Comparing Top-Down and Bottom-up Holographic Defects and Boundaries

William Harvey

University of Victoria

YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

Interfaces & Symmetry

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[Harvey, Jensen, Uzu, 2025]: 2504.13244

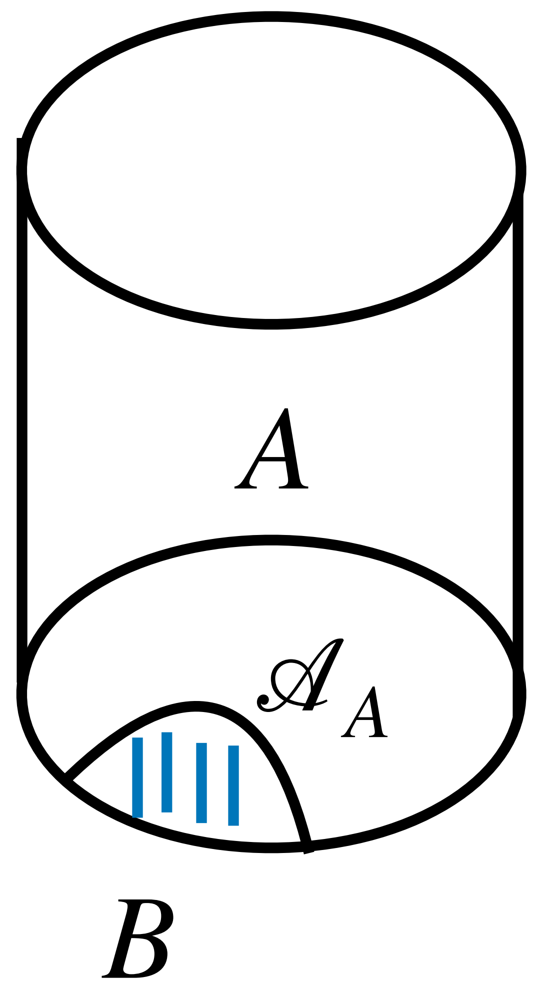
Overview

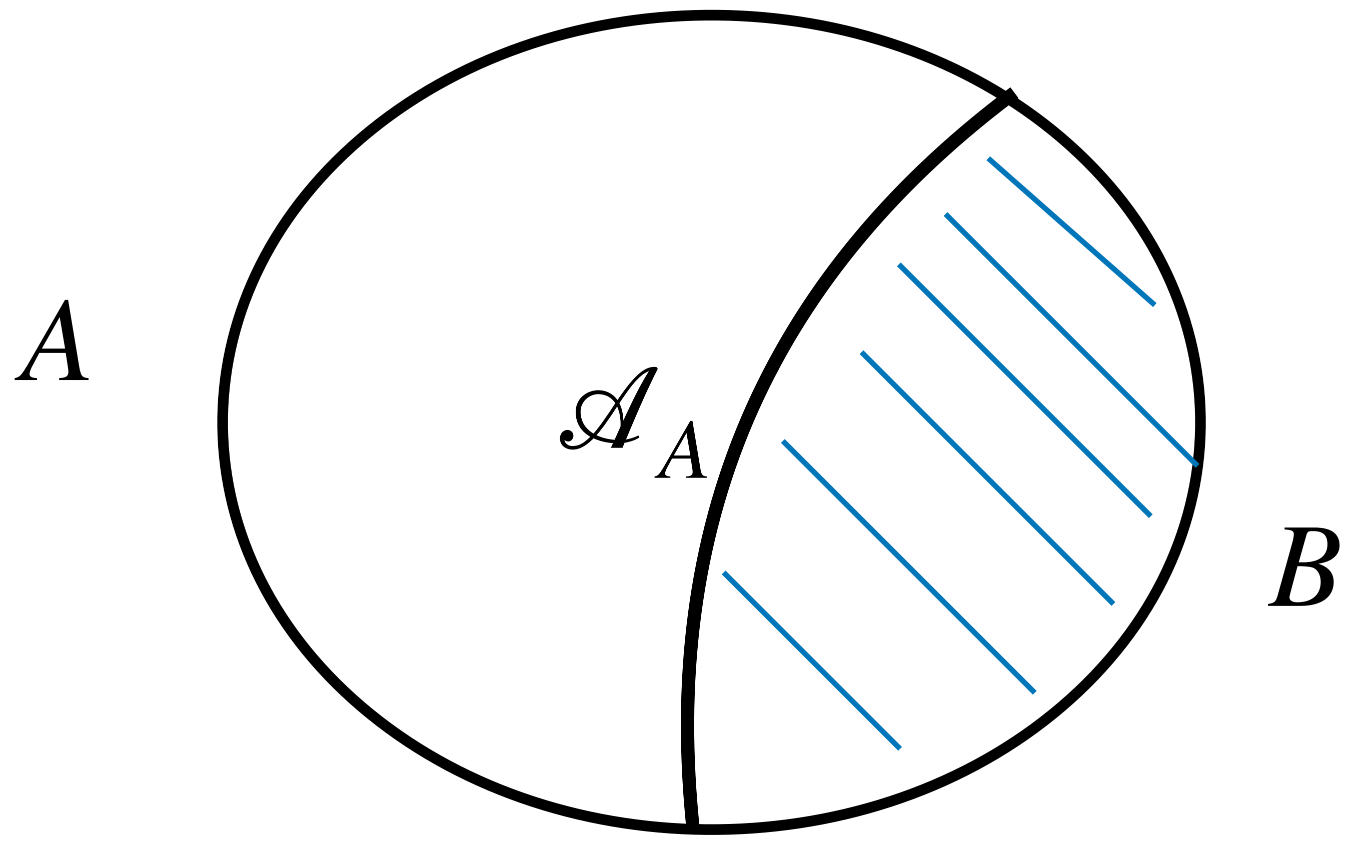
- **Intro:** AdS/B/DCFT & Holographic Entanglement Entropy **[1/5]: [5/5]**
- **Main Set-up:** Bulk locality, ϕ_b, and comparing theories [2/5]: [7/7]
- **Bottom-up models** (*mainly the ETW brane example*) [3/5]: [4/4]
- **Top-down constructions** (*General Procedure*) [4/5]: [4/4]
- **Results:** Defect/Boundary Entropy Comparisons [5/5]: [3/3]

