On TQFTs and Entaglement

Michele Del Zotto

Interfaces and Symmetries workshop

March 4, 2026 — YITP - Kyoto

UPPSALA UNIVERSITET

Center for Geometry and Physics

Vetenskapsrådet

SCGCS

European Research Council
Established by the European Commission

REFERENCES FOR TODAY'S SEMINAR

- MAIN RESULT FROM JOINT WORK WITH PAVEL PUTROV (ICTP) & ABHIJIT GADDE (TFIR)

arXiv:2602 + followups in preparation

- SEE ALSO MIKE HOPKINS LECTURES
  - @ PI IN 2022
  - @ HARVARD IN 2025

  WE ARE WRITING NOTES OF THOSE AS WELL...

- KITAEV, PRESKILL 2006
- LEVIN, WEN 2006

ARE THE FIRST EXAMPLES

COMMERCIAL BREAK

We are organizing a thematic programme at the Institut Henri Poincaré in **Paris** this summer!

Conference **Symmetries 26** on June 22-26, 2026

You are all welcome! There is overlap with the topics of this workshop

https://indico.math.cnrs.fr/event/14721/

EXECUTIVE SUMMARY

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CONSIDER GAPPED QUANTUM SYSTEMS

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GROUND STATES → ENTANGLEMENT PROPERTIES

↓

SPT / TOP. ORDER

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SPT / TOP. ORDER I.E.

TOPOLOGICAL QUANTUM FIELD THEORIES

EXECUTIVE SUMMARY

CONSIDER GAPPED QUANTUM SYSTEMS

GROUND STATES → ENTANGLEMENT PROPERTIES

ARE THERE RELATIONS?

SPT / TOP. ORDER I.E.

TOPOLOGICAL QUANTUM FIELD THEORIES

$\boxed{\text{CLAIM: TQFTs "ARE" ENTANGLEMENT}}$

PLAN OF THIS TALK:

- LIGHTNING REVIEW OF SPTs & TOs
- REVIEW OF MULTIPARTITE ENTANGLEMENT
- MAIN RESULT: THE CLAIM IS TRUE FOR LEVIN-WEN MODELS
- CONCLUSIONS & OUTLOOK

LIGHTNING REVIEW OF SPTs & TOs

CONSIDER QUANTUM SYSTEM

$\mathcal{H}$ - HILBERT SPACE , $\psi \in B_1 \subset \mathcal{H}$ STATES

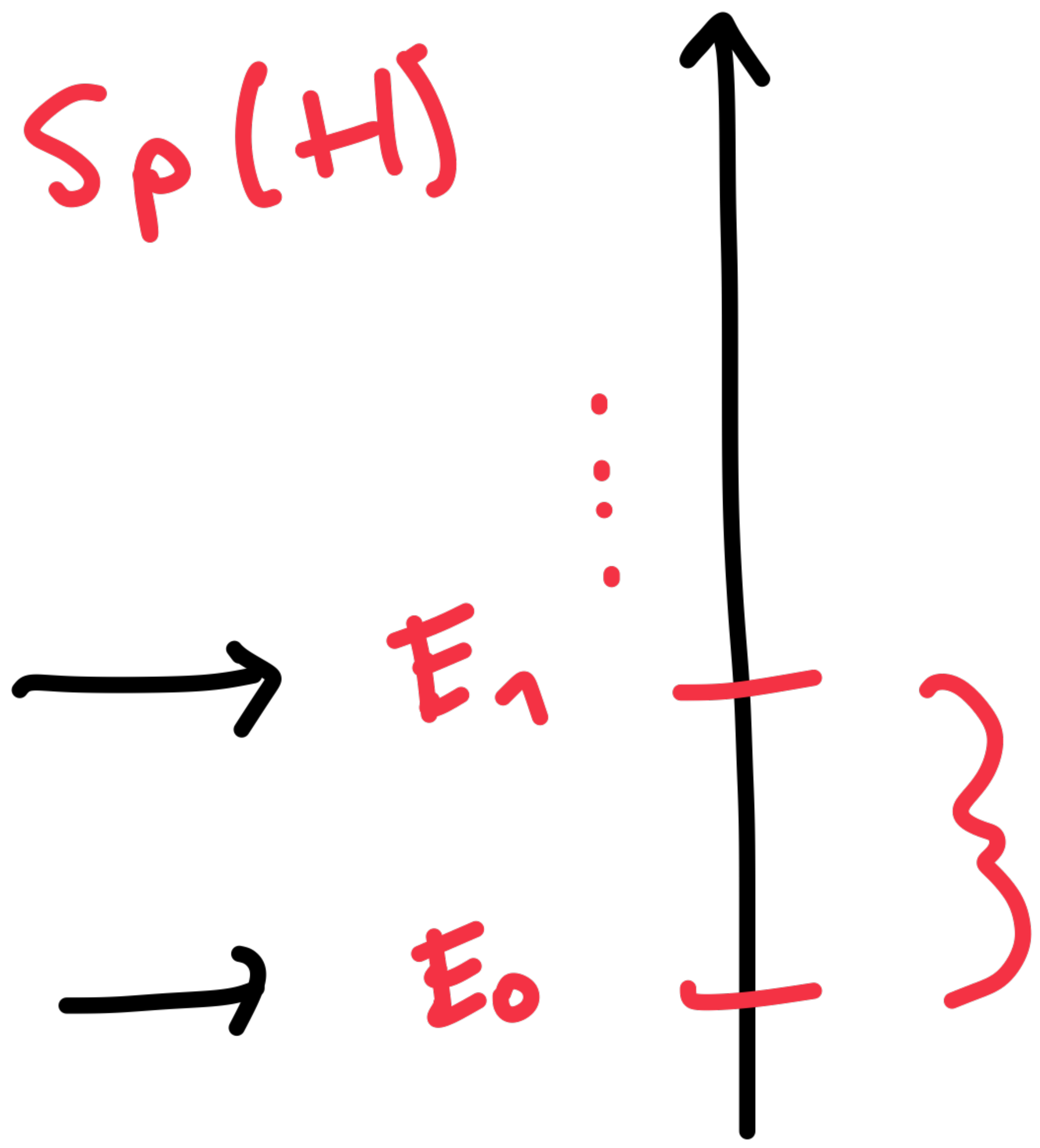
$H \in \mathcal{O}(\mathcal{H})$ HAMILTONIAN : $H^\dagger = H$

$$Sp(H) \subseteq [E_0, +\infty)$$

$$H\psi_i = E_i \psi_i$$

FIRST EXCITED STATE $\longrightarrow$

GROUNDSTATE $\longrightarrow$



$\Delta = E_1 - E_0$

THE GAP

LIGHTNING REVIEW OF SPTs & TOs

ADD PARAMETERS, DEFORMATIONS OF THE HAMILTONIAN

$H_\lambda \in \mathcal{O}(\mathcal{H})$ HAMILTONIAN

λ → PARAMETERS

$Sp(H_\lambda)$

$\vdots$

FIRST EXCITED STATE → $E_1(\lambda)$

$\Delta(\lambda)$ ← THE GAP

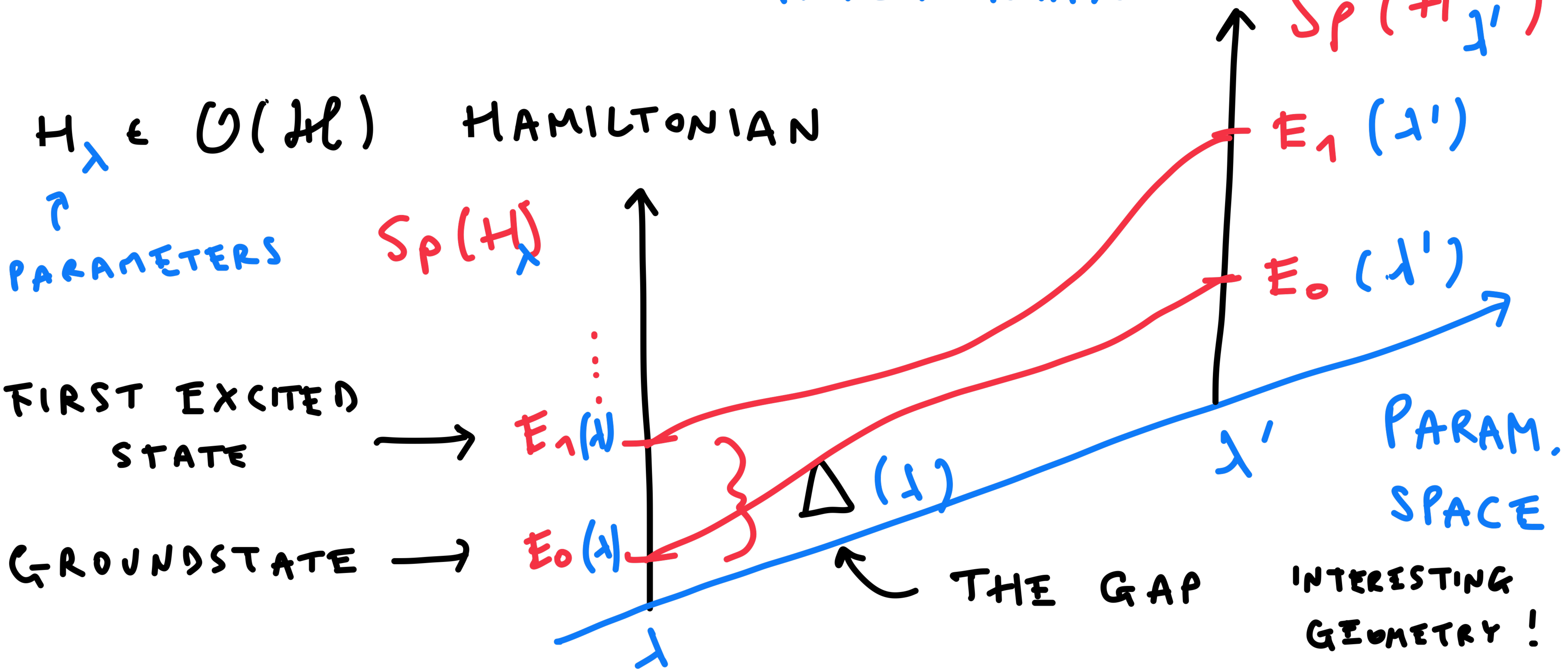
GROUNDSTATE → $E_0(\lambda)$

LIGHTNING REVIEW OF SPTs & TOs

ADD PARAMETERS, DEFORMATIONS OF THE HAMILTONIAN

$H_\lambda \in \mathcal{O}(\mathcal{H})$ HAMILTONIAN

PARAMETERS



LIGHTNING REVIEW OF SPTs & TOs

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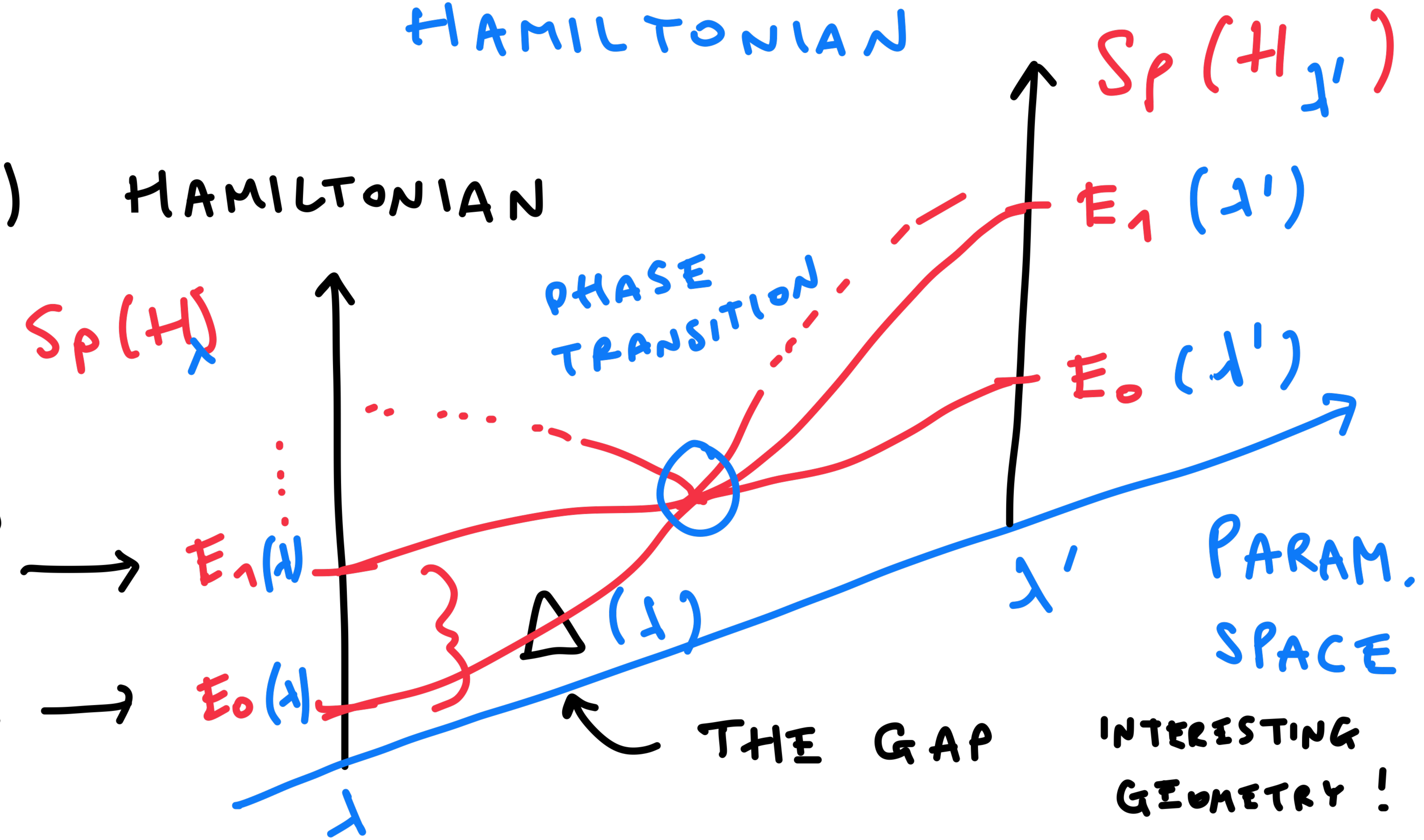
FIRST EXCITED STATE →

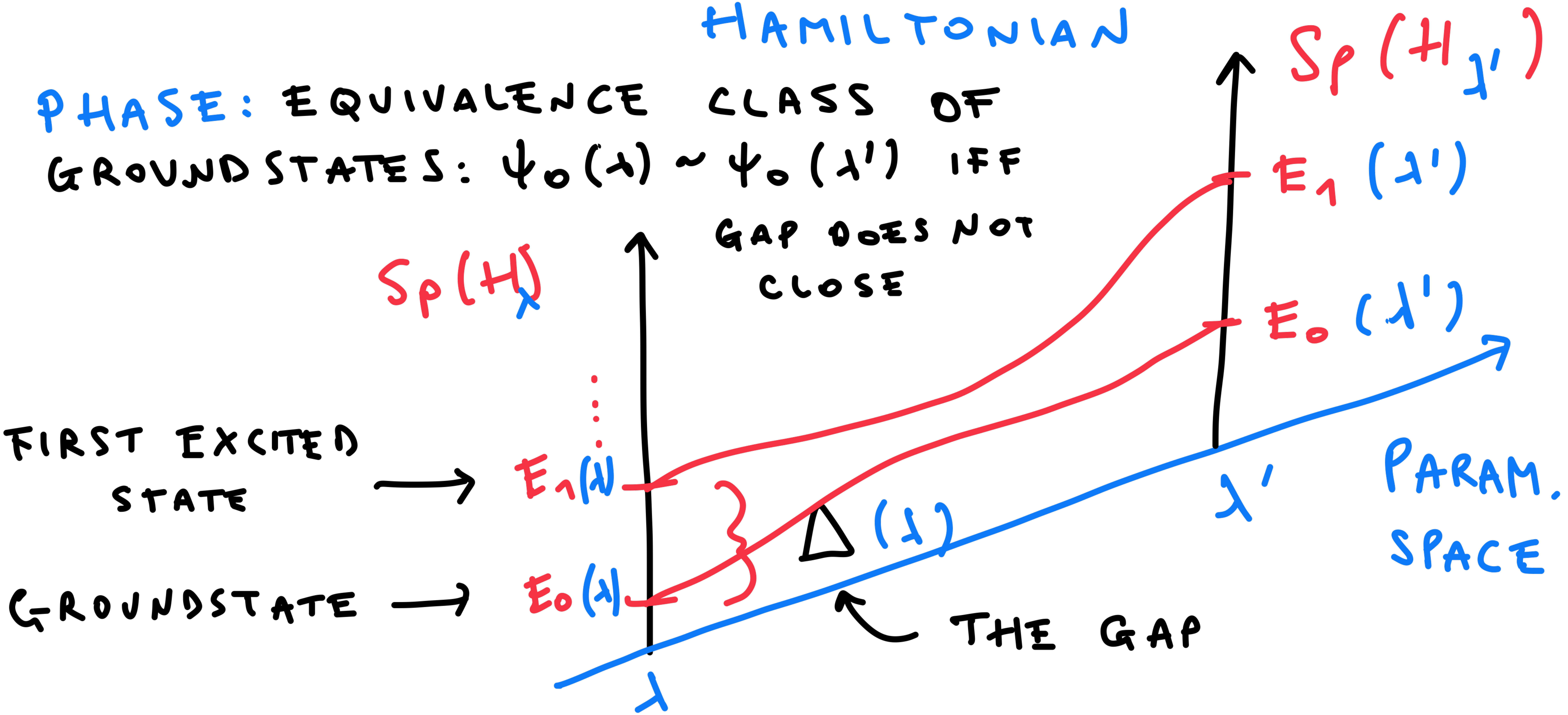
GROUNDSTATE →



LIGHTNING REVIEW OF SPTs & TOs

ADD PARAMETERS, DEFORMATIONS OF THE HAMILTONIAN

PHASE: EQUIVALENCE CLASS OF GROUNDSTATES: $\psi_0(\lambda) \sim \psi_0(\lambda')$ IFF GAP DOES NOT CLOSE



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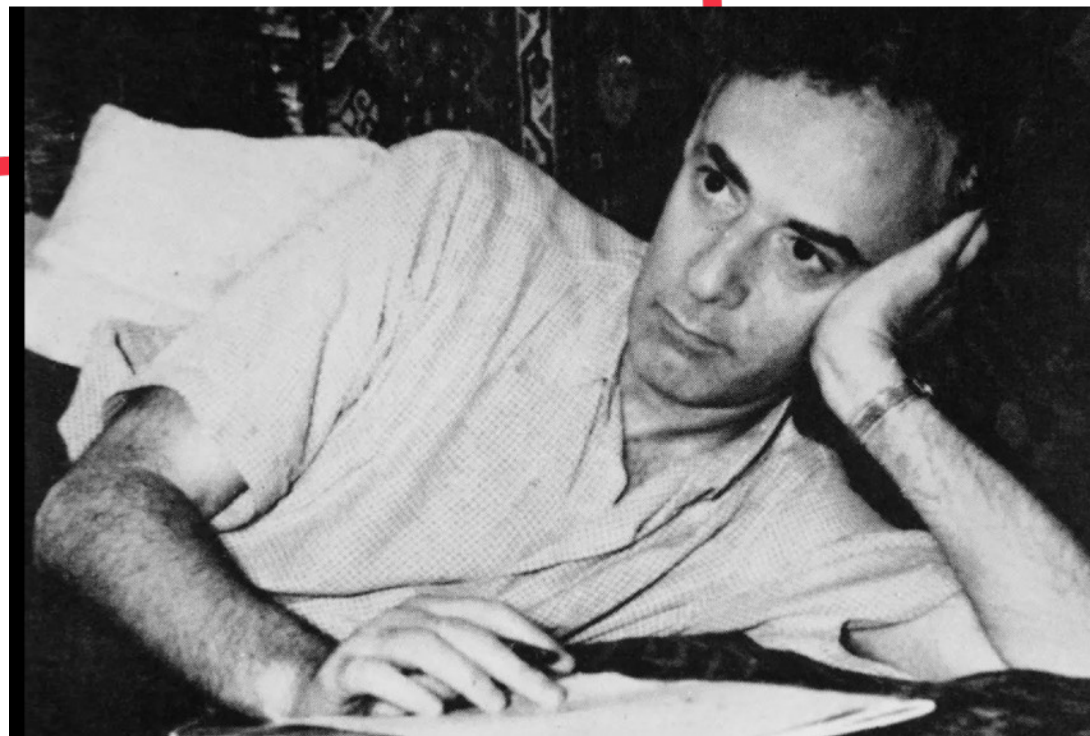
PHASES ARE ORGANIZED BY SYMMETRIES

$$\boxed{\text{THE GAP CANNOT CLOSE UNLESS A SYMMETRY IS VIOLATED}}$$



LIGHTNING REVIEW OF SPTs & TOs

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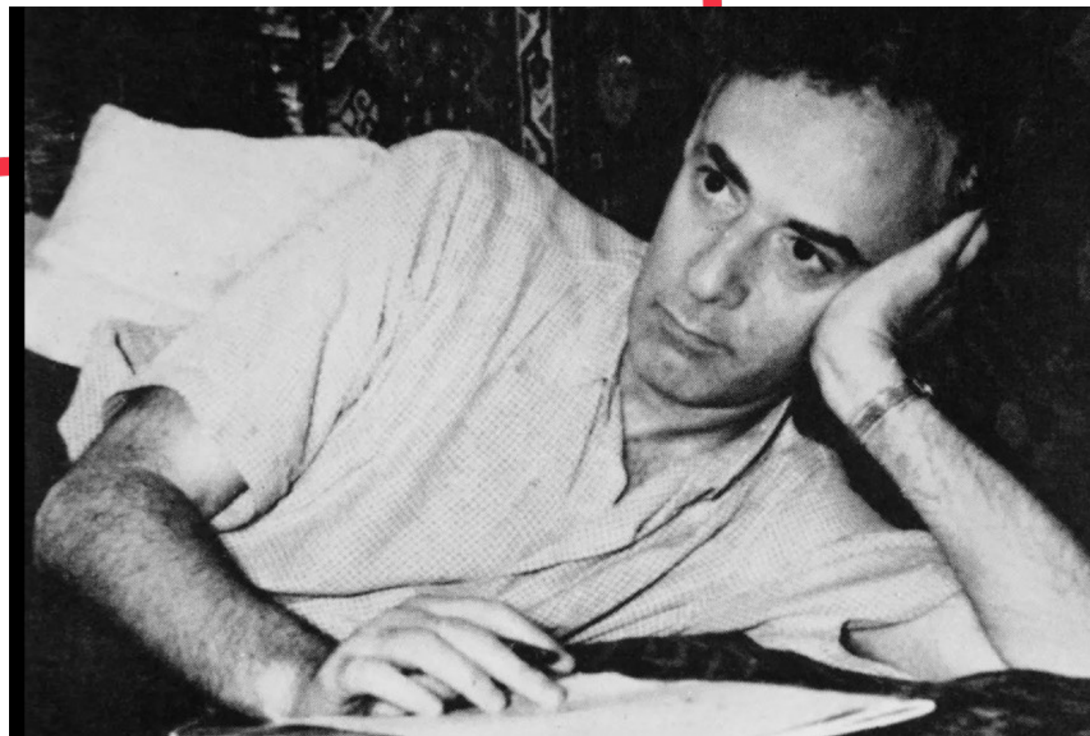
PHASE: EQUIVALENCE CLASS OF GROUNDSTATES $\psi_0(\lambda) \sim \psi_0(\lambda')$ IFF GAP DOES NOT CLOSE

PHASES ARE ORGANIZED BY SYMMETRIES

TWO TYPES:

- SYMMETRY PROTECTED TRIVIAL : SHORT RANGE ENTANGLED
- TOPOLOGICAL ORDER : LONG RANGE ENTANGLED

$\boxed{\text{THE GAP CANNOT CLOSE UNLESS A SYMMETRY IS VIOLATED}}$



LIGHTNING REVIEW OF SPTs & TOs

FROM QFT PERSPECTIVE :

MICROSCOPIC / UV

$\Big\downarrow$ RENORMALIZATION FLOW

MACROSCOPIC / IR

LIGHTNING REVIEW OF SPTs & TOs

FROM QFT PERSPECTIVE :

MICROSCOPIC / UV

RENORMALIZATION FLOW

MACROSCOPIC / IR

FIXED POINTS ARE SCALE INVARIANT

SPACE OF QFTs

LIGHTNING REVIEW OF SPTs & TOs

FROM QFT PERSPECTIVE :

MICROSCOPIC / UV

$\rightsquigarrow$ RENORMALIZATION FLOW

MACROSCOPIC / IR

SCALE INVARIANT

- GAPLESS $\rightsquigarrow$ CFT
- GAPPED $\rightsquigarrow$ TQFT

LIGHTNING REVIEW OF SPTs & TOs

FROM QFT PERSPECTIVE :

MICROSCOPIC / UV

$\rightsquigarrow$ RENORMALIZATION FLOW

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SCALE INVARIANT

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WELL DEFINED MATHEMATICALLY

LIGHTNING REVIEW OF SPTs & TOs

FROM QFT PERSPECTIVE :

MICROSCOPIC / UV

$\rightsquigarrow$ RENORMALIZATION FLOW

MACROSCOPIC / IR

SCALE INVARIANT

RELATION:

GAPPED GROUND STATE

$\Updownarrow$

TQFT

- GAPLESS $\rightsquigarrow$ CFT
- GAPPED $\rightsquigarrow$ TQFT

WELL DEFINED MATHEMATICALLY

LIGHTNING REVIEW OF SPTs & TOs

RELATION:

$$\boxed{\begin{array}{l}\text{GAPPED GROUND STATE} \\ \qquad \updownarrow \\ \qquad\quad \text{TQFT}\end{array}}$$

- GAPPED $\rightsquigarrow$ TQFT

WELL DEFINED MATHEMATICALLY

LIGHTNING REVIEW OF SPTs & TOs

TQFT IS A FUNCTOR

$$Z : \mathrm{Bord}_{\langle \ldots, d-1, d \rangle} \longrightarrow \mathcal{C}$$

$\mathcal{C}$ ← TARGET MULTIFUSION HIGHER CATEGORY

$\mathrm{Bord}_{\langle \ldots, d-1, d \rangle}$ ← HIGHER CATEGORY OF BORDISMS

RELATION:

$$\boxed{\text{GAPPED GROUND STATE} \leftrightarrow \text{TQFT}}$$

LIGHTNING REVIEW OF SPTs & TOs

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TARGET MULTIFUSION HIGHER CATEGORY

HIGHER CATEGORY OF BORDISMS

RELATION:

