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Cosmic Strings for Electromagnetic Duality

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Twist defect

- CFTs with an ordinary internal global symmetry G have many interesting observables beyond those of local operators.

- Twist defects (aka monodromy defects) are codimension-2, extended defects in spacetime associated with a group element $g \in G$. In 3+1d, these are strings in space.

- The defining property of a twist defect is that any local operator Φ undergoes a g-symmetry transformation when going around the defect.

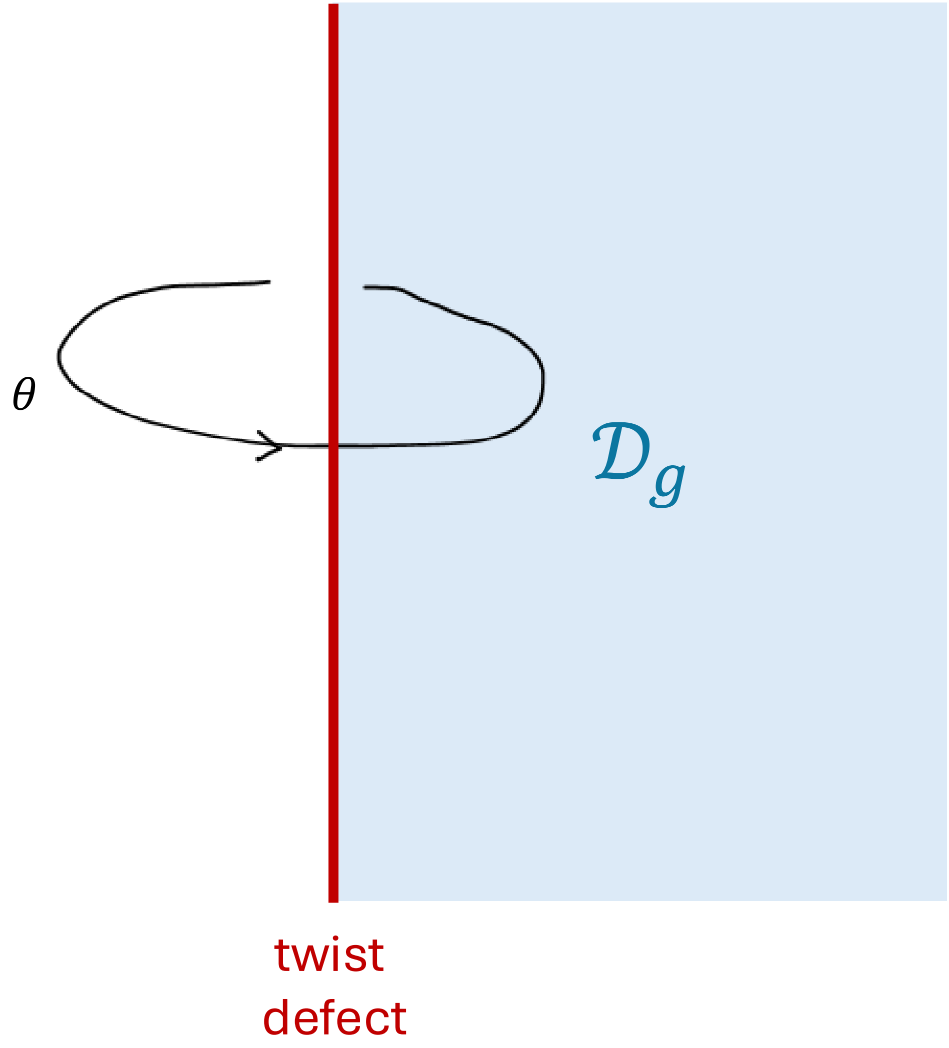
Twist defect

- Let θ be the angular coordinate around the codimension-2 twist defect (shown as the red line).

- As a local operator is taken around the twist defect:

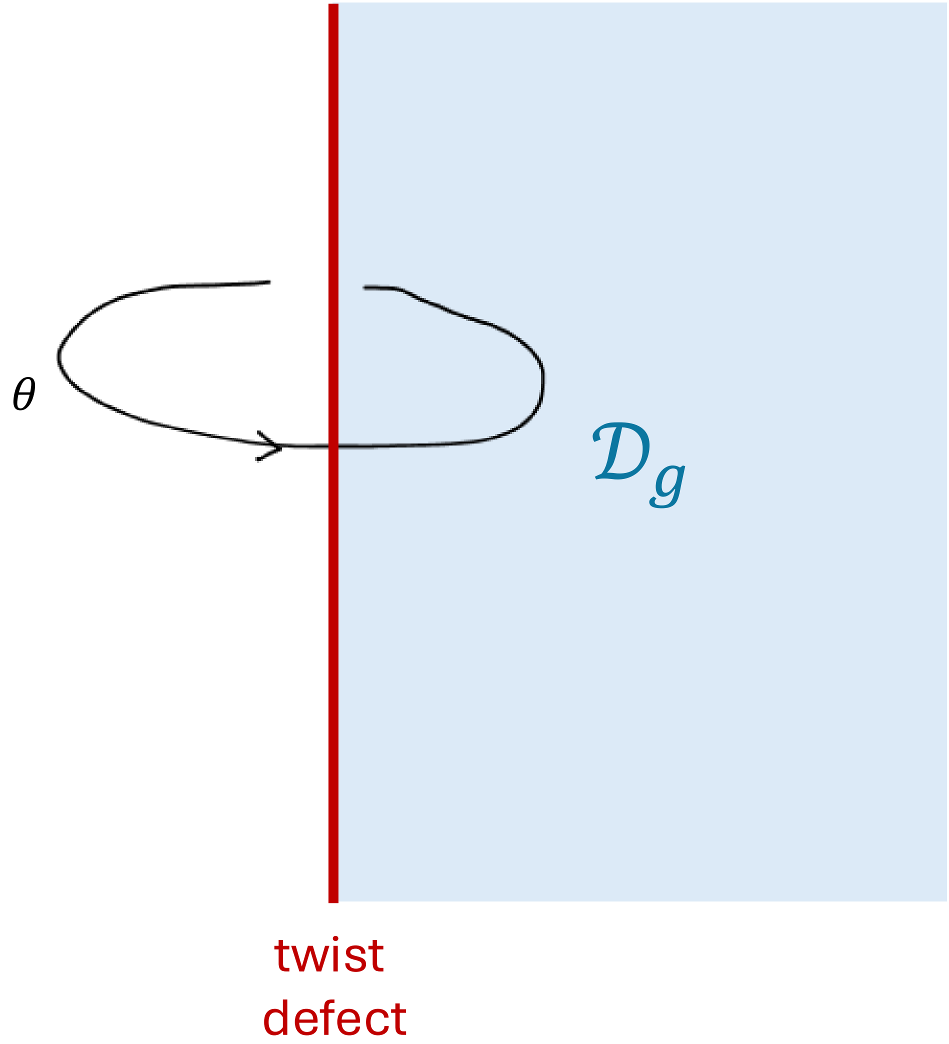
$$\Phi(\theta + 2\pi) = \Phi^g(\theta)$$

Φ^g is the image of Φ under the g-transformation.



