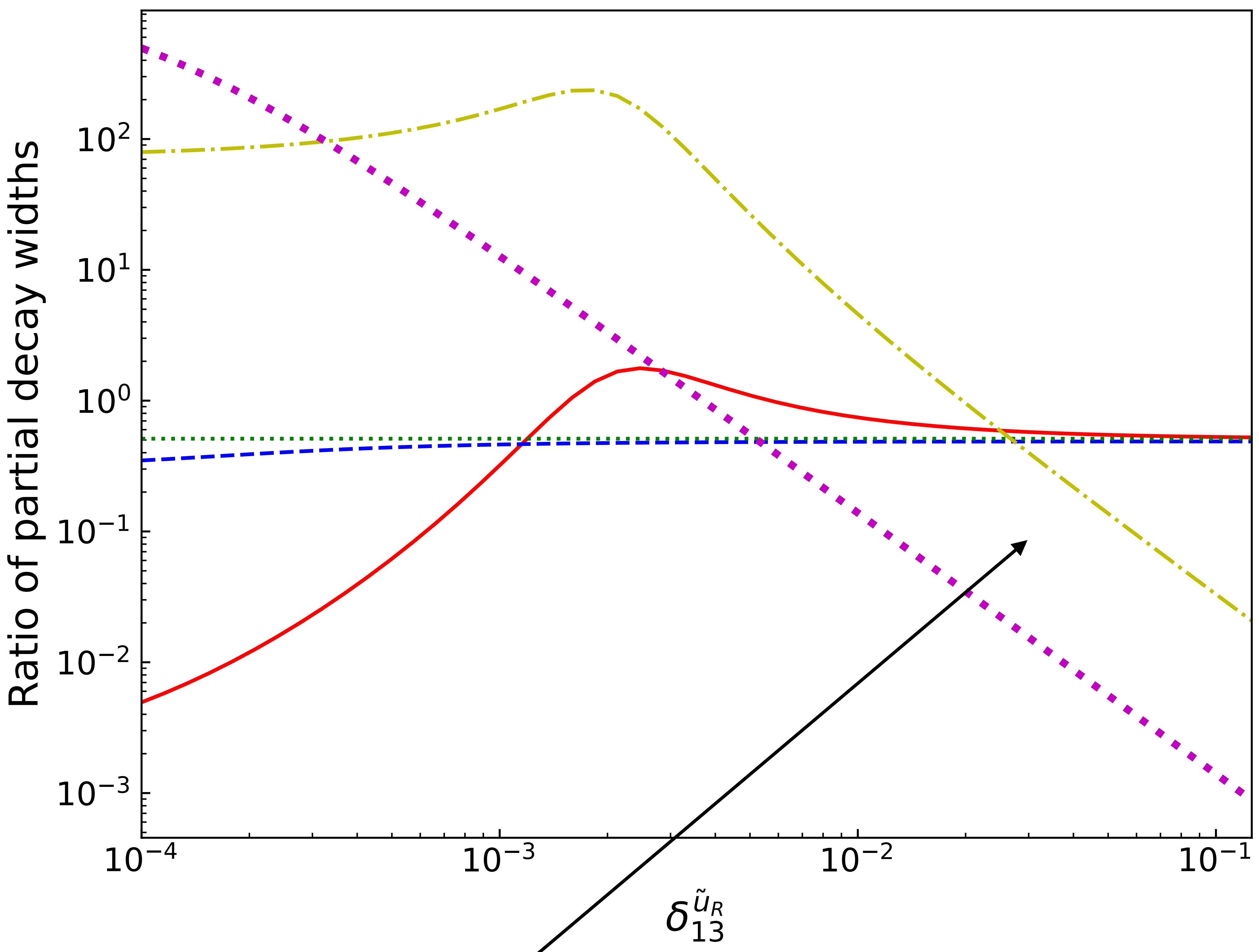
$$\widetilde{m}^2_{\tilde{f}} = M^2_{\text{SUSY}} \begin{pmatrix} 1 & \delta^{\tilde{f}}_{12} & \delta^{\tilde{f}}_{13} \\ \delta^{\tilde{f}*}_{12} & 1 & \delta^{\tilde{f}}_{23} \\ \delta^{\tilde{f}*}_{13} & \delta^{\tilde{f}*}_{23} & 1 \end{pmatrix}$$



- The gluino exchange dominates as $\delta_{13}^{\tilde{u}_R}$ increases, even in $\Gamma(p \to \eta e^+)/\Gamma(p \to \pi^0 e^+)$.

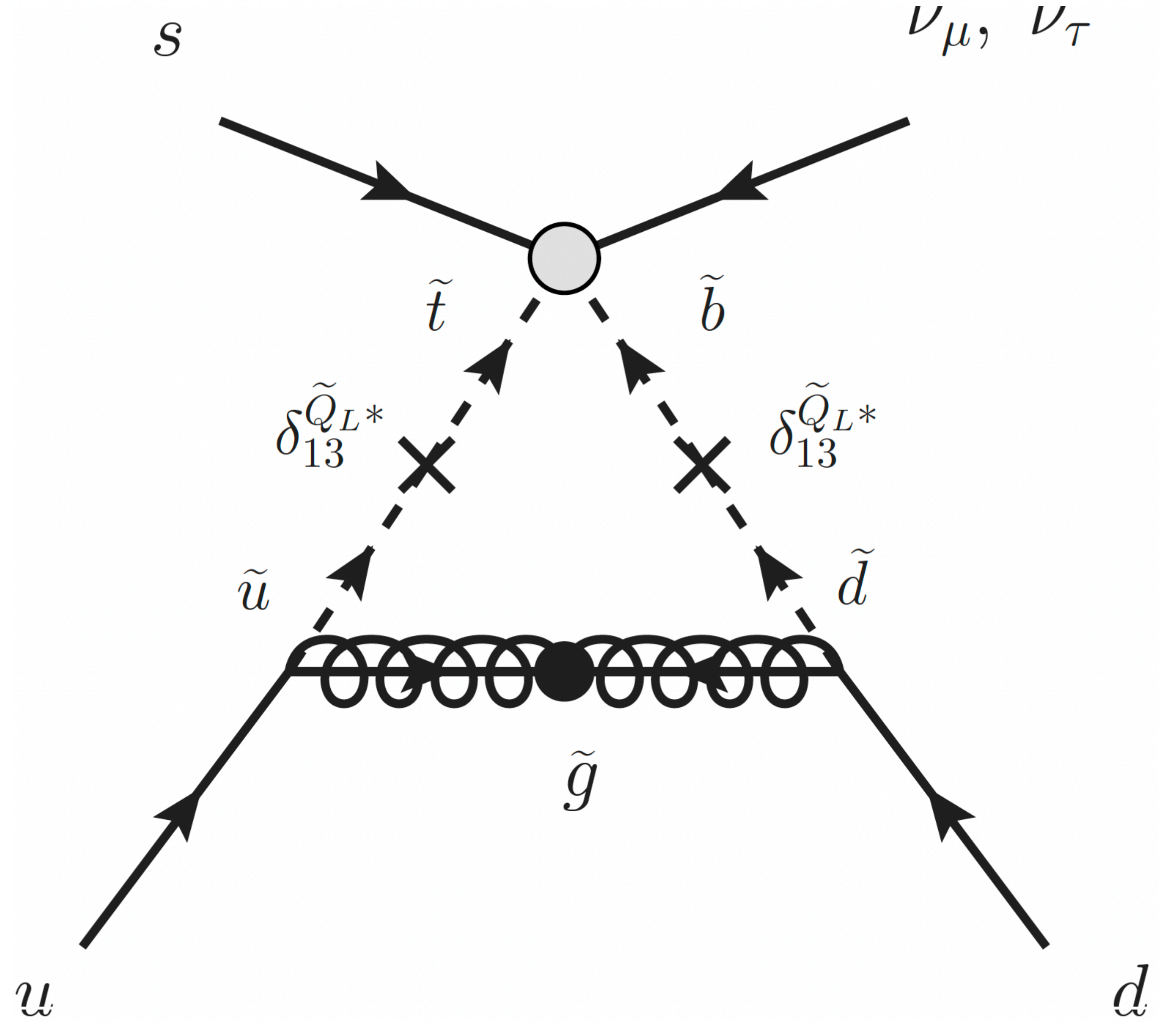


Sfermion flavor violation and ratios



- $M_X = 10^{17}$ GeV
- $M_{H_C} = 10^{16}$ GeV
- $M_1 = 5$ TeV
- $M_2 = 1$ TeV
- $M_3 = 10$ TeV
- $M_{\rm SUSY} = 100$ TeV
- $\tan\beta = 3$
- $\mu_H = 200$ GeV

$$\widetilde{m}^2_{\tilde{f}} = M^2_{\rm SUSY}\begin{pmatrix} 1 & \delta^{\tilde{f}}_{12} & \delta^{\tilde{f}}_{13} \\ \delta^{\tilde{f}*}_{12} & 1 & \delta^{\tilde{f}}_{23} \\ \delta^{\tilde{f}*}_{13} & \delta^{\tilde{f}*}_{23} & 1 \end{pmatrix}$$



- The gluino exchange dominates as $\delta_{13}^{\tilde{u}_R}$ increases, even in $\Gamma(p\to\eta e^+)/\Gamma(p\to\pi^0 e^+)$.
- $\Gamma(n\to\pi^0\bar{\nu})/\Gamma(p\to\pi^0\ell^+)$ is a useful probe of this scenario.

Summary

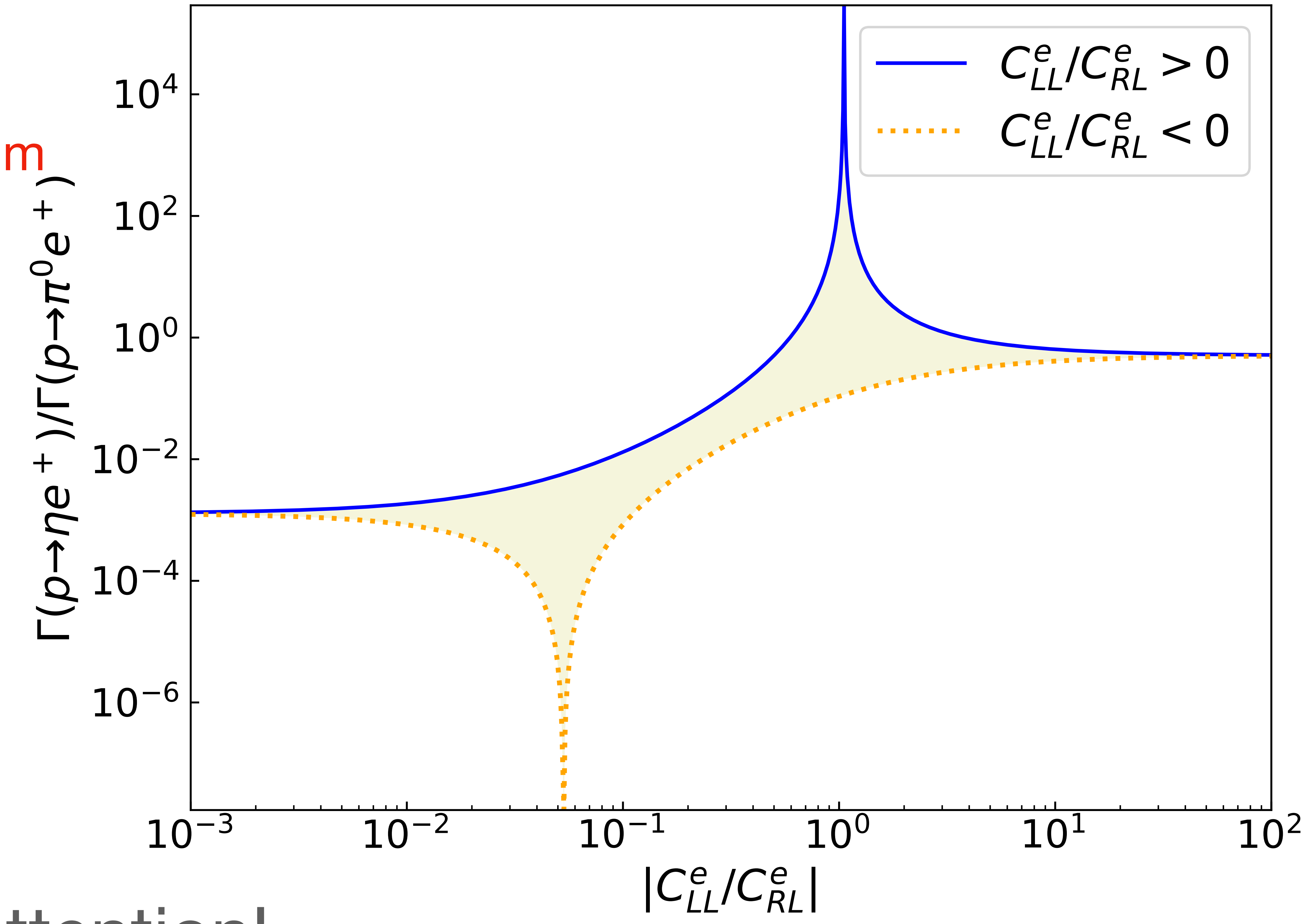
The ratios of nucleon decay branching fractions are sensitive to UV physics.

