Holographic Network, Parallel Universe and Casimir Effect

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Outline

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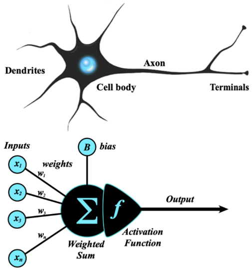
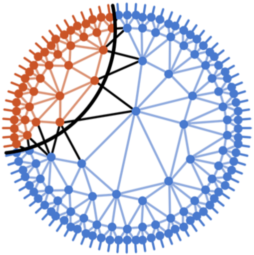
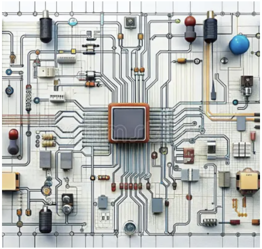
Motivations for Networks

Everything in the universe is interconnected (gravity/entanglement). Networks offer a strong framework for studying these connections.

- Neural networks are driving revolution in artificial intelligence.
- Tensor networks offer insights into entanglement and gravity.
- Circuits etal naturally exhibit network structures.



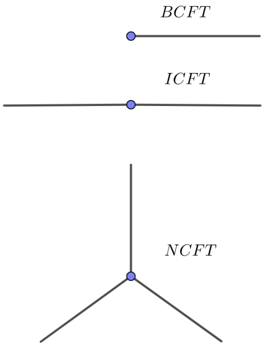


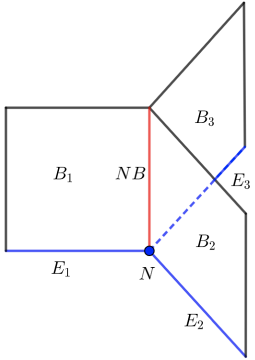


Motivations for Networks

We aim to study the CFT in networks and its gravity dual (AdS/NCFT).

- NCFT describes electron/phonon physics in nanoscale circuits.
- NCFT is a multi-branch generalization of BCFT and ICFT.
- AdS/NCFT is a natural realization of parallel universe.





Outline

1. Background
   - Motivations for Networks
   - NCFT vs BCFT
2. Main Results
   - Gravity Dual of Networks
   - Holographic Entanglement Entropy
   - Parallel Universe
   - Bound of Network Casimir Effect
3. Summary and Outlook

NCFT vs BCFT

NCFT is the multi-branch generalization of BCFT.

- The symmetry group is reduced from $O(d+1,1)$ to $O(d,1)$ for both BCFT and NCFT.
- Boundary conditions for CFT

$$\text{BCFT}: \;\; J_n|_{\text{bdy}} = 0, \;\; T_{na}|_{\text{bdy}} = 0, \tag{1}$$

$$\text{NCFT}: \;\; \sum_m \overset{(m)}{J}_n|_{\text{node}} = 0, \;\; \sum_m \overset{(m)}{T}_{na}|_{\text{node}} = 0 \tag{2}$$

- Boundary conditions for AdS/BCFT

$$\text{AdS/BCFT}: \;\; \left(K_{ij} - K h_{ij}\right)|_{\text{EOW brane}} = -T h_{ij}, \tag{3}$$

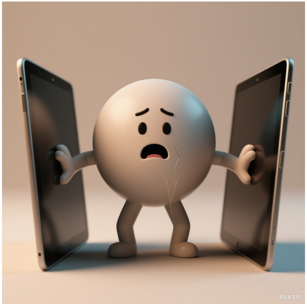
- Junction conditions for AdS/NCFT

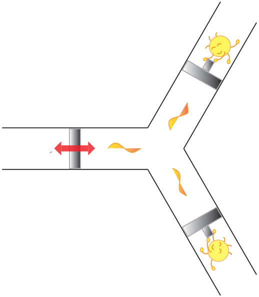
$$\text{AdS/NCFT}: \;\; \sum_m \left(\overset{(m)}{K}_{ij} - \overset{(m)}{K} h_{ij}\right)|_{\text{Net-brane}} = -T h_{ij}, \tag{4}$$

NCFT vs BCFT I

Compared with BCFT, NCFT exhibits numerous new features.

- Casimir force is always attractive for BCFT, harmful for nanodevice.
- Casimir force can be either attractive or repulsive for NCFT [arXiv:2506.20405].