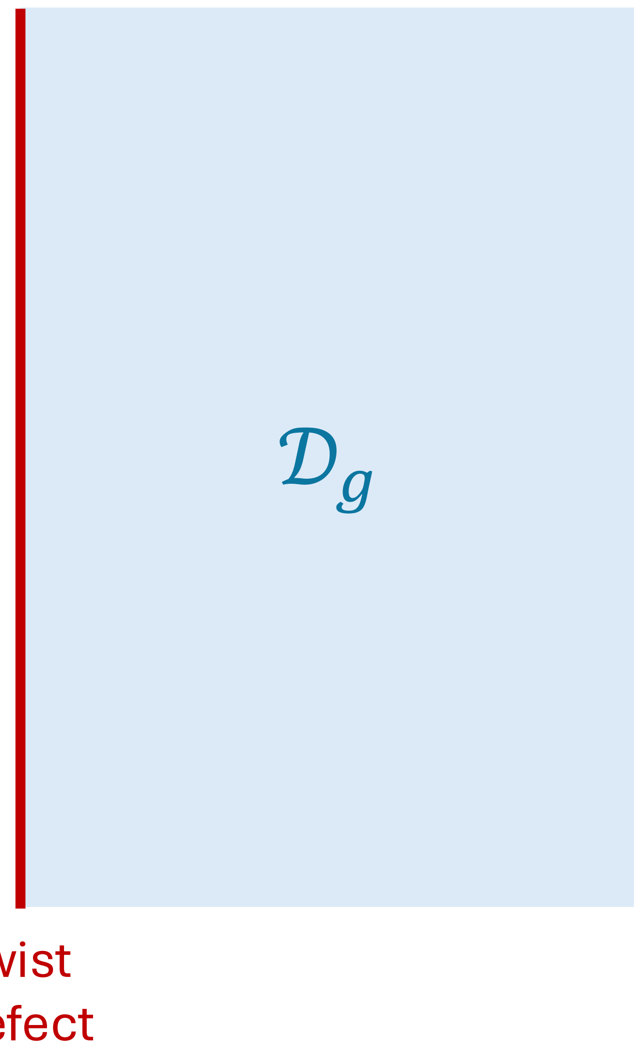
Twist defect and symmetry defect

- Twist defects are not genuine codimension-**2** defects.
- Rather, they are attached to a codimension-**1** defect $\mathcal{D}_g$.
- $\mathcal{D}_g$ is the topological defect associated with the global symmetry element $g \in G$ [Gaiotto-Kapustin-Seiberg-Willett '14].
- In other words, twist defects live at the *end* of the symmetry defect.



Twist defect vs. Symmetry defect

Twist defect	Symmetry defect $\mathcal{D}_g$
codimension-2	codimension-1
conformal defect	topological defect
non-genuine	genuine



Twist defect

- In 1+1d CFT, twist defects are point operators, aka twist fields. They play a crucial role in the computation of the Reyni entropy using the replica trick.
- In fermionic 1+1d CFTs and superstring theory, twist fields for the fermion parity $(-1)^F$ are known as the Ramon operators. They are not genuine local operators because they are not mutually local with respect to the fermion operators.
- Many examples in higher dimensional CFTs. E.g., the twist line defect in the 2+1d Ising CFT has been studied using Monte Carlo [Billo et al. '13] and the conformal bootstrap [Gaiotto-Mazac-Paulos '13]. Monodromy defects in free CFTs [Bianchi-Chalabi-Prochazka-Robinson-Sisti '21].

Electromagnetic duality of Maxwell's equations

- Classical Maxwell's equations in 3+1d without matter:

$$\nabla \cdot \vec{E} = 0 \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \times \vec{B} = \frac{\partial \vec{E}}{\partial t}$$

is invariant under electromagnetic duality:

$$\vec{E} \to \vec{B} \qquad \vec{B} \to -\vec{E}$$

Twist defect for electromagnetic duality
[SHS-Zhong '25]

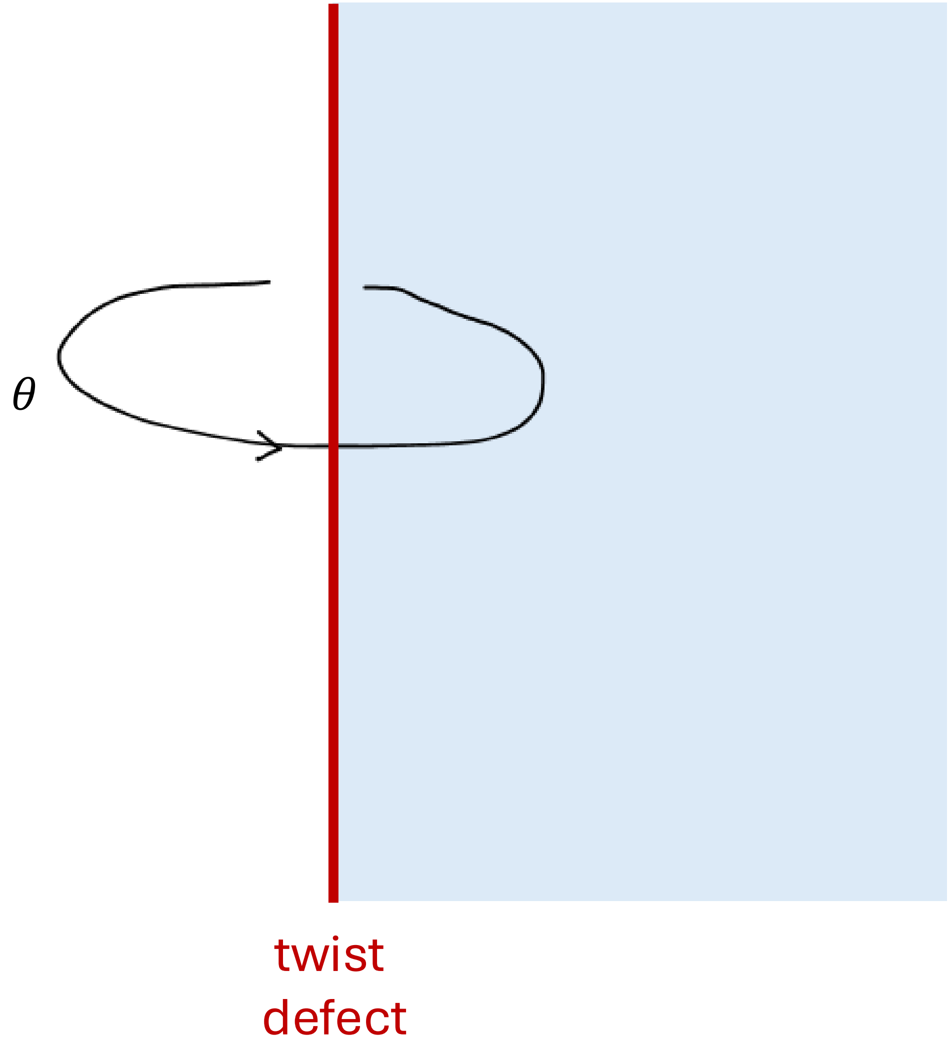
- Does it make sense to consider twist defects for electromagnetic duality?
- This doesn't obviously make sense because

Duality $\neq$ Symmetry

- They would be strings in 3d space, around which the electric and magnetic fields are exchanged:

$$\vec{E}(\theta + 2\pi) = \vec{B}(\theta)$$
$$\vec{B}(\theta + 2\pi) = -\vec{E}(\theta)$$

- In 3+1d spacetime, they are (non-genuine) 2d surface defects.



- It’s not obvious if twist defects for electromagnetic duality makes sense.
- Nonetheless, let’s close our eyes, shut up, and calculate.
- We will come back to this more conceptual point later.