Entanglement Entropy and AdS/CFT

[Ryu, Takayanagi, 2006]

Ryu-Takayanagi Conjecture: The entanglement entropy of a CFT is related to the bulk spacetime analogous to black hole thermodynamics.



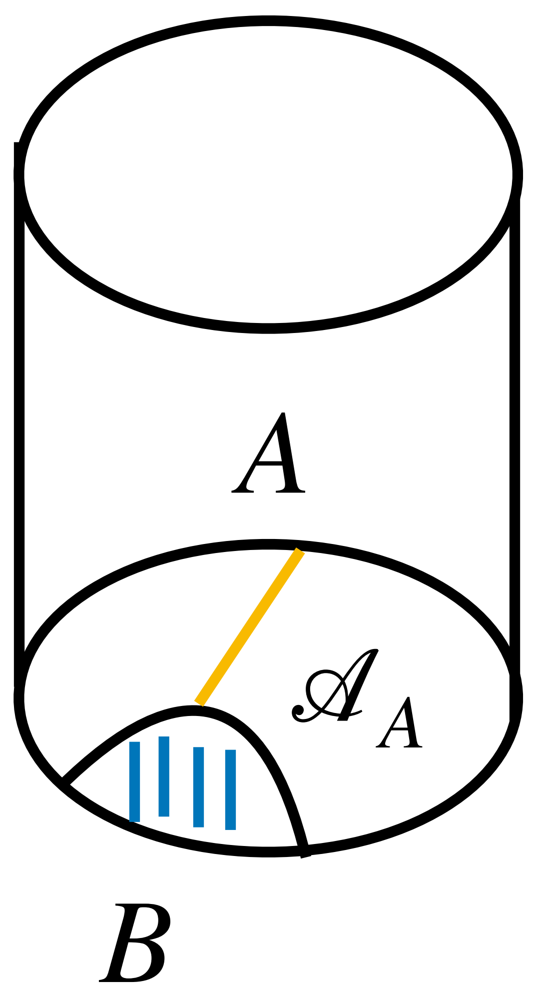


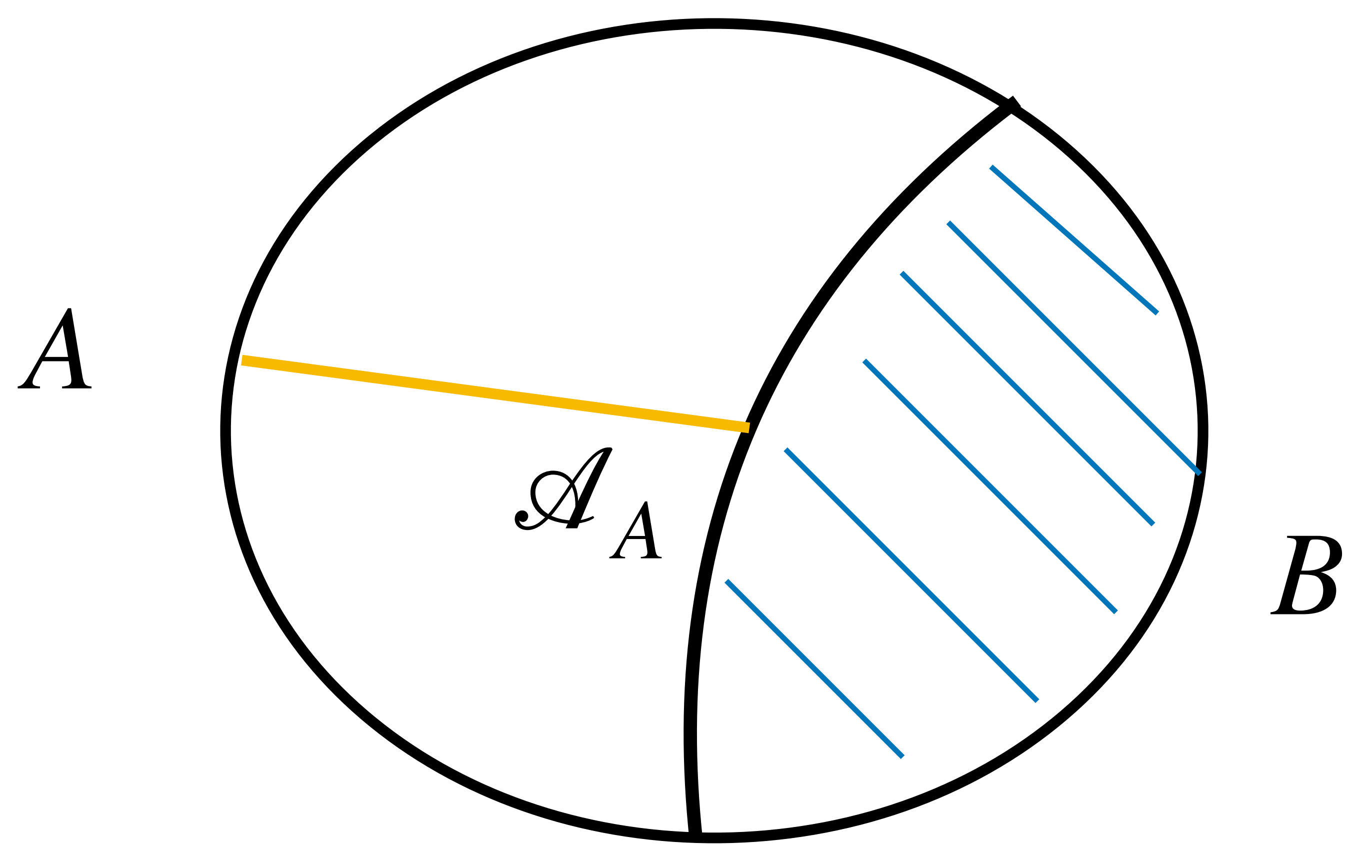
$$\mathcal{S}_{\text{EE}} = \frac{\mathscr{A}_A}{4G_N} \longrightarrow$$

$\mathscr{A}_A$ is the *minimal spatial surface* of the AdS bulk along the boundary that's the same as the one in which the A subsystem (CFT) lives.

