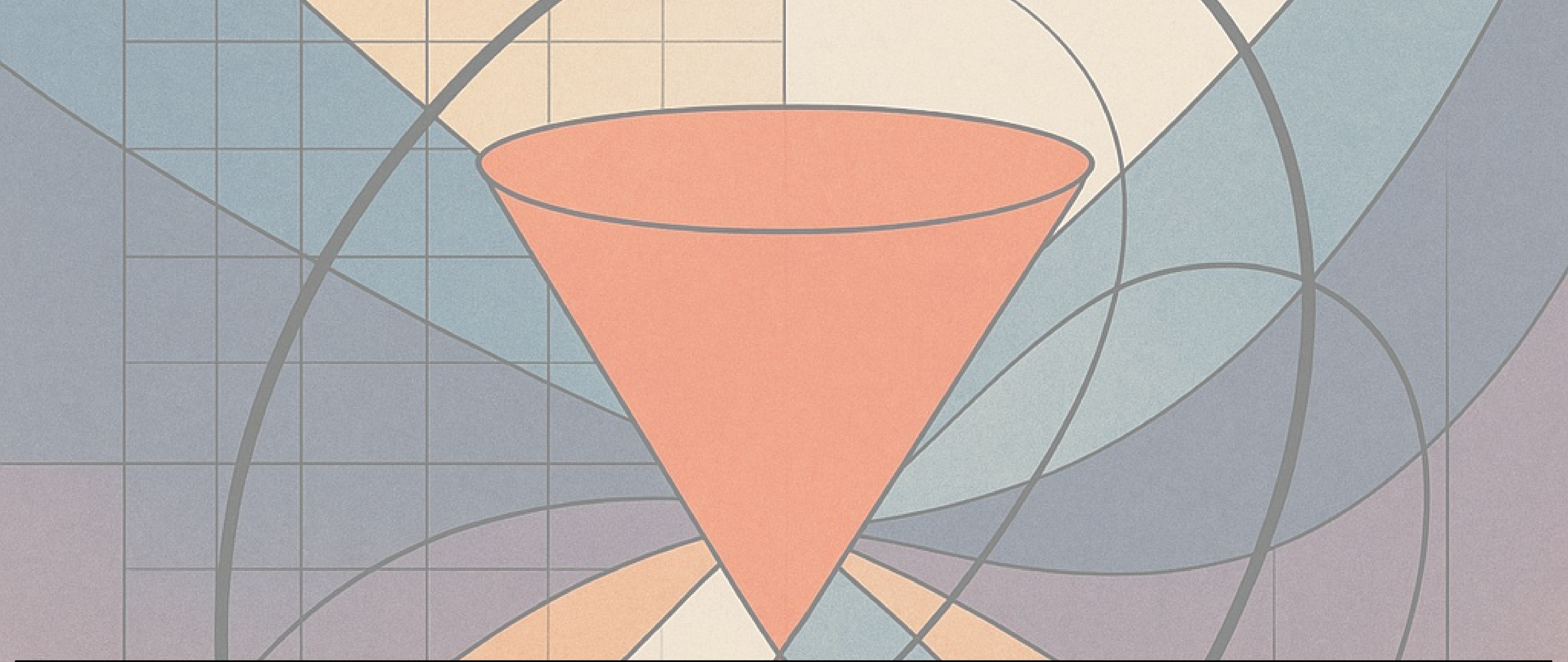

Multipoint Lightcone Bootstrap with a defect

Based on 2602.23428 with Lorenzo Bianchi & Andrea Mattiello

Interfaces & Symmetry, YITP-IAS Kyoto, 02/03/2025

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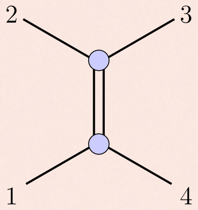
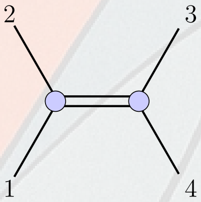
Motivation: Universality in CFTs at Large Spin

- **Conformal bootstrap:** consistency of different OPE expansions to constrain CFT data

$\langle\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle \implies \sum$



$= \sum$



$\implies \{\Delta_\phi, \lambda_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_3}, \dots\}$

- **Lightcone bootstrap:** [Alday, Caron-Huot, Fitzpatrick, Kaplan, Komargodski, Poland, Simmons-Duffin, Zhiboedov, …]
  - CFTs with a twist gap $\implies$ infinite families of GFF-like operators e.g. $[\mathcal{O}_1\mathcal{O}_2]_{m,J} = \text{“}\mathcal{O}_1^m \partial_{\mu_1} \dots \partial_{\mu_J} \mathcal{O}_2\text{”}$
  - Universal large-spin expansion for CFTs!
  - Generalizable to higher number of points [Bercini,Gonçalves,Vieira;2020][Antunes,Costa,Gonçalves,Vilas Boas;2021] [Kaviraj,Mann,**L.Q.**,Schomerus;2022][Harris,Kaviraj,Mann,**L.Q.**,Schomerus;2024]
  - Effective also in two-point functions with a defect! [Lemos,Liendo,Meineri,Sarkar;2017]...

This talk: extension to higher points with a defect!