Chirality structure ↔ UV physics

Mixed-type Pure-type

Mediator, mass spectrum

The ratios of branching fractions



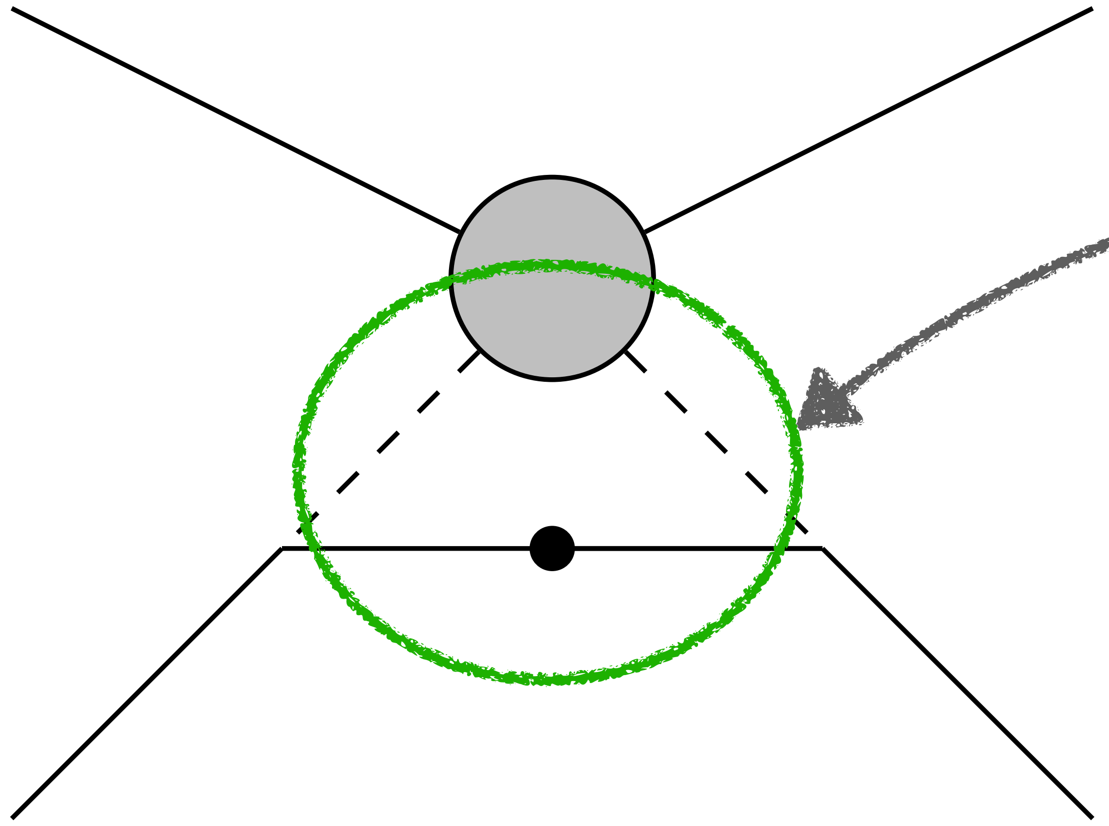
Thank you for your attention!

Backup

SUSY dimension-five nucleon decay operators

P. Nath and R. L. Arnowitt (1988)
J Hisano, H. Murayama and T. Yanagida (1993)

- Higgsino/gaugino-dependence



Loop function:

$$F(M, m_1, m_2) \equiv \int \frac{d^4p}{\pi^2} \frac{i}{(\not{p} - M)(p^2 - m_1^2)(p^2 - m_2^2)}$$

$$= \frac{M}{m_1^2 - m_2^2} \left(\frac{m_1^2}{m_1^2 - M^2} \ln \frac{m_1^2}{M^2} - \frac{m_2^2}{m_2^2 - M^2} \ln \frac{m_2^2}{M^2} \right)$$

$$\to \frac{M}{m_1^2},$$ When $M \ll m_1 \sim m_2$

The dimension-five contribution is proportional to $m_{\tilde{g}}/(M_{H_C} M_{\mathrm{SUSY}}^2)$

Current bounds and future prospects

Decay Mode	Current [years]	HK sensitivity [years]
$p \to \pi^0 e^+$	2.4×10^{34} [25]	7.8×10^{34} [11]
$p \to \pi^0 \mu^+$	1.6×10^{34} [25]	7.7×10^{34} [11]
$p \to \eta e^+$	1.0×10^{34} [26]	4.3×10^{34} [11]
$p \to \eta \mu^+$	4.7×10^{33} [26]	4.9×10^{34} [11]
$p \to \pi^+ \bar{\nu}$	3.9×10^{32} [27]	
$n \to \pi^- e^+$	5.3×10^{33} [26]	2.0×10^{34} [11]
$n \to \pi^- \mu^+$	3.5×10^{33} [26]	1.8×10^{34} [11]
$n \to \pi^0 \bar{\nu}$	1.1×10^{33} [27]	
$n \to \eta \bar{\nu}$	1.6×10^{32} [28]	

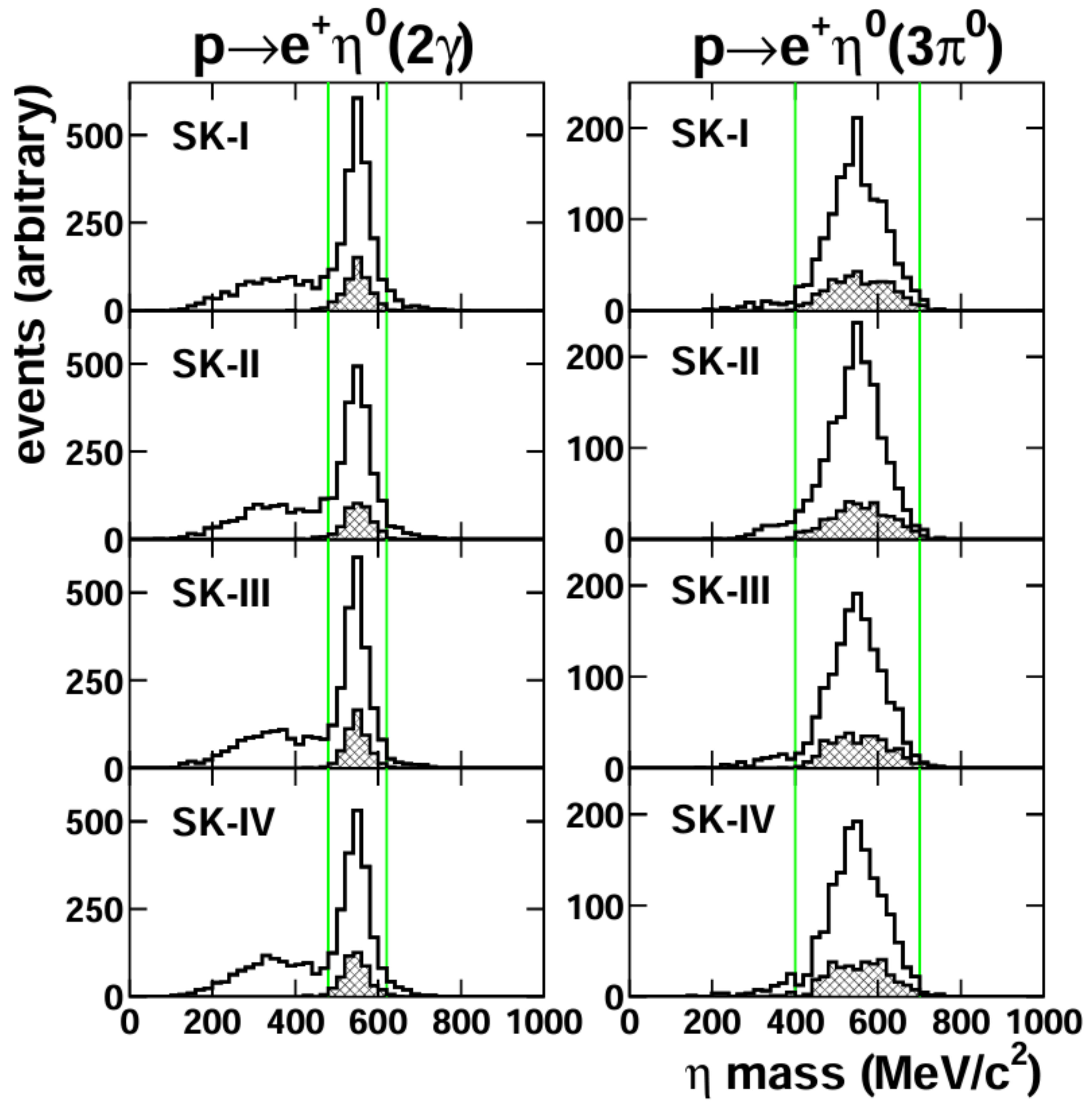
[11]: Hyper-Kamiokande Collaboration [arXiv: 1805.04163]
[25]: Super-Kamiokande Collaboration [arXiv: 2010.16098]
[26]: Super-Kamiokande Collaboration [arXiv: 1705.07221]
[27]: Super-Kamiokande Collaboration [arXiv: 1305.4391]
[28]: C. McGrew et al. (1999)

- Units: 10^{33} years
- 90% CL
- 1.9 Megaton-year exposure is assumed for the prospect.

Event Reconstruction

Super-Kamiokande Collaboration [arXiv: 1705.07221]

- Eta mass reconstruction at SK



- Left: $\eta \to 2\gamma$, branching ratio = 39%
- Right: $\eta \to 3\pi^0$, branching ratio = 33%
- Open histogram: Monte-Carlo events
- Hatched histogram:

  - left: true $\eta \to 2\gamma$
  - right: true $\eta \to 3\pi^0$

Effective interactions for nucleon decay

S. Weinberg (1979)
D. V. Nanopoulos and S. Weinberg (1979)

- Assume the tree-level exchange of a scalar or vector boson
  - Vector boson V_μ can induce $\mathcal{O}^{(1)}_{ijk\ell}$, $\mathcal{O}^{(2)}_{ijk\ell}$

    Renormalizable interaction: $V_\mu \psi_{R\dot{\alpha}} \bar{\sigma}^{\mu\dot{\alpha}\alpha} \chi_{L\alpha}$

    ψ_R include the conjugate of ψ_L
  - Scalar boson S can induce $\mathcal{O}^{(1)}_{ijk\ell}$, $\mathcal{O}^{(2)}_{ijk\ell}$, $\mathcal{O}^{(3)}_{ijk\ell}$, $\mathcal{O}^{(4)}_{ijk\ell}$.

    Renormalizable interactions: $S(\psi_L \chi_L)$, $S(\psi_R \chi_R)$

$$\mathcal{O}^{(1)}_{ijk\ell} \equiv \epsilon_{abc}\epsilon_{\alpha\beta}\left(u^a_{Ri} d^b_{Rj}\right)\left(Q^{c\alpha}_k L^\beta_\ell\right),$$
$$\mathcal{O}^{(2)}_{ijk\ell} \equiv \epsilon_{abc}\epsilon_{\alpha\beta}\left(Q^{a\alpha}_i Q^{b\beta}_j\right)\left(u^c_{Rk} e_{R\ell}\right),$$
$$\mathcal{O}^{(3)}_{ijk\ell} \equiv \epsilon_{abc}\epsilon_{\alpha\beta}\epsilon_{\gamma\delta}\left(Q^{a\alpha}_i Q^{b\gamma}_j\right)\left(Q^{c\delta}_k L^\beta_\ell\right),$$
$$\mathcal{O}^{(4)}_{ijk\ell} \equiv \epsilon_{abc}\left(u^a_{Ri} d^b_{Rj}\right)\left(u^c_{Rk} e_{R\ell}\right),$$

- In non-SUSY GUTs, the gauge boson exchange typically dominates: mixed-type
- In SUSY-GUT, the one-loop contribution can be significant: both can contribute

The direct method

Y. Aoki et al. (2017)
J. S. Yoo, et al. (2022)

- We use hadron matrix elements evaluated by lattice simulations.

I	0		1	
$\chi\chi'$	LR	LL	LR	LL
$W_{p\pi^+\nu}$, $\sqrt{2}W_{p\pi^0\ell}$, $\sqrt{2}W_{n\pi^0\nu}$, $W_{n\pi^-\ell}$	-0.159	0.151	0.169	-0.134
$W_{p\eta\ell}$, $W_{n\eta\nu}$	0.006	0.113	0.044	-0.044

- $W^{RR} = W^{LL}$, $W^{RL} = W^{LR}$
- In units of GeV^2

$$\langle \Pi(\boldsymbol{p}) | \mathcal{O}_{\chi\chi'} | N(\boldsymbol{k}) \rangle = P_{\chi'} \left[W_0^{\mathcal{O}}(q^2) + \underline{\frac{\not{q}}{m_N} W_1^{\mathcal{O}}(q^2)} \right] u_N(\boldsymbol{k})$$

$$\mathcal{O}_{\chi\chi'} = \epsilon_{abc}(q_\chi^a q_\chi'^b) q_{\chi'}''^c$$

- $\mathcal{O}(m_\ell/m_p) \to$ only relevant for anti-muon

Positron channels: $\Gamma(p \to \eta e^+)/\Gamma(p \to \pi^0 e^+)$

- To understand the behavior, let us use the expressions calculated from chiral lagrangian (i.e., the so-called indirect method M. Claudson, M. B. Wise, and L. J. Hall (1982))

