Long Heavy Objects in CFTs: Defects meet Effective Field Theory

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Motivation

Energy

Defect CFT: Captures the low-energy interaction of massive degrees of freedom with gapless excitations.

[Wilson; 1975][Cardy; 1984][Billò, Goncalves, Lauria, Meineri; 2016]

Defect CFT

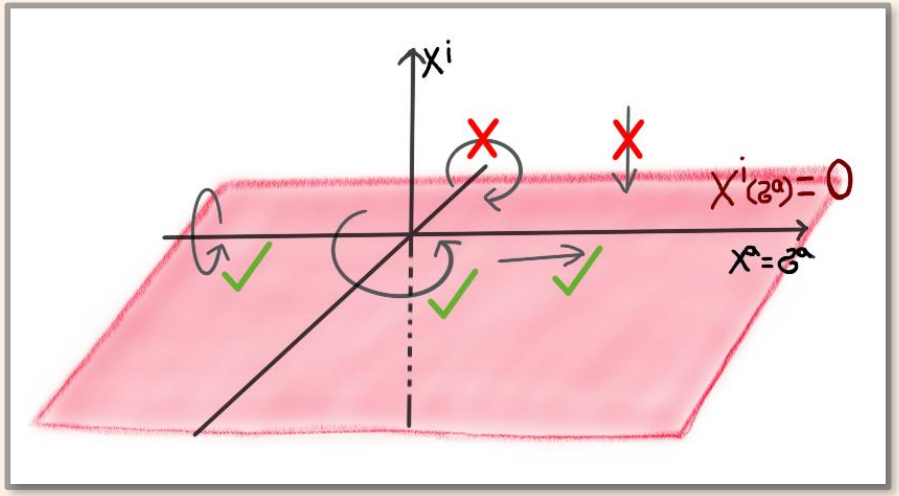
- p-dim. flat defect in d-dim. CFT explicitly breaks part of the conformal symmetry

- Breaking of translational invariance induces the existence of the displacement operator

$$\partial_\mu T^{\mu\nu}\left(\sigma^a, x^j\right) = -\delta^{\nu i}\,\delta^{d-p}\left(x^j\right)\mathrm{D}_i\left(\sigma^a\right)$$

→ fixes $\Delta_{\mathrm{D}} = p+1$

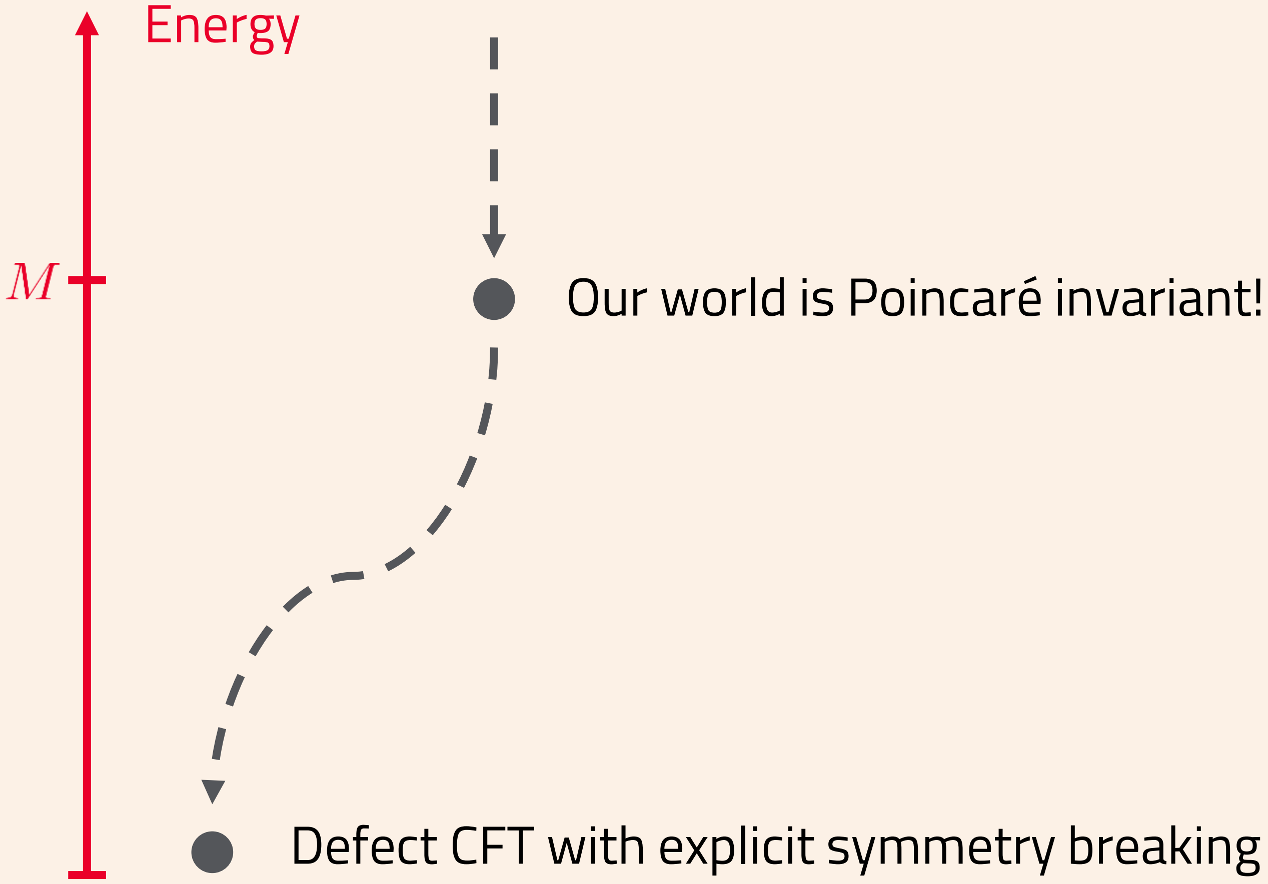
→ fixes $\langle \mathrm{D}_i(\sigma)\mathrm{D}_i(\sigma')\rangle = \dfrac{\delta_{ij}c_D}{|\sigma-\sigma'|^{2p+2}}$