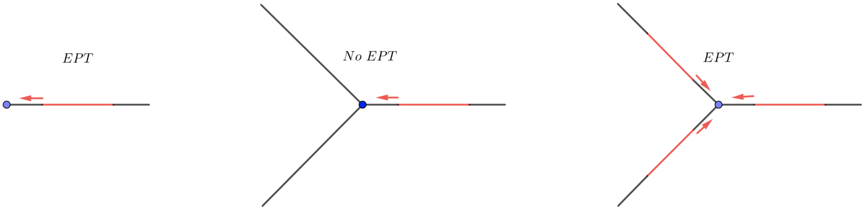




NCFT vs BCFT II

Compared with BCFT, NCFT exhibits numerous new features.

- BCFT: Entanglement phase transition (EPT) when subsystem approaches the boundary.
- NCFT: Entanglement phase transition depends on combinations of subsystems.



Outline

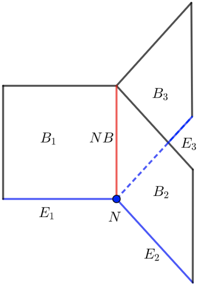
Geometry and Junction Condition

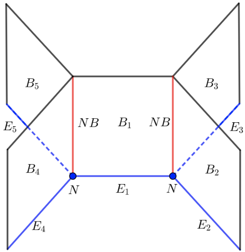
- Action

$$I=\sum_{m}^{p}\int_{B_m} d^{d+1}x\sqrt{|g|}(R-2\Lambda)+2\int_{NB} d^d y\sqrt{|h|}(-T+\sum_{m}^{p}\overset{(m)}{K}),$$

- Junction condition

$$\delta I|_{NB}=-\int_{NB} d^d y\sqrt{|h|}\left[Th_{ij}+\sum_{m}\left(\overset{(m)}{K}_{ij}-\overset{(m)}{K}h_{ij}\right)\right]\delta h^{ij}=0.$$





Proof I: JC leads to conservation at node

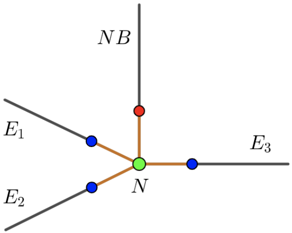
Junction condition on Net-brane leads to energy conservation at node.

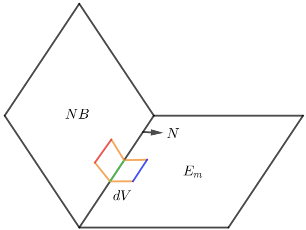
- Conserved current $J_i = T_{ij}\xi^j$ with $\xi^j \sim \delta^j_a$ on edges and Net-brane

$$T_{ij} = 2Kh_{ij} - 2K_{ij} + t^{ct}_{ij}, \quad D^i T_{ij} = 0 \tag{5}$$

- Gauss's law on edges and Net-brane

$$0 = \int_V \nabla_i J^i dV = -\left(\sum_m^p \overset{(m)}{J_E}{}_{n}|_N + \sum_m^p \overset{(m)}{J_{NB}}{}_{n}|_N\right) dS, \tag{6}$$





Proof I: JC leads to conservation at node

- $J_n|_N = T_{ni}\xi^i|_N = T_{na}|_N$ yields

$$\sum_m \overset{(m)}{T}_{E\ na}|_N = -\sum_m \overset{(m)}{T}_{NB\ na}|_N. \tag{7}$$

- Junction condition and asymptotically AdS result in

$$\sum_m \overset{(m)}{T}{}^{CFT}_{na}|_N = \sum_m (2\overset{(m)}{K}_{NB\ na} - 2\overset{(m)}{K}_{NB}\ h_{NB\ na} - t^{ct}_{NB\ na})|_N$$
$$\sim h_{NB\ na}|_N = 0. \tag{8}$$

- Local Killing vector is sufficient for our proof.

$$\xi^j = \delta^j_a + O(y), \quad D_{(i}\xi_{j)} = O(y) \tag{9}$$

Proof II: : JC leads to conservation at node

- The renormalized gravity action is invariant under coordinate transformation $\delta x^\mu = -\xi^\mu$

$$\text{Continuous on brane/node: } : \quad \overset{(m)}{\xi}|_{NB,N} = \xi|_{NB,N},$$

$$\text{Maintain brane/node positions : } \quad \xi^{\hat{n}}|_{NB} = \xi^{n}|_{N} = 0$$