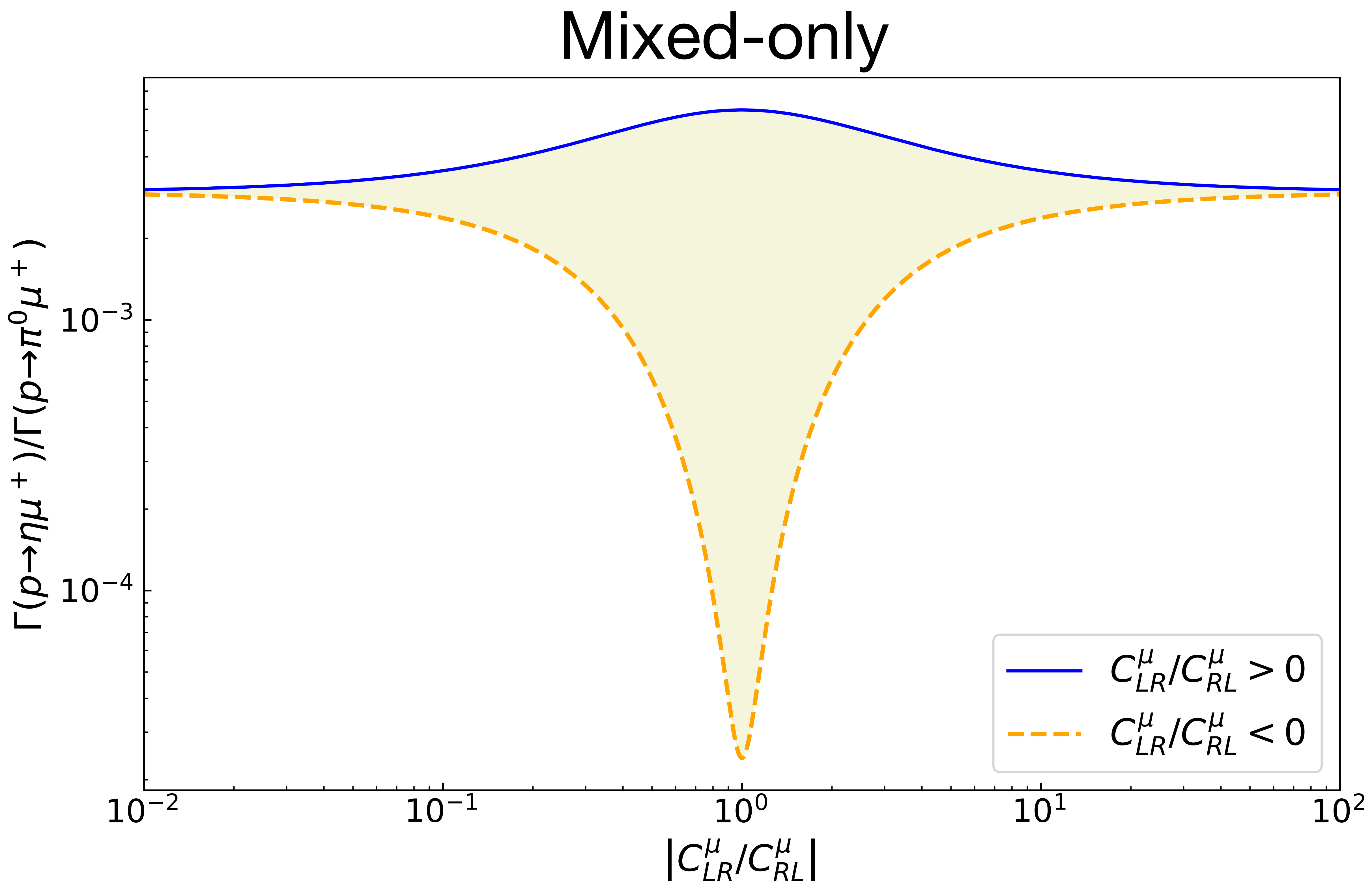
$$\Gamma(p \to \pi^0 e^+) = \frac{m_p}{32\pi}\left(1 - \frac{m_\pi^2}{m_p^2}\right)^2 \frac{(1+D+F)^2\alpha^2}{2f^2}\left[|C^e_{RL} - C^e_{LL}|^2 + |C^e_{LR} - C^e_{RR}|^2\right]$$

$$\Gamma(p \to \eta e^+) = \frac{m_p}{32\pi}\left(1 - \frac{m_\eta^2}{m_p^2}\right)^2 \frac{\alpha^2}{6f^2}\Big[|(1+D-3F)C^e_{RL} + (3-D+3F)C^e_{LL}|^2 + |(1+D-3F)C^e_{LR} + (3-D+3F)C^e_{RR}|^2\Big] ,$$

- $p \to \pi^0 e^+$ is suppressed when $C^e_{LL}/C^e_{RL} = C^e_{RR}/C^e_{LR} = 1$
- $p \to \eta e^+$ is suppressed when $C^e_{LL}/C^e_{RL} = C^e_{RR}/C^e_{LR} = -(1+D-3F)/(3-D+3F) \simeq -0.1$

Chirality structure and branching fractions

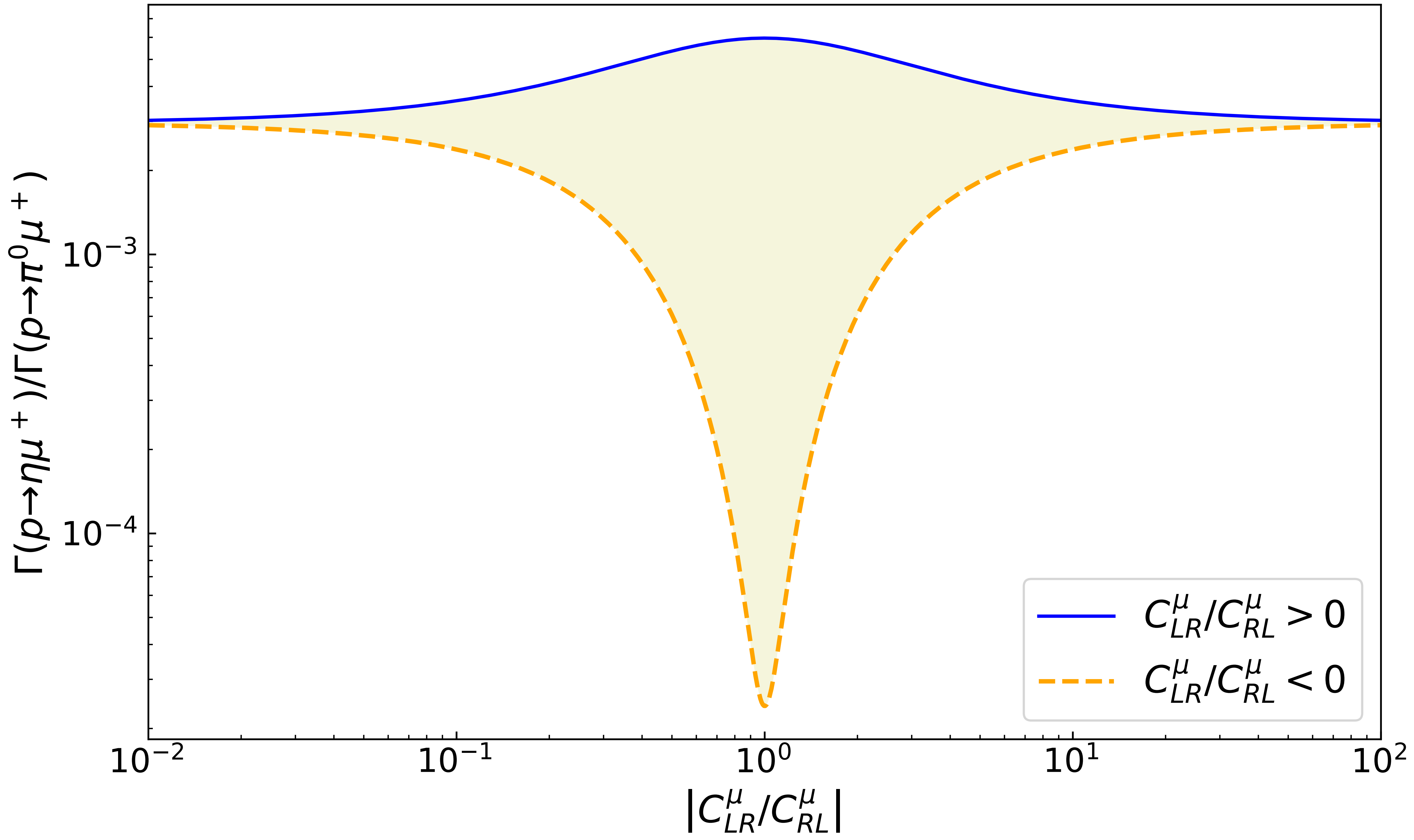
- **Case 3:** mixed- or pure-only case, but with nonzero m_μ.



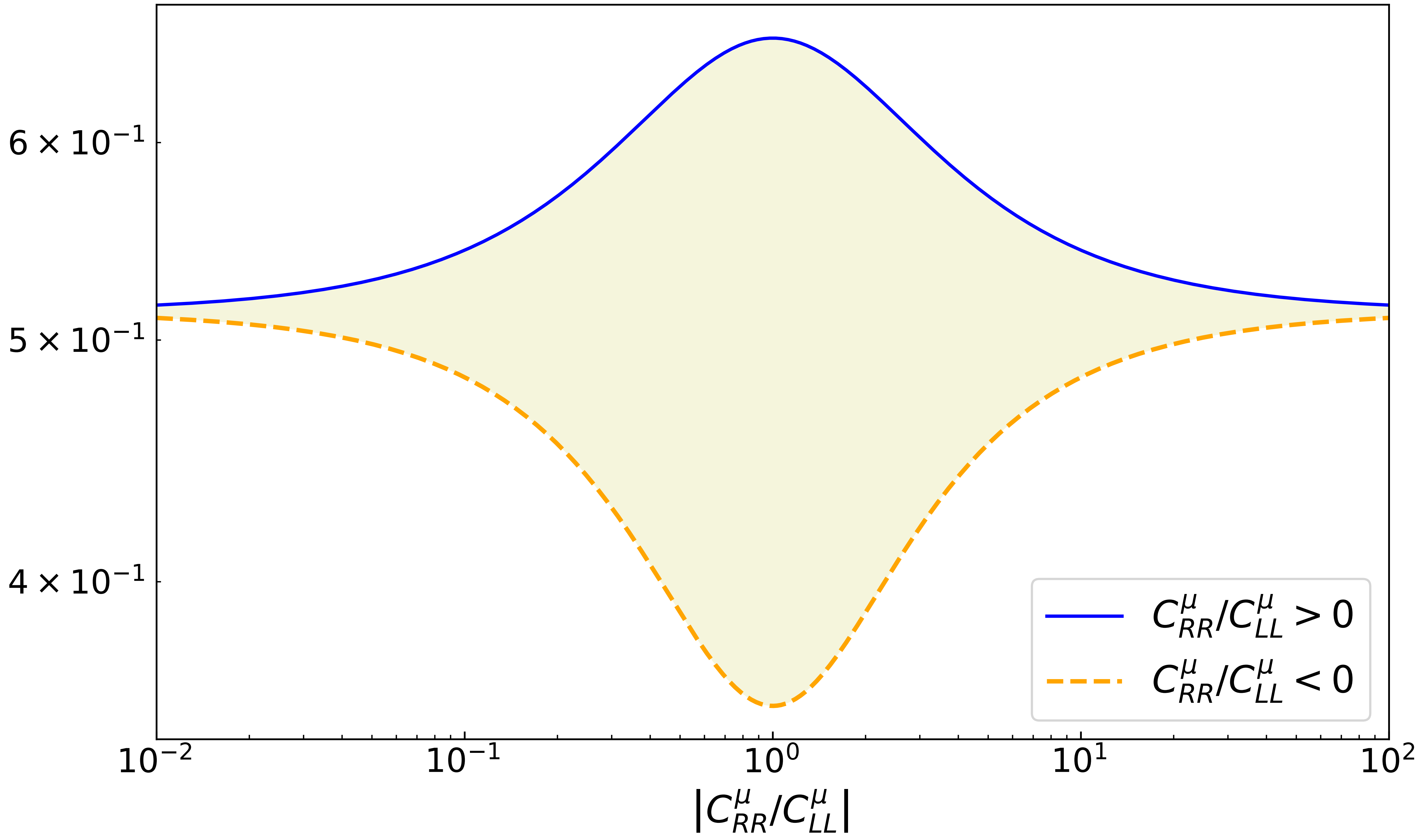
Chirality structure and branching fractions

- **Case 3:** mixed- or pure-only case, but with nonzero m_μ.

Mixed-only



Pure-only



$\Gamma(p \to \eta\mu^+)/\Gamma(p \to \pi^0\mu^+)$ **can distinguish between different mixed/pure operators.**

Anti-muon channels: $\Gamma(p \to \eta\mu^+)/\Gamma(p \to \pi^0\mu^+)$

- For the mixed-only case, up to $\mathcal{O}(m_\mu/m_p)$,

$$\Gamma(p \to \Pi\mu^+) = \frac{m_p}{32\pi}\left(1 - \frac{m_\Pi^2}{m_p^2}\right)^2 \left(W^{LR}_{p\Pi\ell,0}\right)^2 \left[|C^\mu_{RL}|^2 + |C^\mu_{LR}|^2\right] ,$$
$$\times \left[1 + \frac{4m_\mu}{m_p}\left\{\frac{W^{LR}_{p\Pi\ell,1}}{W^{LR}_{p\Pi\ell,0}} + \left(1 - \frac{m_\Pi^2}{m_p^2}\right)^{-1}\right\}\frac{\mathrm{Re}(C^\mu_{RL}C^{\mu*}_{LR})}{|C^\mu_{RL}|^2 + |C^\mu_{LR}|^2}\right]$$

  - The source of the dependence on $|C^e_{LR}/C^e_{RL}|$
  - This effect is enhanced when $|W^{LR}_{p\Pi\mu,1}| \gg |W^{LR}_{p\Pi\mu,0}|$

I	0		1	
$\chi\chi'$	LR	LL	LR	LL
$W_{p\pi^+\nu}$, $\sqrt{2}W_{p\pi^0\ell}$, $\sqrt{2}W_{n\pi^0\nu}$, $W_{n\pi^-\ell}$	−0.159	0.151	0.169	−0.134
$W_{p\eta\ell}$, $W_{n\eta\nu}$	0.006	0.113	0.044	−0.044

Form factors obtained by lattice QCD in units of GeV^2

Y. Aoki et al. (2017)
J. S. Yoo, et al. (2022)

Other channels

- The ratio of neutrino channels is related to that of charged leptons in either mixed- or pure-only cases.

$$\Gamma(N \to \Pi\bar{\nu}) = \frac{m_N}{32\pi}\left(1 - \frac{m_\Pi^2}{m_N^2}\right)^2 \left|W^{RL}_{N\Pi\nu,0}\right|^2 \sum_i |C^{\nu_i}_{RL}|^2$$

$$\frac{\Gamma(n \to \eta\bar{\nu})}{\Gamma(n \to \pi^0\bar{\nu})} = \frac{(1 - m_\eta^2/m_n^2)^2}{(1 - m_\pi^2/m_n^2)^2} \cdot \frac{|W^{XX'}_{n\eta\nu}|^2}{|W^{XX'}_{n\pi^0\nu}|^2} \simeq \frac{\Gamma(p \to \eta e^+)}{\Gamma(p \to \pi^0 e^+)}$$

