(Semi)-universality of CFT at LARGE spin

Sridip Pal
IHES

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Conformal Field Theories (CFTs)
Physics of Phase Transition to Quantum Gravity

CFT observables satisfy various non-perturbative consistency conditions.

→

The observables become highly constrained.

↓ **Conformal Bootstrap**

Emergence of universal features

Conformal Field Theories (CFTs)

The most basic ingredient of a CFT is spectrum of operators.

They are labeled by scaling dimension, spin.

We are interested in learning whether there is any universality in density of operators at large scaling dimension/spin.

Cardy formula

$$\rho(\Delta) \underset{\Delta\to\infty}{\sim} \left(\frac{c}{48\Delta^3}\right)^{1/4} \exp\left[2\pi\sqrt{\frac{c\Delta}{3}}\right]$$

In 1+1 D CFT, the large energy density of states in universal, determined by a single number, central charge.

Smeared Cardy formula

$$(2\delta - 1) \leqslant \liminf_{\Delta\to\infty} \frac{\int_{\Delta-\delta}^{\Delta+\delta} d\Delta' \ \rho(\Delta')}{\left(\frac{c}{48\Delta^3}\right)^{1/4} \exp\left[2\pi\sqrt{\frac{c\Delta}{3}}\right]} \leqslant \limsup_{\Delta\to\infty} \frac{\int_{\Delta-\delta}^{\Delta+\delta} d\Delta' \ \rho(\Delta')}{\left(\frac{c}{48\Delta^3}\right)^{1/4} \exp\left[2\pi\sqrt{\frac{c\Delta}{3}}\right]} \leqslant (2\delta + 1)$$

The precise version requires averaging over window of size δ and putting a lower and upper bound.

[Mukhametzhanov, SP 2020]
[Mukhametzhanov, Zhiboedov 2019]

Smeared Cardy formula

Smeared density at large energy is universal and determined by a single number, central charge.

[Mukhametzhanov, SP 2020]
[Mukhametzhanov, Zhiboedov 2019]

Today we will focus on the large spin sector.

Is there any universality in large spin?

Large spin universality via correlator bootstrap

OPE of two operators produces operators, which in the large spin limit is very close to generalized free field theory.

[Komargodski, Zhiboedov **2012**]
[Fitzpatrick, Kaplan, Poland, Simmons-Duffin **2012**]
[.........]

Rigorous proof

[**SP,** Qiao, Rychkov **2022**]
[Van Rees, **2024**]

Today we will focus on the large spin sector using partition function.

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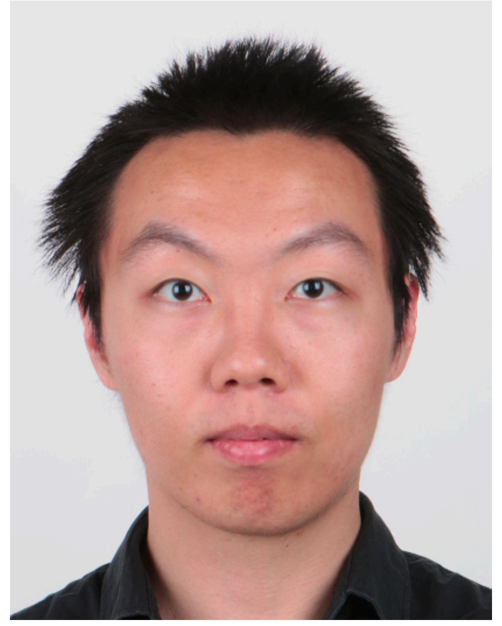
Is there any universality in large spin?

Different from Generalized free field

Plan of the talk:

2505.02897 [hep-th]



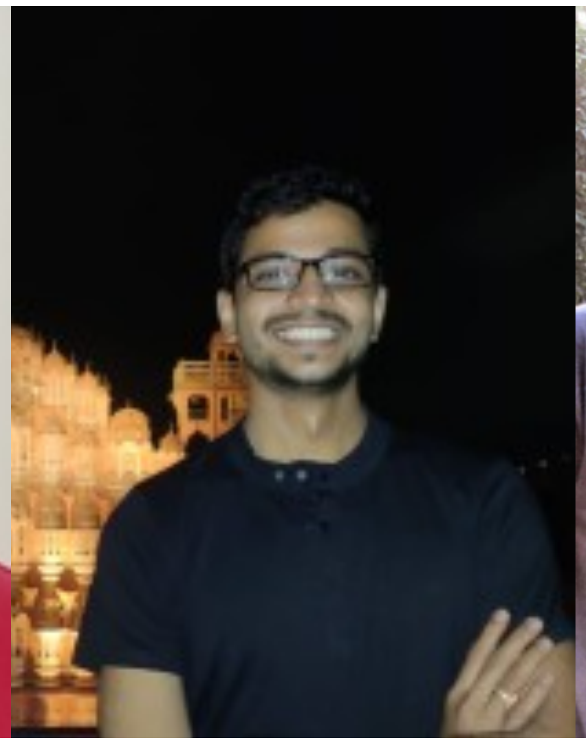
Jiaxin Qiao

Balt C Van Rees

2512.00158 [hep-th]

Shiraz Minwalla

Harsh Anand



Vipul Kumar

Jyotirmoy Mukherjee

Asikur Rahman

Nathan Benjamin

Results:

$$\log\left[\mathrm{tr}\left(e^{-\beta(H-\omega_i J_i)}\right)\right] \underset{\omega_i \to 1}{\sim} \frac{h(\beta)}{\prod(1-\omega_i^2)}$$

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$$h_{2\mathrm{d}}(\beta) = \frac{2\pi c}{12} \times 2\pi\beta^{-1}$$

Universal $c > 1$

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$$h_{2\mathrm{d}}(\beta) = \frac{2\pi c}{12} \times 2\pi\beta^{-1}$$ **Universal** $c > 1$

In higher D, this has an asymptotic expansion in inverse Temp.
Hence **Semi-Universal (related to thermal EFT/hydrodynamics)**

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In microcanonical , it captures the density of states at large spin.

