When do two QFTs admit an interface between them?

YITP-IAS Workshop: Interfaces & Symmetry

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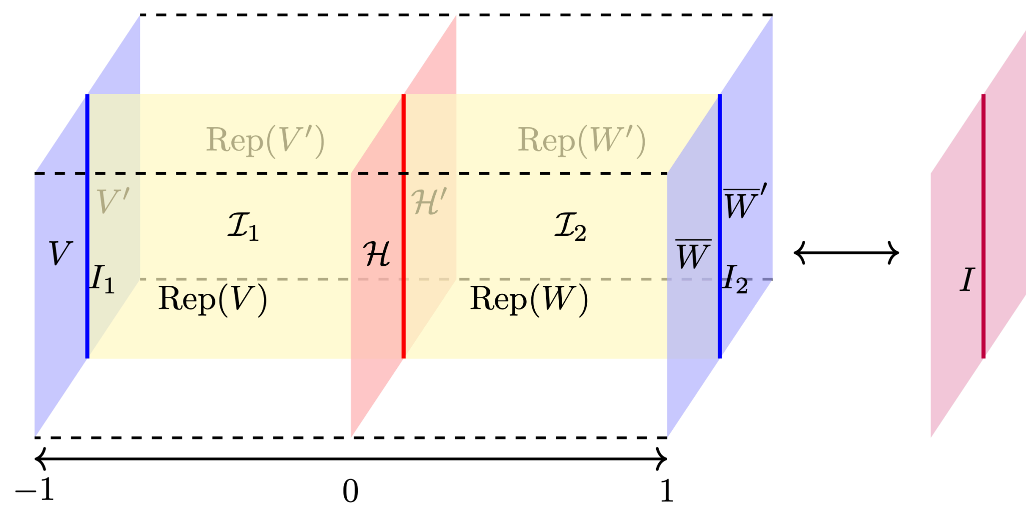
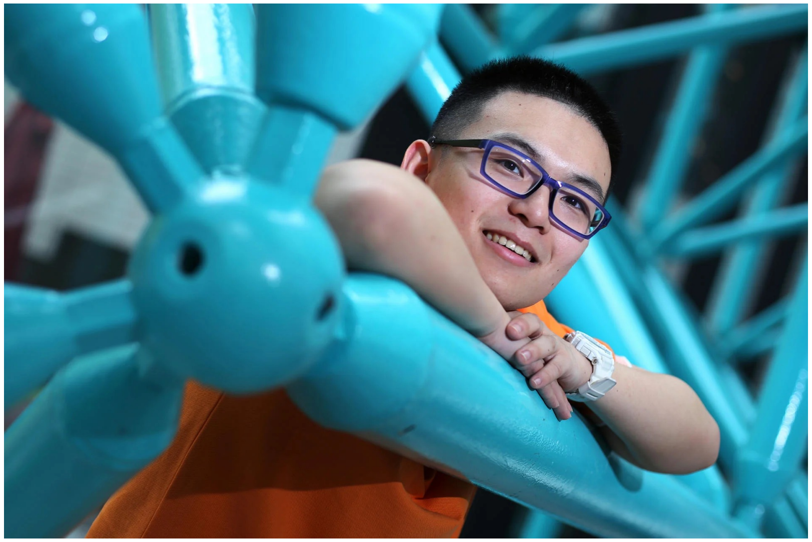


Brandon Rayhaun | Institute for Advanced Study | March 2, 2026

Collaborators



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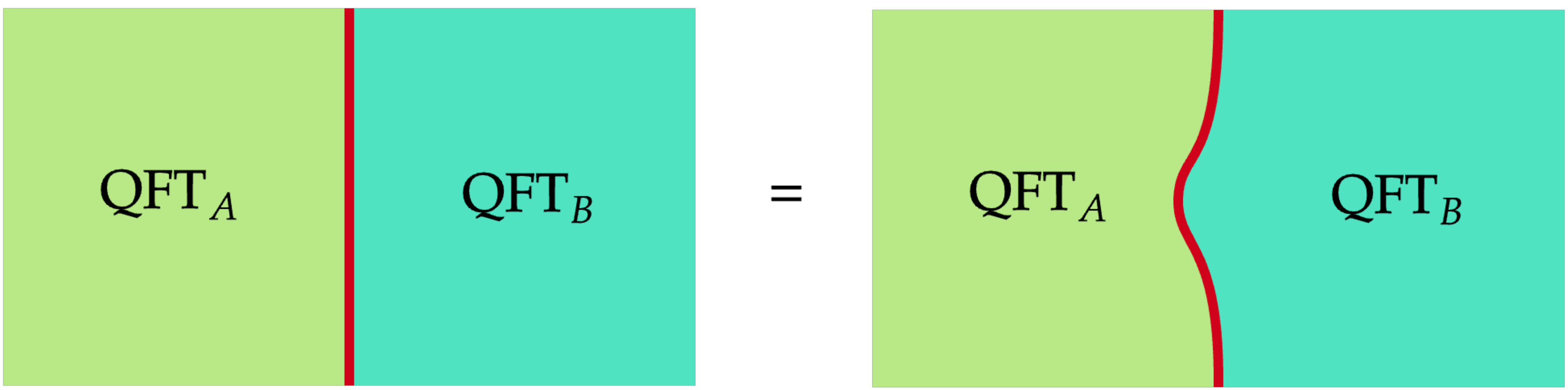
To simplify the question posed in the title…

1. Work in low dimensions (2D CFTs, 3D TQFTs)
2. Require the interface to be topological



Topological interfaces and orbifolding

General expectation:

Existence of a topological interface between QFT_1 and QFT_2

$\Leftrightarrow$

QFT_1 can be obtained from gauging QFT_2 and vice versa.

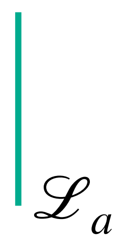
For 3D TQFTs, this is a theorem [Runkel-Mulevicius, Mulevicius].