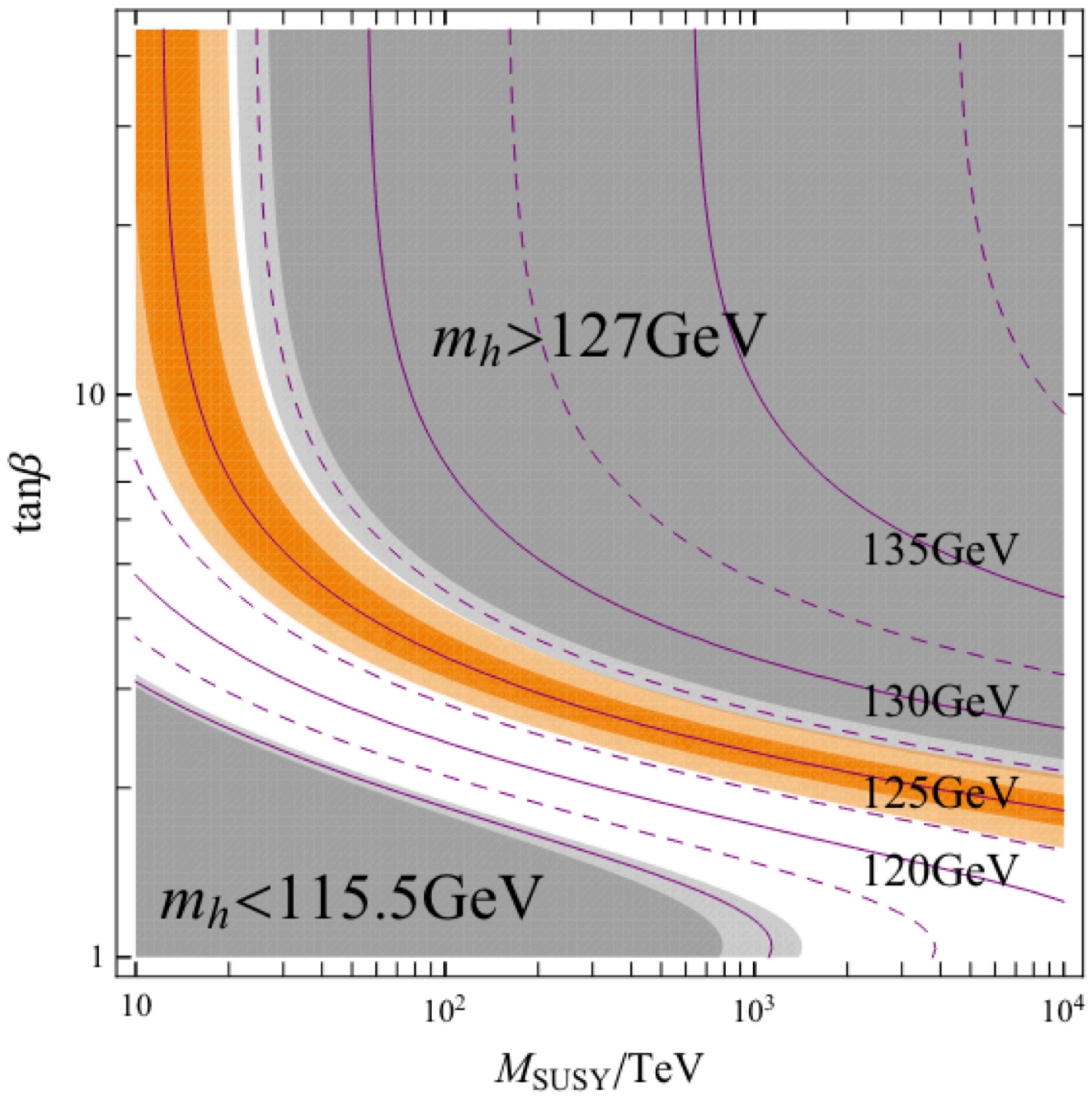
- Due to the presence of more Wilson coefficients (=six for the neutrino ratio), it is difficult to extract information about the Wilson coefficients on generic grounds. The same applies to $\Gamma(n \to \pi^0\bar{\nu})/\Gamma(p \to \pi^0\ell^+)$.

- Those ratios can still become powerful probes when considering specific UV models.

High-scale SUSY

M. Ibe, S. Matsumoto, T. T. Yanagida (2012)

- Relation between SUSY-breaking scale and $\tan\beta$

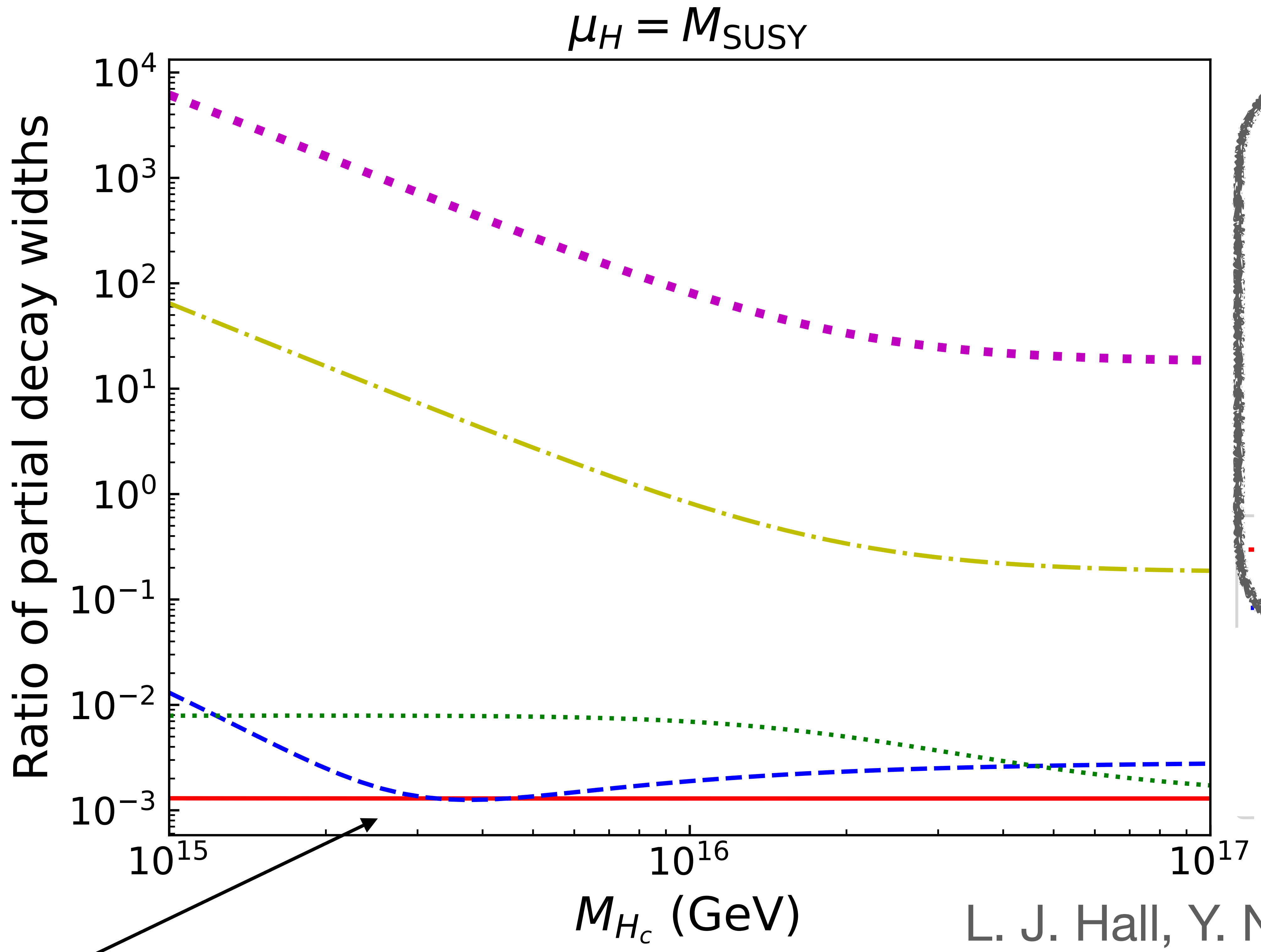


For $M_{\rm SUSY} = 10^2$ TeV,

$\tan\beta \sim 3$ is needed to reproduce

the correct Higgs mass.

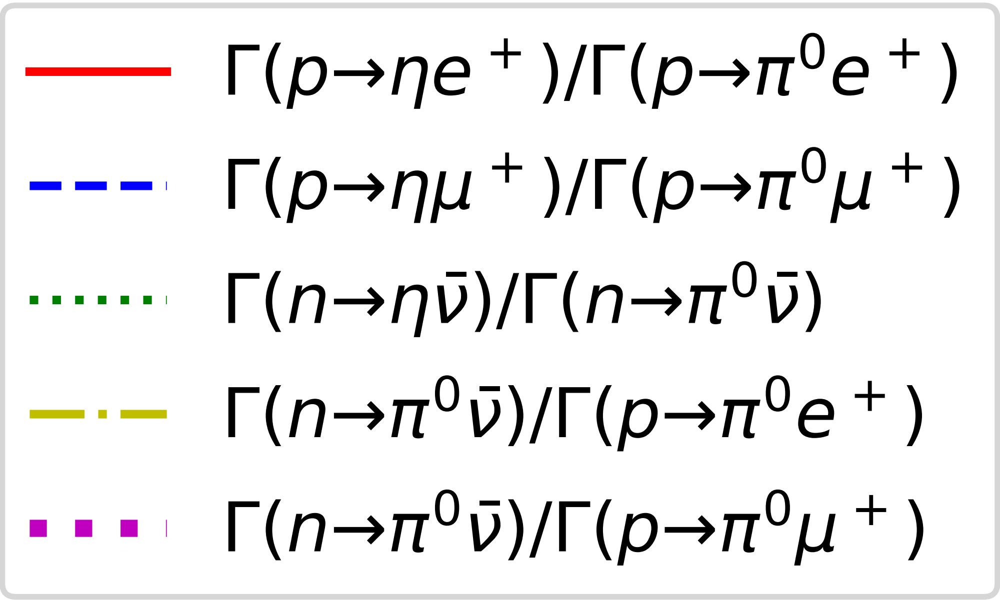
The minimal SU(5) with high-scale SUSY



- $M_X = 10^{16}$ GeV
- $M_2 = 1$ TeV
- $M_3 = 10$ TeV
- $M_{\mathrm{SUSY}} = 100$ TeV
- $\tan\beta = 3$

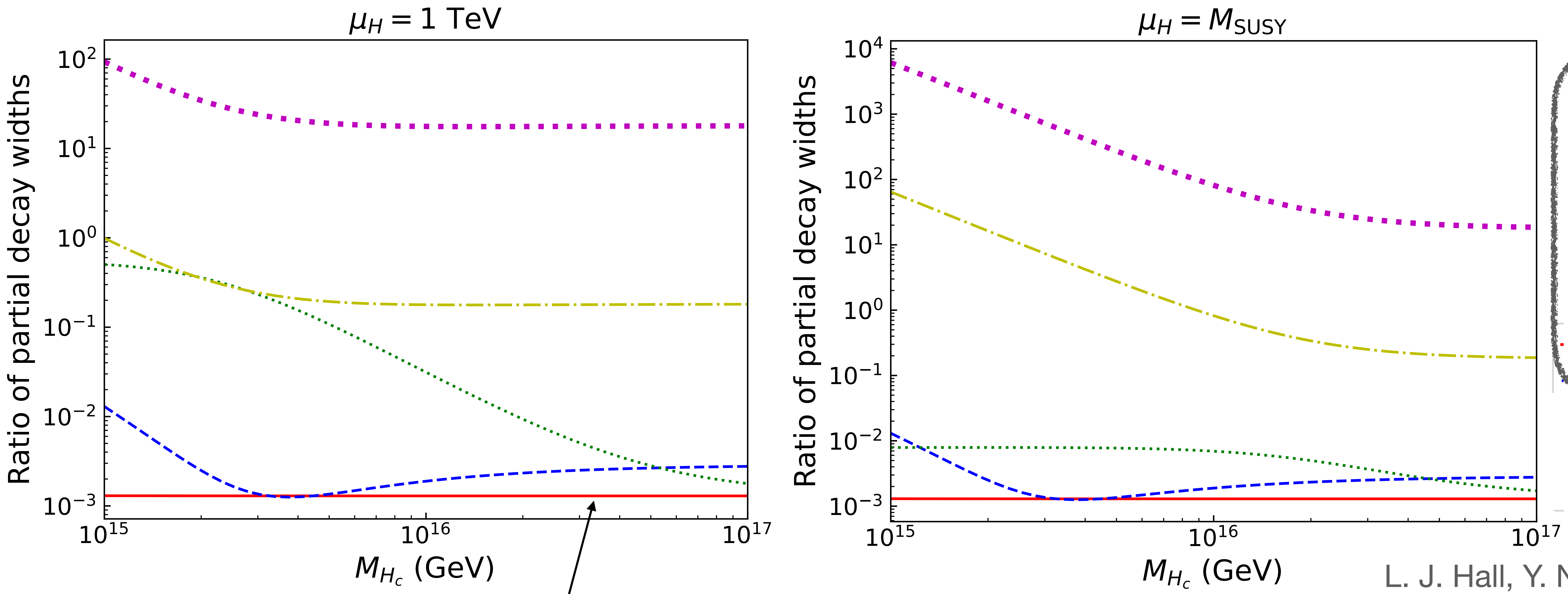
Mini-split type mass spectrum

L. J. Hall, Y. Nomura and S. Shirai (2012)
M. Ibe, S. Matsumoto and T. T. Yanagida (2012)



$\Gamma(p \to \eta e^+)/\Gamma(p \to \pi^0 e^+)$: gauge-boson/higgsino exchange (mixed-type) dominates.