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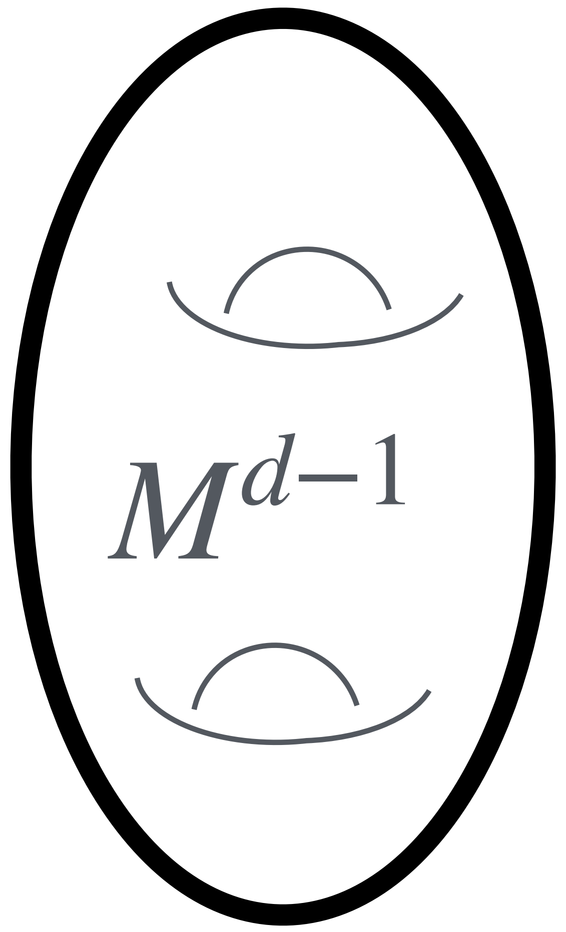
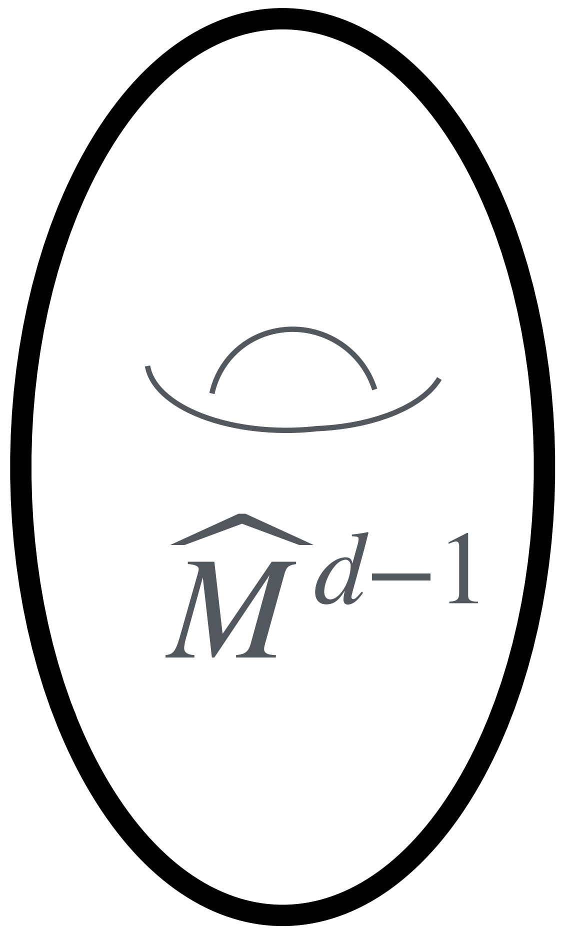


$Z(M^{d-1}) \in \mathrm{Hilb}_{\mathbb{C}}$



$Z(\widehat{M}^{d-1}) \in \mathrm{Hilb}_{\mathbb{C}}$

LIGHTNING REVIEW OF SPTs & TOs

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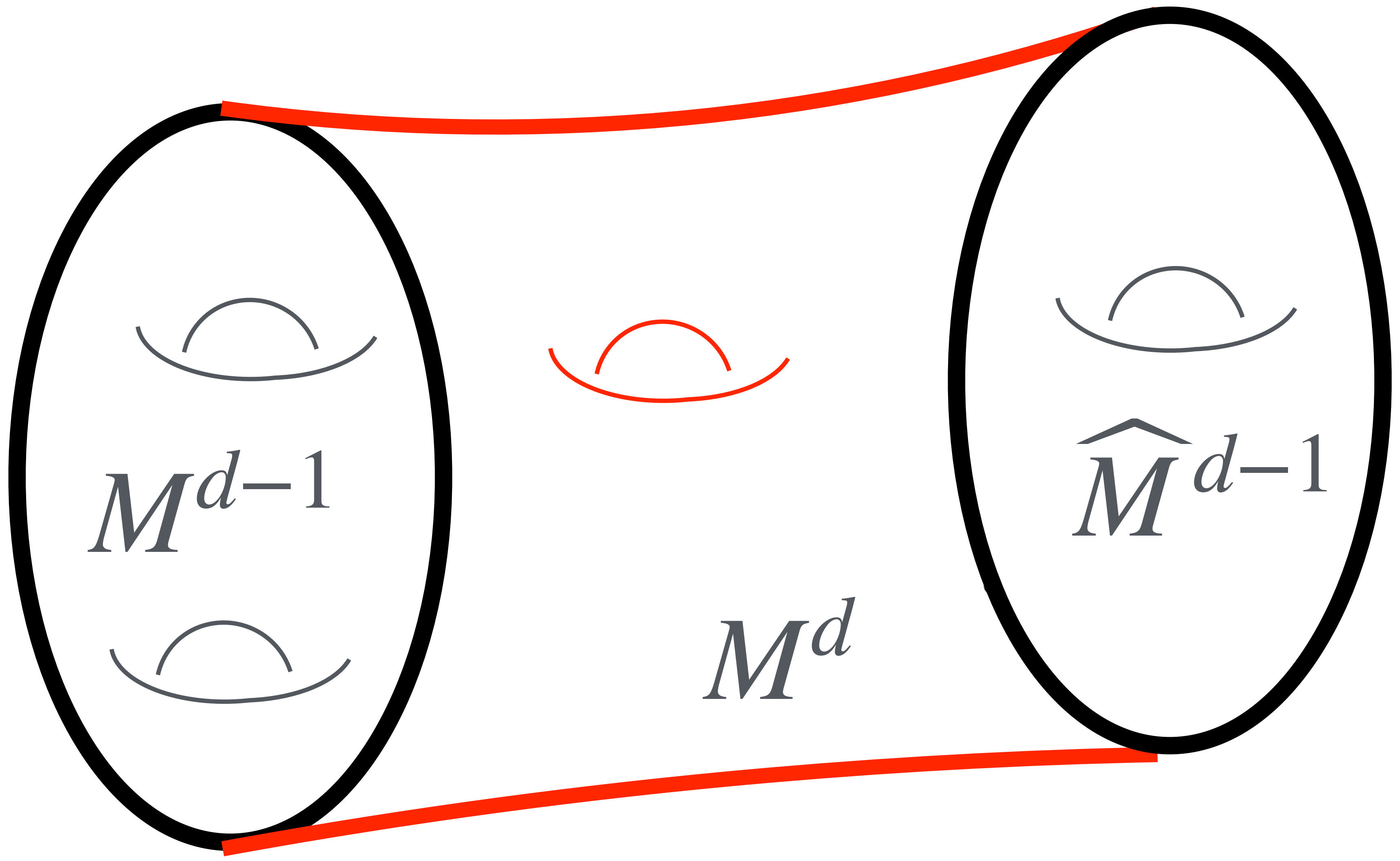

$Z(M^{d-1}) \in \mathrm{Hilb}_{\mathbb{C}}$

$Z(\widehat{M}^{d-1}) \in \mathrm{Hilb}_{\mathbb{C}}$

$Z(M^d) \in \mathrm{Hom}_{\mathbb{C}}(Z(M^{d-1}), Z(\widehat{M}^{d-1}))$

LIGHTNING REVIEW OF SPTs & TOs

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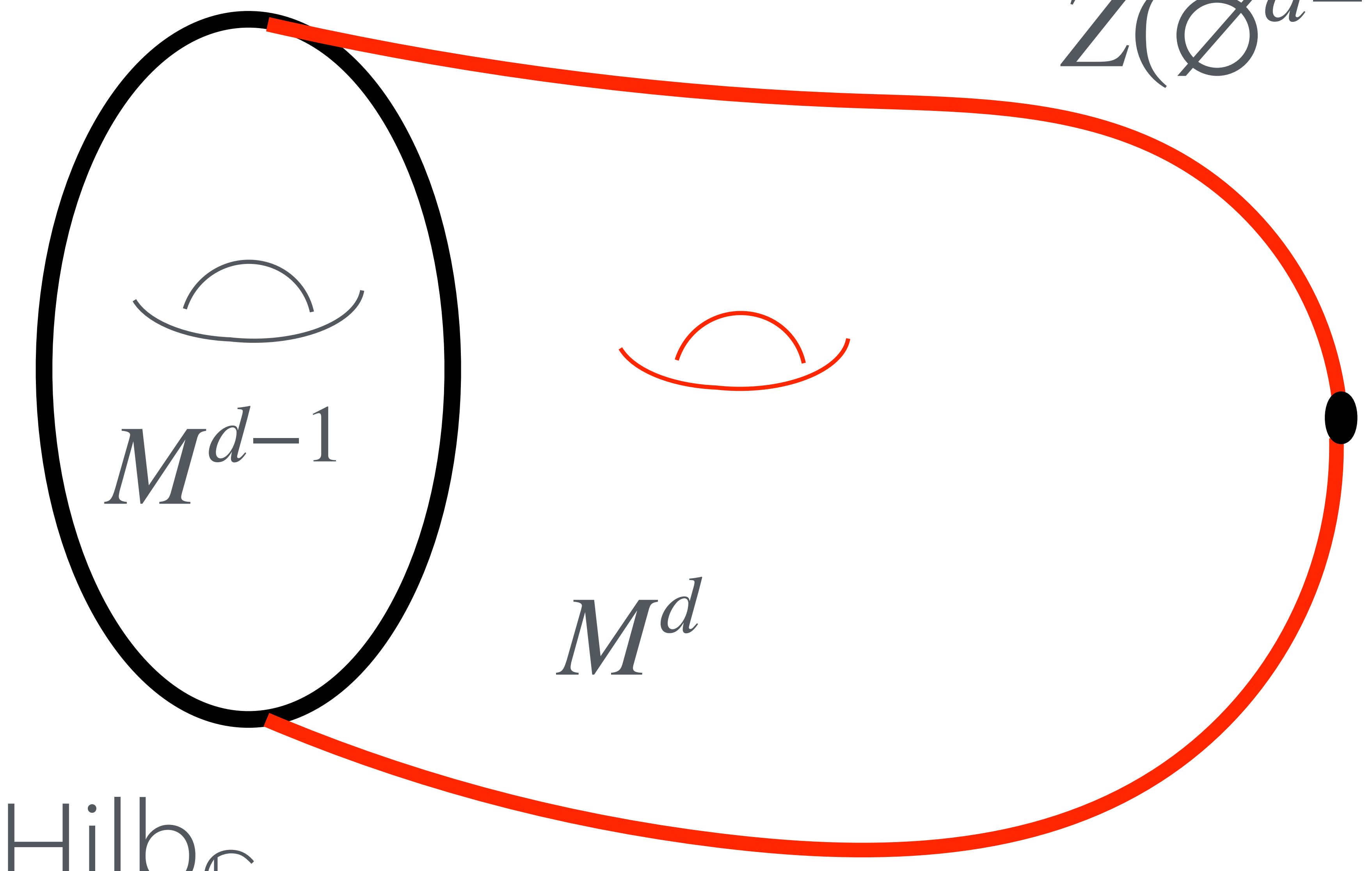
$\mathcal{C}$ ← TARGET MULTIFUSION HIGHER CATEGORY

$\mathrm{Bord}_{\langle \dots, d-1, d \rangle}$ ↑ HIGHER CATEGORY OF BORDISMS

$Z(\varnothing^{d-1}) \simeq \mathbb{C}$



$Z(M^{d-1}) \in \mathrm{Hilb}_{\mathbb{C}}$

RELATION:

GAPPED GROUND STATE ⇕ TQFT

$$Z(M^d) \in \mathrm{Hom}_{\mathbb{C}}(Z(M^{d-1}), \mathbb{C}) \simeq Z(M^{d-1})^*$$

LIGHTNING REVIEW OF SPTs & TOs

TQFT IS A FUNCTOR

$$Z : \mathrm{Bord}_{\langle \dots, d-1, d \rangle} \longrightarrow \mathcal{C}$$

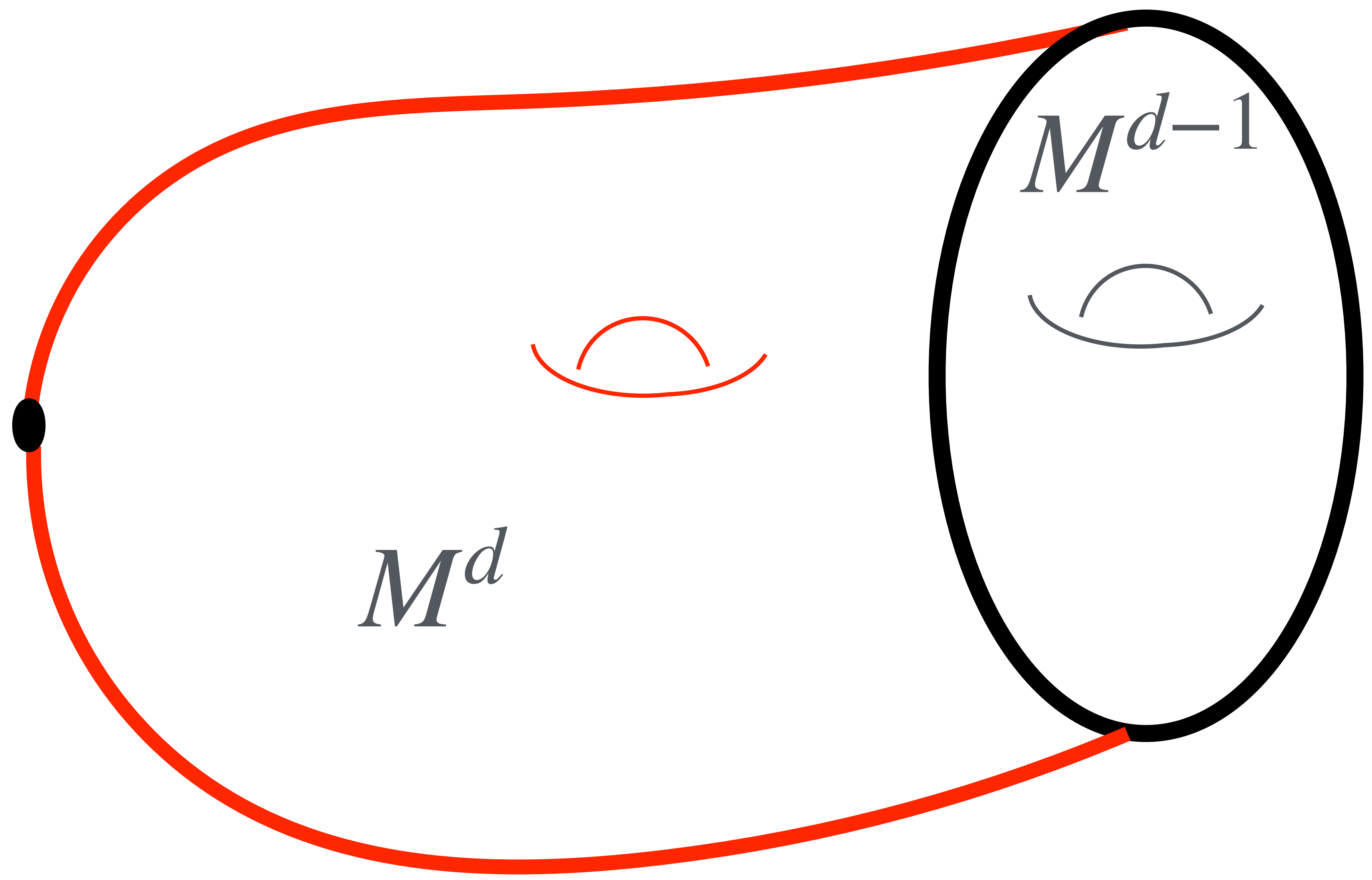
TARGET MULTIFUSION HIGHER CATEGORY

HIGHER CATEGORY OF BORDISMS

RELATION:

GAPPED GROUND STATE $\leftrightarrow$ TQFT

$Z(\emptyset^{d-1}) \simeq \mathbb{C}$



$Z(M^d) \in \mathrm{Hom}_{\mathbb{C}}(\mathbb{C}, Z(M^{d-1}))$

A **state** in $Z(M^{d-1})$

LIGHTNING REVIEW OF SPTs & TOs

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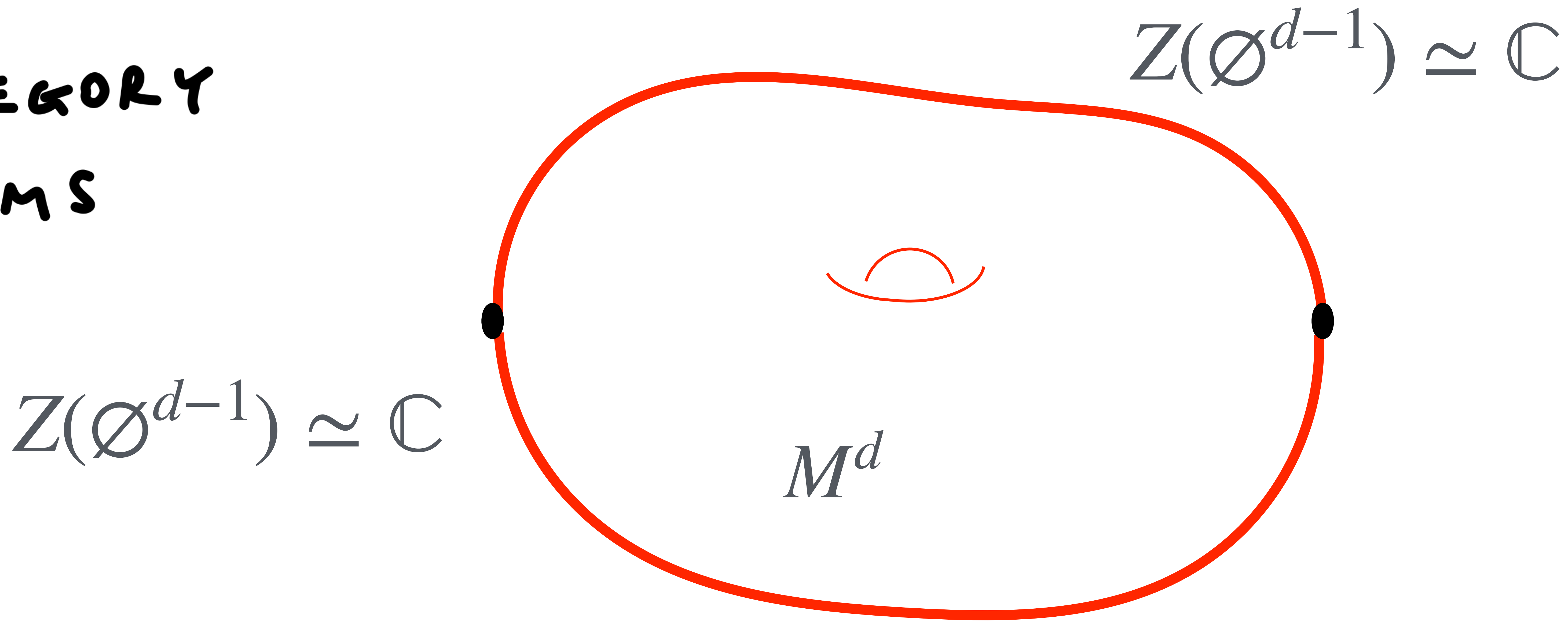

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RELATION:

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LIGHTNING REVIEW OF SPTs & TOs

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TARGET MULTIFUSION HIGHER CATEGORY

HIGHER CATEGORY OF BORDISMS

FOR TODAY:

- $Z(M^{d-1}) \in \mathrm{Hilb}_{\mathbb{C}}$

HILBERT SPACE OF Z ON M^{d-1}

RELATION:

$$\boxed{\begin{array}{c}\text{GAPPED GROUND STATE} \\ \Updownarrow \\ \text{TQFT}\end{array}}$$

LIGHTNING REVIEW OF SPTs & TOs

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  HILBERT SPACE OF Z ON M^{d-1}

- $Z(M^d) \in \mathbb{C}$

  PARTITION FUNCTION OF Z ON M^d COMPACT CLOSED

LIGHTNING REVIEW OF SPTs & TOs

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TQFT

LIGHTNING REVIEW OF SPTs & TOs

FOR TODAY:

- $Z(M^{d-1}) \in \mathrm{Hilb}_{\mathbb{C}}$

  HILBERT SPACE OF Z ON M^{d-1}

IF $\dim Z(M^{d-1}) = 1 \quad \forall M^{d-1}$

$\Rightarrow$ SHORT RANGE ENTANGLED

OTHERWISE, LONG RANGE.

- $Z(M^d) \in \mathbb{C}$

  PARTITION FUNCTION OF Z ON M^d COMPACT CLOSED

GAPPED GROUND STATE

$\Updownarrow$

TQFT

LIGHTNING REVIEW OF SPTs & TOs

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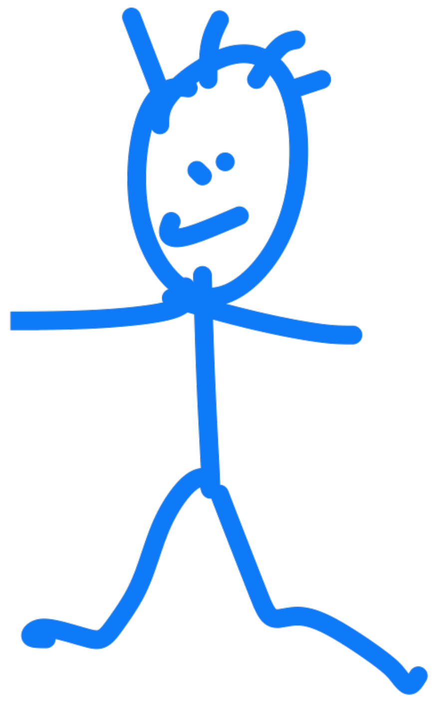
TQFT



BUT WHAT IS THE PHYSICAL INTERPRETATION OF THE TQFT PARTITION FUNCTION?

LIGHTNING REVIEW OF SPTs & TOs

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- $Z(M^{d-1}) \in \mathrm{Hilb}_{\mathbb{C}}$

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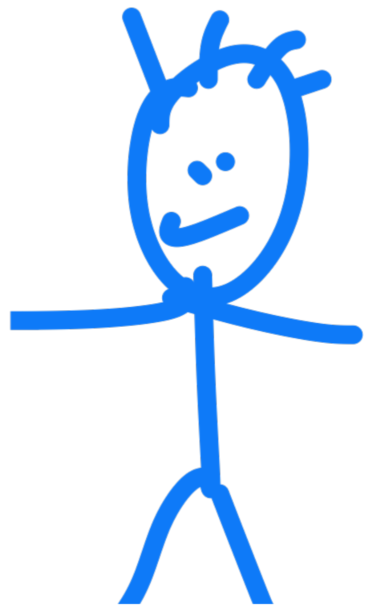
GAPPED GROUND STATE $\Leftrightarrow$ TQFT



BUT WHAT IS THE PHYSICAL INTERPRETATION OF THE TQFT PARTITION FUNCTION?

How can we match infinitely many quantities with only one ground state?

MULTIPARTITE ENTANGLEMENT

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CONSIDER QUANTUM SYSTEM w/ HILBERT SPACE

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \dots \otimes \mathcal{H}_p$$

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p = # PARTIES

MULTIPARTITE ENTANGLEMENT

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$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \dots \otimes \mathcal{H}_p$$

p = # PARTIES

EACH PARTY HAS ORTHONORMAL BASIS

$$\beta_{\alpha_i} \qquad \alpha_i = 1, \dots, \dim \mathcal{H}_i \qquad i = 1, \dots, p$$

$$\forall \Psi \in \mathcal{H}, \quad \Psi = \sum_{\alpha_i} \psi_{\alpha_1 \dots \alpha_p} \beta_{\alpha_1} \otimes \dots \otimes \beta_{\alpha_p}$$

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ENTANGLEMENT: MEASURE OF HOW MUCH Ψ IS NOT A PRODUCT STATE.

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MULTIPARTITE ENTANGLEMENT

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MULTIPARTITE ENTANGLEMENT

WANT: INFO IN Ψ INVARIANT UPON LOCAL UNITARIES

$$\mathcal{H}_1 \otimes \dots \otimes \mathcal{H}_p \circlearrowleft U_1 \otimes \dots \otimes U_p \qquad U_i \in \mathcal{U}(\mathcal{H}_i)$$

$$\Psi = \sum_{\alpha_i} \psi_{\alpha_1 \dots \alpha_p} \beta_{\alpha_1} \otimes \dots \otimes \beta_{\alpha_p}$$

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USE GRAPHICAL CALCULUS:

$$\Psi = \sum_{\alpha_i} \psi_{\alpha_1 \dots \alpha_p} \beta_{\alpha_1} \otimes \dots \otimes \beta_{\alpha_p}$$

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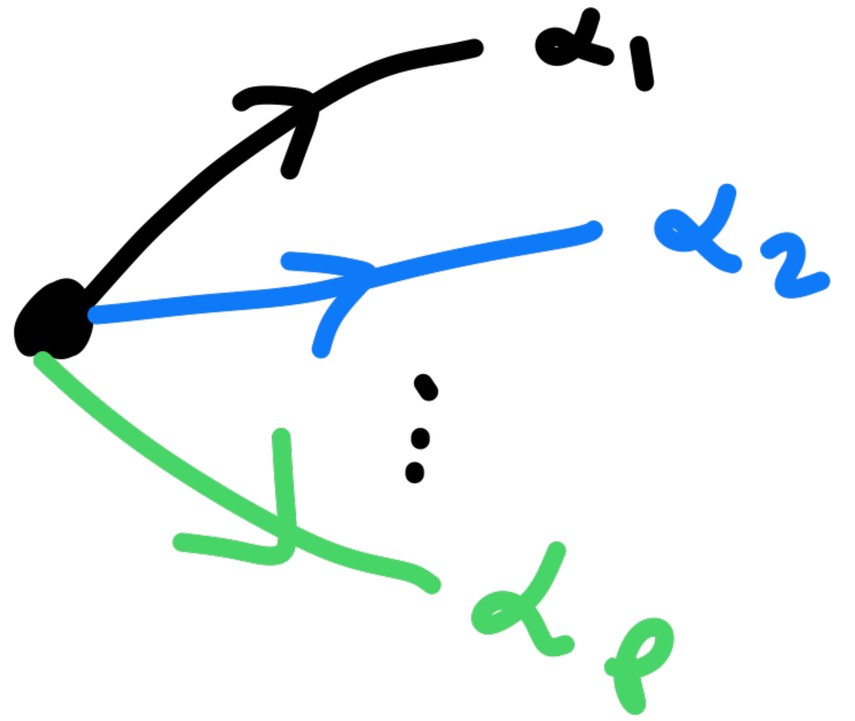

$\psi_{\alpha_1 \dots \alpha_p}$

$$\Psi = \sum_{\alpha_i} \psi_{\alpha_1 \dots \alpha_p} \beta_{\alpha_1} \otimes \dots \otimes \beta_{\alpha_p}$$