In 2D, in leading order, entropy at large spin is twist independent.

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In higher D $\quad S \sim (\prod_{i=1}^{r} J_i)^{1/(r+1)} \times S_{in}\left(\frac{\tau}{(\prod_{i=1}^{r} J_i)^{1/(r+1)}}\right)$

Ratios of spin fixed.

Results:

$$\log\left[\mathrm{tr}\,\left(e^{-\beta(H-\omega_i J_i)}\right)\right] \underset{\omega_i\to 1}{\sim} \frac{h(\beta)}{\prod(1-\omega_i^2)}$$

In 2D, the canonical result is a consequence of modular invariance. [….., Kusuki; Benjamin, Shao, Ooguri, Wang; **SP**, Qiao, Rees]

Nontriviality lies in microcanonical, especially when one aims to understand refined information. [**SP**, Qiao, Rees]

In higher D, without modular invariance, deriving the canonical result requires much more work!! [Anand, Benjamin, Kumar, Minwalla, Mukherjee, **SP**, Rahman]

Plan

- **Quick recap of results in 2D CFT** (in microcanonical)
- **Higher D results**

Quick recap of results in 2D CFT

large spin universality in unitary 2D CFT

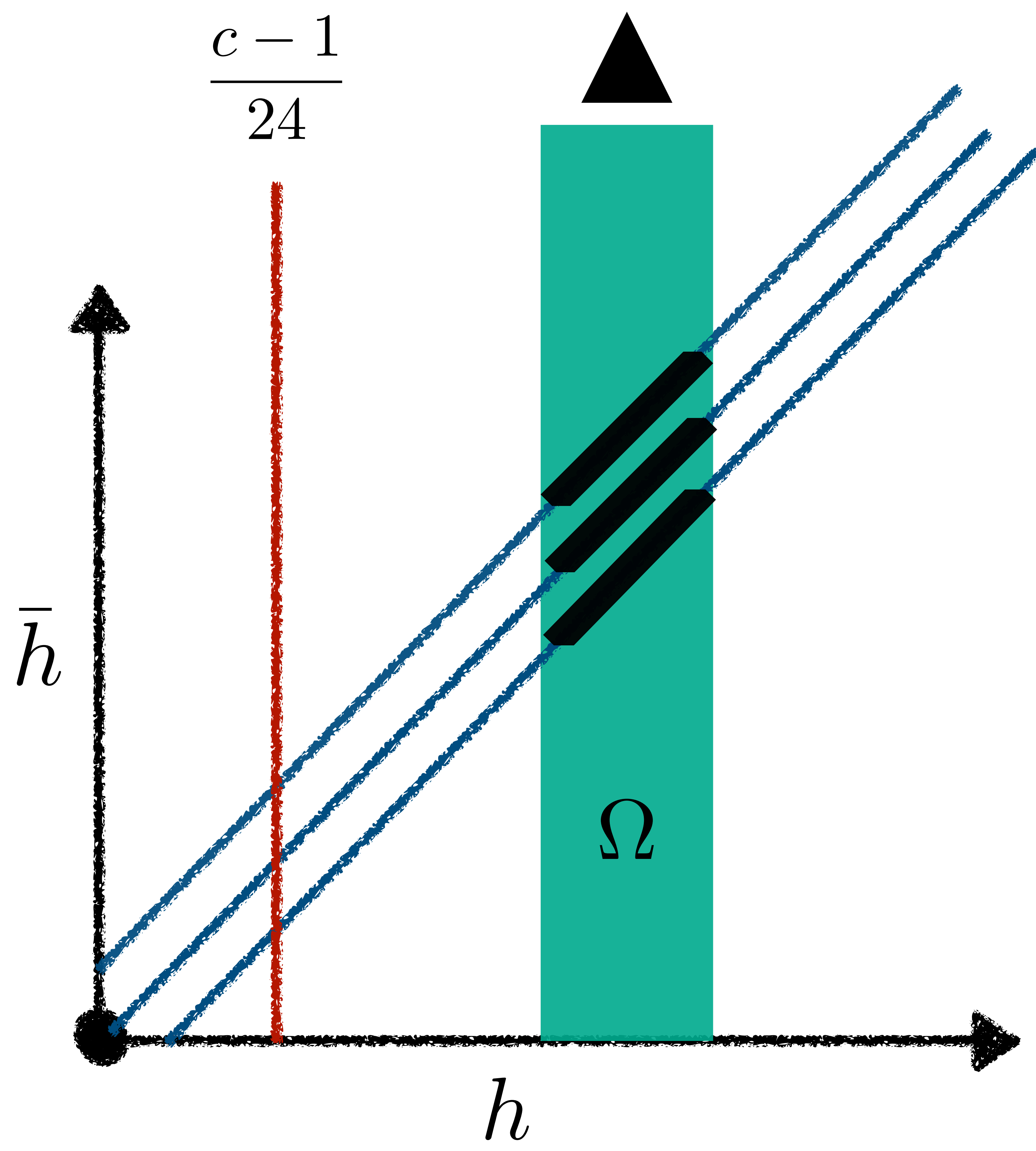
Central charge bigger than 1 & twist gap in the spectrum of Virasoro primaries

[**SP**, J.Qiao, B. Van Rees **2025**]

- Max spacing for spin J operators with bounded twist above $\frac{c-1}{12}$ goes to zero at least as fast as $J^{-1/4+\epsilon}$
- Large spin limt exists [No limsup or liminf]

$$\lim_{J\to\infty} \frac{\int_\Omega d\tau\ \rho_J(\tau)}{\frac{1}{\sqrt{2J}} e^{4\pi\sqrt{\frac{c-1}{24}J}}} = \left(\int_\Omega d\tau\ \rho_0(\tau)\right)$$

- Smoothened entropy formula is non-perturbatively valid.