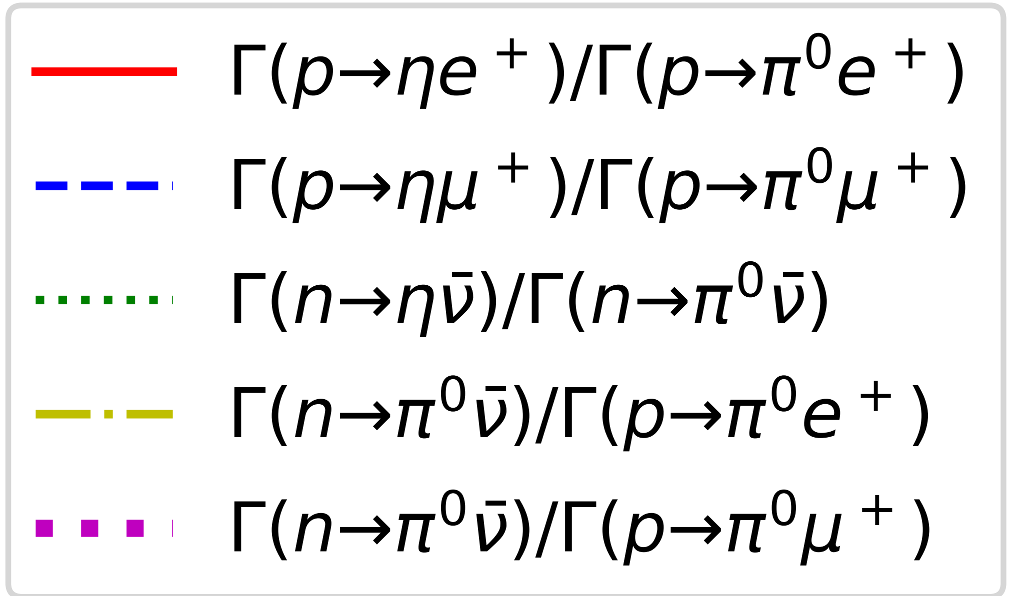
The minimal SU(5) with high-scale SUSY



- $M_X = 10^{16}$ GeV
- $M_2 = 1$ TeV
- $M_3 = 10$ TeV
- $M_{\rm SUSY} = 100$ TeV
- $\tan\beta = 3$

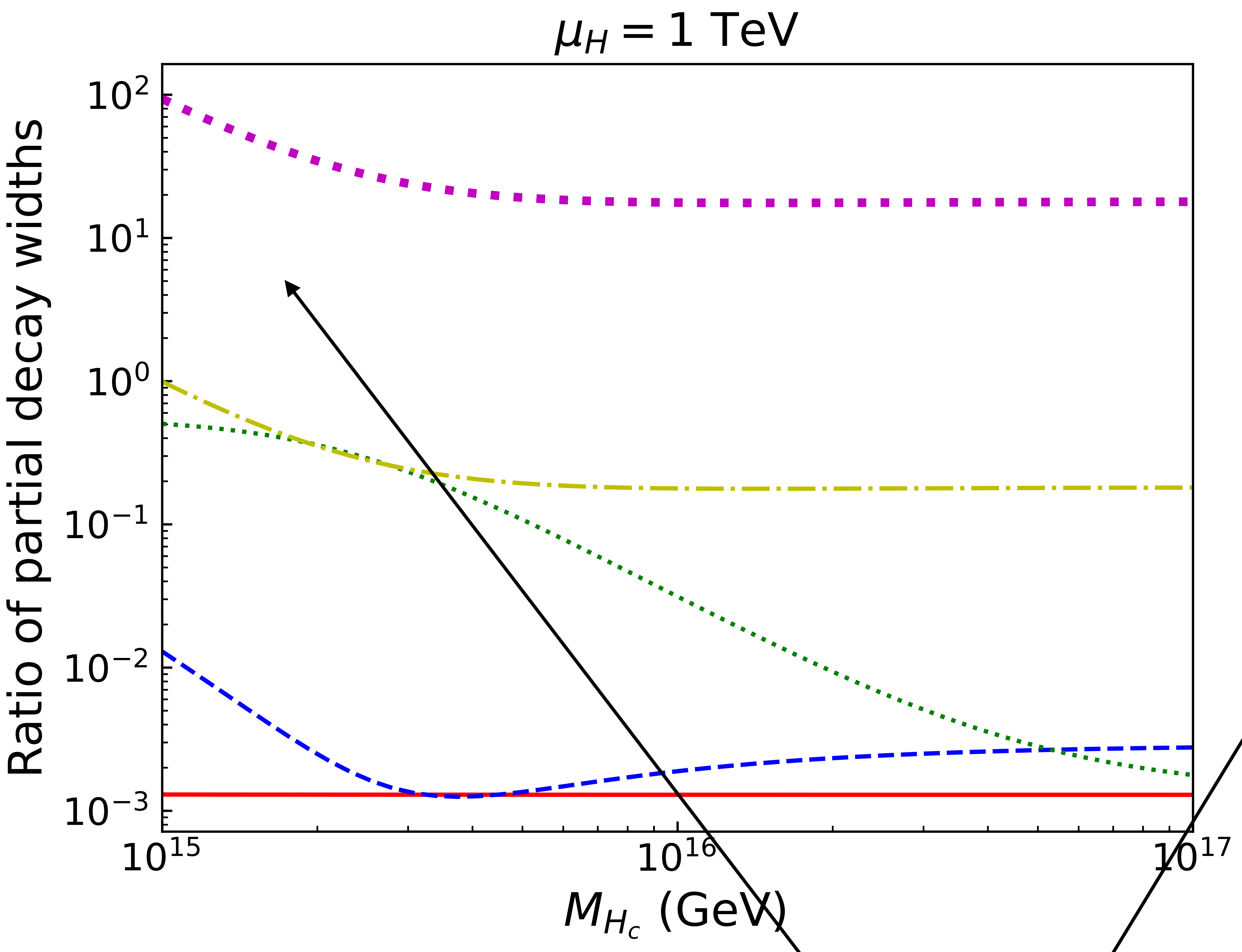
Mini-split type mass spectrum

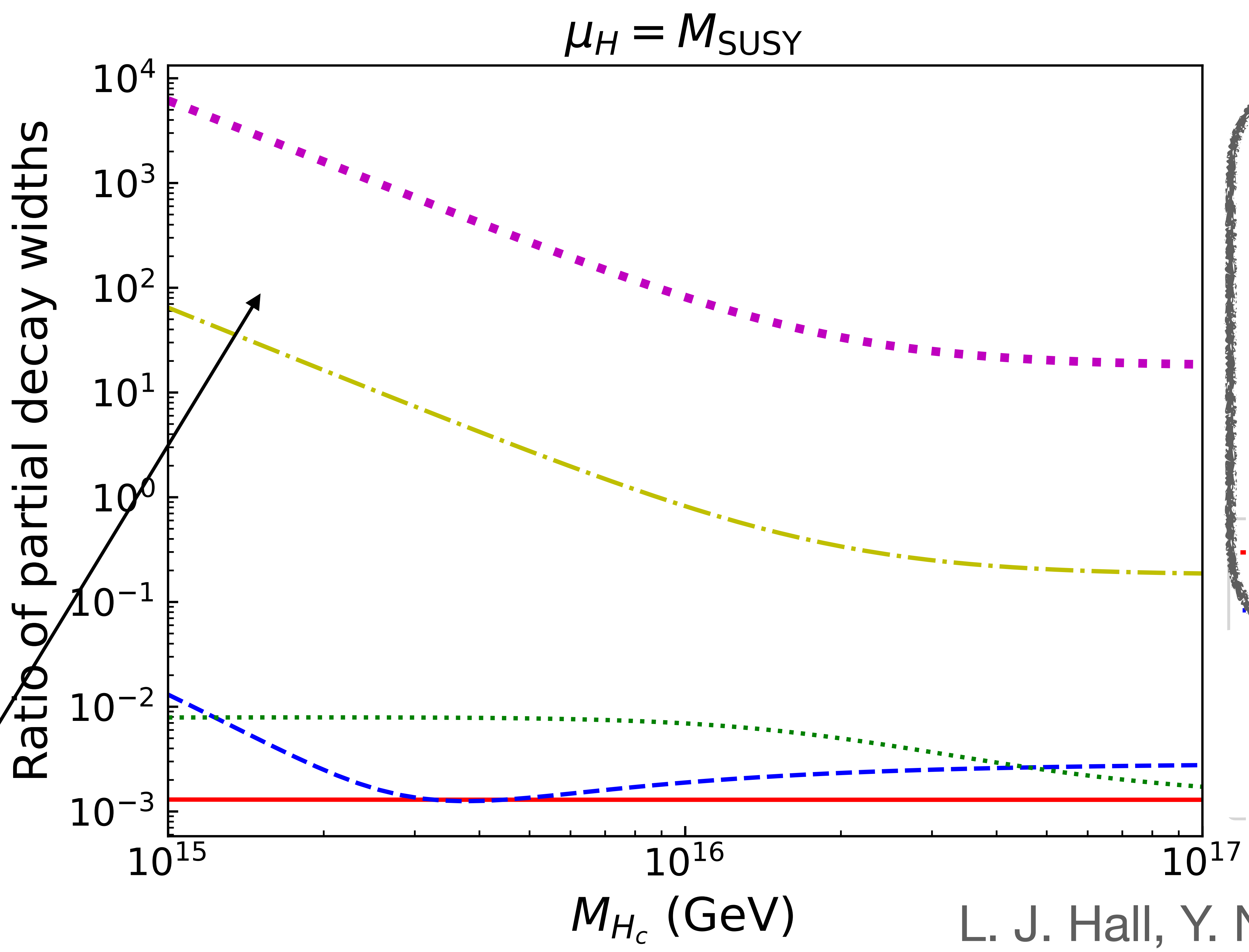
L. J. Hall, Y. Nomura and S. Shirai (2012)
M. Ibe, S. Matsumoto and T. T. Yanagida (2012)



$\Gamma(p\to\eta e^+)/\Gamma(p\to\pi^0 e^+)$: gauge-boson exchange (mixed-type) dominates.

The minimal SU(5) with high-scale SUSY

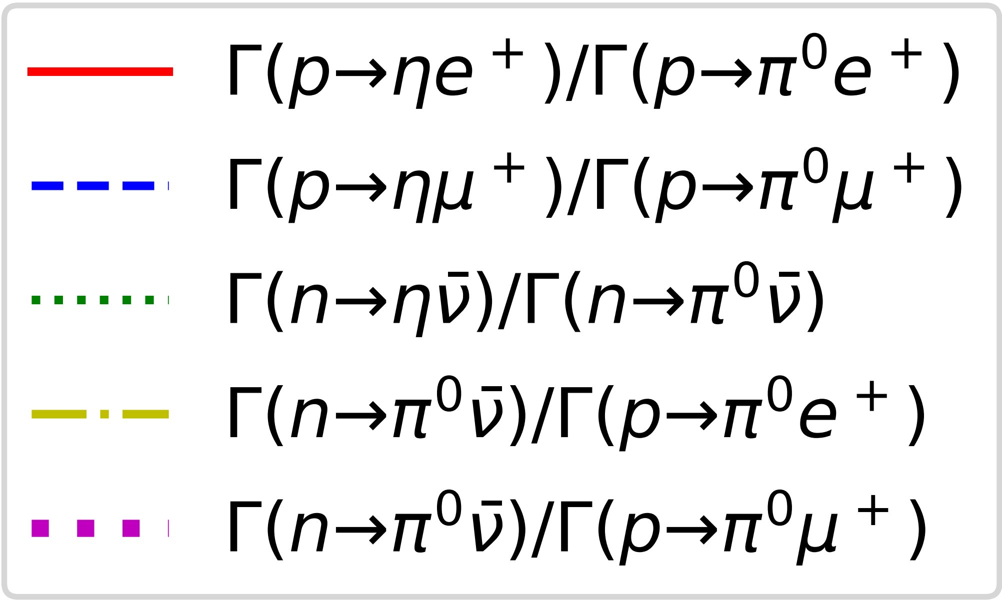




- $M_X = 10^{16}$ GeV
- $M_2 = 1$ TeV
- $M_3 = 10$ TeV
- $M_{\rm SUSY} = 100$ TeV
- $\tan\beta = 3$

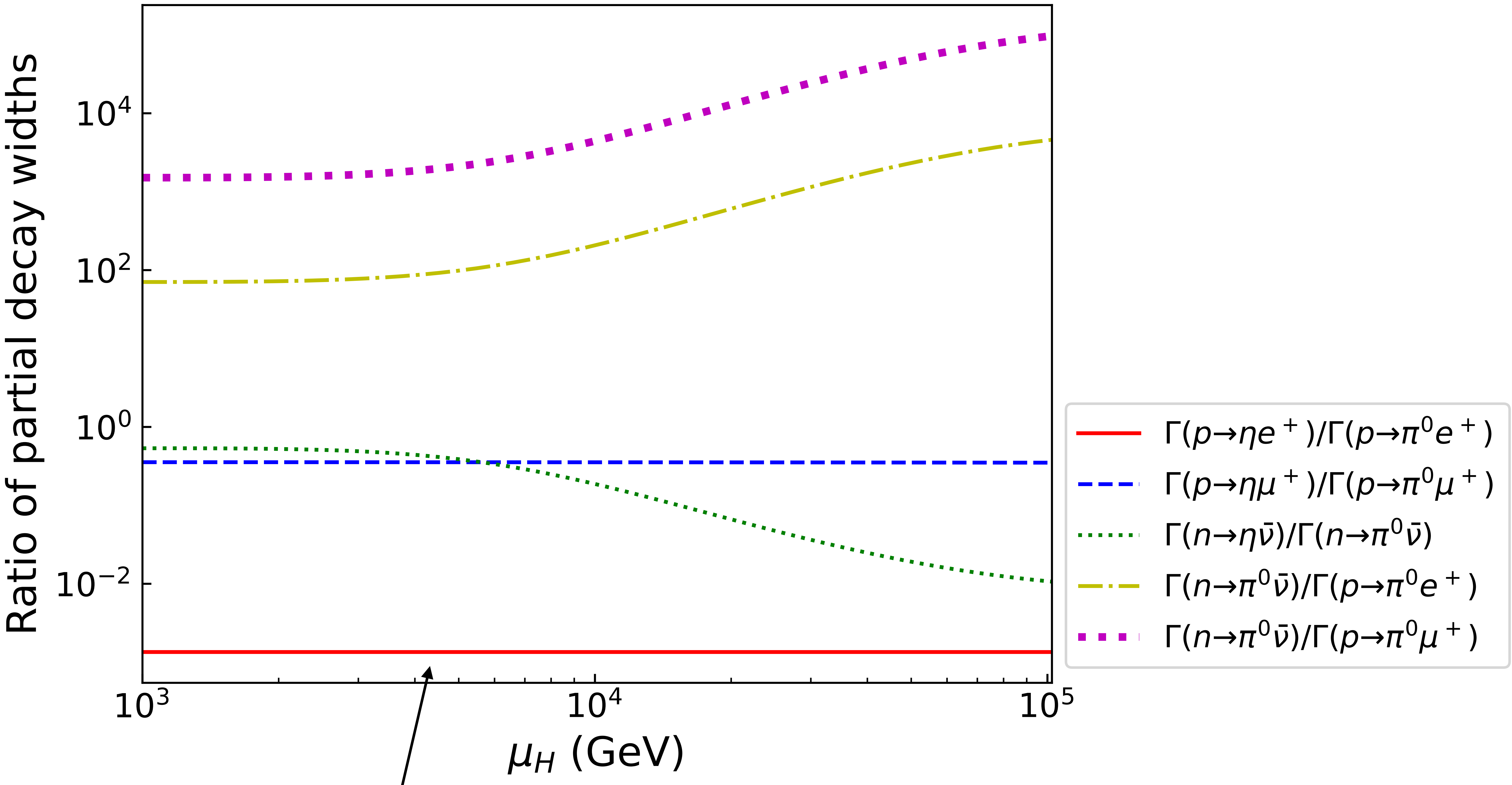
Mini-split type mass spectrum

L. J. Hall, Y. Nomura and S. Shirai (2012)
M. Ibe, S. Matsumoto and T. T. Yanagida (2012)



- $\Gamma(p \to \eta\mu^+)/\Gamma(p \to \pi^0\mu^+)$: wino-exchange contributes (pure-type) as M_{H_C} decreases.
- $\Gamma(n \to \pi^0\bar\nu)/\Gamma(p \to \pi^0 e^+)$ and $\Gamma(n \to \pi^0\bar\nu)/\Gamma(p \to \pi^0\mu^+)$ show the same tendency.