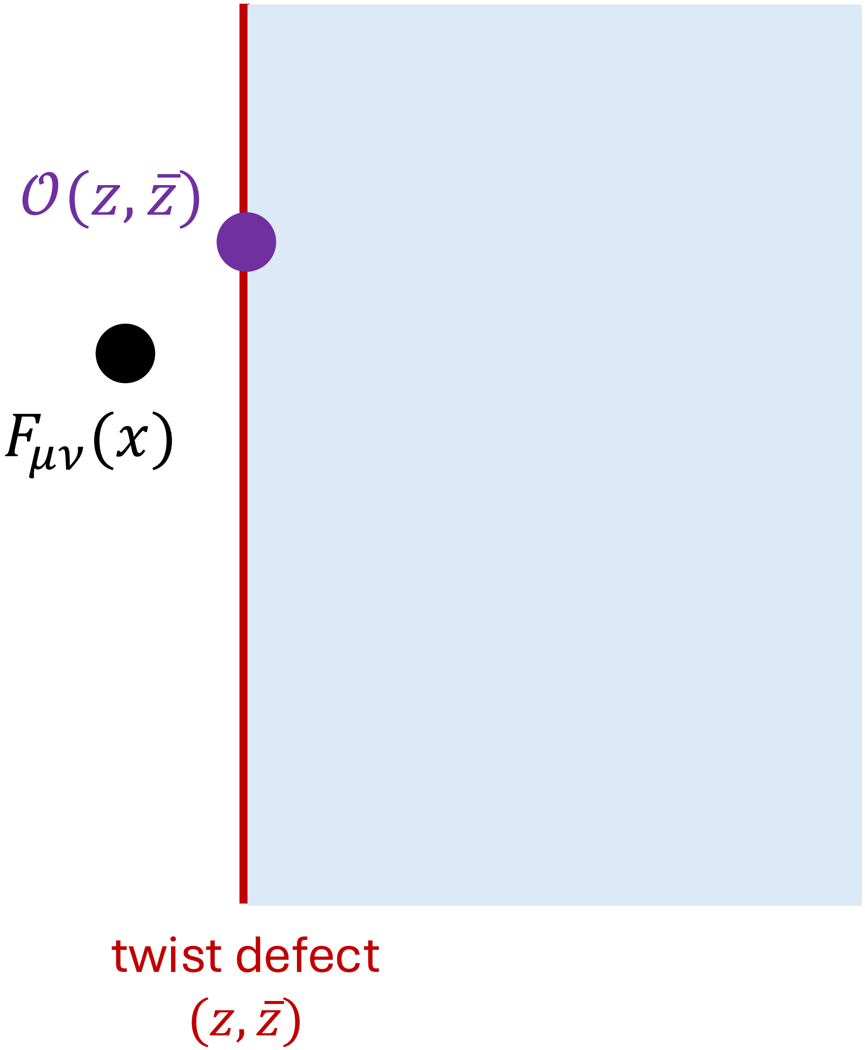
Twist defect for electromagnetic duality

[SHS-Zhong '25]

- The local operators in the bulk are built out of the field strength $F_{\mu\nu}(x)$.
- <u>Goal</u>: Compute the spectrum of defect operators $\mathcal{O}(z,\bar{z})$ living on the twist defect.
- Two calculations, same result:

1. Solve Maxwell's equations on $S^3 \times \mathbb{R}$ in the presence of a twist defect.
2. Analyze $\langle F_{\mu\nu}\mathcal{O}\rangle$ using defect CFT (DCFT) [..., Billo-Goncalves-Lauria-Meineri '16, ...].

The two approaches are related by the operator-state correspondence.



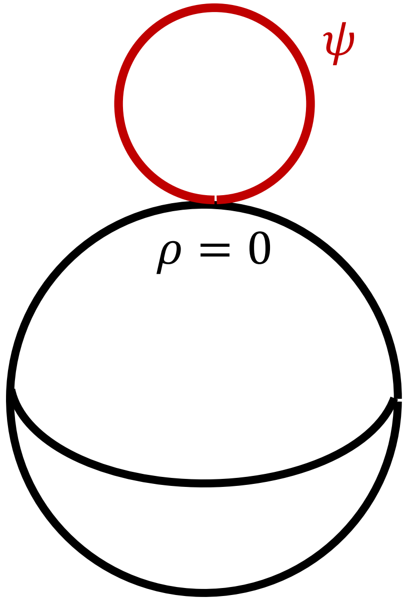
Conformal map

$\mathbb{R}^4$ $\longrightarrow$ $S^3 \times \mathbb{R}$

$$z = \sqrt{1-\rho}\, e^{\tau + i\psi}$$
$$w = \sqrt{\rho}\, e^{\tau + i\theta}$$

$$ds^2 = 2dz d\bar{z} + 2dw d\bar{w}$$

twist defect: $w = \bar{w} = 0$



Hopf coordinates

$$ds^2 = d\tau^2 + (1-\rho)d\psi^2 + \rho d\theta^2 + \frac{d\rho^2}{4\rho(1-\rho)}$$

$$0 \le \rho \le 1,$$
$$0 \le \psi < 2\pi, \qquad 0 \le \theta < 2\pi$$

twist defect: $\rho = 0$ (great circle)

at the north pole of the based S^2

Maxwell's equations on $S^3 \times \mathbb{R}$ with a twist

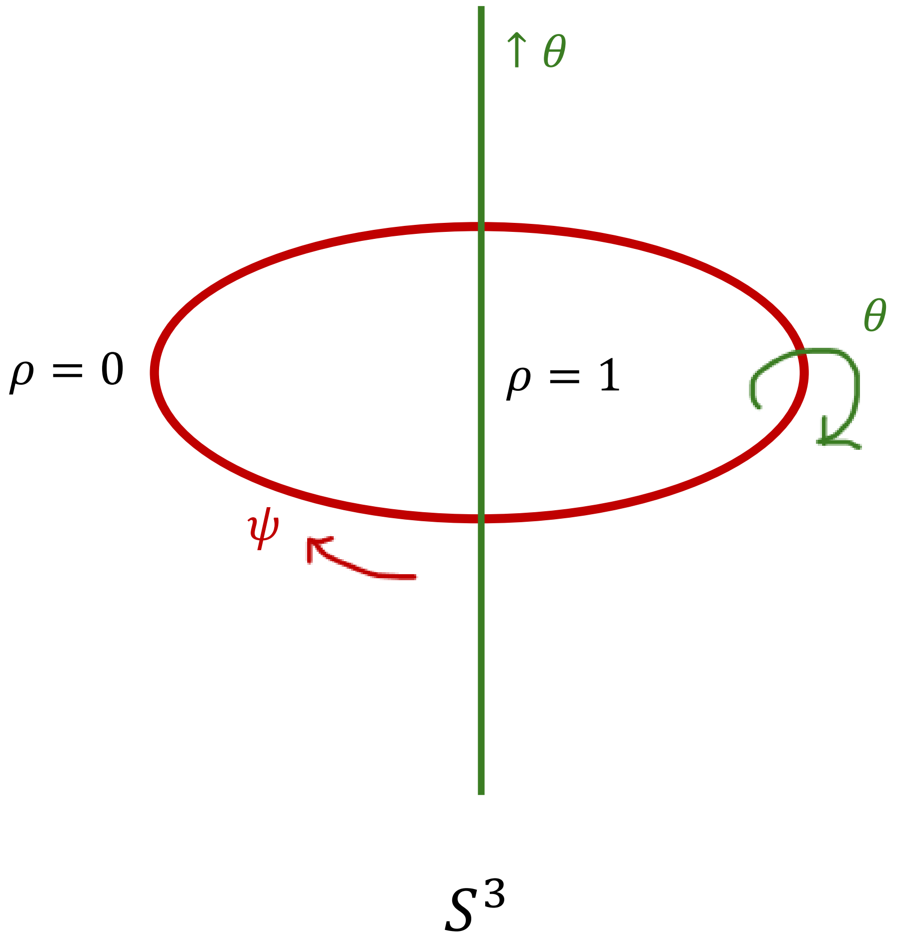
$$ds^2 = -dt^2 + (1-\rho)d\psi^2 + \rho d\theta^2 + \frac{d\rho^2}{4\rho(1-\rho)}$$

- The twist defect extends along the t, ψ direction, localized at $\rho = 0$.

$$F_{\mu\nu}(t, \psi, \theta + 2\pi, \rho) = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}(t, \psi, \theta, \rho)$$

- Note that $\theta \to \theta + 2\pi$ is an order 4 action. The spin quantum number associated with the rotation group **transverse** to twist defect is quantized as

$$s \in \mathbb{Z} + \frac{1}{4}$$



Twist defect for electromagnetic duality

[SHS-Zhong '25]