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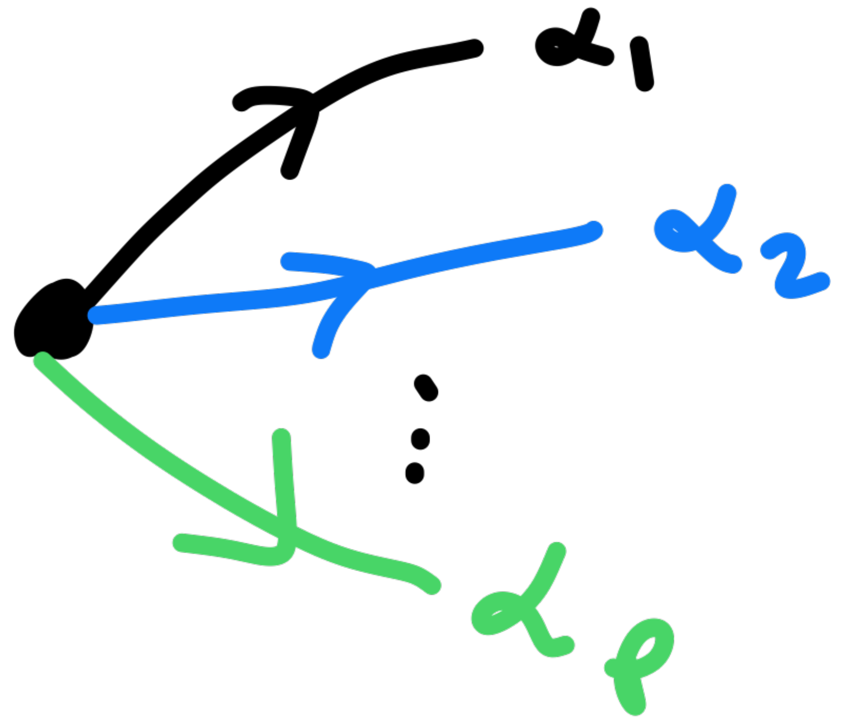
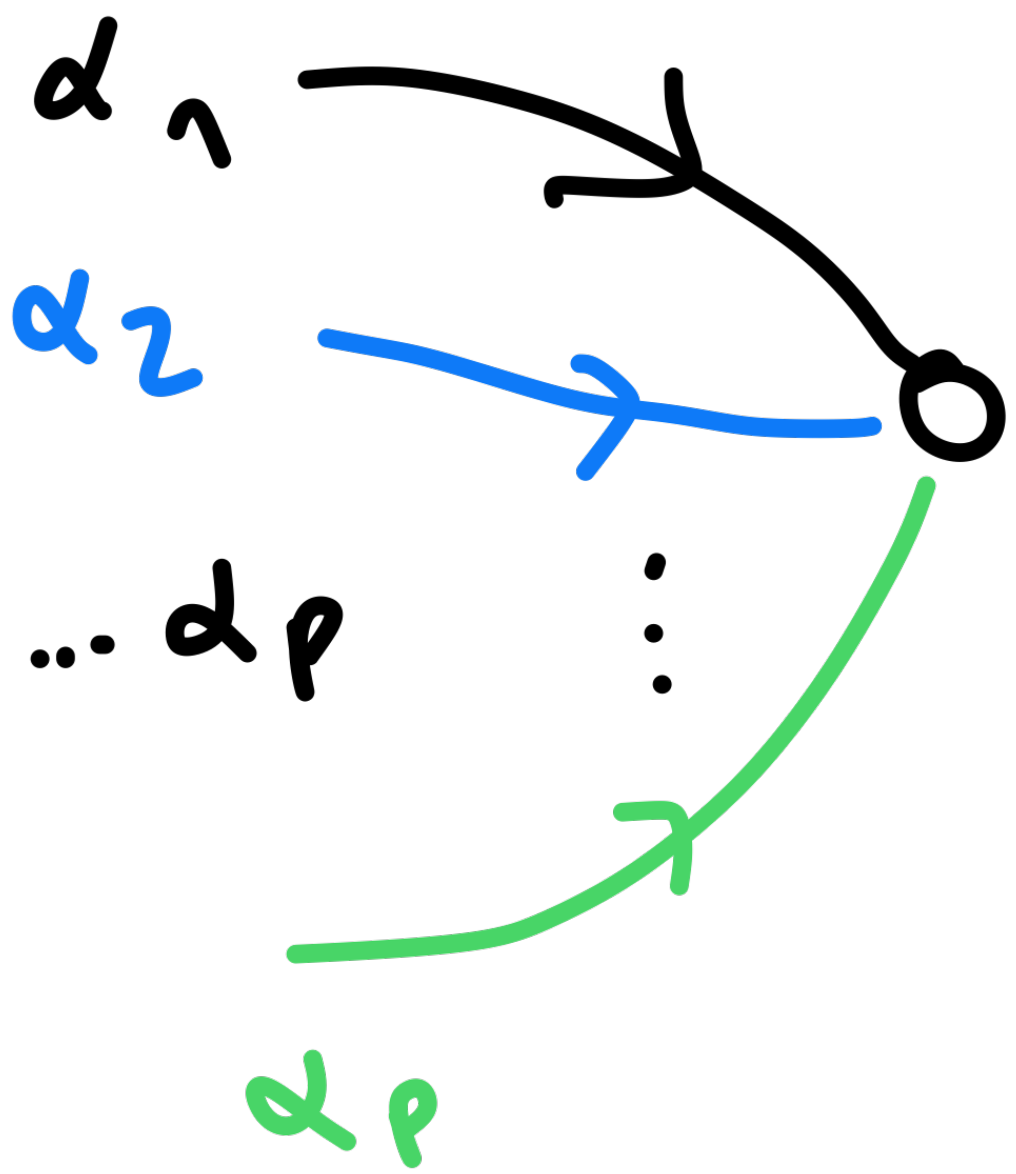


$\psi_{\alpha_1 \cdots \alpha_p} \in \mathcal{H}$



$\bar{\Psi}^{\alpha_1 \cdots \alpha_p} \in \mathcal{H}^*$

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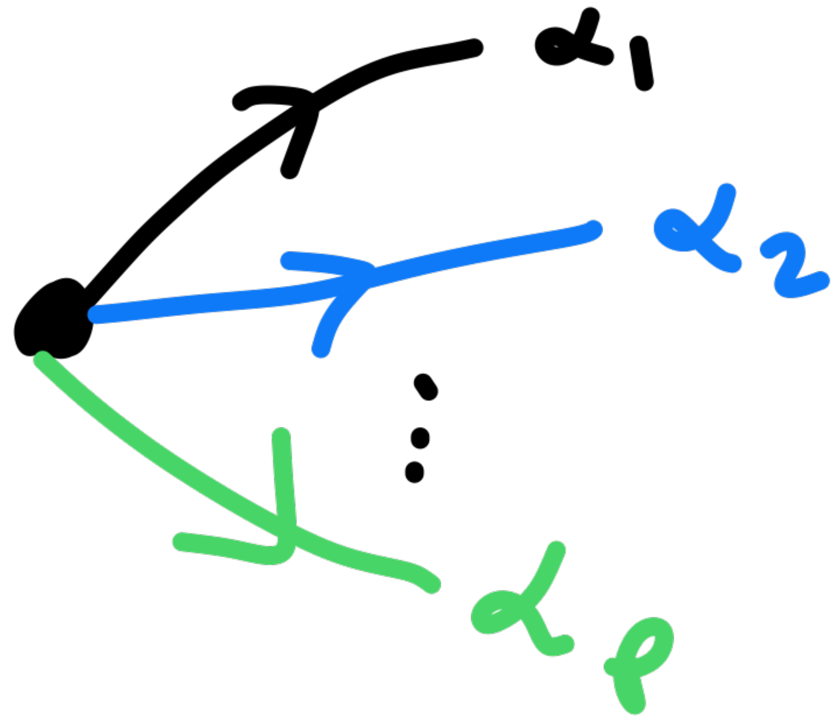

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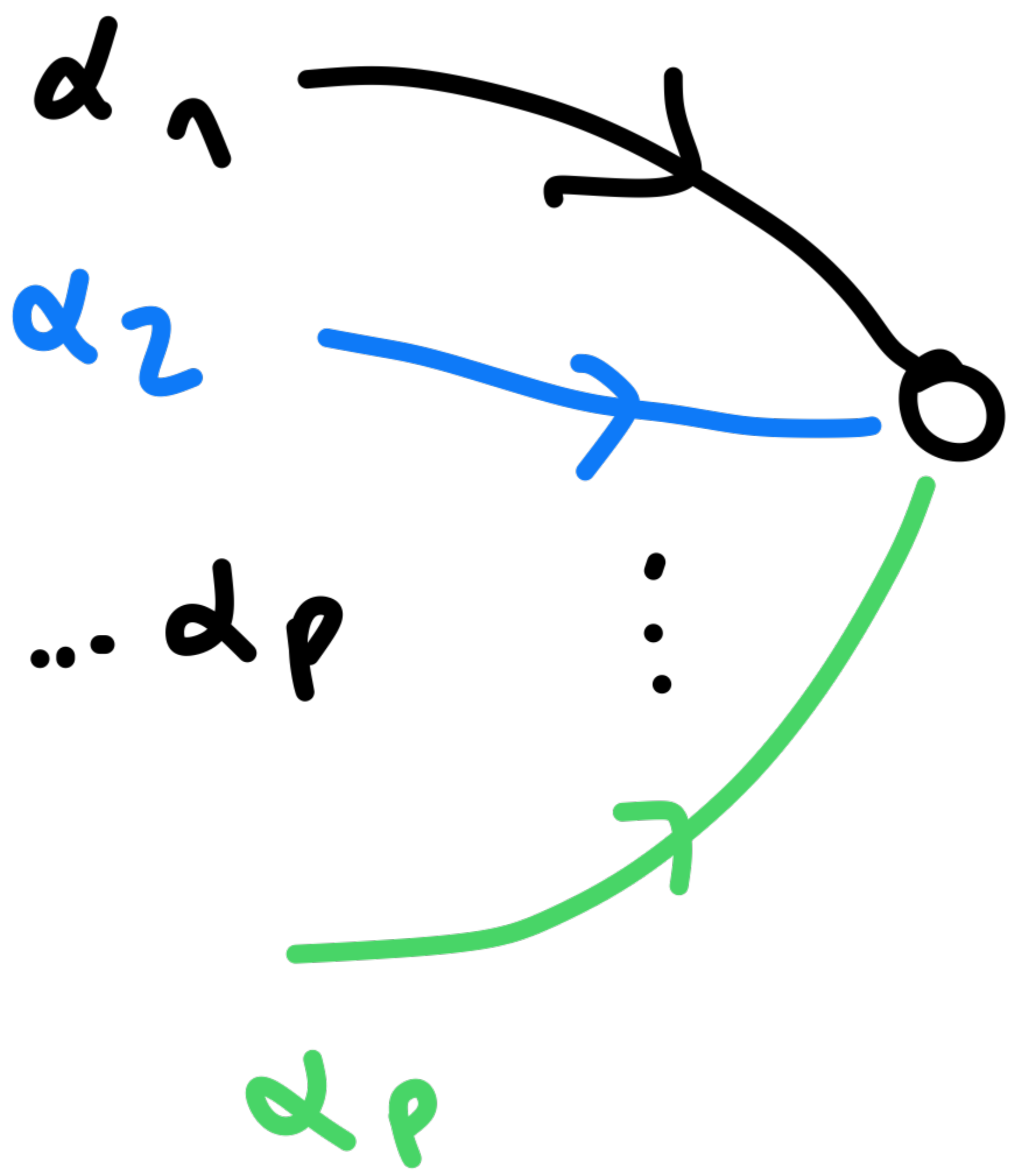

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INVARIANTS OUT OF CONTRACTIONS OF REPLICAS

See Shinsei Ryu's talk

MULTIPARTITE ENTANGLEMENT

EXAMPLE: $P=2$

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$$

$\psi_{\alpha_1 \alpha_2}$ 

$\bar{\psi}^{\alpha_1 \alpha_2}$ 

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$\bullet$ $\circ$ ONE REPLICA

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INVARIANTS OUT OF CONTRACTIONS OF REPLICAS

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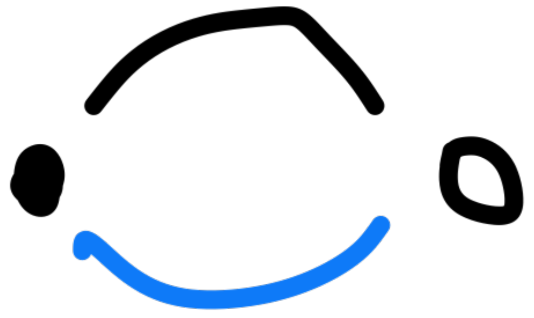 ONE REPLICA

NOT MUCH, JUST $\|\psi\|^2 = \sum_{\alpha_1 \alpha_2} \bar{\psi}^{\alpha_1 \alpha_2} \psi_{\alpha_1 \alpha_2}$

MULTIPARTITE ENTANGLEMENT

EXAMPLE: $P = 2$

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$$

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• ○ TWO REPLICAS

• ○

INVARIANTS OUT OF CONTRACTIONS OF REPLICAS

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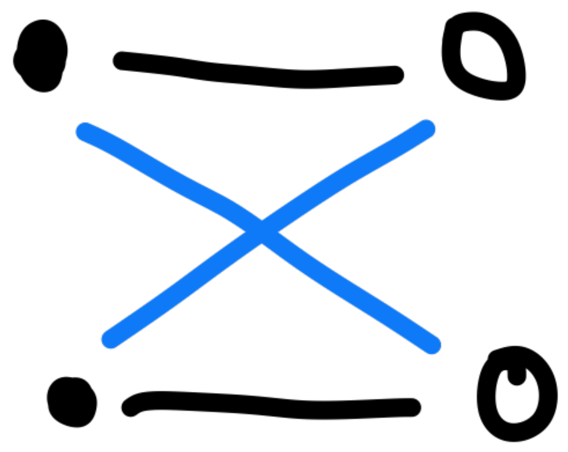

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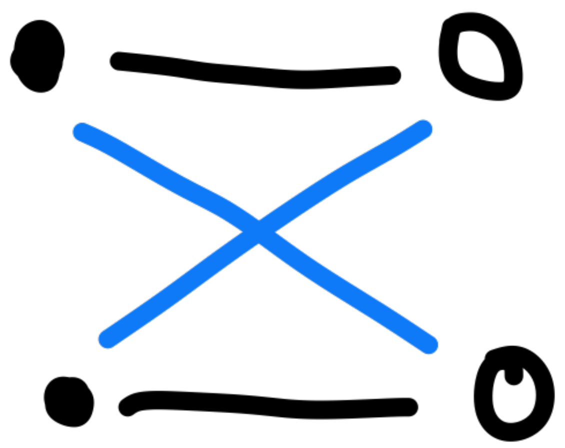

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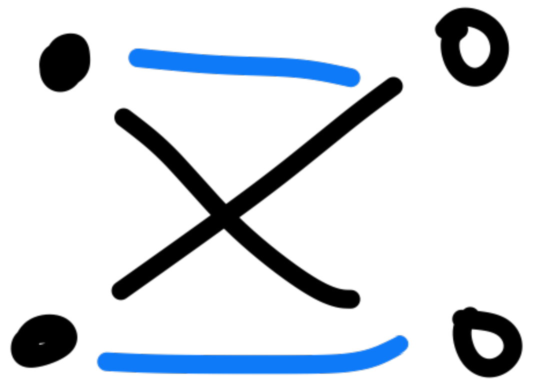

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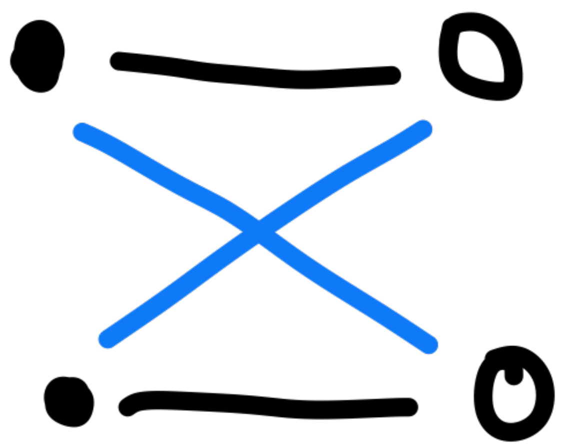
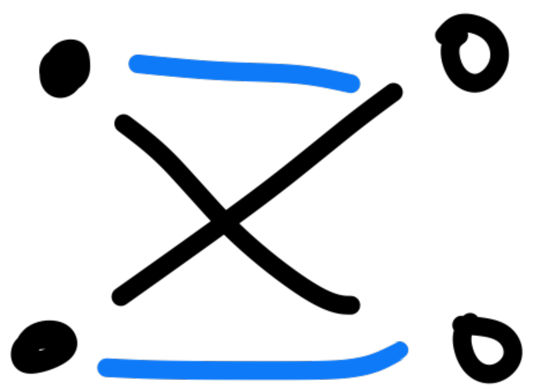
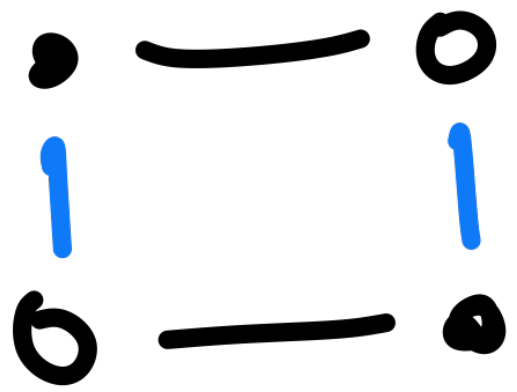
 OR  I.E. 

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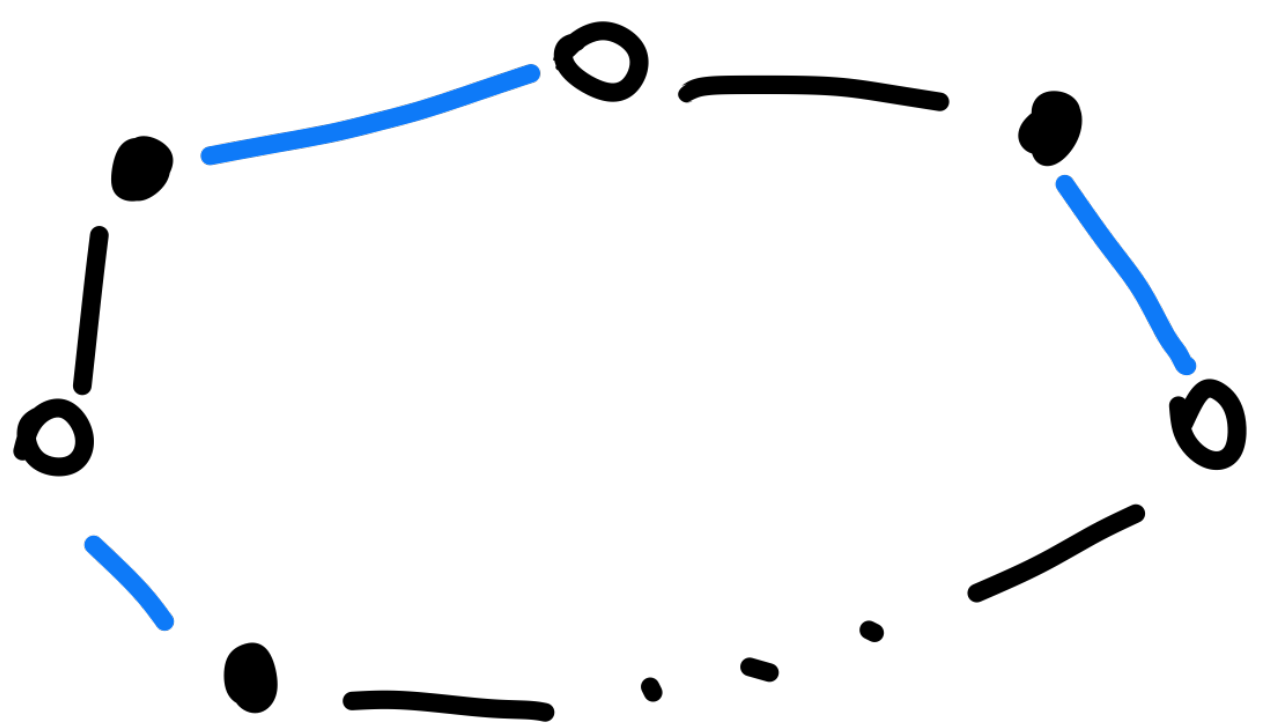

IN FACTS THE ONLY OPTION ARE NECKLACES

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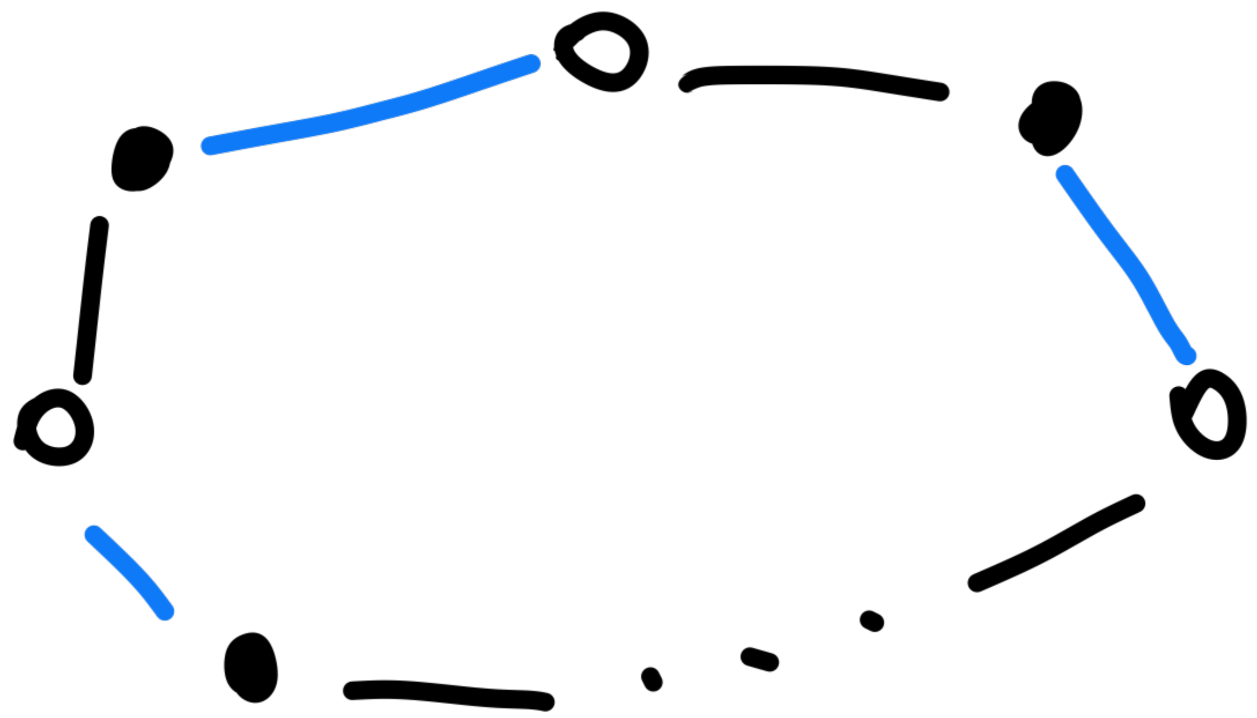
IS THE REDUCED DENSITY MATRIX

$\rho = \mathrm{Tr}_{\mathcal{H}_2}\Pi_\psi$

INVARIANTS OUT OF CONTRACTIONS OF REPLICAS



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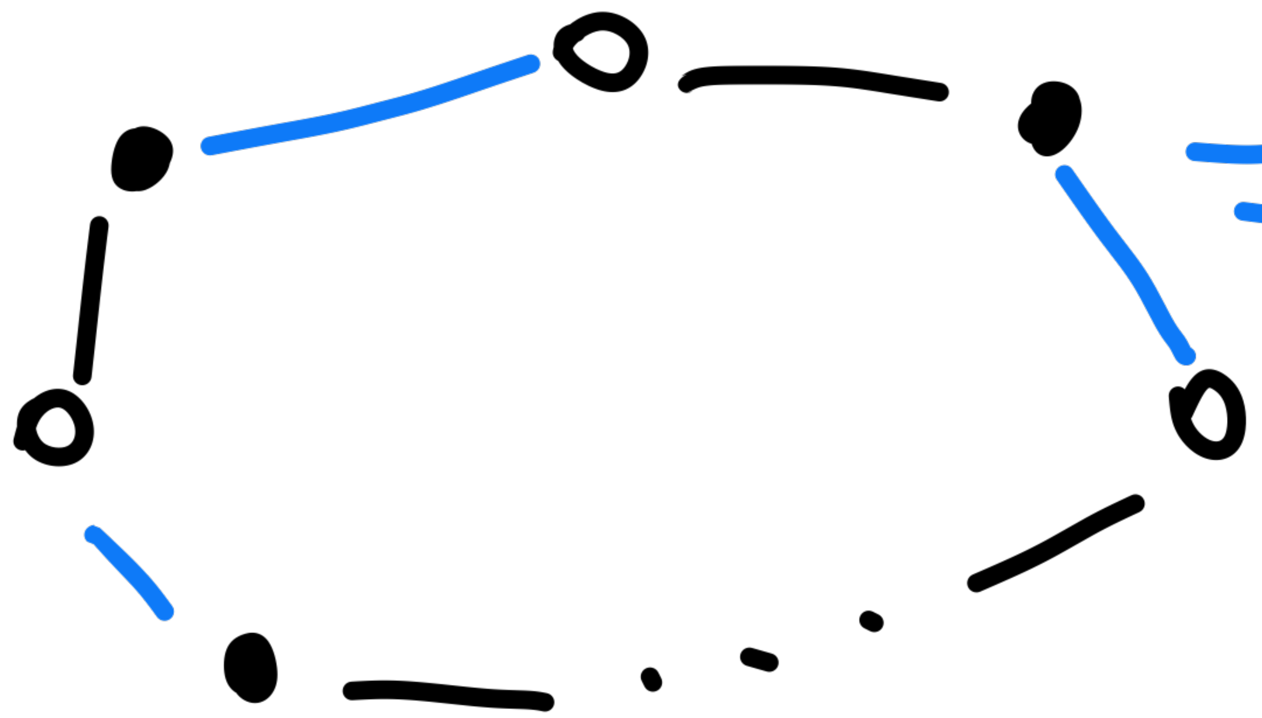

$$= \mathrm{tr}\, \rho^R = W_R^{(2)}$$

R-RÉNYI ENTROPY PARTITION FUNCTION

INVARIANTS OUT OF CONTRACTIONS OF REPLICAS

MULTIPARTITE ENTANGLEMENT

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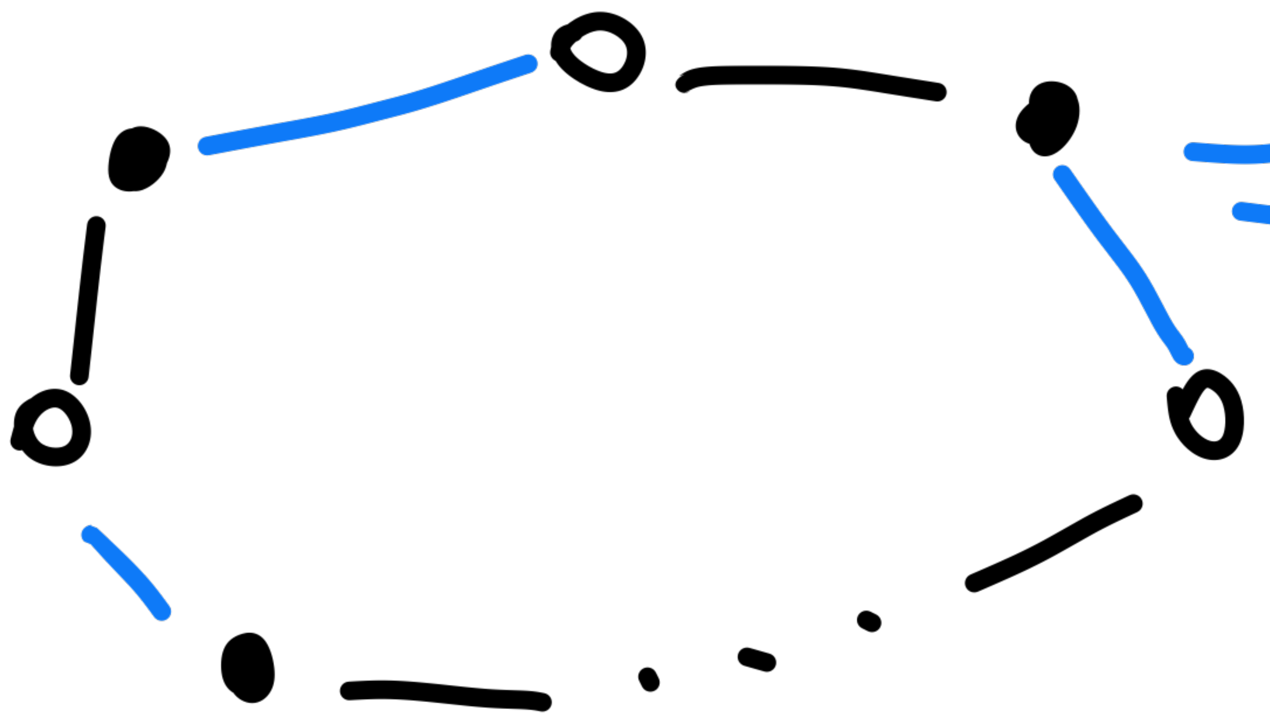