- D_i determines the variation of the path integral under an infinitesimal conformal transformation that does not preserve the position of the defect

$$\delta_\xi S_{\mathrm{DCFT}} = -\int d^p\sigma\,\mathrm{D}_i\xi^i\,, \qquad \text{for } \xi^\mu = (\xi^a, \xi^i) \text{ conf. killing vector}$$

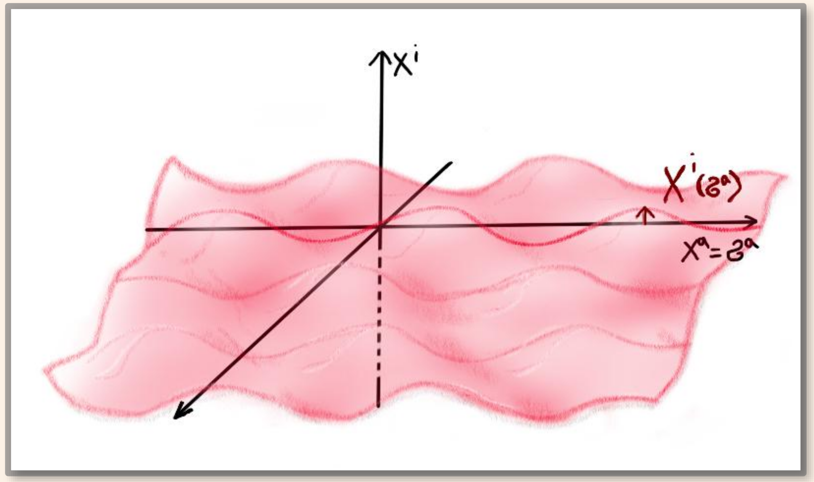
Motivation


Energy
M
Our world is Poincaré invariant!
Defect CFT with explicit symmetry breaking

QFT with Spontaneous Symmetry Breaking

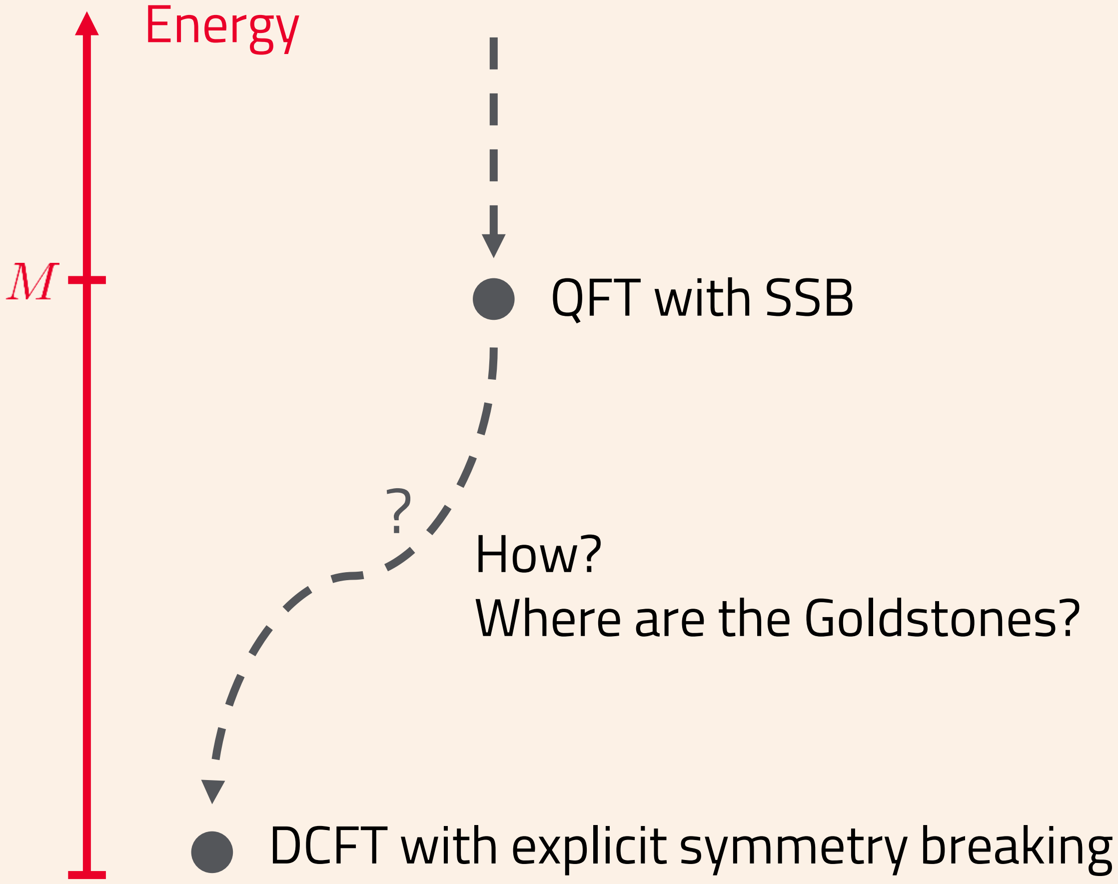
- QFT are by construction Poincaré invariant, any symmetry breaking must be spontaneous.
- SSB leads to the existence of massless Goldstone bosons.
  [Goldstone, Salam, Weinberg; 1962]
- A long, heavy object that spontaneously breaks the Poincaré symmetry supports the propagation of GB on their worldvolume.
  [Luscher; 1981] [Luscher, Symanzik, Weisz; 1991] [Polchinski, Strominger;1991]
- Leading term in the action of the GB is universal and given by the Nambu-Goto action
  [Nambu; 1974] [Goto; 1971]

