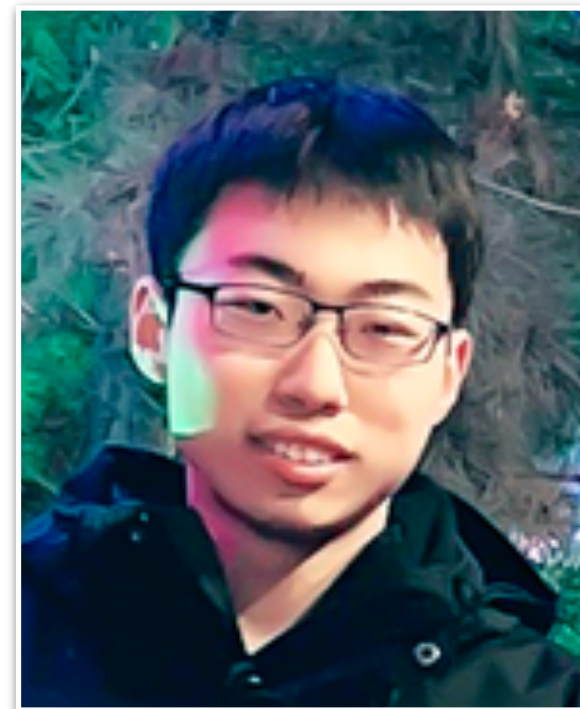


Boundary transitions from a single round of measurements on gapless quantum states

Sara Murciano

Workshop Interfaces & Symmetries, YITP

6 March 2026



Yue Liu



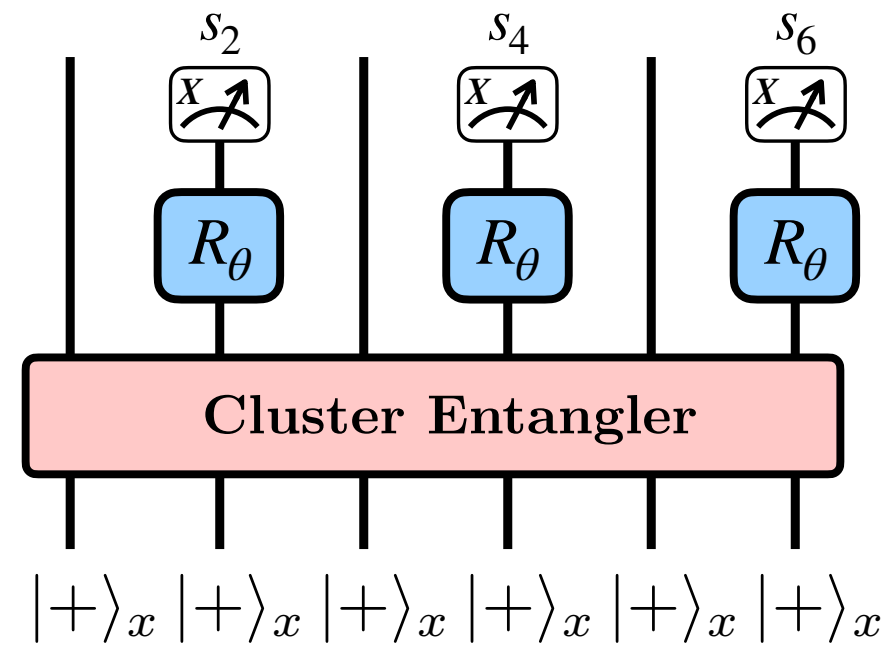
David Mross



Jason Alicea

Measurement-generated novel quantum phenomena

Preparation of long-range entangled states from cluster state



Pirolì, Styliaris, Cirac, Phys. Rev. Lett. 127, 220503 (2021).

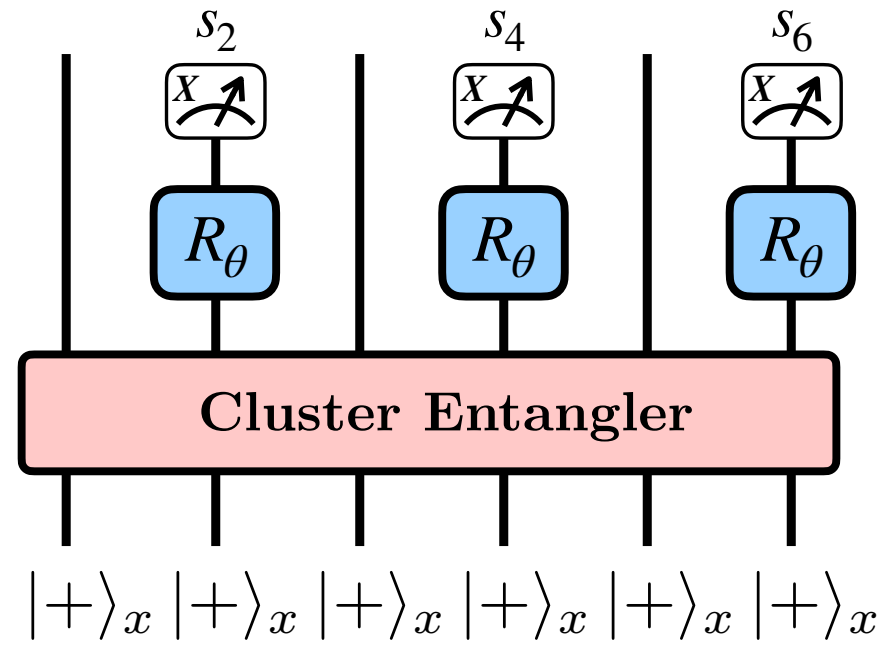
Tantivasadakarn, Thorngren, Vishwanath, Verresen.
Phys. Rev. X 14, 021040 (2024)

Lee, Ji, Bi, Fisher, arXiv:2208.11699, 2022.

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Measurement-altered criticality

1D Tomonaga-Luttinger Liquids

Altered Correlations

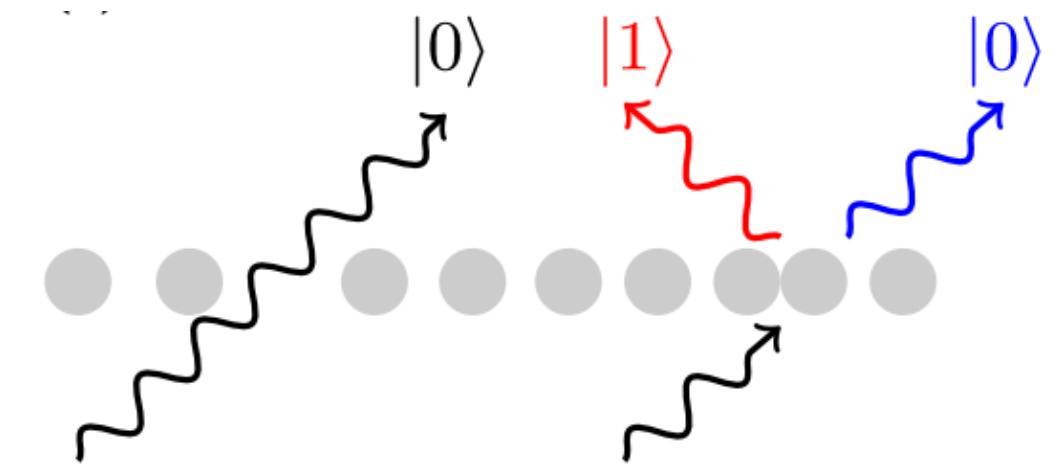
Measurements Conspire Nonlocally to Restructure Critical Quantum States

Samuel J. Garratt¹, Zack Weinstein¹, and Ehud Altman^{1,2}

Altered Entanglement

New critical states induced by measurement

Xinyu Sun¹, Hong Yao^{1,*}, and Shao-Kai Jian^{2,†}



Transverse-Field Ising Model

Nonlocality and entanglement in measured critical quantum Ising chains

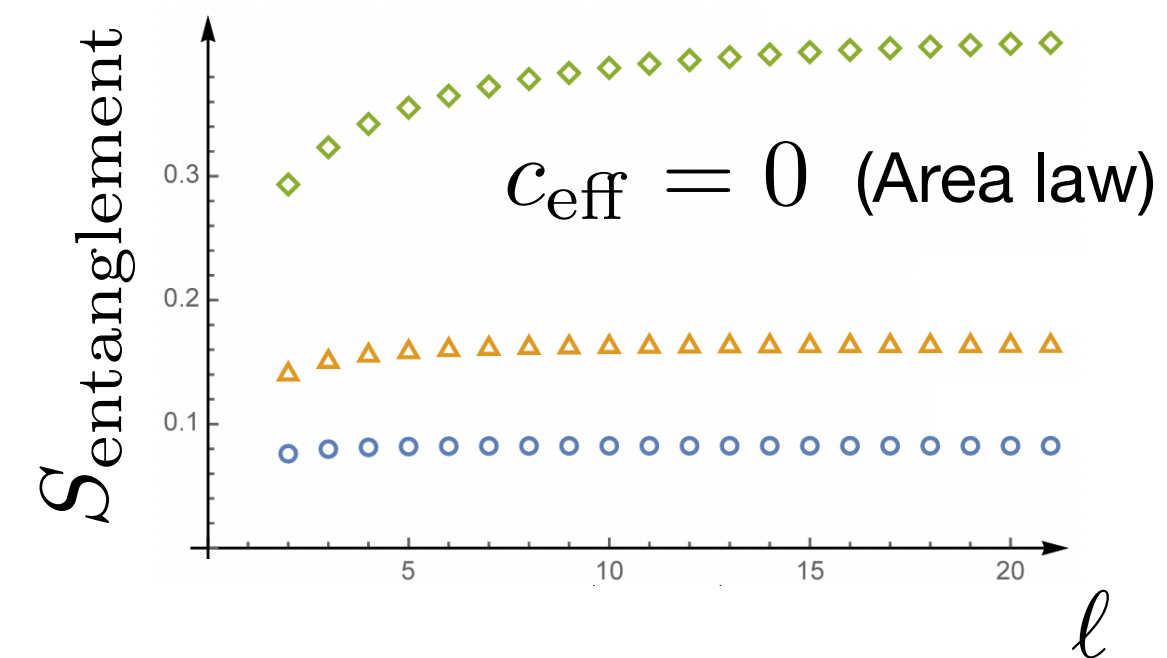
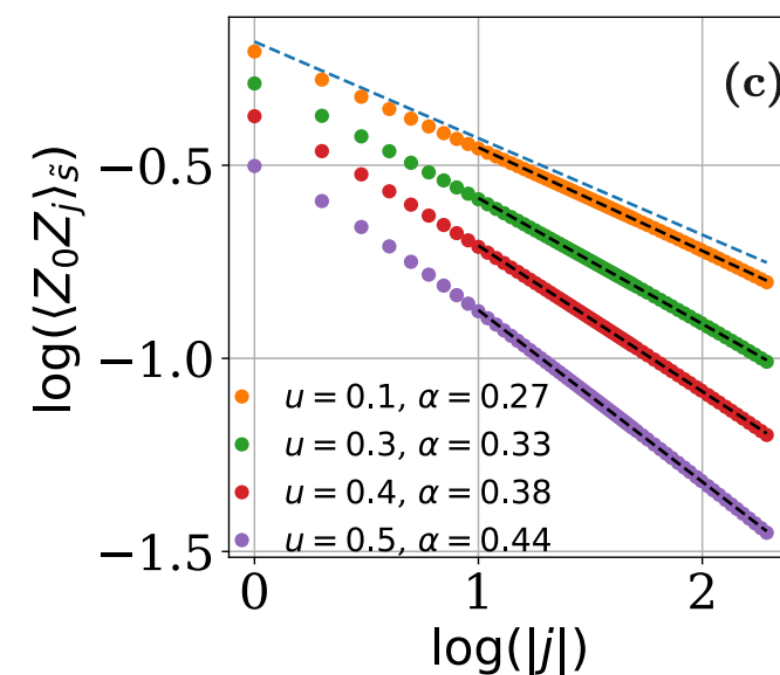
Zack Weinstein^{1,*}, Rohith Sajith^{1,*}, Ehud Altman^{1,2}, and Samuel J. Garratt¹

Entanglement in a one-dimensional critical state after measurements

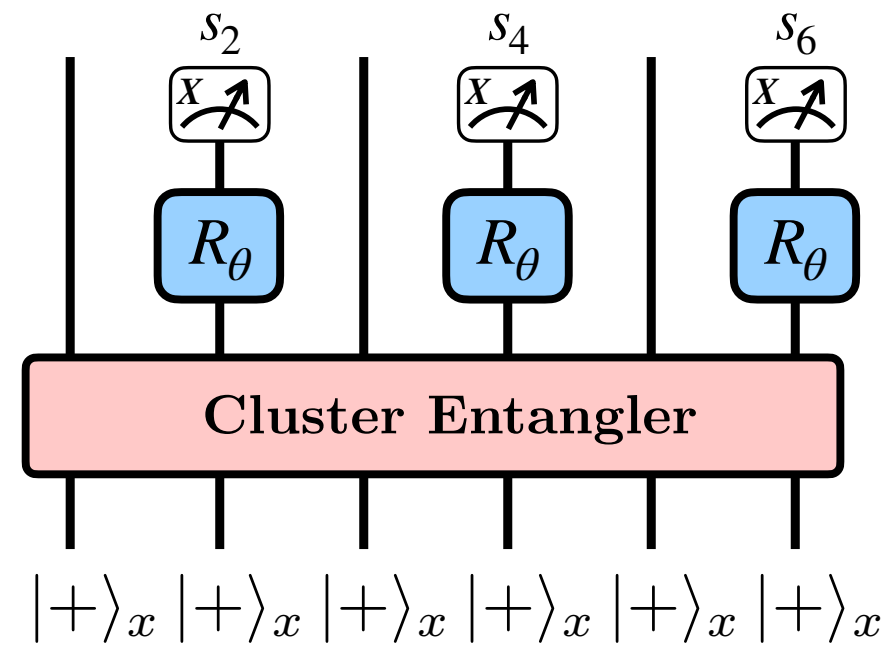
Zhou Yang¹, Dan Mao¹, and Chao-Ming Jian¹

Measurement-altered Ising quantum criticality

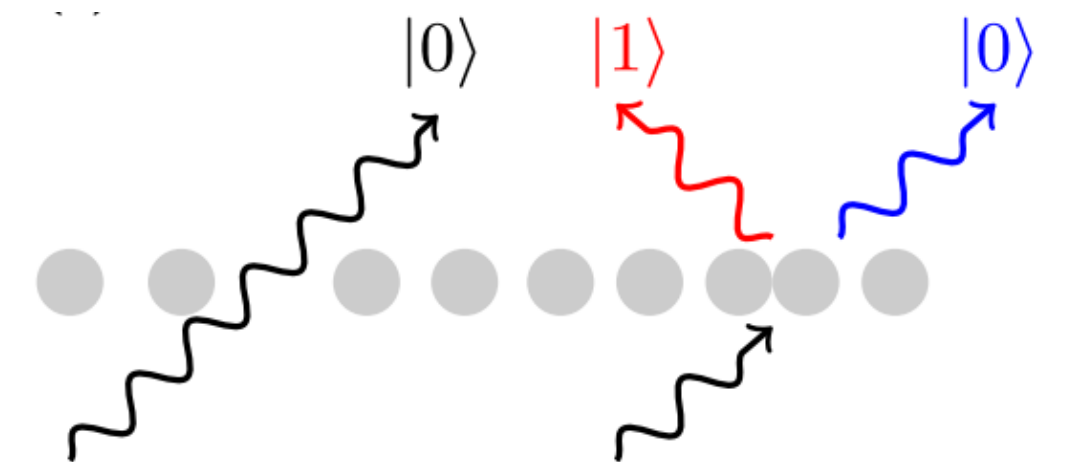
Sara Murciano^{1,2,*}, Pablo Sala^{1,2,*}, Yue Liu¹, Roger S. K. Mong³, and Jason Alicea^{1,2}



Preparation of long-range entangled states from cluster state



Measurements in a gapless parent state of the 1D cluster state



Measurement-altered criticality

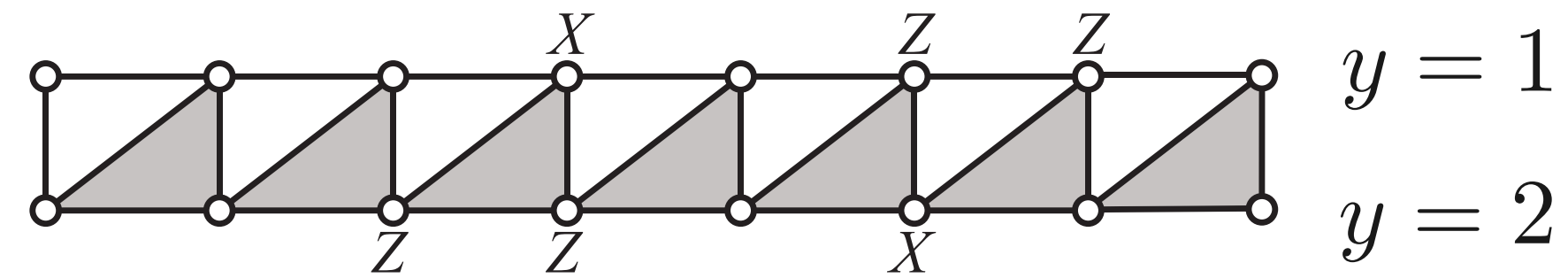
Outline:

- Review about the canonical cluster state
- Gapless parent of the 1D cluster state
- Weak measurements on the gapless parent state
- Conclusions and outlook

Outline:

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Canonical cluster state: a review



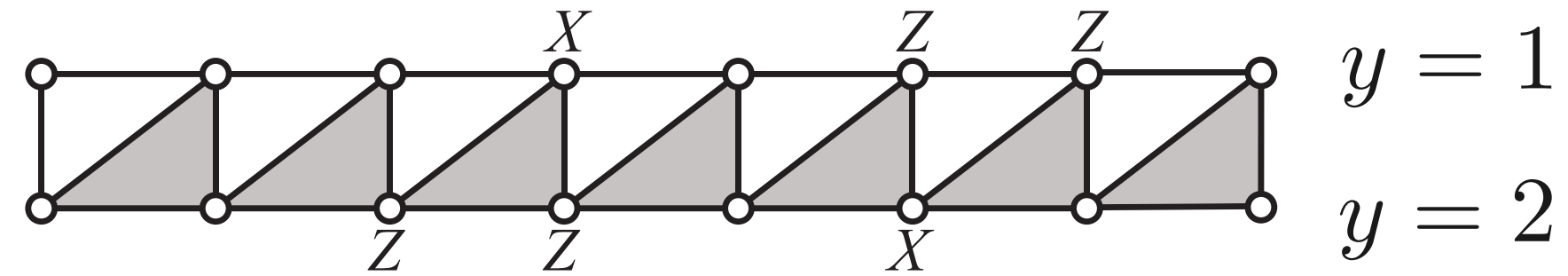
$$H_{\text{cluster}} = - \sum_{j=1}^{N-1} (Z_{j,1} X_{j,2} Z_{j+1,1} + Z_{j,2} X_{j+1,1} Z_{j+1,2})$$

Symmetry Protected Topological (SPT) phase:

$$G_1 = \prod_{j=1}^N X_{j,1}$$

$$G_2 = \prod_{j=1}^N X_{j,2}$$

Canonical cluster state: a review



$$H_{\text{cluster}} = - \sum_{j=1}^{N-1} (Z_{j,1} X_{j,2} Z_{j+1,1} + Z_{j,2} X_{j+1,1} Z_{j+1,2})$$

Symmetry Protected Topological (SPT) phase:

$$G_1 = \prod_{j=1}^N X_{j,1}$$

$$G_2 = \prod_{j=1}^N X_{j,2}$$

$$1 = Z_{j,1} X_{j,2} Z_{j+1,1}$$

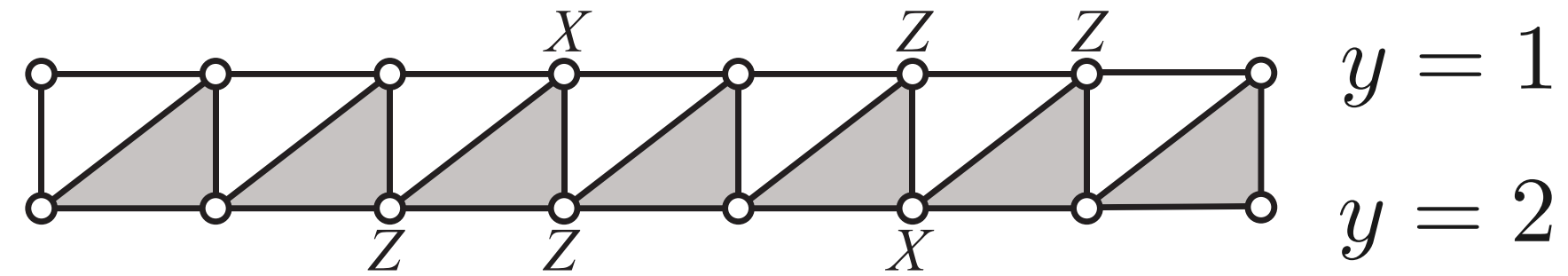
$$Z_{j+1,1} X_{j+1,2} Z_{j+2,1}$$

$$Z_{j+2,1} X_{j+2,2} Z_{j+3,1}$$

⋮

$$Z_{k-1,1} X_{k-1,2} Z_{k,1}$$

Canonical cluster state: a review



$$H_{\text{cluster}} = - \sum_{j=1}^{N-1} (Z_{j,1} X_{j,2} Z_{j+1,1} + Z_{j,2} X_{j+1,1} Z_{j+1,2})$$

Symmetry Protected Topological (SPT) phase:

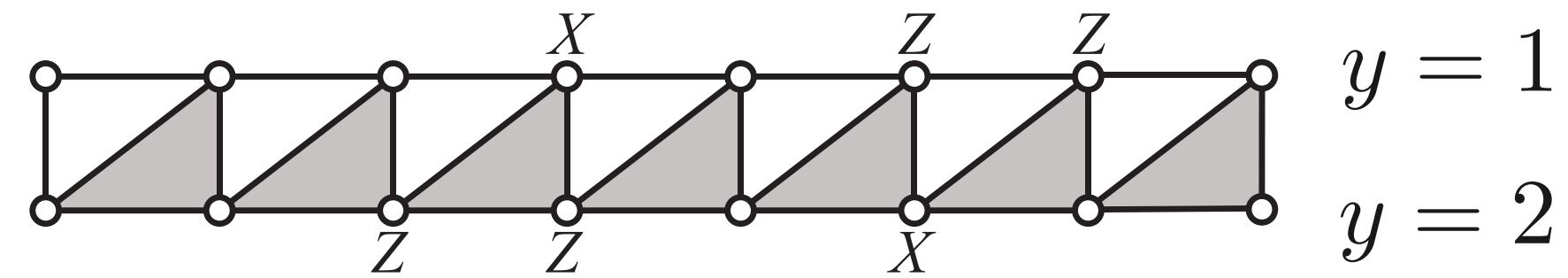
$$G_1 = \prod_{j=1}^N X_{j,1}$$

$$G_2 = \prod_{j=1}^N X_{j,2}$$

String order parameter:

$$\mathcal{S}_{j,k} \equiv \left[\begin{array}{cccc} Z_j & & & Z_k \\ X_j & X_{j+1} & \cdots & X_{k-1} \end{array} \right] = 1$$

Canonical cluster state: a review



$$H_{\text{cluster}} = - \sum_{j=1}^{N-1} (Z_{j,1} X_{j,2} Z_{j+1,1} + Z_{j,2} X_{j+1,1} Z_{j+1,2})$$

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String order parameter:

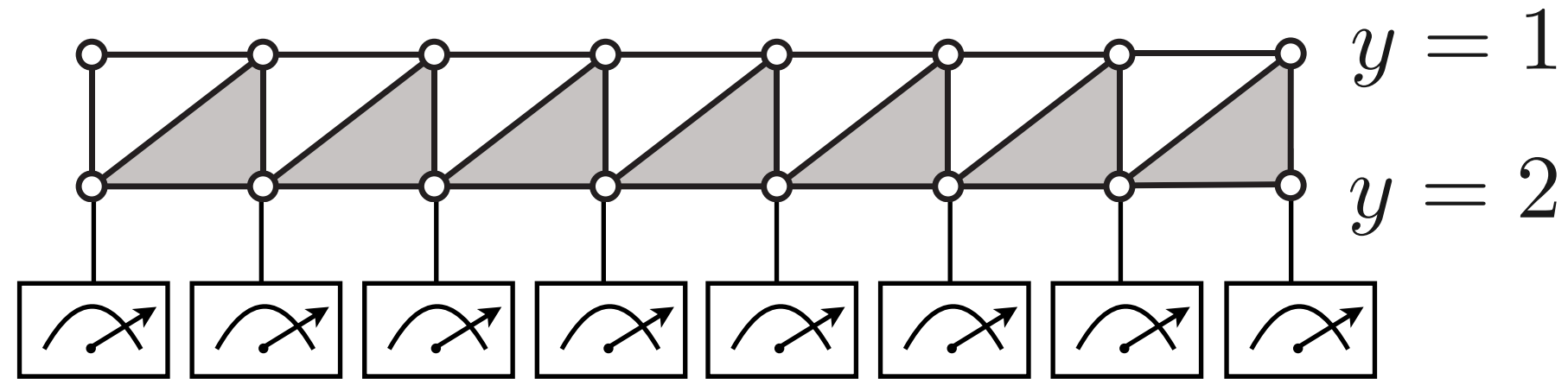
$$\mathcal{S}_{j,k} \equiv \left[\begin{array}{cccc} Z_j & & & Z_k \\ X_j & X_{j+1} & \cdots & X_{k-1} \end{array} \right] = 1$$

but $\langle Z_{j,1} Z_{k,1} \rangle = 0$

short-range!