Lightning review: generalized symmetries in 2D QFTs

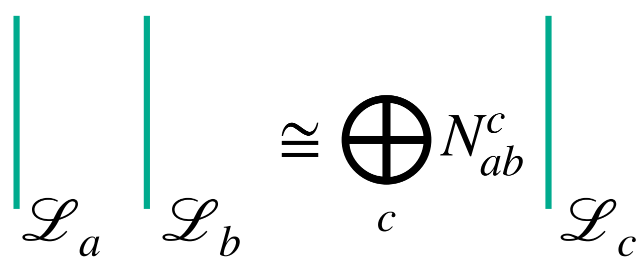
In 2D QFTs, generalized symmetries are implemented by topological lines and junctions between them.

Finite symmetries are described by fusion categories $(\{\mathscr{L}_a\}, N_{ab}^c, F)$:

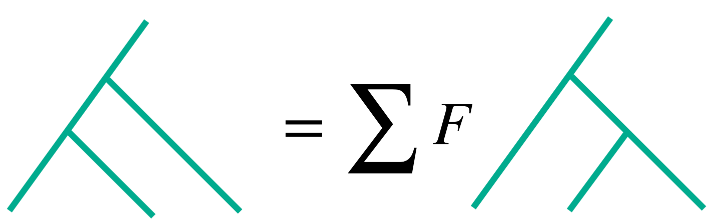
Simple topological lines:



Fusion rules:

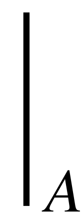


F-symbols:

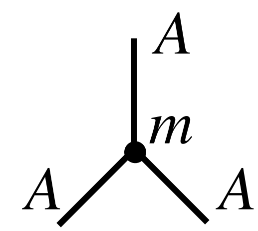


Orbifolds are implemented by algebra objects (A, m, u):

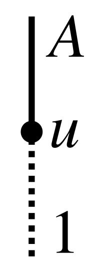
Topological line:



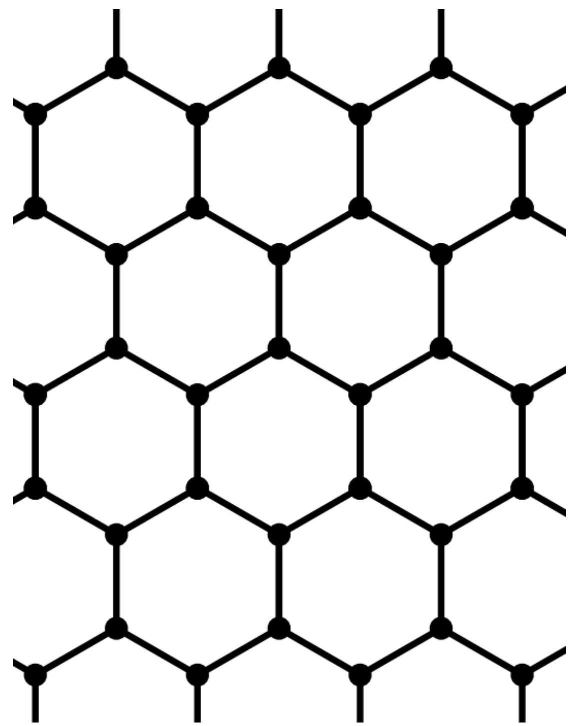
Multiplication junction:



Unit junction:



Gauging implemented by inserting a fine-enough mesh of A in spacetime



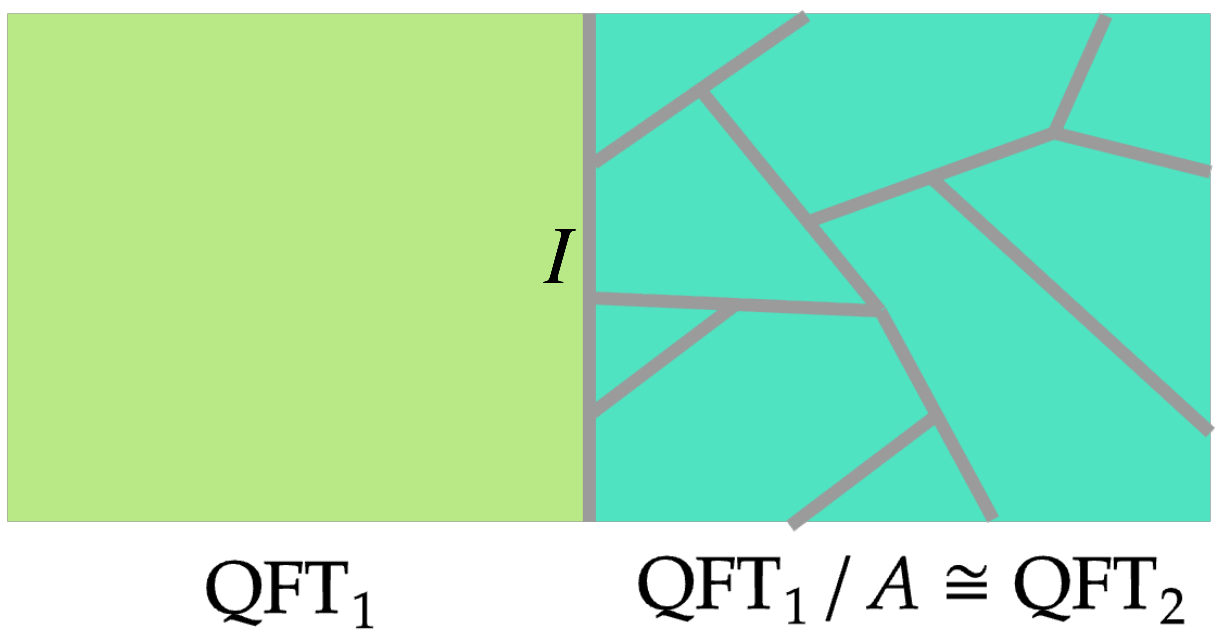
Topological interfaces and orbifolding in 2D