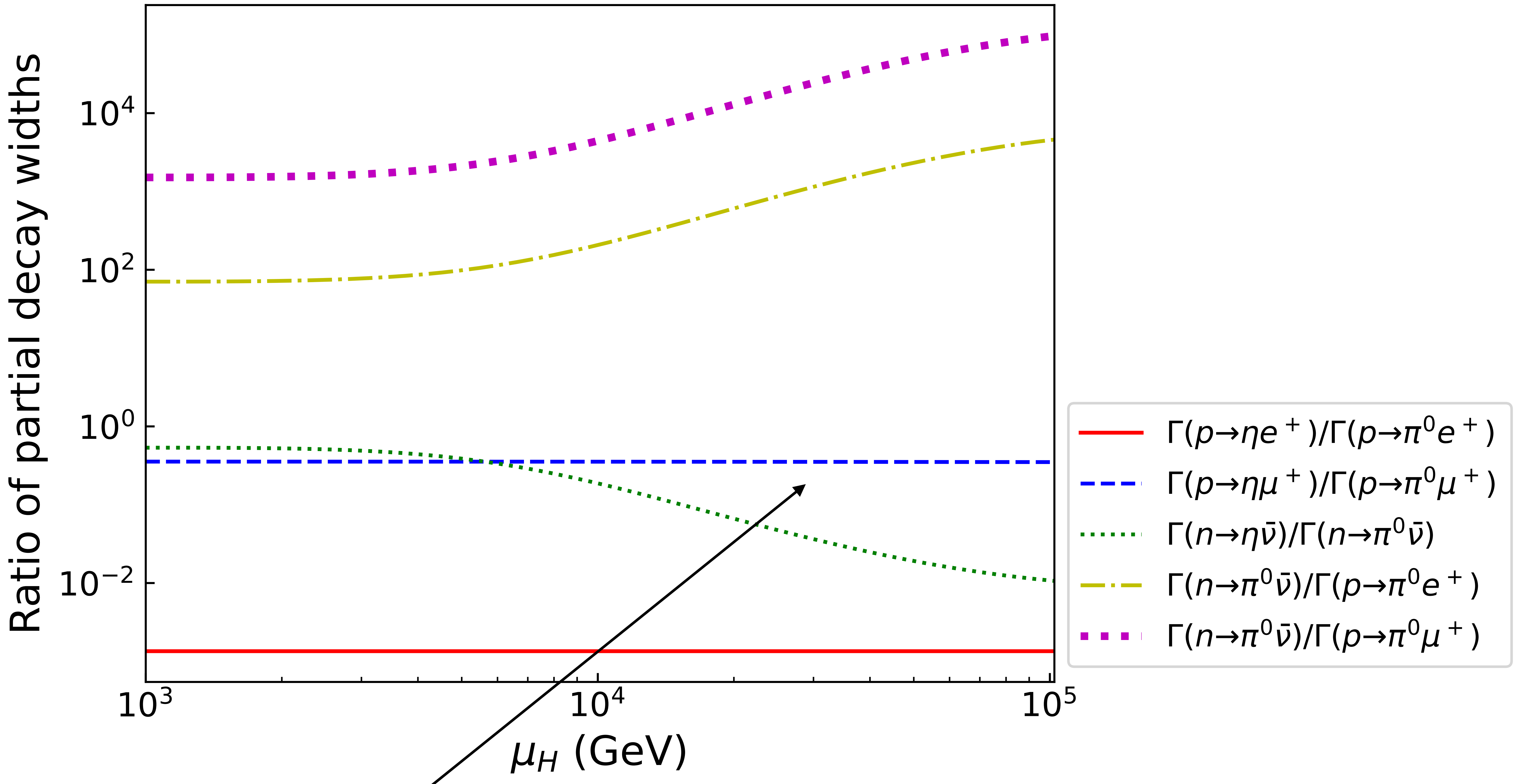
The minimal SU(5) with high-scale SUSY



- $M_X = 10^{17}$ GeV
- $M_{H_C} = 10^{16}$ GeV
- $M_2 = 1$ TeV
- $M_3 = 10$ TeV
- $M_{\rm SUSY} = 100$ TeV
- $\tan\beta = 3$

$\Gamma(p \to \eta e^+)/\Gamma(p \to \pi^0 e^+)$: gauge-boson exchange (mixed-type) dominates even when the GUT gauge boson is heavy.

The minimal SU(5) with high-scale SUSY

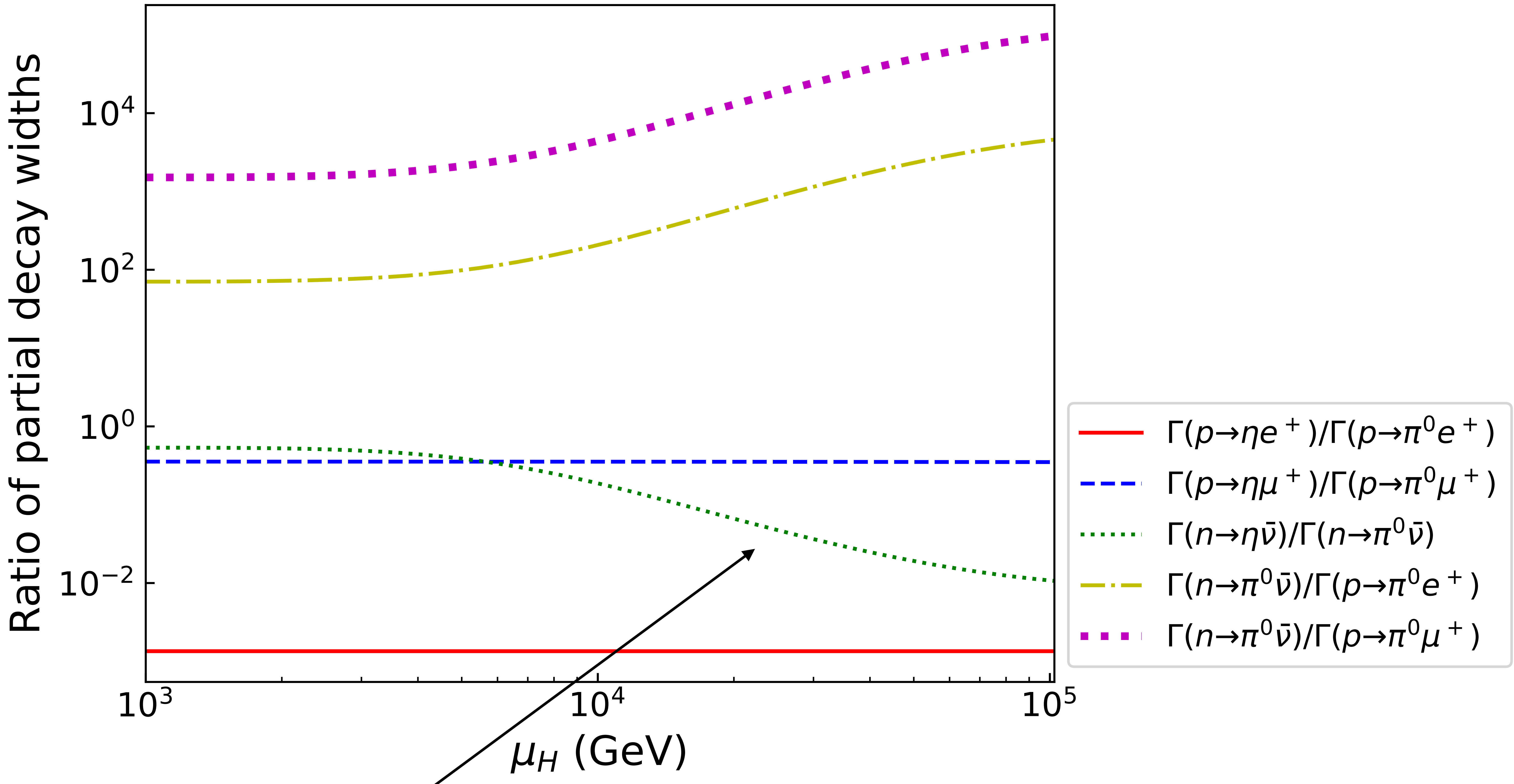


- $M_X = 10^{17}$ GeV
- $M_{H_C} = 10^{16}$ GeV
- $M_2 = 1$ TeV
- $M_3 = 10$ TeV
- $M_{\mathrm{SUSY}} = 100$ TeV
- $\tan\beta = 3$

$\Gamma(p \to \eta\mu^+)/\Gamma(p \to \pi^0\mu^+)$: wino contribution (mixed-type) dominates regardless of the higgsino mass.

→ this ratio is sensitive to dim-5 wino v.s. the GUT gauge boson competition.

The minimal SU(5) with high-scale SUSY

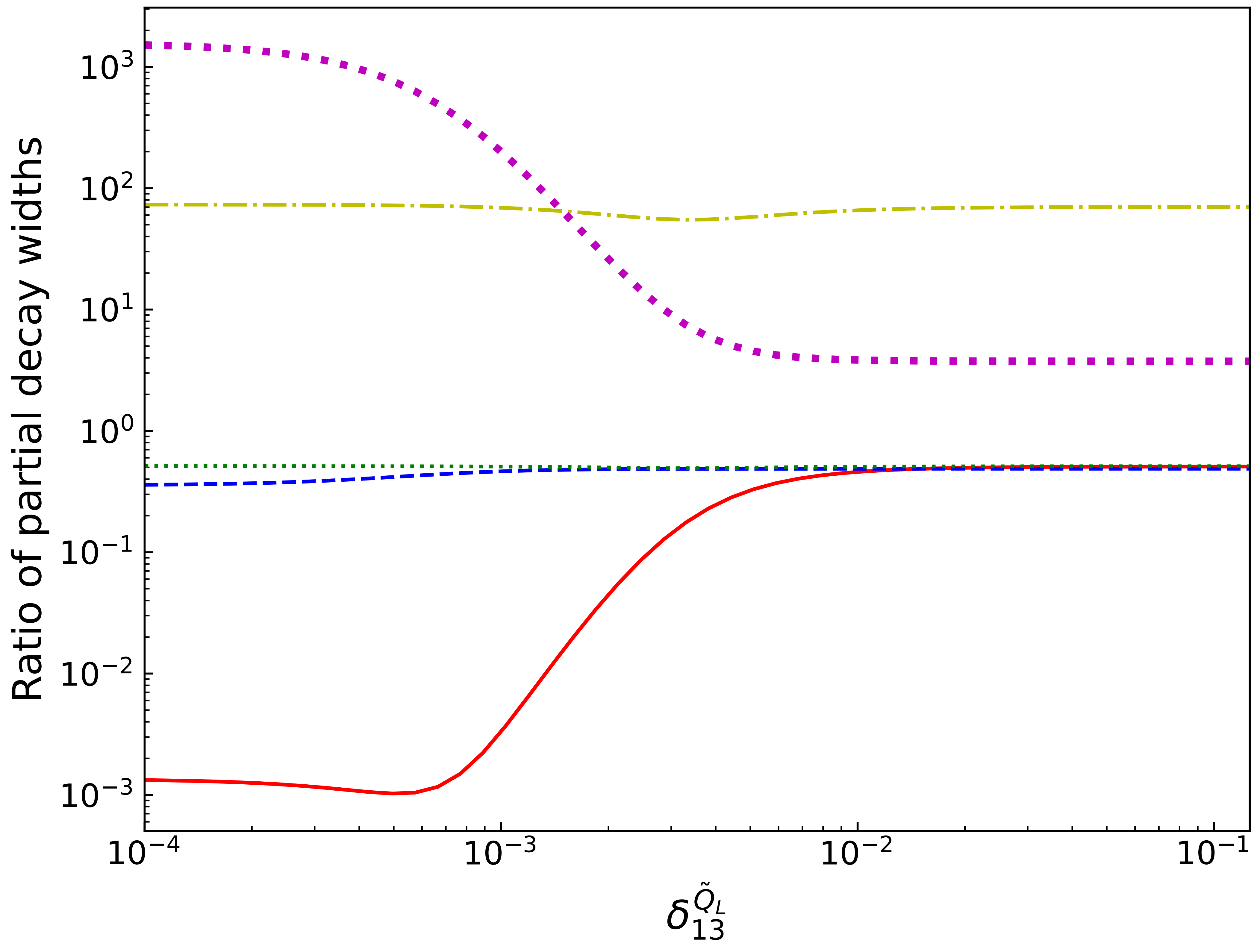


- $M_X = 10^{17}$ GeV
- $M_{H_C} = 10^{16}$ GeV
- $M_2 = 1$ TeV
- $M_3 = 10$ TeV
- $M_{\mathrm{SUSY}} = 100$ TeV
- $\tan\beta = 3$

$\Gamma(n \to \eta\bar{\nu})/\Gamma(n \to \pi^0\bar{\nu})$: higgsino contribution (mixed-type) becomes larger as its mass increases.