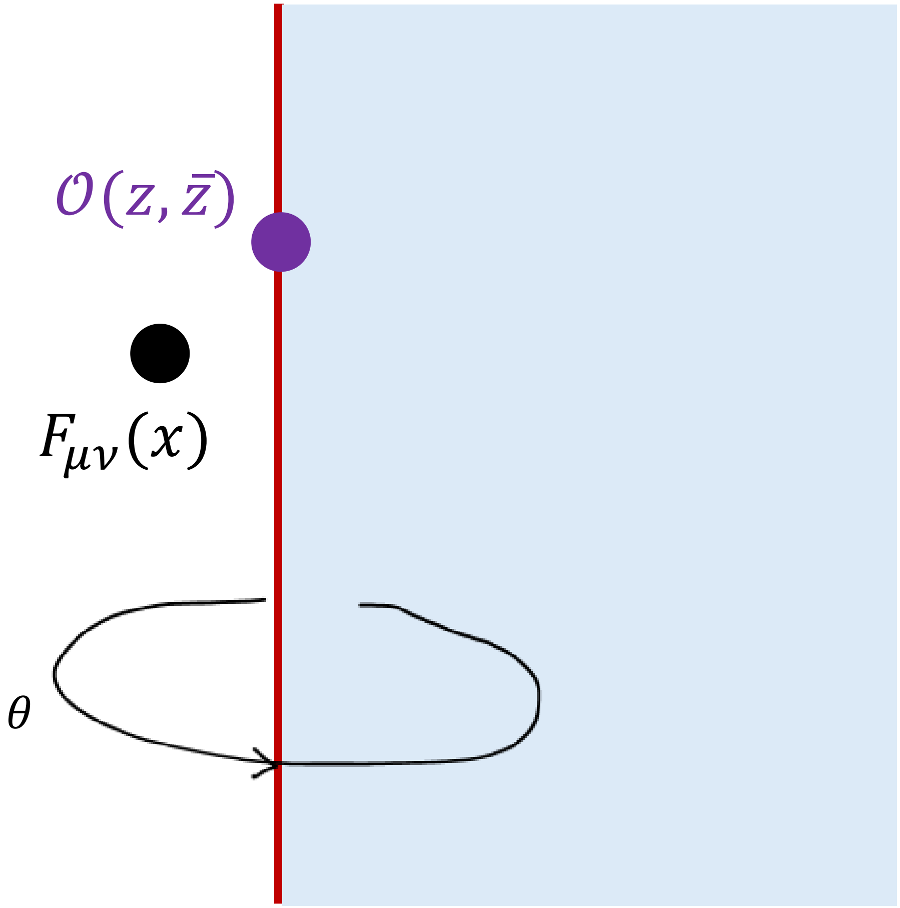
- The spectrum of defect conformal primary operators $\mathcal{O}(z,\bar{z})$ living on the twist defect is

$$(h,\bar{h}) = \left(1+\frac{|s|}{2}, \frac{|s|}{2}\right), \qquad |s| \in \mathbb{N}+\frac{1}{4}$$

$$(h,\bar{h}) = \left(\frac{|s|}{2}, 1+\frac{|s|}{2}\right), \qquad |s| \in \mathbb{N}+\frac{3}{4}$$

$$s \in \mathbb{Z}+\frac{1}{4}$$

- The spectrum is not symmetric under $h \leftrightarrow \bar{h}$; the twist defect is chiral!



twist defect

$(z,\bar{z})$

Twist defect for electromagnetic duality

[SHS-Zhong '25]

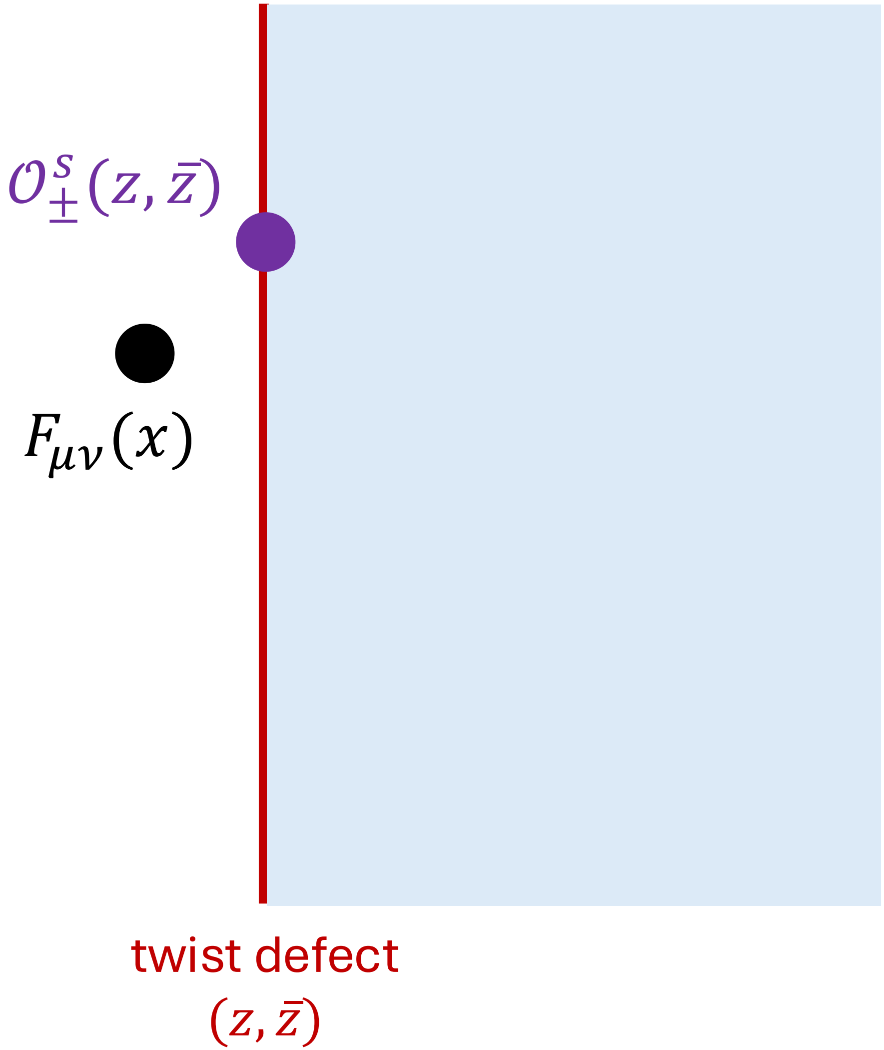
$$\mathcal{O}_+^s:\ (h,\bar{h}) = \left(1+\frac{|s|}{2},\frac{|s|}{2}\right), \qquad |s| \in \mathbb{N}+\frac{1}{4}$$

$$\mathcal{O}_-^s:\ (h,\bar{h}) = \left(\frac{|s|}{2},1+\frac{|s|}{2}\right), \qquad |s| \in \mathbb{N}+\frac{3}{4}$$

- These primaries all have parallel spin

$$h-\bar{h} = \pm 1$$

- There is no energy-momentum tensor; that's fine because it's not a stand-alone 1+1d CFT.
- Composites of these primaries give the displacement operators, which have $\Delta = 3$ and $s = \pm 1$.