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R-RÉNYI ENTROPY

PARTITION FUNCTION

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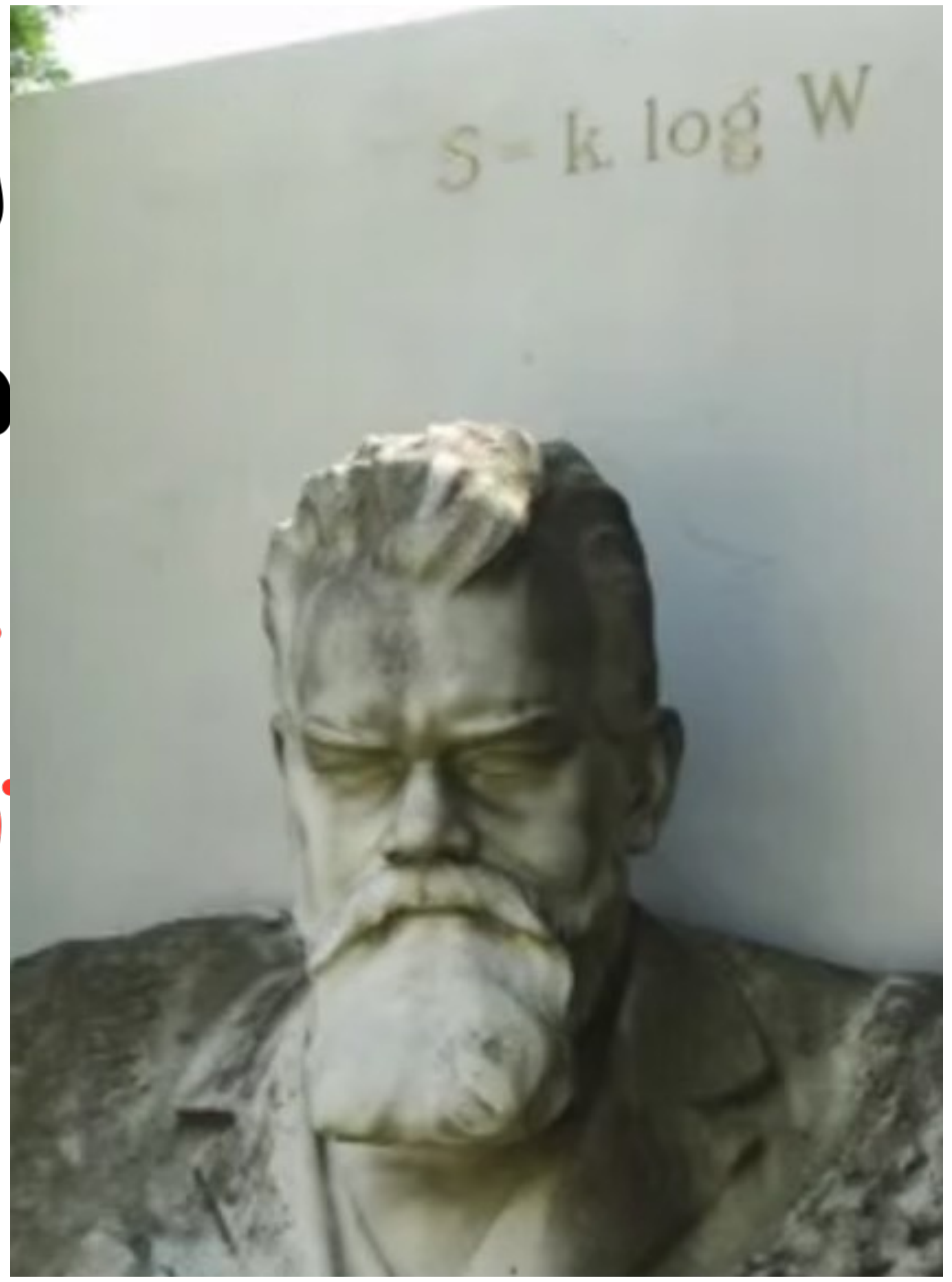

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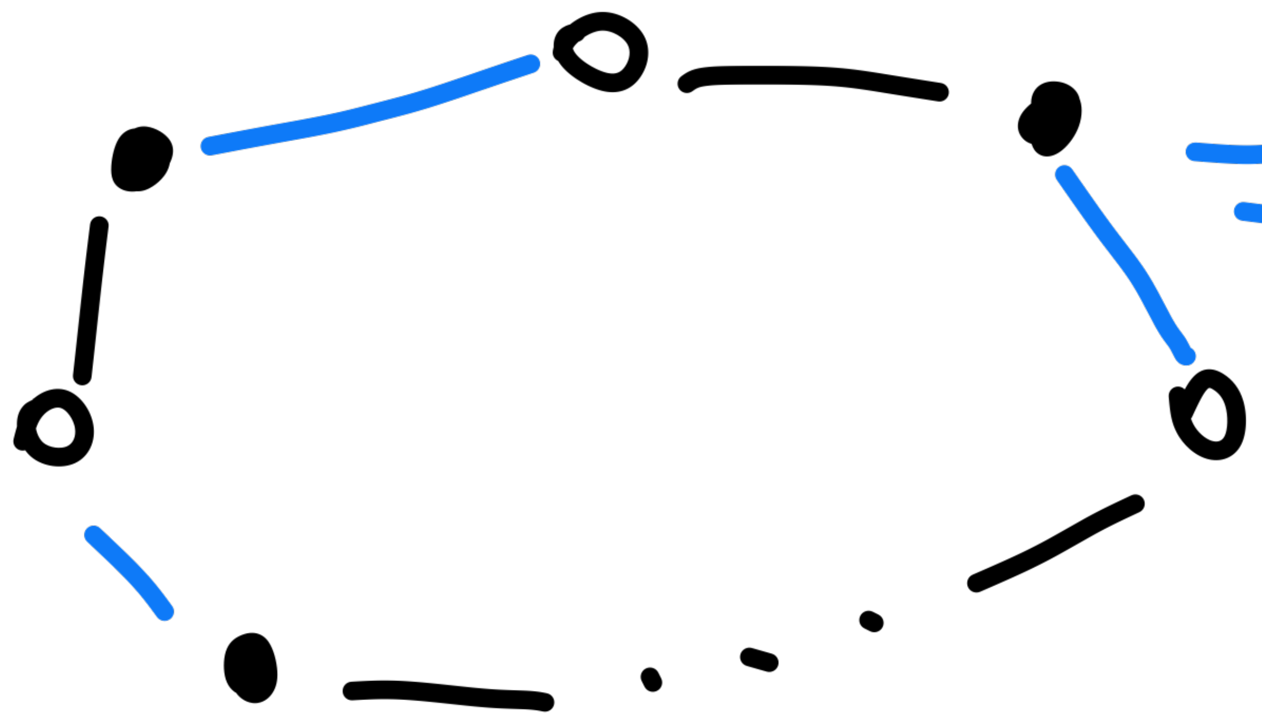

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PARTITION FUNCTION

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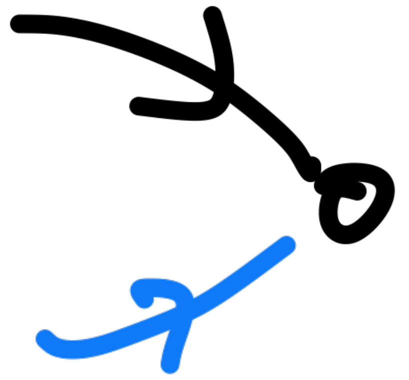

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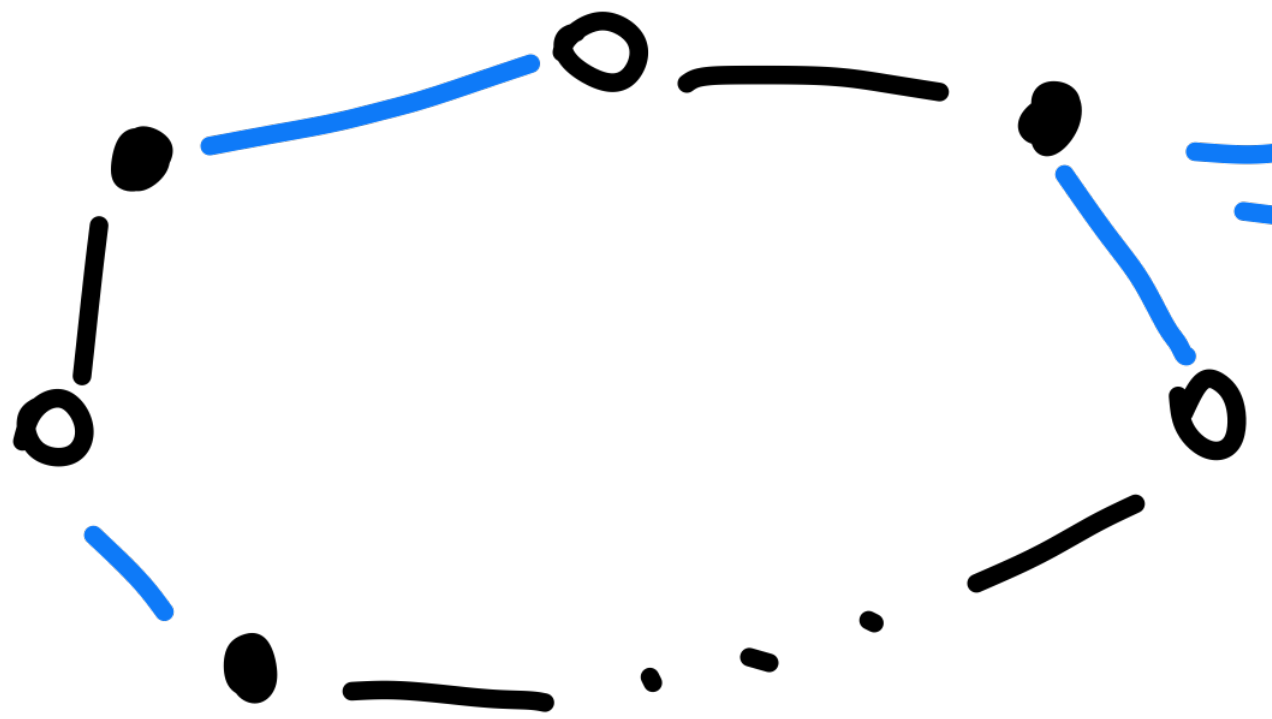

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R-RÉNYI ENTROPY

PARTITION FUNCTION

RMK: $W_R^{(2)}$ ARE THE ONLY BIPARTITE INVARIANTS

$$S_{VN} = \lim_{R\to 1} \frac{1}{1-R} \log W_R^{(2)}$$

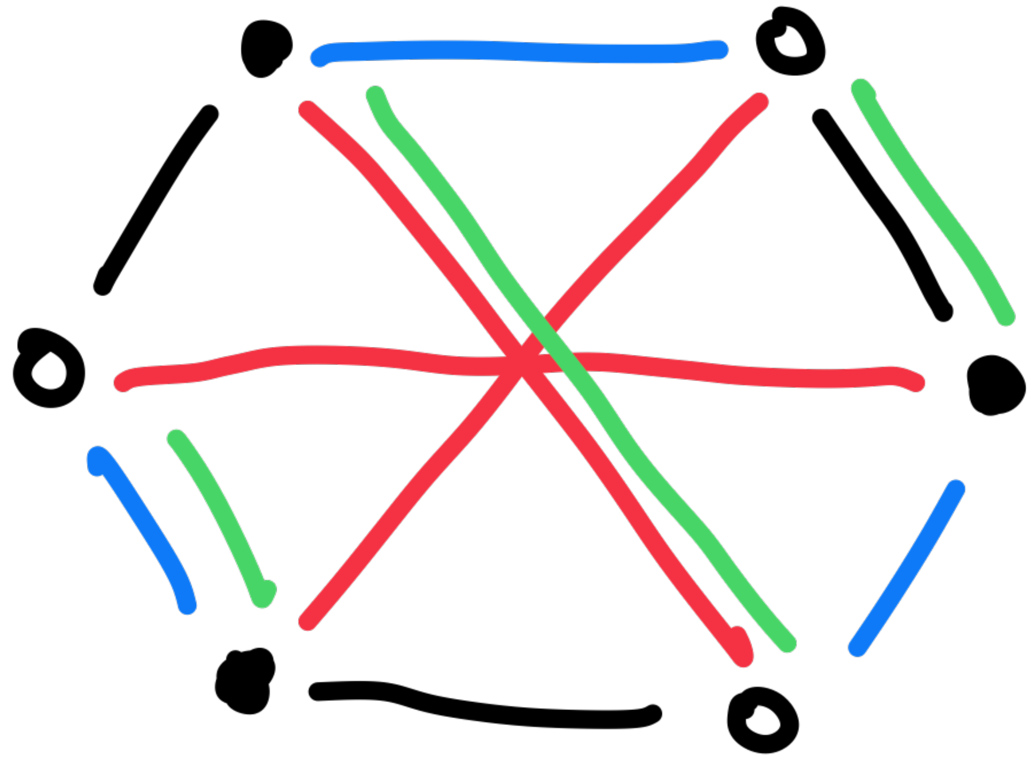
VON NEUMANN ENTROPY

MULTIPARTITE ENTANGLEMENT

IN GENERAL : BI-PARTITE P-VALENT GRAPHS

$\Updownarrow$

P-PARTITE ENTANGLEMENT INVARIANTS



$P = 4 \quad R = 3$

$$\boxed{W_{\Gamma}^{(p)}(\Psi)}$$

RMK 1 : P-PARTITE GENERALIZATIONS OF RENYI ENTROPIES.

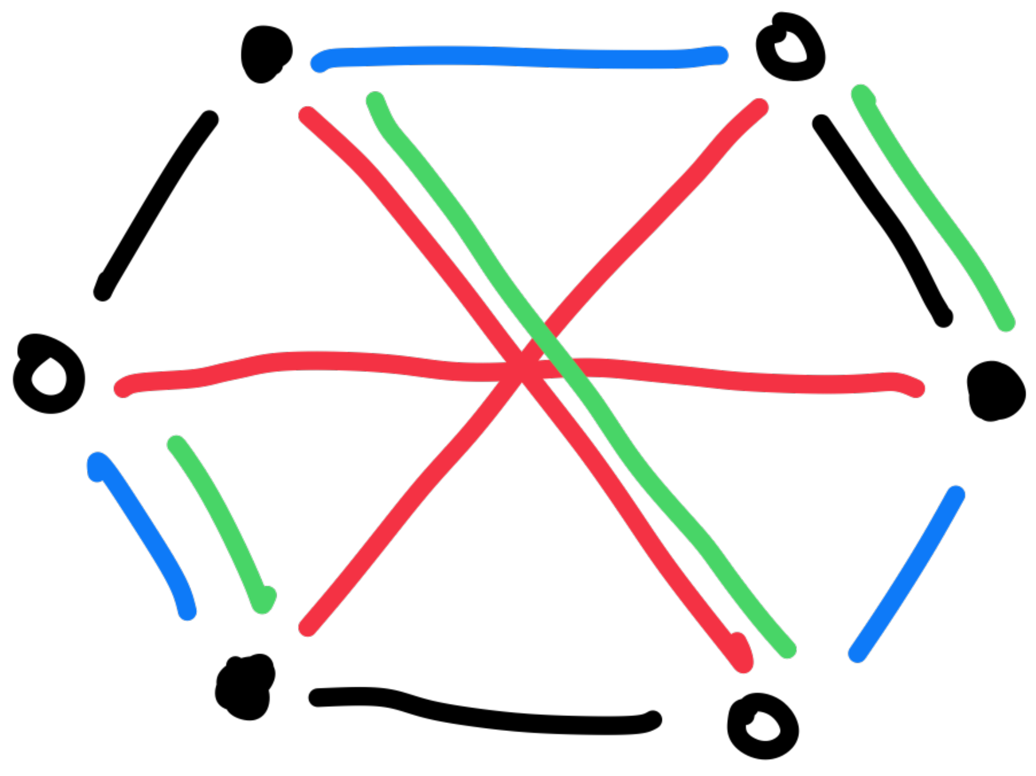
$$\frac{\#\text{ NODES } \Gamma}{2} = \#\text{ REPLICAS}$$

MULTIPARTITE ENTANGLEMENT

IN GENERAL: BI-PARTITE P-VALENT GRAPHS

$\updownarrow$

P-PARTITE ENTANGLEMENT **INVARIANTS**



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RMK 2:

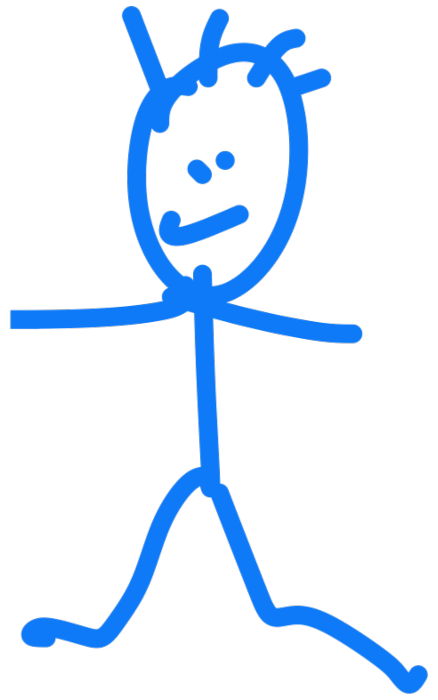
MORE REFINED QUANTITIES ARE OBTAINED IMPOSING FURTHER REQUIREMENTS (VANISHING, ADDITIVITY, ...)

See Ryu's talk for more examples

TQFT FROM ENTANGLEMENT

GAPPED GROUND STATE Ψ_0 $\longleftrightarrow$ TQFT Z

$$\boxed{\log W_{\Gamma}^{(d+1)}(\Psi_0) = \log Z(M_{\Gamma}^{d}) + \dots}$$



WHERE M_{Γ}^{d} IS A d-MANIFOLD ENCODED BY THE GRAPH Γ

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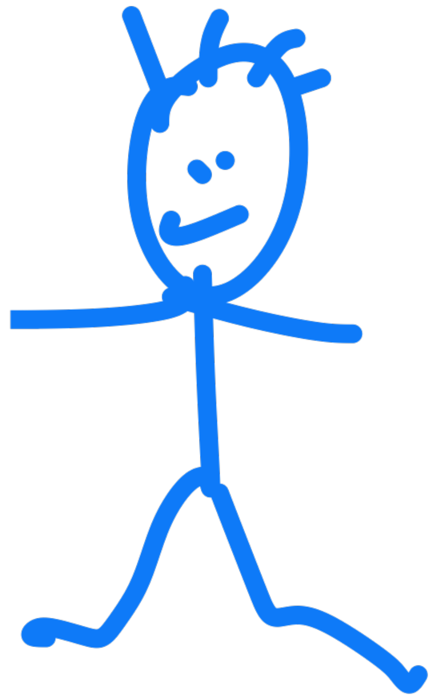

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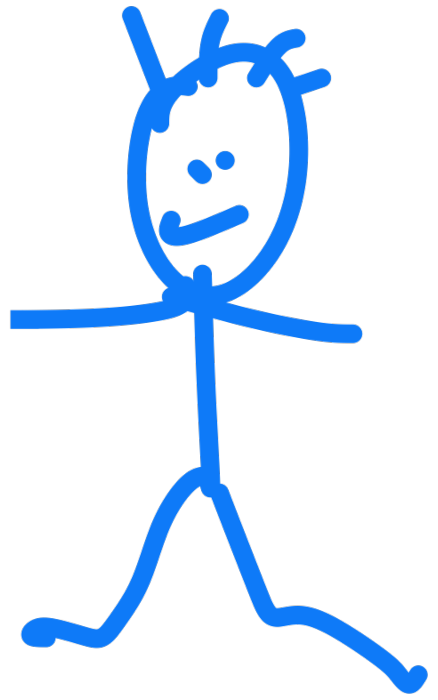

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Unifies many previous results

Kitaev-Preskill 2006, Levin-Wen 2006, ... lots and lots! Also related: Sheffer-Fan-Stern-Berg-Ryu 2025

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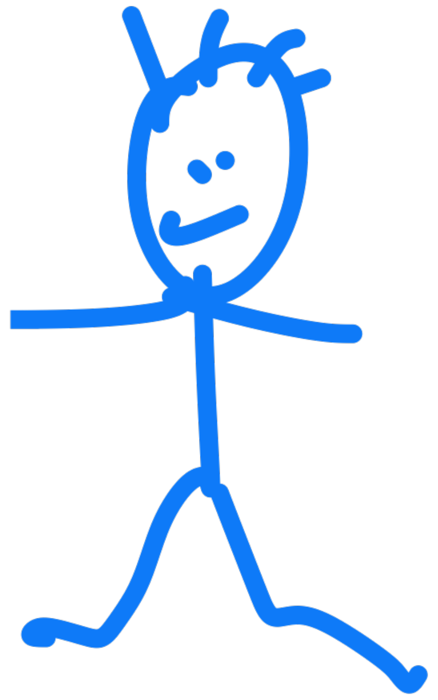

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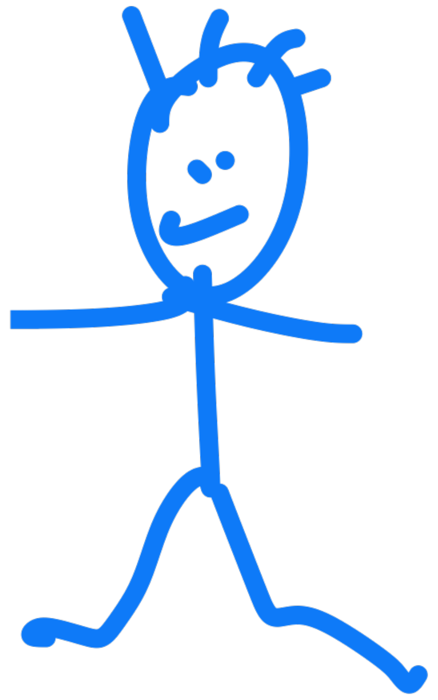

WHERE M^d_{Γ} IS A d-MANIFOLD ENCODED BY THE GRAPH Γ

TO CHECK THIS CLAIM WE NEED A CLASS OF MODELS WHERE WE KNOW Ψ_0 & Z

$\longrightarrow$ LEVIN-WEN MODELS

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GAPPED GROUND STATE Ψ_0 $\leftrightsquigarrow$ TQFT Z

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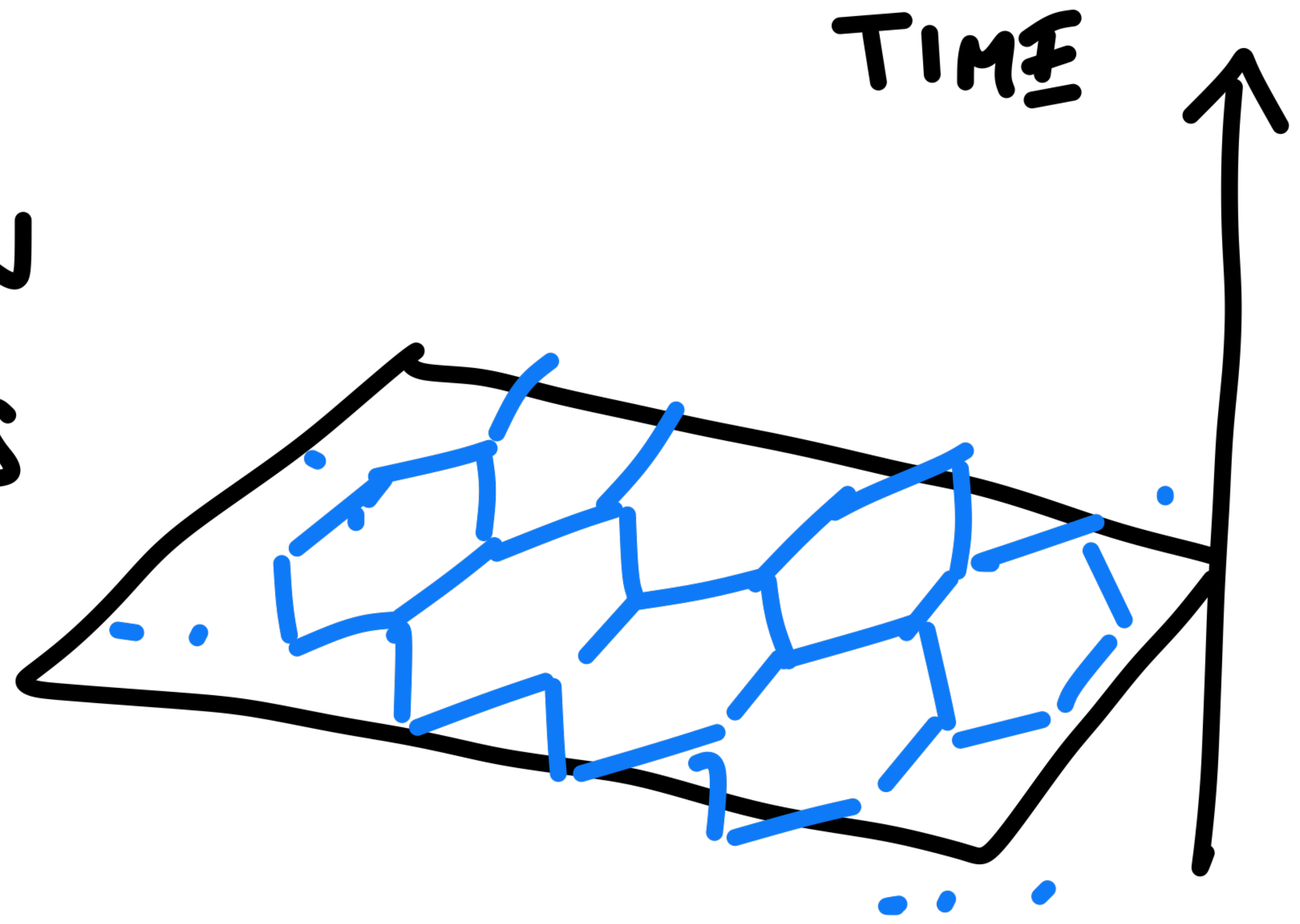
QUESTION: Is this enough to completely determine the IR TQFT?

IDEA: If TQFT is **non-degenerate**, one has **universal construction**. But often the TQFT is degenerate — there are states in $Z(M^{d-1})$ that arise from $Z(M^d)$ decorated with defects, when the TQFT is also unitary one can extend universal construction (McNamara talk @ NYU, November 2025)

LEVIN WEN STRING NET MODELS

LEVIN WEN MODELS ARE HAMILTONIAN LATTICE MODELS IN 2+1 DIMENSIONS



THE INPUT DATA FOR THESE MODELS IS A SPHERICAL FUSION CATEGORY $\mathcal{C}$.