Orbifold $\Longrightarrow$ Interface

Suppose QFT_1 has a fusion category $\mathscr{C}$ of topological lines and an algebra object A of finite quantum dimension such that

$$\mathrm{QFT}_2 \cong \mathrm{QFT}_1 / A.$$

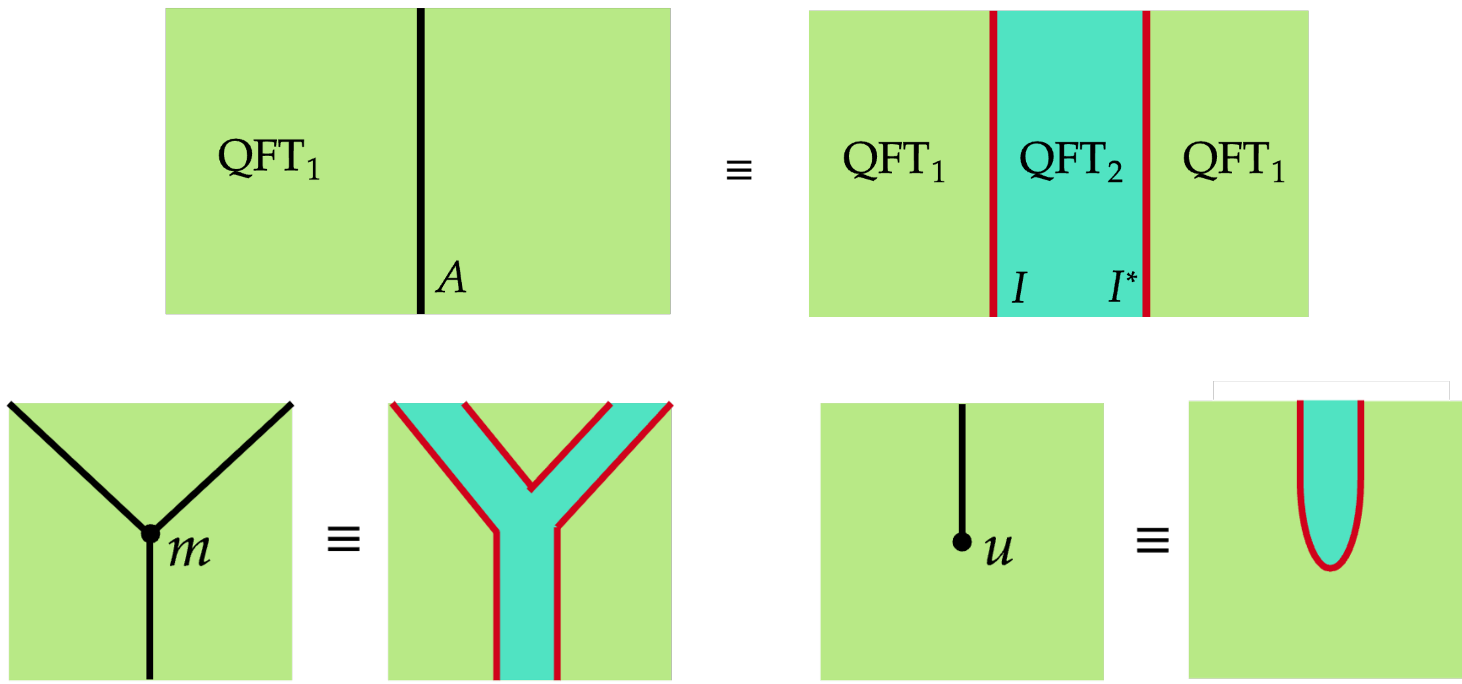
Half space gauging construction produces a topological interface of finite quantum dimension [..., Fröhlich-Fuchs-Runkel-Schweigert, ..., Choi-Cordova-Hsin-Lam-Shao, Kaidi-Ohmori-Zheng, ...]:



Topological interfaces and orbifolding in 2D

Interface $\Longrightarrow$ Orbifold

Given a topological interface I between QFT_1 and QFT_2, can define an algebra object (A, m, u) [Diatlyk-Luo-Wang-Weller]:



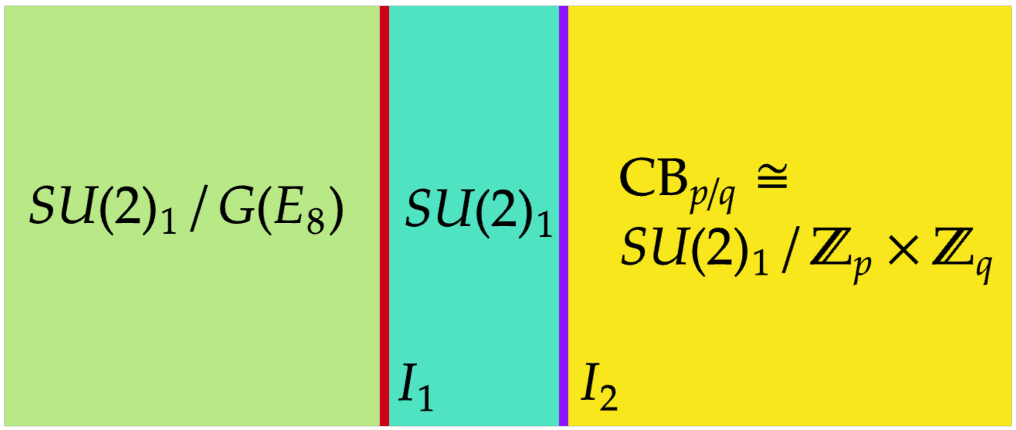
Claim: $\mathrm{QFT}_2 \cong \mathrm{QFT}_1/A$

Warning!

The algebra (A, m, u) obtained from a topological interface of finite quantum dimension is not guaranteed to live inside a *finite-rank* fusion category. [Möller-R]

Example:

1. Consider $SU(2)_1/G(E_8)$.
2. It has a topological interface to any compact boson theory with $R = p/q$. From the correspondence between interfaces and orbifolds, we obtain infinitely many algebra objects $A_{p/q}$ in $SU(2)_1/G(E_8)$.
3. Although $SU(2)_1/G(E_8)$ has infinitely many symmetries, every finite symmetry occurs inside $\mathscr{C}_{\mathrm{Ver}}$, the category of Verlinde lines (= lines which commute with chiral algebra).
4. A fusion category has only finitely many algebra objects, so the $A_{p/q}$ cannot all fit. Infinitely many must occur outside of $\mathscr{C}_{\mathrm{Ver}}$.



Some upshots

- While infinite Lie groups have infinitely many finite subgroups, infinite noninvertible symmetries may have only finitely many finite sub-symmetries.

- While gauging a finite invertible symmetry can always be done within a finite-rank fusion category of topological lines, gauging a finite non-invertible symmetry sometimes can only be done inside of an infinite-rank category of topological lines.