- JC on Net-brane leads to conservation at node

$$\mathcal{L}_\xi I_{\text{ren}} = -\int_{NB} d^d y \sqrt{|h|}\left[T h_{ij} + \sum_m \left(\overset{(m)}{K}_{ij} - \overset{(m)}{K} h_{ij} \right) \right] \mathcal{L}_\xi h_{ij}$$

$$+ \sum_m^p \int_{E_m} d^d y \sqrt{|h|} \frac{1}{2} \overset{(m)}{T}{}^{ij} \mathcal{L}_\xi h_{ij}, \quad \mathcal{L}_\xi h_{ij} = 2 D_{(i}\xi_{j)}, D_i \overset{(m)}{T}{}^{ij} = 0$$

$$= \int_N d^{d-1} y \sqrt{|\sigma|} \sum_m^p \overset{(m)}{T}_{na} \xi^a = 0$$

Perturbative solution I: conservation law at node

- Perturbative Poincaré AdS

$$ds^2 = \frac{dz^2 - dt^2 + d\overset{(m)}{x}{}^2 + \frac{2}{d} f_m(z, \overset{(m)}{x}) dt d\overset{(m)}{x} + \delta_{ab} dy^a dy^b}{z^2}, \quad (10)$$

- Solution

$$f_m(z, \overset{(m)}{x}) = X_m(\overset{(m)}{x}) + c_m z^d,$$

$$\overset{(m)}{T}{}^{\text{CFT}}_{xt} = c_m$$

- Junction condition

$$\cosh(\rho) \sum_m c_m = 0.$$

- Conservation of energy flux at node

$$\sum_m \overset{(m)}{T}{}^{\text{CFT}}_{xt}|_N = \sum_m c_m = 0.$$

Perturbative solution II: gravitational KK modes

Gravitational KK modes combines those of AdS/BCFT with NBC and DBC, corresponding to isolated and transparent modes.

- Perturbative metric

$$ds^2 = dr^2 + \cosh^2(r)\left(\bar{h}^{(0)}_{ij}(y) + \epsilon \overset{(m)}{H}(r)\bar{h}^{(1)}_{ij}(y)\right) dy^i dy^j$$

- EOM on Net-brane

$$\left(\bar{\Box} + 2 - M^2\right)\bar{h}^{(1)}_{ij}(y) = 0,$$

$$\cosh^2(r)\overset{(m)}{H}{}''(r) + d\sinh(r)\cosh(r)\overset{(m)}{H}{}'(r) + M^2\overset{(m)}{H}(r) = 0,$$

- Solutions of KK modes

$$\overset{(m)}{H}(r) = \overset{(m)}{c}H(r) = \overset{(m)}{c}\begin{cases} \mathrm{sech}^{\frac{d}{2}}(r)\ P^{\frac{d}{2}}_{l_M}(-\tanh r), & \text{even } d, \\ \mathrm{sech}^{\frac{d}{2}}(r)\ Q^{\frac{d}{2}}_{l_M}(-\tanh r), & \text{odd } d. \end{cases}$$

Perturbative solution II: gravitational KK modes

- Junction condition and continuity condition

$$\sum_{m=1}^{p} \overset{(m)}{H}{}'(\rho) = 0 \ \rightarrow \ \sum_{m=1}^{p} \overset{(m)}{c} H'(\rho) = 0,$$
$$\overset{(i)}{H}(\rho) = \overset{(j)}{H}(\rho) \ \rightarrow \quad \overset{(i)}{c} H(\rho) = \overset{(j)}{c} H(\rho).$$

- One class of modes obeying Neumann boundary condition

$$\text{NBC}: \ \ H'(\rho) = 0, \ \overset{(i)}{c} = \overset{(j)}{c}, \tag{11}$$

NBC corresponds to isolated mode $\overset{(m)}{T}{}_{na}|_N = 0$, stable $M^2 > 0$

- $(p-1)$ classes of modes satisfying Dirichlet boundary condition

$$\text{DBC}: \ \ H(\rho) = 0, \ \sum_{m=1}^{p} \overset{(m)}{c} = 0. \tag{12}$$

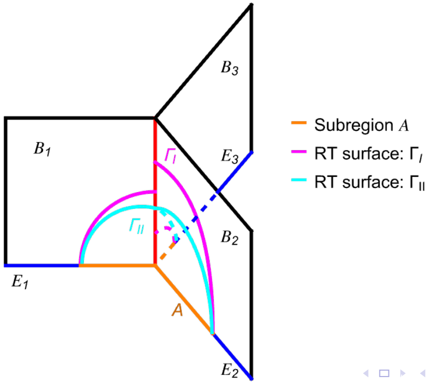
DBC corresponds to transparent mode $\overset{(m)}{T}{}_{na}|_N \neq 0$, stable $M^2 > 0$

Outline

Proposal of HEE

While proposal I leads to smaller HEE, proposal II is the correct one.