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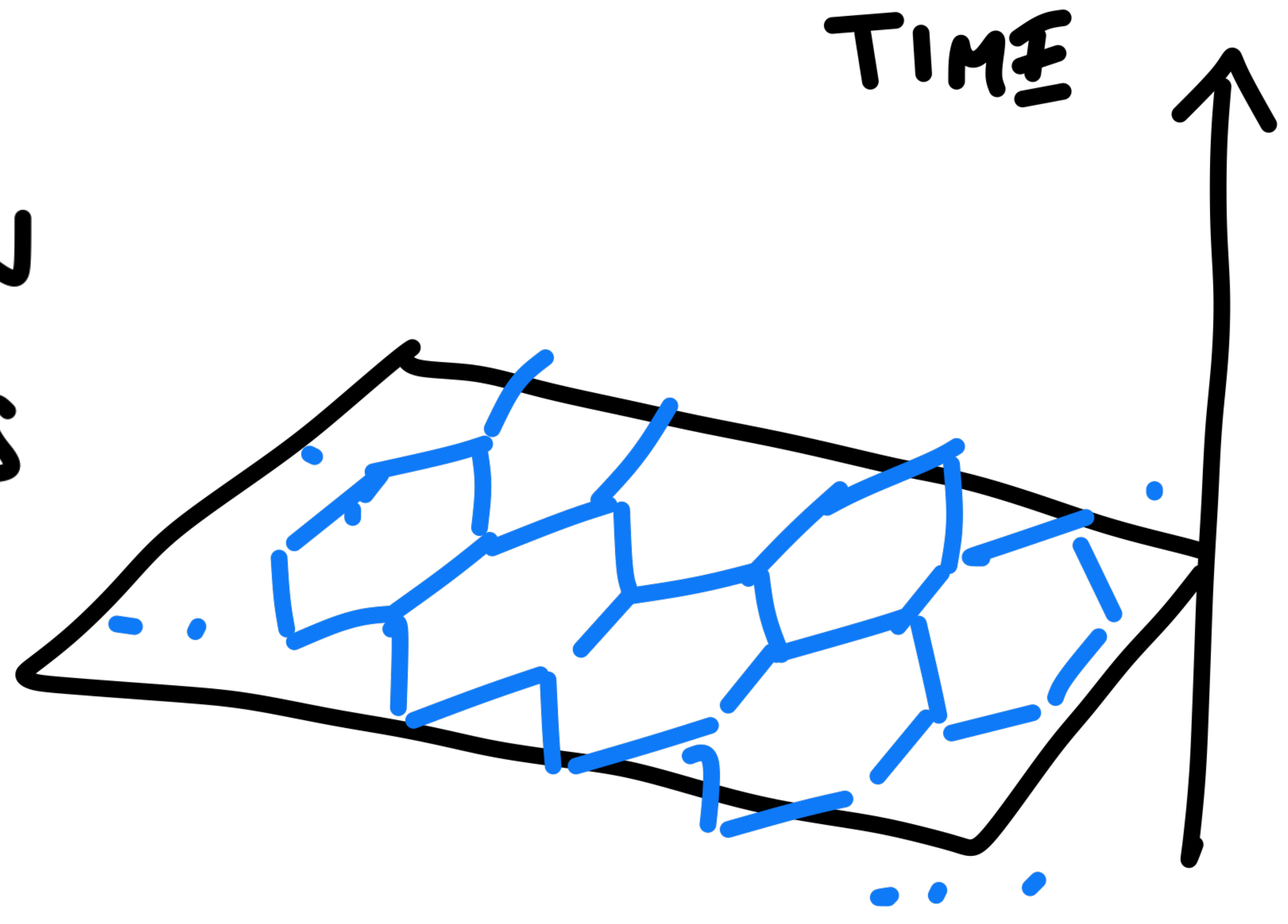

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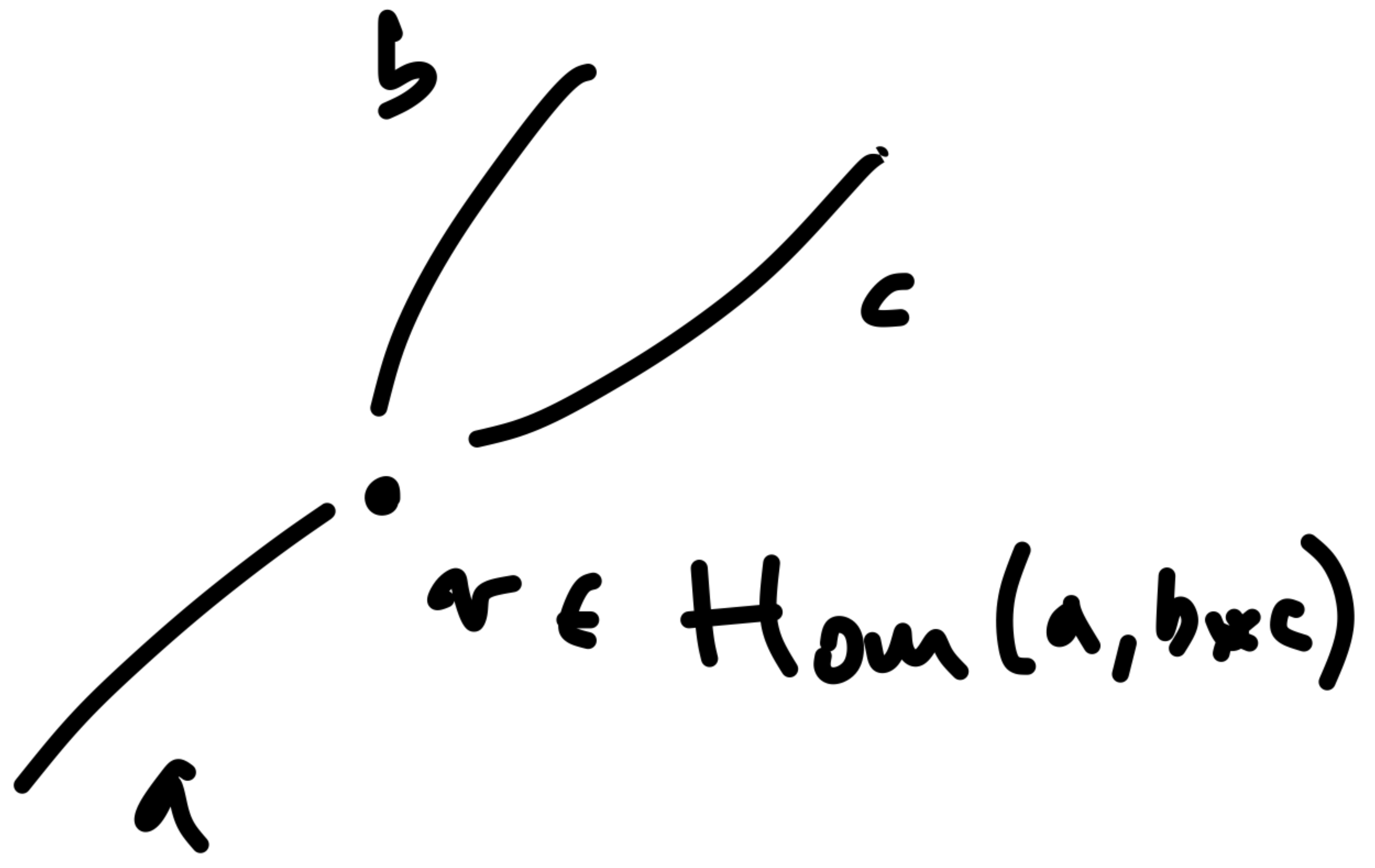

IR LIMIT: TURAEV-VIRO TQFT CORRESPONDING TO $\mathcal{C}$. (KIRILLOV JR + BALSAM 2010)

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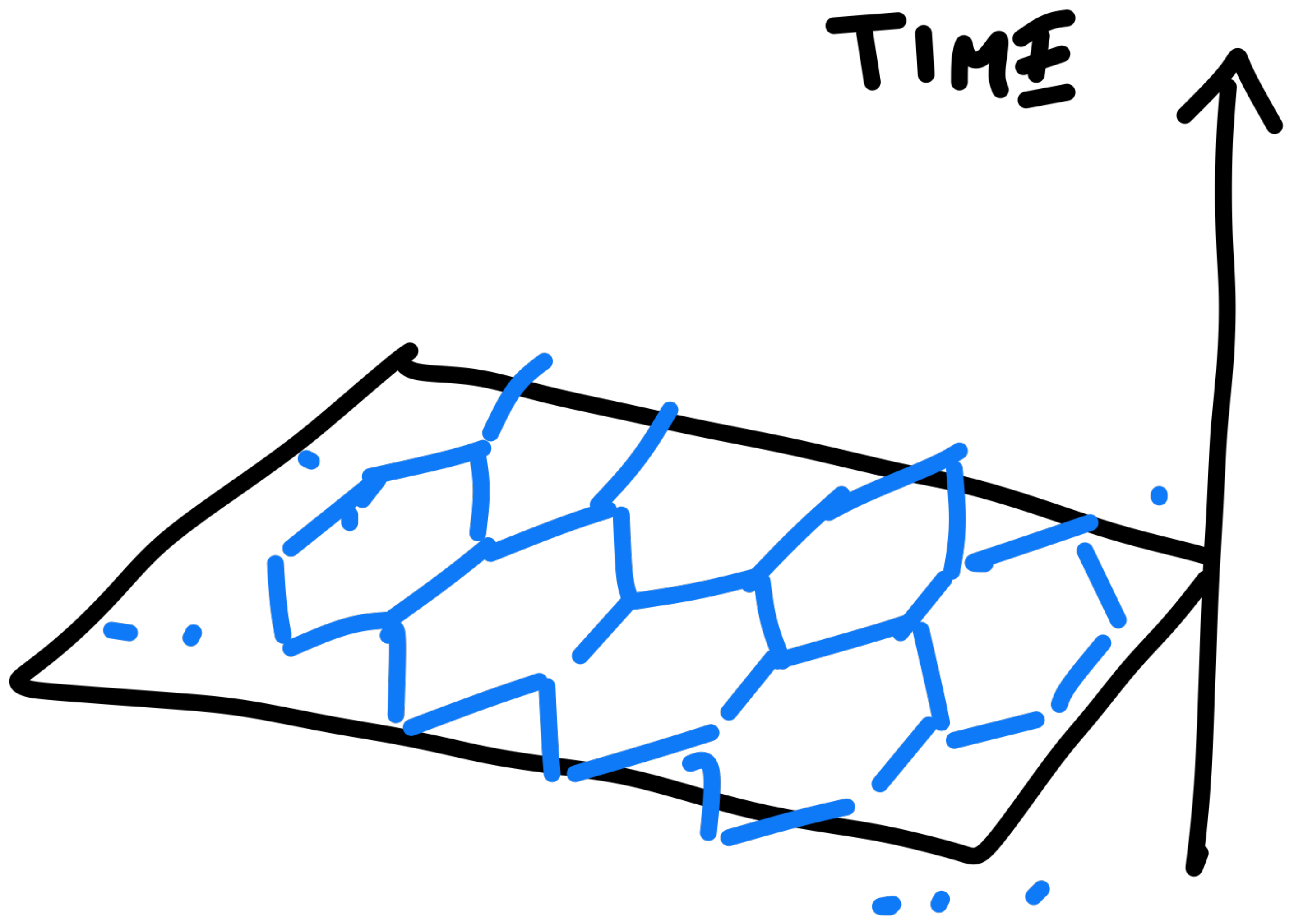

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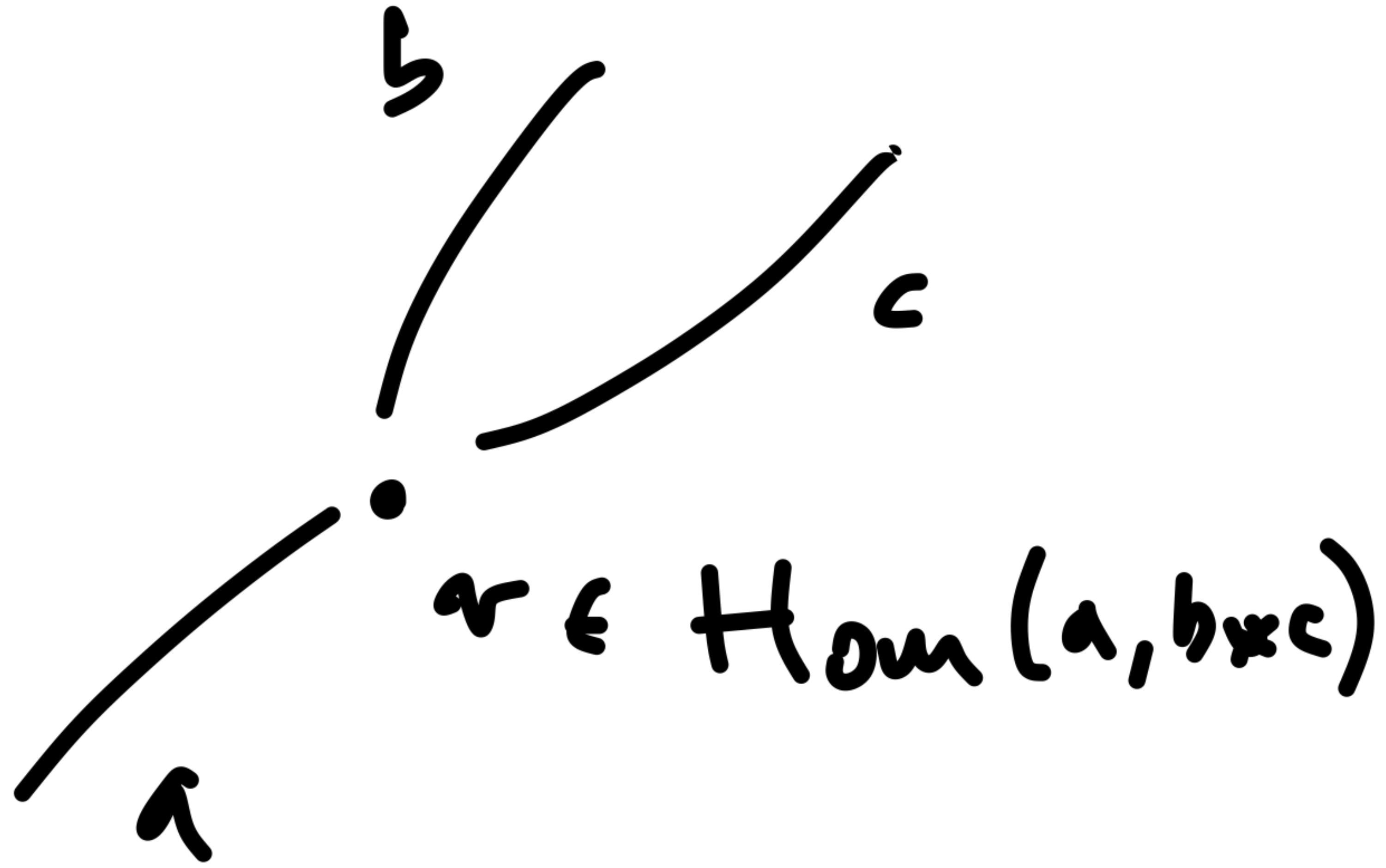

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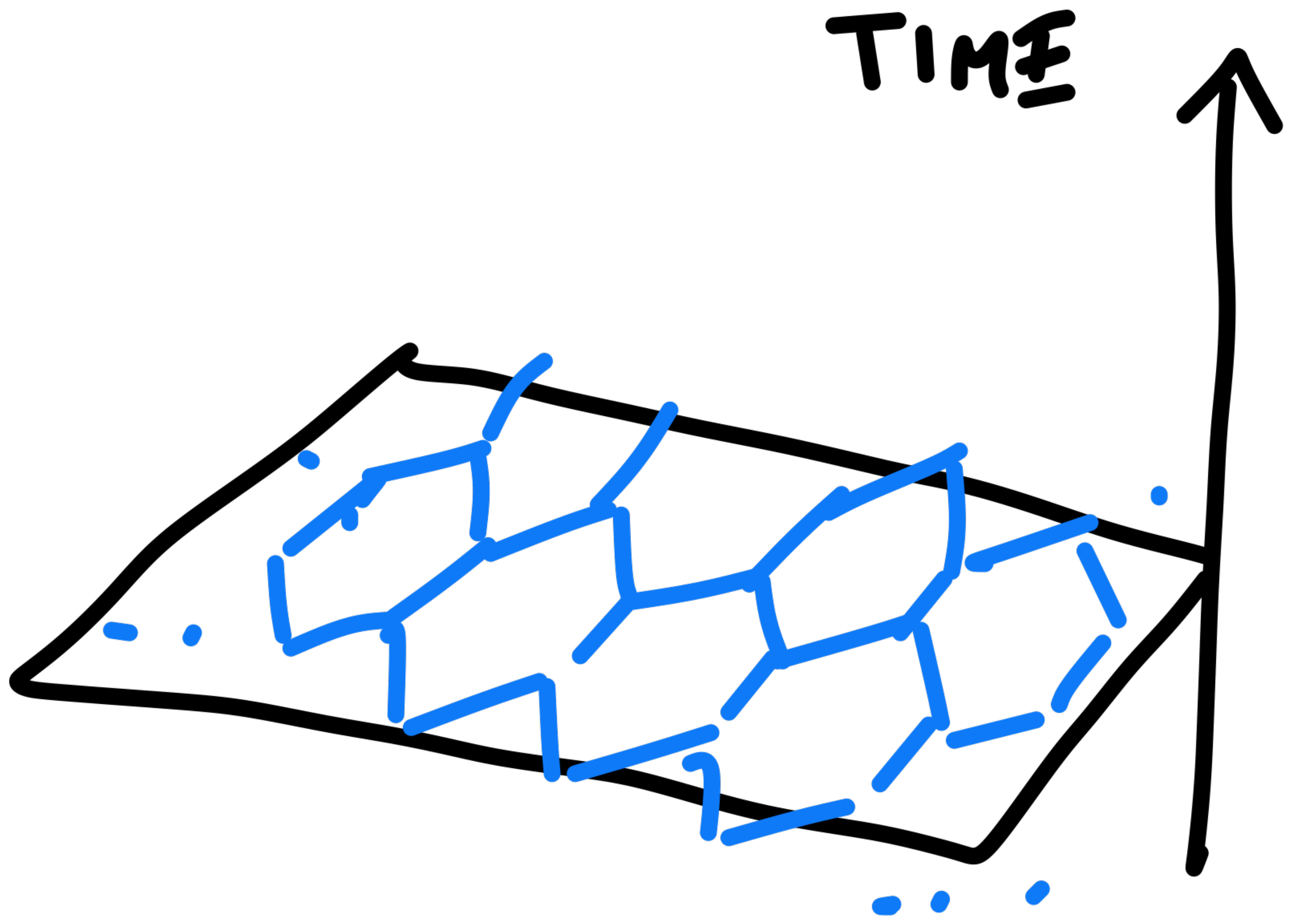

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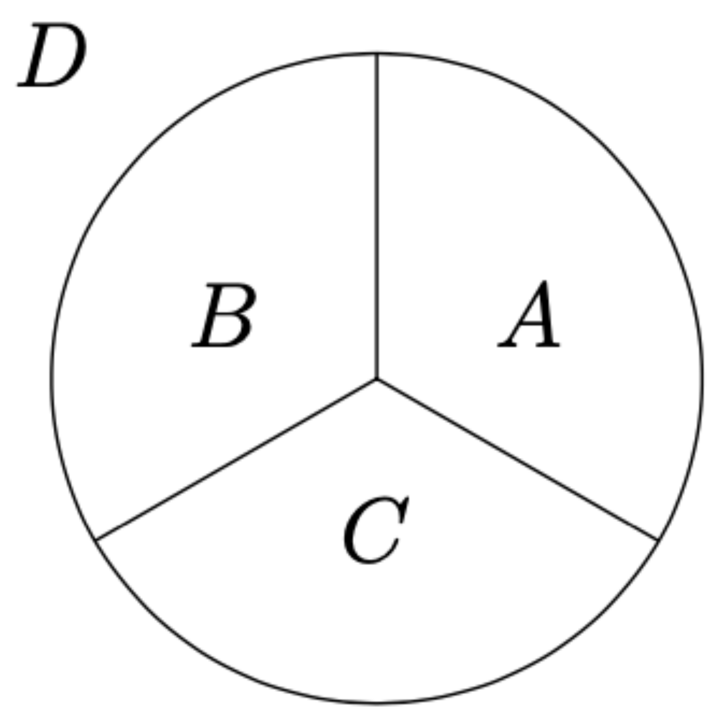

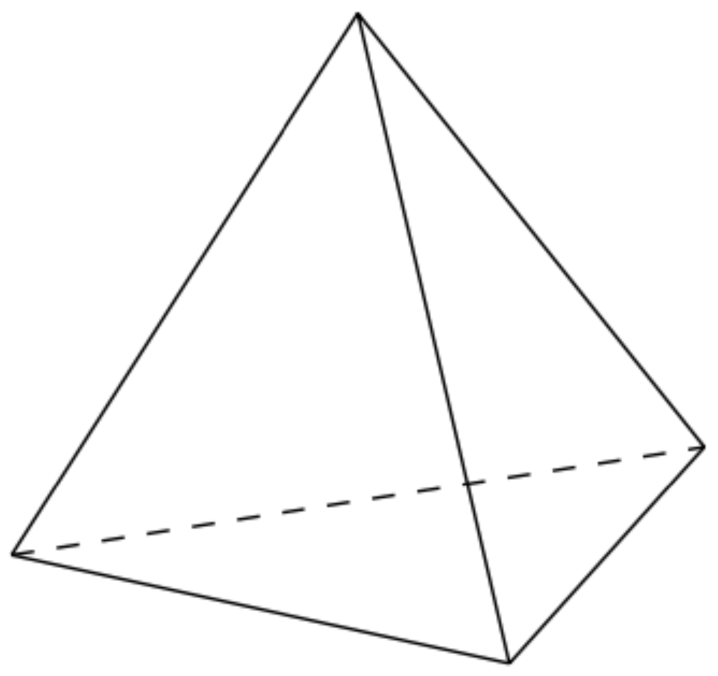

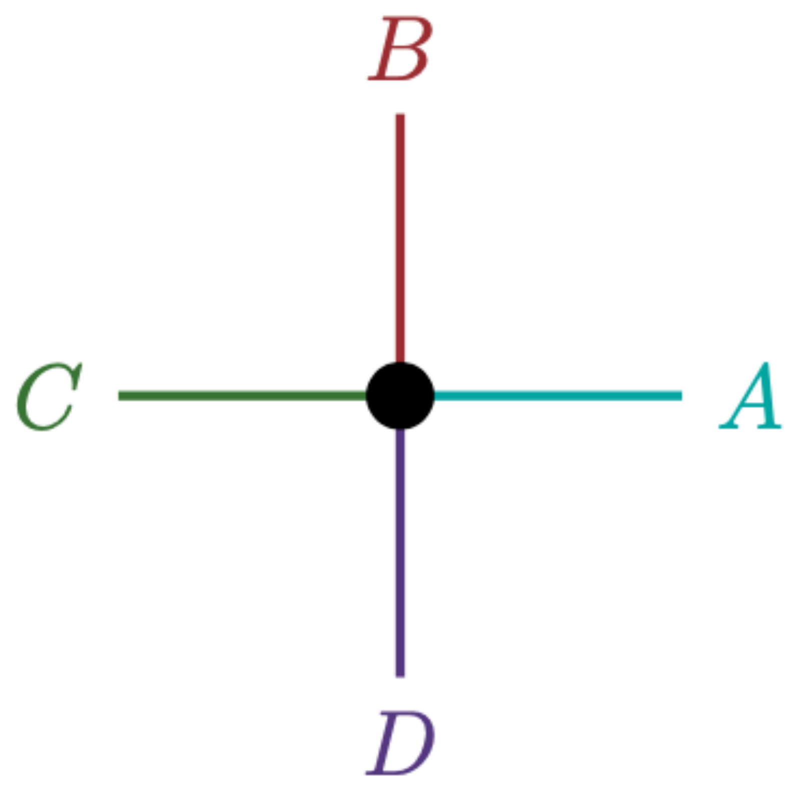

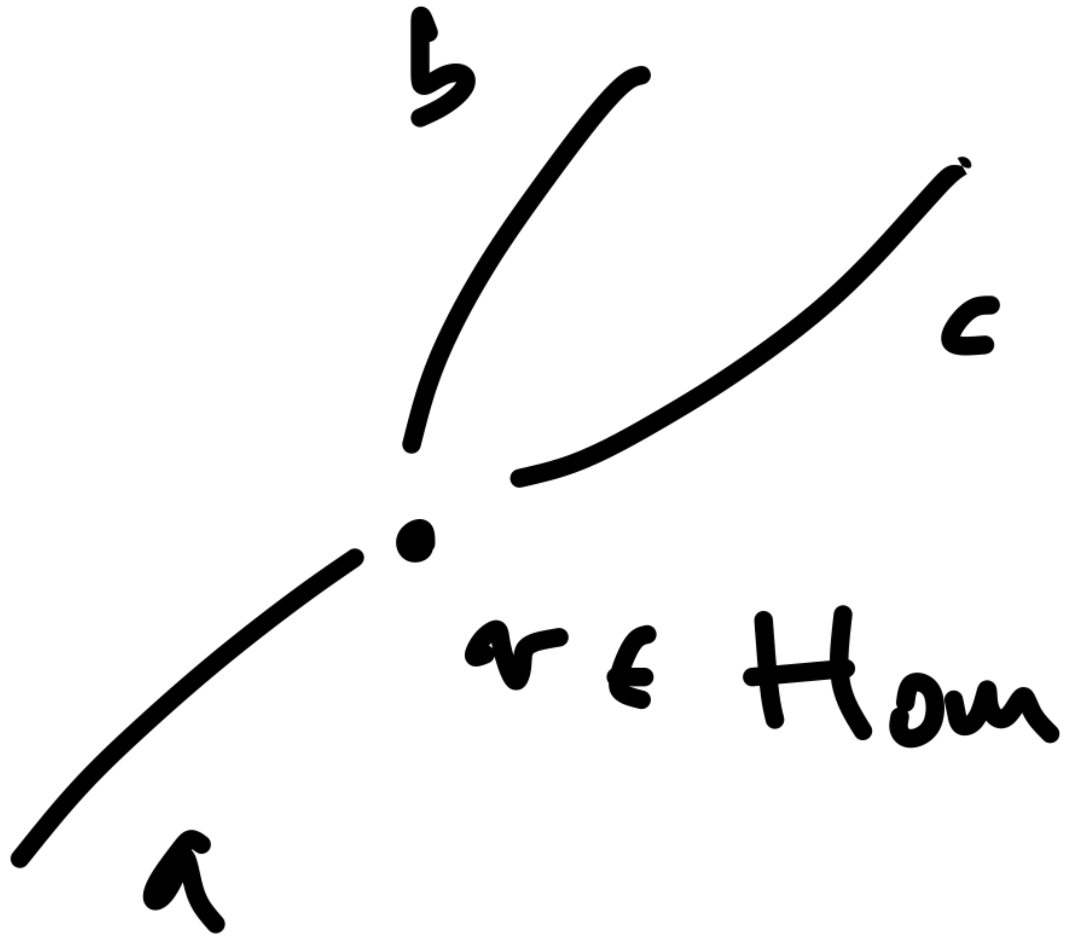

$\alpha \in \mathrm{Hom}(a, b \otimes c)$

4-partite entanglement invariant: as long as the regions are large enough captures IR behavior

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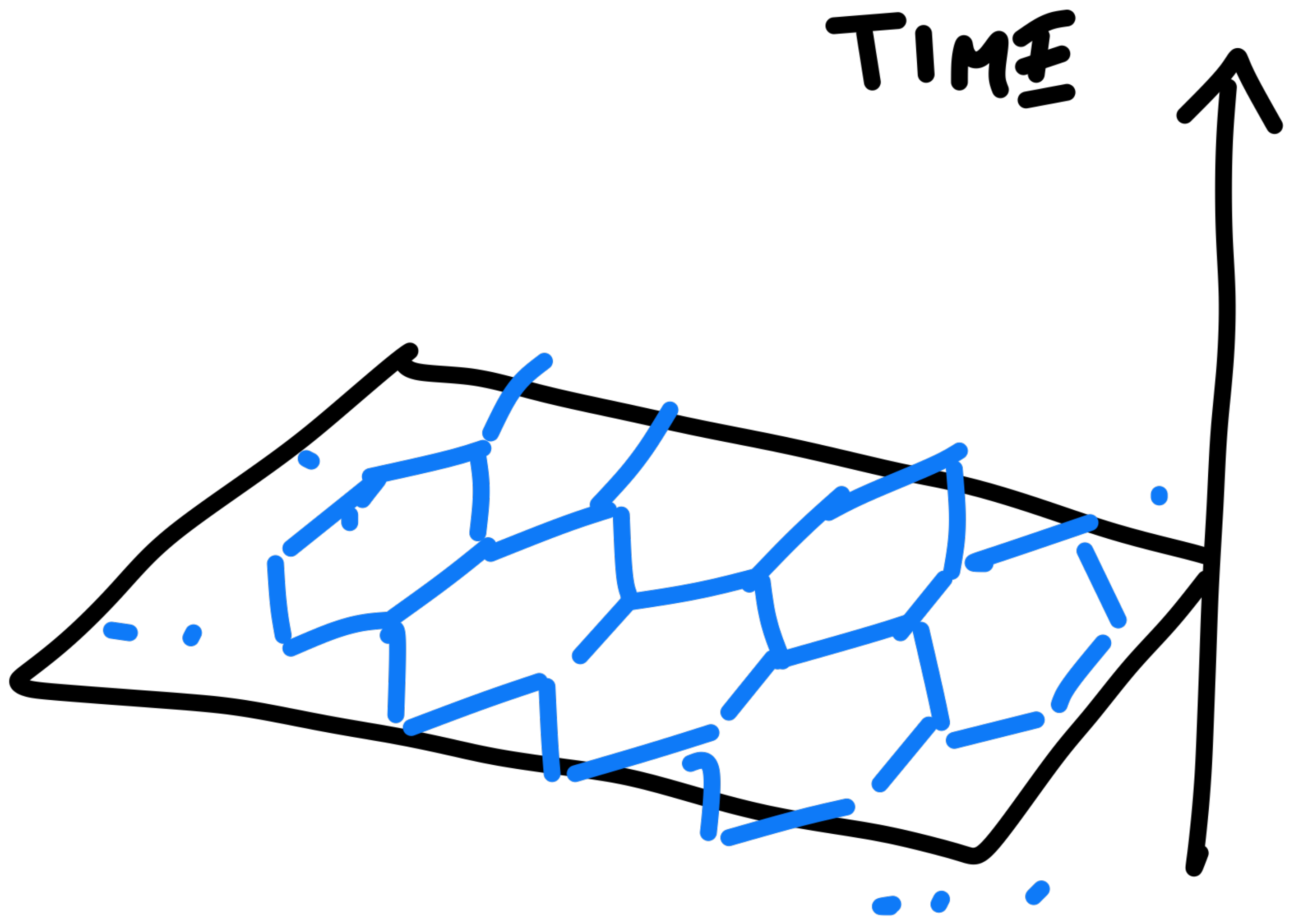