Conversion into a GHZ state

X measurements



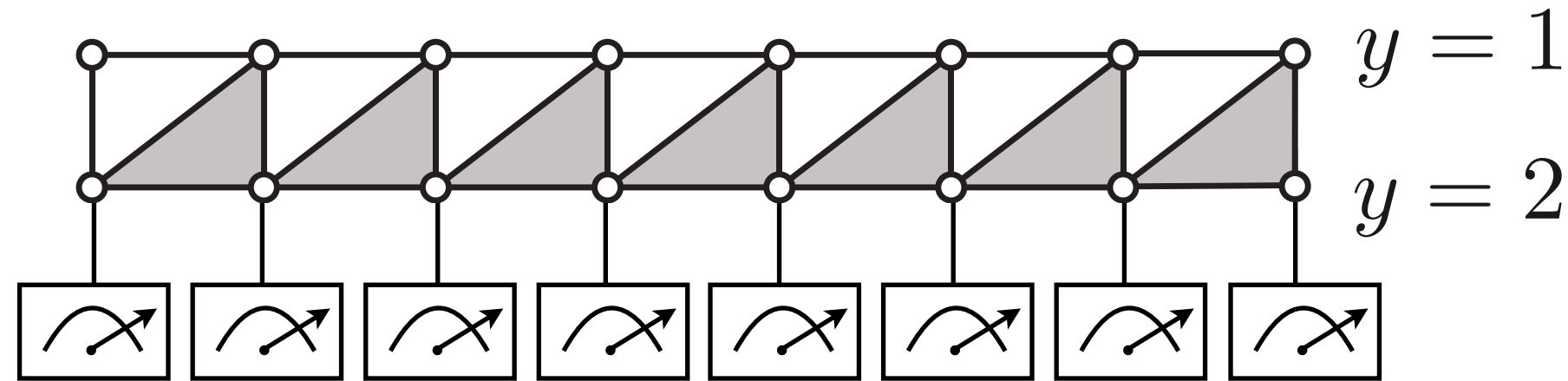
$$H_{\text{cluster}} = - \sum_{j=1}^{N-1} (Z_{j,1} X_{j,2} Z_{j+1,1} + Z_{j,2} X_{j+1,1} Z_{j+1,2})$$

Post-measurement state: $|\psi\rangle_{\mathbf{s}} \propto \mathcal{P}_{\mathbf{s}} |\psi_0\rangle \equiv \prod_{j=1}^N \left(\frac{1+s_j X_{j,2}}{2} \right) |\psi_0\rangle$

$$\langle Z_{j,1} Z_{k,1} \rangle_{\mathbf{s}} = \frac{\langle \psi_0 | \mathcal{P}_{\mathbf{s}} Z_{j,1} Z_{k,1} \mathcal{P}_{\mathbf{s}} \mathcal{S}_{j,k} | \psi_0 \rangle}{\langle \psi_0 | \mathcal{P}_{\mathbf{s}} | \psi_0 \rangle} = s_j \cdots s_{k-1}$$

Conversion into a GHZ state

X measurements



$$H_{\text{cluster}} = - \sum_{j=1}^{N-1} (Z_{j,1} X_{j,2} Z_{j+1,1} + Z_{j,2} X_{j+1,1} Z_{j+1,2})$$

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controlled unitary feedback:
 $U_{\mathbf{s}} |\psi\rangle_{\mathbf{s}}$

$$s_j = 1$$

$$\sum_{\mathbf{s}} p_{\mathbf{s}} \langle Z_{j,1} Z_{k,1} \rangle_{\mathbf{s}} s_j \cdots s_{k-1} = 1$$

Reminder:

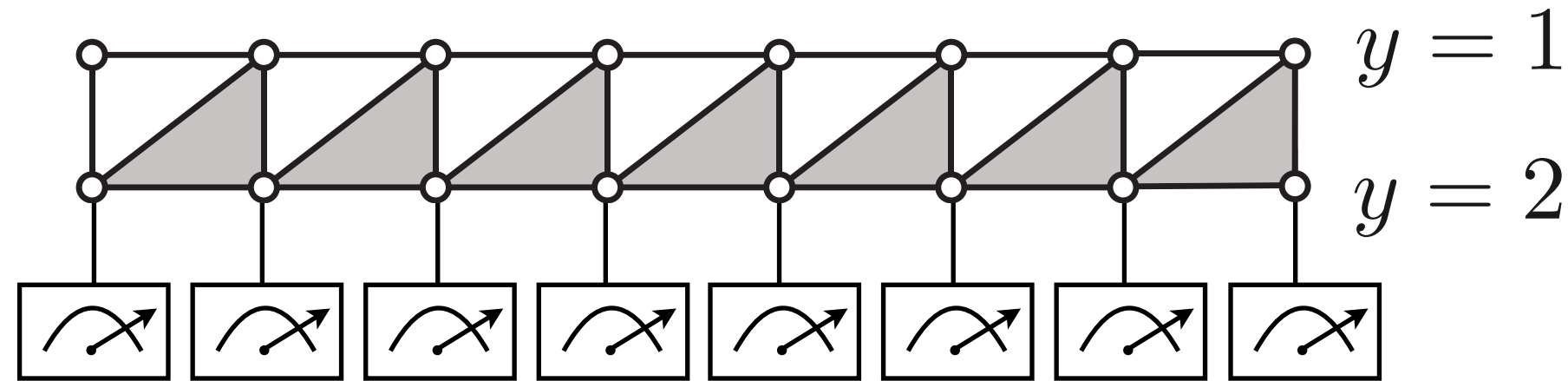
$$\sum_{\mathbf{s}} p_{\mathbf{s}} \langle Z_{j,1} Z_{k,1} \rangle_{\mathbf{s}} = \langle Z_{j,1} Z_{k,1} \rangle_0$$

**correlations
characteristic
of a GHZ state**

$$|\text{GHZ}\rangle = \frac{1}{\sqrt{2}} (|\uparrow \cdots \uparrow\rangle + |\downarrow \cdots \downarrow\rangle)$$

Conversion into a GHZ state

X measurements



$$H_{\text{cluster}} = - \sum_{j=1}^{N-1} (Z_{j,1} X_{j,2} Z_{j+1,1} + Z_{j,2} X_{j+1,1} Z_{j+1,2})$$

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long-range order!

$$\langle Z_{j,1} Z_{k,1} \rangle_{\mathbf{s}} = \frac{\langle \psi_0 | \mathcal{P}_{\mathbf{s}} Z_{j,1} Z_{k,1} \mathcal{P}_{\mathbf{s}} \mathcal{S}_{j,k} | \psi_0 \rangle}{\langle \psi_0 | \mathcal{P}_{\mathbf{s}} | \psi_0 \rangle} = s_j \cdots s_{k-1}$$

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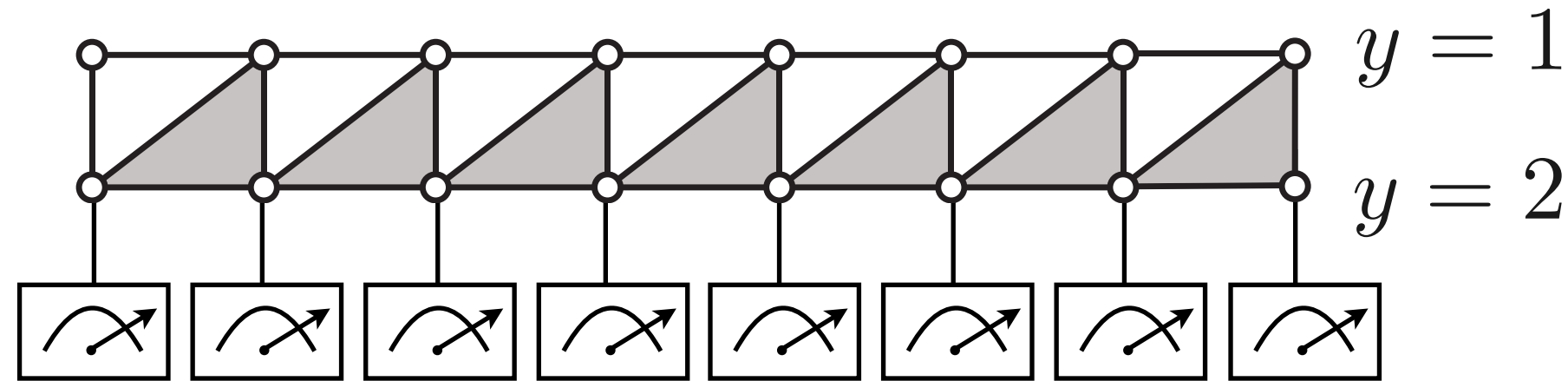
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**correlations
characteristic
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$$|\text{GHZ}\rangle = \frac{1}{\sqrt{2}} (|\uparrow \cdots \uparrow\rangle + |\downarrow \cdots \downarrow\rangle)$$

Conversion into a GHZ state: is it robust?

weak 'tilted' measurement



$$H_{\text{cluster}} = - \sum_{j=1}^{N-1} (Z_{j,1} X_{j,2} Z_{j+1,1} + Z_{j,2} X_{j+1,1} Z_{j+1,2})$$

Post-measurement state: $|\psi\rangle_{\mathbf{s}} \propto e^{2\beta \sum_j s_j (\cos \omega X_{j,2} + \sin \omega Z_{j,2})} |\psi_0\rangle$ β : measurement strength
 ω : measurement angle

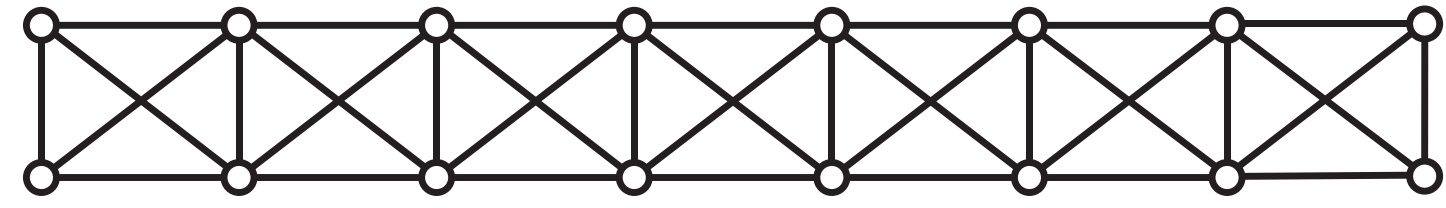
$$\langle Z_{j,1} Z_{k,1} \rangle_{\text{uni}} \approx (\tanh 2\beta \cos \omega)^{|j-k|} \equiv e^{-|j-k|/\xi}$$

long-range order only appears when
the measurement is **projective**
and when the **tilt angle is $\omega = 0$**

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Gapless parent of the cluster state

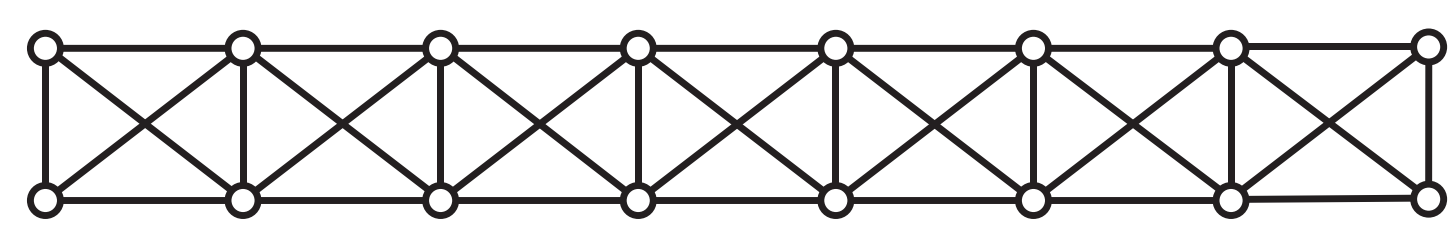


$y = 1$

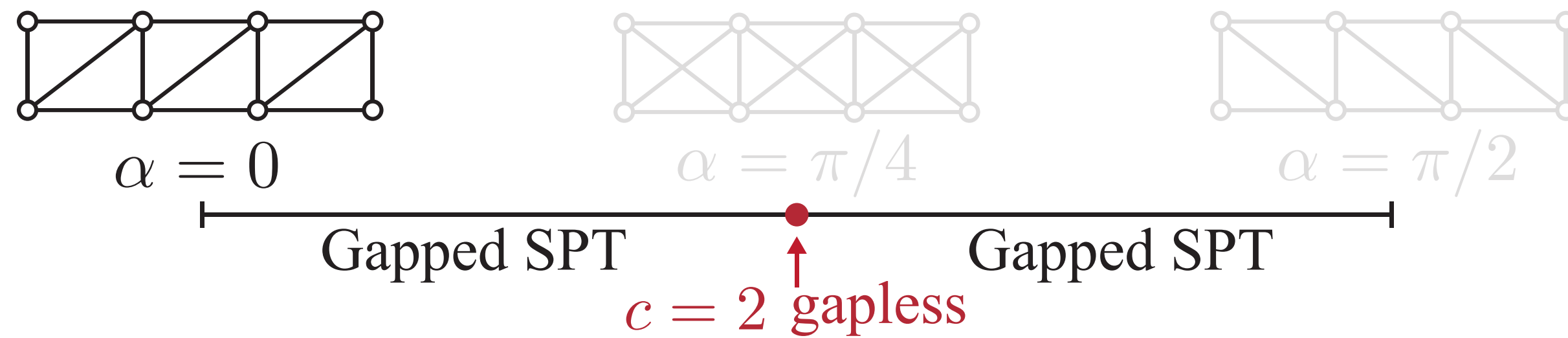
$y = 2$

$$H = - \sum_{j=1}^{N-1} [\cos \alpha (Z_{j,1} X_{j,2} Z_{j+1,1} + Z_{j,2} X_{j+1,1} Z_{j+1,2}) \\ + \sin \alpha (Z_{j,2} X_{j,1} Z_{j+1,2} + Z_{j,1} X_{j+1,2} Z_{j+1,1})].$$

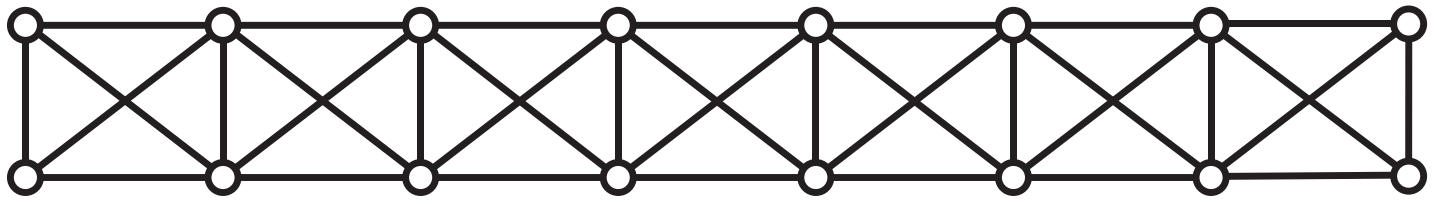
Gapless parent of the cluster state



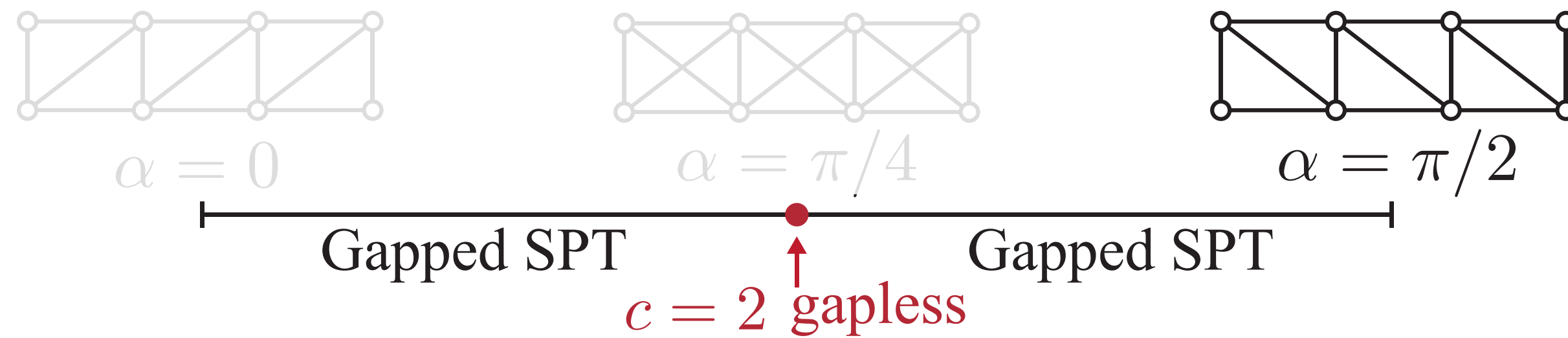
$$\begin{aligned}
 H = - \sum_{j=1}^{N-1} & [\cos \alpha (Z_{j,1} X_{j,2} Z_{j+1,1} + Z_{j,2} X_{j+1,1} Z_{j+1,2}) \\
 & + \sin \alpha (Z_{j,2} X_{j,1} Z_{j+1,2} + Z_{j,1} X_{j+1,2} Z_{j+1,1})].
 \end{aligned}$$



Gapless parent of the cluster state



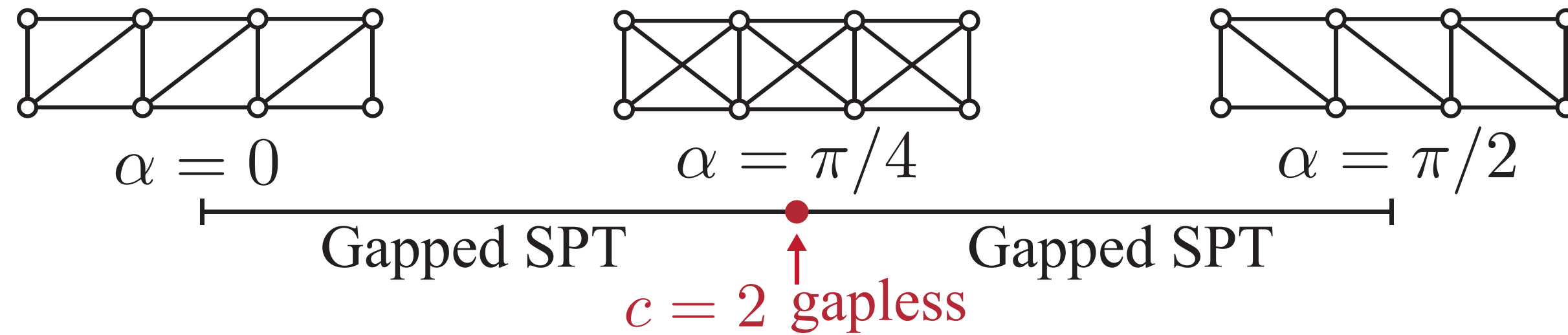
$$\begin{aligned}
 & y = 1 \\
 & y = 2
 \end{aligned}
 \quad H = - \sum_{j=1}^{N-1} [\cos \alpha (Z_{j,1} X_{j,2} Z_{j+1,1} + Z_{j,2} X_{j+1,1} Z_{j+1,2}) \\
 + \sin \alpha (Z_{j,2} X_{j,1} Z_{j+1,2} + Z_{j,1} X_{j+1,2} Z_{j+1,1})].$$



Gapless parent of the cluster state

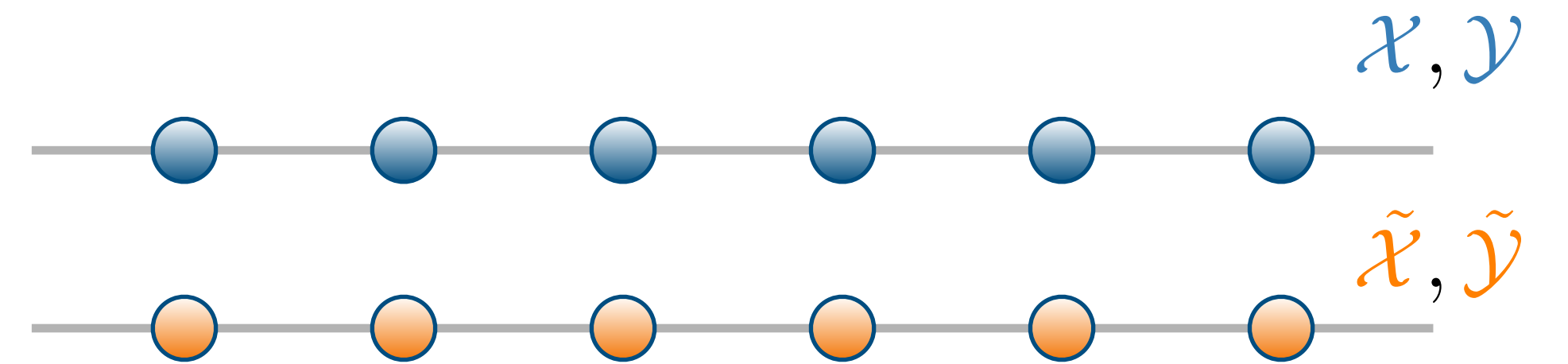
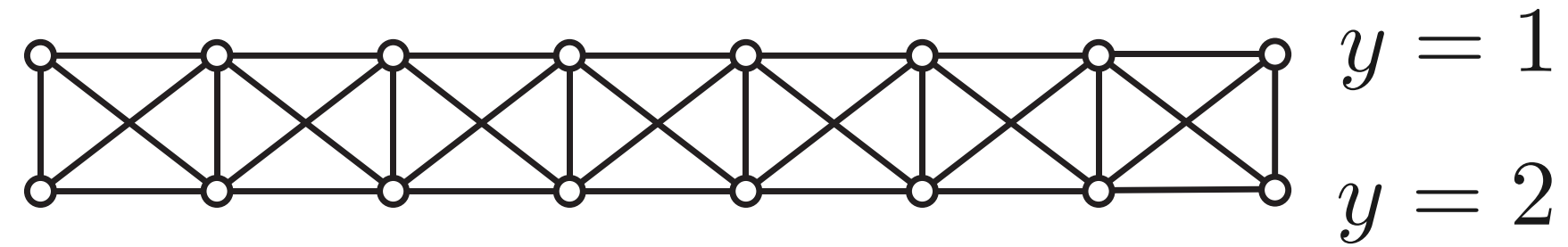
$y = 1$
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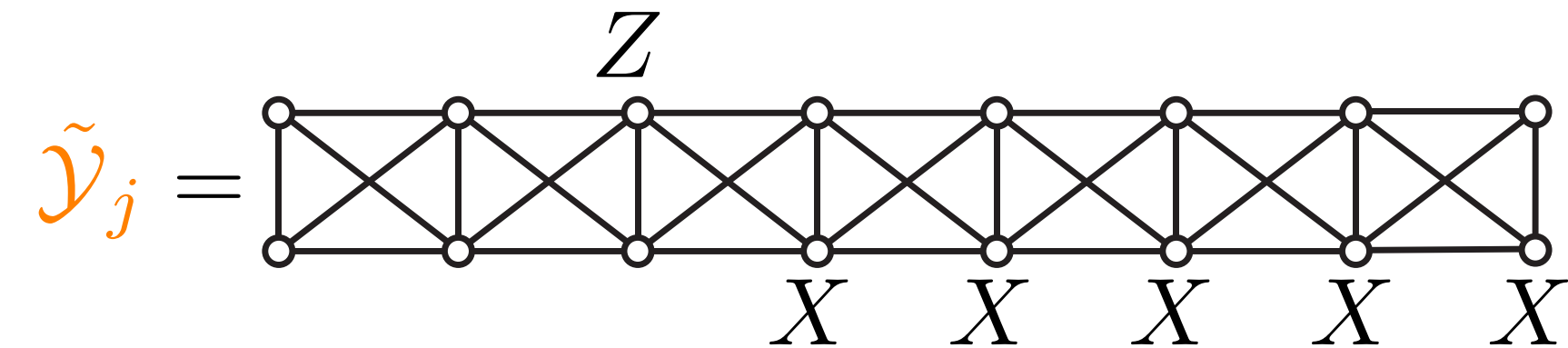
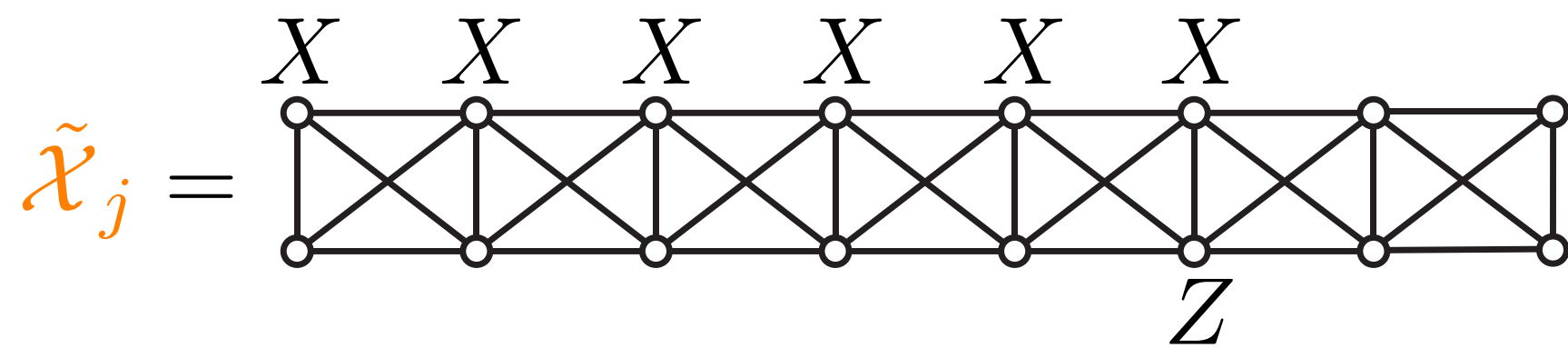
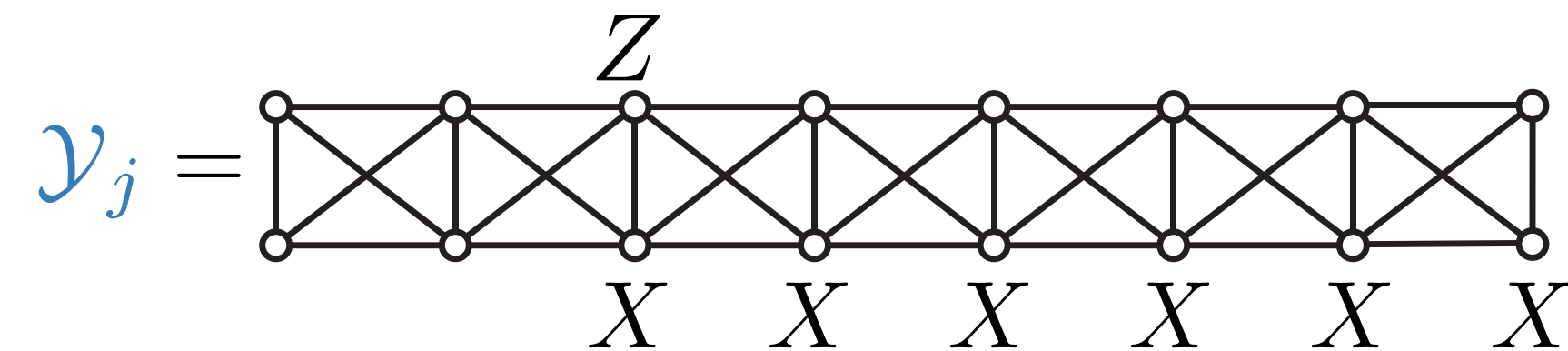
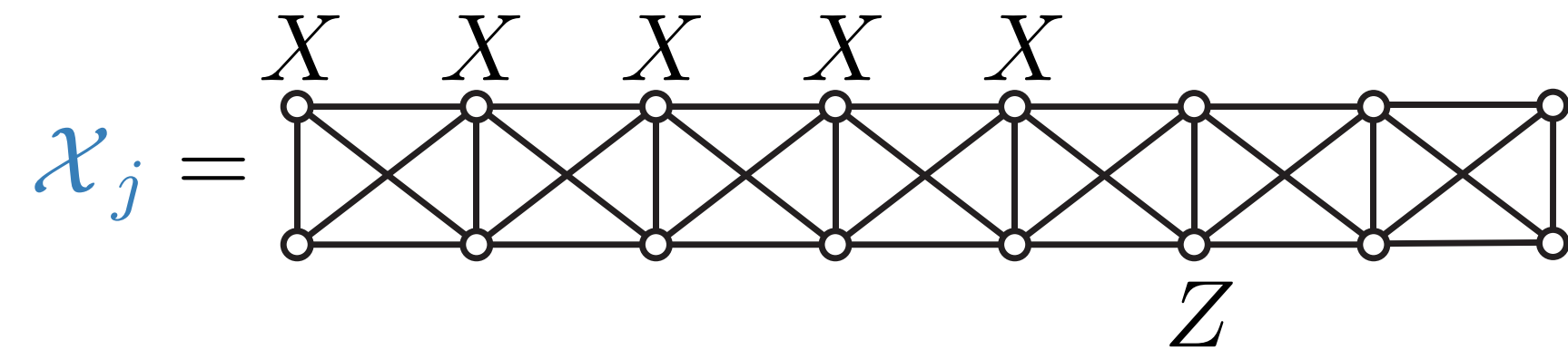