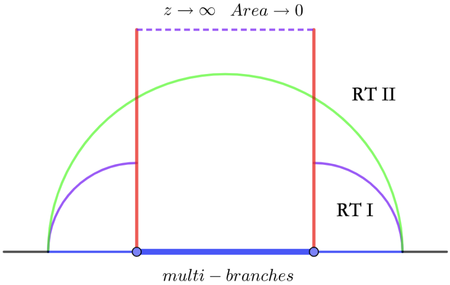
- Disconnected proposal I: inspired by AdS/BCFT, RT surfaces are perpendicular to Net-brane
- Connected proposal II: RT surfaces must be interconnected through the same interaction on Net-brane



Why connected RT surface?

RT surfaces intersect at the same point on Net-brane for connected subsystems within the network

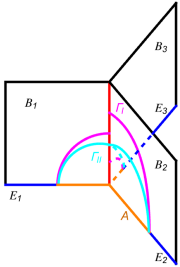
- Naturally, connected subsystem corresponds to connected RT surface.
- Both proposal I and proposal II obeys strong subadditivity of entanglement entropy.
- Proposal I is independent of the internal edge, which is unphysical.

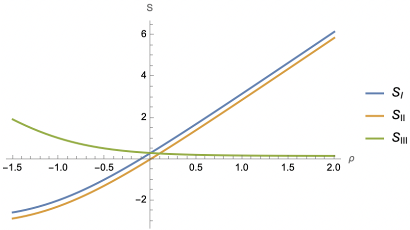


Network entropy

We propose several natural definitions of network entropy.

- Proposal I: $S_{\rm I}=S_{\rm NCFT}-S_{\rm CFT}$, $S_{\rm CFT}=S_{\rm BCFT}(0)$, obeying g-theorem $S_{\rm I}|_{\rm IR}\leq S_{\rm I}|_{\rm UV}$
- Proposal II: $S_{\rm II}=S_{\rm NCFT}(\rho)-S_{\rm NCFT}(0)$, obeying g-theorem $S_{\rm II}|_{\rm IR}\leq S_{\rm II}|_{\rm UV}$
- Proposal III: $S_{\rm III}=S_{\rm NCFT}-S_{\rm BCFT}\geq 0$, requiring entanglement to assemble network from edges.

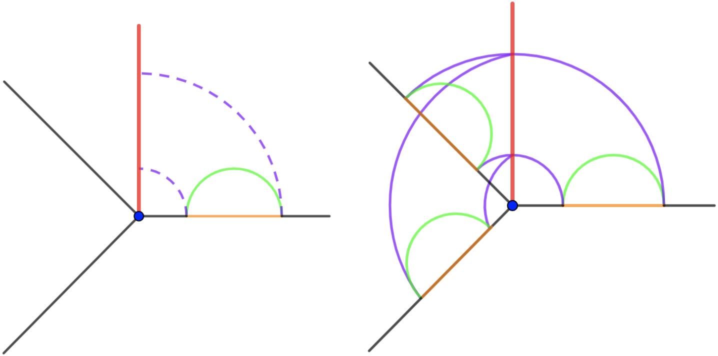




Entanglement Phase Transition (EPT)

Entanglement phase transition (EPT) depends on subsystem combinations.

- No EPT when a single subsystem approaches a node.
- EPT appears when muliti-subsystems approach a node.



Shortest path problem I

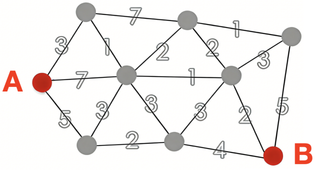
The shortest path problem seeks to determine shortest path between two points within networks. equivalent to finding the shortest distance in bulk.

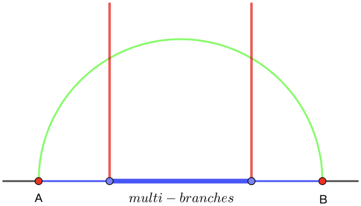
- Bulk: multi-Euclidean AdS_2 glued by tensionless Net-branes

$$d\overset{(m)}{s}{}^2 = \frac{dz^2 + d\overset{(m)}{x}{}^2}{z^2},\ 0 \le \overset{(m)}{x} \le L_m,$$

- Every loop-free path from A to B is dual to a single AdS_2

$$L_{AB} = 2\log(\frac{l_{AB}}{\epsilon})$$





Shortest path problem II

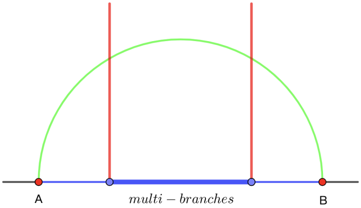
The shortest path problem is equivalent to calculating the holographic two-point correlators of massive operators.

