Analytic bootstrap for holographic surface defects

Based on work in progress with D. Bonomi, M. Ferragatta, V. Forini.
Related to some older results with D. Bonomi, E. De Sabbata and A. Gimenez-Grau
(2205.09775,2212.02524,2312.05221)

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March 2^{nd}, 2026
"Interfaces & Symmetry"

Context and motivation

General idea

Use symmetries and consistency of a defect conformal field theory to obtain results for the quantum gravitational interaction of gravitons and branes in AdS.

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- The context is that of conformal defects, i.e. extended probes of a conformal field theory.
- Many interesting examples: Wilson and 't Hooft lines, boundaries, interfaces, twist operator, surface defects, ...
- They probe phases of the theory that cannot be probed by local operators.

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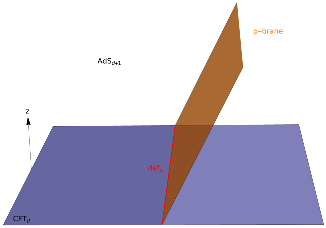
Use symmetries and consistency of a defect conformal field theory to obtain results for the quantum gravitational interaction of gravitons and branes in AdS.

- The context is that of conformal defects, i.e. extended probes of a conformal field theory.
- Many interesting examples: Wilson and 't Hooft lines, boundaries, interfaces, twist operator, surface defects, ...
- They probe phases of the theory that cannot be probed by local operators.
- Through holography, they are related to branes and we can use them to study extended objects in quantum gravity.
- For surface defects holography actually computes some exact results.



Outline

1. The observables
   - Defect conformal field theories
   - Defect CFT data
2. The tools
   - Inversion formulae
   - Dispersion relations
3. Applications
   - Surface defects in $\mathcal{N} = 4$ SYM
4. Results

Section 1

The observables

Defect conformal field theories

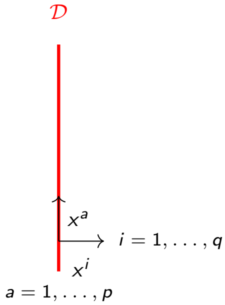
- In a d-dimensional CFT a conformal defect is supported on a p-dimensional submanifold (codimension $q = d - p$).
- For a planar defect we have

$$x^\mu = (x_\parallel^a, x_\perp^i),$$

with the defect localized at $x_\perp = 0$.

- The global conformal symmetry is reduced as

$$SO(d+1,1) \longrightarrow SO(p+1,1) \times SO(q),$$



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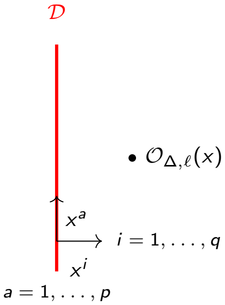

- Bulk operators $\mathcal{O}_{\Delta,\ell}$ with dimension Δ and spin ℓ and descendants $\partial_{\mu_1} \ldots \partial_{\mu_n} \mathcal{O}_{\Delta,\ell}$

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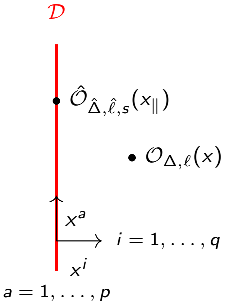
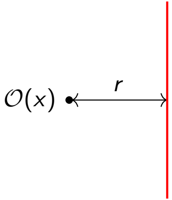


- Bulk operators $\mathcal{O}_{\Delta,\ell}$ with dimension Δ and spin ℓ and descendants $\partial_{\mu_1} \ldots \partial_{\mu_n} \mathcal{O}_{\Delta,\ell}$
- Defect operators $\hat{\mathcal{O}}_{\hat{\Delta},\hat{\ell},s}$ with dimension $\hat{\Delta}$, parallel spin $\hat{\ell}$ and orthogonal spin s and descendants $\partial_{a_1} \ldots \partial_{a_n} \hat{\mathcal{O}}_{\hat{\Delta},\hat{\ell},s}$.