interchain reflection
symmetry

Gapless parent of the cluster state



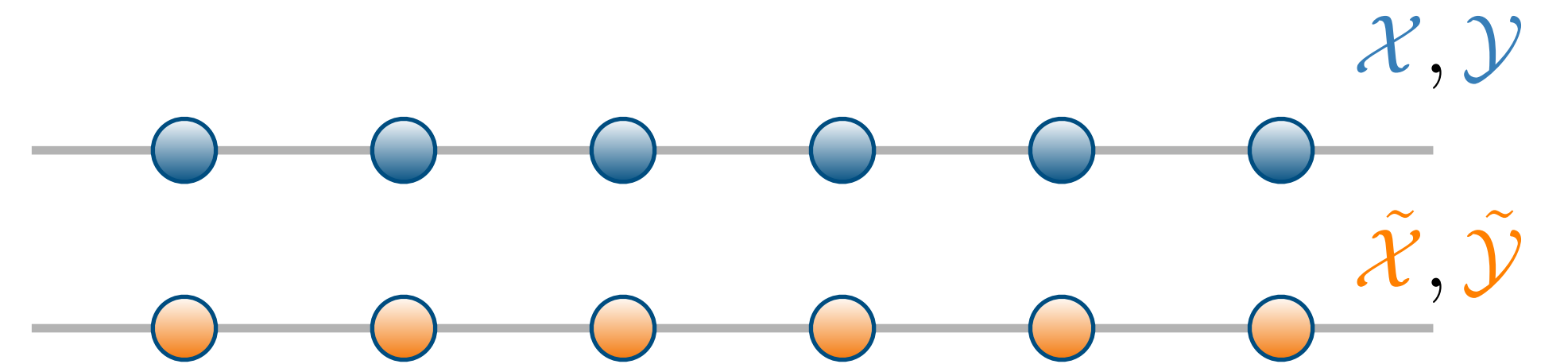
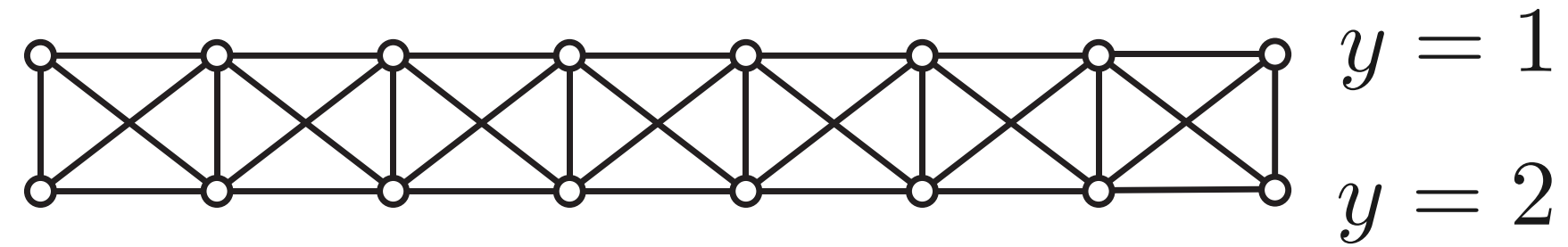
New set of Pauli operators:



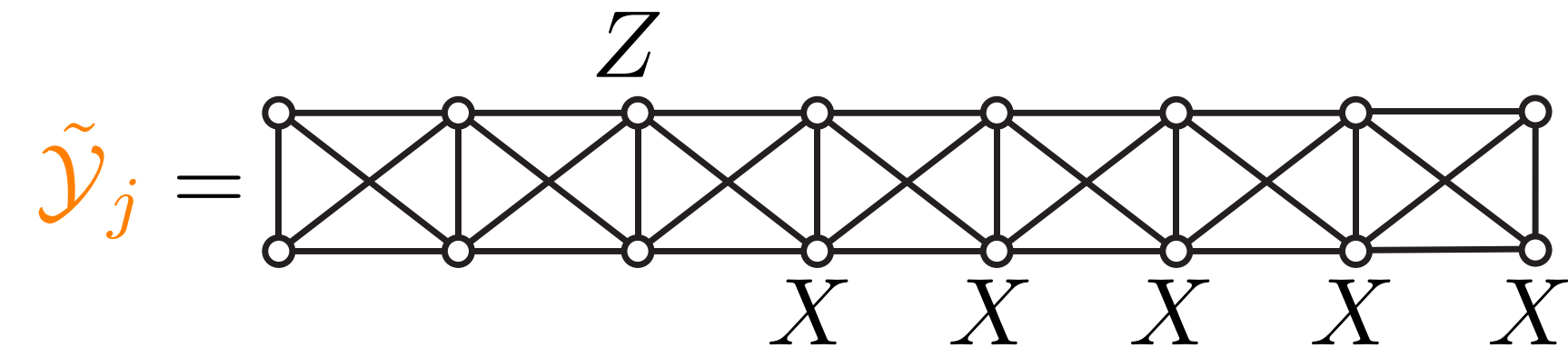
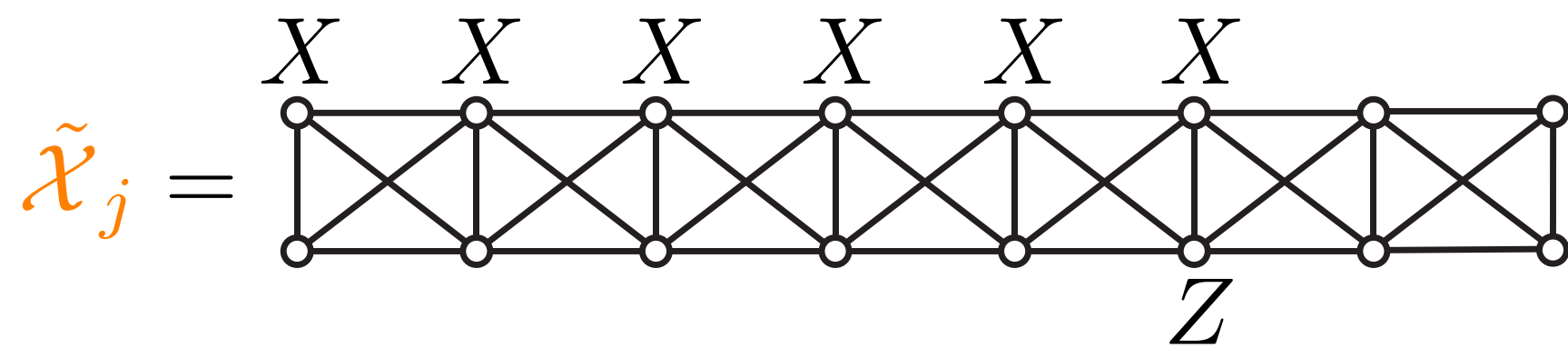
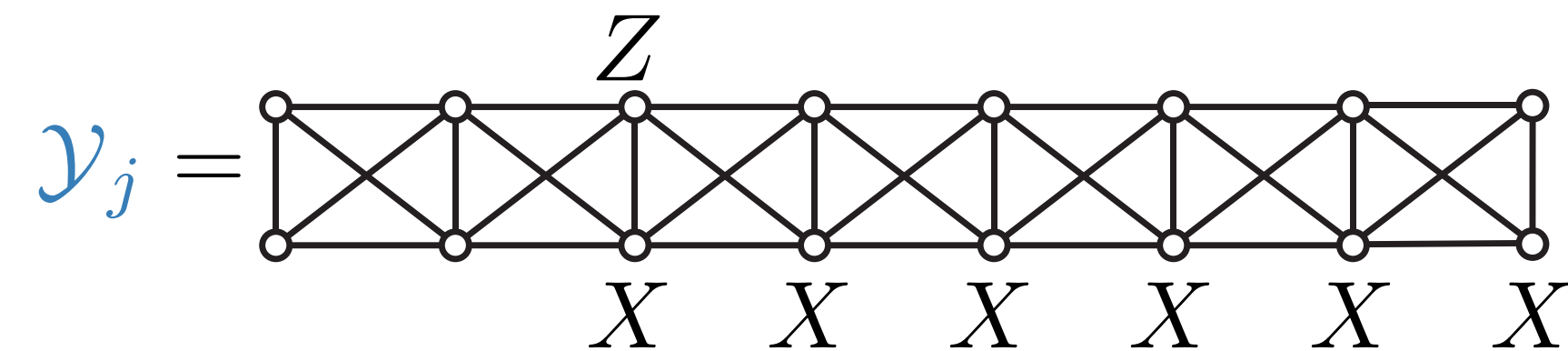
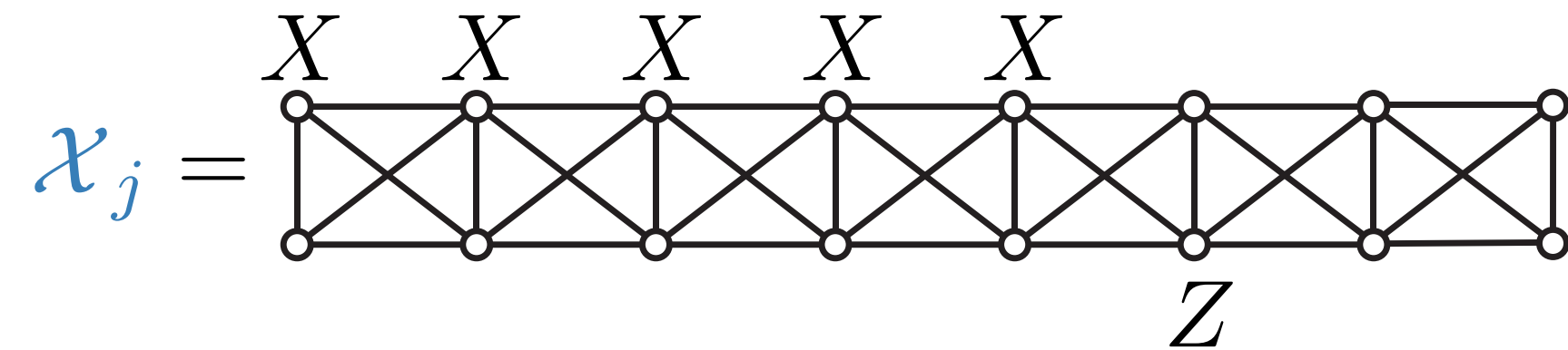
$$H = - \sum_{j=1}^{N-1} [(\sin \alpha \mathcal{X}_j \mathcal{X}_{j+1} + \cos \alpha \mathcal{Y}_j \mathcal{Y}_{j+1}) + (\cos \alpha \tilde{\mathcal{X}}_j \tilde{\mathcal{X}}_{j+1} + \sin \alpha \tilde{\mathcal{Y}}_j \tilde{\mathcal{Y}}_{j+1})]$$

two decoupled $\mathcal{X}\mathcal{Y}$ models
 $\alpha = \pi/4$: gapless system
with $c = 2$

Gapless parent of the cluster state



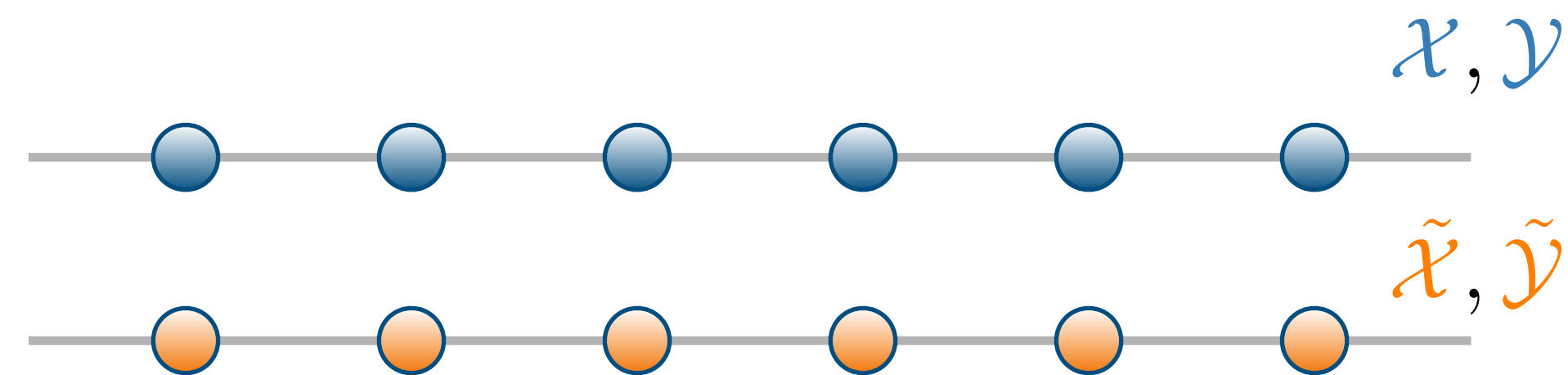
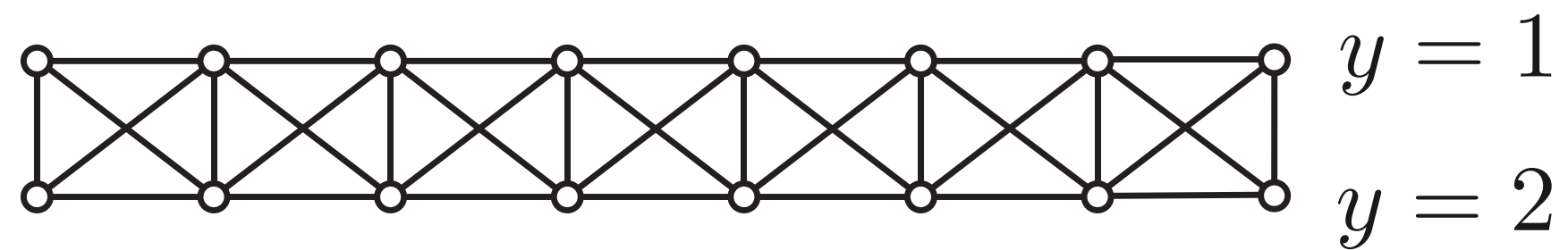
New set of Pauli operators:



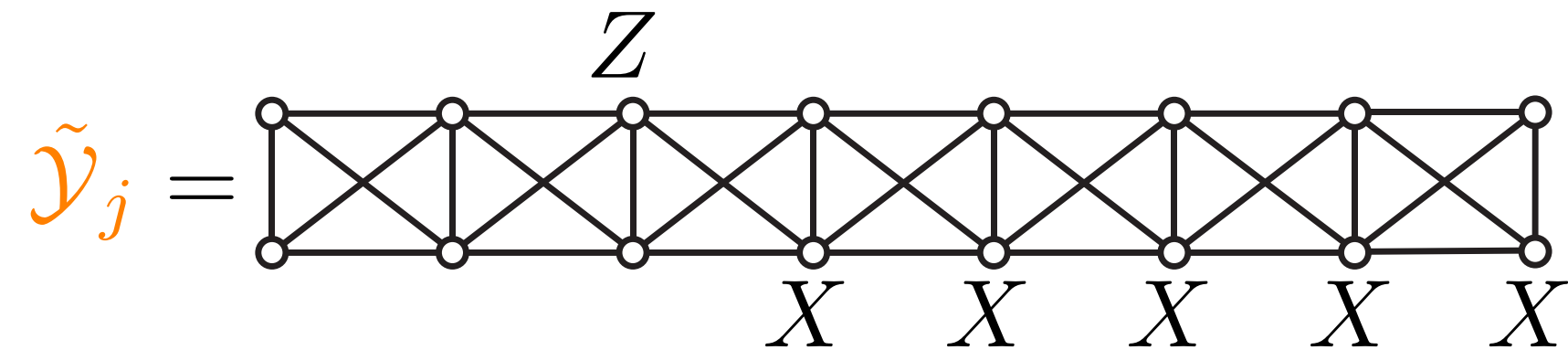
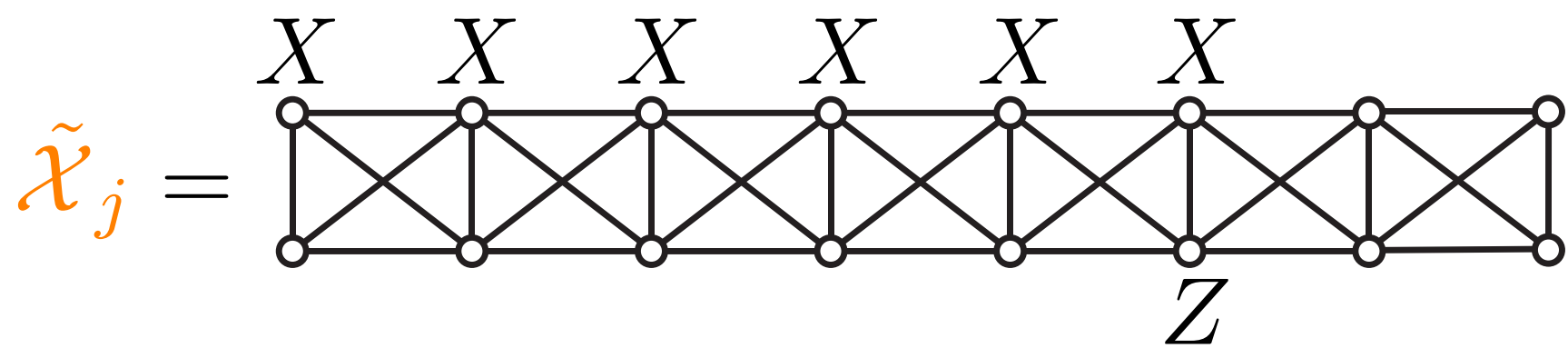
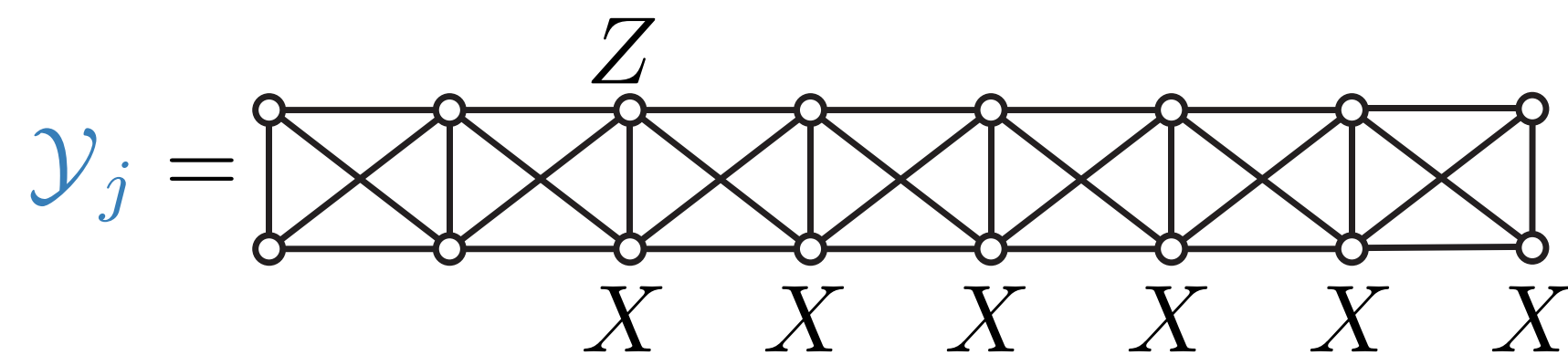
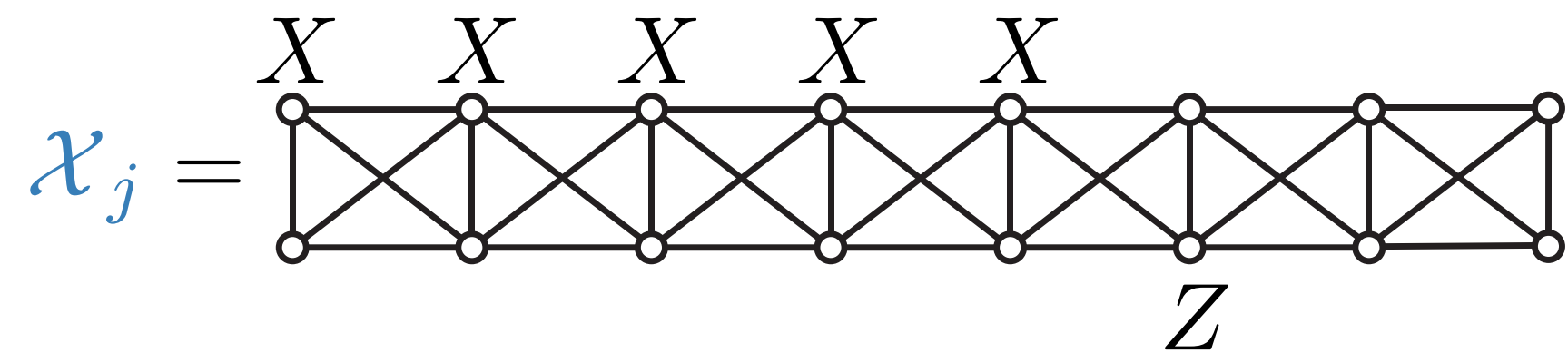
$$H = - \sum_{j=1}^{N-1} [(\sin \alpha x_j x_{j+1} + \cos \alpha y_j y_{j+1}) + (\cos \alpha \tilde{x}_j \tilde{x}_{j+1} + \sin \alpha \tilde{y}_j \tilde{y}_{j+1})]$$

$\alpha = 0$

Gapless parent of the cluster state



New set of Pauli operators:



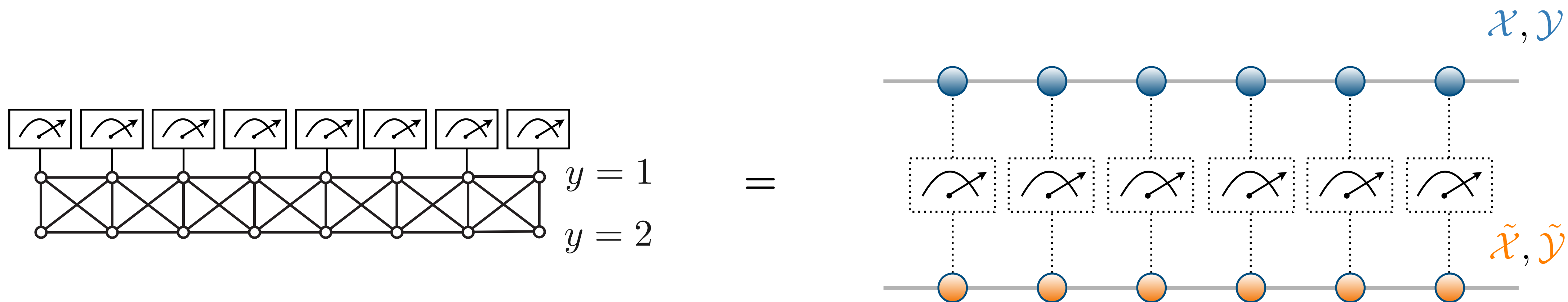
$$H_{\Delta} = - \sum_j \left[(\mathcal{X}_j \mathcal{X}_{j+1} + \mathcal{Y}_j \mathcal{Y}_{j+1} + \Delta \mathcal{Z}_j \mathcal{Z}_{j+1}) \right.$$

$\alpha = \pi/4$ + symmetry-preserving terms generating $\mathcal{Z}\mathcal{Z}$ -type operators

$$\left. + (\tilde{\mathcal{X}}_j \tilde{\mathcal{X}}_{j+1} + \tilde{\mathcal{Y}}_j \tilde{\mathcal{Y}}_{j+1} + \Delta \tilde{\mathcal{Z}}_j \tilde{\mathcal{Z}}_{j+1}) \right]$$

Gapless parent of the cluster state

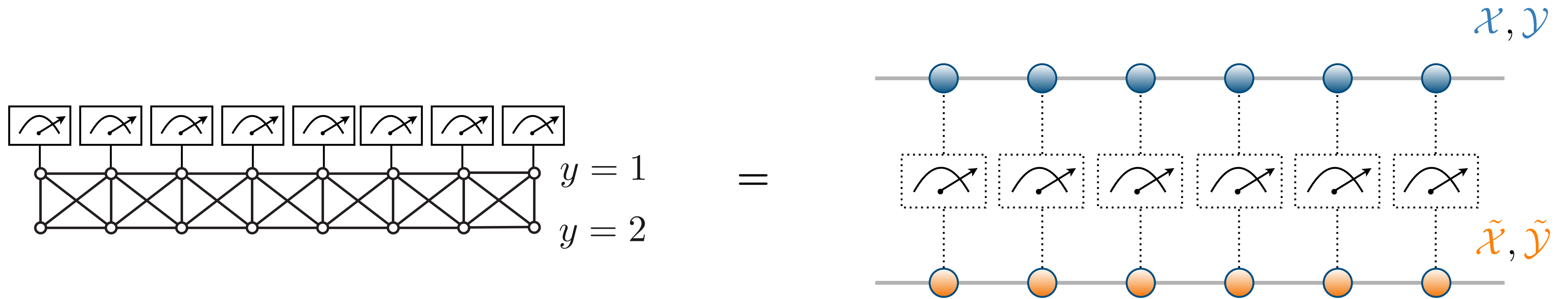
Original	$X_{j,1}$	$X_{j,2}$	$Z_{j,1}Z_{j+1,1}$	$Z_{j,1}$
New	$\mathcal{X}_j\tilde{\mathcal{X}}_j$	$\mathcal{Y}_j\tilde{\mathcal{Y}}_j$	$\mathcal{Y}_j\tilde{\mathcal{Y}}_{j+1}$	$\begin{pmatrix} \mathcal{Y}_{j+1} & \cdots & \mathcal{Y}_N \\ \tilde{\mathcal{Y}}_j & \tilde{\mathcal{Y}}_{j+1} & \cdots & \tilde{\mathcal{Y}}_N \end{pmatrix}$



$$H_{\Delta} = - \sum_j \left[(\mathcal{X}_j\mathcal{X}_{j+1} + \mathcal{Y}_j\mathcal{Y}_{j+1} + \Delta\mathcal{Z}_j\mathcal{Z}_{j+1}) + (\tilde{\mathcal{X}}_j\tilde{\mathcal{X}}_{j+1} + \tilde{\mathcal{Y}}_j\tilde{\mathcal{Y}}_{j+1} + \Delta\tilde{\mathcal{Z}}_j\tilde{\mathcal{Z}}_{j+1}) \right]$$

Gapless parent of the cluster state

Original	$X_{j,1}$	$X_{j,2}$	$Z_{j,1}Z_{j+1,1}$	$Z_{j,1}$
New	$\mathcal{X}_j \tilde{\mathcal{X}}_j$	$\mathcal{Y}_j \tilde{\mathcal{Y}}_j$	$\mathcal{Y}_j \tilde{\mathcal{Y}}_{j+1}$	$\begin{pmatrix} \mathcal{Y}_{j+1} & \cdots & \mathcal{Y}_N \\ \tilde{\mathcal{Y}}_j & \tilde{\mathcal{Y}}_{j+1} & \cdots & \tilde{\mathcal{Y}}_N \end{pmatrix}$



Can we still prepare long-range order from this gapless Hamiltonian?