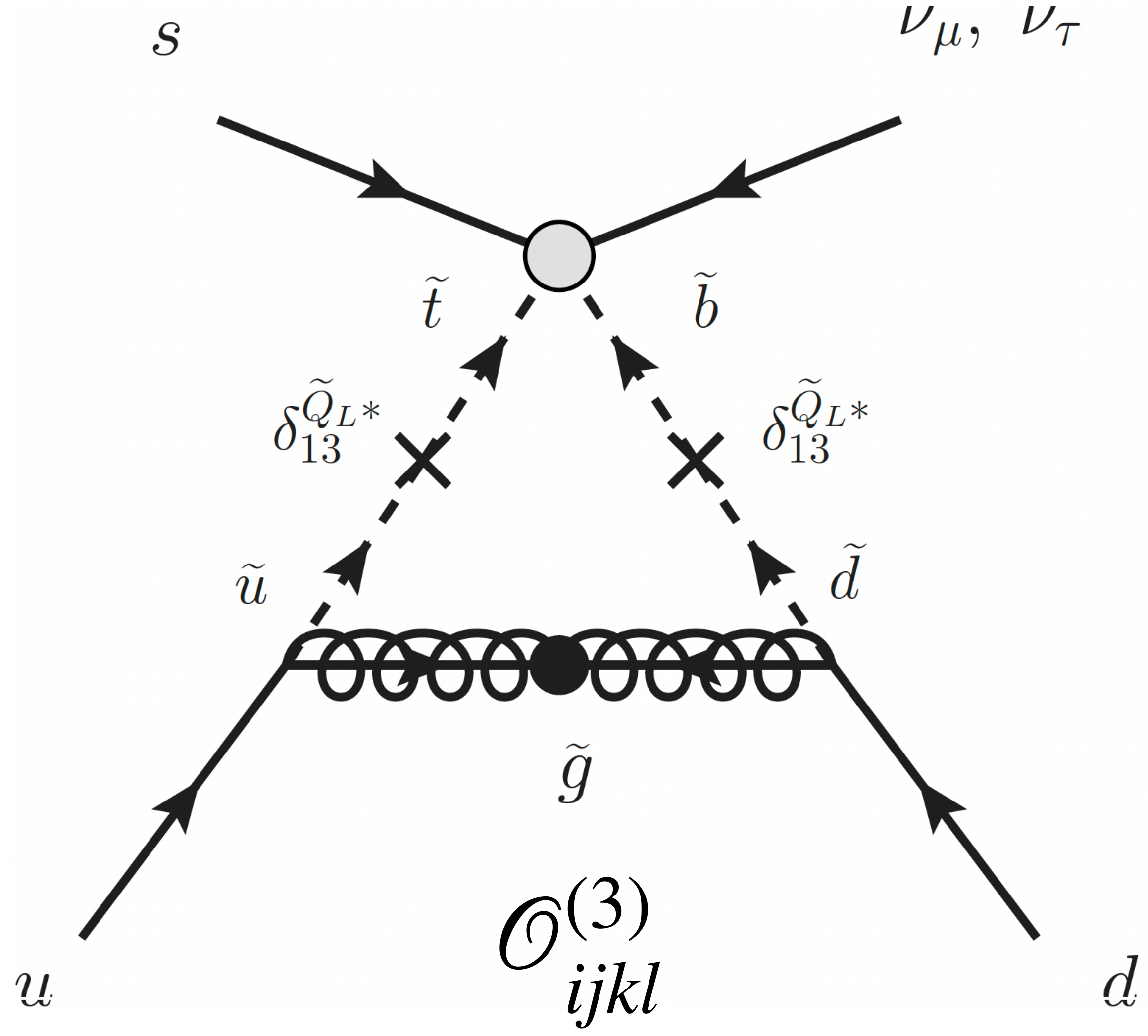
→ this ratio is most useful to discriminate the higgsino contribution from that of wino.

Sfermion flavor violation and ratios



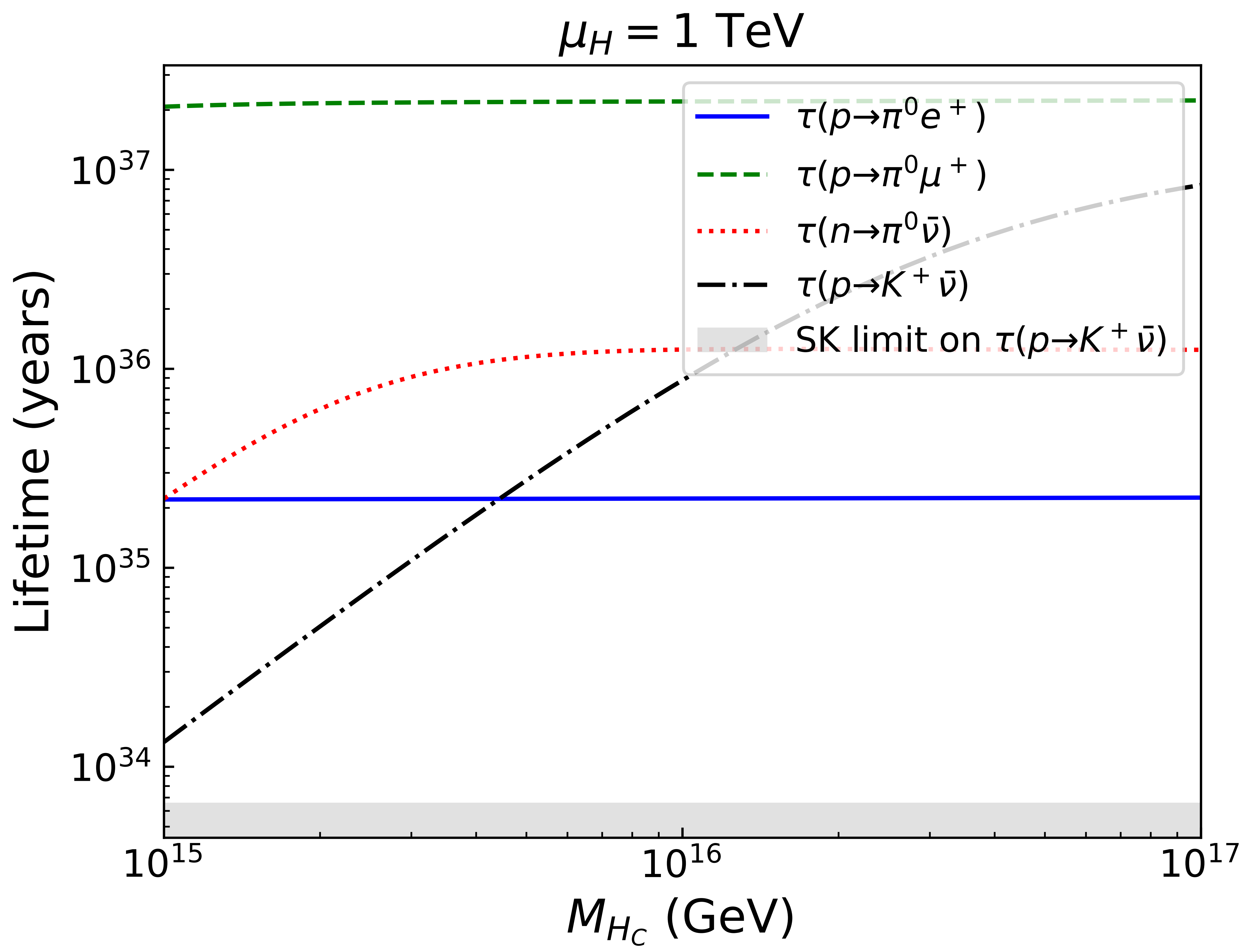
- $M_X = 10^{17}$ GeV
- $M_{H_C} = 10^{16}$ GeV
- $M_1 = 5$ TeV
- $M_2 = 1$ TeV
- $M_3 = 10$ TeV
- $M_{\rm SUSY} = 100$ TeV
- $\tan\beta = 3$
- $\mu_H = 200$ GeV

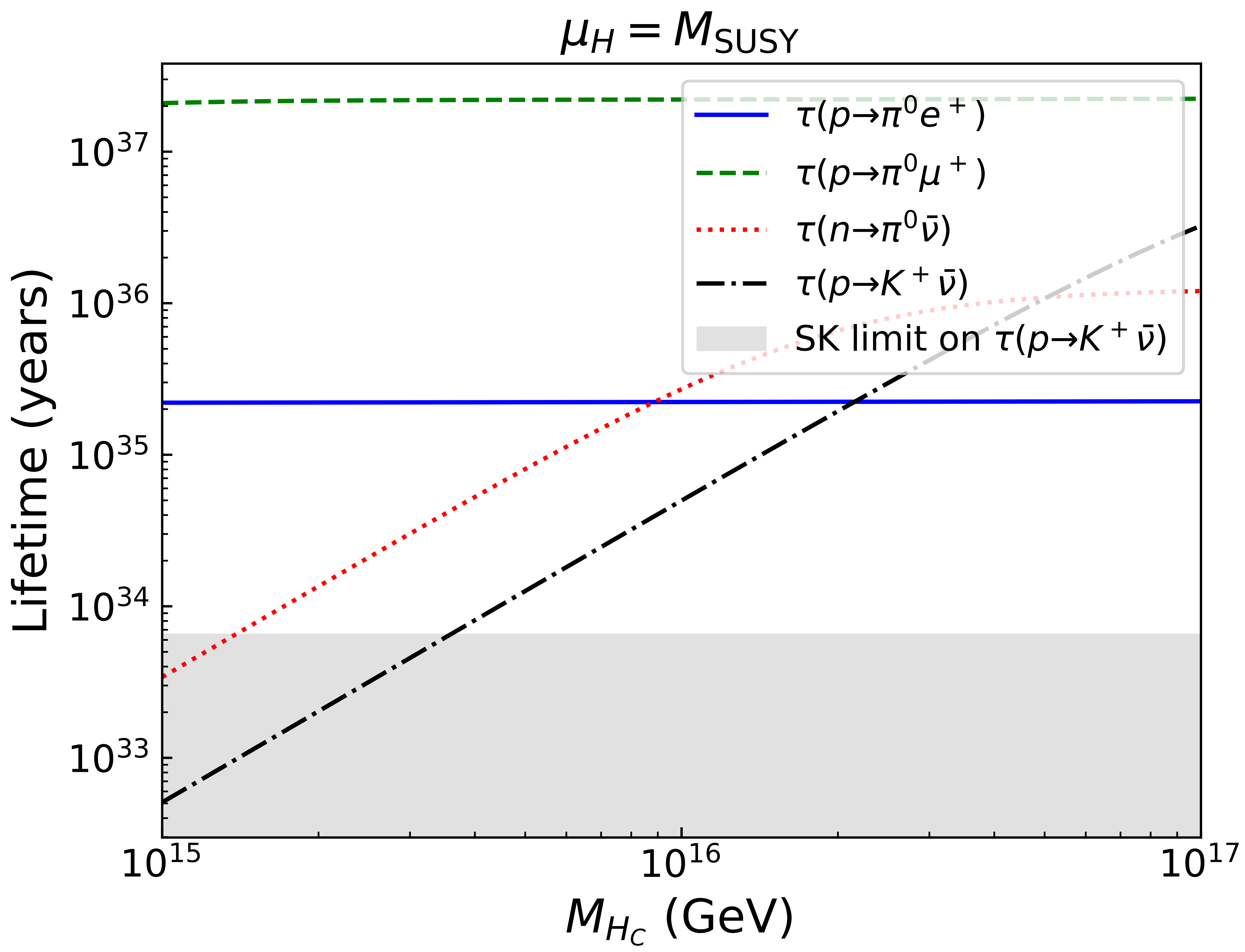
$$\widetilde{m}^2_{\tilde{f}} = M^2_{\rm SUSY}\begin{pmatrix} 1 & \delta_{12}^{\tilde{f}} & \delta_{13}^{\tilde{f}} \\ \delta_{12}^{\tilde{f}*} & 1 & \delta_{23}^{\tilde{f}} \\ \delta_{13}^{\tilde{f}*} & \delta_{23}^{\tilde{f}*} & 1 \end{pmatrix}$$



- The gluino exchange dominates as $\delta_{13}^{\tilde{Q}_L}$ increases, even in $\Gamma(p\to\eta e^+)/\Gamma(p\to\pi^0 e^+)$.
- $\Gamma(n\to\pi^0\bar{\nu})/\Gamma(p\to\pi^0\mu^+)$ is a useful probe of this scenario.