DCFT [Billò, Goncalves, Lauria, Meineri, 2016]



- One-point functions

$$\langle \mathcal{O}(x) \rangle_D \equiv \frac{\langle \mathcal{O}(x) D \rangle}{\langle D \rangle} = \frac{a_{\mathcal{O}}}{r^{\Delta}}$$

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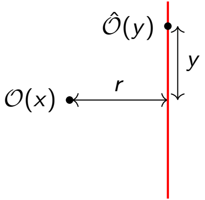

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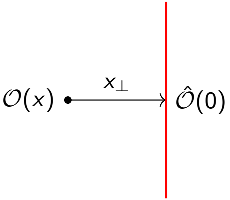

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$$\mathcal{O}(x) = \sum_{\text{def. prim.}} b_{\mathcal{O}\hat{\mathcal{O}}} |x_\perp|^{\hat{\Delta} - \Delta} \left(\hat{\mathcal{O}}(0) \underbrace{+ \text{def. desc.}}_{\text{all fixed}} \right)$$

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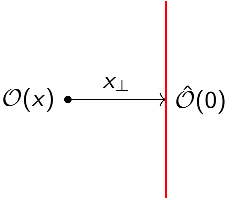

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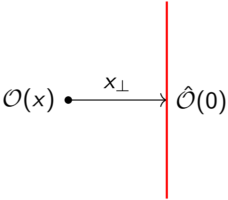

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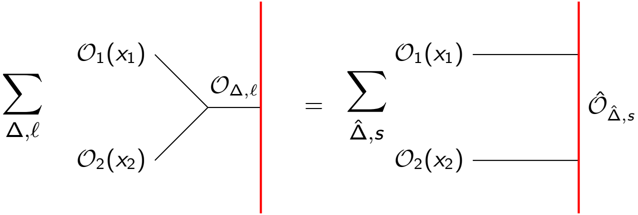
- The naive set of defect CFT data is $\{a_{\mathcal{O}}, b_{\mathcal{O}\hat{\mathcal{O}}}, \hat{\Delta}_{\hat{\mathcal{O}}}, \hat{c}_{\hat{\mathcal{O}}_1 \hat{\mathcal{O}}_2 \hat{\mathcal{O}}_3}\}$.

DCFT

- Defect crossing



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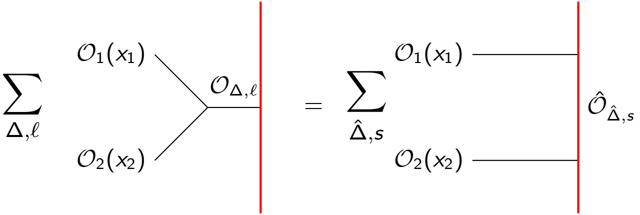


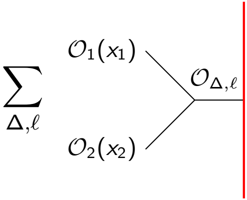
- Main player: bulk two-point function

$$\langle \phi(x)\phi(y)\rangle_D = \frac{F(z,\bar{z})}{|x_\perp|^{\Delta_\phi}|y_\perp|^{\Delta_\phi}}$$

with cross-ratios $\frac{(x-y)^2}{|x_\perp||y_\perp|} = \frac{(1-z)(1-\bar{z})}{\sqrt{z\bar{z}}}$, $\frac{x\cdot y}{|x_\perp||y_\perp|} = \frac{z+\bar{z}}{2\sqrt{z\bar{z}}}$.

OPE channels

Bulk channel

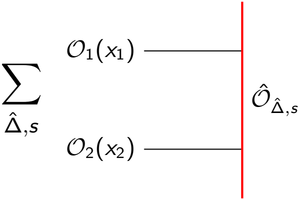


$$= \left(\frac{\sqrt{z\bar{z}}}{(1-z)(1-\bar{z})}\right)^{\Delta_\phi} \sum_{\Delta,\ell} \lambda_{\phi\phi\mathcal{O}}\, a_{\mathcal{O}}\, f_{\Delta,\ell}(z,\bar{z})$$

$\downarrow$

bulk conformal blocks
fixed by conformal invariance

Defect channel



$$= \sum_{\hat{\Delta},s} b^2_{\phi\hat{\mathcal{O}}}\, \hat{f}_{\hat{\Delta},s}(z,\bar{z})$$