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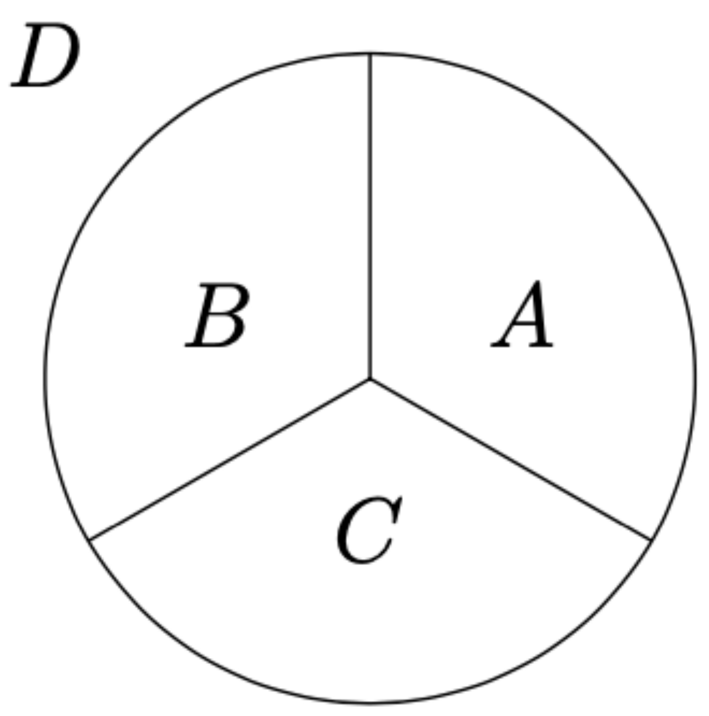

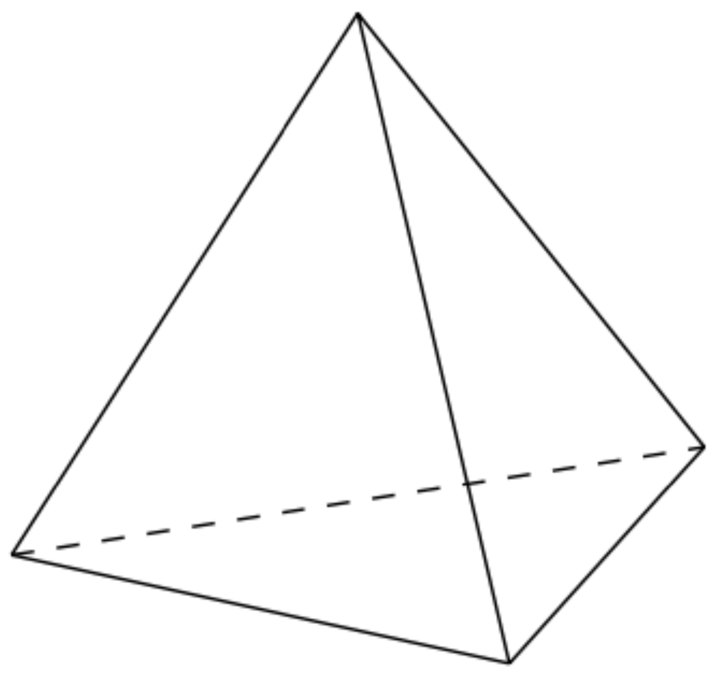

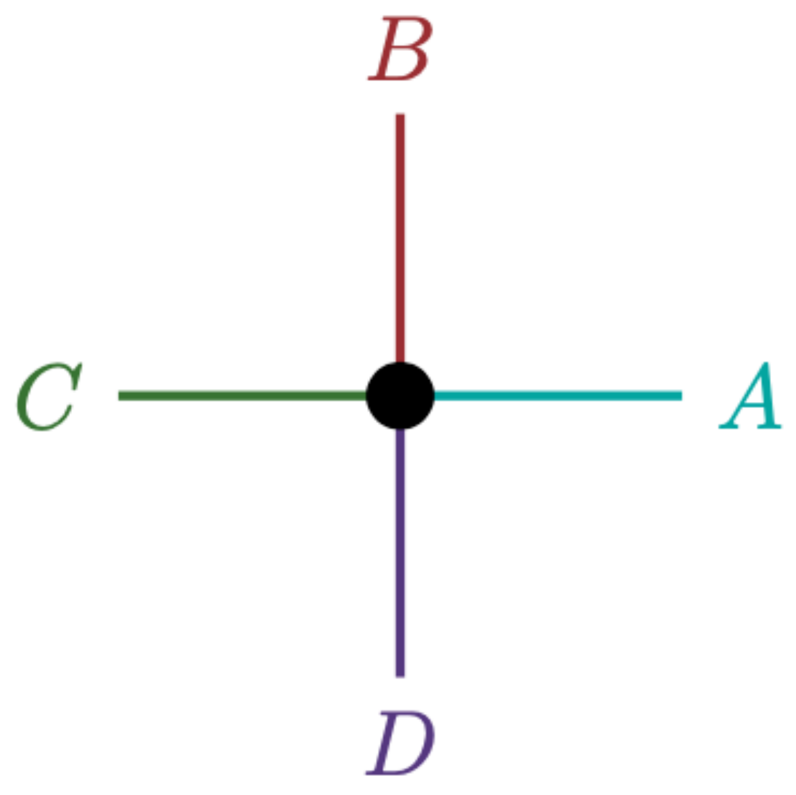

This relation generalizes in d-dimensions: (d+1)-parties maps to d-symplex

4-partite entanglement invariant: as long as the regions are large enough captures IR behavior

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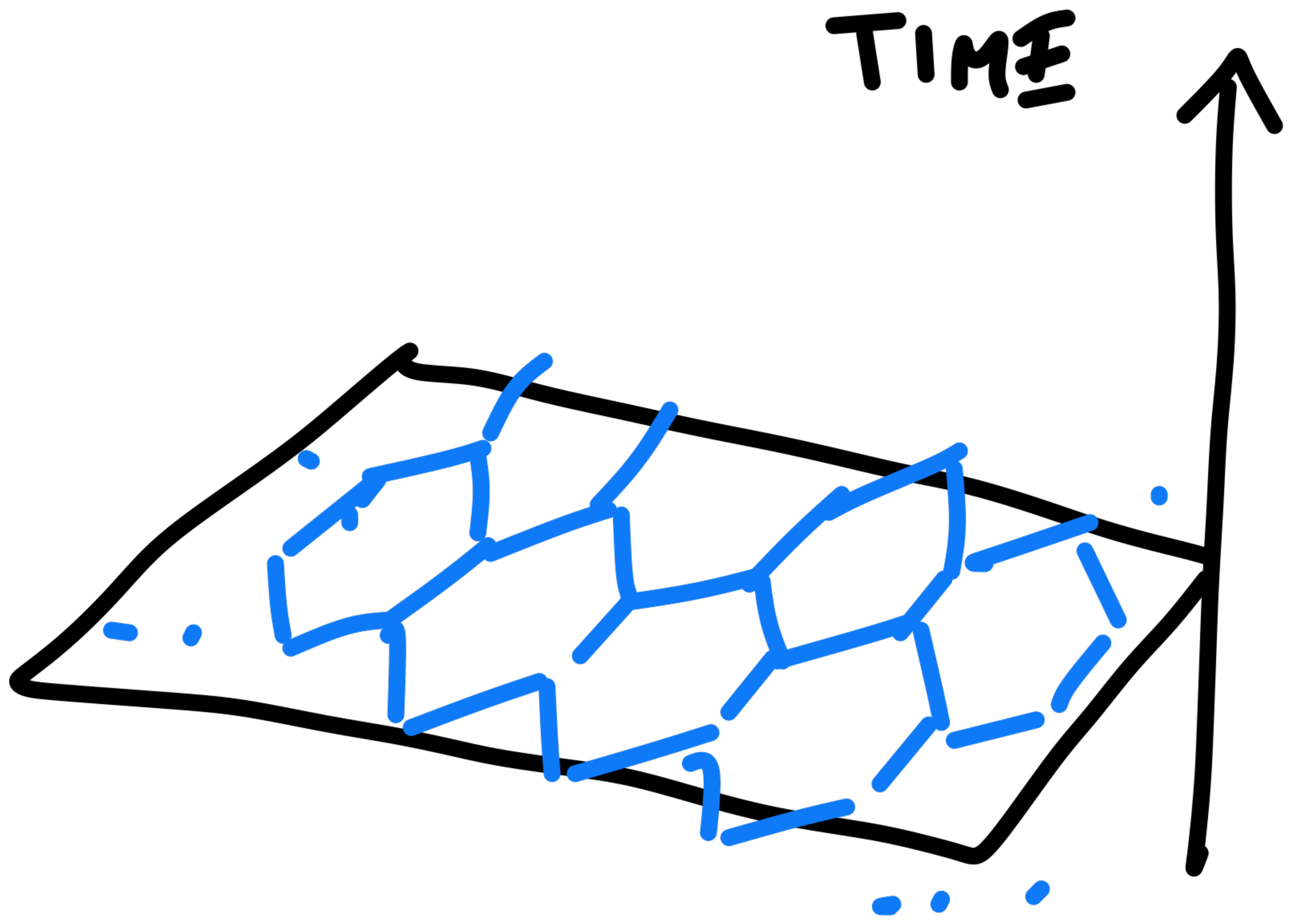

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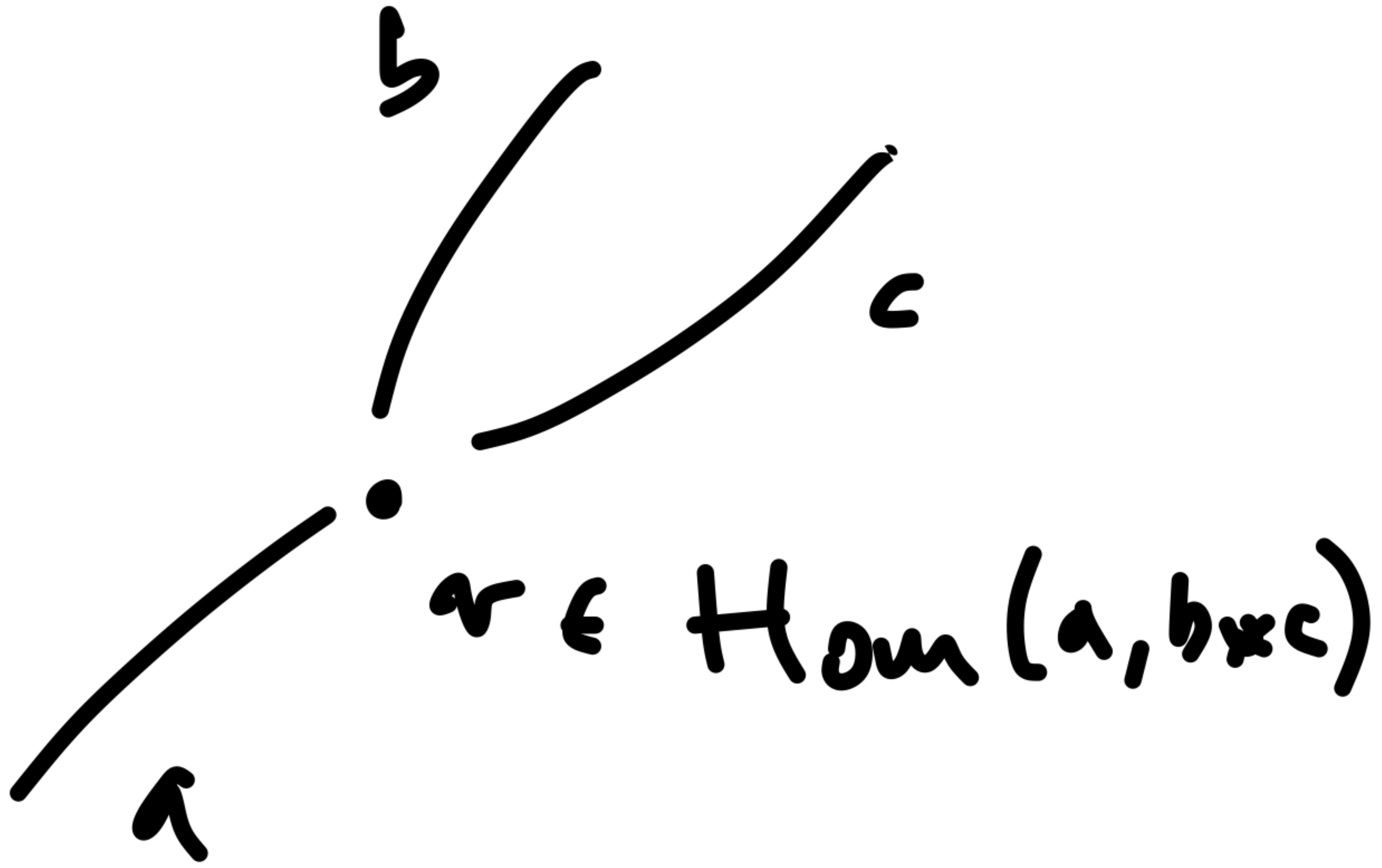
WE CAN SHOW THAT



$$\log W_\Gamma^{(4)}(\Psi_{LW}) = \log Z_{TV}(M_\Gamma^{(3)}) + O(e^{-cL})$$

HOLDS FOR ALL THESE MODELS!

LEVIN WEN STRING NET MODELS

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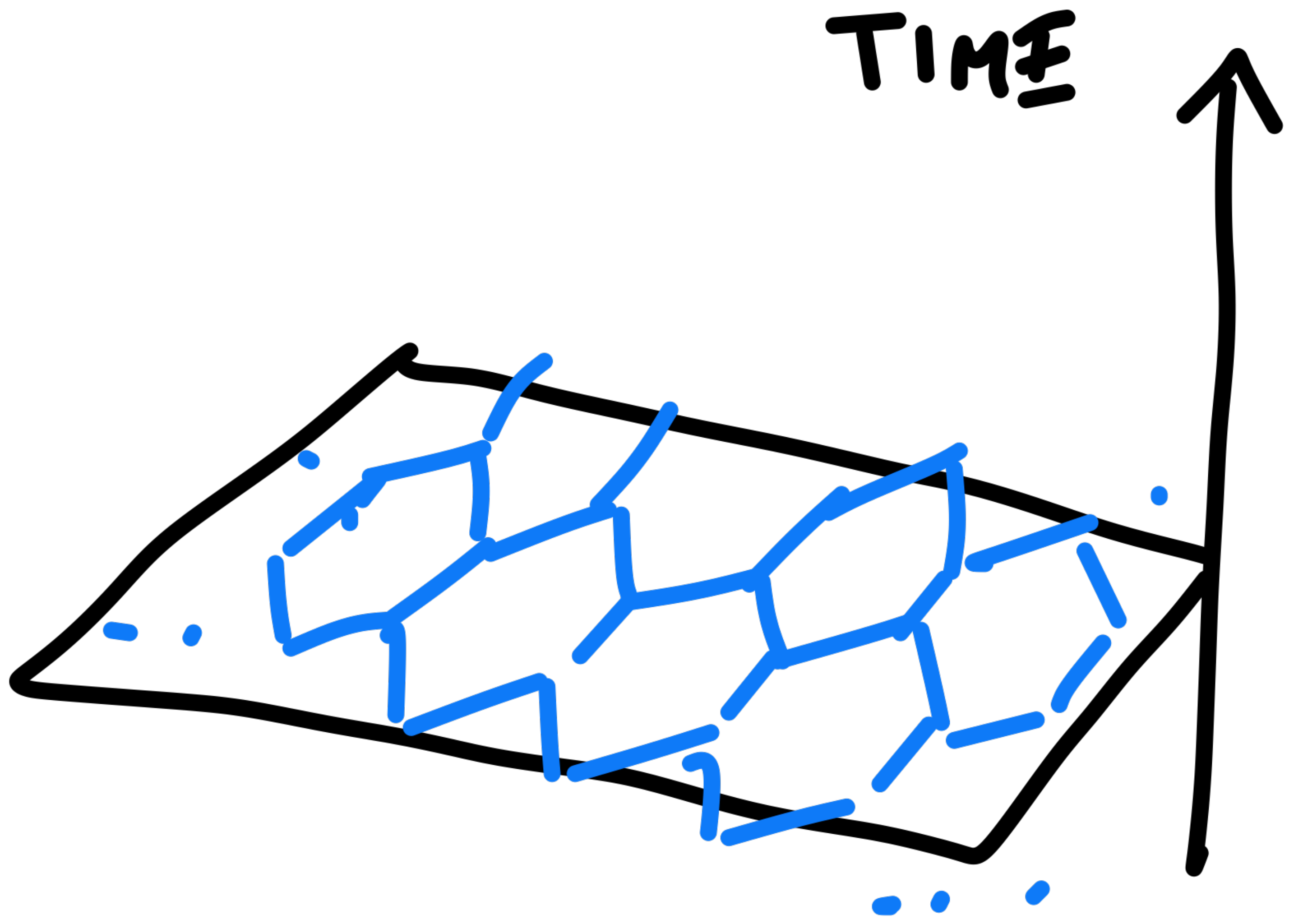

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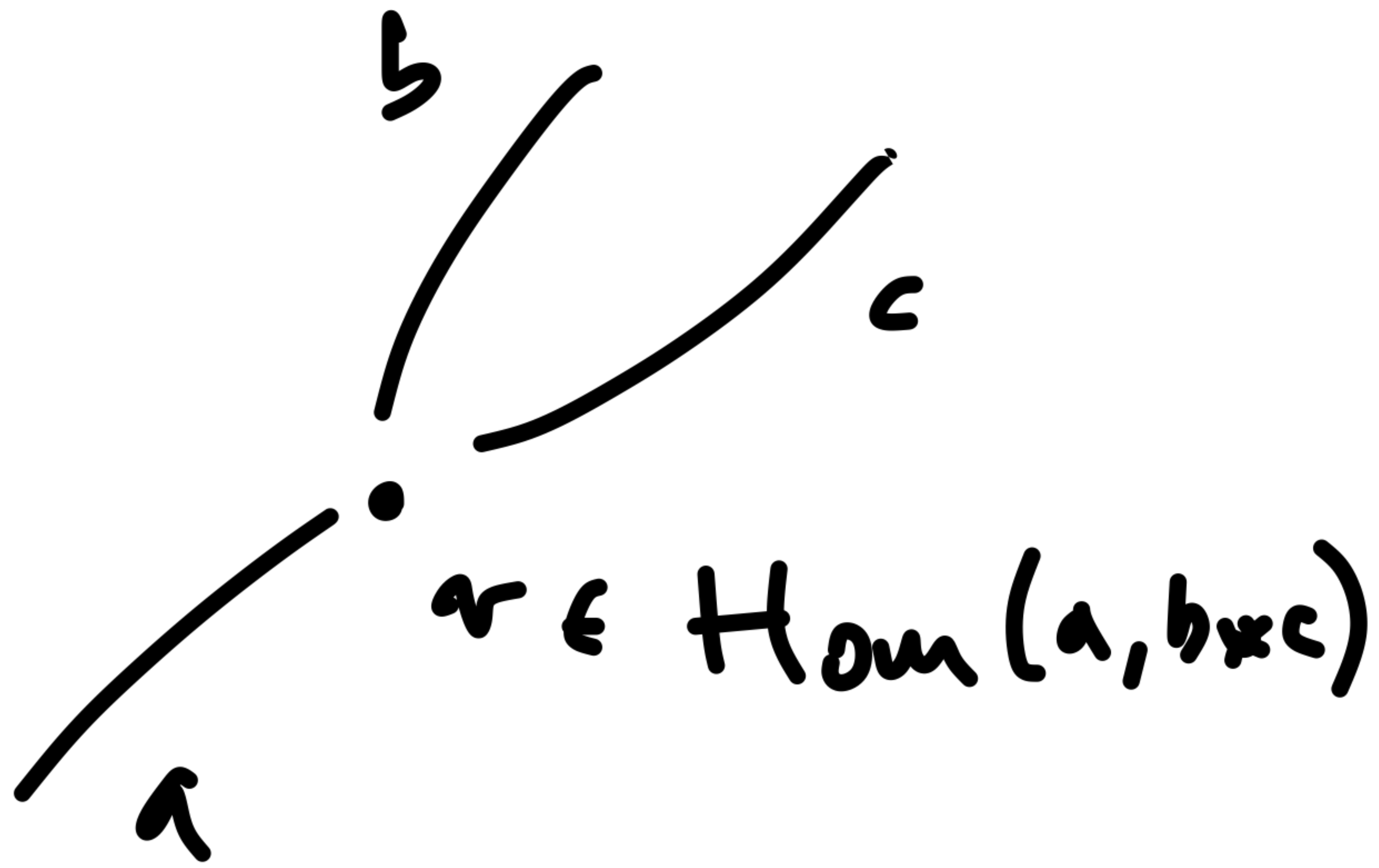
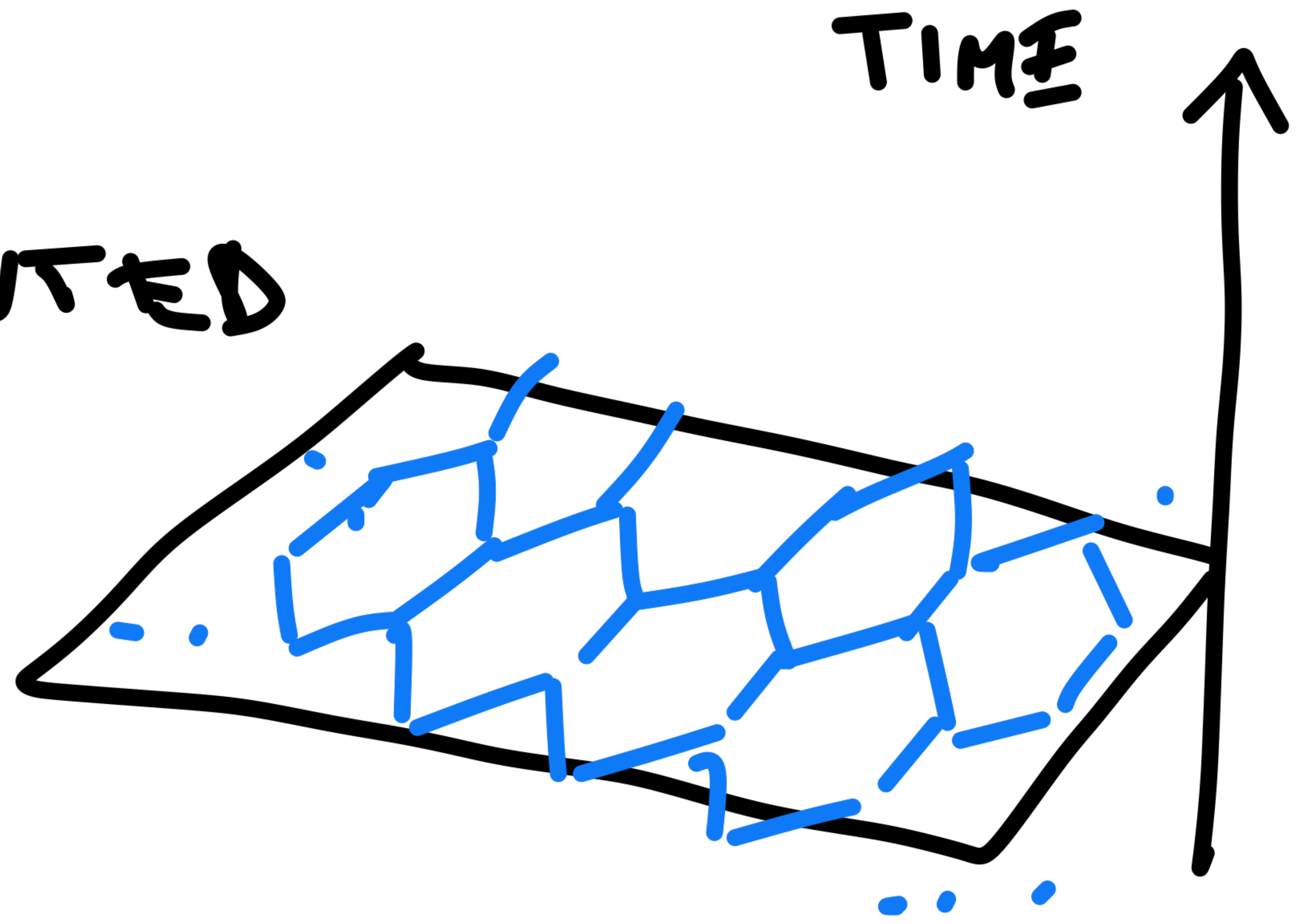


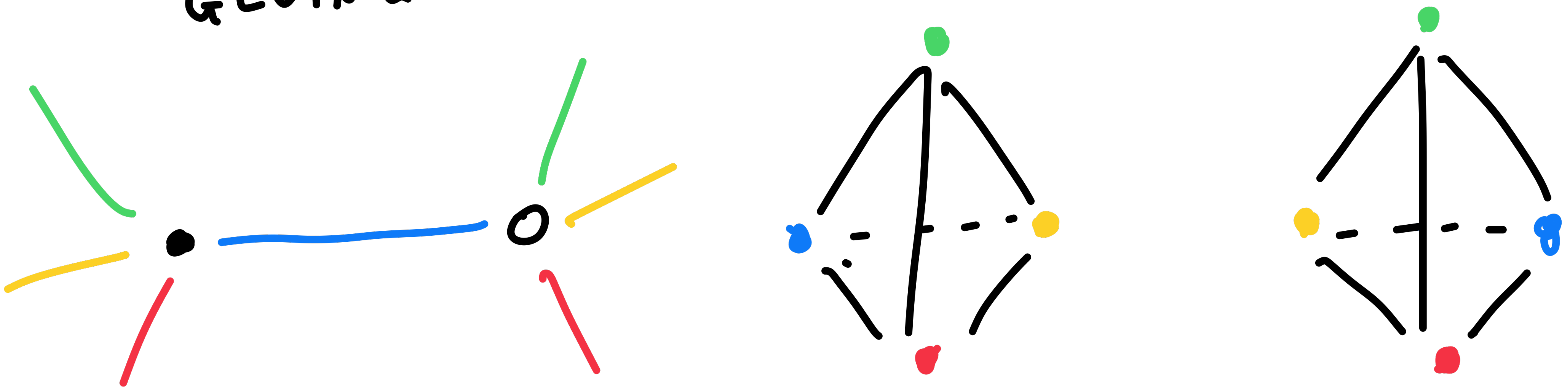
HOLDS FOR ALL THESE MODELS! DERIVATION IS VERY TECHNICAL