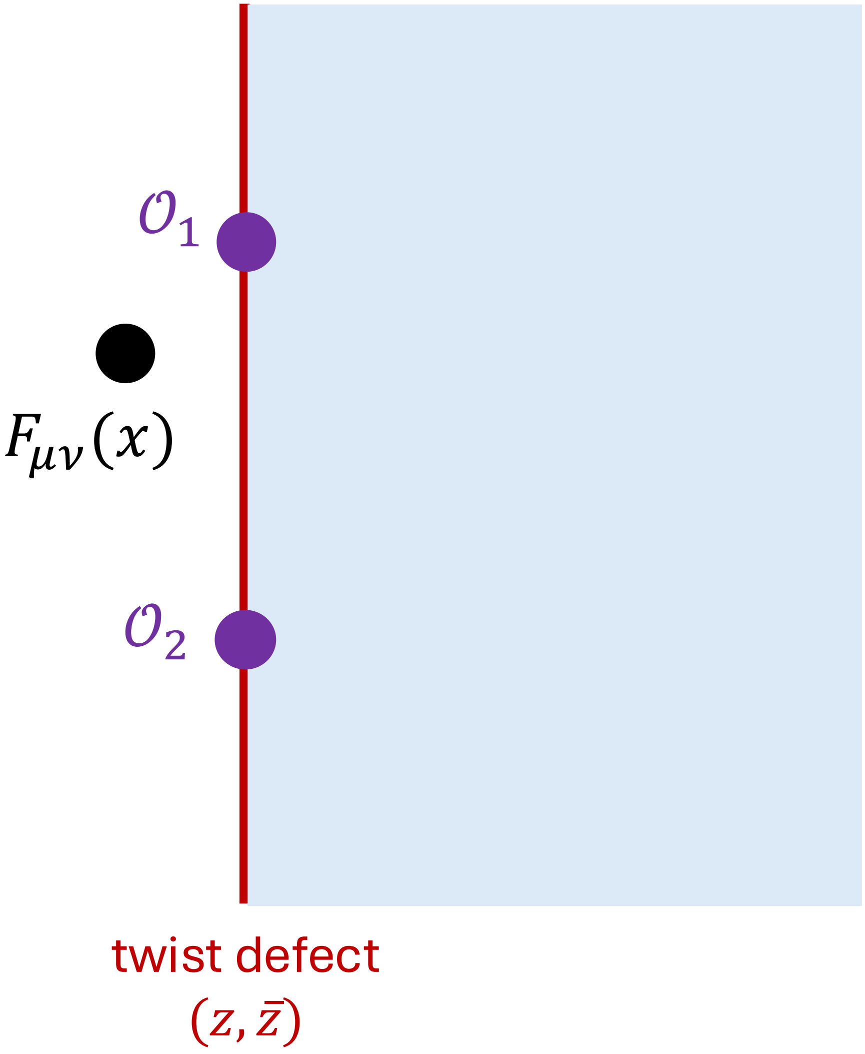
Twist defect for electromagnetic duality

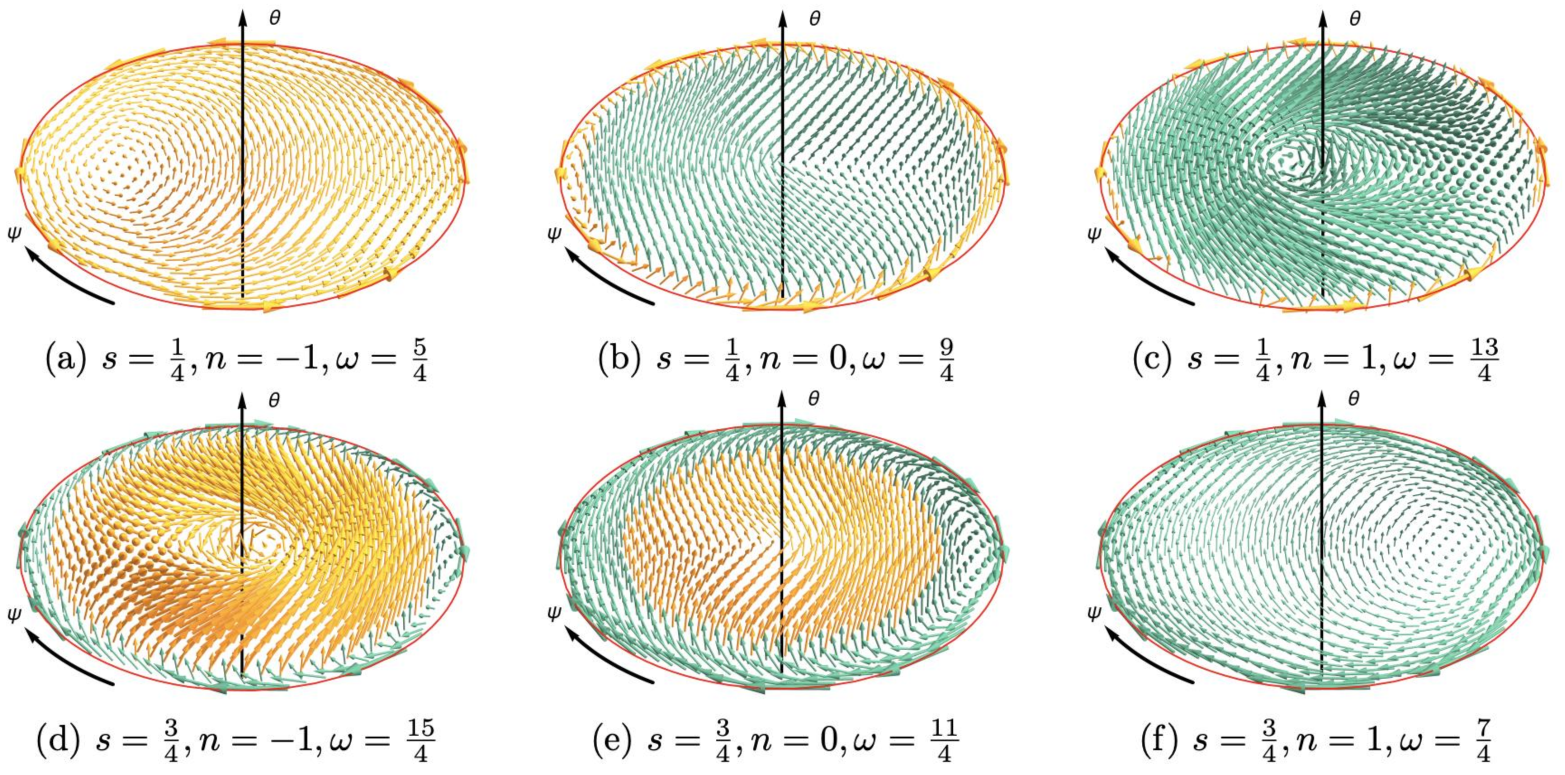
[SHS-Zhong '25]

$$\mathcal{O}_+^s:\ (h,\bar{h}) = \left(1+\frac{|s|}{2},\frac{|s|}{2}\right), \qquad |s| \in \mathbb{N}+\frac{1}{4}$$

$$\mathcal{O}_-^s:\ (h,\bar{h}) = \left(\frac{|s|}{2},1+\frac{|s|}{2}\right), \qquad |s| \in \mathbb{N}+\frac{3}{4}$$

- Following [Herzog-Shrestha '22], conformal symmetry and unitarity of $\langle F_{\mu\nu}\mathcal{O}\rangle$ determine the same spectrum of defect operators.
- Moreover, consideration of the three-point function $\langle F_{\mu\nu}\mathcal{O}_1\mathcal{O}_2\rangle$ shows that these defect operators form a generalized free field sector.
- In other words, their correlation functions are computed by Wick's theorem.





(a) $s=\frac{1}{4}, n=-1, \omega=\frac{5}{4}$ (b) $s=\frac{1}{4}, n=0, \omega=\frac{9}{4}$ (c) $s=\frac{1}{4}, n=1, \omega=\frac{13}{4}$

(d) $s=\frac{3}{4}, n=-1, \omega=\frac{15}{4}$ (e) $s=\frac{3}{4}, n=0, \omega=\frac{11}{4}$ (f) $s=\frac{3}{4}, n=1, \omega=\frac{7}{4}$

Figure 4: Poynting vector $\vec{S}$ for the electromagnetic waves in the presence of a conformal twist defect. We color those Poynting vectors with positive energy flux density in the ψ-direction $S_\psi > 0$ in green, and those with negative $S_\psi < 0$ in yellow. Given an $|s| \in \mathbb{N} + \frac{1}{4}$ or $|s| \in \mathbb{N} + \frac{3}{4}$, the energy flux density near the twist defect flows in a fixed direction along the twist defect, shown as the red circle, independent of n and ω. (Here we only display the Poynting vector on a $\theta = $ const slice.)