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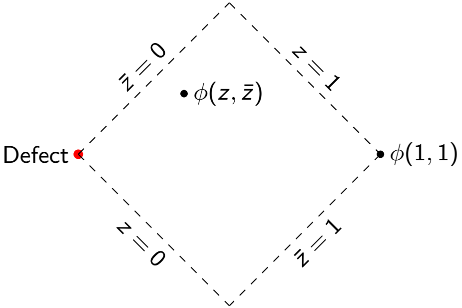
Correlator in Lorentzian



$z, \bar{z} \to 0$ defect OPE
$z, \bar{z} \to 1$ bulk OPE
$z \to 1$ or $\bar{z} \to 1$ bulk lightcone OPE
$z \to 0$ or $\bar{z} \to 0$ defect lightcone OPE
$\bar{z} \to 0$ and $z \to 1$ double lightcone limit

Bulk conformal blocks in the lightcone limit $\bar{z} \to 1$

$$f_{\Delta,\ell}(z,\bar{z}) = (1-\bar{z})^{\frac{\Delta-\ell}{2}} f_{\Delta+\ell}(z) + \dots$$

Section 2

The tools

Inversion formulae

General idea

Extract the CFT data or reconstruct the full correlator starting from its discontinuity.

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Extract the CFT data or reconstruct the full correlator starting from its discontinuity.

- Old idea for scattering matrices imported from optics [Kramers, Kronig, 1926].
- More recently, a Lorenztian inversion formula and a dispersion relation have been derived for CFT four-point function. They involve a double discontinuity (dDisc)[Caron-Huot, 2017; Carmi, Caron-Huot, 2019] (there is also a dispersion relation with a single Disc [Bissi, Dey, Hansen, 2019]).
- For defects, we have two inversion formulae.

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Defect inversion formula

It extracts the defect CFT data from a discontinuity. [Lemos, Liendo, Meineri, Sarkar '17]

$$F(z,\bar{z}) = \sum_{\hat{\Delta},s} b^2_{\phi\hat{\mathcal{O}}} \hat{f}_{\hat{\Delta},s}(z,\bar{z}) \qquad \rightarrow \qquad b_{\hat{\Delta},s} = \int_0^1 d^2z\, K_{\hat{\Delta},s}(z,\bar{z})\, \mathrm{Disc}F(z,\bar{z})$$

$$\mathrm{Disc}F(z,\bar{z}) = F(z,\bar{z}+i\epsilon) - F(z,\bar{z}-i\epsilon) \qquad \bar{z} > 1$$

and $b_{\hat{\Delta},s}$ encodes the CFT data in its poles and residues.

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- For defects, we have two inversion formulae.

Bulk inversion formula

It extracts the bulk CFT data from a double discontinuity. [Liendo, Linke, Schomerus '19]

$$F(z,\bar{z}) \sim \sum_{\Delta,\ell} \lambda_{\phi\phi\mathcal{O}}\, a_{\mathcal{O}}\, f_{\Delta,\ell}(z,\bar{z}) \;\rightarrow\; c_{\Delta,\ell} = \int_0^1 d^2z\, K_{\Delta,\ell}(z,\bar{z})\,\mathrm{dDisc}\left[\left(\frac{(1-z)(1-\bar{z})}{\sqrt{z\bar{z}}}\right)^{\Delta_\phi} F(z,\bar{z})\right]$$

$$\mathrm{dDisc}_{\bar{z}=0} F(z,\bar{z}) = F(z,\bar{z}) - \frac{1}{2}F^{\circlearrowleft}(z,\bar{z}) - \frac{1}{2}F^{\circlearrowright}(z,\bar{z})$$

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Dispersion relations

Disc dispersion relation

It reconstructs the full correlator from the discontinuity [LB, Bonomi '22], [Barrat, Gimenez-Grau, Liendo '22]

$$F(r,w) = \int_0^r \frac{dw'}{2\pi i}\left(\frac{1}{w'-w} + \frac{1}{w'-\frac{1}{w}} - \frac{1}{w'}\right)\mathrm{Disc}F(r,w')$$

$$z = rw \qquad \bar{z} = \frac{r}{w}$$

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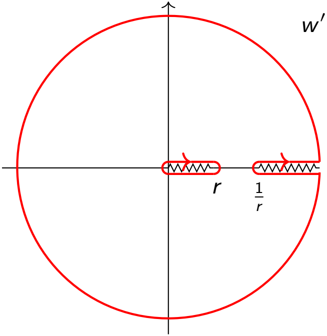
Why is this useful?