Conformal Defects: Basics

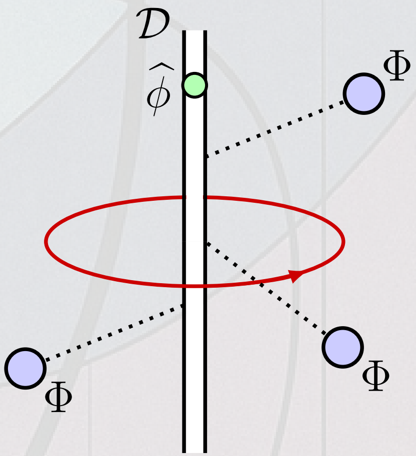
- Conformal defects break conformal symmetry to:

$$SO(d+1,1)_{\text{conf}} \longrightarrow SO(p+1,1)_{\|} \times SO(d-p)_{\perp}$$

- **Bulk fields:** $\Phi \Rightarrow \{\Delta, J\}$ **Defect fields:** $\hat{\phi} \Rightarrow \{\hat{\Delta}, j, s\}$

- One-point functions and bulk-to-defect functions:

$$\langle \Phi(x) \rangle_{\mathcal{D}} \equiv \frac{\langle \Phi(x)\mathcal{D}\rangle}{\langle \mathcal{D}\rangle} = \frac{a_\Phi}{r^{\Delta_\Phi}} \qquad \left\langle \Phi(x)\widehat{\phi}(y)\right\rangle_{\mathcal{D}} = \frac{b_{\Phi\widehat{\phi}}}{r^{\Delta-\widehat{\Delta}}(r^2+y^2)^{\widehat{\Delta}}}$$

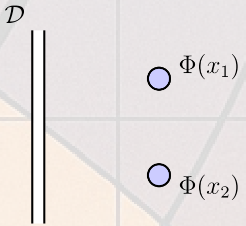


- $\exists$ defect OPE $\quad \Phi(x) \overset{\text{near } \mathcal{D}}{=} \sum_{\text{def. prim.}} b_{\Phi\widehat{\mathcal{O}}} |x_\perp|^{\widehat{\Delta}-\Delta} \Big(\widehat{\mathcal{O}}(0) + \underbrace{\text{def. desc.}}_{\text{all fixed}}\Big)$

$\Downarrow$

$\exists$ **crossing symmetry** for $\left\{a_{\mathcal{O}}, b_{\mathcal{O},\mathcal{O}'}, \widehat{\Delta}_{\widehat{\mathcal{O}}}, \widehat{\lambda}_{\widehat{\mathcal{O}}_1\widehat{\mathcal{O}}_2\widehat{\mathcal{O}}_3}\right\}$!

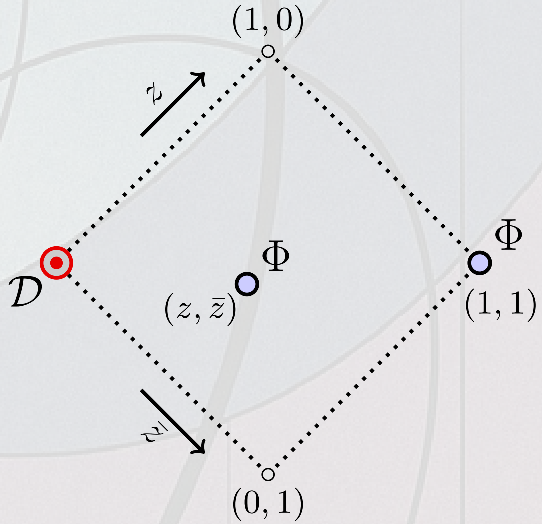
$\longrightarrow$ **Simplest example:** Bulk-Bulk two-point function $\longrightarrow$



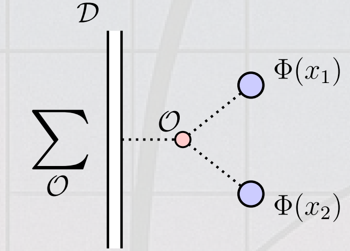
Review: The Bulk-Bulk OPE expansions

Correlator of two bulk scalar fields in presence of defect:

$$\langle \Phi(x_1)\Phi(x_2)\rangle_{\mathcal{D}} \simeq \mathcal{G}(z,\bar{z}) \qquad z, \bar{z} : \text{cross ratios}$$



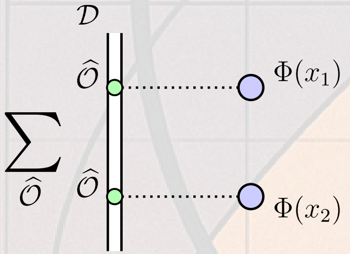
Conformal block expansion either via bulk OPE (Bulk channel):



$$\mathcal{G}(z,\bar{z}) = \sum_{\Delta,J} \lambda_{\Phi\Phi\mathcal{O}}\, a_{\mathcal{O}}\, G_{\Delta,J}(z,\bar{z})$$

Bulk-channel conformal blocks

Or with the Defect OPE (Defect channel):