- Every loop-free path from A to B is dual to a single AdS_2

$$L_{AB} = 2\log(\frac{l_{AB}}{\epsilon})$$

- Two-point function of operators dual to massive particles is determined by the proper distance in bulk

$$\langle O(A)O(B)\rangle \sim e^{-ML_{AB}}, \tag{13}$$

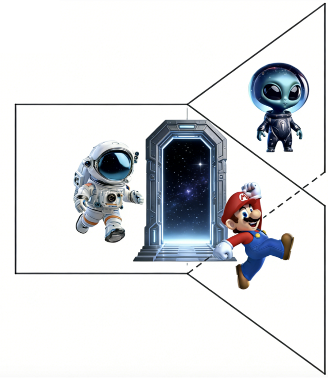


Outline

Junction as Stargate

AdS/NCFT can be regarded as traversable parallel universes.

- Geometry, Physical laws can differ in different parallel universe.
- Possible to travel from one universe to another in a probabilistic sense.



Connect different Universes

- Threefold universe

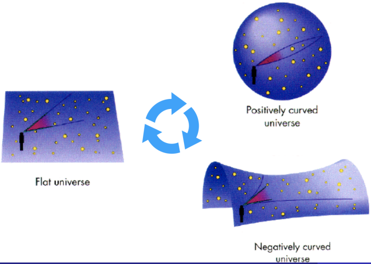
$$ds^2 = -(1+\kappa r^2)dt^2 + \frac{dr^2}{1+\kappa r^2} + r^2 d\Omega^2, \quad 0 \le r \le r_N, \qquad (14)$$

where $\kappa = 0, -1, 1$ for flat, dS and AdS.

- Junction condition at Net-brane $r = r_N$

$$\sum_{m=1}^{3} (\overset{(m)}{K}_{ij} - \overset{(m)}{K} h_{ij}) = 8\pi G_N T_{brane\ ij}, \qquad (15)$$

where $T_{brane\ ij}$ obey both weak and null energy conditions.



Connect different gravity theories

- Gauss-Bonnet gravity

$$I_{\rm GB} = \sum_m^p \int_{B_m} d^{d+1}x\sqrt{|g|}\Big(R + d(d-1) + \overset{(m)}{\alpha}\ \mathcal{L}_{\rm GB}(\bar{R})\Big)$$
$$+2\int_{NB} d^d y\sqrt{|h|}\Big(\sum_m^p \overset{(m)}{\mathcal{K}}_{GB} - T\Big), \quad T = p(d-1)\tanh(\rho)$$

- Junction condition at Net-brane $r = \rho$

$$\sum_m \overset{(m)}{K}_{ij} - \overset{(m)}{K} h_{ij} + \frac{2\overset{(m)}{\alpha}}{1 + 2\overset{(m)}{\alpha}(d-1)(d-2)}\left(\overset{(m)}{Q}_{ij} - \frac{1}{3}\overset{(m)}{Q} h_{ij}\right) = -T h_{ij}.$$

- Ghost-free and tachyon-free conditions for gravitational KK modes

$$\sum_m^p \frac{4\overset{(m)}{\alpha}(d-3)\tanh(\rho)}{1 + 2\overset{(m)}{\alpha}(d-1)(d-2)} \geq 0 \qquad (16)$$

Outline

Bound of Casimir effect for BCFTs

- Casimir effect of a strip

$$\langle T_{nn} \rangle = -(d-1)\frac{\kappa_1}{L^d}, \tag{17}$$

where n denotes the normal direction.

- Displacement operator $D = T_{nn}|_{\text{bdy}}$

$$\langle T_{nn}(y) T_{nn}(0) \rangle = \frac{C_D}{|y|^{2d}}. \tag{18}$$

- Conjecture: AdS/BCFT with minimal brane tension imposes a lower bound of Casimir effect [Miao, 2025].

$$(-\frac{\kappa_1}{C_D}) \geq \lim_{T \to -(d-1)} (-\frac{\kappa_1}{C_D})|_{\text{holo}} = \frac{-2^{d-2} d \pi^{d-\frac{1}{2}} \Gamma\left(\frac{d-1}{2}\right) \Gamma\left(\frac{1}{d}\right)^d}{\Gamma(d+2)\left(d\Gamma\left(\frac{1}{2}+\frac{1}{d}\right)\right)^d} \tag{19}$$

where T is the brane tension.