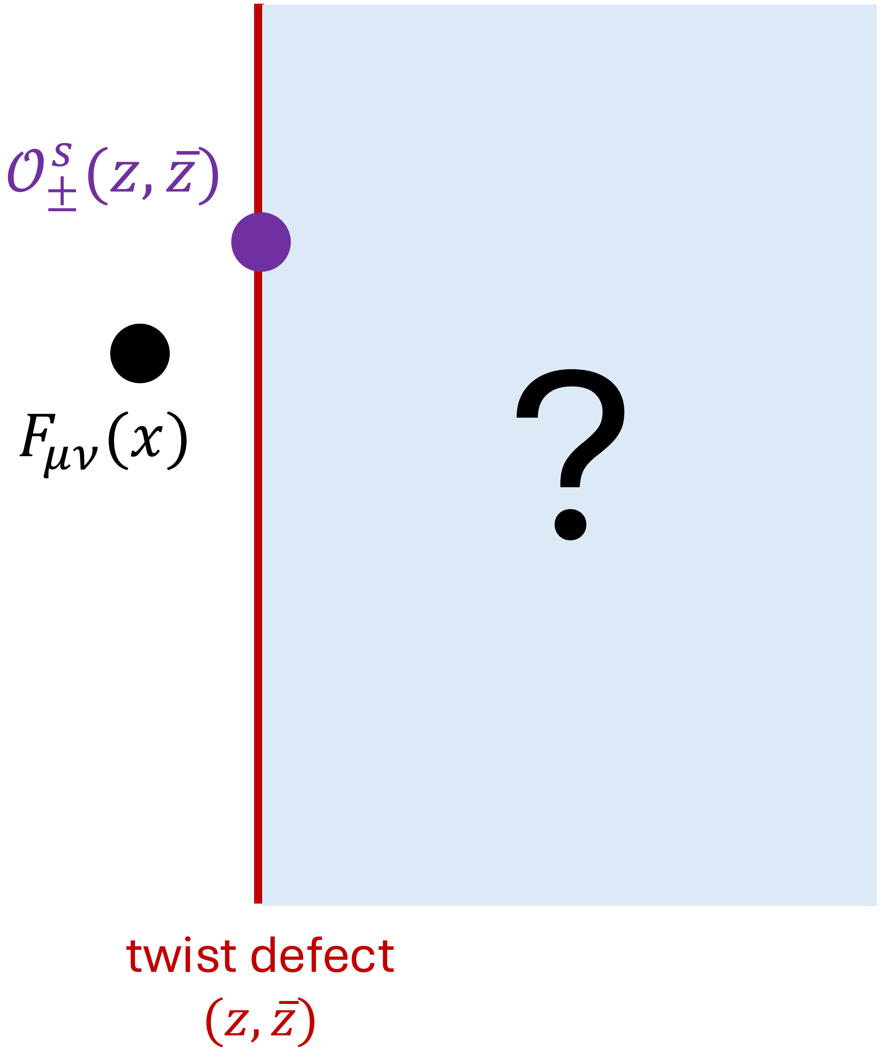
Twist defect for electromagnetic duality

[SHS-Zhong '25]

$$\mathcal{O}_{+}^{s}\colon\ (h,\bar{h}) = \left(1+\frac{|s|}{2},\frac{|s|}{2}\right), \qquad |s| \in \mathbb{N}+\frac{1}{4}$$

$$\mathcal{O}_{-}^{s}\colon\ (h,\bar{h}) = \left(\frac{|s|}{2},1+\frac{|s|}{2}\right), \qquad |s| \in \mathbb{N}+\frac{3}{4}$$

- Is that all there is?
- Is it a twist defect for duality, or for symmetry?



Duality vs. symmetry

- Duality states that two seemingly different descriptions (e.g., Lagrangians) in fact describe the same quantum system. Exact dualities include electromagnetic duality for 3+1d free Maxwell theory and N=4 super Yang-Mills, as well as T-duality in 1+1d compact boson field theory.
- Under such a duality, the abstract operator content is preserved, but the labeling changes – for instance, the electric and magnetic fields are exchanged.

- Global symmetries are intrinsic to the quantum system. They are realized by conserved operators or equivalently by topological defects, and act by mapping one operator to another within the same description.

Electromagnetic duality of the quantum theory

- Classical Maxwell's equations are invariant under electromagnetic duality.
- Quantum mechanically, the overall normalization of the action matters

$$\mathcal{L} = -\frac{1}{4e^2} F^{\mu\nu} F_{\mu\nu}$$

where the gauge fields are normalized to have quantized magnetic fluxes over any two-surface Σ:

$$\oint_\Sigma F \in 2\pi\mathbb{Z} \quad \text{(Dirac's quantization)}$$

- Spectrum of line operators and partition functions [Witten '95] depend on e^2.
- Electromagnetic duality not only exchanges the electric and magnetic fields, but also changes the electric coupling constant:

$$e^2 \to \frac{2\pi}{e^2}$$

Duality vs. symmetry

- Duality can sometimes lead to a global symmetry, but not always.
- For instance, at the self-dual coupling where

$$e^2 = 2\pi$$

electromagnetic duality maps the Lagrangian back to itself, and the duality transformation becomes a $\mathbb{Z}_4$ global symmetry.

- This symmetry is implemented by a codimension-1 topological defect [Gaiotto-Witten '08, Kapustin-Tikhonov '09]

$$\frac{i}{2\pi}\int_M A_- dA_+$$

where $A_\pm$ are the gauge fields on the two sides of the defect.

Non-invertible symmetry for electromagnetic duality
[Choi-Cordova-Hsin-Lam-SHS ‘21]

- It was realized in [Choi-Cordova-Hsin-Lam-SHS ‘21] that when

$$e^2 = \frac{2\pi}{N} \text{ for any positive integer } N$$

the electromagnetic duality is also associated with a topological defect:

$$\mathcal{D}_N(M): \frac{iN}{2\pi}\int_M A_- dA_+$$

supported on a codimension-1 manifold M. When M is space, this is a conserved operator.

- Variation of the total action gives the duality transformation across the defect

$$\frac{1}{2e^2}\int_{x<0} F_- \wedge\star F_- + \frac{iN}{2\pi}\int_{x=0} A_- \wedge dA_+ + \frac{1}{2e^2}\int_{x>0} F_+ \wedge\star F_+$$

$$\Rightarrow\ F^+ = i\star F^-$$

Non-invertible symmetry for electromagnetic duality

[Choi-Cordova-Hsin-Lam-SHS ‘21]

$$\mathcal{D}_N(M) \colon \frac{iN}{2\pi}\int_M A_- dA_+$$

- For $N > 1$, this describe a non-invertible topological defect.
- When M is space, it is not a unitary operator; it’s non-invertible

$$\mathcal{D}_N(M) \times \mathcal{D}_N^\dagger(M) = \frac{1}{N}\sum_{\Sigma \in H^2(M,\mathbb{Z}_N)} \exp\left(i\oint_\Sigma \star F\right)$$