- The discontinuity is easier to compute than the full correlator (e.g. optical theorem)
- The discontinuity for $w'\to r$ (i.e. $\bar{z}\to 1$) is controlled by the bulk lightcone OPE.
- In perturbation theory around generalized free field theories, the Disc kills many bulk operators.

Low-spin ambiguities

- Both the Lorentzian inversion formula and the dispersion relation are derived by contour deformation arguments.
- Particularly simple for the dispersion relation



$w' \to r$ $\bar{z} \to 1$ bulk OPE

$w' \to 0$ $z \to 0$ and $\bar{z} \to \infty$, no OPE

- The cuts are related by the $w' \leftrightarrow 1/w'$ symmetry.
- To neglect the small and big circle we need to know the behaviour of the correlator for $w \to 0$. If it's not good enough we have ambiguities related to low s opeators in the defect channel.

Section 3

Applications

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Holography

Large-N factorization and sparse spectrum, very good control on the the Disc.

- Wilson lines in $\mathcal{N} = 4$ SYM [Barrat, Gimenez-Grau, Liendo, 2021].
- Surface defects in $\mathcal{N} = (2, 0)$ in 6d [Chen, Gimenez-Grau, Zhou, 2023].
- Surface defects in $\mathcal{N} = 4$ SYM [LB,Bonomi, Ferragatta, Forini, 2026].

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$O(N)$ model in ϵ expansion

At leading order one bulk operator has anomalous dimension and contributes to the Disc.

- Localized magnetic field in $O(N)$ model [LB, Bonomi, De Sabbata, 2022; Gimenez-Grau, 2022].
- Spin impurity in the $O(3)$ model [LB, Bonomi, De Sabbata, Gimenez-Grau, 2023].

$\mathcal{N} = 4$ Super Yang–Mills

Superconformal gauge theory with maximal supersymmetry in 4d.

- Conformal symmetry $SO(1,5)$

$$(P_\mu, M_{\mu\nu}, K_\mu, D)$$

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- Full superconformal symmetry

$$PSU(2,2|4)$$

Field content (all in adjoint of the gauge group)

$$A_\mu, \qquad \lambda^A_\alpha, \qquad \phi^I \ (I = 1, \dots, 6)$$

$\frac{1}{2}$ BPS Surface Defects in $\mathcal{N} = 4$ Super Yang–Mills

- Preserved supersymmetry: $\frac{1}{2}$ BPS $\Rightarrow$ 8 supercharges

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- Central extension C must be preserved

$$C = M_\perp + r$$

- Outer authorphism $J = M_\perp - r$ can be broken!

Explicit realization: Gukov Witten defects [Gukov, Witten, 2006]

Disorder-type defect in the gauge theory

Impose a prescribed singularity on the surface (in polar coordinates (ρ, θ) for the orthogonal directions):

$$A \sim \alpha\, d\theta, \qquad \Phi \sim \frac{\beta + i\gamma}{\rho} e^{-i\theta}, \qquad \bar{\Phi} \sim \frac{\beta - i\gamma}{\rho} e^{i\theta}$$

plus a 2d theta-like parameter η. Here $\alpha, \beta, \gamma, \eta$ are vectors of defect parameters (how many depends on how the gauge group is broken). $\Phi = \phi_5 + i\phi_6$ is charged under r.

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Probe brane description

A half-BPS surface defect is dual to a probe D3-brane with worldvolume:

$$\mathrm{AdS}_3 \times S^1 \subset \mathrm{AdS}_5 \times S^5.$$

$$ds^2_{AdS_5} = \cosh^2 u\; ds^2_{AdS_3} + du^2 + \sinh^2 u\; d\psi \qquad ds^2_{S^5} = \cos^2\theta\; ds^2_{S^3} + d\theta^2 + \sin^2\theta\; d\varphi^2$$

$$ds^2_{D3} = \cosh^2 u_0 d^2_{AdS_3} + d(\sinh^2 u_0 \psi + \varphi)^2$$

The parameter u_0 is free. In general, it mixes AdS_5 and S^5.