- The equivalence relation

$$\mathrm{QFT}_1 \sim \mathrm{QFT}_2 \Leftrightarrow \mathrm{QFT}_1/A \cong \mathrm{QFT}_2 \text{ for some } A$$

  is only an equivalence relation if one allows algebras A which do not sit inside any finite-rank fusion category symmetry of QFT_1.

When can two 2D CFTs be related by orbifolding fusion category symmetries?

Assume CFT_1 and CFT_2 are both *rational* to make life simpler.*

Let TQFT_i be the 3D topological order which hosts the maximal (say, left-moving) chiral algebra of CFT_i on its boundary. (E.g. for a WZW model, the TQFT is Chern-Simons theory.)

- One obvious necessary condition:

(1) $$c_L(\mathrm{CFT}_1) = c_L(\mathrm{CFT}_2) \text{ and } c_R(\mathrm{CFT}_1) = c_R(\mathrm{CFT}_2)$$

- Claim [Möller-R]: If CFT_1 and CFT_2 are related by orbifolding (a sequence of) fusion category symmetries, then TQFT_1 is related by orbifolding/ interfaces to TQFT_2. Mathematically, the MTCs underlying the two TQFTs are Witt equivalent:

(2) $$\mathrm{TQFT}_1 \sim_{\mathrm{Witt}} \mathrm{TQFT}_2$$

- Conjecture: (1) and (2) are sufficient to guarantee that CFT_1 and CFT_2 are related by orbifolding fusion category symmetries.

* Although we assume rationality, we do not require that the symmetries we orbifold commute with the maximal chiral algebra.

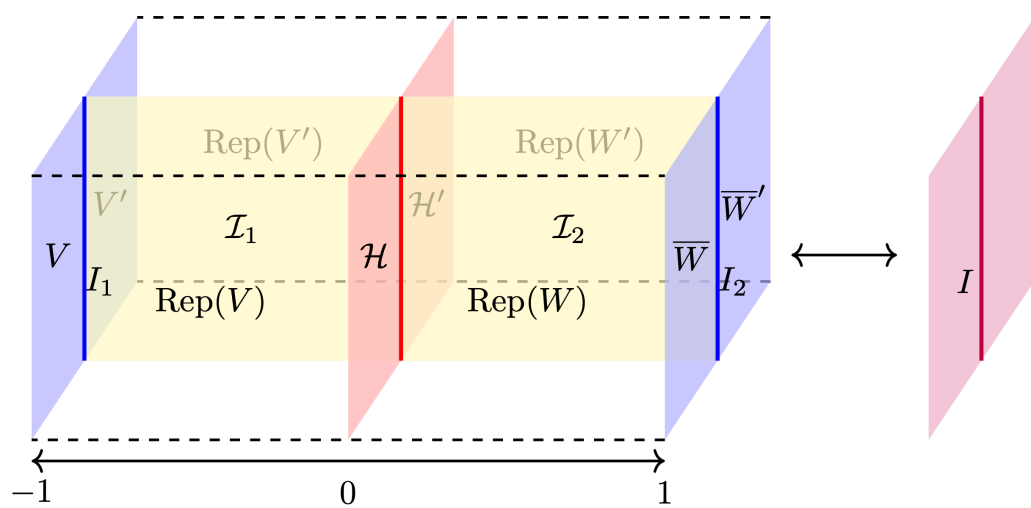
Quick example

$\mathrm{CFT}_1 \equiv SU(2)_1$ WZW model $\Longrightarrow \mathrm{TQFT}_1 = SU(2)_1$ Chern-Simons.

$\mathrm{CFT}_2 \equiv$ bosonized Dirac fermion $\Longrightarrow \mathrm{TQFT}_2 = U(1)_4$ Chern-Simons.

$U(1)_4$ and $SU(2)_1$ TQFTs do not admit a gapped interface between them. Thus, impossible to reach CFT_2 from CFT_1 by iteratively orbifolding fusion category symmetries.

"Proof":



So, finite orbifolding is not enough to completely traverse the conformal manifold. What about infinite gauging?

The usual infinite gauging takes you off the conformal manifold. E.g. gauging the $SO(3)$ symmetry of the $SU(2)_1$ WZW model gaps the theory. Not what we want.

Instead, one should "flat gauge". E.g. in the case of gauging continuous invertible symmetries, only sum over flat connections in the path integral.

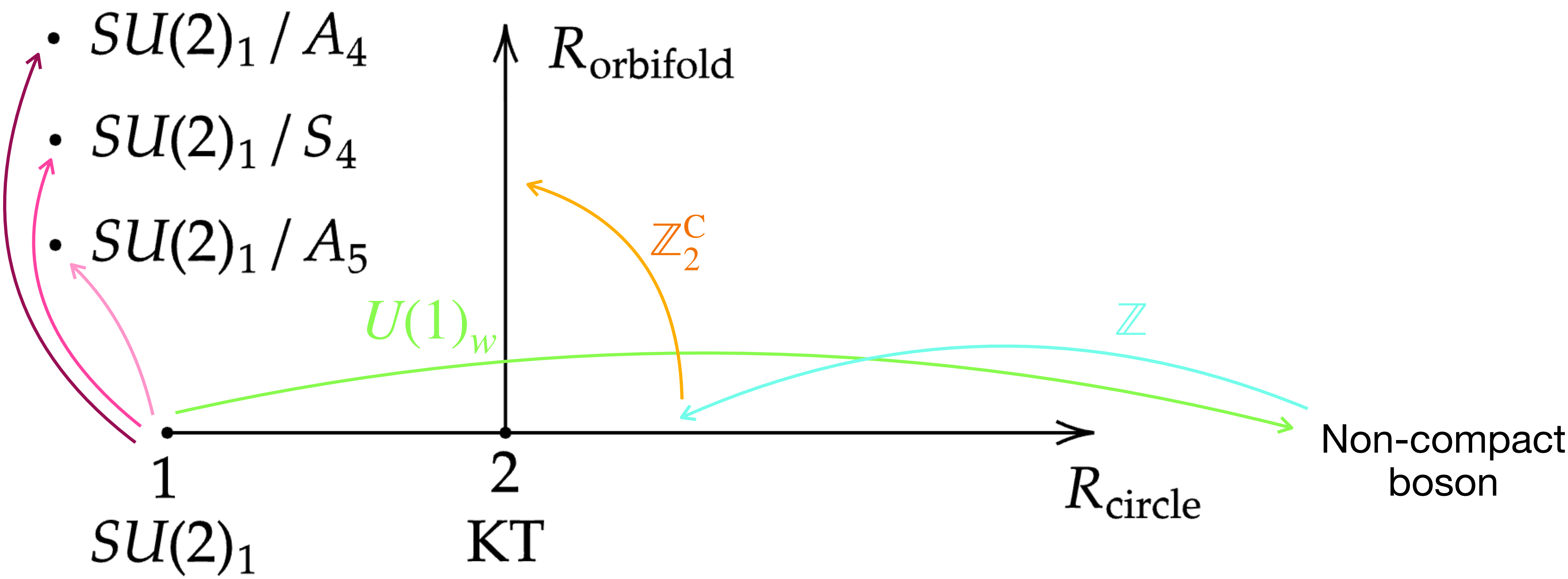
Speculation (effective gauging hypothesis): Any two 2D CFTs with the same central charges are related by (possibly infinite) flat gauging?