Now what do we do in higher dimension?

$$\log\left[\mathrm{tr}\,\left(e^{-\beta(H-\omega_i J_i)}\right)\right] \underset{\omega_i\to 1}{\sim} \frac{h(\beta)}{\prod(1-\omega_i^2)}$$

Thermal effective field theory / Hydrodynamics

[Bhattacharya, Lahiri, Loganayagam, Minwalla **2007**]
[Banerjee, Bhattacharya, Bhattacharya, Jain, Minwalla, Sharma **2012**]
[Jensen, Kaminski, Kovtun, Meyer, Ritz, Arom, **2012**]

Reincarnated in recent times in context of deriving Cardy-like formula

[Benjamin, Lee, Ooguri, Simmons-Duffin **2023**]
[see also Shaghoulian, **2015**]

More applications….

[Kang, Lee, Ooguri **2022**]
[Benjamin, Lee, **SP**, Simmons-Duffin, Xu **2024**]
[Kusuki, Ooguri, **SP** **2025**]
[Xu, Simmons-Duffin **2025**]

Defects…….

[Ditakyk, Khanchandani, Popov, Wang **2024**]
[Kravchuk, Radcliffe, Sinha **2024**]
[Cuomo, He, Komargodski **2024**]

Thermal effective field theory / Hydrodynamics

An effective field theory description to systematically capture high temperature physics of a CFT.

$$\log Z = \sum_k a_k \, \beta^{-k+r} + O\left(e^{-\#/\beta}\right)$$

We will come back to finite temperature, $\omega \to 1$ later.

Effective field theory:

$$\mathrm{tr}\ \left(e^{-\beta(H-\omega_i J_i)}\right)$$

- Doing a path integral on twisted $S^1_\beta \times S_{d-1}$
- We write the metric in KK form and do KK reduction.
  Write effective action in terms of background fields on S_{d-1}
- Thermal EFT / Hydrodynamics expansion $\beta \to 0$
- Think of a fluid at inverse temp β

$$ds^2 = g_{ij}dx^i dx^j + e^{2\phi}(d\tau - A)^2$$

- We want to understand $Z[\Sigma_{d-1} \times S^1_\beta]$ in $\beta \to 0$ limit.

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KK-reduction; thermal mass gap $\sim 1/\beta$

$$Z_{\text{gapped}}[g_{ij}, A_i, \phi]$$

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$$Z_{\text{gapped}}[g_{ij}, A_i, \phi]$$

Weyl invariance to rescale the metric

$$Z_{\text{gapped}}[\hat{g}_{ij}, A_i, 0]\,, \qquad \hat{g} = e^{-2\phi} g$$

Upto weyl anomaly pieces

$$Z_{\text{gapped}}[\hat{g}_{ij}, A_i, 0] \;=\; e^{-S[\hat{g}_{ij}, A_i] + \text{Weyl anomaly}}$$

- Write down all possible terms consistent with symmetry

$$S[\hat{g}, A] = \int d^{d-1}x \sqrt{\hat{g}} \left(-f + c_1 \hat{R} + c_2 F^2 + \cdots \right)$$

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- In 2D only single term survives since 1D is Ricci flat and gauge connection is flat too.

$$S = \frac{2\pi}{\beta} f$$

- We identify f with central charge $f = \frac{2\pi c}{12}$

$$S[\hat{g}, A] = \int d^{d-1}x\sqrt{\hat{g}}\left(-f + c_1\hat{R} + c_2F^2 + \cdots\right)$$

$$\beta^2 \qquad \beta^2$$

This is a systematic high temperature expansion

What about $\omega \to 1$ limit?

$$\log\left[\mathrm{tr}\ \left(e^{-\beta(H-\omega_i J_i)}\right)\right] \underset{\omega_i \to 1}{\sim} \frac{h(\beta)}{\prod(1-\omega_i^2)}$$

Coming back to $\omega_i \to 1$ the big picture idea is following:

- Rapidly rotating fluid $\omega_i \to 1$
- Comoving temperature is high and comoving volume is large.
- Use that to deduce universality.

2D story revisited- intuitive arguments

$$\log \mathrm{tr}\ e^{-\beta(H-\omega J)} \underset{\omega\to 1}{\sim} \frac{L}{\beta(1-\omega^2)}\frac{2\pi c}{12}$$

Path integral on twisted $S^1_\beta \times S^1_L$

$$ds^2 = d\tau^2 + dx^2\,, (\tau, x) \sim (\tau+\beta, x - i\omega\beta)$$

$$x' = x + i\omega\beta$$

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$$ds^2 = (1-\omega^2)(d\tau - \frac{i\omega}{1-\omega^2}dx')^2 + \frac{1}{1-\omega^2}dx'^2\,, (\tau, x') \sim (\tau+\beta, x')$$

$$\beta \to (1-\omega^2)^{1/2}\beta\,, \quad L \to L\frac{1}{(1-\omega^2)^{1/2}}$$

$$\frac{L}{\beta} \to \frac{L}{\beta(1-\omega^2)}$$

3D story- intuitive arguments

$$\log \mathrm{tr}\ e^{-\beta(H-\omega J)} \underset{\omega\to 1}{\sim} \frac{h(\beta)}{(1-\omega^2)}$$

Path integral on twisted $S^1_\beta \times S^2$

$$ds^2 = d\tau^2 + d\theta^2 + \sin^2\theta d\phi^2\,, (\tau,\phi) \sim (\tau+\beta, \phi - i\omega\beta)$$

$$\phi' = \phi + i\omega\beta$$

$$ds^2 = (1-\omega^2\sin^2\theta)(d\tau - \frac{i\omega\sin^2\theta}{1-\omega^2\sin^2\theta}d\phi')^2 + d\theta^2 + \frac{\sin^2\theta}{1-\omega^2\sin^2\theta}d\phi'^2\,, (\tau,\phi') \sim (\tau+\beta,\phi')$$

$$ds^2_{weyl} = (d\tau - \frac{i\omega\sin^2\theta}{1-\omega^2\sin^2\theta}d\phi')^2 + \frac{1}{(1-\omega^2\sin^2\theta)}d\theta^2 + \frac{\sin^2\theta}{(1-\omega^2\sin^2\theta)^2}d\phi'^2\,, (\tau,\phi') \sim (\tau+\beta,\phi')$$