Chiral Primary Operators in $\mathcal{N}=4$ SYM

$$\mathcal{O}_k(x,Y) = Y^{I_1}\cdots Y^{I_k}\,\mathrm{Tr}(\phi_{I_1}\cdots\phi_{I_k}), \qquad I_i = 1,\dots,6$$

Y is a complex null polarization vector projecting on the symm. traceless representation.

$$Y\cdot Y = 0.$$

- Protected 1/2-BPS operators.

$$\Delta = k.$$

- Transform in the $[0,k,0]$ representation of $SU(4)_R$.

Chiral Primary Operators in $\mathcal{N} = 4$ SYM

$$\mathcal{O}_k(x, Y) = Y^{I_1} \cdots Y^{I_k} \mathrm{Tr}(\phi_{I_1} \cdots \phi_{I_k}), \qquad I_i = 1, \ldots, 6$$

Y is a complex null polarization vector projecting on the symm. traceless representation.

$$Y \cdot Y = 0.$$

- Protected 1/2-BPS operators.

$$\Delta = k.$$

- Transform in the $[0, k, 0]$ representation of $SU(4)_R$.
- We will focus on $k = 2$. Under $SO(6)_6 \to SU(2) \times SU(2) \times U(1)$

$$[0, 2, 0] \to (0, 0)_0 \oplus (0, 0)_{\pm 2} \oplus (1/2, 1/2)_{\pm 1} \oplus (1, 1)_0$$

- One-point function with surface known exactly from localization. [Drukker, Gomis, Matsuura, 2008; Choi, Gomis, Garcia, 2024]

$$\langle \mathcal{O}_{2,2} \rangle_D = \frac{1}{z^2} \frac{4\pi^2}{\sqrt{2\lambda}} \sum_{I=1}^{M} N_I \left(\beta_I + i\gamma_I\right)^2$$

$$\langle \mathcal{O}_{2,0} \rangle_D = \frac{1}{|z|^2} \frac{8\pi^2}{\sqrt{6\lambda}} \left(\sum_{I=1}^{M} N_I \left((\beta_I^2 + \gamma_I^2) - \frac{\lambda}{4\pi^2} \frac{N - N_I}{2N} \right) \right).$$

They coincide with the holographic result!

Two-point function

$$\langle \mathcal{O}_2(P_1,Y_1)\,\mathcal{O}_2(P_2,Y_2)\rangle_D = \frac{(Y_1\circ Y_1)(Y_2\circ Y_2)}{(P_1\circ P_1)(P_2\circ P_2)}\,\mathcal{F}(z,\bar z,\alpha,\bar\alpha,\eta).$$

- Two spacetime and two R-symmetry cross-ratios (perfect symmetry)

$$\frac{(1-z)(1-\bar z)}{\sqrt{z\bar z}} = -\frac{2P_1\cdot P_2}{(P_1\circ P_1)^{1/2}(P_2\circ P_2)^{1/2}}, \qquad \frac{(1-\alpha)(1-\bar\alpha)}{\sqrt{\alpha\bar\alpha}} = -\frac{2Y_1\cdot Y_2}{(Y_1\circ Y_1)^{1/2}(Y_2\circ Y_2)^{1/2}}$$

$$\frac{z+\bar z}{2\sqrt{z\bar z}} = \frac{P_1\circ P_2}{(P_1\circ P_1)^{1/2}(P_2\circ P_2)^{1/2}} \qquad \frac{\alpha+\bar\alpha}{2\sqrt{\alpha\bar\alpha}} = \frac{Y_1\circ Y_2}{(Y_1\circ Y_1)^{1/2}(Y_2\circ Y_2)^{1/2}}$$

- One additional cross-ratio η associated to the broken $U(1)_J$. Absent if J is preserved (e.g. for $\beta,\gamma\to 0$ in Gukov Witten).