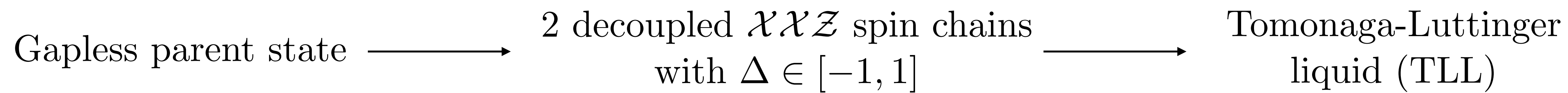
Is it robust far from projective measurements and is there a privileged measurement direction?

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Outline:

- Review about the canonical cluster state
- Gapless parent of the 1D cluster state
- Weak measurements on the gapless parent state
- Conclusions and outlook

Tomonaga-Luttinger liquid



Tomonaga-Luttinger liquid

Gapless parent state \longrightarrow 2 decoupled $\mathcal{X}\mathcal{X}\mathcal{Z}$ spin chains with $\Delta \in [-1, 1]$ \longrightarrow Tomonaga-Luttinger liquid (TLL)

$$S_{\text{TLL}} = \frac{K}{2\pi} \int_{x,\tau} [(\partial_x \theta)^2 + (\partial_\tau \theta)^2]$$

\downarrow
 phase

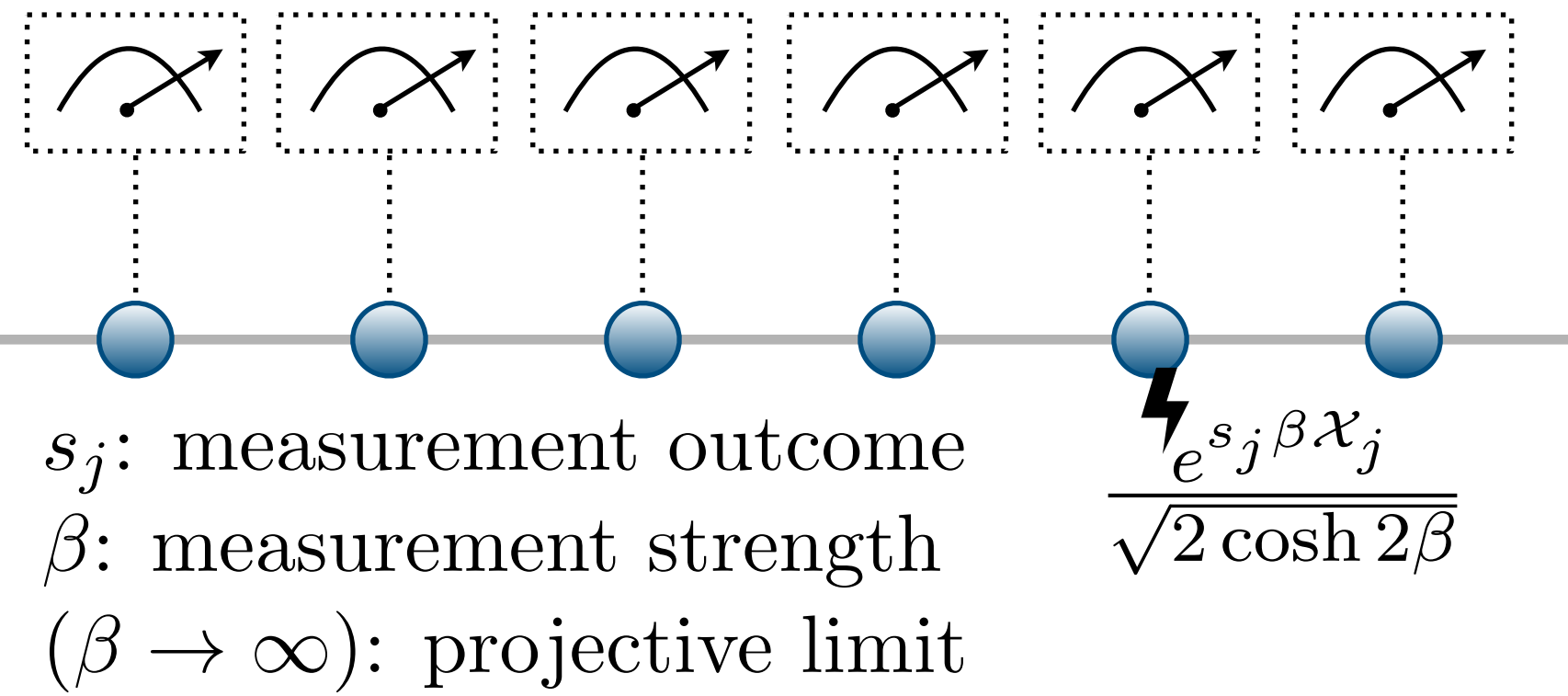
$$K = \frac{\pi}{2 \arccos \Delta}$$

Luttinger parameter

Operator/Field	Power-law correlations
$\mathcal{X}_j + i\mathcal{Y}_j \sim$ density $e^{-i\theta} + ic_1(-1)^j \left[e^{-i(2\phi+\theta)} - e^{i(2\phi-\theta)} \right]$	$\langle \mathcal{X}_0 \mathcal{X}_j \rangle \sim j ^{-\frac{1}{2K}}$
	$\langle \mathcal{Y}_0 \mathcal{Y}_j \rangle \sim j ^{-\frac{1}{2K}}$
$\mathcal{Z}_j \sim -\frac{2}{\pi} \partial_x \phi + c_2(-1)^j \cos(2\phi)$	$C_1 j ^{-2} + C_2 (-1)^j j ^{-2K}$

Entanglement
$\rho_A = \text{Tr}_B \psi_c\rangle \langle \psi_c \implies$ $S_A^{(n)} = \frac{1}{1-n} \log \text{Tr}[\rho_A^n] \sim c \frac{n+1}{6n} \log \ell$
with central charge $c = 1$

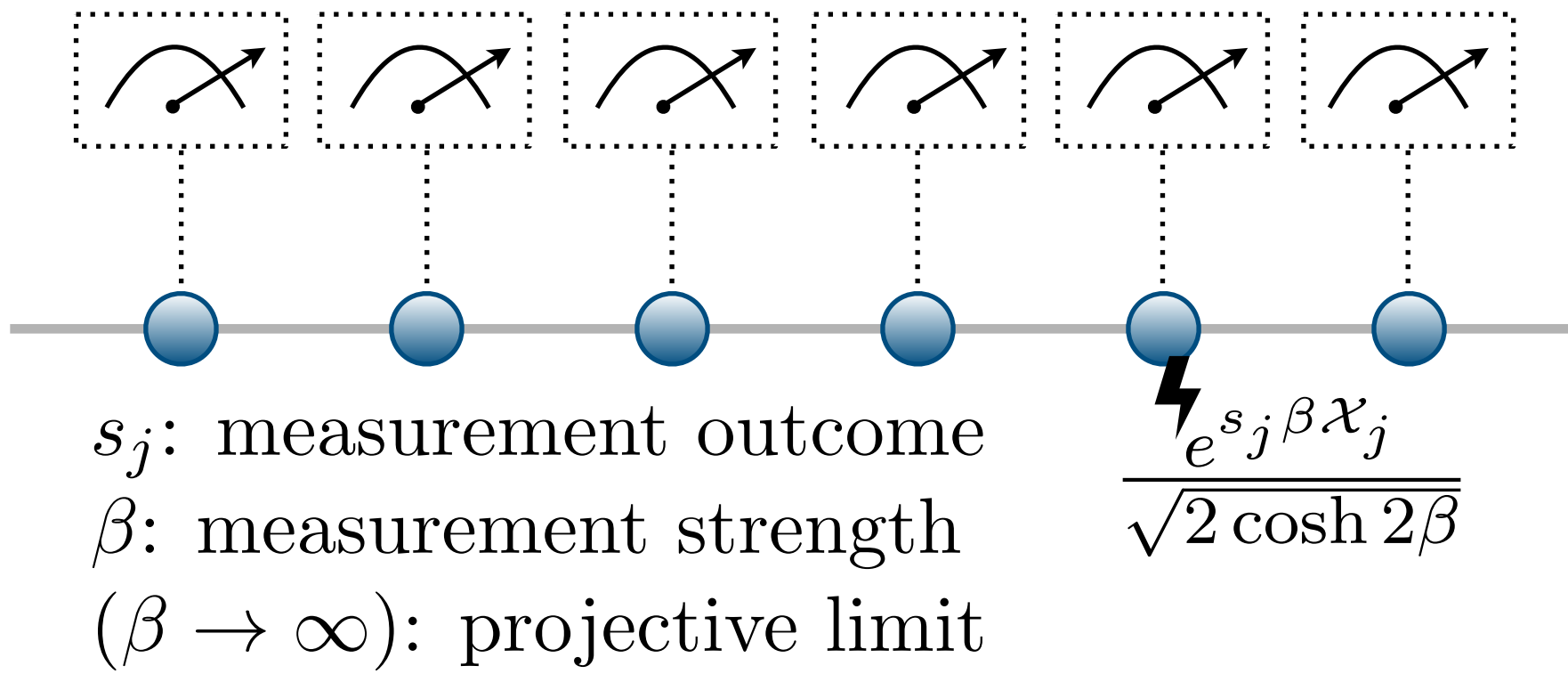
Weak measurement on a single XXZ spin chain



For Z-basis measurements, see:
S. J. Garratt, Z. Weinstein, E. Altman,
Phys. Rev. X 13, 021026 (2023).

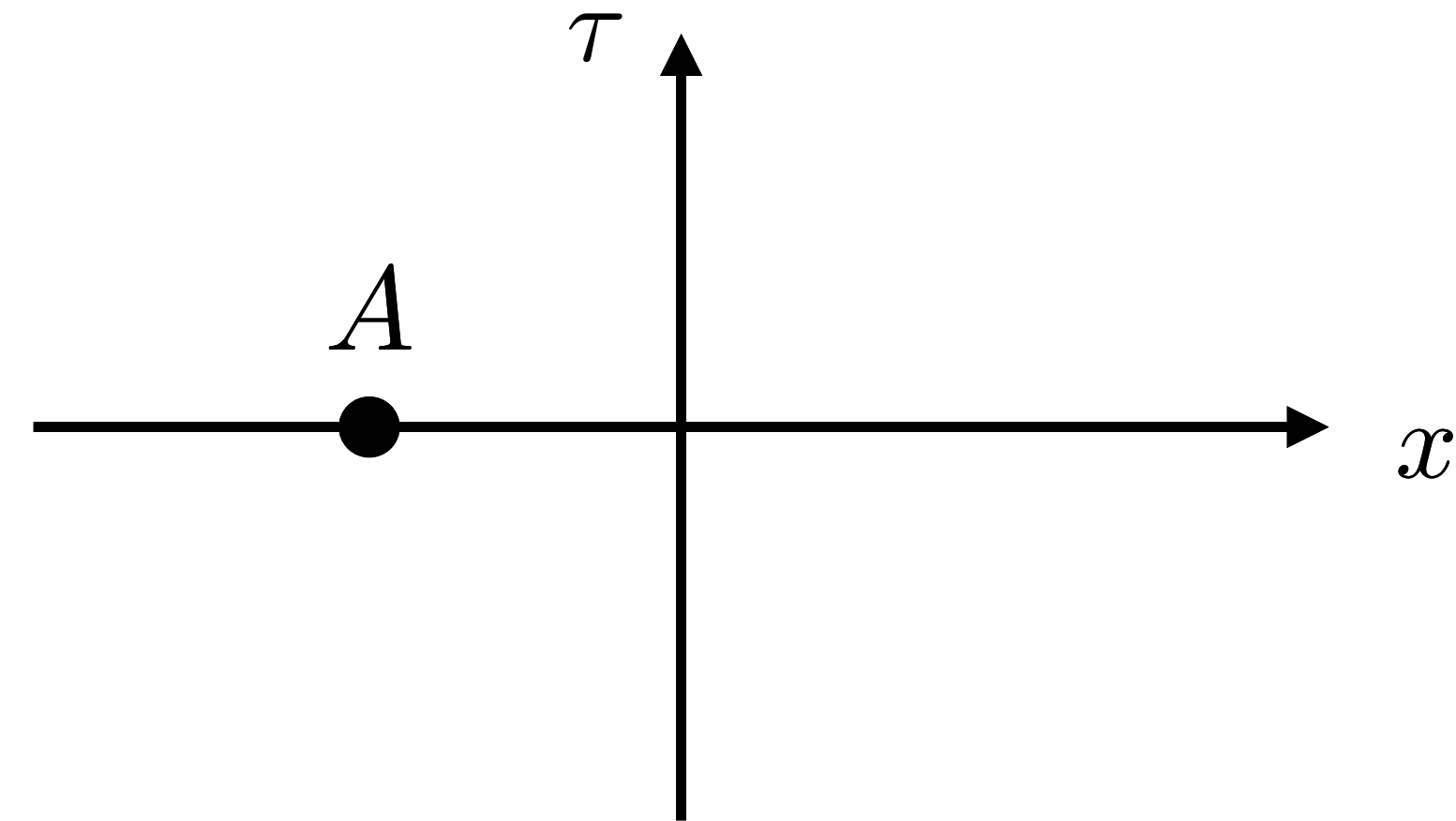
Post-measurement state: $|\psi_{\mathbf{s}}\rangle = \frac{1}{\mathcal{N}} e^{\beta \sum_j s_j \chi_j} |\psi_c\rangle$

Weak measurement on a single XXZ spin chain

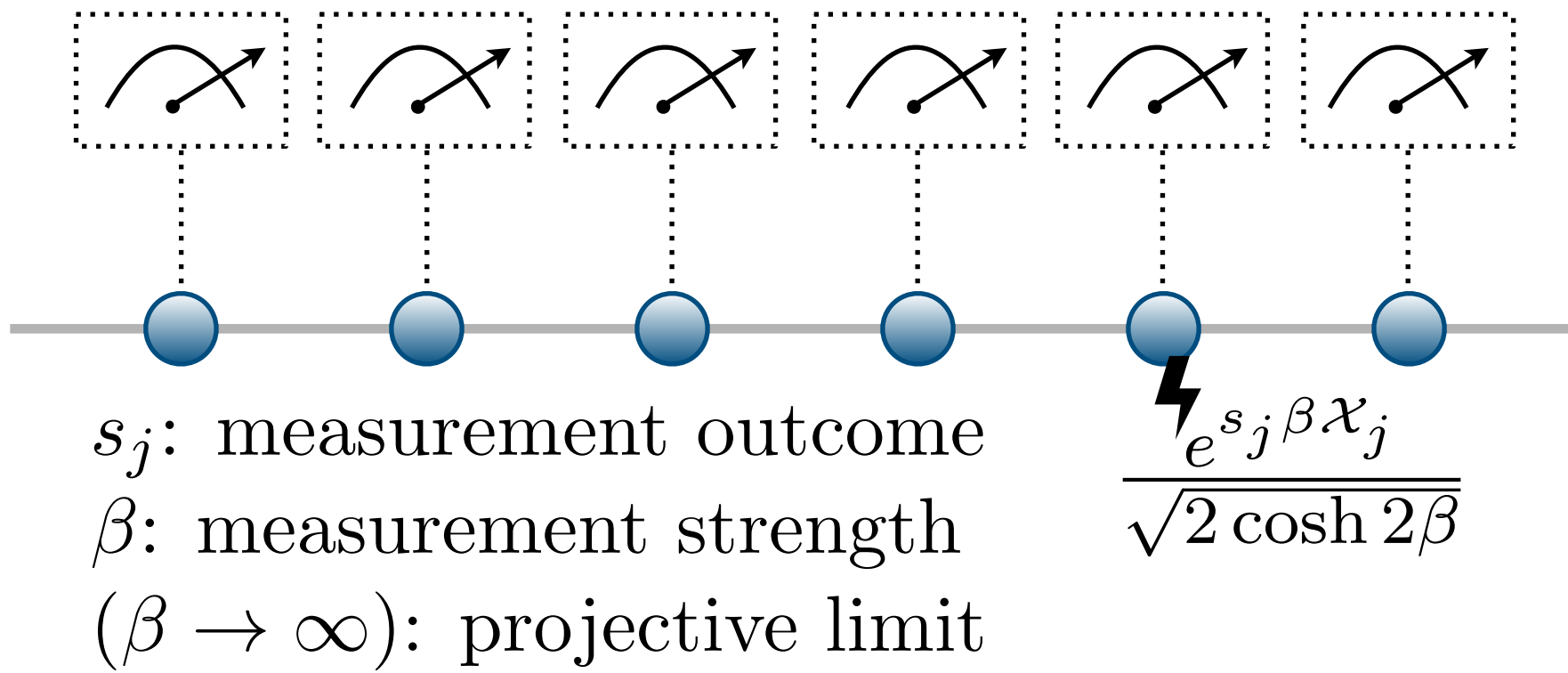


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$$\langle \psi_c | A | \psi_c \rangle \rightarrow \int \mathcal{D}\theta e^{-S_c} A_{\text{cont}}(x, \tau = 0)$$



Weak measurement on a single XXZ spin chain

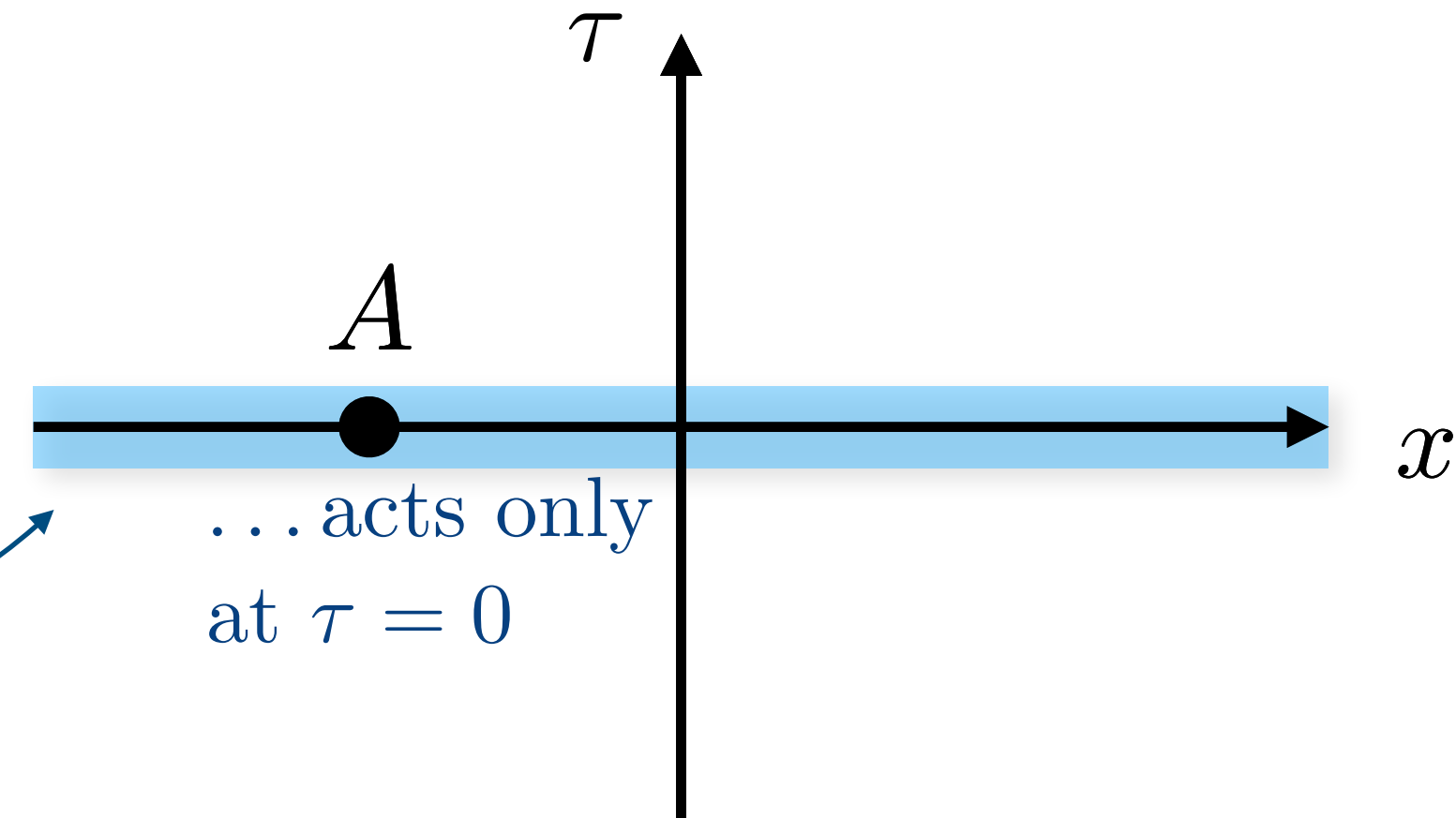


Post-measurement state: $|\psi_{\mathbf{s}}\rangle = \frac{1}{\mathcal{N}} e^{\beta \sum_j s_j \chi_j} |\psi_c\rangle$

$$\mathcal{M} \sim e^{\beta \sum_j O_j + \dots}$$

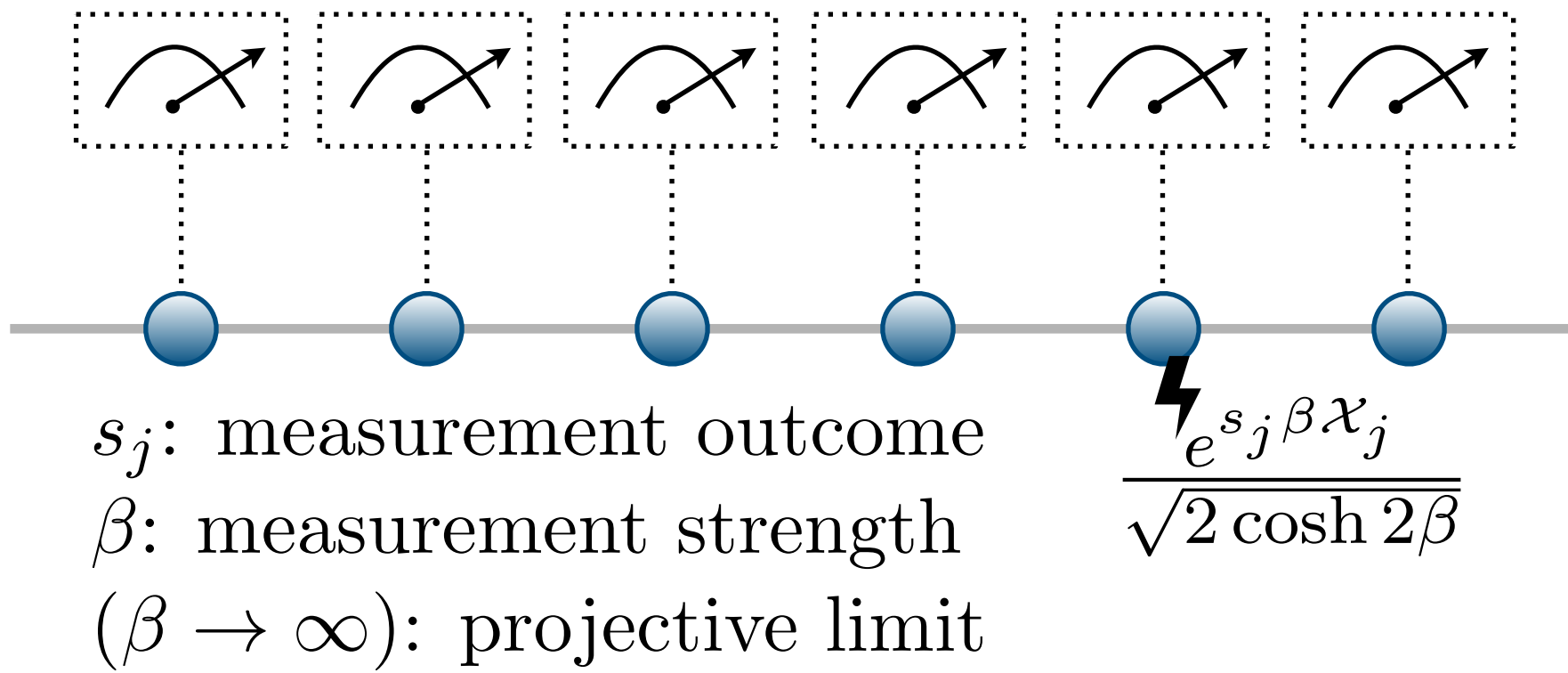
$$\langle \psi_c | \mathcal{M} A \mathcal{M} | \psi_c \rangle \rightarrow \int \mathcal{D}\theta e^{-S_c - S_{\text{meas}}} A_{\text{cont}}(x, \tau = 0)$$

Correction from weak measurement...