$$S_{\text{Goldstones}} = M^p \int_{\mathcal{D}} d^p\sigma \sqrt{\det\left(\partial_a X^\mu(\sigma)\partial_b X_\mu(\sigma)\right)}$$
$$\approx M^p \int d^p\sigma \left(1 + \frac{1}{2}\partial_a X^i \partial^a X_i + \dots\right)$$



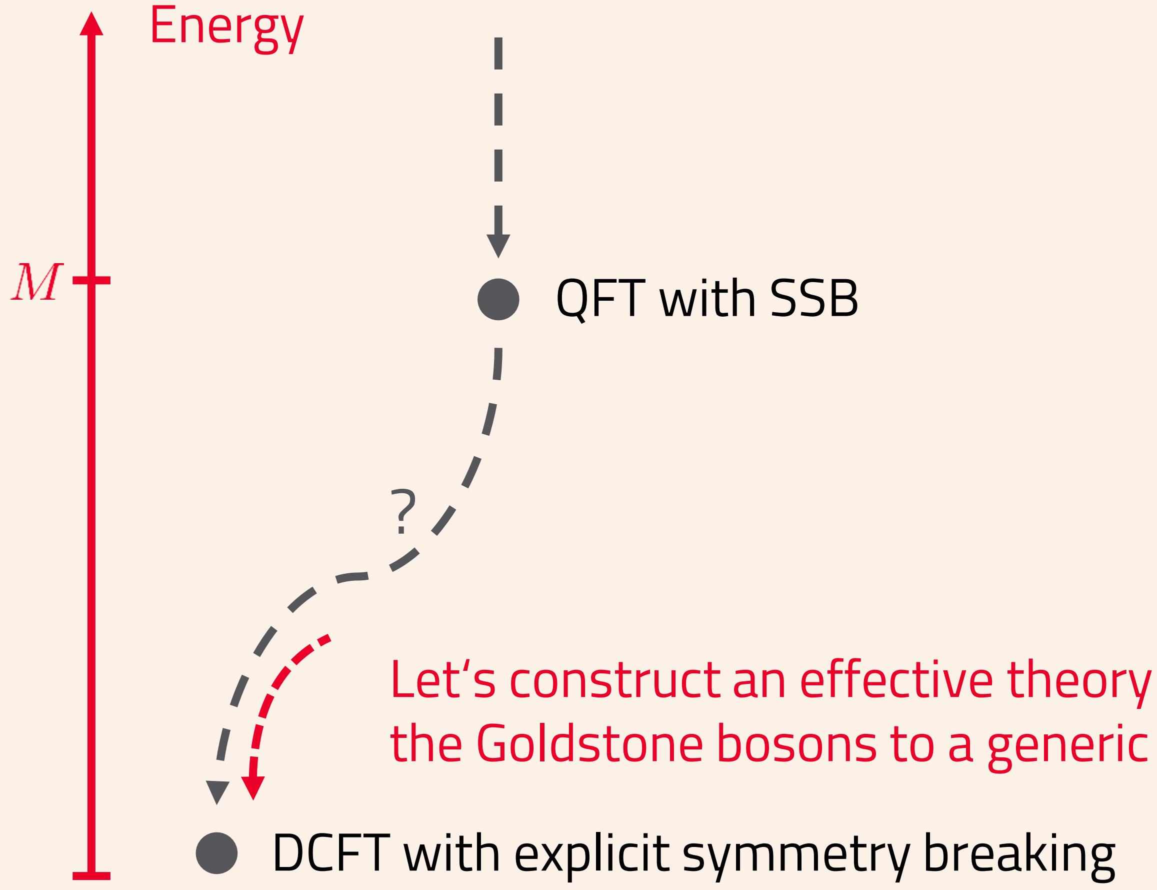
- Dependence on the system enters only through M.