Electromagnetic duality is implemented by a

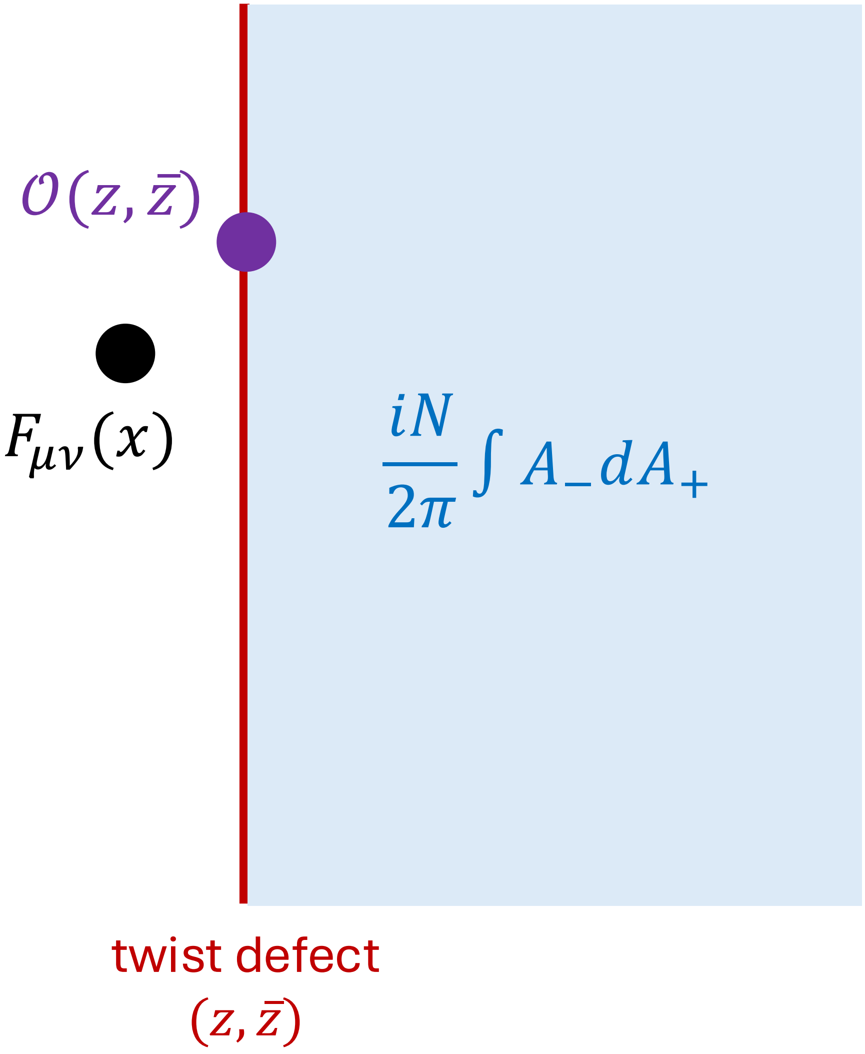
non-invertible global symmetry at $e^2 = 2\pi/N$.

- Generalization to $\frac{e^2}{2\pi} \in \mathbb{Q}^+$ [Niro-Roumpedakis-Sela ‘22, Cordova-Ohmori ‘23].

The end of non-invertible symmetries

[SHS-Zhong '25]

- At $e^2 = 2\pi/N$, our twist defect is associated with the non-invertible global symmetry for the electromagnetic duality.
- The twist defect is a conformal surface defect living at the **end** of the codimension-1 topological defect $\mathcal{D}_N$.
- In 1+1d CFT, twist fields for non-invertible symmetries have been studied extensively. In Ising CFT, they have $(h, \bar{h}) = (\frac{1}{16}, 0)$ etc.



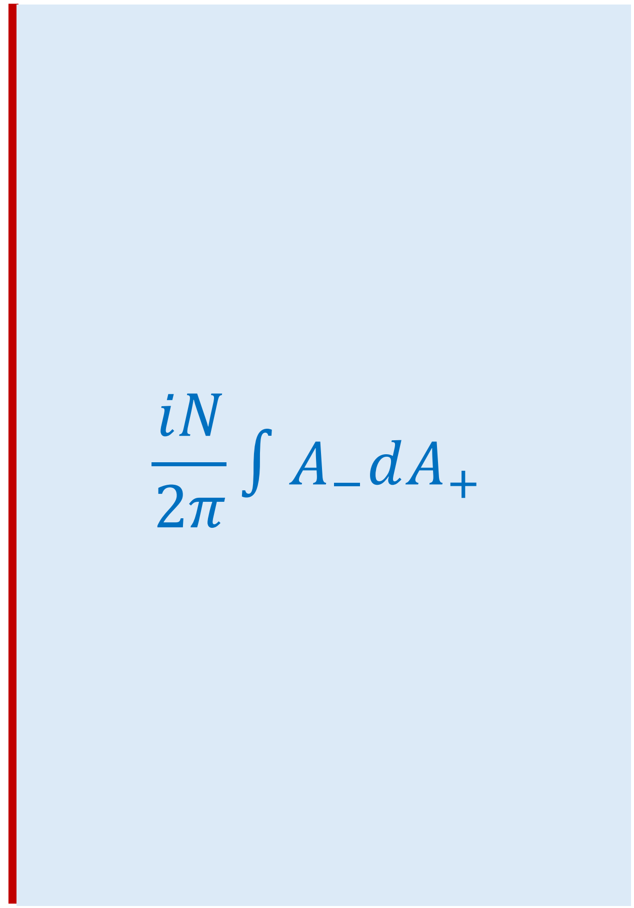
Chiral boson

[SHS-Zhong '25]

- Since the non-invertible topological defect $\mathcal{D}_N$ carries a Chern-Simons term, there must be an anomaly-inflow to its end locus - the twist defect.

- The simplest scenario is to have a 1+1d chiral boson ϕ sector on the twist defect, in addition to the generalized free field sector.

$$-\frac{N}{2\pi}\int dt d\psi \left[(\partial_\psi \phi)^2 + \partial_t \phi \partial_\psi \phi\right] \,, \quad \phi \sim \phi + 2\pi$$



twist defect

(t, ψ)

Twist defect = GFF+ chiral boson
[SHS-Zhong '25]

- We identify two sectors of the twist defect for this non-invertible symmetry:

1. Generalized free field:

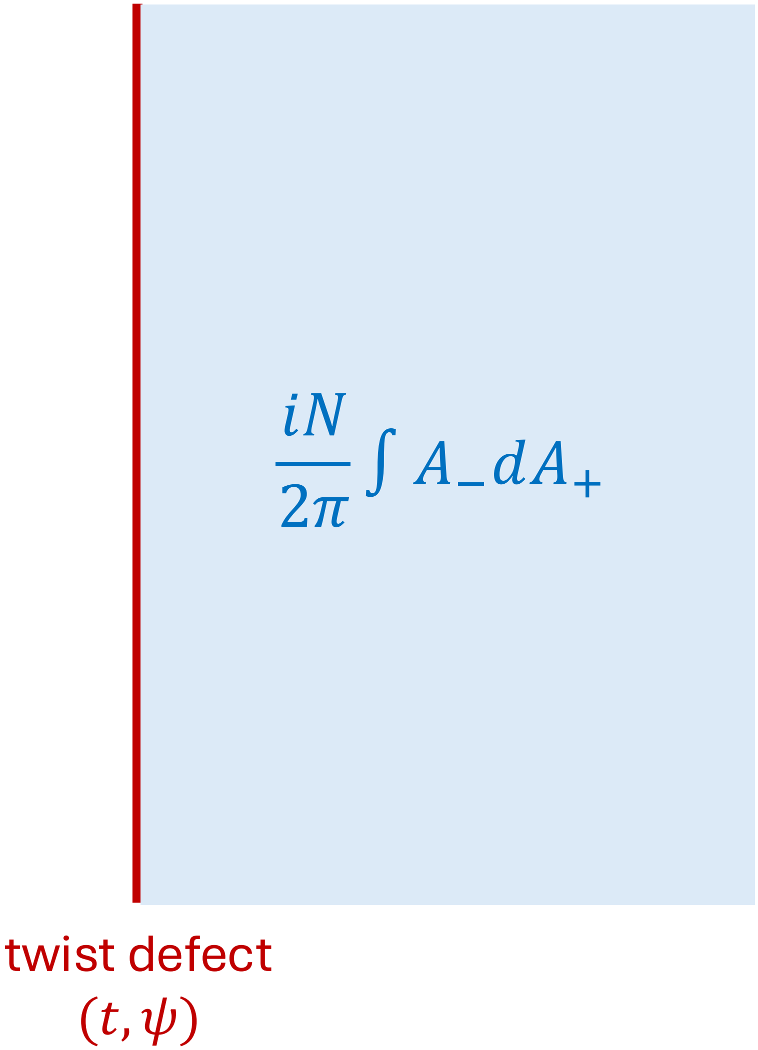
$$\mathcal{O}_+^s:\ (h,\bar{h}) = \left(1+\frac{|s|}{2}, \frac{|s|}{2}\right), \qquad |s| \in \mathbb{N}+\frac{1}{4}$$

$$\mathcal{O}_-^s:\ (h,\bar{h}) = \left(\frac{|s|}{2}, 1+\frac{|s|}{2}\right), \qquad |s| \in \mathbb{N}+\frac{3}{4}$$

2. Chiral boson

$$-\frac{N}{2\pi}\int dt d\psi \left[\left(\partial_\psi \phi\right)^2 + \partial_t \phi \partial_\psi \phi\right]$$

- This twist defect is very stable –no relevant defect deformation.