Weak measurement \sim
 “imaginary time”
 quantum impurity!

Weak measurement on a single XXZ spin chain



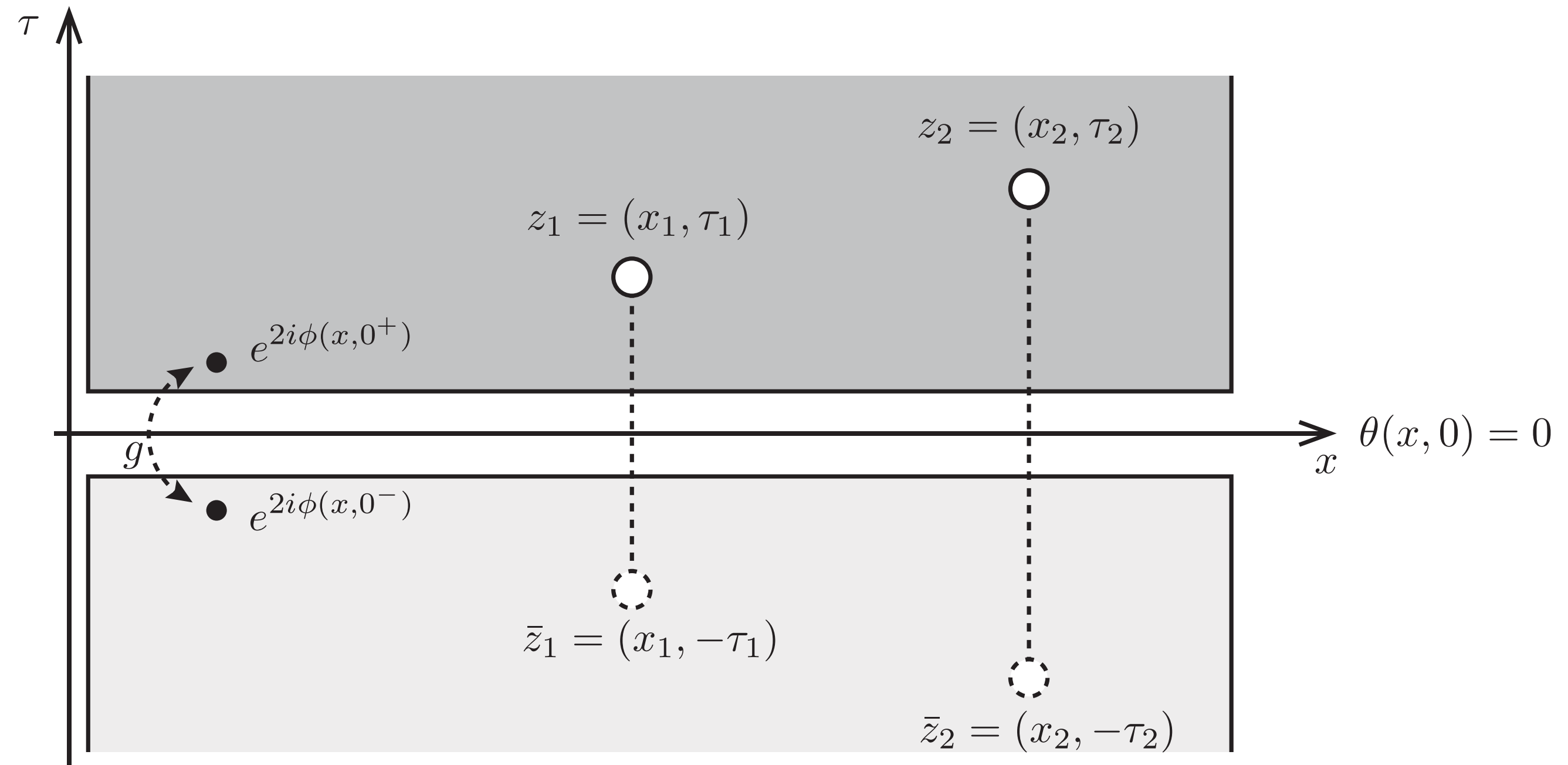
Post-measurement state: $|\psi_{\text{uni}}\rangle = \frac{1}{\mathcal{N}} e^{\beta \sum_j \chi_j} |\psi_c\rangle$

$$S' = S_{\text{TLL}} + \beta \int_x \cos \theta|_{\tau=0}$$

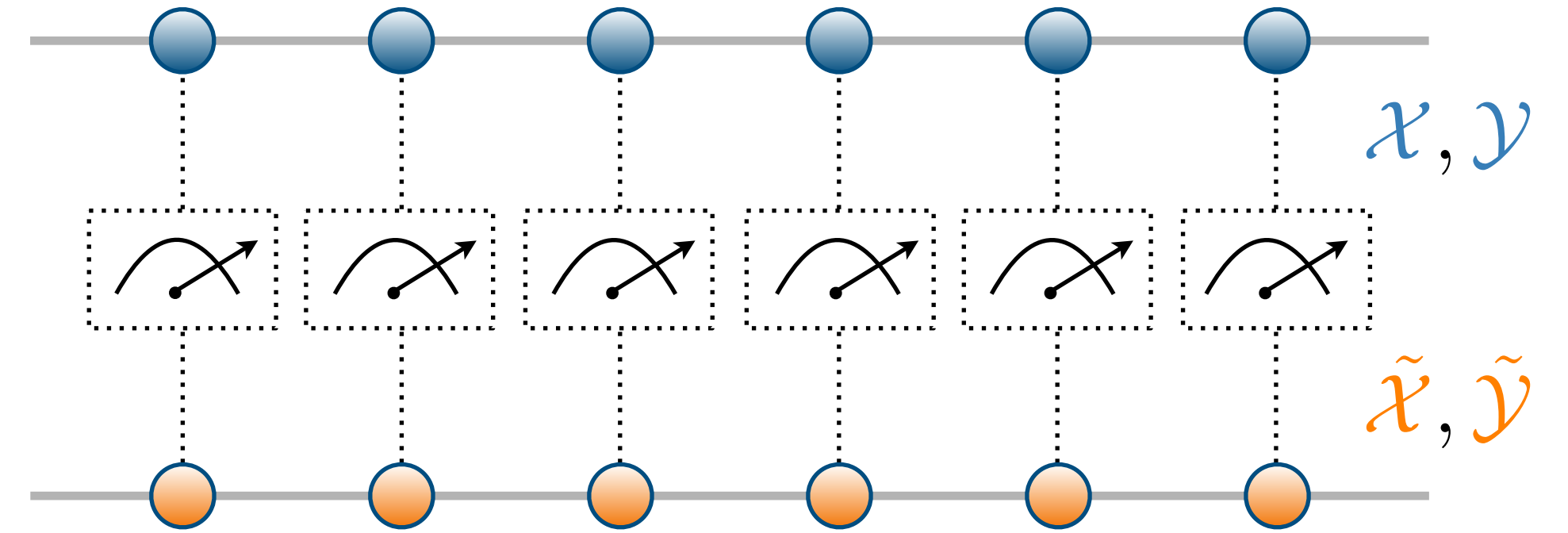
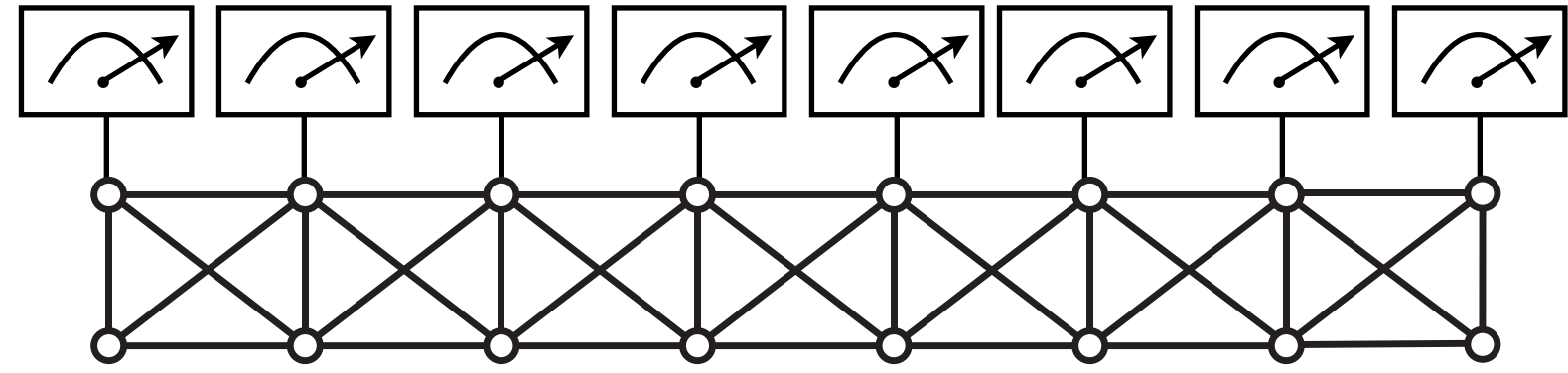
“Impurity” action is **RELEVANT** for $K > 1/4$.
 How are critical properties modified?

$$\langle \mathcal{X}_0 \mathcal{X}_x \rangle_c \propto x^{-1/(2K)} \rightarrow x^{-\min(4, 8K)}$$

$$\langle \mathcal{Y}_0 \mathcal{Y}_x \rangle_c \propto x^{-1/(2K)} \rightarrow x^{-2}$$



Weak measurement on the gapless parent state (X-basis)

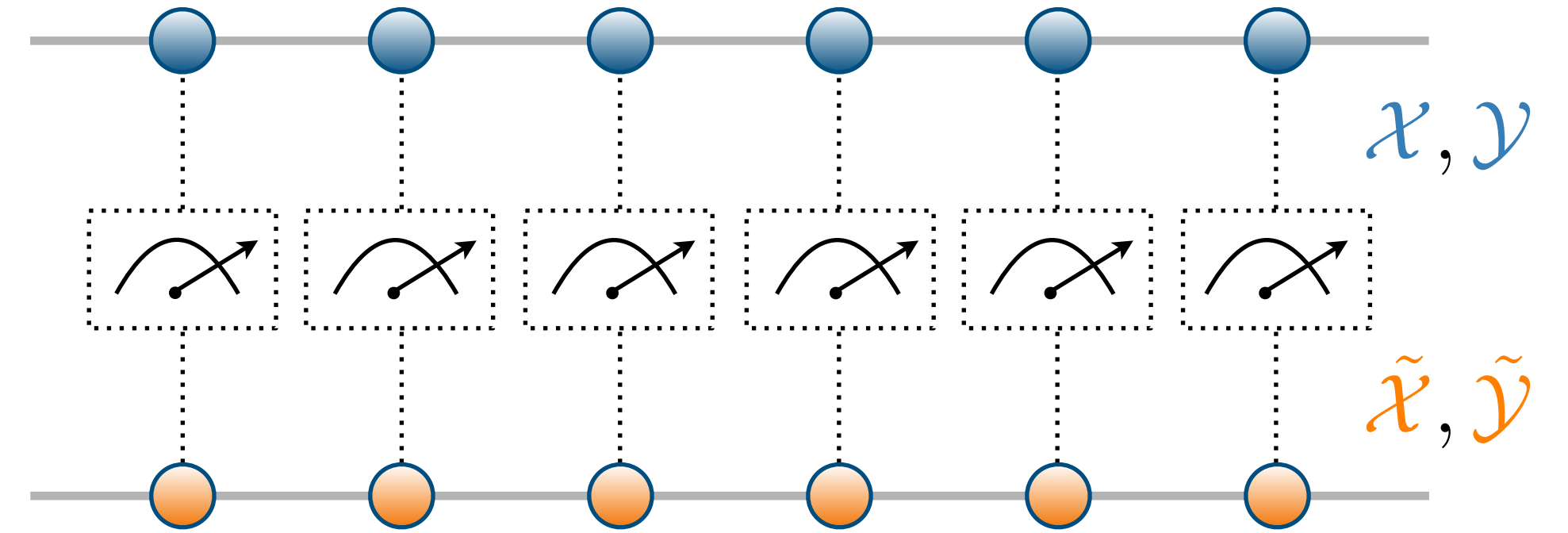
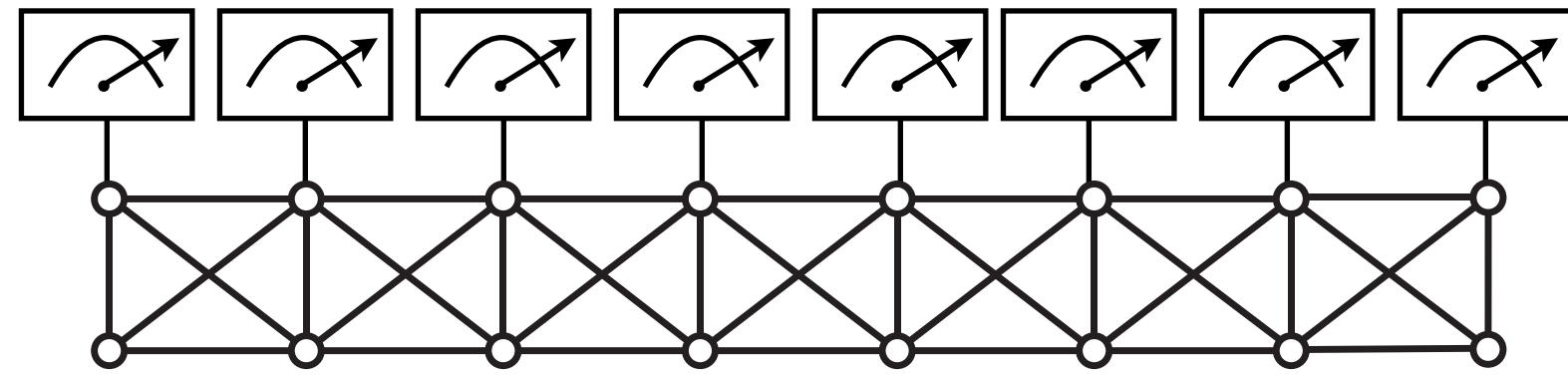


Post-measurement state: $|\psi_{\text{uni}}\rangle = \frac{1}{\mathcal{N}} e^{\beta \sum_j X_{j,1}} |\psi_{\Delta}\rangle \rightarrow \frac{1}{\mathcal{N}} e^{\beta \sum_j x_j \tilde{x}_j} |\psi_c\rangle |\tilde{\psi}_c\rangle$

$$S' = S_{\text{TLL}}[\theta] + S_{\text{TLL}}[\tilde{\theta}] + \beta \int_x \cos(\theta) \cos(\tilde{\theta}) |_{\tau=0}$$

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Weak measurement on the gapless parent state (X-basis)



Post-measurement state: $|\psi_{\text{uni}}\rangle = \frac{1}{\mathcal{N}} e^{\beta \sum_j X_{j,1}} |\psi_{\Delta}\rangle \rightarrow \frac{1}{\mathcal{N}} e^{\beta \sum_j x_j \tilde{x}_j} |\psi_c\rangle |\tilde{\psi}_c\rangle$

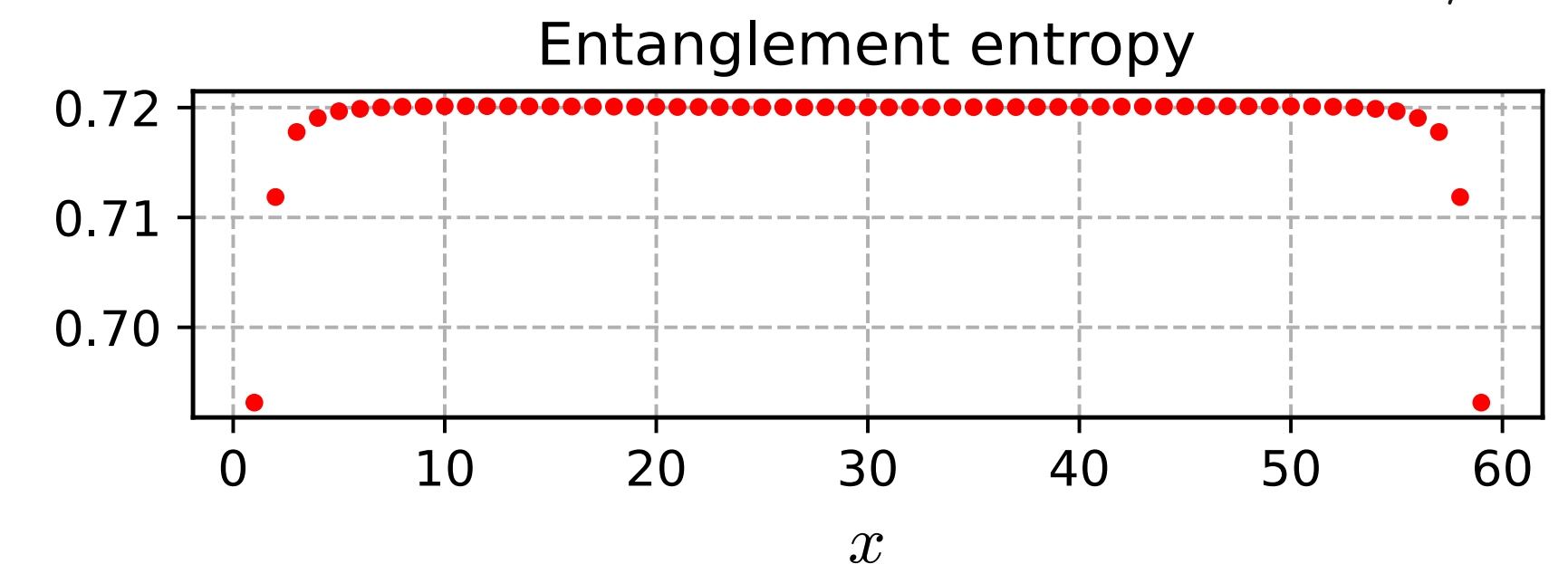
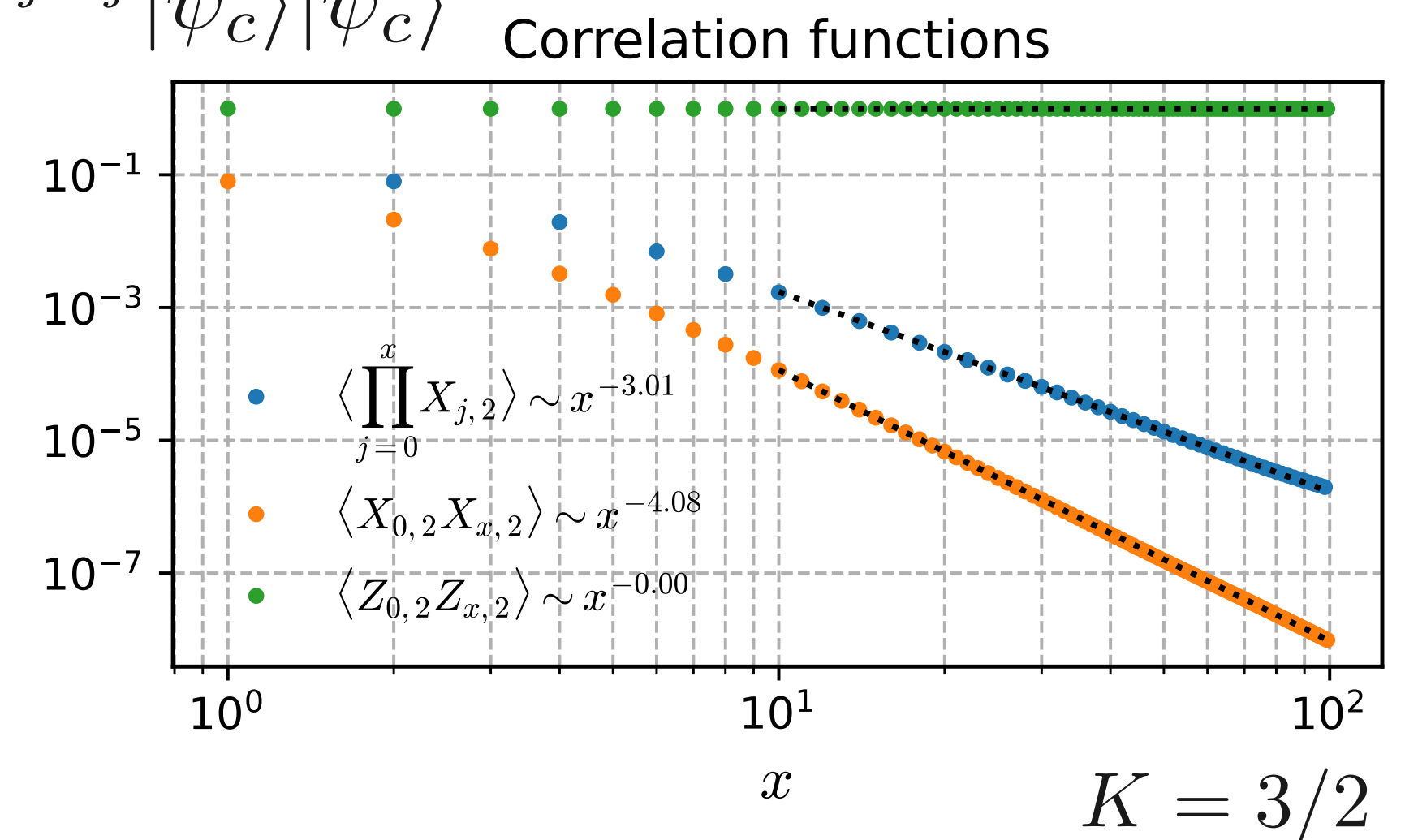
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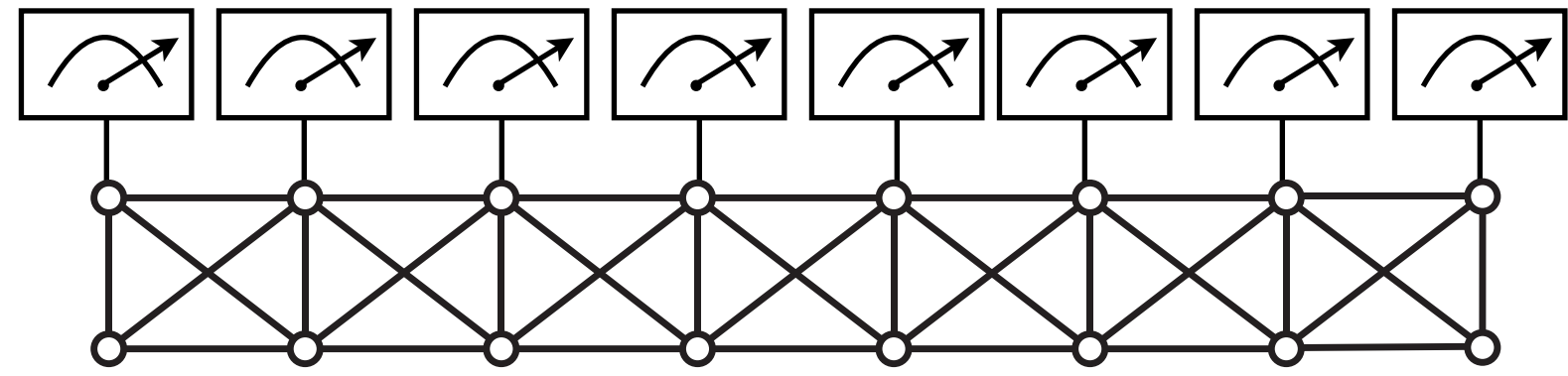
$$\langle X_{0,2} X_{x,2} \rangle_{\text{uni}} \sim |x|^{-\min(4, 4K)}$$

order parameter $\langle Z_{0,2} Z_{x,2} \rangle_{\text{uni}} \sim \text{const.}$

disorder parameter $\langle \prod_{i=j}^k X_{i,2} \rangle_{\text{uni}} \sim |j - k|^{-2K}$