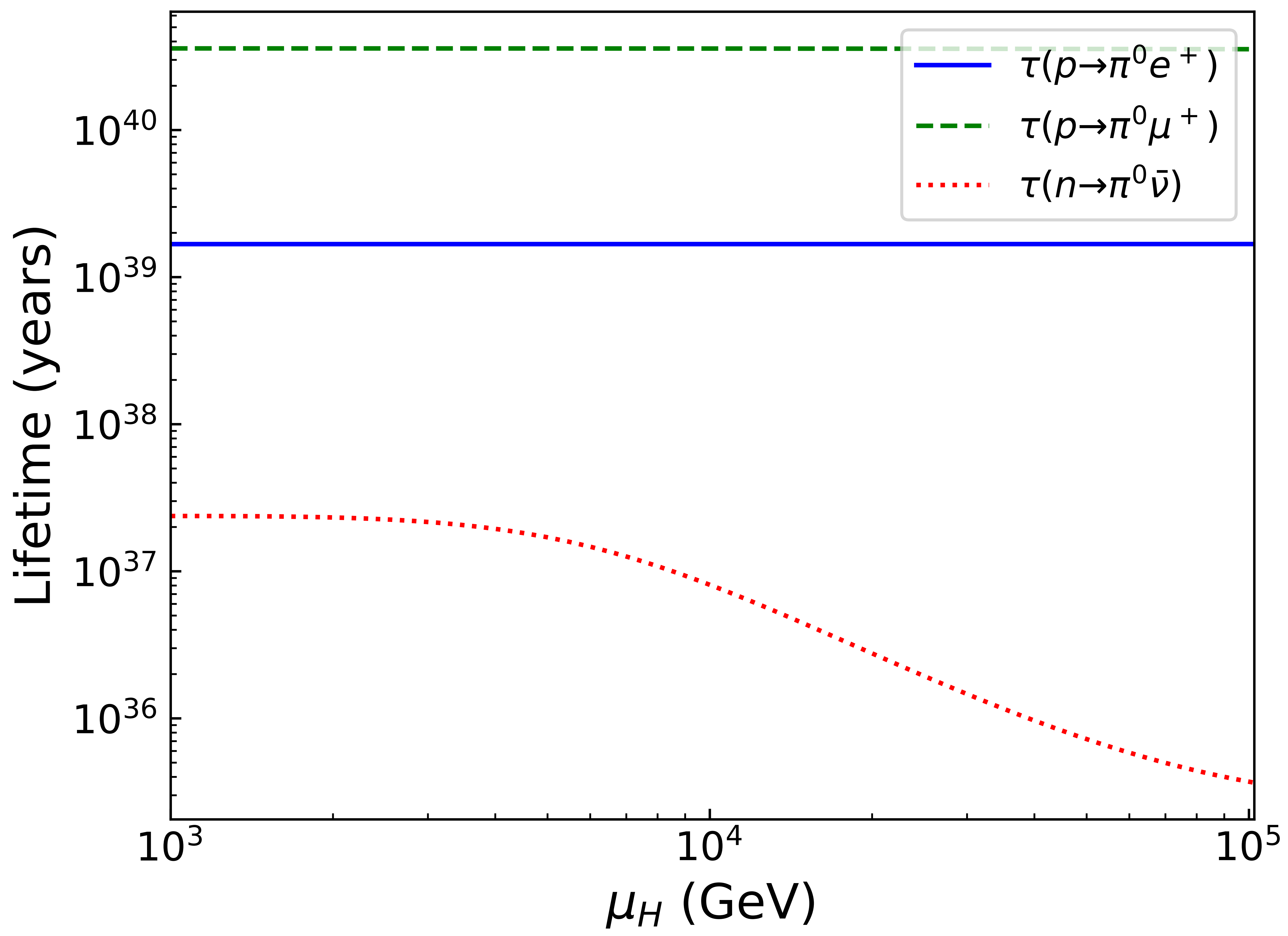
The minimal SU(5) with high-scale SUSY





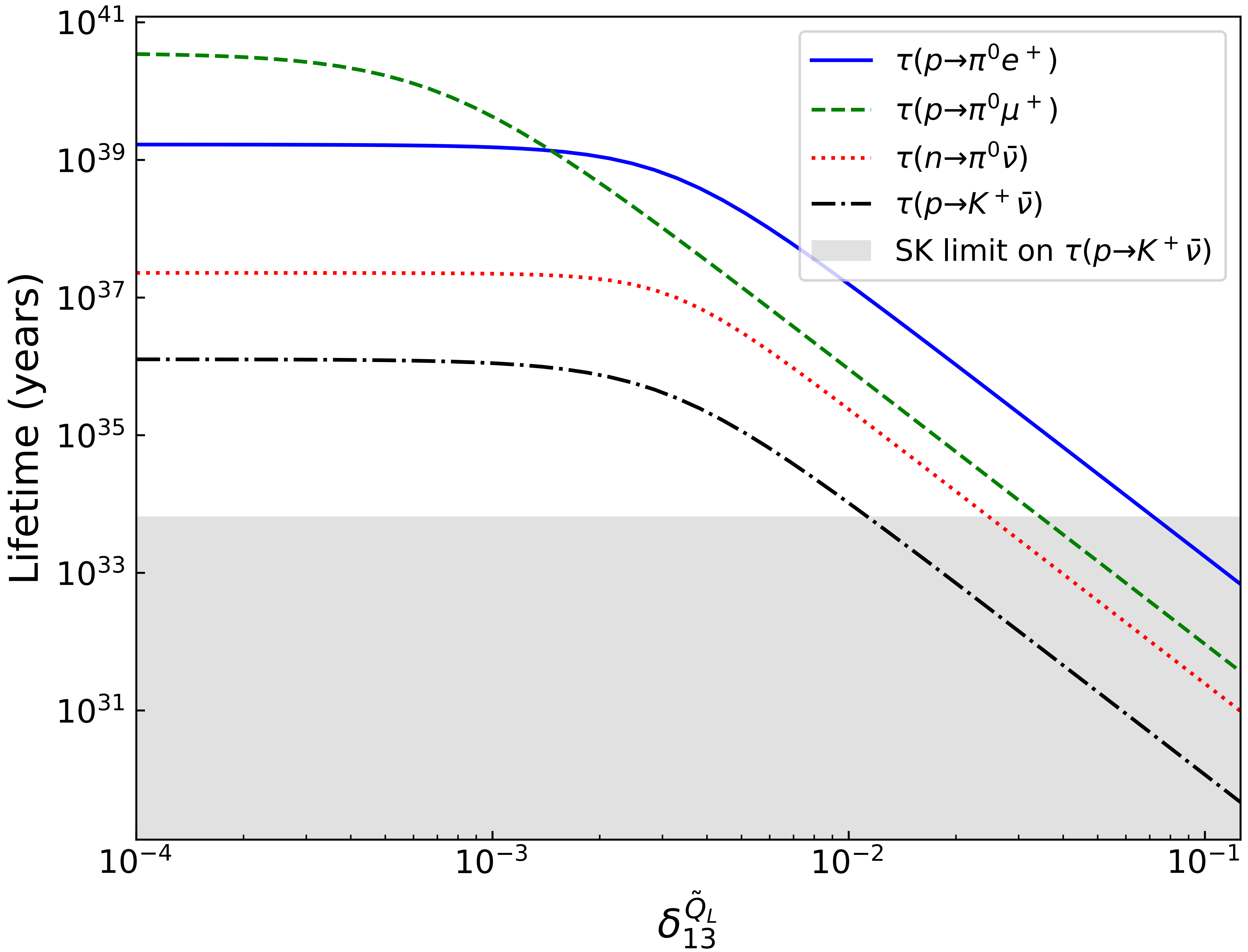
- $M_X = 10^{16}$ GeV, $M_2 = 1$ TeV, $M_3 = 10$ TeV, $M_{\rm SUSY} = 100$ TeV, $\tan\beta = 3$

The minimal SU(5) with high-scale SUSY



- $M_X = 10^{17}$ GeV, $M_{H_C} = 10^{16}$ GeV, $M_2 = 1$ TeV, $M_3 = 10$ TeV, $M_{\rm SUSY} = 100$ TeV, $\tan\beta = 3$

Sfermion flavor violation



In the presence of flavor violation,

- various channels could be accessible by upcoming experiments, or
- even ruled out by current experimental limits for large $\delta_{13}^{\tilde{f}}$

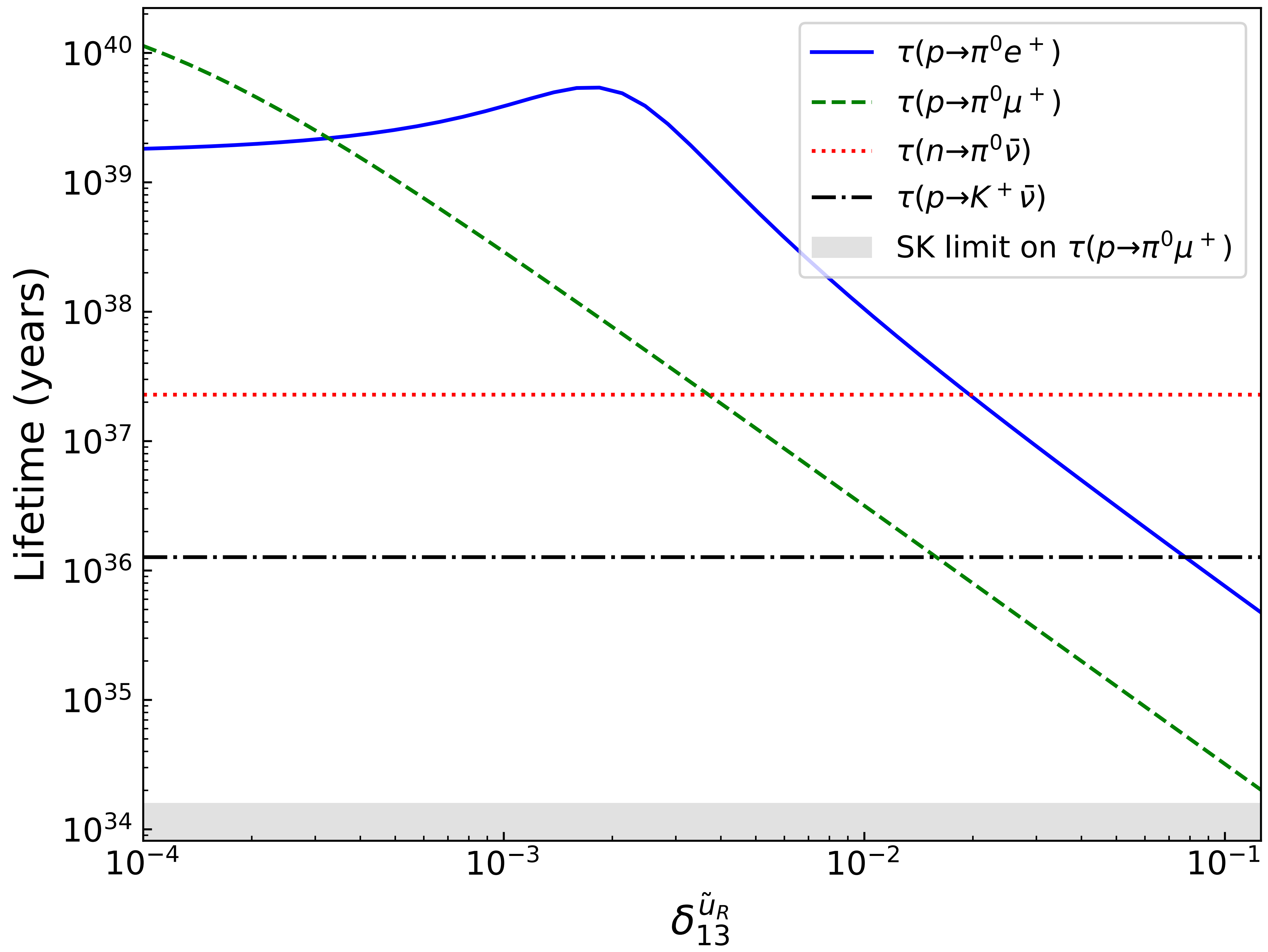
→ can readily be avoided if M_{SUSY} or M_{H_C} is larger

$$M_X = 10^{17}\ \text{GeV},\ M_{H_C} = 10^{16}\ \text{GeV},\ \mu_H = 200\ \text{GeV},$$

$$M_1 = 5\ \text{TeV},\ M_2 = 1\ \text{TeV},\ M_3 = 10\ \text{TeV},\ M_{\mathrm{SUSY}} = 100\ \text{TeV},\ \tan\beta = 3$$

$$\widetilde{m}^2_{\tilde{f}} = M^2_{\mathrm{SUSY}} \begin{pmatrix} 1 & \delta_{12}^{\tilde{f}} & \delta_{13}^{\tilde{f}} \\ \delta_{12}^{\tilde{f}*} & 1 & \delta_{23}^{\tilde{f}} \\ \delta_{13}^{\tilde{f}*} & \delta_{23}^{\tilde{f}*} & 1 \end{pmatrix}$$

Sfermion flavor violation



In the presence of flavor violation,

- various channels could be accessible by upcoming experiments, or
- even ruled out by current experimental limits for large $\delta_{13}^{\tilde{f}}$

→ can readily be avoided if $M_{\rm SUSY}$ or M_{H_C} is larger

$$\widetilde{m}^2_{\tilde{f}} = M^2_{\rm SUSY}\begin{pmatrix} 1 & \delta_{12}^{\tilde{f}} & \delta_{13}^{\tilde{f}} \\ \delta_{12}^{\tilde{f}*} & 1 & \delta_{23}^{\tilde{f}} \\ \delta_{13}^{\tilde{f}*} & \delta_{23}^{\tilde{f}*} & 1 \end{pmatrix}$$

$M_X = 10^{17}$ GeV, $M_{H_C} = 10^{16}$ GeV, $\mu_H = 200$ GeV,

$M_1 = 5$ TeV, $M_2 = 1$ TeV, $M_3 = 10$ TeV, $M_{\rm SUSY} = 100$ TeV, $\tan\beta = 3$

Uncertainties in hadron matrix elements

- As we have seen in Aoki san's talk, the uncertainties in the nucleon decay matrix elements obtained by the lattice QCD calculation are currently $\sim 10\,\%$.
- **We expect the uncertainties to be significantly reduced if, instead of the matrix elements themselves, the ratios of them are estimated directly by the calculation.** This is because each matrix element is not independent, but correlated with each other.
- At leading order in the chiral perturbation,

$$W^{LR}_{p\pi^+\nu,0} = \frac{1+D+F}{f}\alpha \ , \qquad W^{LR}_{p\pi^+\nu,1} = -\frac{2(D+F)}{f}\alpha \ ,$$
$$W^{LL}_{p\pi^+\nu,0} = \frac{1+D+F}{f}\beta \ , \qquad W^{LL}_{p\pi^+\nu,1} = -\frac{2(D+F)}{f}\beta \ ,$$
$$W^{LR}_{p\eta\ell,0} = -\frac{1+D-3F}{\sqrt{6}f}\alpha \ , \qquad W^{LR}_{p\eta\ell,1} = \frac{2(D-3F)}{\sqrt{6}f}\alpha \ ,$$
$$W^{LL}_{p\eta\ell,0} = \frac{3-D+3F}{\sqrt{6}f}\beta \ , \qquad W^{LL}_{p\eta\ell,1} = \frac{2(D-3F)}{\sqrt{6}f}\beta \ ,$$

$$\langle 0|\epsilon_{abc}(u^a_R d^b_R)u^c_L|p\rangle \equiv \alpha P_L u_p \ ,$$
$$\langle 0|\epsilon_{abc}(u^a_L d^b_L)u^c_L|p\rangle \equiv \beta P_L u_p \ ,$$

$$\alpha = -0.01257(111)\ \mathrm{GeV}^3 \ ,$$
$$\beta = 0.01269(107)\ \mathrm{GeV}^3 \ .$$

$\alpha \simeq -\beta$ to $\sim 1\,\%$

If we take their ratios, f, α, β will cancel out.

Comments on $W^{\chi\chi'}_{p\eta\ell,1}$

- In previous lattice simulations, the values of $W^{\chi\chi'}_{p\eta\ell,1}(0)$ were not estimated. However, the following combination is evaluated in Y. Aoki et al. (2017).

$$W^{\chi\chi'}_{p\eta\ell,\mu} \equiv W^{\chi\chi'}_{p\eta\ell,0} + \frac{m_\mu}{m_N} W^{\chi\chi'}_{p\eta\ell,1}$$

- We extract $W^{\chi\chi'}_{p\eta\ell,1}(0)$ from $W^{\chi\chi'}_{p\eta\ell,\mu}(m_\mu^2)$ by using $W^{\chi\chi'}_{p\eta\ell,0}(0)$, neglecting m_μ^2 dependence.