Motivation



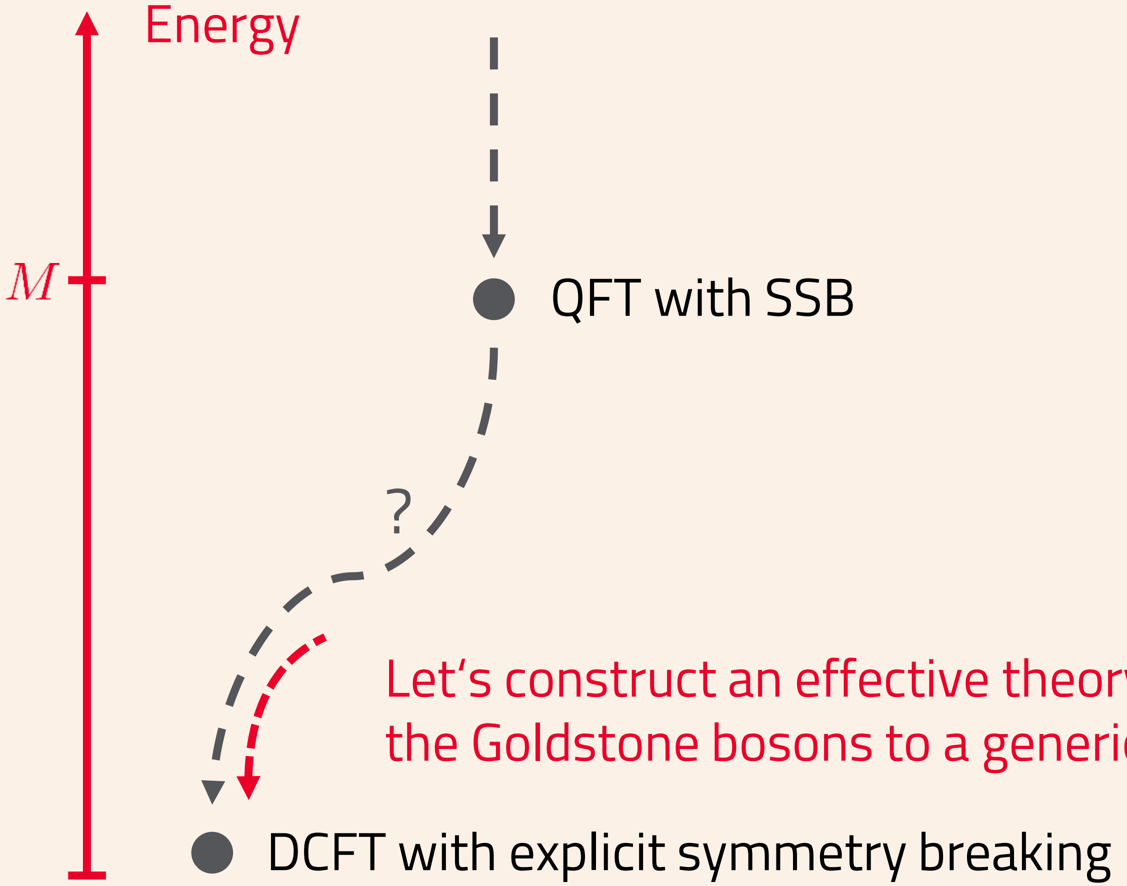
Energy
M
QFT with SSB
?
How?
Where are the Goldstones?
DCFT with explicit symmetry breaking

Motivation



Let's construct an effective theory that describes the interaction of the Goldstone bosons to a generic DCFT!

Motivation

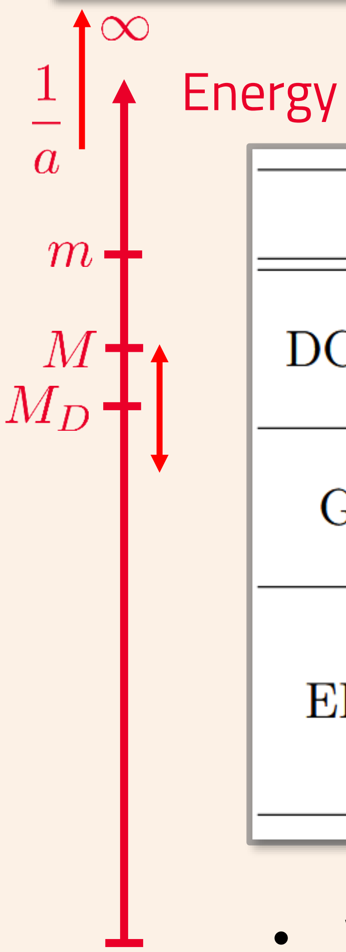


Related Work:

[Horn, Nicolis, Penco; 2015] [Goon, Hinterbichler, Trodden; 2011] [de Rahm, Tolley; 2011]....
[Metlitski; 2022] [Krishnan, Metlitski; 2024]

Let's construct an effective theory that describes the interaction of the Goldstone bosons to a generic DCFT!

EFT-Basics



	Degrees of Freedom	Scale	Symmetry
DCFT	$\hat{O}_{\Delta,j,s}$	$\frac{1}{a}$: UV scale	$SO(p+1,1) \times SO(d-p)$ is realized linearly.
GB	X^i	M m: UV scale	Coset $ISO(d)/\left(ISO(p) \times SO(d-p)\right)$ is realized non-linearly.
EFT		$M_D = M c_D^{-\frac{1}{p}}$	Restore Poincaré symmetry! Scale invariance is only reached in the IR and not restored.