Singular region comes around the equator, set $\theta = \pi/2$

3D story- intuitive arguments

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Singular region comes around the equator, set $\theta = \pi/2$

$$ds^2_{weyl;\text{eff}} = (d\tau - \frac{i\omega}{1-\omega^2}d\phi')^2 + \frac{1}{(1-\omega^2)^2}d\phi'^2$$

In angular direction, volume blows up, thermal EFT is applicable.

$$\log \text{tr } e^{-\beta(H-\omega J)} \underset{\omega\to 1}{\sim} \frac{h(\beta)}{(1-\omega^2)}$$

Technically what we do is following:

Thermal EFT/ Hydrodynamics expansion $\beta \to 0$

Take the $\omega_i \to 1$ limit term by term

Term by term we have $a_n \times \frac{\beta^n}{\prod(1-\omega_i^2)}$ **[proven in all order]**

Resum them [needs to do it via Borel summation]

[this is involved mathematically- story for another time]

Checks:

- We check these against holography and free scalar field theory in all dimensions.

$$\log\left[\mathrm{tr}\ \left(e^{-\beta(H-\omega_i J_i)}\right)\right] \underset{\omega_i\to 1}{\sim} \frac{h(\beta)}{\prod(1-\omega_i^2)}$$

$$h_{\text{free}}^{d=4}(\beta) = \frac{\pi^4}{180\beta^3} - \frac{\pi^2}{72\beta} + \frac{3\zeta(3)}{8\pi^2} - \frac{7\beta}{1440} + \sum_{n=1}^{\infty}\sum_{d|n}\frac{(-1)^{d+1}}{\pi^2 d^3} e^{-\frac{2\pi^2 n}{\beta}}$$

$$h_{BH}(\beta) = \frac{N^2}{64}\left(\frac{2\pi}{\beta^3}\left(2\beta^2+\pi^2\right)^{\frac{3}{2}} + \frac{2\pi^4}{\beta^3} - \frac{10\pi^2}{\beta} - \beta\right)$$

Rotating blackholes in AdS

Fine Prints I:

- Non-perturbative terms do not spoil things.

Generically, we expect terms like $(1-\omega^2)^{-\#}\, e^{-\frac{a}{\beta(1-\omega^2)}}$

Hence they go away unless there is fine tuning.

Fine Prints I:

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↳ Generically, we expect terms like $(1-\omega^2)^{-\#}\, e^{-\frac{a}{\beta(1-\omega^2)}}$

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In free theory, there is such fine tuning, however conjecure checks out.

Fine Prints II:

- High temperature and omega to 1 limit commutes

→ In 2D it is obvious if we write in terms of $\beta_{L,R} = \beta(1 \pm \omega)$

→ In higher D, it follows from uniformity of omega to 1 limit along with the generic argument for absence of non-perturbative term.

[related to Borel summation story]

Fine Prints III:

$$\mathrm{tr}\ \left(e^{-\beta(H-\omega_i J_i)}\right)$$

- In 2D, unitarity guarantees the trace is well defined when

$$\mathrm{Re}(\beta) > 0,\ \ |\omega| < 1$$

- In higher D, unitarity bounds, alone are not enough.
- In 4D ANEC [Cordova, Diab 2017] is sufficient. Higher D ??

- Black holes, grey galaxy and gas phase: transitions among them

[Minwalla et al. ….series of papers]

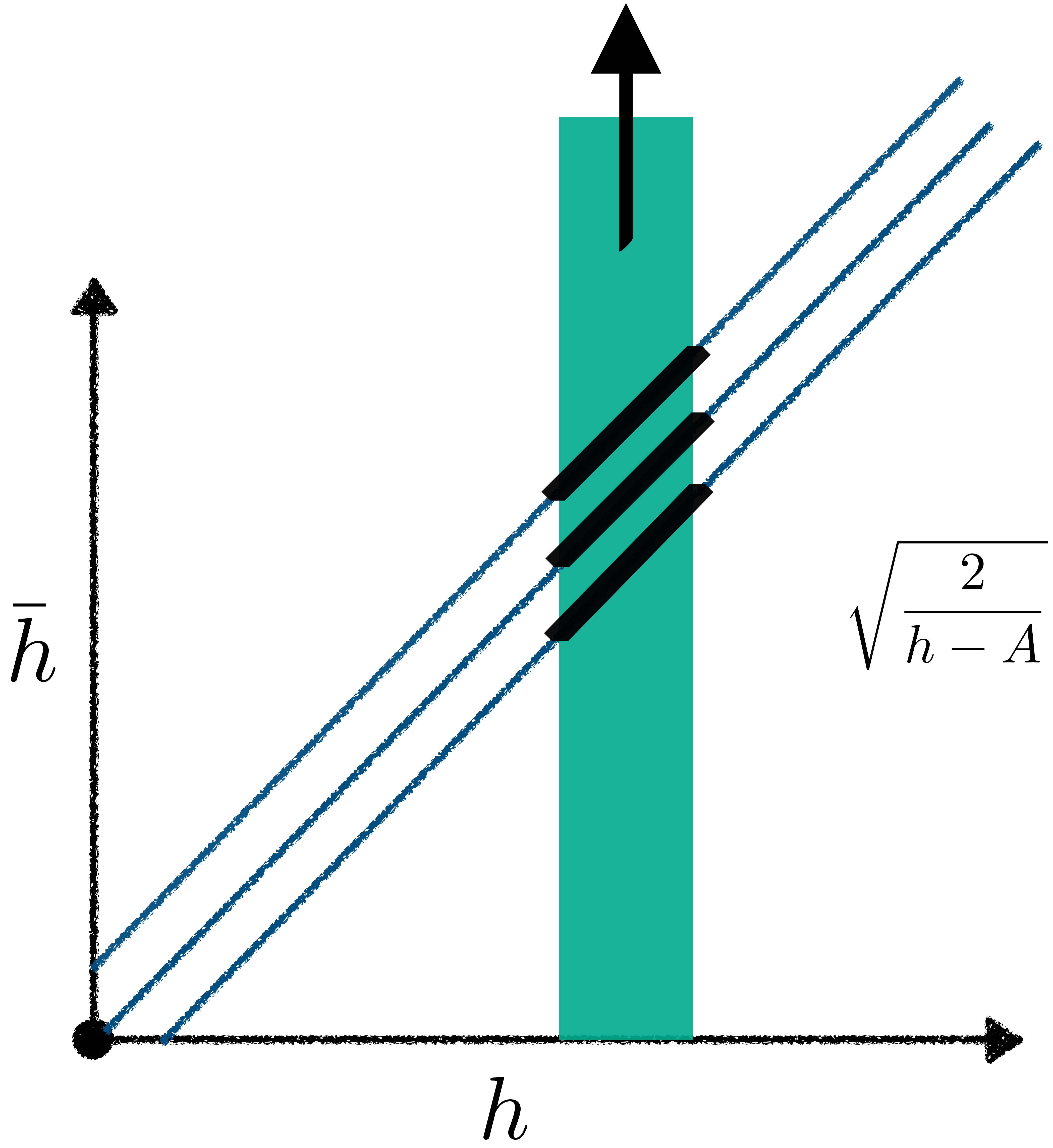
↳ In all phases, our conjecture to be true.