Motivation 1: a rational analog

Let V be a *rational* chiral algebra. Any two CFTs containing V as part of their chiral algebra are related by gauging Verlinde lines, i.e. the category of topological lines which commute with V. [Fuchs-Runkel-Schweigert]

Formally taking V to be the Virasoro algebra recovers the speculation.

Motivation 2: experimental evidence at $c = 1$ [..., Lam, Choi-Lam-R]



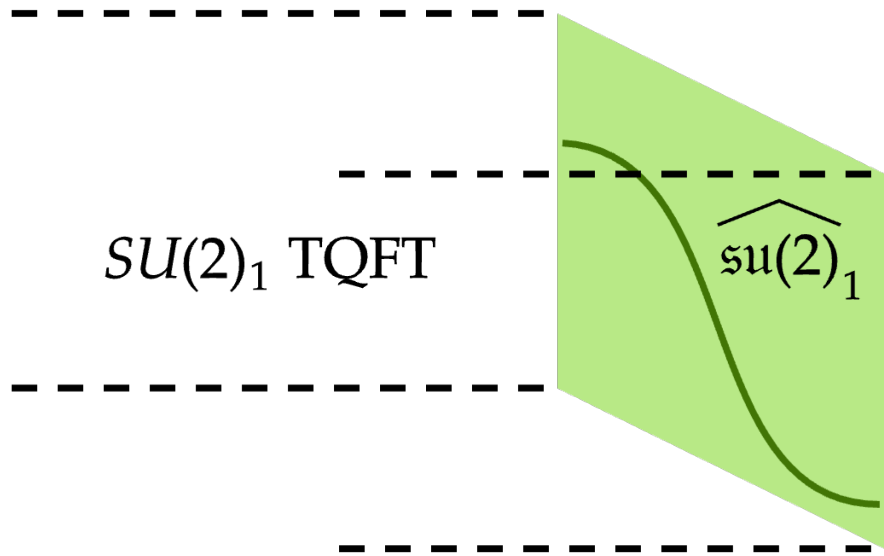
An eventual aspiration would be to prove the effective gauging hypothesis and use it to classify CFTs at a fixed central charge.

At general c, this likely won't simplify things much, because generic $c \geq 1$ CFTs have horrible noninvertible categories of topological line operators with infinite quantum dimension.

However, at $c = 1$ a small miracle happens…

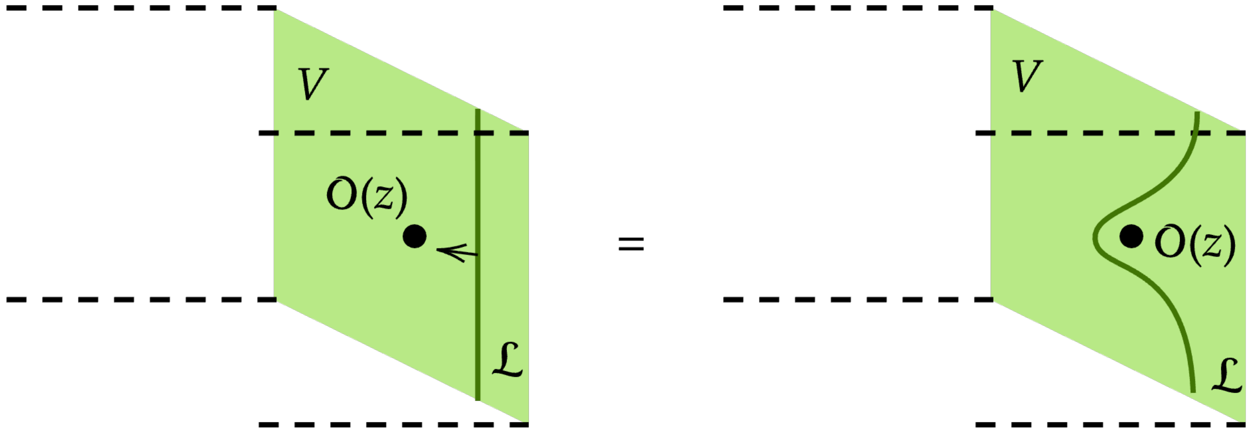
$SU(2)_1$ WZW model has no noninvertible symmetries

First, solve easier problem: determine the topological lines of the $\widehat{\mathfrak{su}(2)}_1$ chiral algebra



We know that we have at least an $SU(2)$ worth of lines coming from exponentiating the spin-1 Noether currents. Are there more?