LEVIN WEN STRING NET MODELS



FACT: ALL SMOOTH COMPACT ORIENTED CLOSED 3-MANIFOLDS ARE GRAPHICALLY ENCODED MANIFOLDS

GLUING = CONTRACTIONS



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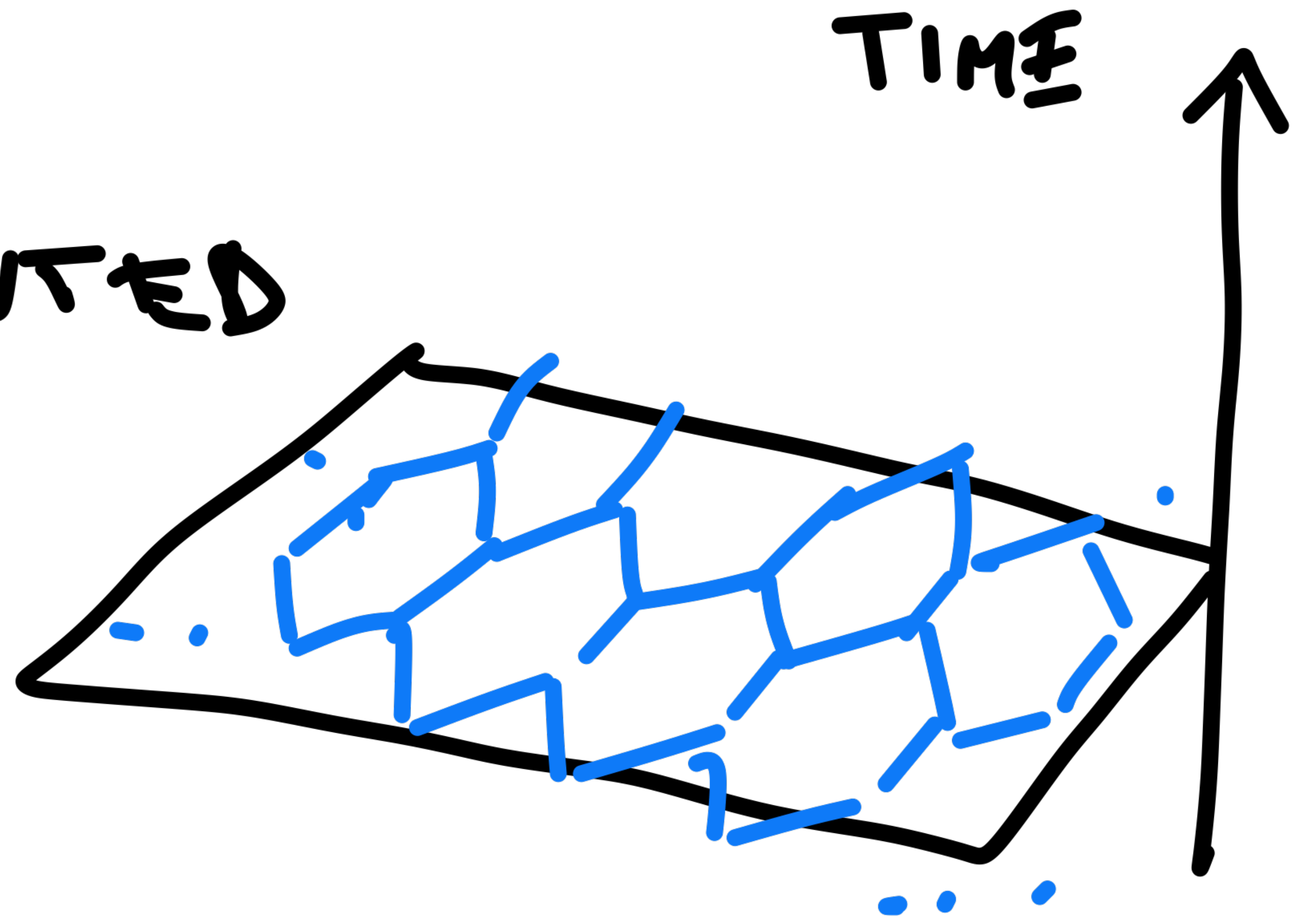

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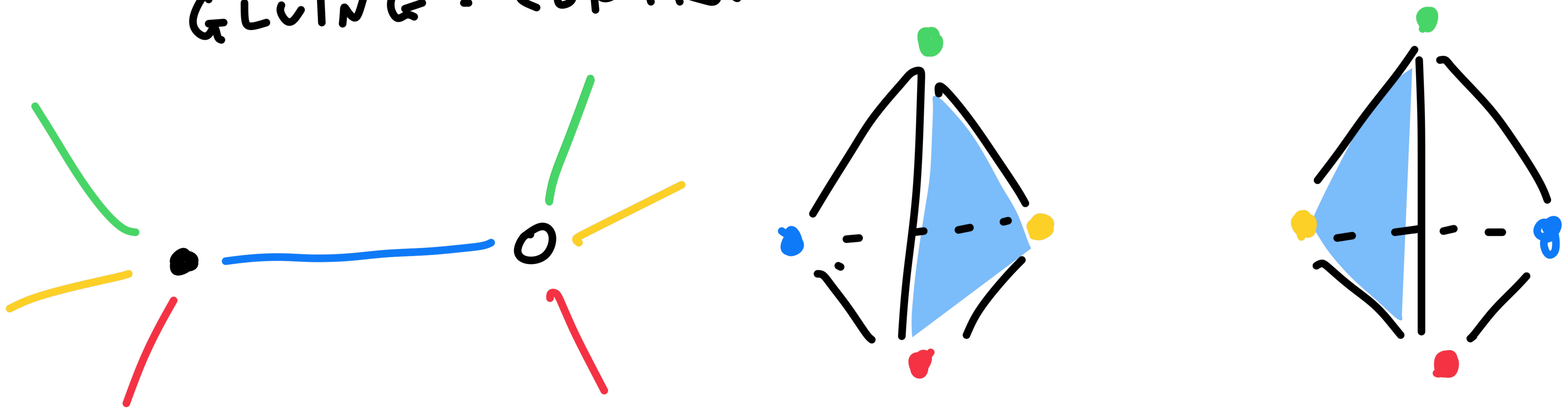
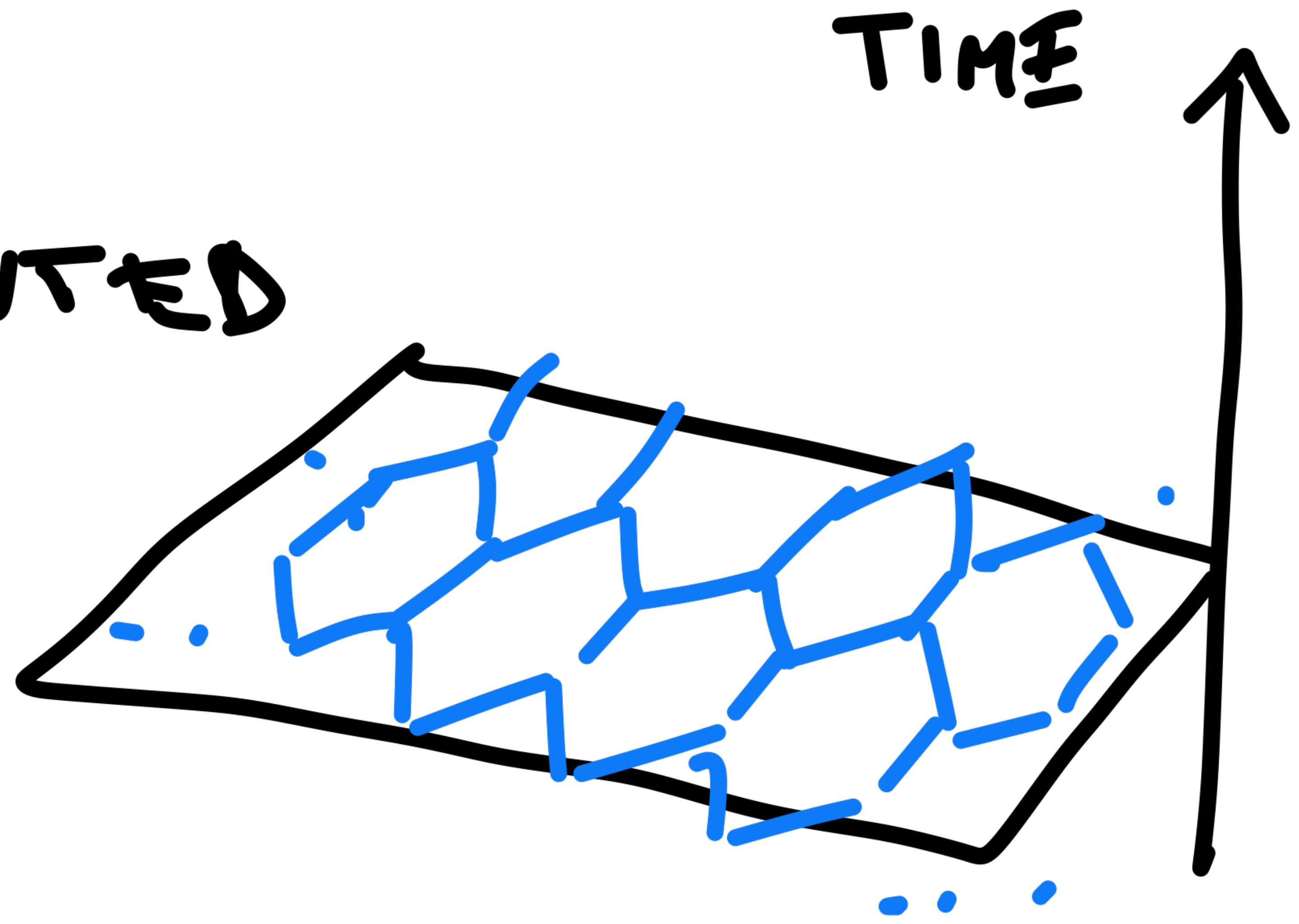
GLUING = CONTRACTION

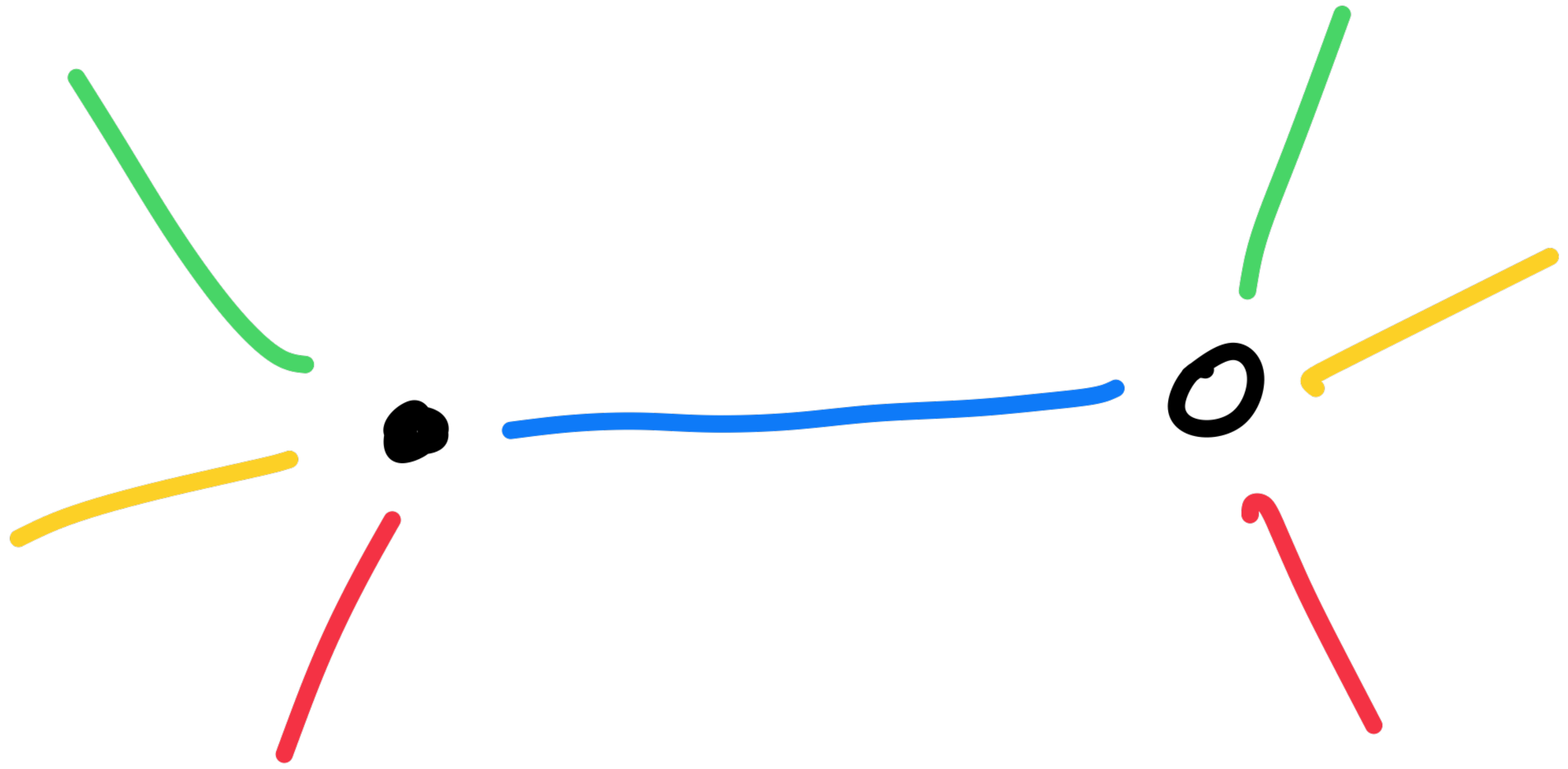


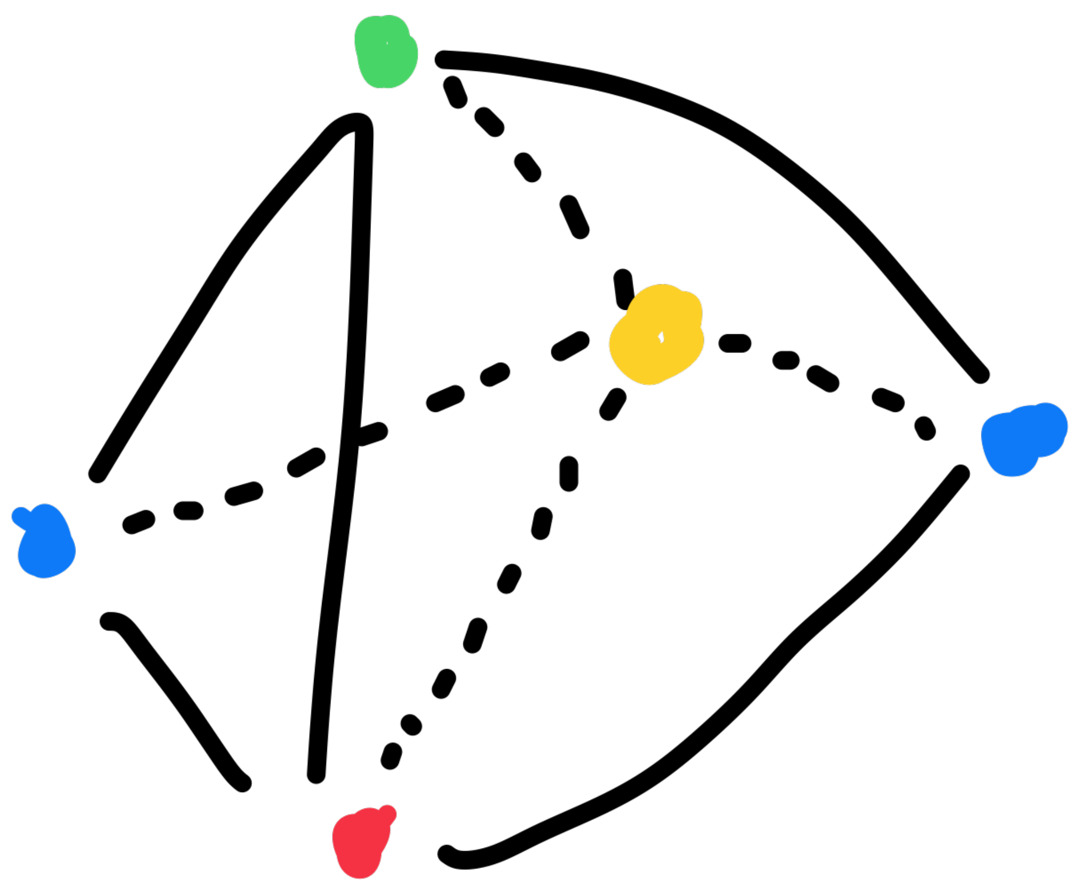
IDENTIFY

LEVIN WEN STRING NET MODELS

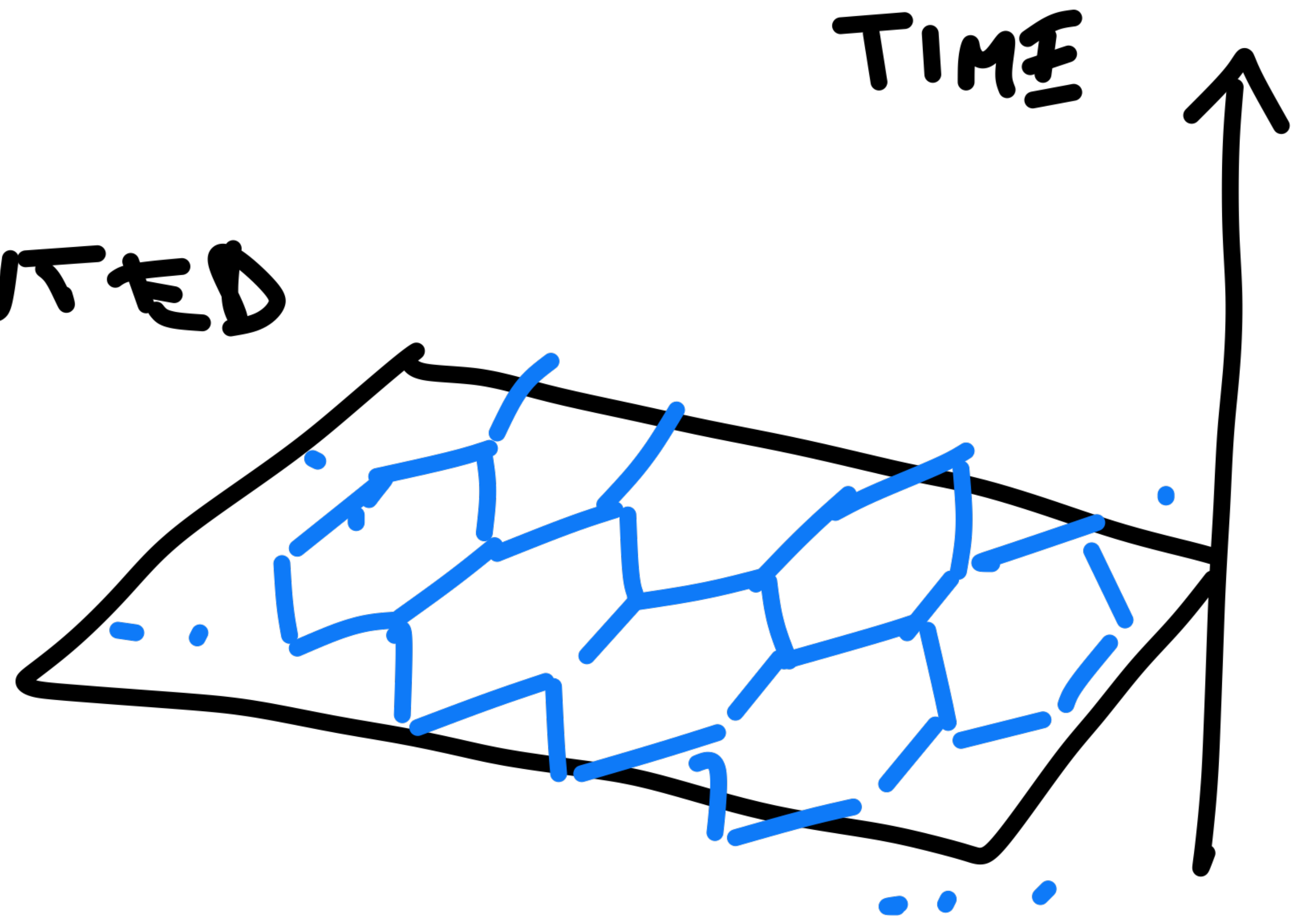


FACT: ALL SMOOTH COMPACT ORIENTED CLOSED 3-MANIFOLDS ARE GRAPHICALLY ENCODED MANIFOLDS



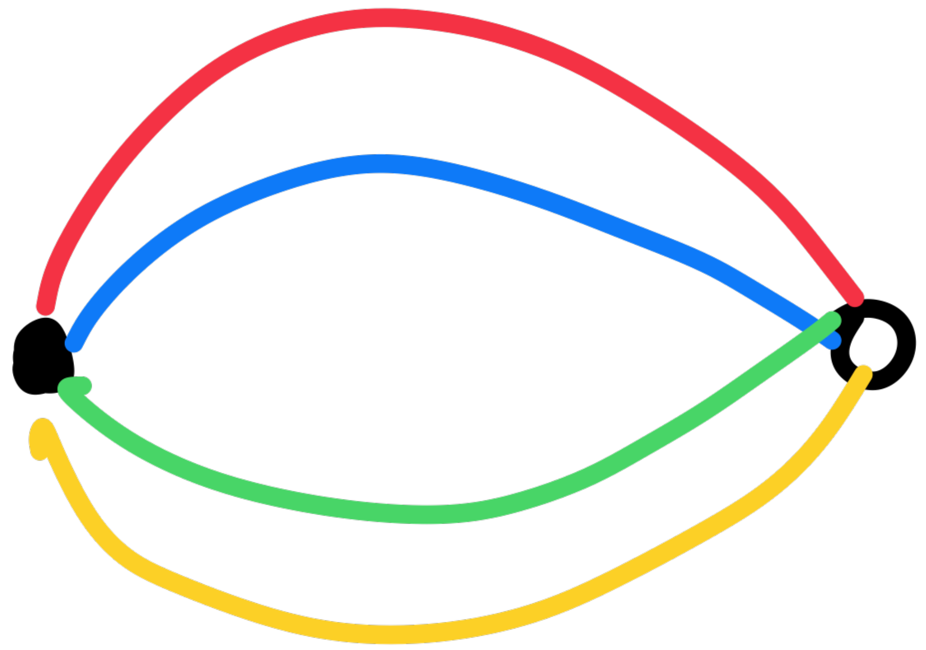


LEVIN WEN STRING NET MODELS



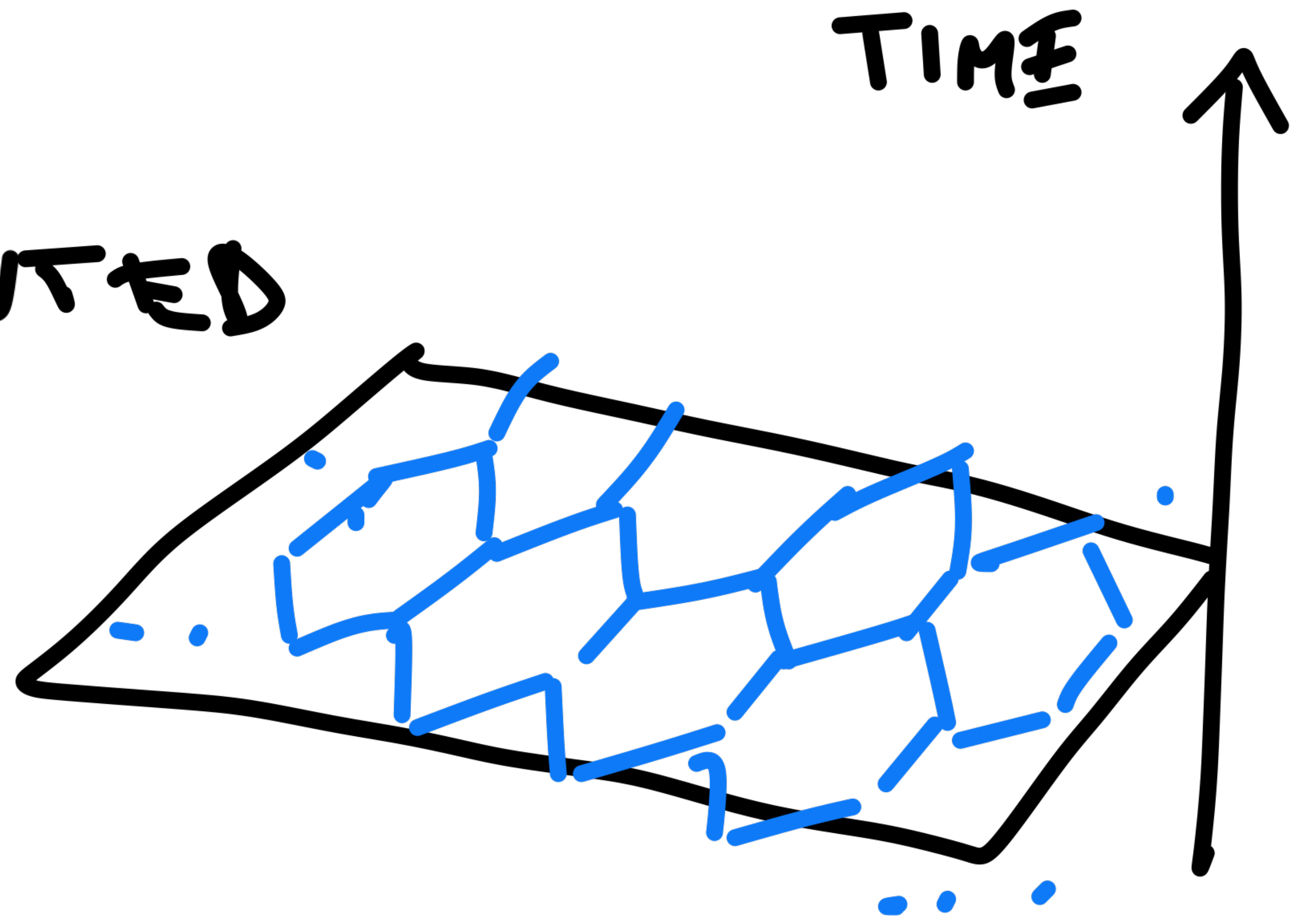
FACT: ALL SMOOTH COMPACT ORIENTED CLOSED 3-MANIFOLDS ARE GRAPHICALLY ENCODED MANIFOLDS

e.g. THIS



GIVES S^3

LEVIN WEN STRING NET MODELS



FACT: ALL SMOOTH COMPACT ORIENTED CLOSED 3-MANIFOLDS ARE GRAPHICALLY ENCODED MANIFOLDS

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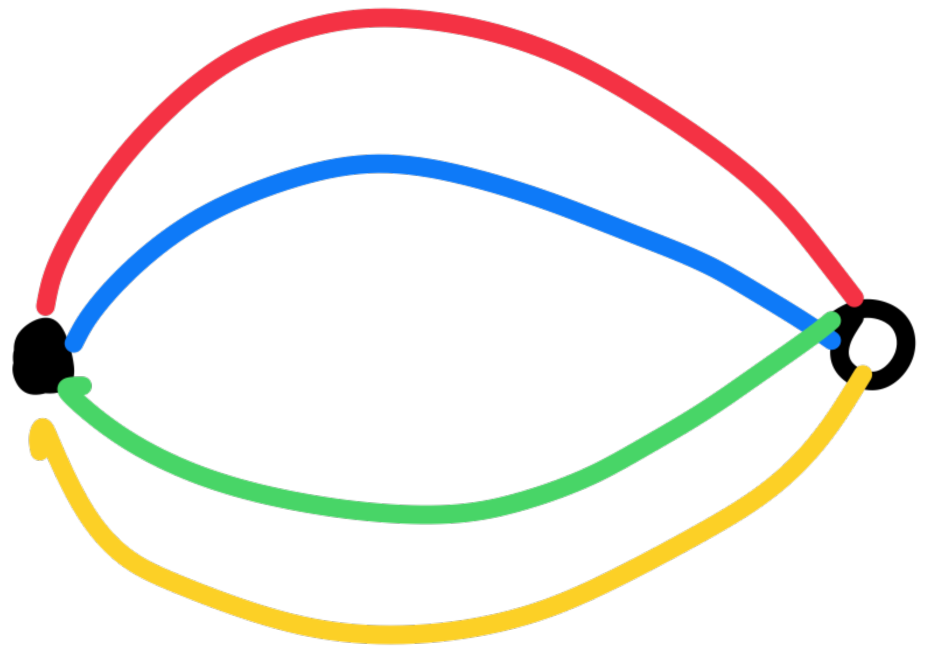
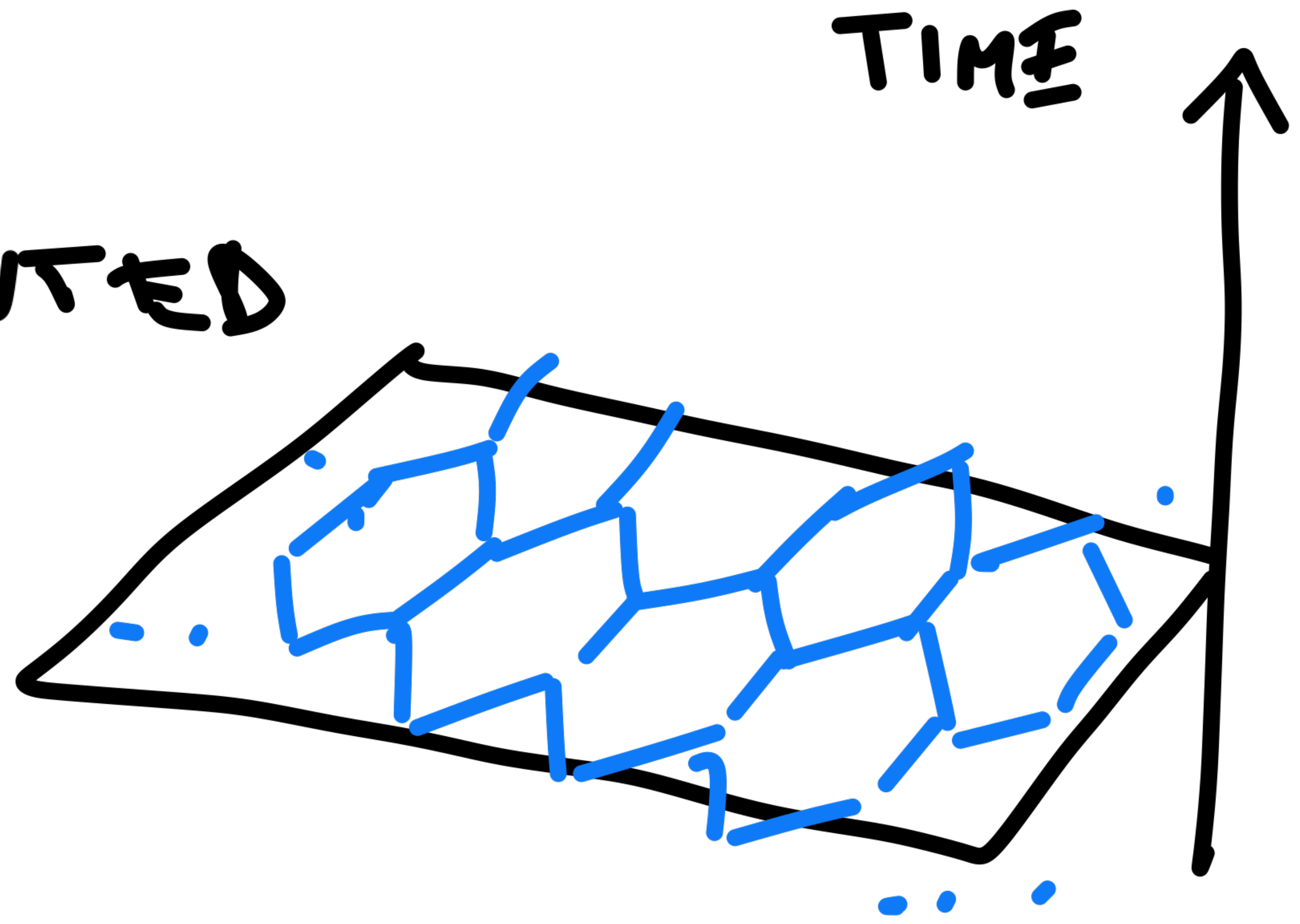