OPEN questions:

- Are there more checks on our conjecture ?
- Is there an argument without resorting to high T expansion and resummation ?
- In 2D, large c Cardy formula is due to [Hartman Keller Stoica **2014**] [Dey, Qiao, **SP** **2024**]
  Is there a large N analogue in higher D ?
- Can we make microcanonical result precise ? [very challenging!!]
- Can we connect back to usual lightcone bootstrap regime ?
- Is there a similar story with charge, index ?

[THANK YOU]

[**SP**, J.Qiao, B. Van Rees 2025]

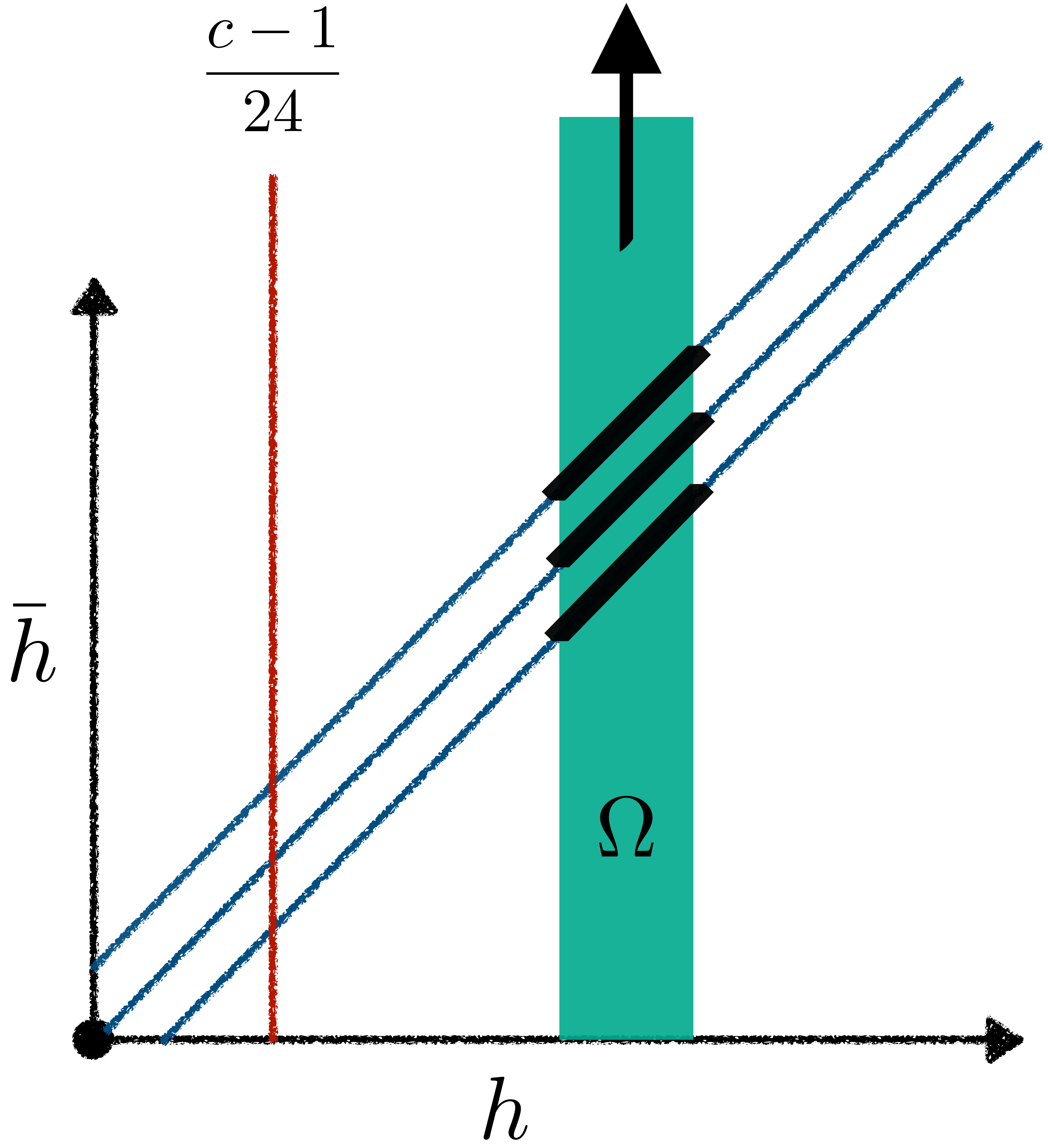


$$\lim_{J\to\infty} \frac{\int_\Omega \; d\Delta' \rho_J(\Delta')}{\frac{1}{\sqrt{2J}} e^{4\pi\sqrt{AJ}}} = \left(\int_\Omega \; dh \; \rho_0(h) \right)$$