- Validity of the EFT requires $\frac{1}{a}, m \gg M_D$

Leading Order EFT

- Recall: $\delta_\xi S_{\rm DCFT} = -\int d^p\sigma \, {\rm D}_i \xi^i \, ,$ for $\xi^\mu = (\xi^a, \xi^i)$ conf. killing vector
- Transverse translations: $P_j X^i = \delta^i_j$

  Rotations changing the location of the defect: $L_{ak} X^i = -\delta^i_k \sigma_a - X_k \partial_a X^i$

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  → Restoration of Poincaré symmetry mandates $S_{\text{Coupling}} = \int d^p\sigma \, \mathrm{D}_i X^i$
- Leading order EFT:

$$\begin{aligned} S_{\text{EFT}} &= S_{\text{DCFT}} + S_{\text{Goldstones}} + S_{\text{Coupling}} \\ &= S_{\text{DCFT}} + M^p \int d^p\sigma + M^p \int d^p\sigma \, \frac{1}{2} \partial^a X^j \partial_a X_j + \int d^p\sigma \, \mathrm{D}_i X^i + \mathcal{O}\left(X^3\right) \end{aligned}$$

Leading Order EFT

- Leading order EFT:

$$S_{\text{EFT}} = S_{\text{DCFT}} + S_{\text{Goldstones}} + S_{\text{Coupling}}$$

$$= S_{\text{DCFT}} + M^p \int d^p\sigma + M^p \int d^p\sigma \, \frac{1}{2}\partial^a X^j \partial_a X_j + \int d^p\sigma \, \mathrm{D}_i X^i + \mathcal{O}\left(X^3\right)$$

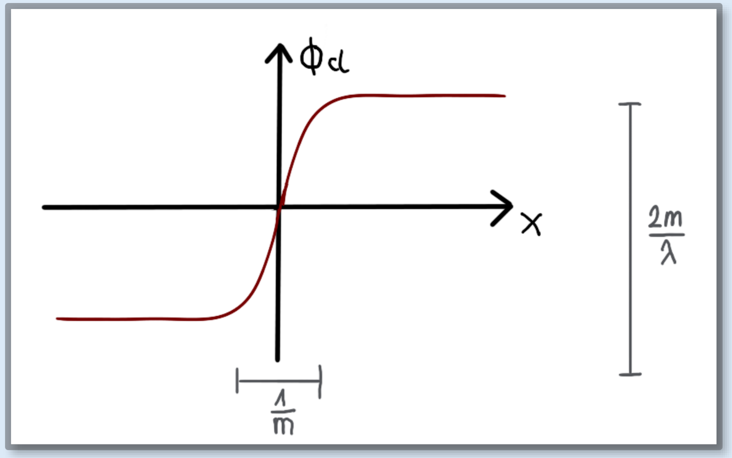
- Define canonically normalized field $\mathcal{X}^i = M^{\frac{p}{2}} X^i$ and normalized operator $\hat{\mathrm{D}}_i = c_D^{-\frac{1}{2}} \mathrm{D}_i$

$$S_{\text{EFT}} = S_{\text{DCFT}} + M^p \int d^p\sigma + \int d^p\sigma \, \frac{1}{2}\partial^a \mathcal{X}^j \partial_a \mathcal{X}_j + \left(c_D^{\frac{1}{p}} M^{-1}\right)^{\frac{p}{2}} \int d^p\sigma \, \hat{\mathrm{D}}_i \mathcal{X}^i + \mathcal{O}\left(\frac{1}{M^p}\right)$$

- Coupling is irrelevant.
- The Goldstones decouple in the IR!

Example: Soliton in 2-dimensional ϕ^4-Theory

$$S = \int dt dx \, \frac{1}{2}(\partial_\mu \phi)^2 + \frac{\lambda}{4}\left(\phi^2 - \frac{m^2}{\lambda}\right)^2 + \frac{1}{2}(\partial_\mu \psi)^2 - \frac{u}{2}\frac{\sqrt{\lambda}}{m}(\partial_\mu \psi)^2 \phi$$



- Classical kink solution $\phi_{\text{cl}}(x)$ is localized at $x_0 = 0$
  → SSB and existence of a Goldstone boson
- The kink has finite static energy setting the scale $M = E[\phi_{\text{cl}}]$
- m sets the scale at which massive modes start propagating on the defect.
- $\frac{1}{a} = \frac{m^2}{u\sqrt{\lambda}}$ sets the scale at which ϕ gives loop corrections to ψ observables

Example: Soliton in 2-dimensional ϕ^4-Theory

$$S = \int dt dx \, \frac{1}{2}(\partial_\mu \phi)^2 + \frac{\lambda}{4}\left(\phi^2 - \frac{m^2}{\lambda}\right)^2 + \frac{1}{2}(\partial_\mu \psi)^2 - \frac{u}{2}\frac{\sqrt{\lambda}}{m}(\partial_\mu \psi)^2 \phi$$

- Is there a non-trivial DCFT in the IR? What is it?
  → Shift symmetry protects ψ from getting a mass
  → Taking the zero energy limit $\phi(t,x) \approx \phi_{\rm cl}(x) \xrightarrow{m|x| \gg 1} \frac{m}{\sqrt{\lambda}} \mathrm{sign}(x)$ gives DCFT of free boson with compact boson interface [Bachas,…; 2002]

$$S_{\rm DCFT}[\varphi] = \frac{1+u}{2}\int_{x<0} d^2x \, (\partial_\mu \psi)^2 + \frac{1-u}{2}\int_{x>0} d^2x \, (\partial_\mu \psi)^2$$

$$\mathrm{D} = \left[\frac{1+u}{2} \, (\partial_\mu \psi)^2 \Big|_{0-} - \frac{1-u}{2} \, (\partial_\mu \psi)^2 \Big|_{0+} \right]$$

- What is c_D?
  → $c_D = u^2$

Example: Soliton in 2-dimensional ϕ^4-Theory

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- How to obtain the EFT with the Goldstone bosons?