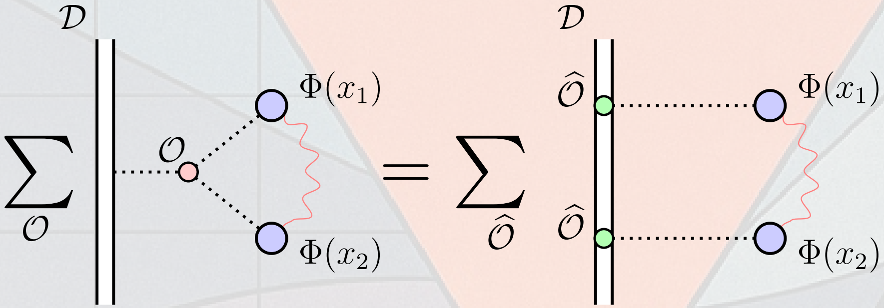


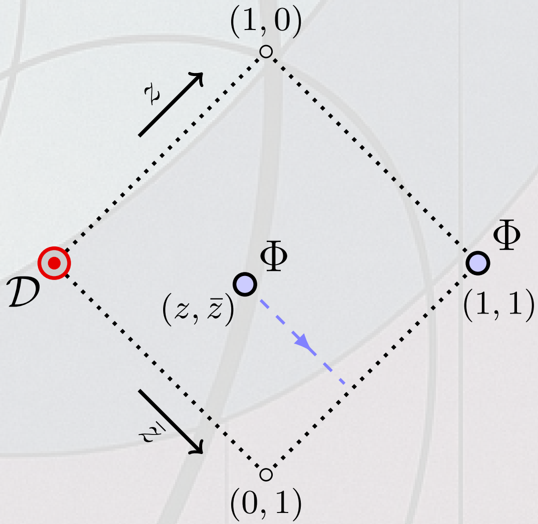
$$\mathcal{G}(z,\bar{z}) = \sum_{\widehat{\Delta},s} b^2_{\Phi\widehat{\mathcal{O}}}\, g_{\widehat{\Delta},s}(z,\bar{z})$$

Defect-channel conformal blocks

Review: The Bulk-Bulk Lightcone Bootstrap

LC bootstrap: consider crossing in the $x_{12}^2 \to 0$ limit $\implies \bar{z} \to 1$

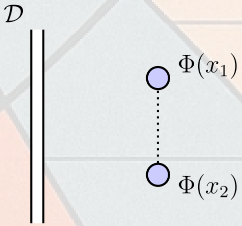




Bulk expansion organized by twist $2h = \Delta - J$

Universal lowest twist exchange: **identity** $\mathbb{1}$

$$\mathcal{G}(z,\bar{z}) \overset{\bar{z}\to 1}{\approx} G_{\mathbb{1}} \overset{\bar{z}\to 1}{\approx} \left(\frac{z}{(1-z)(1-\bar{z})}\right)^{\Delta_\phi}$$



What does this imply on the defect channel?

Review: The Bulk-Bulk Lightcone Bootstrap

Acting with the defect Casimir operators on the crossing equation, we understand the relevant **scaling of spins**:

On the bulk side:

$$\frac{\mathcal{C}^2_{\perp,1}(G_{\mathbb{1}})}{G_{\mathbb{1}}} \propto (1-\bar{z})^{-2}$$

On the defect side (block-by-block):

$$\frac{\mathcal{C}^2_{\perp,1}\big(g_{\widehat{\Delta},s}\big)}{g_{\widehat{\Delta},s}} = s(s+d-p-2)$$

LC limit in bulk channel $\Longrightarrow$ large transverse spin in defect channel $s \sim (1-\bar{z})^{-1}$

Defect conformal blocks in the relevant limit are:

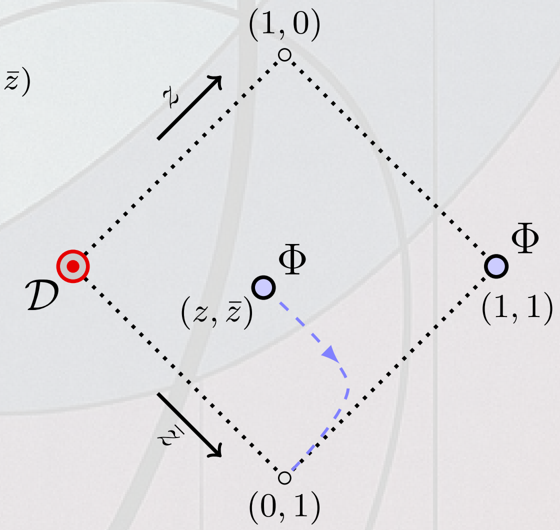
$$g_{\widehat{\Delta},s}(z,\bar{z}) \overset{\text{LC+LS}}{\approx} z^{\widehat{h}}(1-z)^{1-\frac{d}{2}}e^{-s(1-\bar{z})} \qquad \widehat{h} = \frac{\widehat{\Delta}-s}{2}$$

These have to be summed with bulk-to-defect coefficients over large values of transverse spin

Review: The Bulk-Bulk Lightcone Bootstrap

The **crossing equation** in the lightcone limit becomes

$$\left(\frac{z}{(1-z)(1-\bar{z})}\right)^{\Delta_\Phi} = \sum_{\widehat{h}} \int_0^\infty \mathrm{d}s\, b^2_{\Phi\widehat{\mathcal{O}}}\, z^{\widehat{h}} (1-z)^{1-\frac{d}{2}} e^{-s(1-\bar{z})}$$



Simplify analysis, expand around **double-lightcone limit** $\bar{z}-1 \ll z \ll 1$