Holographic bound in AdS$_4$/BCFT$_3$ (Einstein gravity)

- Holographic Casimir coefficient (Takayanagi et al 2011)

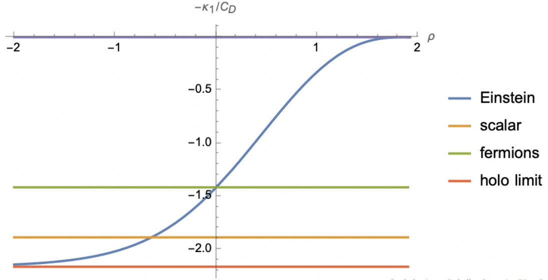
$$\kappa_1 = \text{complicated function of brane tension} \tag{20}$$

- Holographic displacement operator (Chu and Miao 2018 2019)

$$C_D = \frac{32}{\pi\left(2\tan^{-1}\left(\tanh\left(\frac{\rho}{2}\right)\right) + \frac{\pi}{2}\right)}, \tag{21}$$

where $T = 2\tanh(\rho)$ is the brane tension.

- The smaller the brane tension, the smaller the ratio $(-\kappa_1/C_D)$



Comments on the Bound

- Physical meaning I: Casimir energy per degree of freedom has a lower bound.
- Physical meaning II: The lower bound is given by the holographic BCFT with minimal boundary degree of freedom.
- Universal holographic bound, independent of gravity models (Einstein, DGP, Gauss-Bonnet)
- Applies to general boundary shapes such as wedges.
- For 2d BCFTs, we can prove the bound, which is saturated by any BCFT with the same boundary conditions on the two strip boundaries.
- For 3d BCFTs, it is a conjecture that has passed many tests.

Table: $(-\kappa_1/C_D)$ for various 3d CFTs

fermion	scalar	Ising	O(2)	O(3)	holography
-1.42	-1.89	-2.12	-1.83	-1.62	-2.17

Bound of Casimir effect for 2d NCFTs

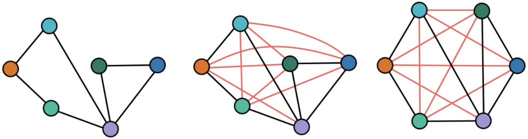
- Bound on Casimir energy for 2d BCFTs in a strip

$$W_{\text{strip}} \geq -\frac{\pi c}{24L} \tag{22}$$

- Bound on Casimir energy for 2d NCFTs in general networks

$$W_{\text{network}} \geq -\frac{\pi c}{24}\sum_{i}^{p}\frac{1}{L_i}, \tag{23}$$

where c is the central charge, L_i is the length of edge E_i.



Bound of Casimir effect for 2d NCFTs

- Bound on Casimir energy for 2d NCFTs in general networks

$$W_{\text{network}} \geq -\frac{\pi c}{24} \sum_{i}^{p} \frac{1}{L_i}, \tag{24}$$

where c is the central charge, L_i is the length of edge E_i.

- Casimir effect is a boundary effect, meaning that the higher the reflection coefficient of the boundary, the stronger the Casimir effect.
- BCFT has the largest reflection coefficient $R = 1$.
- Thus, the lower bound of networks is given by the Casimir energy of p isolated strips with identical boundary conditions.
- Pass tests by 2d free scalars and $AdS_3/NCFT_2$.
- Energy is required to assemble a network from its edges.

$$W_{\text{bind}} = W_{\text{NCFT}} - W_{\text{BCFT}} \geq 0. \tag{25}$$

Summary and Outlook

Summary:

- We propose the gravity dual of CFTs in networks.
- We prove junction condition on Net-brane leads to energy conservation at the node.
- We propose rules to calculate HEE in AdS/NCFT and define a positive network entropy.
- The parallel universes in AdS/NCFT is traversable and obeys null energy condition.
- We argue the bound of Casimir energy for NCFTs is given by sum of that of isolated BCFTs.
- Entanglement and energy are required to assemble a network from its edges.

Outlook:

- Holographic dynamical networks to mimic circuit?
- How does AI emerge from networks, and what is its gravity dual?

Thanks!

Welcome to new world of holographic network!