Casimir momentum for the twist defect

[SHS-Zhong ‘25]

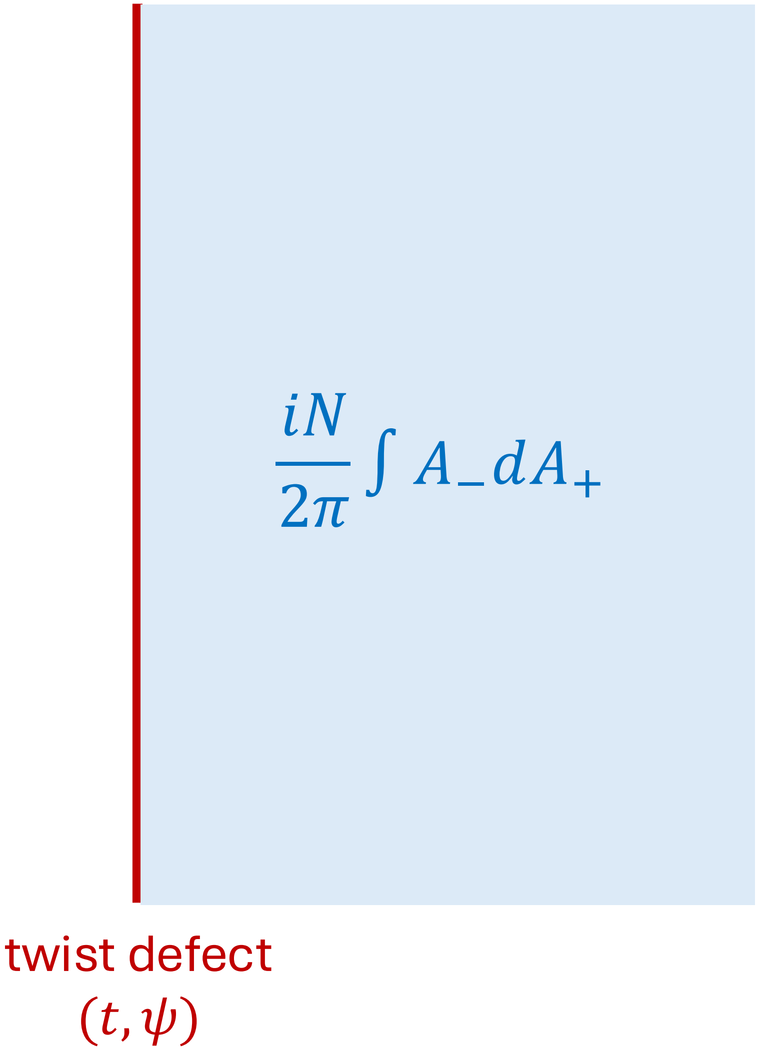
- In a stand-alone 1+1d CFT, the energy and momentum of the ground state are fixed by the left and right central charges:

$$E_0 = -\frac{c_L + c_R}{24}, \qquad P_0 = \frac{c_L - c_R}{24}$$

- The Casimir momentum P_0 is related to the gravitational anomaly [AlvarezGaume-Witten ‘84].
- The Casimir momentum for our twist defect receives contribution from both the generalized free field and chiral boson sectors:

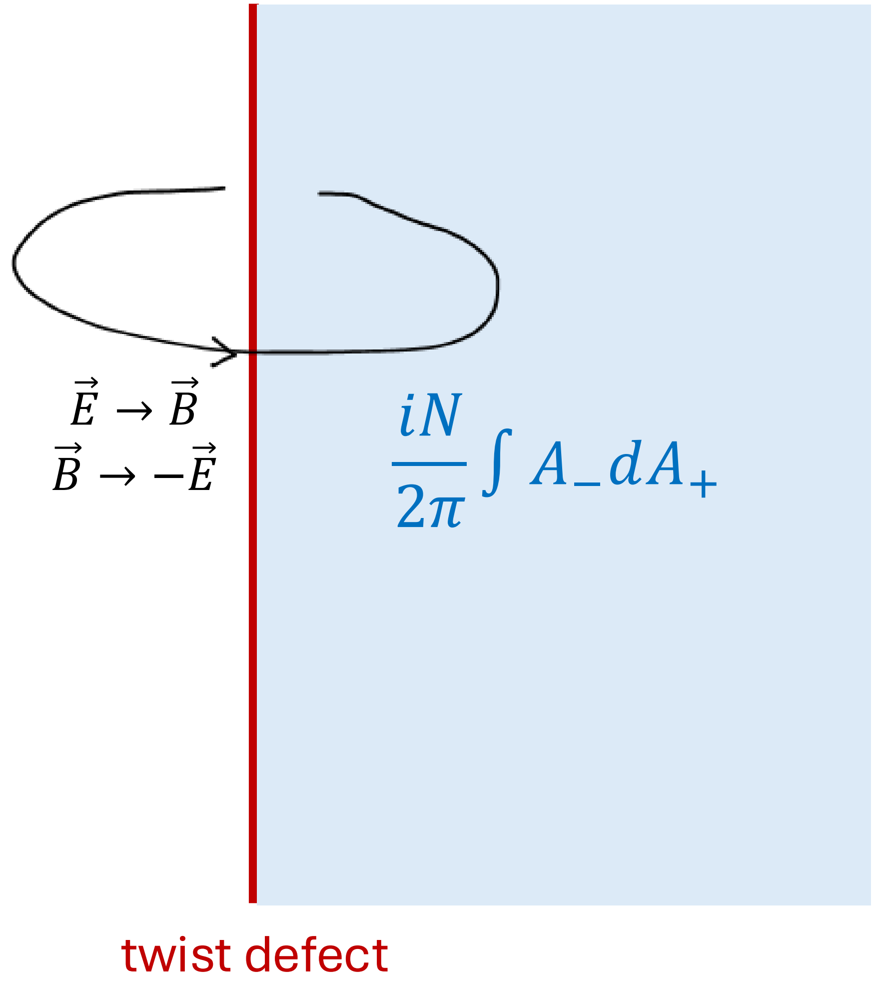
$$P_0 = P_{GFF} + P_{chiral} = \frac{11}{192} - \frac{1}{24} = \frac{1}{64}$$

- Relation to defect gravitational anomaly?



Summary

- We analyze the twist defect for electromagnetic duality in free Maxwell theory.
- It is a conformal surface defect living at the end of the codimension-1 topological defect for a non-invertible global symmetry.
- We identify two sectors of defect operators: (1) generalized free field and (2) chiral boson.
- The first sector is obtained by solving the classical wave equations.
- The second sector is obtained from anomaly-inflow.



Outlook

- What's the b-function for our twist defect? It quantifies the number of dof on the defect.
- There are similar non-invertible global symmetries in 3+1d N=4 super Yang-Mills theory associated with S-duality [Kaidi-Ohmori-Zheng '21,…]. Study their twist defects.
- Can these twist defects exist as cosmic strings in our universe? Even if they exist, they must be extremely rare to avoid producing too many magnetic monopoles…