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- One additional cross-ratio η associated to the broken $U(1)_J$. Absent if J is preserved (e.g. for $\beta, \gamma \to 0$ in Gukov Witten).
- The correlator is organized in charged sectors

$$\mathcal{F}(z, \bar{z}, \alpha, \bar{\alpha}) = \sum_{q=-2}^{2} e^{2iq\eta}\, \mathcal{F}_q(z, \bar{z}, \alpha, \bar{\alpha})$$

Superconformal Ward Identities

- Supersymmetry imposes non-trivial constraints on the function $\mathcal{F}$ [Holguin, Kawai, 2025]

$$\begin{aligned} \left(\partial_z + \partial_\alpha\right) \mathcal{F}_q \left(z, \bar{z}, \alpha, \bar{\alpha}\right)\big|_{\alpha \to z} &= 0\,, \\ \left(\partial_{\bar{z}} + \partial_{\bar{\alpha}}\right) \mathcal{F}_q \left(z, \bar{z}, \alpha, \bar{\alpha}\right)\big|_{\bar{\alpha} \to \bar{z}} &= 0\,, \end{aligned}$$

- They are identical to those for $\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle$ without the defect.

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- They are identical to those for $\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle$ without the defect.
- The solution is

$$
\begin{aligned}
\mathcal{F}(z, \bar{z}, \alpha, \bar{\alpha}) &= \mathcal{C} + (z - \alpha)(z - 1/\bar{\alpha})(\bar{z} - 1/\alpha)(\bar{z} - \bar{\alpha})\, F_q(z, \bar{z}) \\
&+ \frac{(\bar{\alpha} z - 1)(\alpha \bar{z} - 1)(f_q(z, \alpha) + f_q(\bar{z}, \bar{\alpha})) - (z - \alpha)(\bar{z} - \bar{\alpha})(f_q(z, \frac{1}{\bar{\alpha}}) + f_q(\frac{1}{\bar{z}}, \alpha))}{(\alpha \bar{\alpha} - 1)(z \bar{z} - 1)}
\end{aligned}
$$

- We use analytic bootstrap to compute the functions $F_q(z, \bar{z})$ and $f_q(z, \alpha)$ perturbatively at large N.

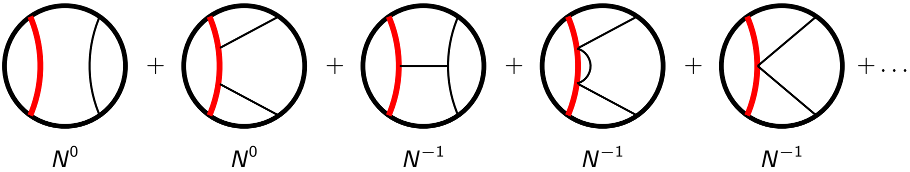
Section 4

Results

Results [LB, Bonomi, Ferragatta, Forini, 2026]

Large-N expansion

$$\langle \mathcal{O}_2\, \mathcal{O}_2 \rangle_D = \underbrace{\langle \mathcal{O}_2\, \mathcal{O}_2 \rangle + \langle \mathcal{O}_2 \rangle \langle \mathcal{O}_2 \rangle}_{O(N^0)} + \underbrace{\langle \mathcal{O}_2\, \mathcal{O}_2 \rangle_{\text{conn}}}_{O(1/N)}$$



$$F_q(z,\bar{z}) = F_q^{(0)}(z,\bar{z}) + \frac{1}{N} F_q^{(1)}(z,\bar{z})$$

Results [LB, Bonomi, Ferragatta, Forini, 2026]

Strategy

- Use the bulk OPE to compute the Disc of the correlator at leading order

$$\mathcal{O}_2 \times \mathcal{O}_2 \subset \mathbb{1} + \mathcal{B}_{[1,0,1]} + \mathcal{B}_{[2,0,2]} + \mathcal{B}_{[0,2,0]} + \mathcal{B}_{[0,4,0]} + \sum_{\ell} \mathcal{C}_{[1,0,1],\ell} + \sum_{\ell} \mathcal{C}_{[0,2,0],\ell} + \sum_{\ell} \mathcal{A}^{\Delta}_{[0,0,0],\ell}$$

- Find the result using the dispersion relation
- Fix potential ambiguities requiring consistency (physical singularities, SCWI, defect block expansion, . . .)

$$F(z,\bar{z}) = \left(\frac{\sqrt{z\bar{z}}}{(1-z)(1-\bar{z})}\right)^2 \sum_{\Delta,\ell} \lambda_{\phi\phi\mathcal{O}}\, a_{\mathcal{O}}\, f_{\Delta,\ell}(z,\bar{z})$$

$$f_{\Delta,\ell}(z,\bar{z}) = (1-\bar{z})^{\frac{\Delta-\ell}{2}} f_{\Delta+\ell}(z) + \dots$$

To have a Disc we need $\frac{\Delta-\ell}{2} - 2$ non-integer or negative (delta-function discontinuity).