Let $\mathscr{C}$ be a category of unitary topological line operators on a unitary chiral algebra V. Define $V^{\mathscr{C}}$ to be the local operators in V which are transparent to all lines $\mathscr{L} \in \mathscr{C}$:



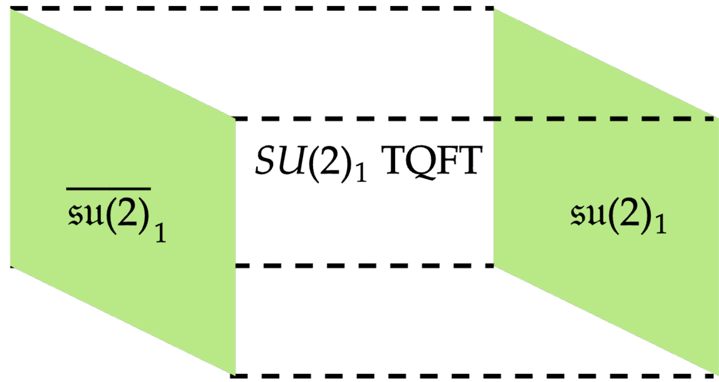
Claim [Gannon-R] (from reinterpreting theorems about conformal nets [Bischoff-Vecchio-Giorgetti]): Suppose that $\mathscr{C}$ contains all the lines coming from the bulk. Then $\mathscr{C}$ is the full category of unitary topological line operators of V if and only if

$$V^{\mathscr{C}} = \mathrm{Vir}_{c(V)}.$$

For the case $V = \widehat{\mathfrak{su}(2)}_1$, one can calculate that $V^{SU(2)} = \mathrm{Vir}_{c=1}$. Thus, no additional lines.

Gluing left- and right-moving symmetries

The full $SU(2)_1$ WZW model is obtained by compactifying $SU(2)_1$ Chern-Simons theory on an interval [Kapustin-Saulina]:



Claim [Gannon-R]: The full category of unitary topological lines of the diagonal CFT built on a rational chiral algebra V is given by the relative Deligne product

$$\mathscr{C} \boxtimes_{\mathscr{B}} \mathscr{C}^{\text{op}}$$

where $\mathscr{B}$ is the category of lines in the bulk.

When $\mathscr{C} \sim G$ and $\mathscr{B} \sim H < G$ are invertible, then the relative Deligne product reduces to the amalgamated product,

$$G \times G/\Delta(H).$$

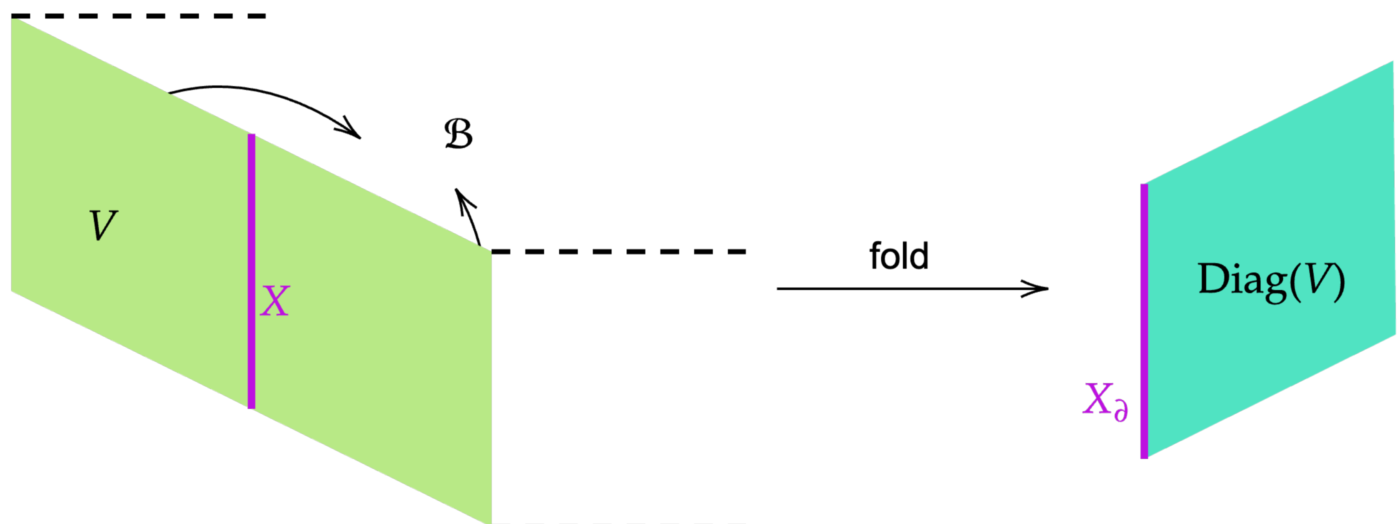
So the full category of lines of $SU(2)_1$ WZW is

$$SO(4) \cong SU(2)_L \times SU(2)_R/\mathbb{Z}_2.$$

Aside on boundary conditions

Studying topological lines of a chiral algebra V has other interesting corollaries.

By folding, they are the same as boundary conditions of the diagonal CFT built on V!



In $SU(2)_1$ WZW, proves that boundaries are one-to-one with elements of $SU(2)$.

Interesting general consequence: $g_{X_\partial} = \dim(X) g_1 \geq g_1$, so the identity Cardy boundary condition has the lowest g-function amongst all boundary conditions.

Anyways, the effective gauging hypothesis + the fact that the full symmetry category of $SU(2)_1$ WZW model is $SO(4)$ leads to the intriguing possibility that $c = 1$ CFTs correspond to ways of flat gauging $SO(4)$.

For finite groups G, the possible ways of gauging are in correspondence with pairs (H, ψ), with H a non-anomalous subgroup of G and $\psi \in H^2(H, U(1))$ a choice of discrete torsion. Perhaps something similar is true even when G is infinite.

Indeed, as we'll see shortly, flat gauging infinite symmetries is qualitatively very similar to gauging finite symmetries, at least when the symmetry is invertible.

Solving CFTs from interfaces/orbifolds

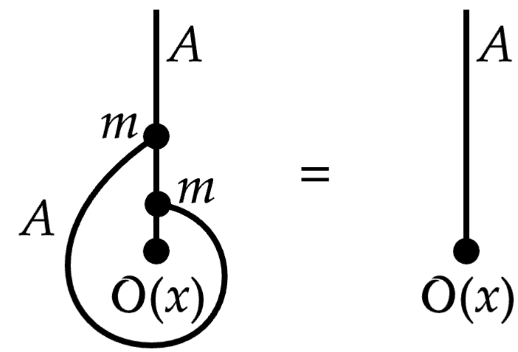
Suppose that CFT_1 and CFT_2 admit a topological interface between them. Equivalently, suppose that $\mathrm{CFT}_2 \cong \mathrm{CFT}_1/A$.

An orbifold/interface relation allows you to transfer all of your knowledge about CFT_1 to CFT_2.