Power matching in z : $\widehat{h} = h_\Phi + m \implies$ **transverse derivative ops.**

$$[\Phi]_{m,s} = \Box_\perp^m \partial_\perp^{i_1} \ldots \partial_\perp^{i_s} \Phi$$

Integral in s done via representation of Gamma $\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} \mathrm{d}t$

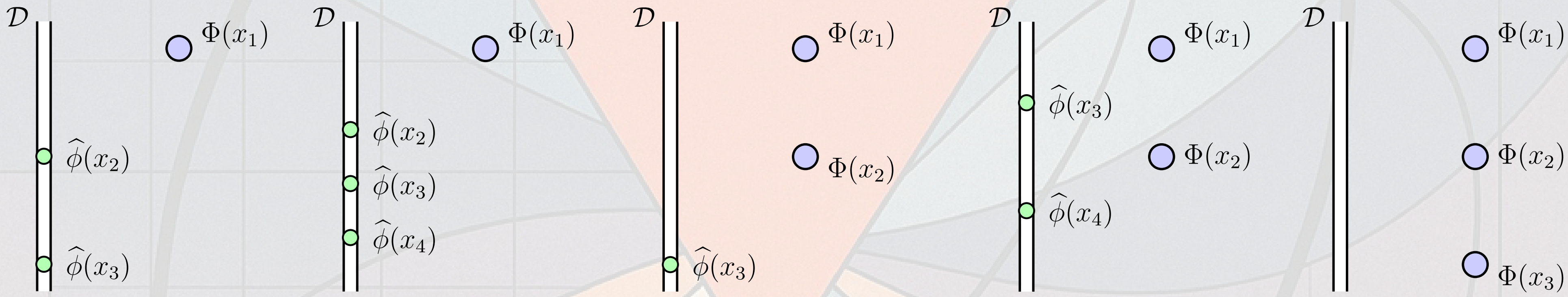
$$\Downarrow$$

Universal Asymptotics of $b^2_{\Phi[\Phi]_{m,s}} = s^{\Delta_\Phi - 1}\left(\frac{1}{\Gamma(\Delta_\Phi)}\binom{m-\frac{d}{2}+\Delta_\Phi}{m} + \mathrm{O}(s^{-\cdots})\right)$

To go beyond asymptotics:
Lorentzian inversion formula
[Lemos,Liendo,Meineri,Sarkar;2017]

Bootstrapping Higher-Point Defect Correlators

- How about higher number of points? Many possible avenues:



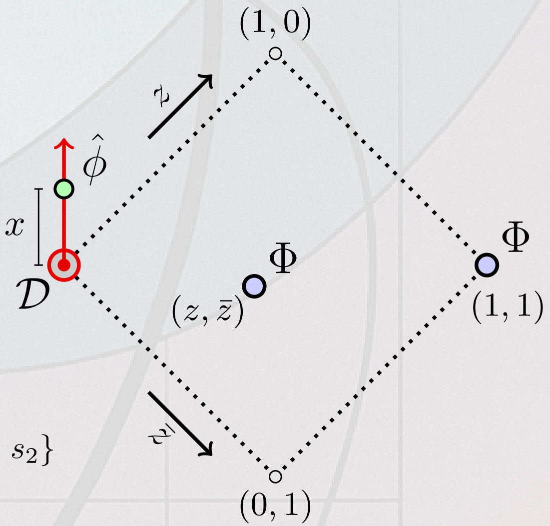
1 cross ratio	3 cross ratios	3 cross ratios	5 cross ratios	6 cross ratios
No crossing!	Only crossing inside the defect!	Simplest with bulk-defect crossing		

No Lorentzian inversion formula available yet $\Longrightarrow$ Lightcone bootstrap!

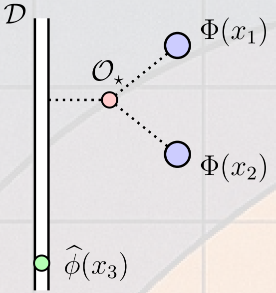
The Bulk-Bulk-Defect Correlator

- Consider two bulk fields and one defect field $\left\langle \Phi(x_1)\Phi(x_2)\widehat{\phi}(x_3)\right\rangle_{\mathcal{D}} \simeq \mathcal{G}(z,\bar{z},x)$

- Cross-ratios as in the bulk-bulk case, with addition of x s.t. $x_3 = \left(x, \vec{0}_{p-1}, \vec{0}_{d-p}\right)$

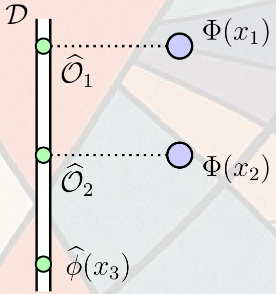


- Expansions like bulk-bulk case, with one "spectator" in the defect



Labels:
$\{\Delta_{\mathcal{O}_\star}, J_{\mathcal{O}_\star}, n\}$

Tensor structures



Labels:
$\{\Delta_{\widehat{\mathcal{O}}_1}, \Delta_{\widehat{\mathcal{O}}_2}, s_1 = s_2\}$

- No identity exchange allowed in the bulk channel $\longrightarrow$ Similar to five-point LC bootstrap: no GFF contribution!