Results [LB, Bonomi, Ferragatta, Forini, 2026]

Zero-charge sector

Only one block gives Disc and after fixing the ambiguity we get

$$F_0^{\text{conn}}(z,\bar{z}) = \lambda_{222} a_{\mathcal{O}_2} \frac{1 - z^2\bar{z}^2 + (2z\bar{z} - (1-z)(1-\bar{z})\sqrt{z\bar{z}})\log(z\bar{z})}{2(1-z)(1-\bar{z})(1-z\bar{z})^3}$$

$$f_0^{\text{conn}}(z,\alpha) = \lambda_{222} a_{\mathcal{O}_2} \frac{(\alpha-1)(\alpha+z)}{2\alpha(z-1)}$$

- The result depends on a single piece of bulk data.
- Valid for any 1/2 BPS rotation-preserving surface defect in $\mathcal{N}=4$ SYM at large N.

Results [LB, Bonomi, Ferragatta, Forini, 2026]

- For the charged sectors we had to compute the (super)blocks. Similar to the spinning defects case, but simpler [Kobayashi, Nishioka, 2018].
- Seemingly puzzling fact: only charged operators can get a one-point function ($M_\perp + r$ is preserved).

$$\mathcal{O}_2 \times \mathcal{O}_2 \subset \mathbb{1} + \mathcal{B}_{[1,0,1]} + \mathcal{B}_{[2,0,2]} + \mathcal{B}_{[0,2,0]} + \mathcal{B}_{[0,4,0]} + \sum_\ell \mathcal{C}_{[1,0,1],\ell} + \sum_\ell \mathcal{C}_{[0,2,0],\ell} + \sum_\ell \mathcal{A}^{\Delta}_{[0,0,0],\ell}$$

- No contribution from the long supemultiplet and it is a non-perturbative fact!

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- No contribution from the long supemultiplet and it is a non-perturbative fact!
- Only the $[0,2,0]$ multiplet contributes to the Disc and it integrates to itself in the dispersion relation.
- The result is a single bulk conformal block and it is consistent with supersymmetry.

Charged sectors

$$\mathcal{F}^{\text{conn}}_{\pm 1}(z,\bar{z},\alpha,\bar{\alpha}) = \lambda_{222} a^{(\pm 2)}_{\mathcal{O}_2} \frac{(1-\alpha)(1-\bar{\alpha})}{\sqrt{\alpha\bar{\alpha}}} \frac{\sqrt{z\bar{z}}}{(1-z)(1-\bar{z})} \qquad \mathcal{F}^{\text{conn}}_{\pm 2}(z,\bar{z},\alpha,\bar{\alpha}) = 0$$

Conjecture: this result is true non-perturbatively.

Check at weak coupling

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There is a weak coupling result for this correlator in the literature [Holguin, Kawai, 2025]

$$\mathcal{F}^{\text{conn}}_{2}(z,\bar{z},\alpha,\bar{\alpha}) = \lambda_{222}\, a^{(\pm 2)}_{\mathcal{O}_2} \frac{(1-\alpha)(1-\bar{\alpha})}{\sqrt{\alpha\bar{\alpha}}} f(z,\bar{z})$$

$$f(z,\bar{z}) = \sum_s \left(\frac{z}{\bar{z}}\right)^{s/2} \left(\frac{z\bar{z}+1}{\sqrt{z\bar{z}}}\right)^{-|s|-1} {}_2F_1\left(\frac{|s|}{2}+1, \frac{|s|+1}{2}; |s|+1; \frac{4z\bar{z}}{(z\bar{z}+1)^2}\right)$$

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We checked that

$$f(z,\bar z) = \frac{\sqrt{z\bar z}}{(1-z)(1-\bar z)}$$

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- We used analytic bootstrap to compute defect two-point functions of chiral primary operators in the presence of a surface operator.
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Future directions

- It looks like holography knows a lot about these defects. Can we find other exact holographic results?
- Higher-point correlation functions involving also defect operators.
- Integrated correlators and interplay with localization.
- Understand the rigid limit and possible interplay with integrability see also Adam's talk

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THANK YOU!