  → Taking $\phi(t,x) \approx \phi_{\mathrm{cl}}(x - X_0(t))$ reproduces the EFT

$$S_{\mathrm{EFT}} = S_{\mathrm{DCFT}} + M \int dt + \int dt \, \frac{1}{2} M \left(\partial_t X_0(t)\right)^2$$

$$+ \int dt \left[\frac{1+u}{2} \left(\partial_\mu \psi\right)^2\Big|_{0^-} - \frac{1-u}{2}\left(\partial_\mu \psi\right)^2\Big|_{0^+}\right] X_0(t) + \dots$$

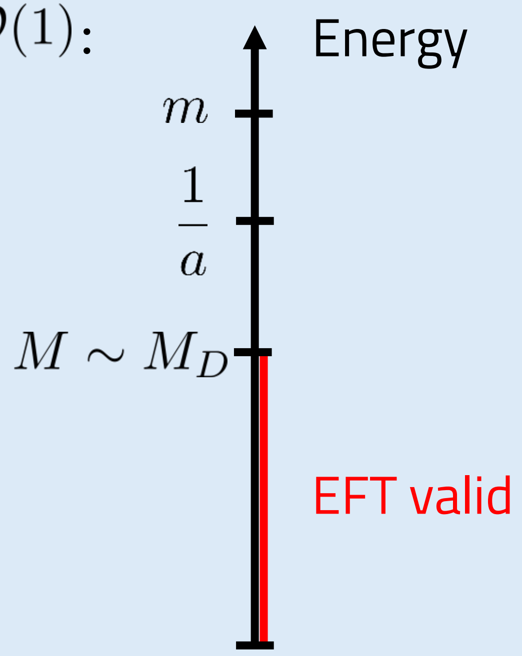
$$\overset{\mathcal{X}=\sqrt{M}X_0\,,\,\hat{\mathrm{D}}=c_D^{-\frac{1}{2}}\mathrm{D}}{=} S_{\mathrm{DCFT}} + M \int dt + \int dt \, \frac{1}{2}\left(\partial_t \mathcal{X}(t)\right)^2 + \frac{c_D}{\sqrt{M}} \int dt \, \hat{\mathrm{D}}_i \mathcal{X}^i + \dots$$

Example: Soliton in 2-dimensional ϕ^4-Theory

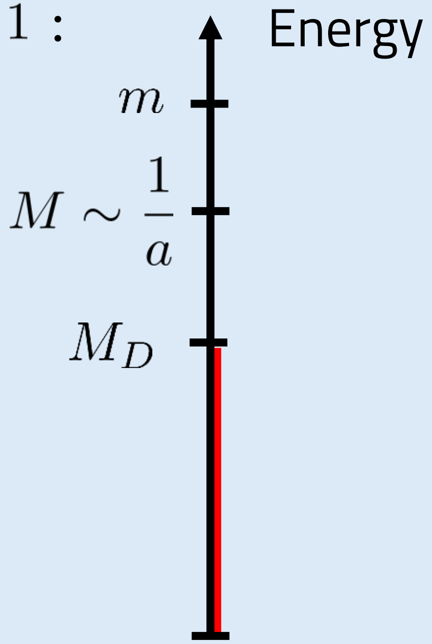
$$S_{\mathrm{EFT}} = S_{\mathrm{DCFT}} + M\int dt + \int dt\,\frac{1}{2}\left(\partial_t \mathcal{X}(t)\right)^2 + \frac{c_D}{\sqrt{M}}\int dt\,\hat{\mathrm{D}}_i \mathcal{X}^i + \ldots$$

- How can $\frac{1}{a} \gg M_D \Leftrightarrow \frac{m}{u\sqrt{\lambda}} \ll 1$ be satisfied?

→ $\frac{m}{\sqrt{\lambda}} \ll 1$ and $u = \mathcal{O}(1)$:



or $m \sim \sqrt{\lambda}$ and $\frac{1}{u} \ll 1$:



Are We Done?

$$S_{\mathrm{EFT}} = S_{\mathrm{DCFT}} + M^p \int d^p\sigma + M^p \int d^p\sigma \, \frac{1}{2} \partial^a X^j \partial_a X_j + \int d^p\sigma \, \mathrm{D}_i X^i + \mathcal{O}\left(X^3\right)$$

- Is the Poincaré invariance fully restored?
- If not, what is missing?

Are We Done?

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- Is the Poincaré invariance fully restored? -No!
- If not, what is missing? - We don't know...