$$\sqrt{\frac{2}{h-A}}[\cosh(4\pi\sqrt{A(h-A)}) - \cosh(4\pi\sqrt{(A-1)(h-A)})]\Theta(h-A)$$

$$A := \frac{c-1}{24}$$

[**SP**, J.Qiao, B. Van Rees **2025**]



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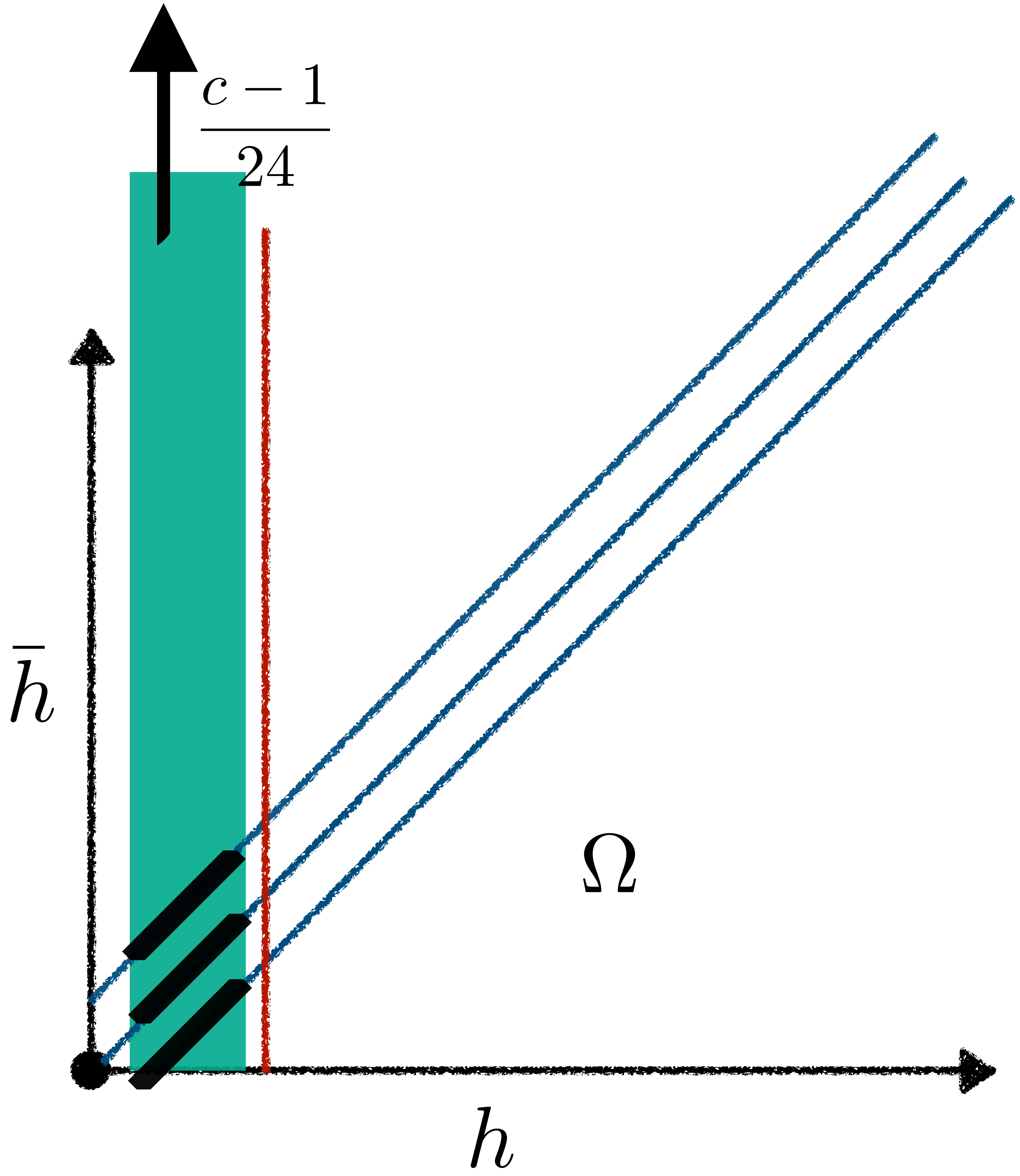
RHS is non-zero

[**SP**, J.Qiao, B. Van Rees **2025**]