[Kaviraj,Mann,**L.Q.**,Schomerus;2022]

The Bulk-Channel Analysis

Bulk-channel conformal blocks: lightcone OPE! [Ferrara,Grillo,Gatto;1972]

$$\Phi_1(x_1)\Phi_2(x_2) \approx \sum_{\mathcal{O}} (x_{12}^2)^{h_{\mathcal{O}}-h_1-h_2} \frac{\lambda_{\Phi_1\Phi_2\mathcal{O}}}{\mathrm{B}(\bar{h}_{\mathcal{O}}+h_{12},\bar{h}_{\mathcal{O}}+h_{21})} \int_0^1 \frac{\mathrm{d}t}{t^{1-\bar{h}_{\mathcal{O}}+h_{12}}} (1-t)^{h_{12}+\bar{h}_{\mathcal{O}}-1} \mathcal{O}\left(x_2 - t x_{21}, x_{21}\right) + \dots$$

They can be determined as Lauricella functions. Leading contribution in lightcone limit:

BBD Bulk-channel in LC limit

$$\left\langle \Phi(x_1)\Phi(x_2)\widehat{\phi}(x_3)\right\rangle_{\mathcal{D}} \overset{x_{12}^2\to 0}{\underset{\text{frame}}{\simeq}} \sum_{\mathcal{O},n} \lambda_{\Phi\Phi\mathcal{O}}\, b^{(n)}_{\mathcal{O}\hat{\Phi}} (1-\bar{z})^{h_\star - 2h_\Phi} z^{-\bar{h}+h_{\hat{\phi}}} (1-z)^{\bar{h}-2h_\Phi} (x^2-z)^n (x^2+z)^{-n-2h_{\hat{\phi}}}$$
$$\times F_D\left(\bar{h}, -2n, \bar{h}-\widehat{h}_{\widehat{\phi}}, 2n+2\widehat{h}_{\widehat{\phi}}, 2\bar{h}; \frac{z-1}{z-x^2}, \frac{z-1}{z}, \frac{z-1}{z+x^2}\right)$$

$$F_D(a,b_1,b_2,b_3,c;x_1,x_2,x_3) = \sum_{(n_1,n_2,n_3)\in\mathbb{N}_0^3} \frac{(a)_{n_1+n_2+n_3}\prod_{i=1}^3 (b_i)_{n_i}}{(c)_{n_1+n_2+n_3}\prod_{i=1}^3 n_i!} x_1^{n_1} x_2^{n_2} x_3^{n_3}$$

Assume: Theory with a twist gap and one leading twist field $\mathcal{O}_\star$ $\Longrightarrow$ defect side?

The Defect-Channel Analysis

As in the bulk-bulk case, Casimir operators tell the relevant scaling of transverse spin

$$\frac{\mathcal{C}^2_{\perp,1}(G_{\mathcal{O}_\star})}{G_{\mathcal{O}_\star}} \propto (1-\bar{z})^{-2}, \qquad \frac{\mathcal{C}^2_{\perp,1}\big(g_{\widehat{h}_1,\widehat{h}_2,s}\big)}{g_{\widehat{h}_1,\widehat{h}_2,s}} = \frac{\mathcal{C}^2_{\perp,2}\big(g_{\widehat{h}_1,\widehat{h}_2,s}\big)}{g_{\widehat{h}_1,\widehat{h}_2,s}} = s(s+d-p-2) \quad \Longrightarrow \quad \boxed{s \propto (1-\bar{z})^{-1}}$$

The defect blocks are known in full generality as Appell functions [Burić,Schomerus;2020] :

$$g_{\hat{h}_1,\hat{h}_2,s}(v_1,v_2,\cos\varphi) = \Omega\, v_1^{\hat{h}_2+\frac{1}{2}(s-\hat{h}_{\hat{\phi}})} v_2^{\hat{h}_1+\frac{1}{2}(s-\hat{h}_{\hat{\phi}})} F_4(\ldots,v_1,v_2)\, C_s^{\left(\frac{q-2}{2}\right)}(\cos\varphi)$$

We can work out the expression of blocks in the relevant scaling

$$g_{\hat{h}_1,\hat{h}_2,s}(z,\bar{z},x) \underset{\bar{z}\to 1}{\overset{s\to\infty}{\simeq}} 2^{-s} e^{-s(1-\bar{z})} x^{-2(\hat{h}_{\hat{\phi}}+\hat{h}_1-\hat{h}_2)} (1+x^2)^{\hat{h}_{\hat{\phi}}-2\hat{h}_1+2\hat{h}_2} z^{\hat{h}_2} (x^2+z)^{\hat{h}_{\hat{\phi}}+2\hat{h}_1-2\hat{h}_2} (1-z)^{1-\frac{d}{2}} (1+z)^{-\hat{h}_{\hat{\phi}}}$$

Integrating over spins with appropriate coefficients should reproduce the bulk channel

Problem: extremely hard! Three-variable Lauricella on the other side!