Are We Done?

$$S_{\rm EFT} = S_{\rm DCFT} + M^p \int d^p\sigma + M^p \int d^p\sigma \, \frac{1}{2} \partial^a X^j \partial_a X_j + \int d^p\sigma \, {\rm D}_i X^i + \mathcal{O}\left(X^3\right)$$

- That depends on the transformation laws of the displacement operator under the non-linearly realized symmetry, which we don't know

$$\delta_{P^j} S_{\rm EFT} = 0 + \int d^p\sigma \, (P^j {\rm D}_i) X^i$$

$$\delta_{L^{ak}} S_{\rm EFT} = 0 - \int d^p\sigma {\rm D}_i X^k \partial^a X^i + \int d^p\sigma \, (P^{ak} {\rm D}_i) X^i$$

Are We Done?

1st Constraint:

$$[P^j, P^k]\,\mathrm{D}_i = 0$$

$$[P^a, P^b]\,\mathrm{D}_i = 0$$

$$[P^j, L^{ak}]\,\mathrm{D}_i = \delta^{kj} P^a \mathrm{D}_i$$

$$[P^b, L^{ak}]\,\mathrm{D}_i = \delta^{ab} P^k \mathrm{D}_i$$

$$[L^{ak}, L^{bj}]\,\mathrm{D}_i = \delta^{ab} L^{kj} \mathrm{D}_i + \delta^{kj} L^{ab} \mathrm{D}_i$$

2nd Constraint:

$$\delta_{P^j} S_{\mathrm{EFT}} \overset{!}{=} 0 + \text{total derivative}$$

$$\delta_{L^{ak}} S_{\mathrm{EFT}} \overset{!}{=} 0 + \text{total derivative}$$

Are We Done?

1st Constraint:

$$[P^j, P^k]\,\mathrm{D}_i = 0$$
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$$\delta_{P^j} S_{\mathrm{EFT}} \overset{!}{=} 0 + \text{total derivative}$$
$$\delta_{L^{ak}} S_{\mathrm{EFT}} \overset{!}{=} 0 + \text{total derivative}$$

No way with only the leading order coupling term!
We need more terms in the EFT.

Are We Done?

1st Constraint:

$$[P^j, P^k]\, \mathrm{D}_i = 0$$

$$[P^a, P^b]\, \mathrm{D}_i = 0$$

$$[P^j, L^{ak}]\, \mathrm{D}_i = \delta^{kj} P^a \mathrm{D}_i$$

$$[P^b, L^{ak}]\, \mathrm{D}_i = \delta^{ab} P^k \mathrm{D}_i$$

$$[L^{ak}, L^{bj}]\, \mathrm{D}_i = \delta^{ab} L^{kj} \mathrm{D}_i + \delta^{kj} L^{ab} \mathrm{D}_i$$

2nd Constraint:

$$\delta_{P^j} S_{\mathrm{EFT}} \stackrel{!}{=} 0 + \text{total derivative}$$

$$\delta_{L^{ak}} S_{\mathrm{EFT}} \stackrel{!}{=} 0 + \text{total derivative}$$

Not enough to fix all the coefficients...

$$S_{\mathrm{EFT}} = S_{\mathrm{DCFT}} + M^p \int d^p\sigma + M^p \int d^p\sigma\, \frac{1}{2} \partial^a X^j \partial_a X_j + \int d^p\sigma\, \mathrm{D}_i X^i + \,?$$

Are We Done?

$$S_{\mathrm{EFT}} = S_{\mathrm{DCFT}} + M^p \int d^p\sigma + M^p \int d^p\sigma \, \frac{1}{2} \partial^a X^j \partial_a X_j + \int d^p\sigma \, \mathrm{D}_i X^i + \mathcal{O}\left(X^3\right)$$

- What else can be there?

Are We Done?

$$S_{\text{EFT}} = S_{\text{DCFT}} + M^p \int d^p\sigma + M^p \int d^p\sigma \, \frac{1}{2}\partial^a X^j \partial_a X_j + \int d^p\sigma \, \mathrm{D}_i X^i + \mathcal{O}\left(X^3\right)$$

- What else can be there?

- EFT must contain all possible terms

$$S = \int d^p\sigma \, \sqrt{\det\left(g_{ab}\right)} \, \mathcal{F}\left[g_{ab}, \nabla_a \partial_b X^i, \hat{O}_{\hat{\Delta},j,s}\right]$$

- Existence is not guaranteed
- Coupling is not fixed by symmetry requirements and runs
- Scaling dimension is not protected

Are We Done?

- EFT must contain all possible terms

$$
\begin{aligned}
S_{\mathrm{EFT}} &= S_{\mathrm{DCFT}} + M^p \int d^p\sigma \sqrt{\det(g_{ab})} - \int d^p\sigma\, \mathrm{D}_i X^i \\
&\quad + \alpha M^{p-\Delta_\alpha} \int d^p\sigma \sqrt{\det(g_{ab})}\, \hat{O} + \beta M^{p-\Delta_\beta} \int d^p\sigma \sqrt{\det(g_{ab})}\, \hat{O}_{ia} \nabla^a X^i + \ldots \\
&= S_{\mathrm{DCFT}} + M^p \int d^p\sigma + \int d^p\sigma\, \frac{1}{2} \partial^a \mathcal{X}^j \partial_a \mathcal{X}_j - \sqrt{c_D} M^{-\frac{p}{2}} \int d^p\sigma\, \hat{\mathrm{D}}_i \mathcal{X}^i \\
&\quad + \alpha M^{p-\Delta_\alpha} \int d^p\sigma\, \hat{O} + \alpha M^{-\Delta_\alpha} \int d^p\sigma\, \frac{1}{2} \partial^a \mathcal{X}^j \partial_a \mathcal{X}_j \hat{O} + \beta M^{\frac{p}{2}-\Delta_\beta} \int d^p\sigma\, \hat{O}_{ia} \nabla^a \mathcal{X}^i + \ldots
\end{aligned}
$$

Finally done!