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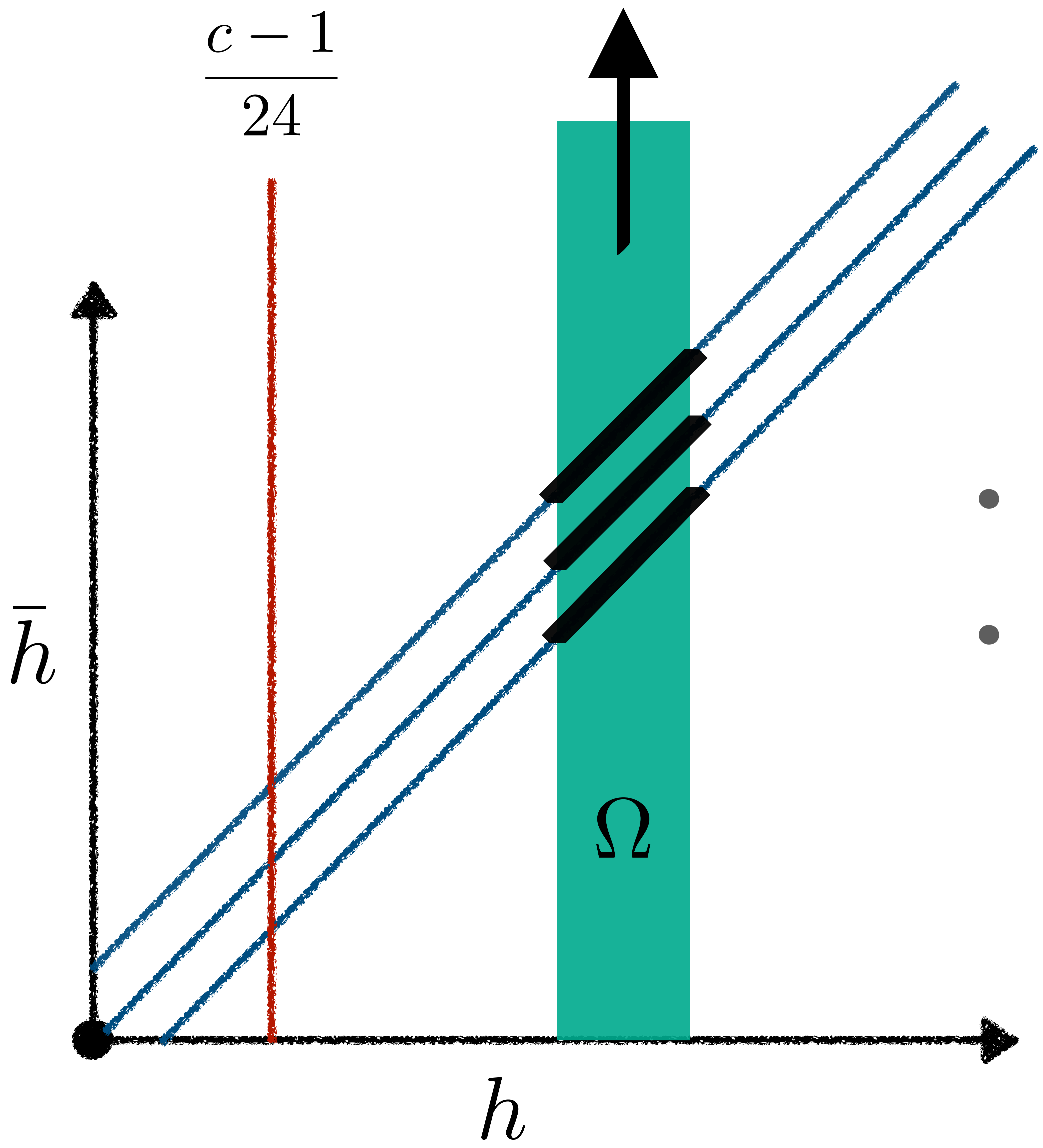
RHS is zero

Growth below the threshold is strictly slower.

[**SP**, J.Qiao, B. Van Rees **2025**]



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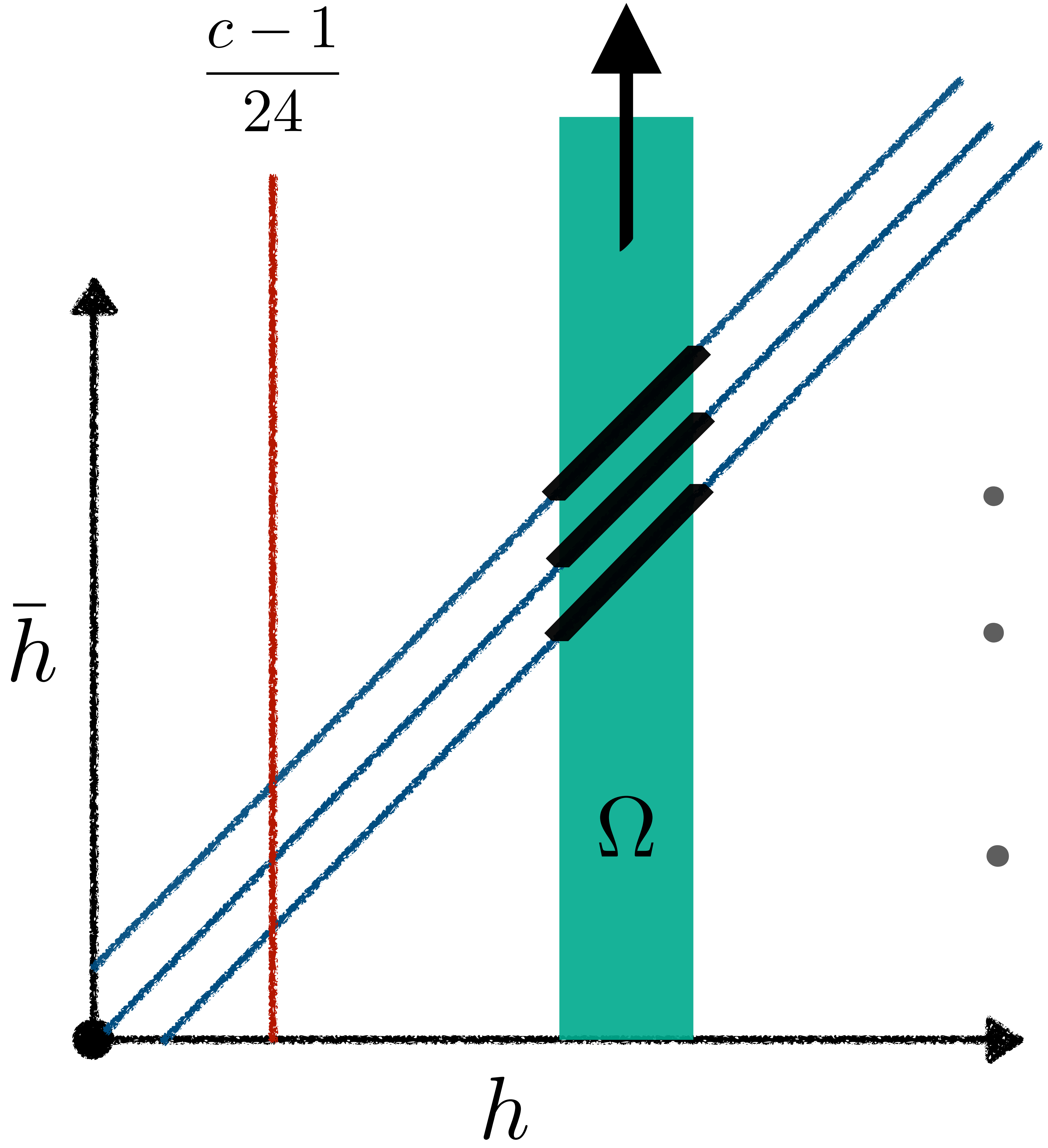
- RHS is non-zero irrespective of the width of strip.
- The maximum spacing of states with a fixed spin J goes to 0 as J becomes large.