The Crossing Equation

Crossing equation in LC limit $\bar{z} \to 1$ (hard):

$$\sum_n \lambda_{\Phi\Phi\mathcal{O}_\star} b^{(n)}_{\mathcal{O}_\star \hat{\Phi}} (1-\bar{z})^{h_\star - 2h_\Phi} \frac{z^{-\bar{h}_\star + h_{\hat{\phi}}}}{(1-z)^{2h_\Phi - \bar{h}_\star}} \frac{(x^2-z)^n}{(x^2+z)^{n+2h_{\hat{\phi}}}} F_D\left(\bar{h}_\star, -2n, \bar{h}_\star - \hat{h}_{\hat{\phi}}, 2n + 2\hat{h}_{\hat{\phi}}, 2\bar{h}_\star; \frac{z-1}{z-x^2}, \frac{z-1}{z}, \frac{z-1}{z+x^2}\right)$$

$$= \sum_{\hat{h}_1, \hat{h}_2} \int_0^\infty \mathrm{d}s\, b_{\Phi\hat{\mathcal{O}}_1} b_{\Phi\hat{\mathcal{O}}_2} \lambda_{\hat{\mathcal{O}}_1\hat{\mathcal{O}}_2\hat{\phi}} 2^{-s} e^{-s(1-\bar{z})} x^{-2(\hat{h}_{\hat{\phi}} + \hat{h}_1 - \hat{h}_2)} (1+x^2)^{\hat{h}_{\hat{\phi}} - 2\hat{h}_1 + 2\hat{h}_2} z^{\hat{h}_2} (x^2+z)^{\hat{h}_{\hat{\phi}} + 2\hat{h}_1 - 2\hat{h}_2} (1-z)^{1-\frac{d}{2}} (1+z)^{-\hat{h}_{\hat{\phi}}}$$

The $\bar{z}$ dependence can be matched. It fixes the scaling in spin of the RHS terms:

$$(1-\bar{z})^{h_\star - 2h_\Phi} = \int_0^\infty \mathrm{d}s\, b_{\Phi\hat{\mathcal{O}}_1} b_{\Phi\hat{\mathcal{O}}_2} \lambda_{\hat{\mathcal{O}}_1\hat{\mathcal{O}}_2\hat{\phi}} 2^{-s} e^{-s(1-\bar{z})} \implies \boxed{b_{\Phi\hat{\mathcal{O}}_1} b_{\Phi\hat{\mathcal{O}}_2} \lambda_{\hat{\mathcal{O}}_1\hat{\mathcal{O}}_2\hat{\phi}} \simeq C_{\hat{\mathcal{O}}_1\hat{\mathcal{O}}_2} \frac{2^s s^{2h_\Phi - h_\star - 1}}{\Gamma(2h_\Phi - h_\star)}}$$

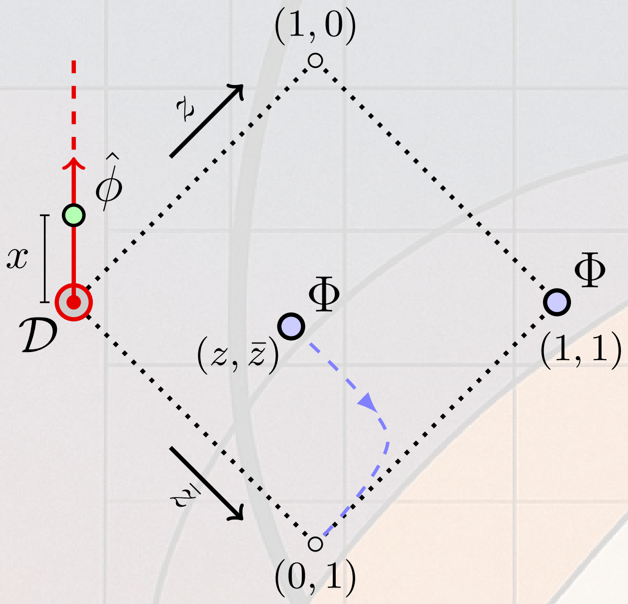
We now have to understand which twists to sum over, and what are the $C_{\hat{\mathcal{O}}_1\hat{\mathcal{O}}_2}$ coefficients

We use two different simplifying limits: $z \to 0$ or $x \to \infty$

Simplifying limit 1: double-lightcone limit

Consider the double-lightcone limit $1 - \bar{z} \ll z \ll 1$, crossing becomes:

$$\sum_n \lambda_{\Phi\Phi\mathcal{O}_\star} b^{(n)}_{\mathcal{O}_\star\hat{\phi}} z^{h_\Phi} x^{-2\hat{h}_{\hat{\phi}}} F_1\left(\hat{h}_{\hat{\phi}}, -2n, 2(\hat{h}_{\hat{\phi}}+n), \bar{h}_\star + \hat{h}_{\hat{\phi}}; x^{-2}, -x^{-2}\right) = \sum_{\hat{h}_1,\hat{h}_2} C_{\widehat{\mathcal{O}}_1\widehat{\mathcal{O}}_2} z^{\hat{h}_2} x^{2\hat{h}_2 - 2\hat{h}_1} \left(1 + \frac{1}{x^2}\right)^{2\hat{h}_2 - 2\hat{h}_1}$$



Only contribution from specific twists!

$$\hat{h}_2 = h_\Phi \quad \Longrightarrow \quad \widehat{\mathcal{O}}_2 = [\Phi]_{0,s} = \Box^0_\perp \partial^{i_1}_\perp \dots \partial^{i_s}_\perp \Phi \,,$$

$$\hat{h}_1 = h_\Phi + \hat{h}_{\hat{\phi}} + m \quad \Longrightarrow \quad \widehat{\mathcal{O}}_1 = [\hat{\phi}\Phi]_{m,0,s} = \hat{\phi}\Box^m_\parallel \Box^0_\perp \partial^{i_1}_\perp \dots \partial^{i_s}_\perp \Phi + (\dots) \,,$$