GIVES S^3

$\longrightarrow$ REPRODUCE KITAEV-PRESKILL 2006

LEVIN WEN STRING NET MODELS



FACT: ALL SMOOTH COMPACT ORIENTED CLOSED 3-MANIFOLDS ARE GRAPHICALLY ENCODED MANIFOLDS

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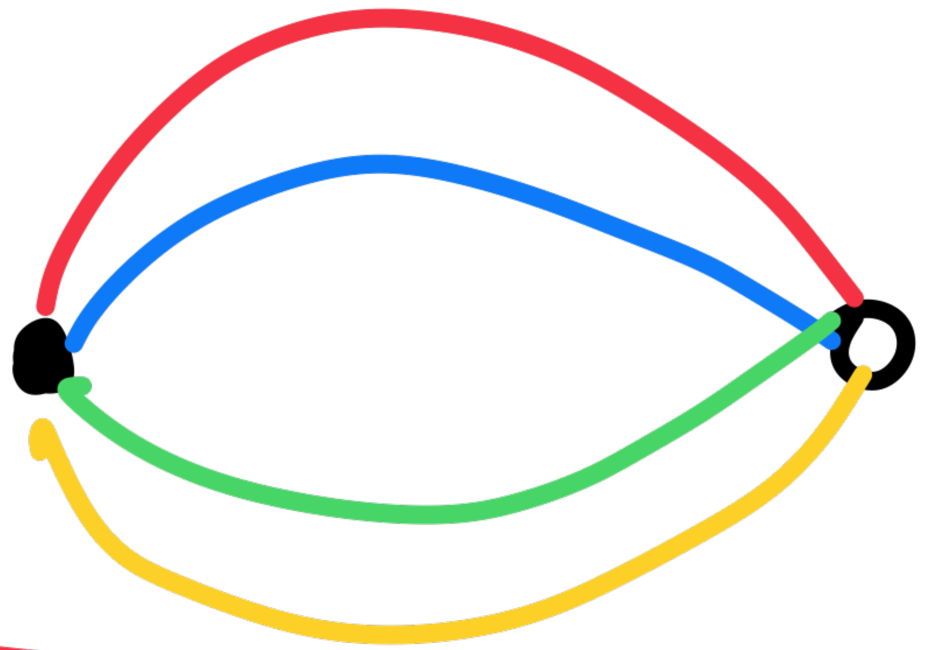
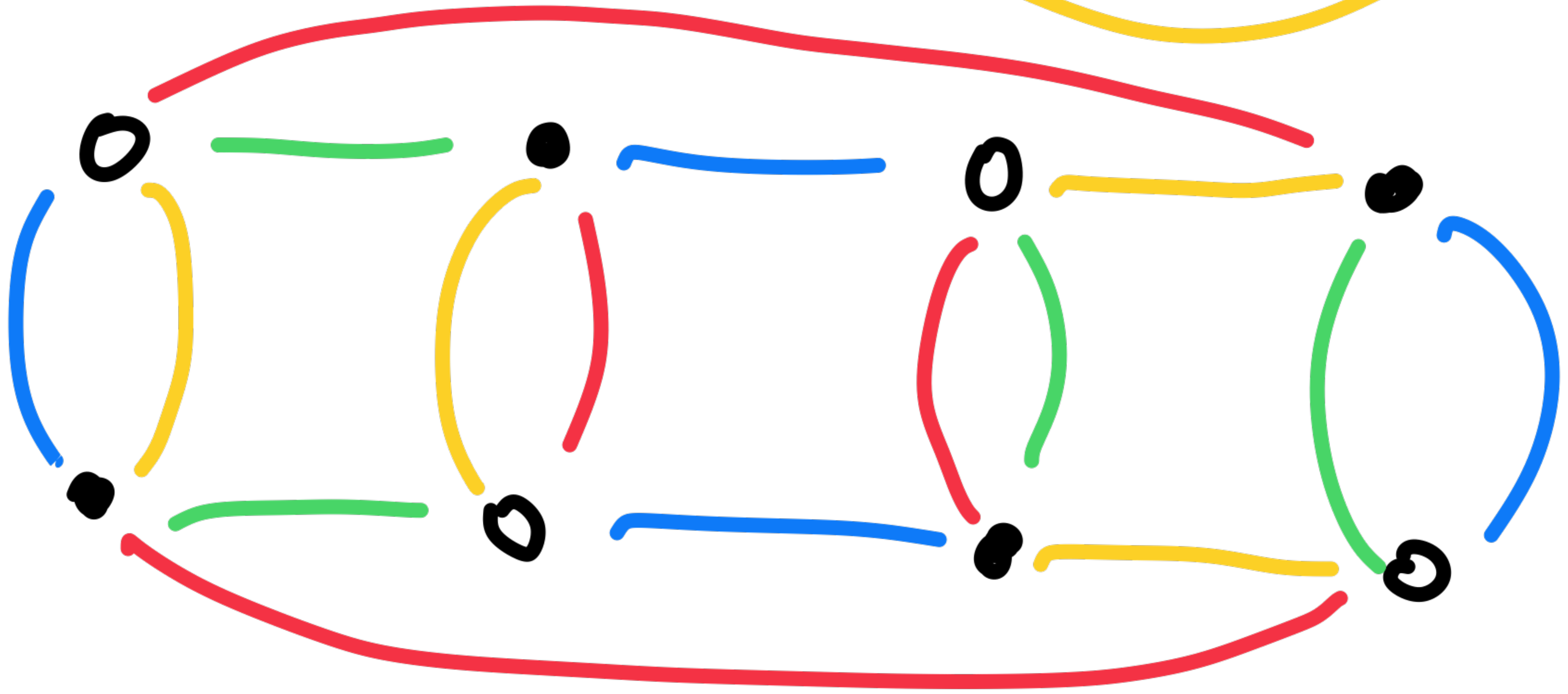


GIVES S^3

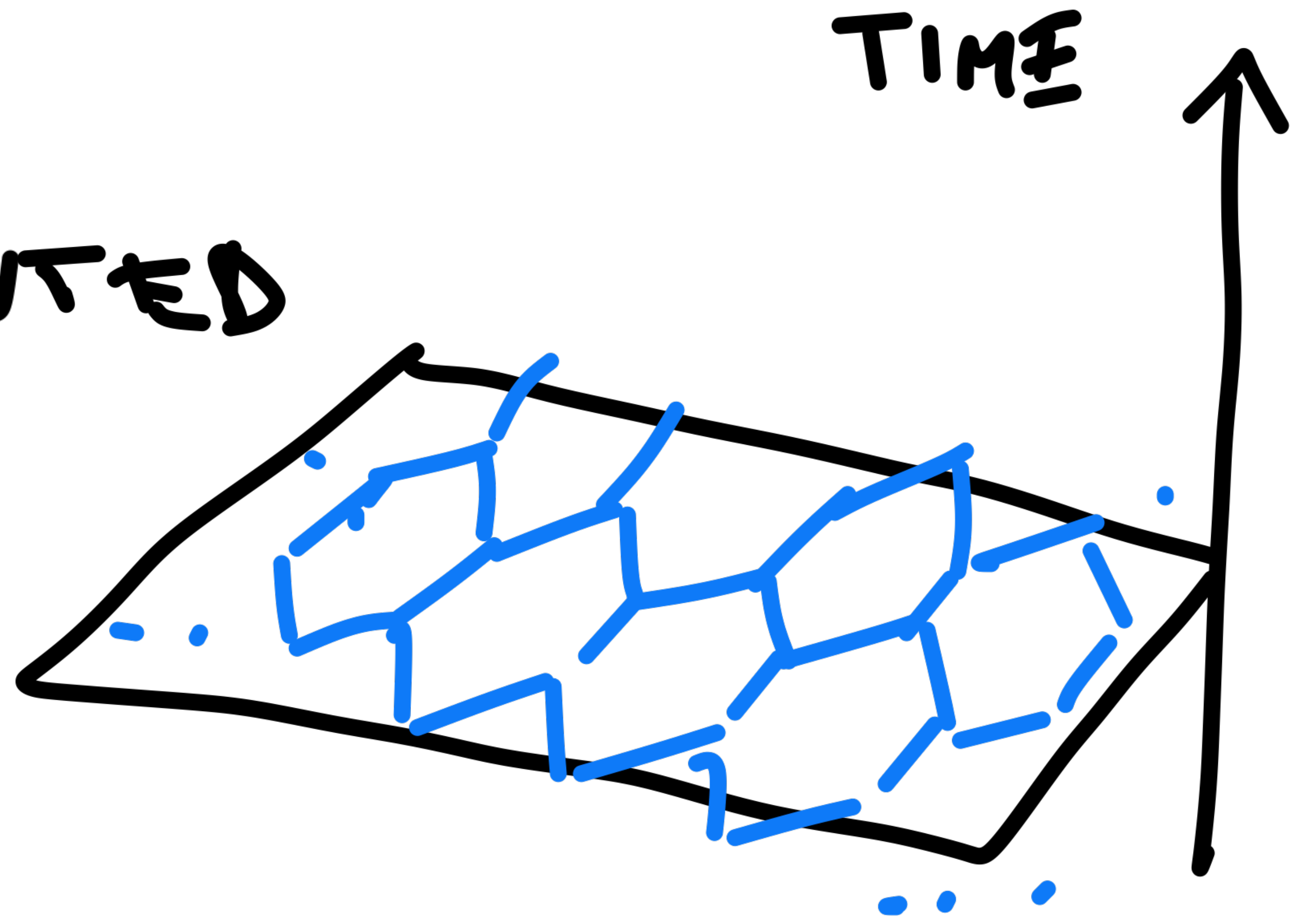
→ REPRODUCE KITAEV-PRESKILL 2006



IS $S^1 \times S^2$

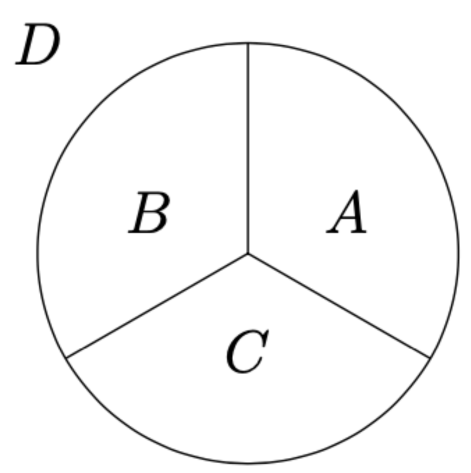
* CRISTOFORI GAGLIARDI PEZZOLA ...

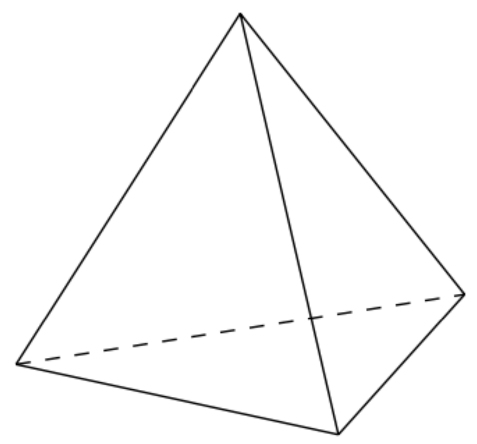
LEVIN WEN STRING NET MODELS

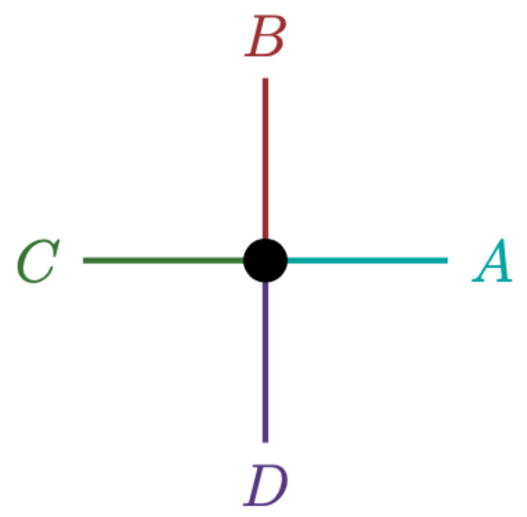


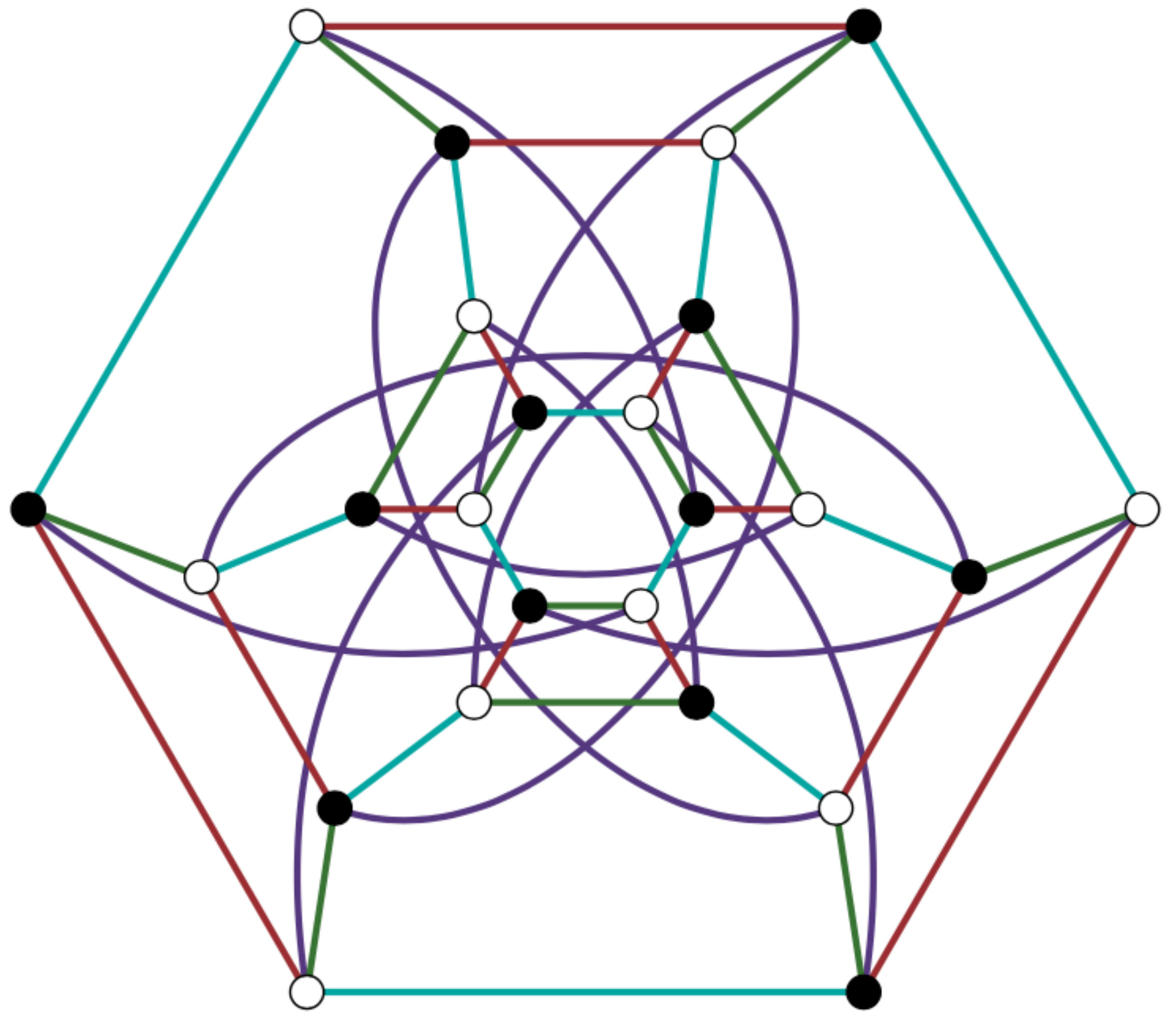
FACT: ALL SMOOTH COMPACT ORIENTED CLOSED 3-MANIFOLDS ARE GRAPHICALLY ENCODED MANIFOLDS

$\Rightarrow$ ENTANGLEMENT DETERMINES ALL PARTITION FUNCTIONS!

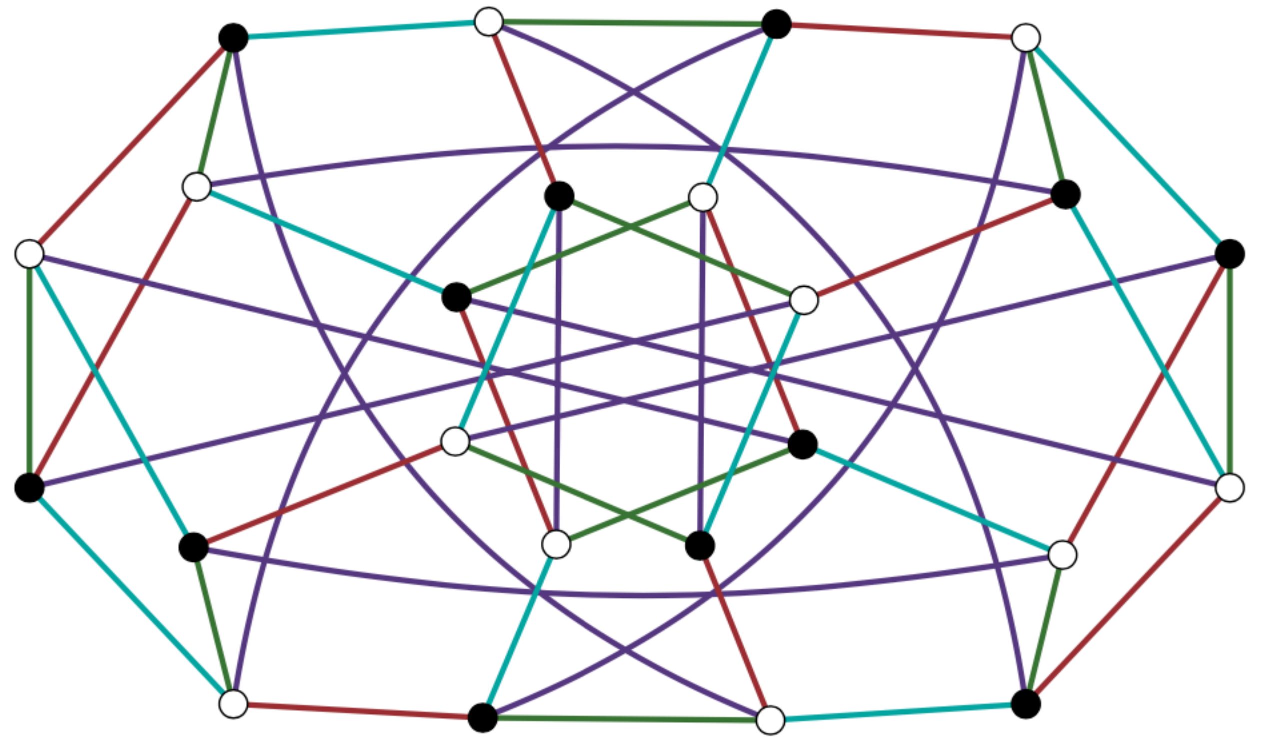








$\mathbb{T}^3$



$\mathbb{S}^3/E_6$

CONCLUSIONS AND OUTLOOK

I HAVE DISCUSSED SOME ASPECTS OF THE INTERPLAY OF TQFT AND ENTANGLEMENT, AND I HAVE EXPLAINED A RECENT RELATED RESULT.

IDEA: GROUND STATE ψ_0

TQFT

QUANTUM INFO / ENTANGLEMENT

CONCLUSIONS AND OUTLOOK

I HAVE DISCUSSED SOME ASPECTS OF THE INTERPLAY OF TQFT AND ENTANGLEMENT, AND I HAVE EXPLAINED A RECENT RELATED RESULT.

IDEA:

GROUND STATE

ψ_0

TQFT

QUANTUM INFO / ENTANGLEMENT

TQFT CAN TEACH US MORE ABOUT ENTANGLEMENT AND VICEVERSA!

CONCLUSIONS AND OUTLOOK

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IDEA:

GROUND STATE

ψ_0

$\swarrow$ TQFT

$\searrow$ QUANTUM INFO / ENTANGLEMENT

Q1:

QUANTUM COMPUTATION: LOGICAL OPERATIONS IN TERMS OF EMERGENT SYMMETRIES

$\longrightarrow$ ENDOMORPHISMS OF THE TQFT !!

CONCLUSIONS AND OUTLOOK

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IDEA: GROUND STATE

ψ_0

TQFT

QUANTUM INFO / ENTANGLEMENT

Q1: QUANTUM COMPUTATION: LOGICAL OPERATIONS IN TERMS OF EMERGENT SYMMETRIES $\longrightarrow$ ENDOMORPHISMS OF THE TQFT !!

APPLICATION OF HIGHER LINEAR ALGEBRA ?

Relation to Sakura's talk

CONCLUSIONS AND OUTLOOK

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IDEA: GROUND STATE ψ_0

TQFT

QUANTUM INFO / ENTANGLEMENT

Q2:

$Z(M^{d-1})$

$Z(M^d)$

HAVE ENTANGLEMENT INTERPRETATION.

CONCLUSIONS AND OUTLOOK

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IDEA:

GROUND STATE ψ_0

TQFT ⟵ ψ_0 ⟶ QUANTUM INFO / ENTANGLEMENT

Q2:

$Z(M^{d-1})$, $Z(M^d)$ HAVE ENTANGLEMENT INTERPRETATION.

HOW ABOUT $Z(M^{d-2})$, $Z(M^{d-3})$ ... ARE THERE HIGHER NOTIONS OF ENTANGLEMENT?

CONCLUSIONS AND OUTLOOK

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IDEA: GROUND STATE ψ_0

TQFT ⟵ ψ_0 ⟶ QUANTUM INFO / ENTANGLEMENT

Q3: HOW ABOUT EXPLICIT EXAMPLES IN HIGHER DIM'S?

CONCLUSIONS AND OUTLOOK

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IDEA: GROUND STATE ψ_0

TQFT ⟵ GROUND STATE ψ_0 ⟶ QUANTUM INFO / ENTANGLEMENT

~~Q3:~~ HOW ABOUT EXPLICIT EXAMPLES IN HIGHER DIM'S?

~~Q4:~~ WHAT DOES ENTANGLEMENT TEACH ABOUT THE CLASSIFICATION OF PHASES OF QUANTUM MATTER?

CONCLUSIONS AND OUTLOOK

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IDEA:

GROUND STATE

ψ_0

TQFT

QUANTUM INFO / ENTANGLEMENT

SUMMARY: DEEP FRUITFUL INTERPLAY BETWEEN GEOMETRY, TOPOLOGY & QUANTUM INFO: ARE TQFTS ENTANGLEMENT?

CONCLUSIONS AND OUTLOOK

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IDEA:

GROUND STATE

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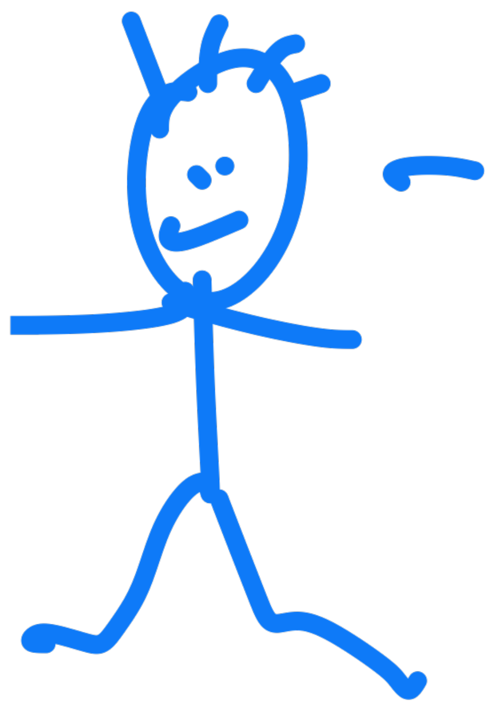

- THANKS!

TQFT

QUANTUM INFO / ENTANGLEMENT

SUMMARY: DEEP FRUITFUL INTERPLAY BETWEEN GEOMETRY, TOPOLOGY & QUANTUM INFO: ARE TQFTs ENTANGLEMENT?