[**SP**, J.Qiao, B. Van Rees **2025**]



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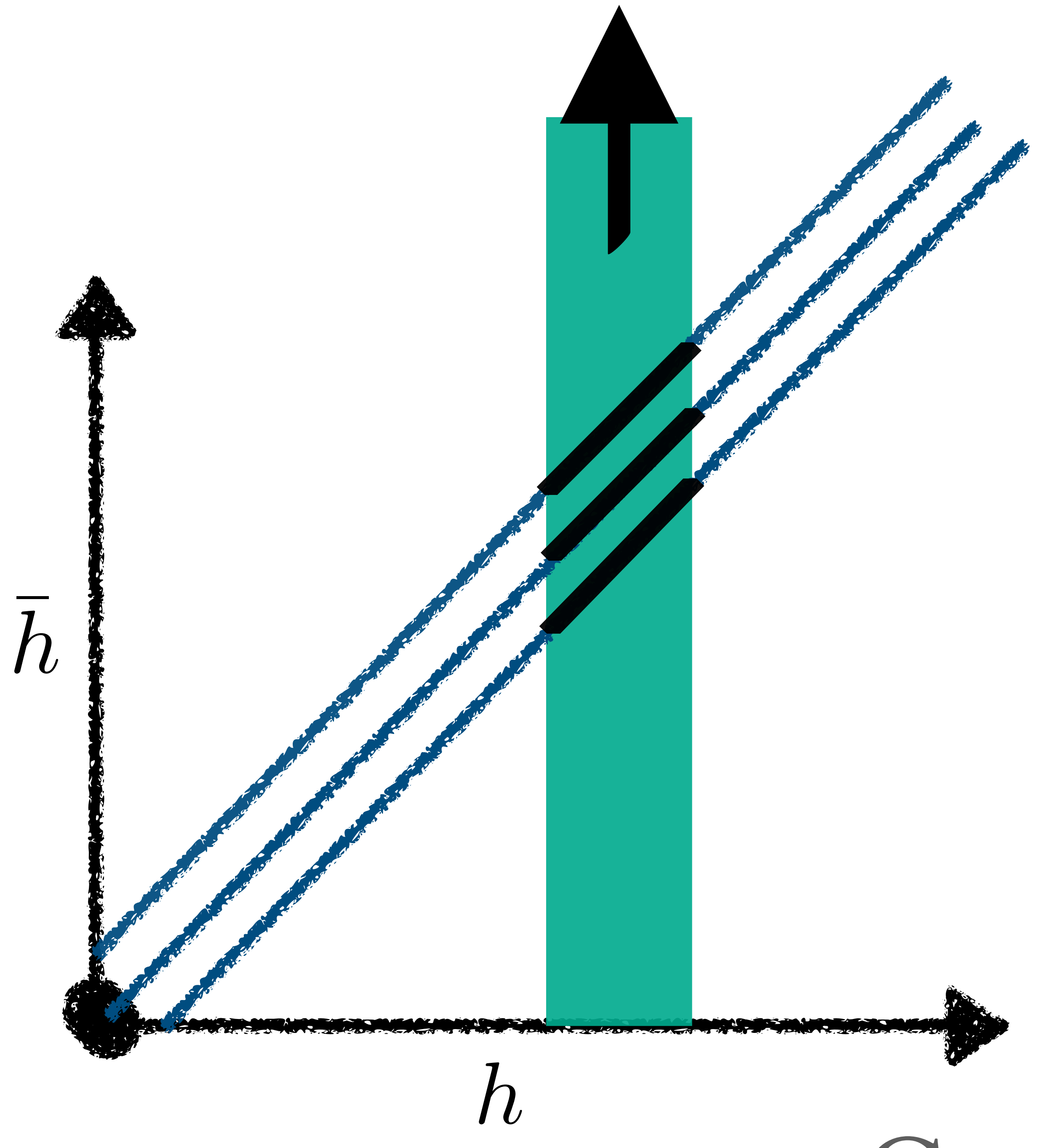
- RHS is non-zero irrespective of the width of strip.
- The maximum spacing of states with a fixed spin J goes to 0 as J becomes large.
- Entropy is universal upto order one in J

[SP, J.Qiao, B. Van Rees 2025]

$$\int_\Omega d\Delta' \rho_J(\Delta') \sim \frac{1}{\sqrt{2J}} e^{4\pi\sqrt{AJ}} \left(\int_\Omega dh\ \rho_0(h) \right)$$

- We smoothen things a bit and estimate the error.



$$\frac{\int dh' \varphi_{h_*,\delta}(h') \left(\rho_J(J+2h') - \rho_{J,vac}(J+2h') \right)}{\frac{1}{\sqrt{2J}} \exp\left(4\pi\sqrt{AJ}\right)}$$

Compactly supported, smooth function, support on $(h_* - \delta, h_* + \delta)$