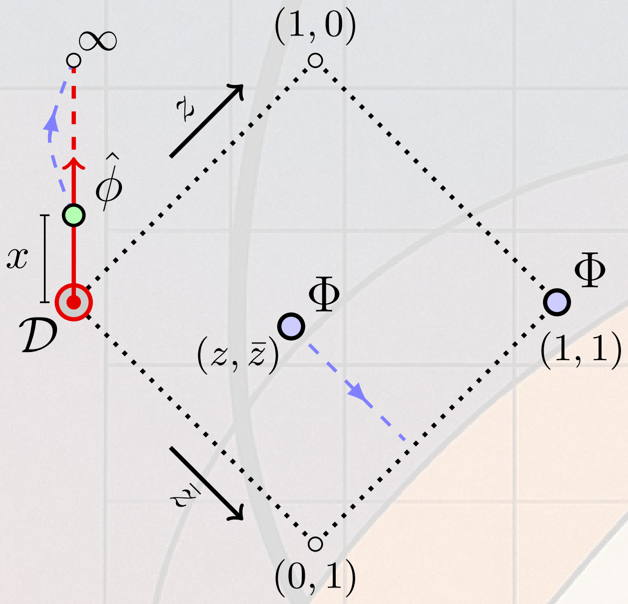
Fix bulk-to-defect coefficients of **double-twist operators** in terms of known data:

$$C_{[\hat{\phi}\Phi]_{m,0,s}[\Phi]_{0,s}} = \lambda_{\Phi\Phi\mathcal{O}_\star} \sum_{j=0}^{m} \frac{\hat{h}_{\hat{\phi}} + j}{\hat{h}_{\hat{\phi}} + m} \binom{2\hat{h}_{\hat{\phi}} + 2m}{m - j} \frac{(-1)^j \, \Gamma(2\bar{h}_\star)\Gamma(\hat{h}_{\hat{\phi}} + j)}{j! \, \Gamma(\bar{h}_\star)\Gamma(\bar{h}_\star + \hat{h}_{\hat{\phi}} + j)} \times \sum_{n=0}^{J/2} b^{(n)}_{\mathcal{O}_\star\hat{\phi}} \left(2\hat{h}_{\hat{\phi}} + 2n\right)_j {}_2F_1(-j, -2n, -2\hat{h}_{\hat{\phi}} - j - 2n + 1; -1)$$

Simplifying limit 2: $\hat{\phi}$ at large separation

Consider now the large separation limit for the defect operator $1-\bar{z} \ll x^{-1} \ll 1$:

$$\frac{\lambda_{\Phi\Phi\mathcal{O}_\star} z^{h_\Phi} \sum_n b^{(n)}_{\mathcal{O}_\star \hat{\phi}}}{(1-z)^{-\bar{h}_\star+2h_\Phi-\frac{d}{2}+1}} \frac{\Gamma(2\bar{h}_\star)}{\Gamma(\bar{h}_\star)} \left[\frac{\Gamma(\hat{h}_{\hat{\phi}})\,{}_2F_1\left(\bar{h}_\star, \bar{h}_\star - \hat{h}_{\hat{\phi}}, 1-\hat{h}_{\hat{\phi}}; z\right)}{\Gamma\left(\bar{h}_\star + \hat{h}_{\hat{\phi}}\right)} + \frac{\Gamma(-\hat{h}_{\hat{\phi}}) z^{\hat{h}_{\hat{\phi}}}\,{}_2F_1\left(\bar{h}_\star, \bar{h}_\star + \hat{h}_{\hat{\phi}}, 1+\hat{h}_{\hat{\phi}}; z\right)}{\Gamma\left(\bar{h}_\star - \hat{h}_{\hat{\phi}}\right)} \right] = \sum_{\hat{h}_2} C_{\hat{\mathcal{O}}_1 \hat{\mathcal{O}}_2} z^{\hat{h}_2}$$



Two different families of contributions!

$$\text{(I)} \implies \begin{array}{lll} \hat{h}_2 = h_\Phi + m & \implies & \hat{\mathcal{O}}_2 = [\Phi]_{m,s} = \Box_\perp^m \partial_\perp^{i_1} \dots \partial_\perp^{i_s} \Phi \,, \\ \hat{h}_1 = h_\Phi + \hat{h}_{\hat{\phi}} + m & \implies & \hat{\mathcal{O}}_1 = [\hat{\phi}\Phi]_{0,m,s} = \hat{\phi} \Box_\parallel^0 \Box_\perp^m \partial_\perp^{i_1} \dots \partial_\perp^{i_s} \Phi \,, \end{array}$$

$$\text{(II)} \implies \begin{array}{lll} \hat{h}_2 = h_\Phi + \hat{h}_{\hat{\phi}} + m & \implies & \hat{\mathcal{O}}_2 = [\hat{\phi}\Phi]_{0,m,s} = \hat{\phi} \Box_\parallel^0 \Box_\perp^m \partial_\perp^{i_1} \dots \partial_\perp^{i_s} \Phi \,, \\ \hat{h}_1 = h_\Phi + 2\hat{h}_{\hat{\phi}} + m & \implies & \hat{\mathcal{O}}_1 = [\hat{\phi}\hat{\phi}\Phi]_{\vec{0},m,s} = \hat{\phi}^2 \Box_\parallel^0 \Box_\perp^m \partial_\perp^{i_1} \dots \partial_\perp^{i_s} \Phi \,. \end{array}$$

we can see bulk-to-defect data of **triple-twist operators**!