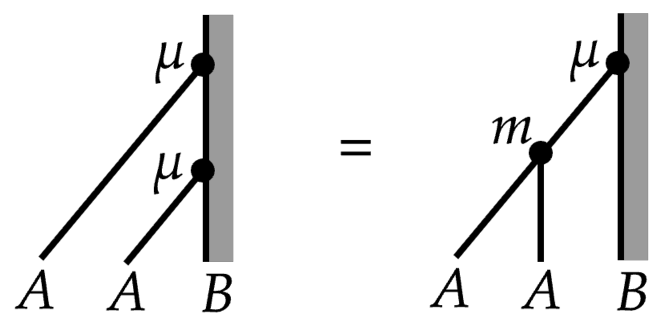
Solving CFTs from interfaces/orbifolds

Slogan: *Objects in CFT_2 are objects in CFT_1 on which the mesh of algebra A can consistently end.*

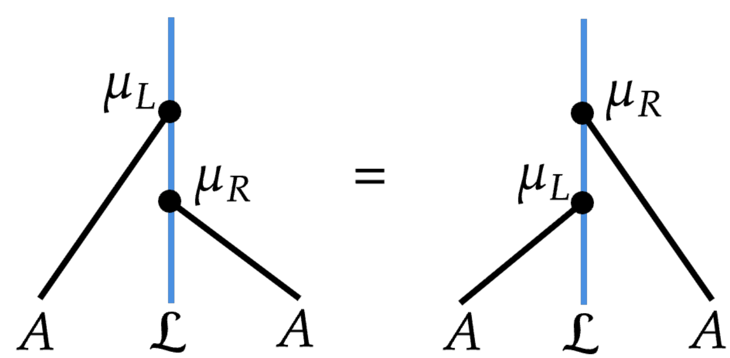
Local operators in CFT_2 are A-twisted local operators in CFT_1 invariant under lassos:



Boundary conditions in CFT_2 are pairs (B, μ) with B a boundary in CFT_1 and μ a junction specifying how the mesh ends. (A-module)



Topological lines in CFT_2 are triples $(\mathscr{L}, \mu_L, \mu_R)$ with $\mathscr{L}$ a line in CFT_1 and μ_L, μ_R junctions specifying how the mesh ends from the left and right. (A-A-bimodule)



“Quantum” symmetries of QFTs obtained from invertible gauging

Suppose CFT_1 has symmetry group G with ’t Hooft anomaly $\omega \in H^3(G, U(1))$, and CFT_2 is obtained by gauging a non-anomalous subgroup $H < G$ with discrete torsion $\psi \in H^2(H, U(1))$,

$$\mathrm{CFT}_2 = \mathrm{CFT}_1 / A(H, \psi)$$

with $A(H, \psi) = \bigoplus_{h \in H} h$ and junction $m : A \otimes A \to A$ determined by ψ.

The “dual” symmetry category $\mathscr{C}(G, \omega, H, \psi)$ of CFT_2 has simple lines labeled by pairs [Ostrik]

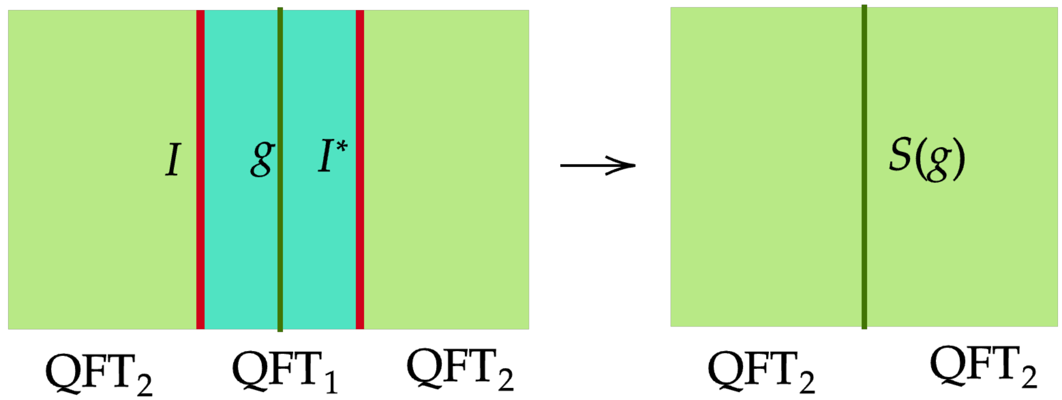
$$([g], \rho)$$

where $[g] \in H \backslash G / H$ and ρ is an irreducible projective representation of $H \cap gHg^{-1}$ with 2-cocycle ψ^g.

$$\psi^g(h, h') = \psi(h, h')\psi(g^{-1}h'^{-1}g, g^{-1}, h^{-1}g)\omega(hh'g, g^{-1}h'^{-1}g, g^{-1}h^{-1}g)^{-1}\omega(h, h', g)\omega(h, h'g, g^{-1}h'^{-1}g)$$

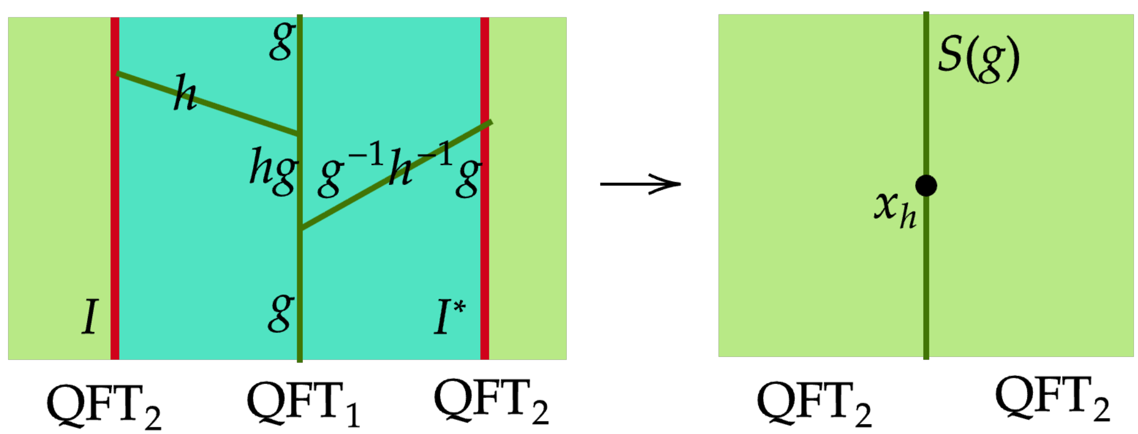
$\mathscr{C}(G, \omega, H, \psi)$ from sandwich functor