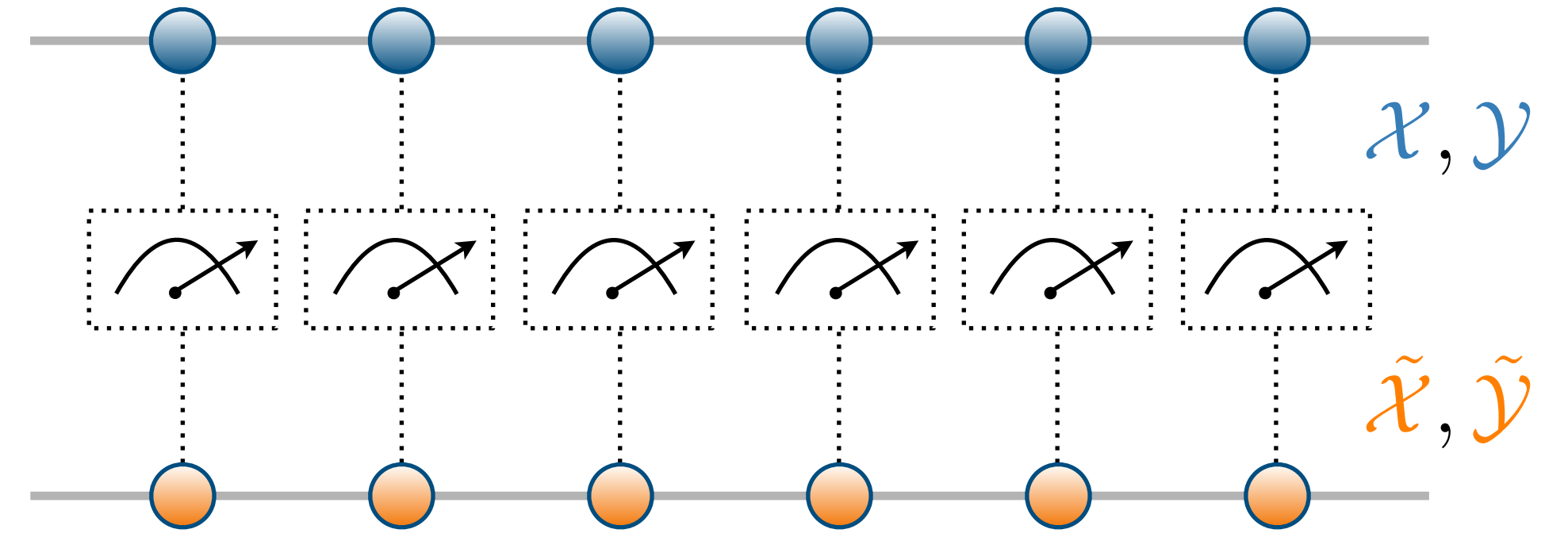


A decoding protocol



Reminder:

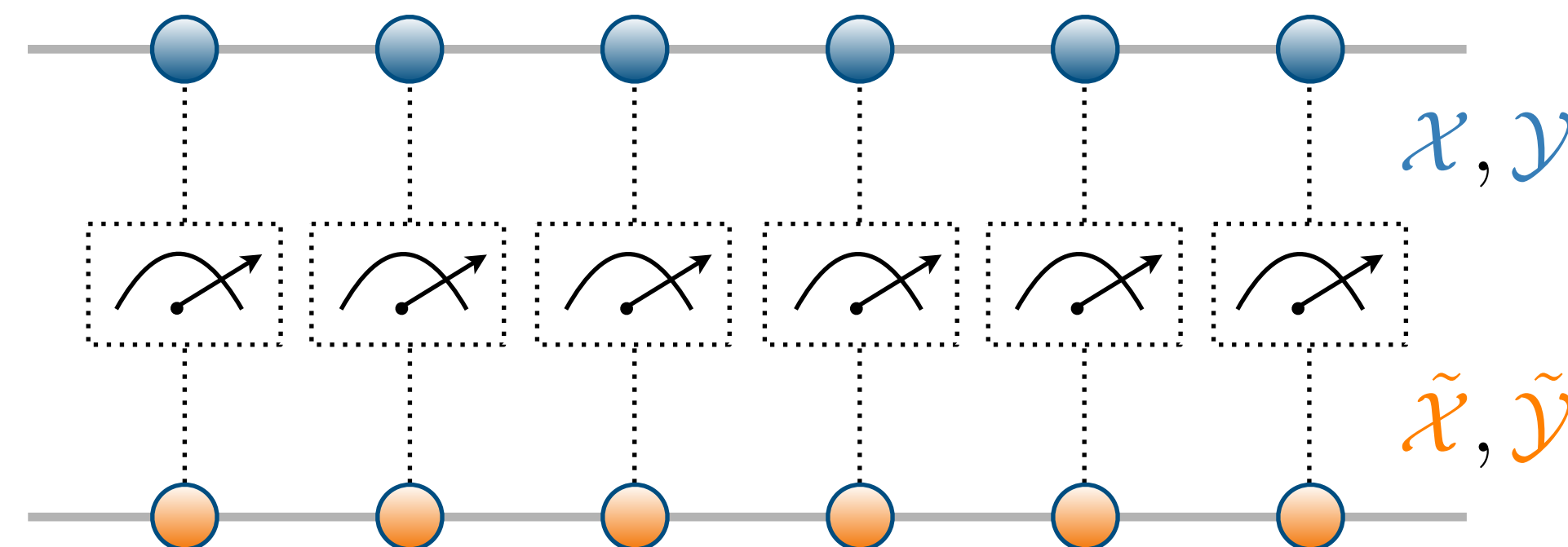
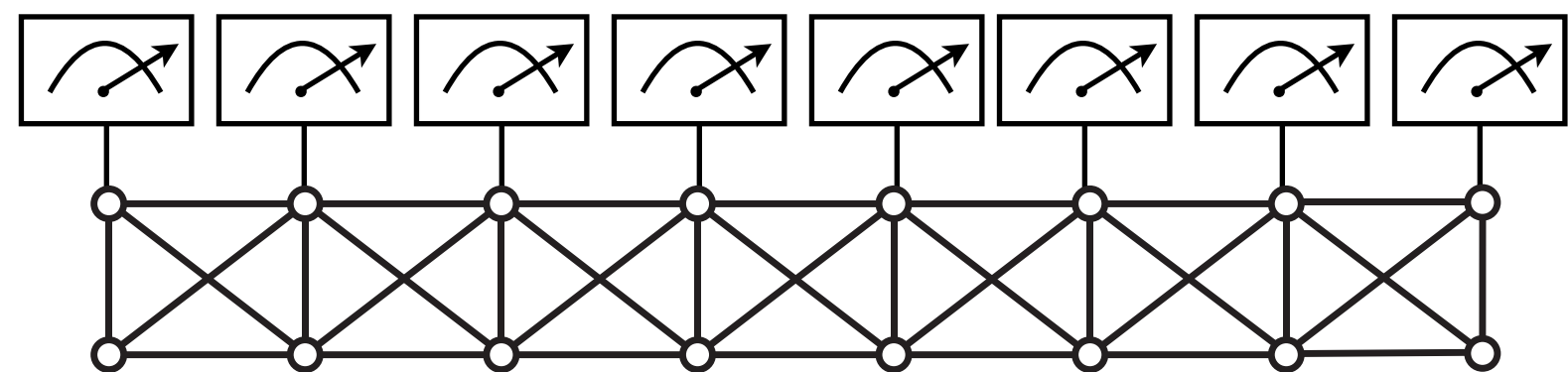
$$\sum_{\mathbf{s}} p_{\mathbf{s}} \langle Z_{j,2} Z_{k,2} \rangle_{\mathbf{s}} = \langle Z_{j,2} Z_{k,2} \rangle_0 \sim |j - k|^{-1/2}$$



$$\sum_{\mathbf{s}} p_{\mathbf{s}} \langle Z_{j,2} Z_{k,2} \rangle_{\mathbf{s}} s_j s_{j+1} \cdots s_{k-1} \sim |j - k|^{-1/(2K)}$$

**Same decoding protocol used for canonical cluster state
yields power-law behavior (not GHZ)!**

Weak measurement on the gapless parent state (Z-basis)

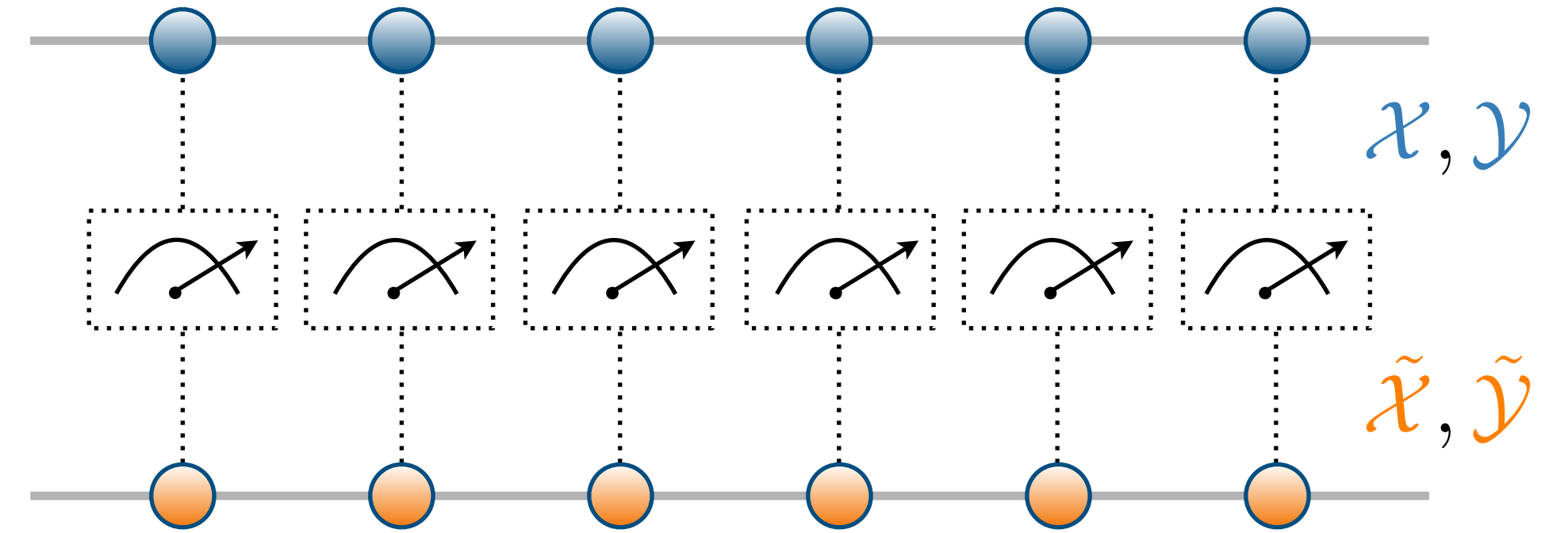
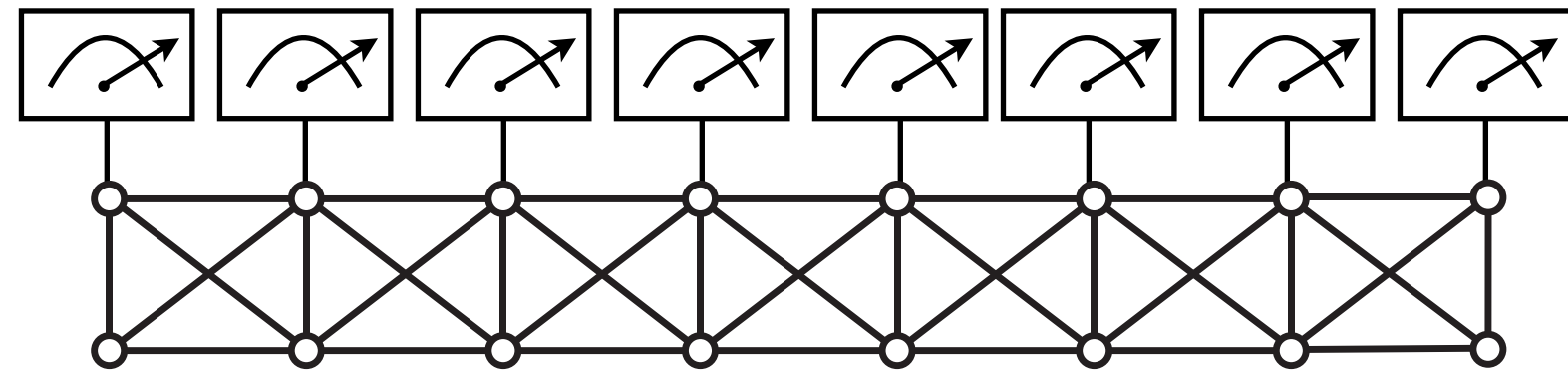


Post-measurement state: $|\psi_{\text{uni}}\rangle = \frac{1}{\mathcal{N}} e^{\beta \sum_j Z_{j,1}} |\psi_{\Delta}\rangle \rightarrow Z_{j,1} = \begin{pmatrix} \mathcal{Y}_{j+1} & \cdots & \mathcal{Y}_N \\ \tilde{\mathcal{Y}}_j & \tilde{\mathcal{Y}}_{j+1} & \cdots & \tilde{\mathcal{Y}}_N \end{pmatrix}$

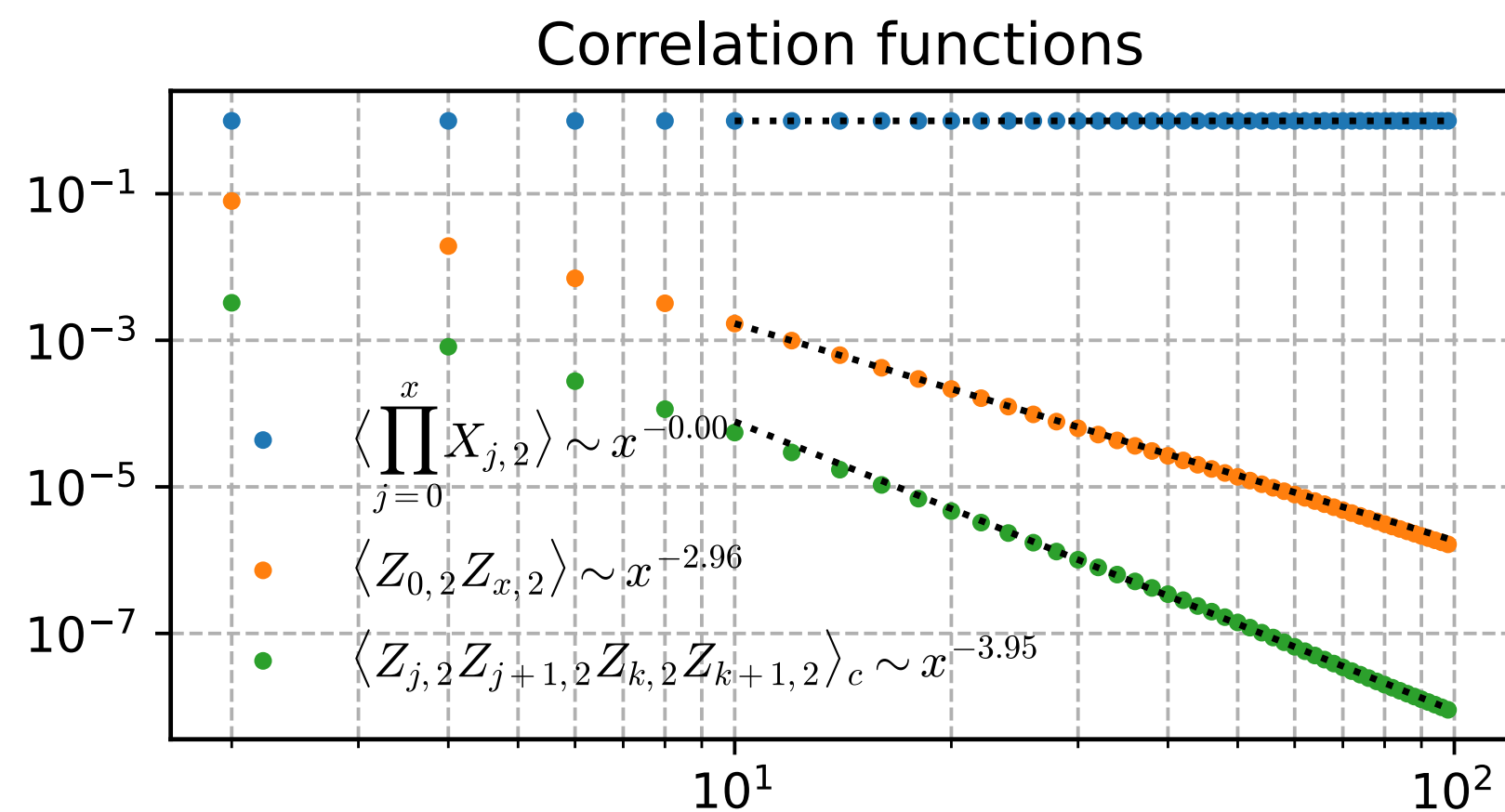


Numerically, we find that the scaling dimension of $Z_{j,1}$ is $1/4$, so the measurement is always relevant.

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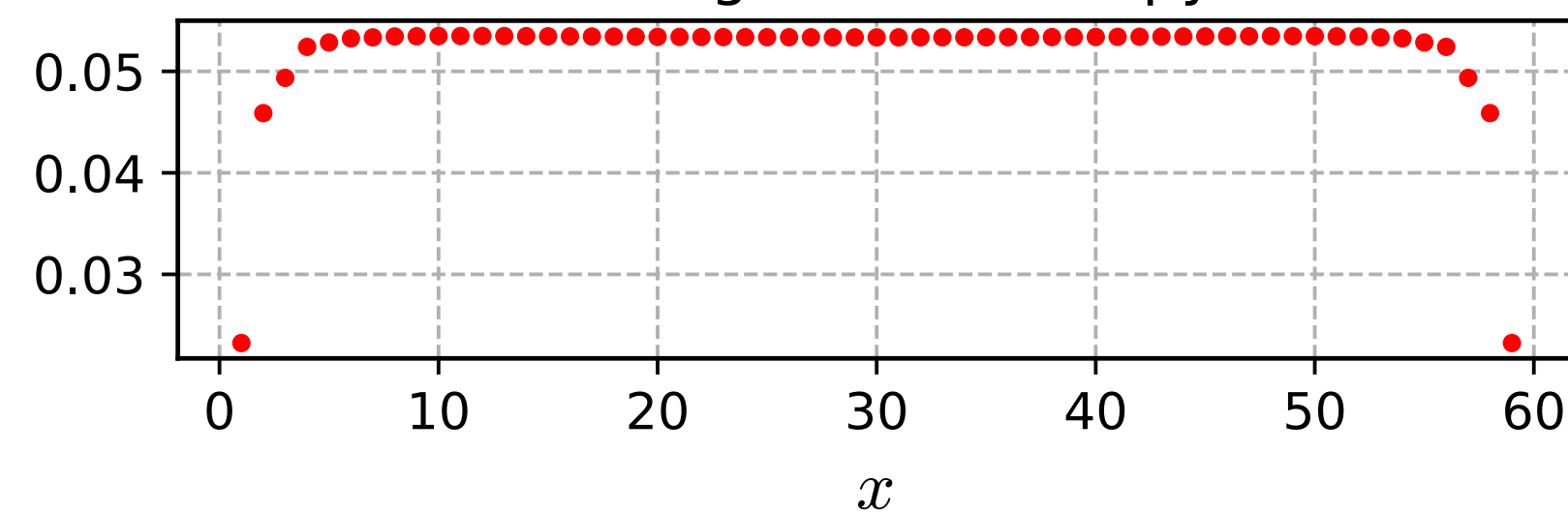


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$$K = 3/2$$

Entanglement entropy



Numerically, we find that the scaling dimension of $Z_{j,1}$ is $1/4$, so the measurement is always relevant.

$$\langle X_{0,2} X_{x,2} \rangle_{\text{uni}} \sim |x|^{-\min(4, 4K)}$$

order parameter $\langle Z_{0,2} Z_{x,2} \rangle_{\text{uni}} \sim |x|^{-2K}$

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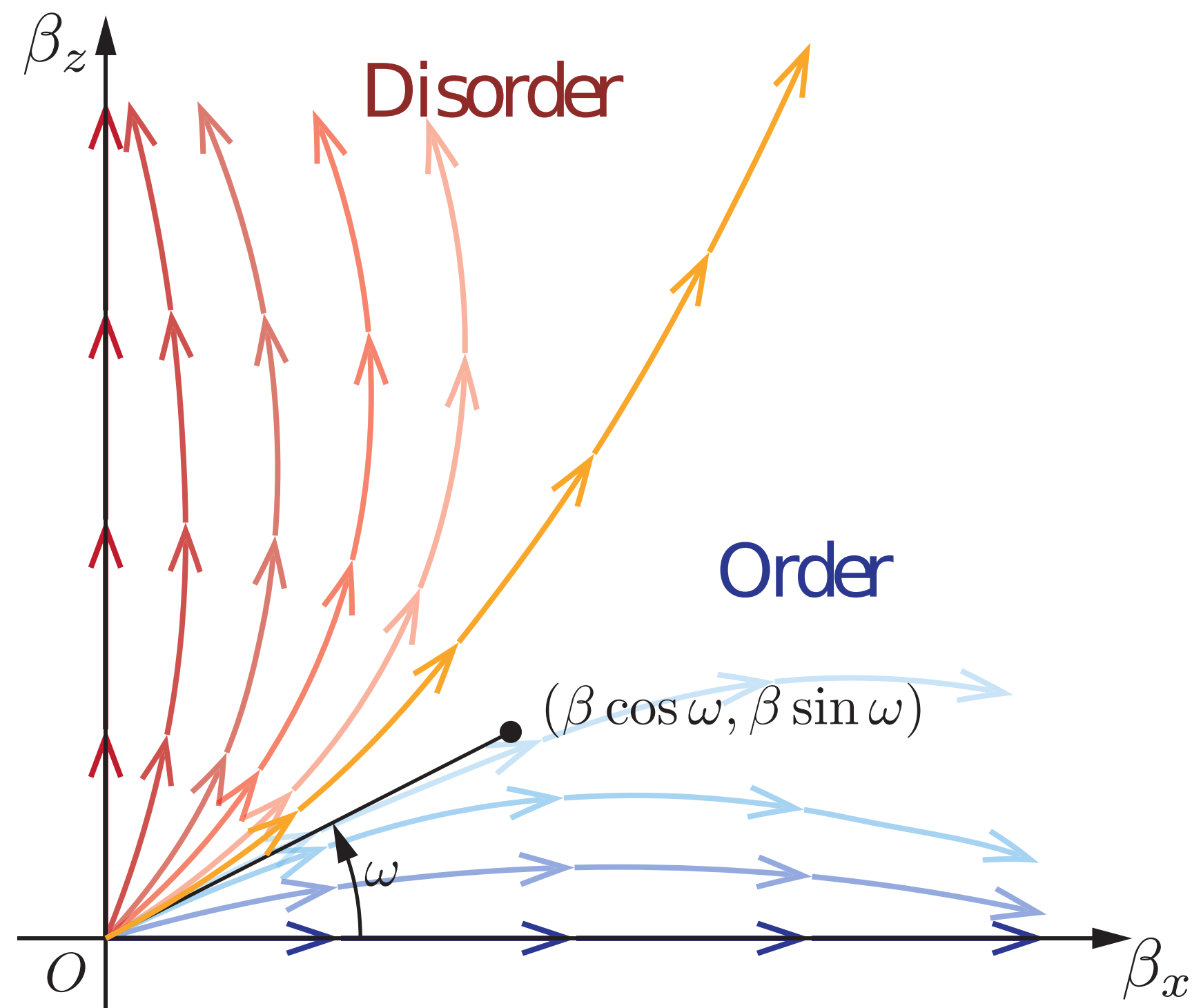
Tilted weak measurements

Post-measurement state: $|\psi_{\text{uni}}\rangle = \frac{1}{\mathcal{N}} e^{\beta \sum_j [\cos(\omega) X_{j,1} + \sin(\omega) Z_{j,1}]} |\psi_{\Delta}\rangle, \quad \omega \in [0, \frac{\pi}{2}]$

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$\omega = 0, \pi/2$ are two stable fixed points for $K > 1$ \Rightarrow intermediate fixed point for some critical value ω_c