Simplifying limit 2: $\hat{\phi}$ at large separation

The results in the $1-\bar{z} \ll x^{-1} \ll 1$ limit for the two families are:

First family:

$$C_{[\hat{\phi}\Phi]_{0,m,s}[\Phi]_{m,s}} = \left(\lambda_{\Phi\Phi\mathcal{O}_\star}\sum_n b^{(n)}_{\mathcal{O}_\star\hat{\phi}}\right)\frac{\Gamma(2\bar{h}_\star)}{\Gamma(\bar{h}_\star)}\frac{\Gamma\left(\hat{h}_{\hat{\phi}}\right)}{\Gamma\left(\bar{h}_\star+\hat{h}_{\hat{\phi}}\right)}\sum_{k=0}^{m}\frac{(-1)^k\left(\bar{h}_\star-2h_\Phi+\frac{d}{2}-k\right)_k\left(\bar{h}_\star\right)_{m-k}\left(\bar{h}_\star-\hat{h}_{\hat{\phi}}\right)_{m-k}}{k!(m-k)!\left(1-\hat{h}_{\hat{\phi}}\right)_{m-k}}$$

Second family:

$$C_{[\hat{\phi}\hat{\phi}\Phi]_{\vec{0},m,s}[\hat{\phi}\Phi]_{0,m,s}} = \left(\lambda_{\Phi\Phi\mathcal{O}_\star}\sum_n b^{(n)}_{\mathcal{O}_\star\hat{\phi}}\right)\frac{\Gamma(2\bar{h}_\star)}{\Gamma(\bar{h}_\star)}\frac{\Gamma\left(-\hat{h}_{\hat{\phi}}\right)}{\Gamma\left(\bar{h}_\star-\hat{h}_{\hat{\phi}}\right)}\sum_{k=0}^{m}\frac{(-1)^k\left(\bar{h}_\star-2h_\Phi+\frac{d}{2}-k\right)_k\left(\bar{h}_\star\right)_{m-k}\left(\bar{h}_\star+\hat{h}_{\hat{\phi}}\right)_{m-k}}{k!(m-k)!\left(1+\hat{h}_{\hat{\phi}}\right)_{m-k}}$$

these determine the asymptotics of bulk-to-defect coefficients via $b_{\Phi\hat{\mathcal{O}}_1}b_{\Phi\hat{\mathcal{O}}_2}\lambda_{\hat{\mathcal{O}}_1\hat{\mathcal{O}}_2\hat{\phi}} \simeq C_{\hat{\mathcal{O}}_1\hat{\mathcal{O}}_2}\frac{2^s s^{2h_\Phi-h_\star-1}}{\Gamma(2h_\Phi-h_\star)}$

Example in N=4 SYM

Example: correlator of two 20’ operators and the tilt operator in $\mathcal{N}$=4 SYM with a line defect

$$\left\langle \mathcal{O}_{20'}(x_1, Y_1)\mathcal{O}_{20'}(x_2, Y_2)\hat{\phi}(x_\parallel, y) \right\rangle$$

The lowest-twist contribution in $\mathcal{O}_{20'} \times \mathcal{O}_{20'}$ comes from the stress tensor supermultiplet.

Focusing on the superprimary, we get some simple closed-form expressions for both families:

First family:

$$C_{[\hat{\phi}\mathcal{O}_{20'}]_{m_1,m_2,s}[\mathcal{O}_{20'}]_{m_2,s}} = 2\lambda_{\mathcal{O}_{20'}} b_{\mathcal{O}_{20'}\hat{\phi}} \frac{(-1)^{m_1}}{2m_1+1}\,, \quad m_1 \leq m_2$$

Second family:

$$C_{[\hat{\phi}\hat{\phi}\mathcal{O}_{20'}]_{m_1,m_2,s}[\hat{\phi}\mathcal{O}_{20'}]_{0,m_2,s}} = 2\lambda_{\mathcal{O}_{20'}} b_{\mathcal{O}_{20'}\hat{\phi}} \frac{(-1)^{m_1+1}}{2m_1+1}\,, \quad m_1 \leq m_2$$

While for $m_1 > m_2$ the coefficients are more complicated but can be expressed in closed forms for small m_2

Conclusions and outlook

- Lightcone bootstrap with defects is **effective at higher points**!
- No need of general blocks, just in special limits!
- Coupling of leading twist $\mathcal{O}_\star$ in $\Phi \times \Phi$ with $\hat{\phi} \implies$ new constraints on CFT data!
- Correction to GFF for coeffs of **double-twists** $b_{\Phi[\hat{\phi}\Phi]_{m_1,m_2,s}}$ and **triple-twists** $b_{\Phi[\hat{\phi}\hat{\phi}\Phi]_{m_1,m_2,s}}$
- Higher(-er) points? BBB correlators: cyclicity and parallel $\leftrightarrow$ transverse symmetry ?
- Simpler kinematics with defects $\implies$ new bulk insights ?
- **Lorentzian inversion formula** and **dispersion relations** for BBD correlators ?