Perturbative Correction to Observables

$$\langle O \rangle_{\text{EFT}} = \left\langle O e^{-S_{\text{EFT}}} \right\rangle_{\text{DCFT}}$$
$$= \langle O \rangle_{\text{DCFT}} - \frac{1}{M^{\frac{p}{2}}} \int d^p\sigma \, \left\langle O \mathrm{D}_i(\sigma) \mathcal{X}^i(\sigma) \right\rangle_c + \dots$$

- Does our proposed coupling term really give the leading correction?

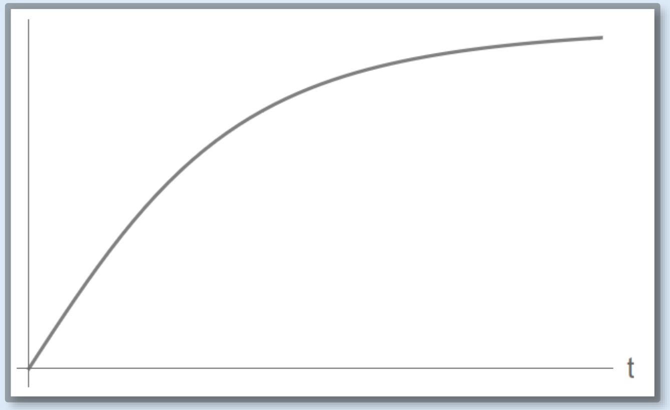
Observable	Displacement Op. $\sqrt{c_D} M^{-\frac{p}{2}} \int \hat{\mathrm{D}}_i \, \mathcal{X}^i$	Defect Scalar $\alpha M^{p-\Delta} \int \sqrt{g} \, \hat{O}$	Defect Vector $\beta M^{\frac{p}{2}-\Delta} \int \sqrt{g} \, \hat{O}_{ia} \nabla^a \, \mathcal{X}^i$	Goldstones $\int \sqrt{g}$
$\langle O \, \mathcal{X}_j \rangle$	$\sqrt{c_D} M^{-\frac{p}{2}}$		$\beta M^{\frac{p}{2}-\Delta_\beta}$	
$\langle O \rangle$	$c_D M^{-p}$	$\alpha M^{p-\Delta_\alpha}$	$\beta M^{p-2\Delta_\beta}$	
$\langle \mathcal{X}^2 \rangle$	$c_D M^{-p}$		$\beta M^{p-2\Delta_\beta}$	M^{-p}

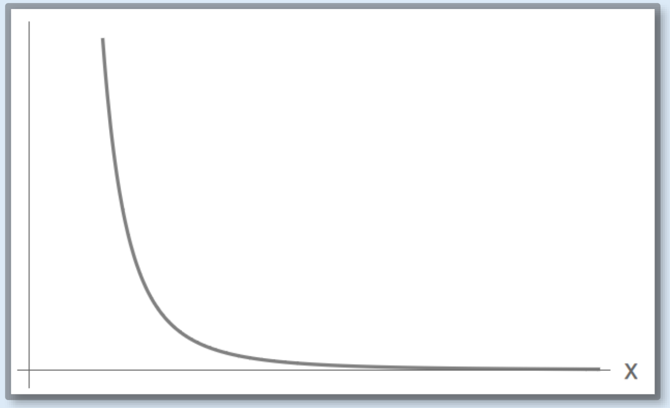
Example: Soliton in 2-dimensional ϕ^4-Theory

- Bulk scalar: $(\partial_\mu \psi(0, x<0))^2$ with $\Delta = 2$
- Interaction of the scalar with the defect:

$$\left\langle (\partial_\mu \psi(0, x<0))^2 \, \partial_t \mathcal{X}(t) \right\rangle = \frac{1}{\sqrt{M}} \int dt' \left\langle (\partial_\mu \psi(0, x<0))^2 \, \mathrm{D}(t') \right\rangle \langle \partial_t \mathcal{X}(t) \mathcal{X}(t') \rangle$$

$$= \frac{u}{4(1+u)} \sqrt{\frac{c_D}{M}} \frac{\frac{t|x|}{t^2+x^2} + \arctan\left(\frac{t}{|x|}\right)}{|x|^3} + \mathcal{O}\left(\frac{1}{M}\right)$$





Summary and Outlook

- The EFT for fluctuating defects extends both the DCFT framework and the effective string theory framework.
- Charges associated to symmetries broken by the defect are conserved (spontaneously broken) in the EFT: a new handle on recently derived constraints on defect CFT data?
  [Gabai, Sever, Zhong; 2025][Girault, Paulos, van Vliet; 2025][Drukker, Kong, Kravchuck;2025]
- Study more sophisticated examples: supersymmetry, holography…
- Test EFT predictions on the lattice

Thank You !