We can obtain lines in QFT_2 from half-space gauging interfaces:



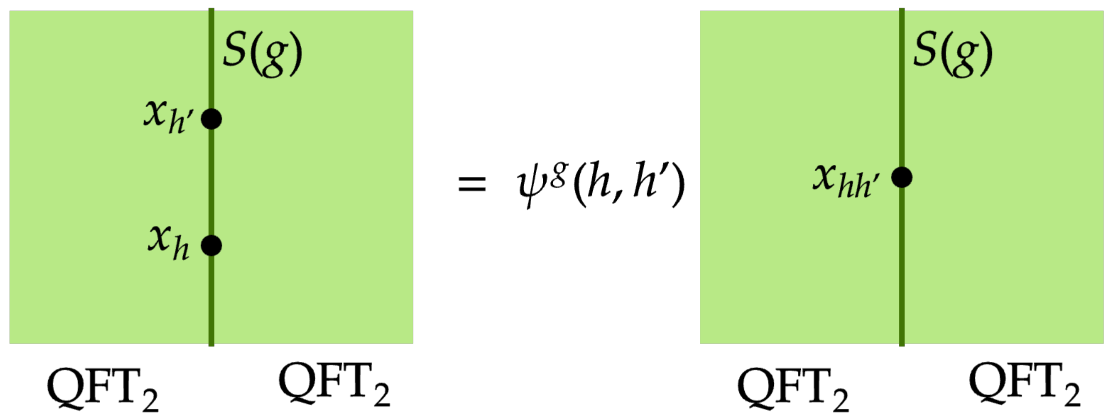
The line $S(g)$ only depends on the double coset HgH.

$S(g)$ may not be an elementary line: can construct topological point operator on it for any $h \in H \cap gHg^{-1}$.



$\mathscr{C}(G, \omega, H, \psi)$ from sandwich functor

It is straightforward to calculate the fusion of these point operators:



This precisely means that $S(g)$ decomposes into elementary lines as

$$S(g) \cong \bigoplus_{\rho} \dim(\rho) \cdot S^{\rho}(g)$$

where the direct sum is over projective irreps with 2-cocycle ψ^g. So we recover Ostrik's calculation of the elementary lines in $\mathscr{C}(G, \omega, H, \psi)$.

An infinite example

Every known $c = 1$ CFT is obtained by gauging $SU(2)_1$ WZW model, so this technology allows for the complete classification of topological lines in all $c = 1$ theories. [Choi-Lam-R, Gannon-R]

Take $\mathrm{CFT}_1 = SU(2)_1$ WZW model with
$G = SO(4) \cong SU(2)_L \times SU(2)_R / \mathbb{Z}_2$.

Gaberdiel and Suchanek gave evidence that flat gauging the diagonal $H = SO(3)$ subgroup (with $\psi = 1$) yields a $c \to 1$ limit of Liouville theory sometimes called the Runkel-Watts theory (RW).

Spiritually, RW is a diagonal CFT built on the $c = 1$ Virasoro algebra. Thus, topological lines corresponds to irreps of $\mathrm{Vir}_{c=1}$:

ZZ topological lines: $\mathscr{L}_{p^2}$ with $p \in \frac{1}{2}\mathbb{Z}_{\geq 0}$

FZZT topological lines: $\mathscr{L}_{p^2}$ with $p \in \mathbb{R}_{\geq 0} \,/\, \frac{1}{2}\mathbb{Z}_{\geq 0}$

Runkel-Watts lines from $SO(3)$ flat gauging

Ostrik/sandwiches tell us that lines $S^{\rho}(g)$ in Runkel-Watts are parametrized by $[g] \in SO(3)\backslash SO(4)/SO(3)$ and ρ a (ψ^{g}-projective) irrep of $SO(3) \cap gSO(3)g^{-1}$.

Can take $g_{\phi} = \left(\begin{pmatrix} e^{i\phi} & 0 \\ 0 & e^{-i\phi} \end{pmatrix}, 1 \right)$ with $\phi \in [0,\pi]$ as representatives of double cosets.

$$SO(3) \cap g_{\phi} SO(3) g_{\phi}^{-1} = \begin{cases} SO(3), & \text{if } \phi = 0,\pi \\ SO(2), & \text{if } \phi \neq 0,\pi \end{cases}$$

$\psi^{g_{\phi}} = 1$ unless $\phi = \pi$, in which case it is the non-trivial class in $H^2(SO(3), U(1)) \cong \mathbb{Z}_2$.

Thus, lines of orbifolded theory are: Match onto lines of RW as:

$S^{j}(g_{\phi=0})$ for $j \in \mathbb{Z}_{\geq 0}$ $\longrightarrow$ $\mathcal{L}_{j^2}$

$S^{j}(g_{\phi=\pi})$ for $j \in \mathbb{Z}_{\geq 0} + 1/2$ $\longrightarrow$ $\mathcal{L}_{j^2}$

$S^{n}(g_{\phi\neq 0,\pi})$ for $n \in \mathbb{Z}$ $\longrightarrow$ $\mathcal{L}_{(\phi/2\pi+n)^2}$

Summary

- There is a tight relationship between interfaces and orbifolds.
- When restricting to finite orbifolds/interfaces of finite quantum dimension, there can be obstructions to connecting theories.
- There is some evidence that upgrading to infinite flat gauging/interfaces of infinite quantum dimension makes these obstructions go away. (Effective gauging hypothesis.) This would rephrase the classification of 2D CFTs in the language of global symmetries.
- When there exists a topological interface/orbifold relation between two CFTs, one can solve many features of one in terms of the corresponding features of the other (spectrum of local operators, boundaries, topological lines, …). The infinite case seems to behave qualitatively similarly to the finite case.
- These observations are particularly powerful at $c = 1$ because the $SU(2)_1$ WZW model has only invertible symmetries. Thus, it suggests that the classification and complete solution of $c = 1$ CFTs may ultimately be expressible in group theoretical language.
    - E.g. classification of all topological lines and boundaries in the known $c = 1$ theories is work in progress with Terry Gannon.

ありがとうございます!