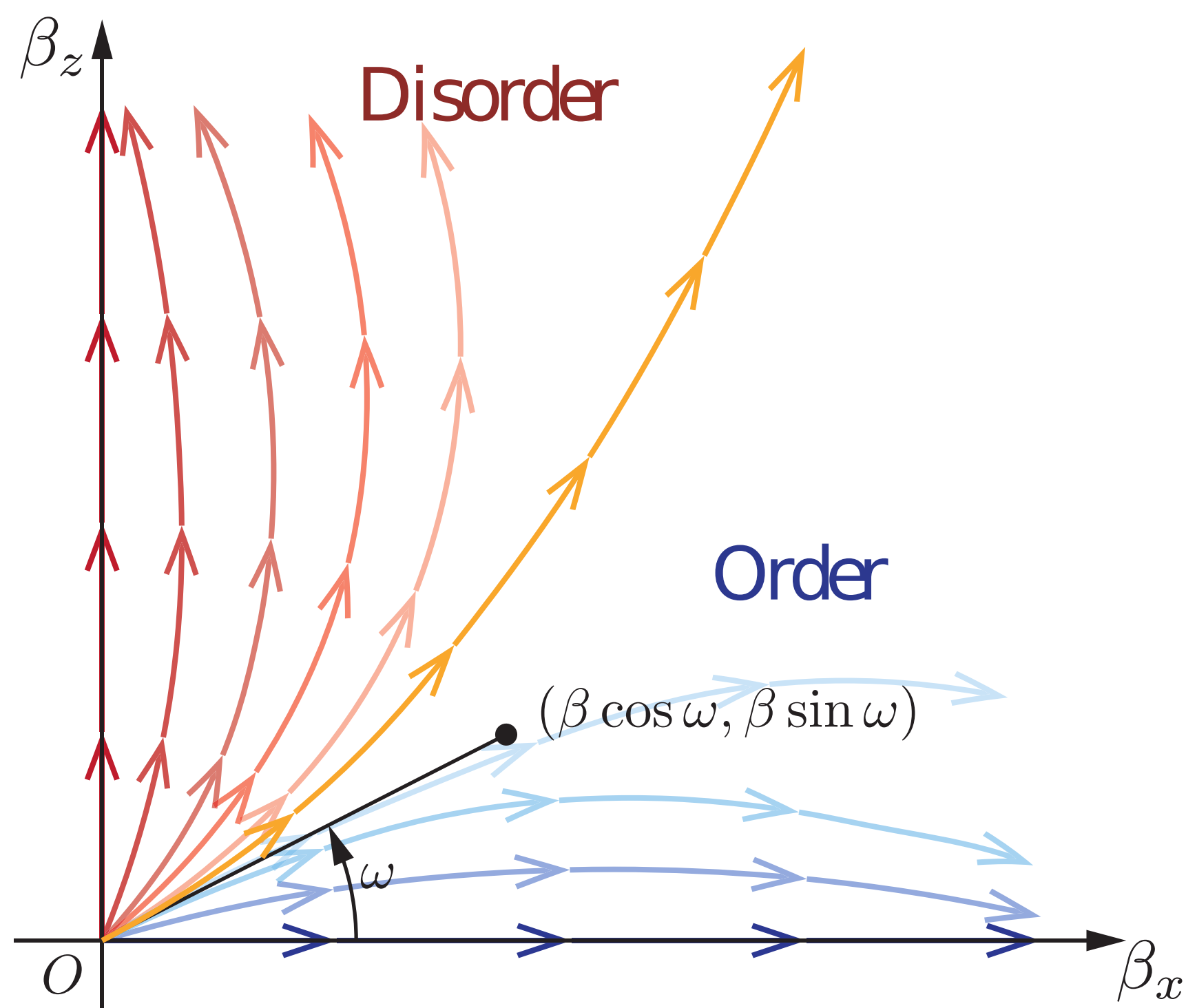


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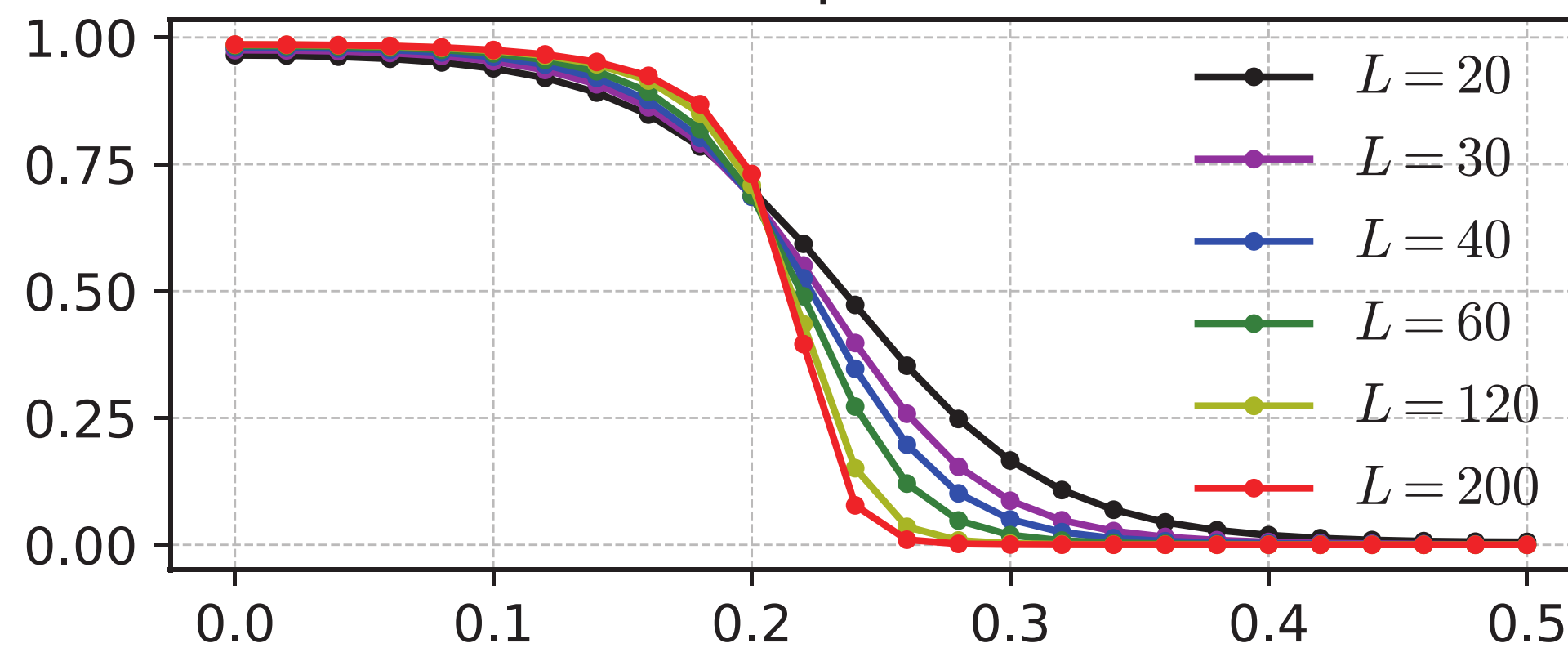
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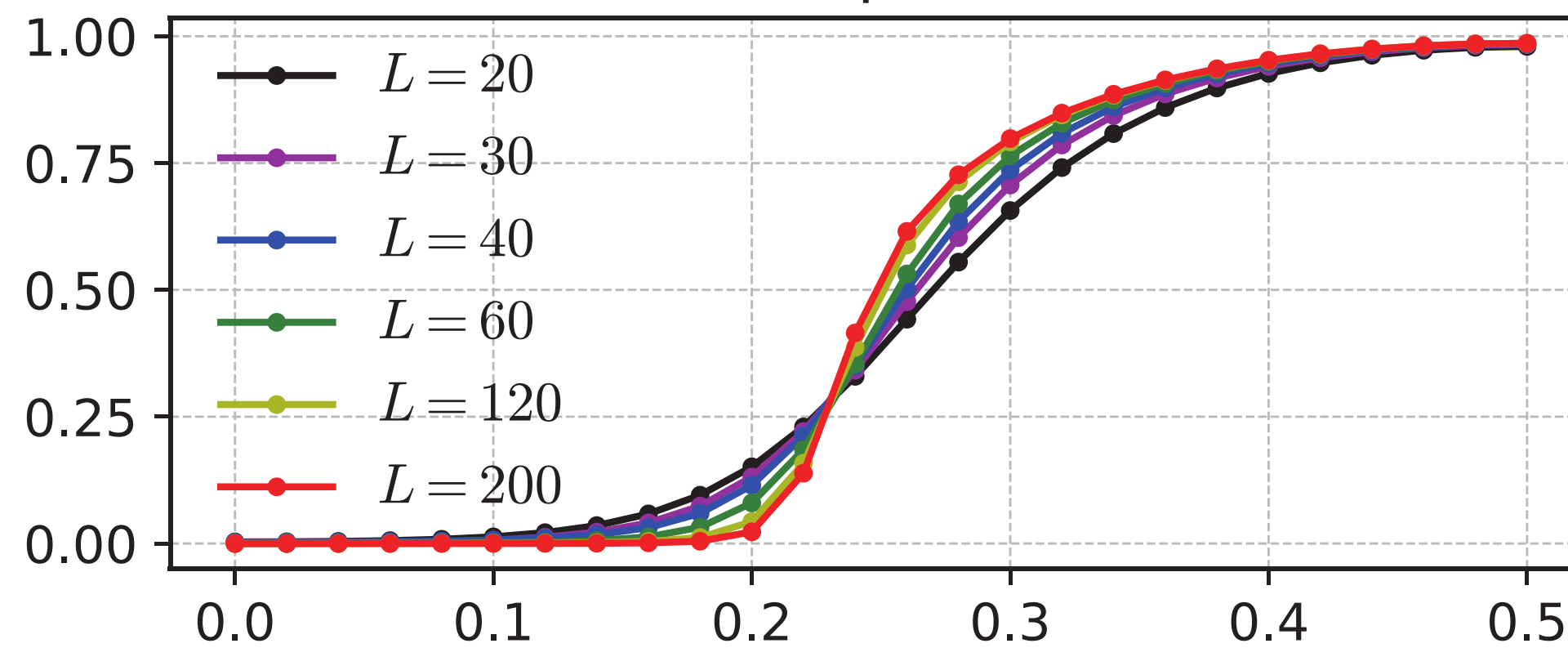
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Order parameter



Disorder parameter



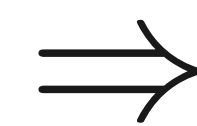
$K = 3/2$

ω/π

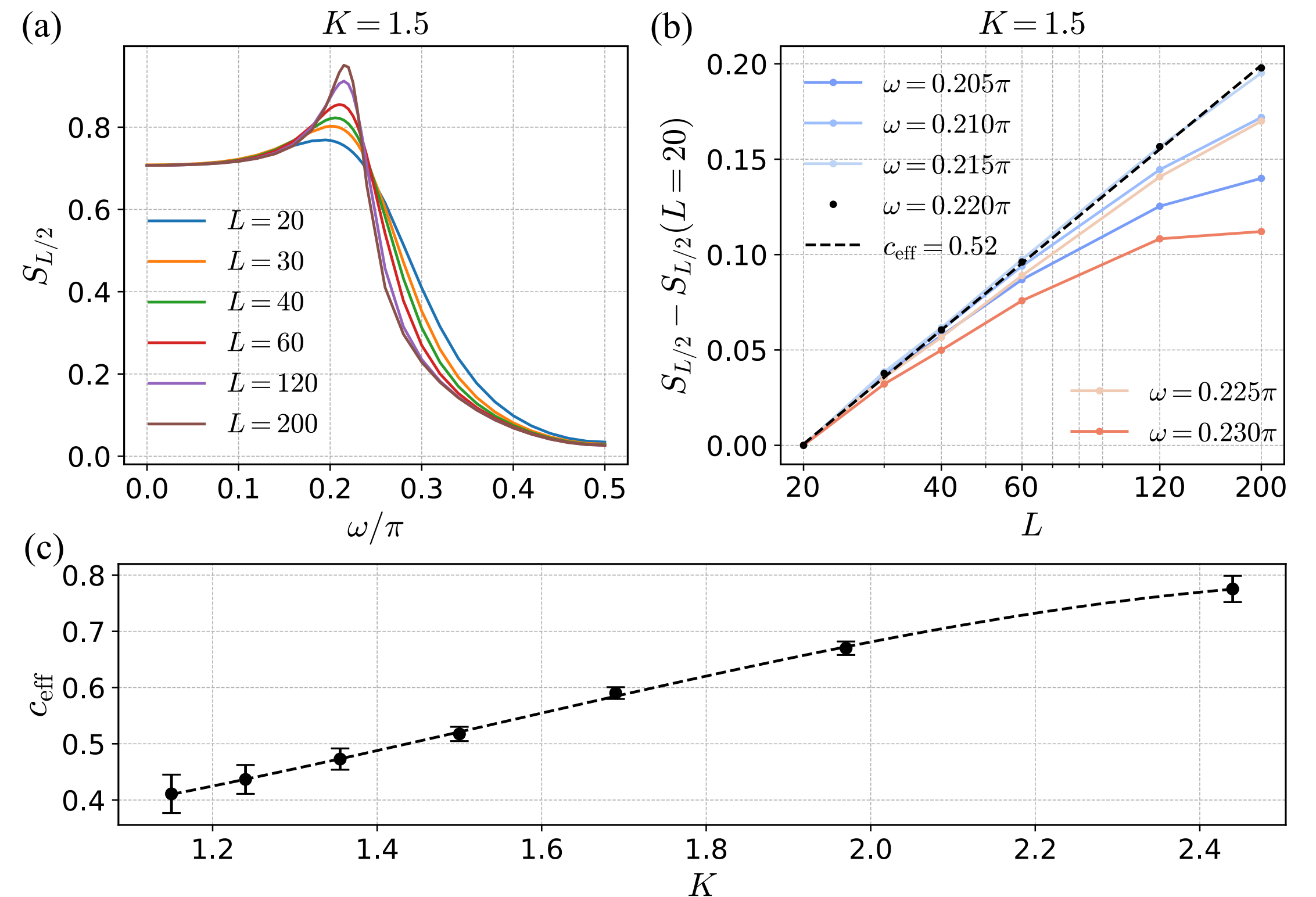
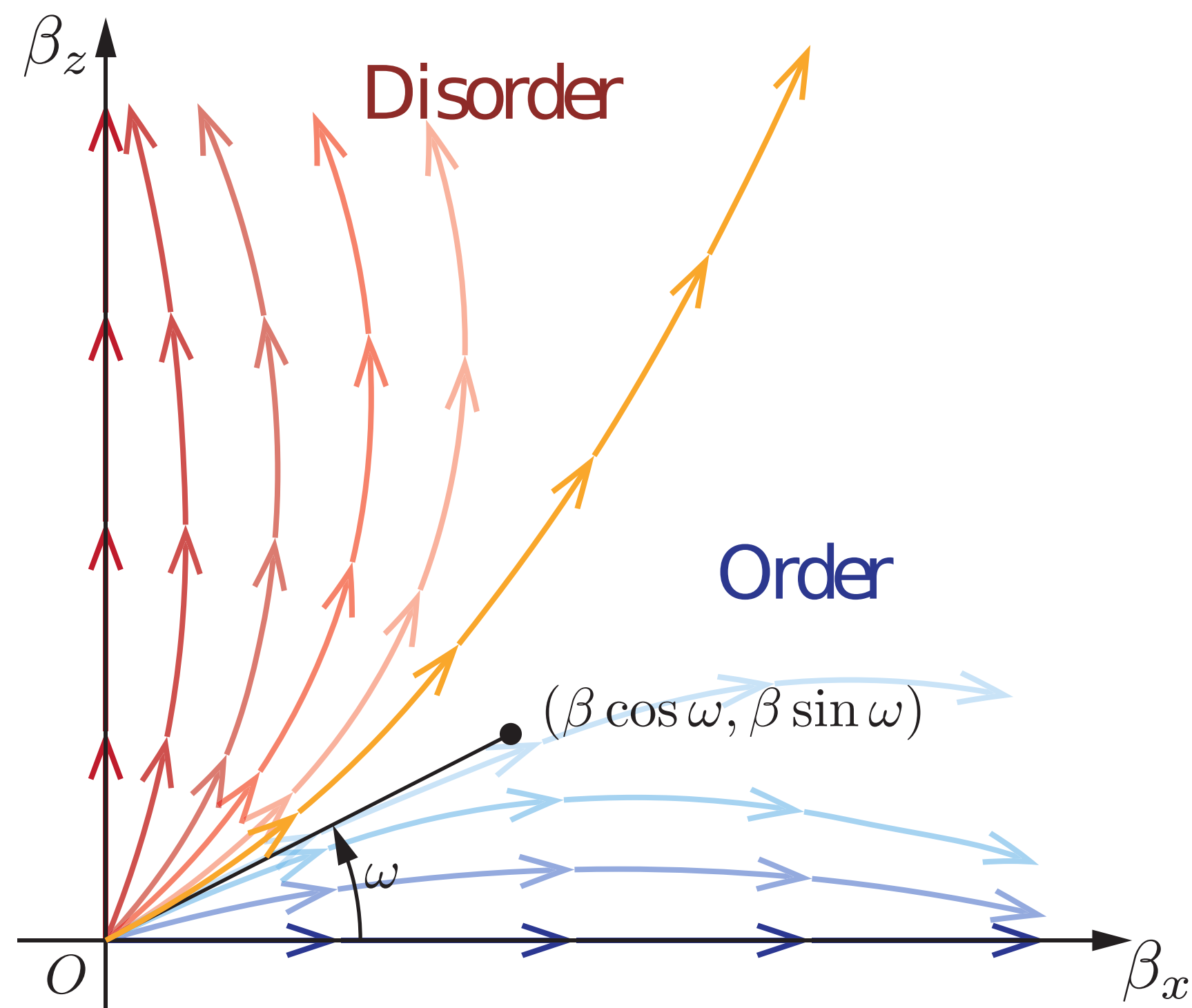
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intermediate fixed point for some critical value ω_c



Tilted weak measurements (symmetry-preserving)

Post-measurement state: $|\psi_{\text{uni}}\rangle = \frac{1}{\mathcal{N}} e^{\beta \sum_j [\cos(\omega) X_{j,1} + \sin(\omega) Z_{j,1} Z_{j+1,1}]} |\psi_{\Delta}\rangle$, $\omega \in [0, \frac{\pi}{2}]$

$\omega = 0, \pi/2$ are two stable fixed point for $K > 1/2$ \implies intermediate fixed point for some critical value ω_c

$$S' = S_{\text{TLL}}[\theta_+] + S_{\text{TLL}}[\theta_-] + \beta \int_x \left[\sin\left(\frac{\pi}{4} - \omega\right) \cos(\theta_+) |_{\tau=0} + \cos\left(\frac{\pi}{4} - \omega\right) \cos(\theta_-) |_{\tau=0} \right]$$

Tilted weak measurements (symmetry-preserving)

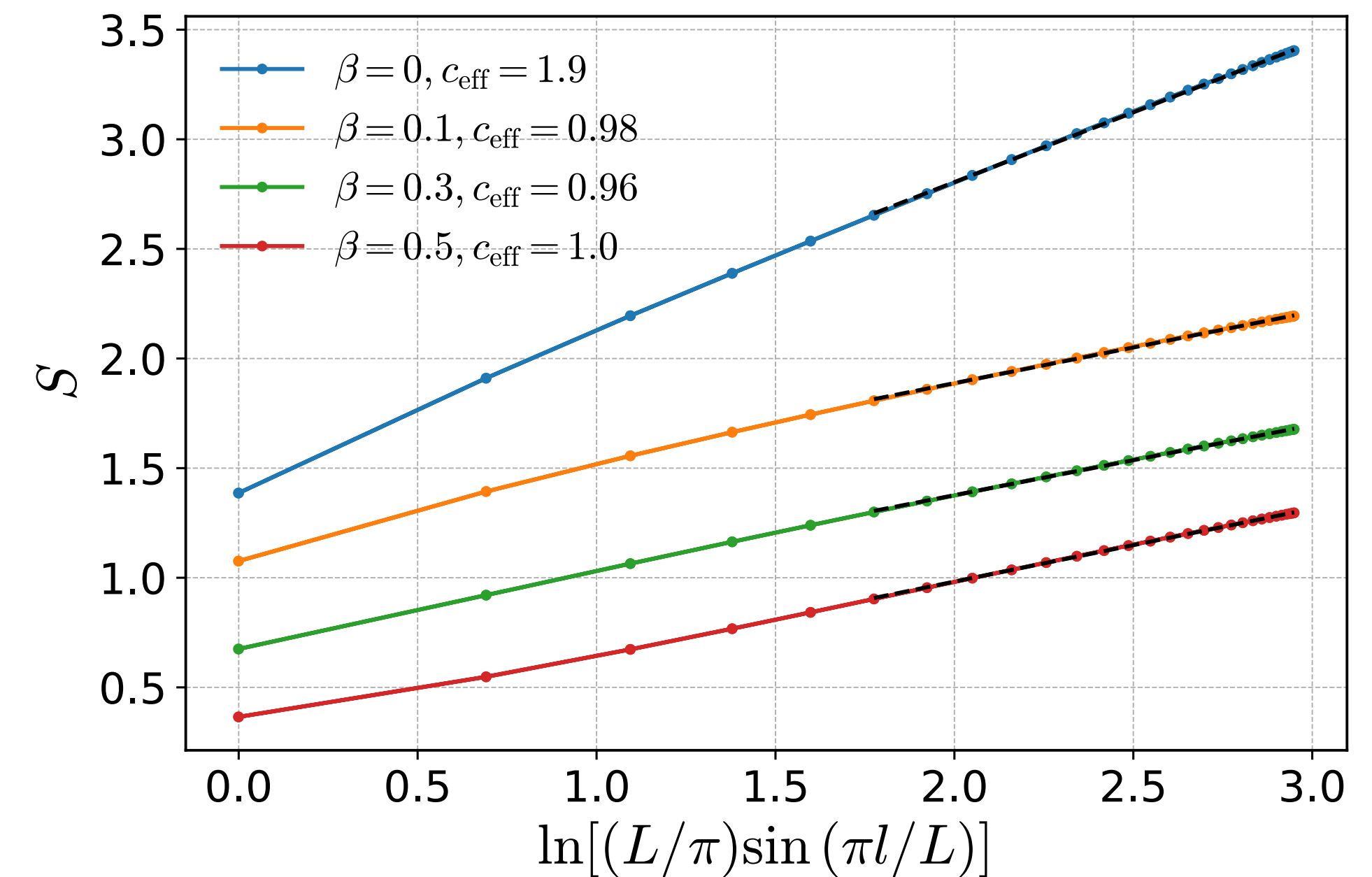
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intermediate fixed point for some critical value ω_c

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$\omega = \pi/4$: intermediate fixed point, with a gapless sector θ_+



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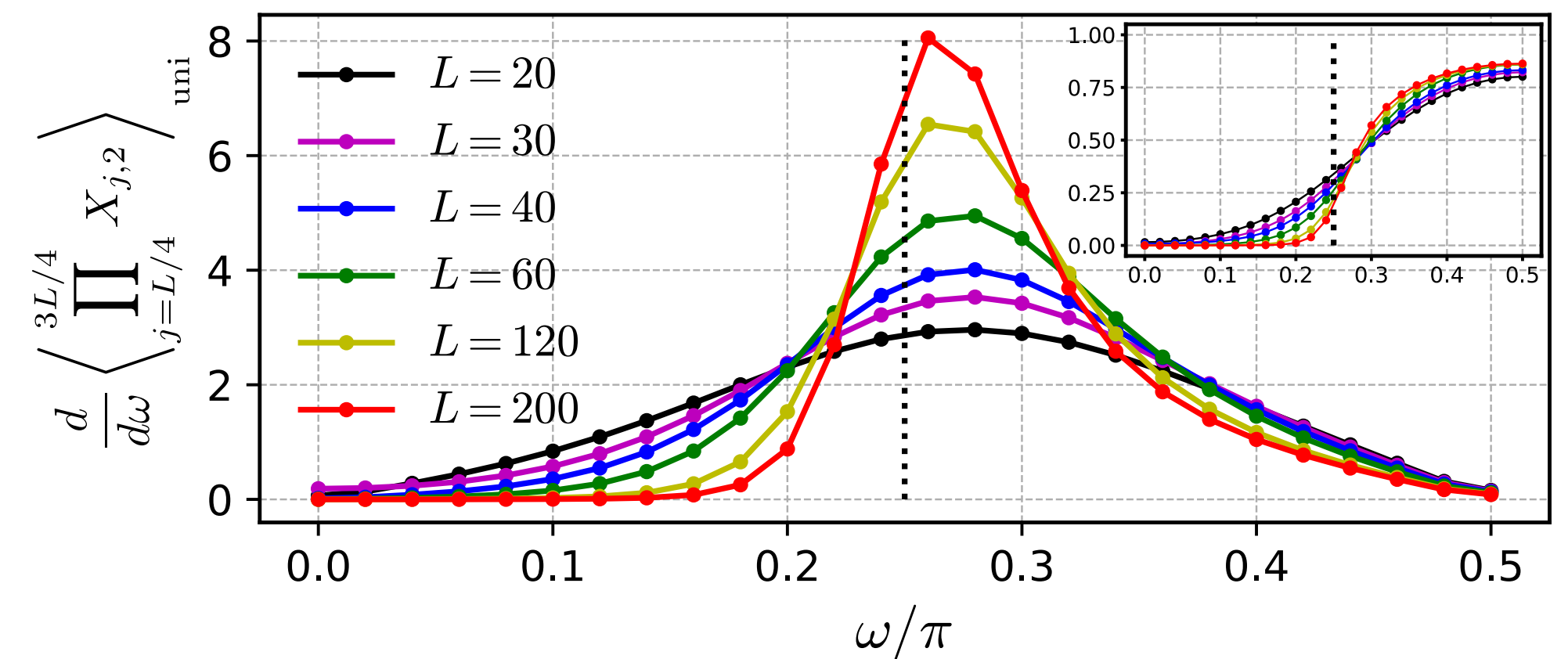
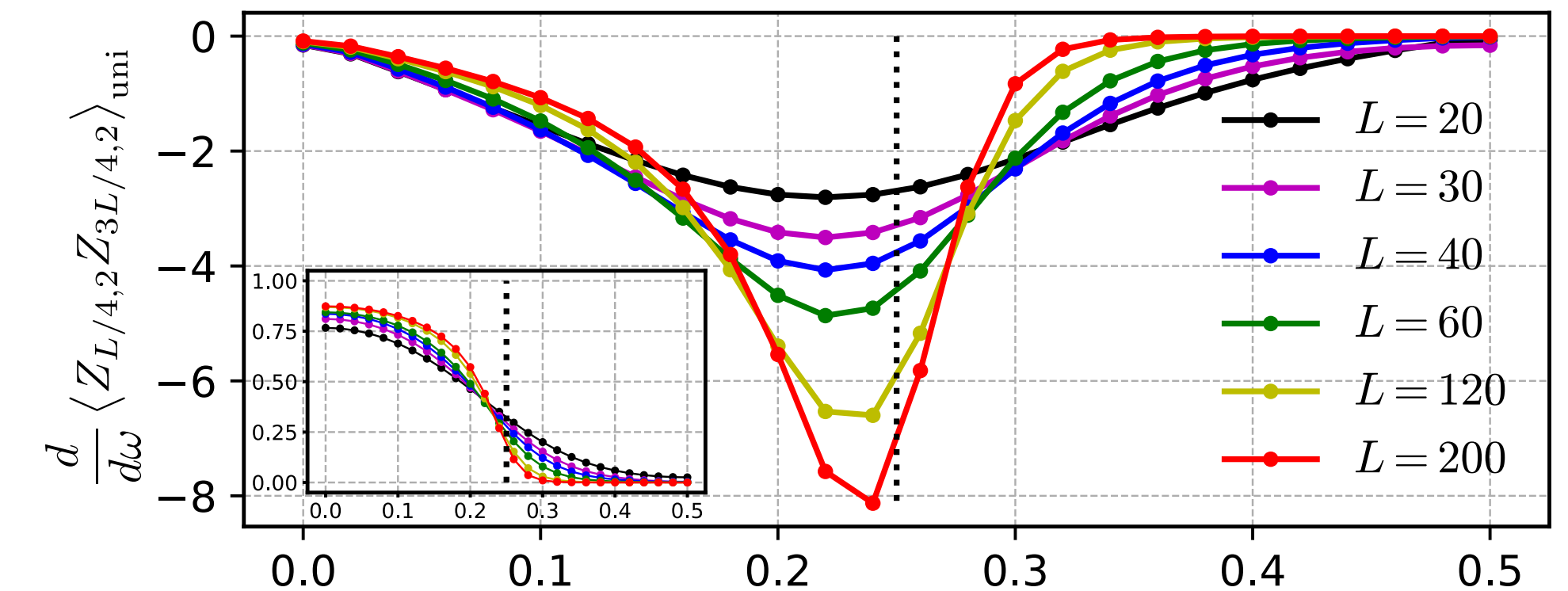
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Outline:

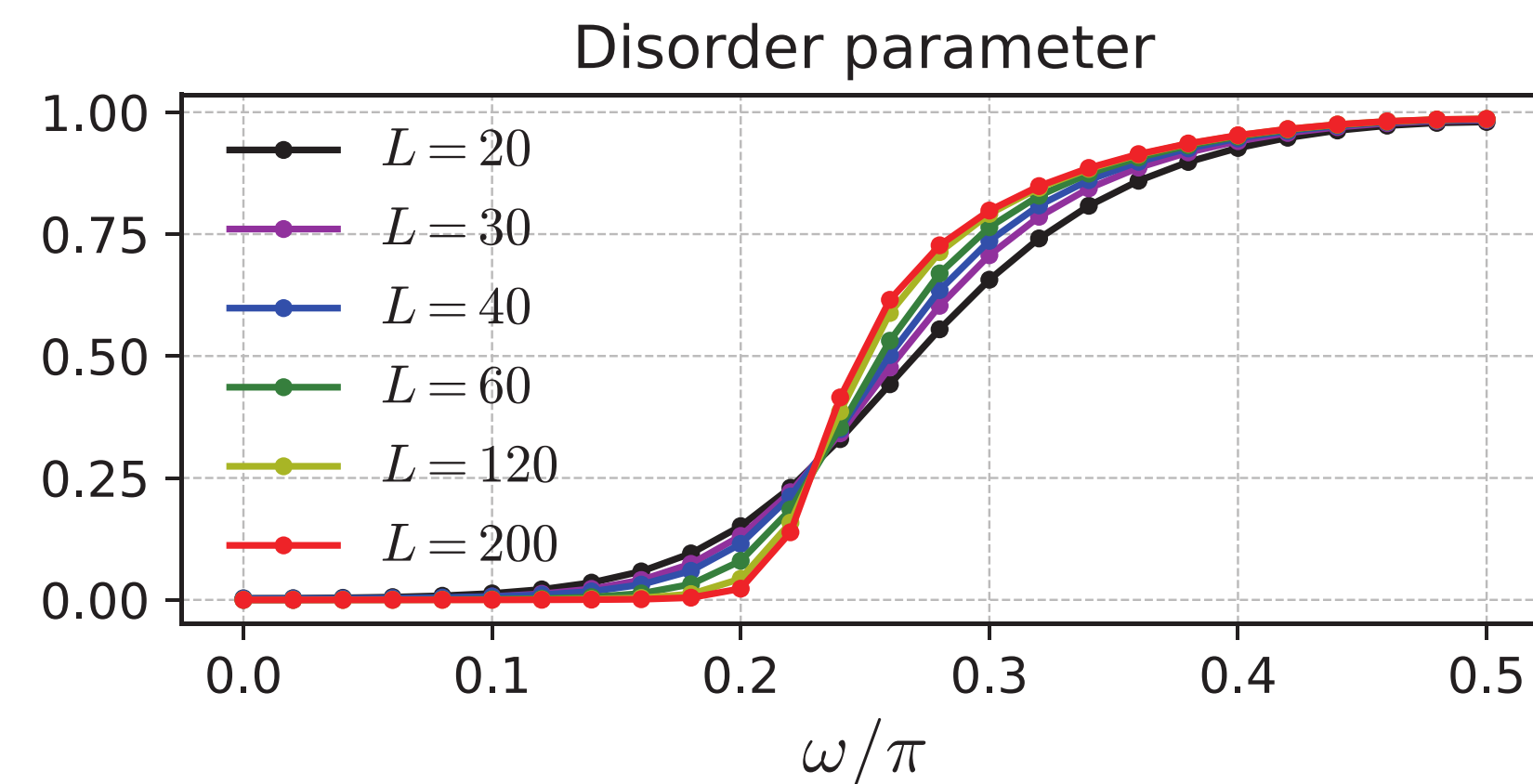
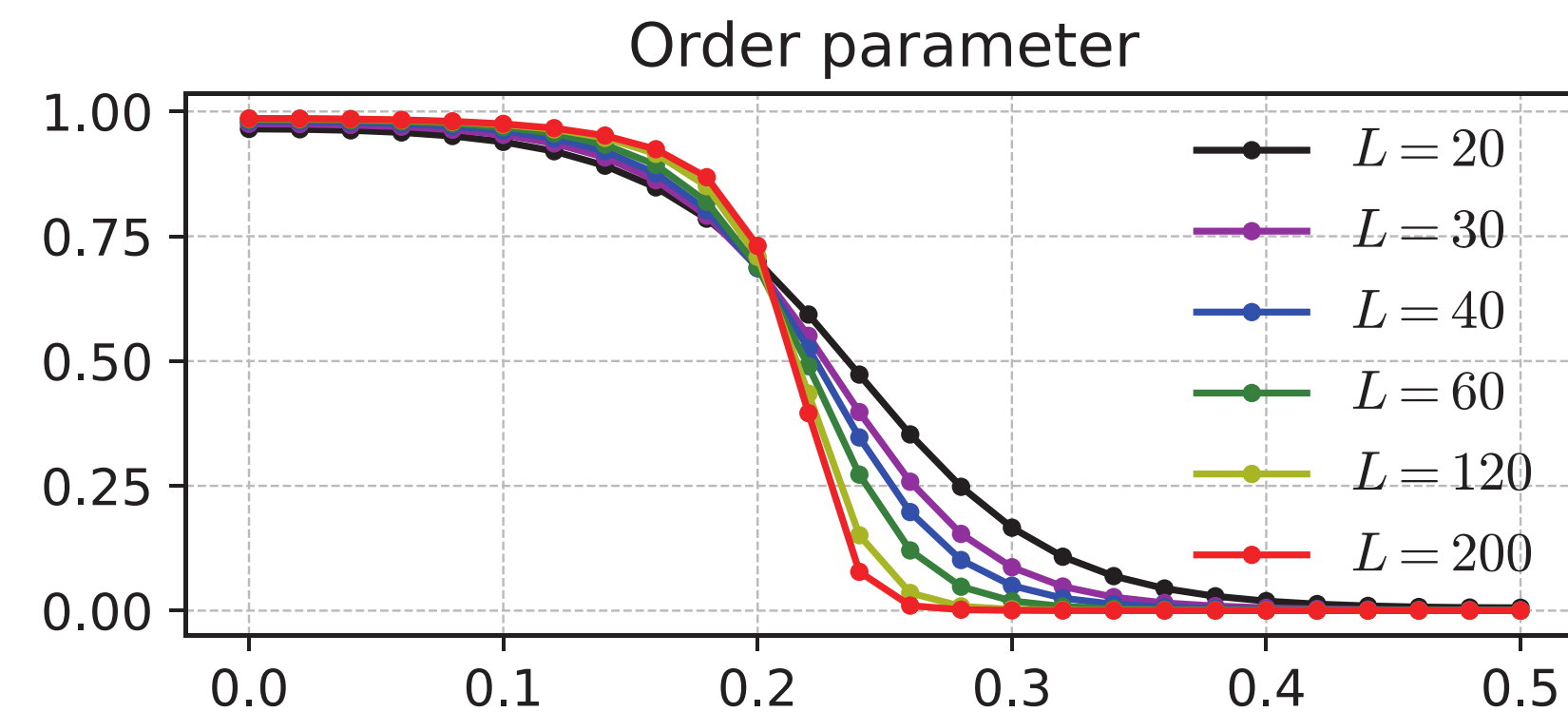
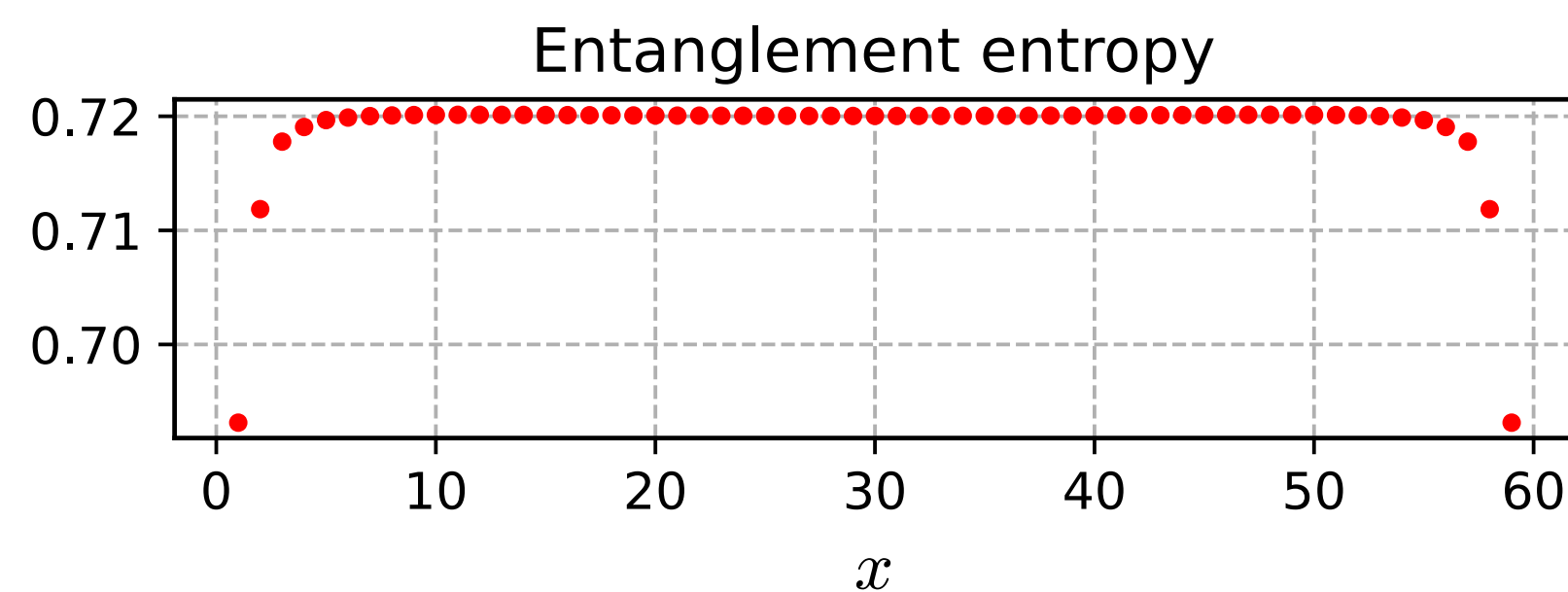
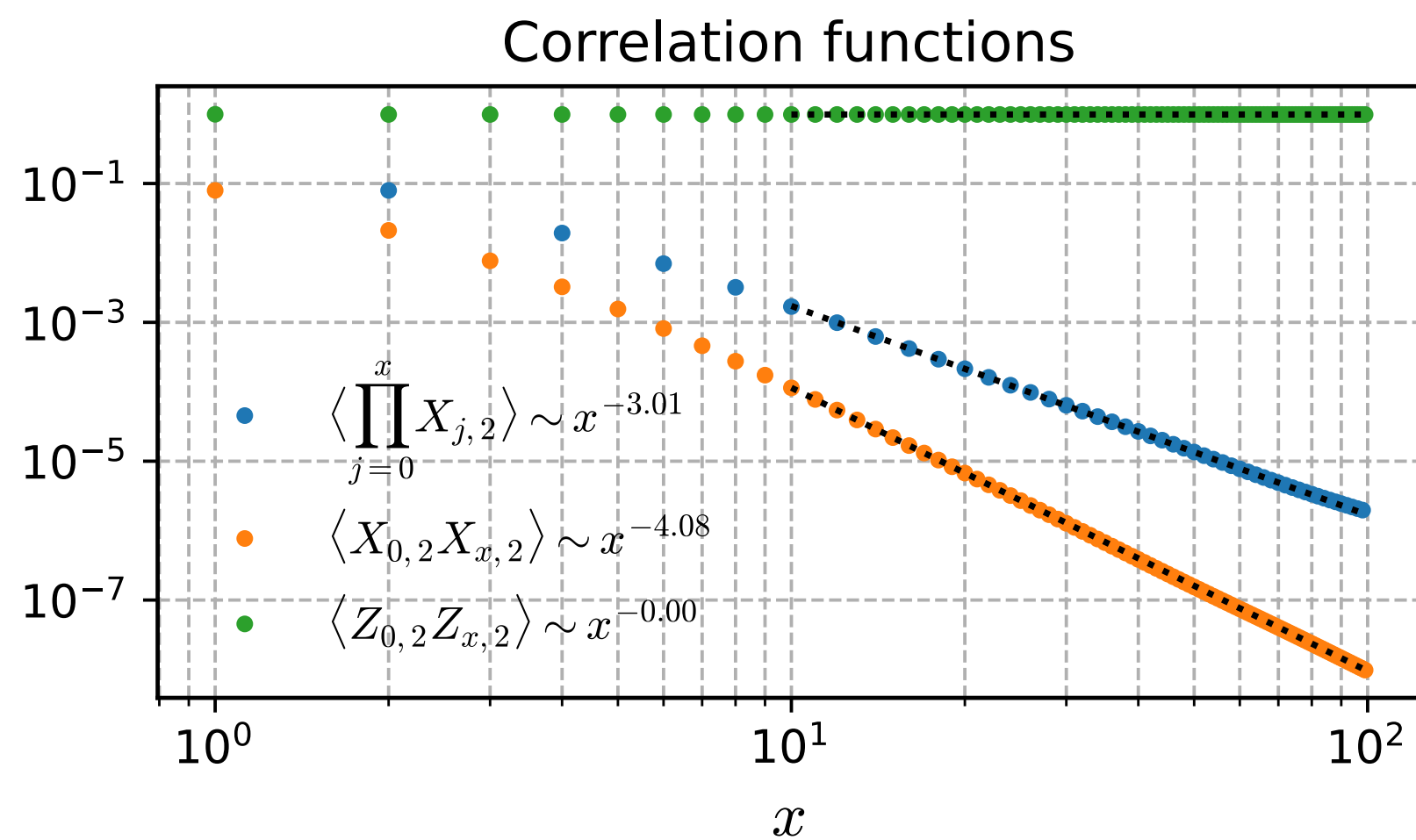
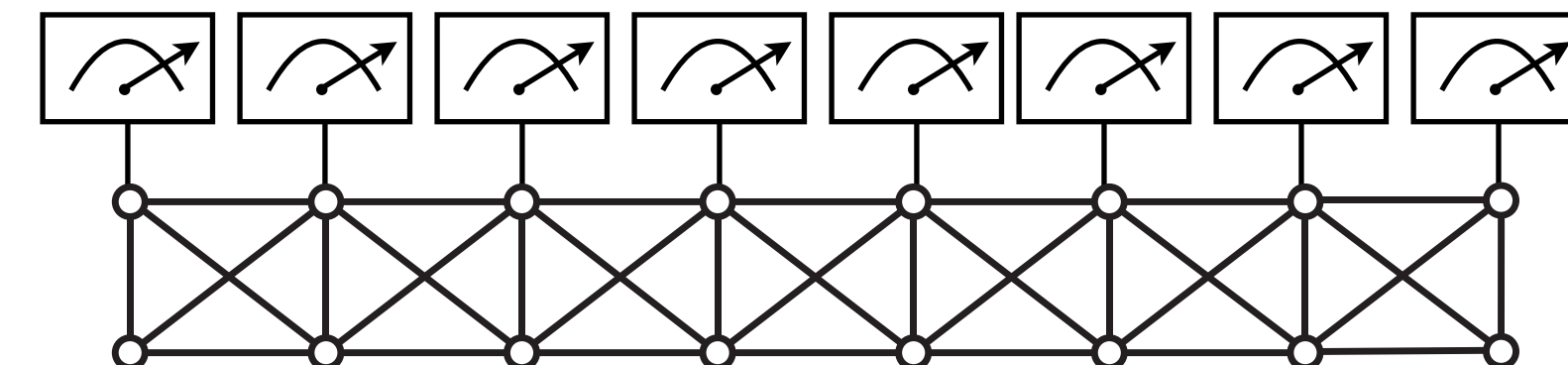
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Conclusions...

Can we still prepare long-range order from this gapless Hamiltonian?

Is it robust far from projective measurements and is there a privileged measurement direction?

Is the decoding protocol still valid?



REVIEW

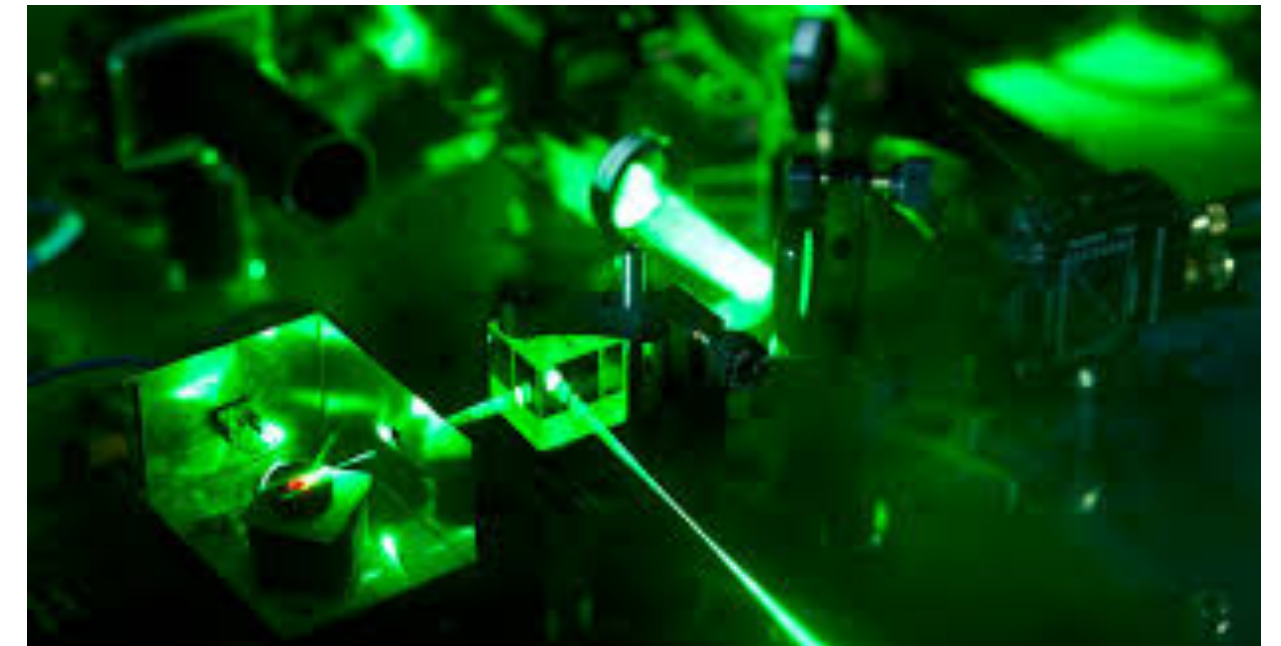
Quantum-Enhanced Measurements: Beating the Standard Quantum Limit

Vittorio Giovannetti,¹ Seth Lloyd,^{2*} Lorenzo Maccone³

Quantum mechanics, through the Heisenberg uncertainty principle, imposes limits on the precision of measurement. Conventional measurement techniques typically fail to reach these limits. Conventional bounds to the precision of measurements such as the shot noise limit or the standard quantum limit are not as fundamental as the Heisenberg limits and can be beaten using quantum strategies that employ “quantum tricks” such as squeezing and entanglement.

Roadmap in criticality & quantum sensing:

- Measurements and quantum critical metrological gain:
application of states with long-range order and power-law decay?



REVIEW

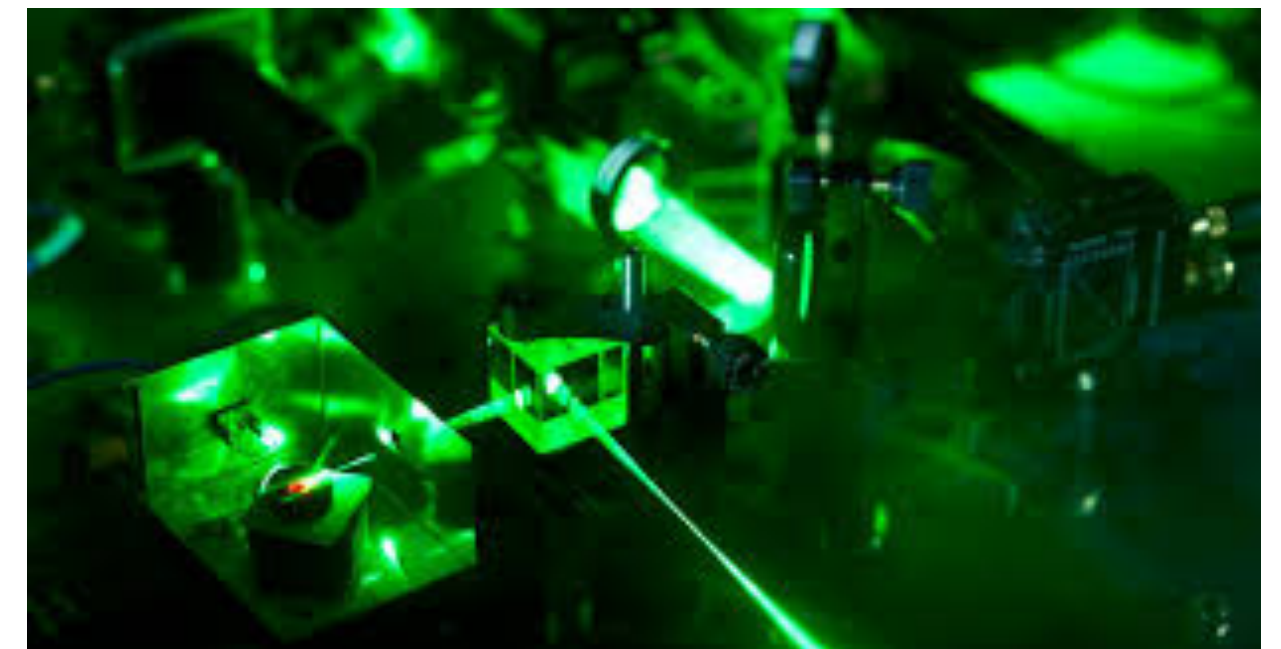
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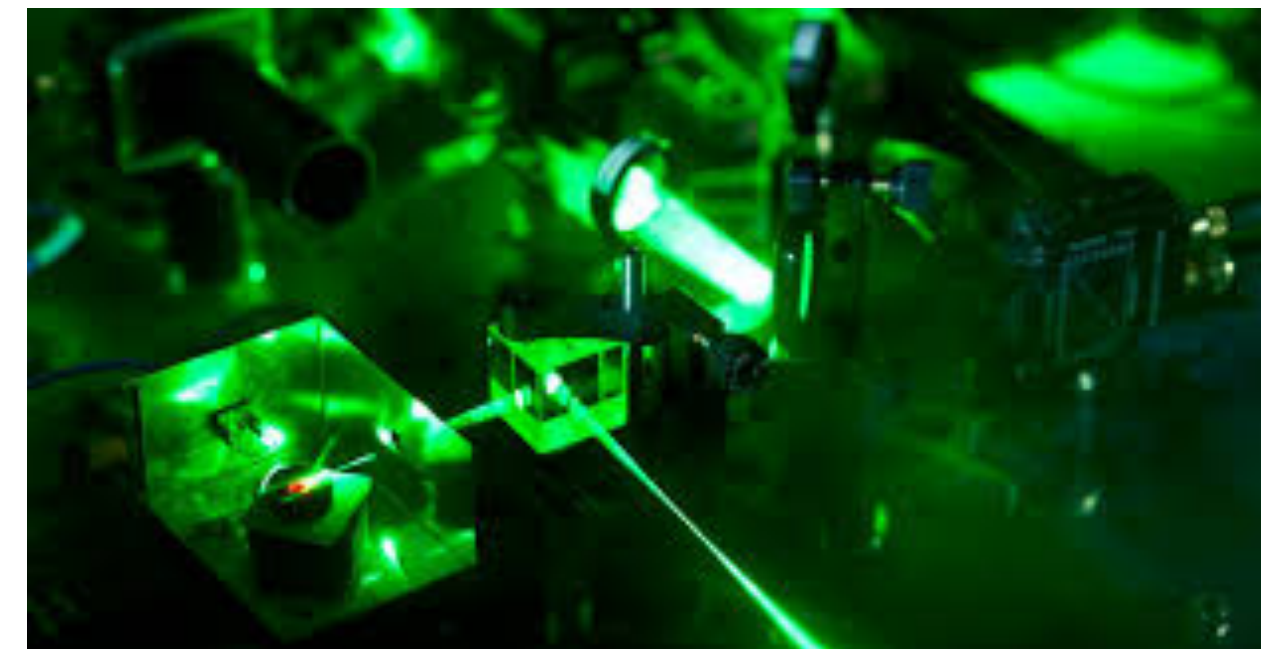
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- Measurements and quantum critical metrological gain: application of states with long-range order and power-law decay?
- Use measurements to produce different critical states?
- Theory side: Higher-dimensions, critical (but non-conformal) field theories, fuzzy sphere ...



Thanks!