Entanglement Entropy and AdS/B/DCFT

Consider a $(d+1)-$dimensional gravity dual of a flat-space $d-$dimensional CFT with a flat defect (DCFT) or boundary (BCFT).





The B/DCFT lives on Minkowski: $ds^2 = -dt^2 + dx_\perp^2 + d\mathbf{x}^2$

The defect/boundary exists at $x_\perp = 0$ and preserves the $SO(d-1,2)$ subgroup of the full $SO(d,2)$ conformal symmetry.

[1/5]: [2/5]

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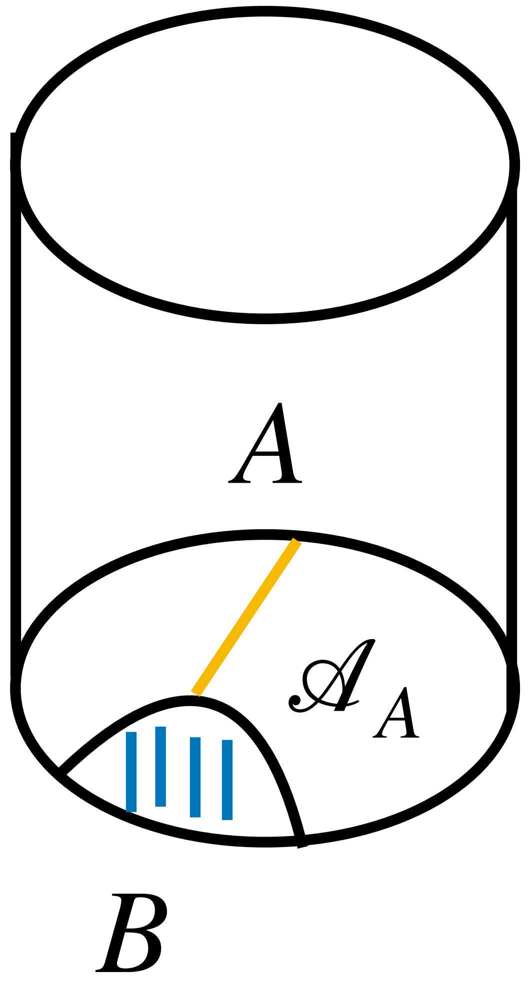
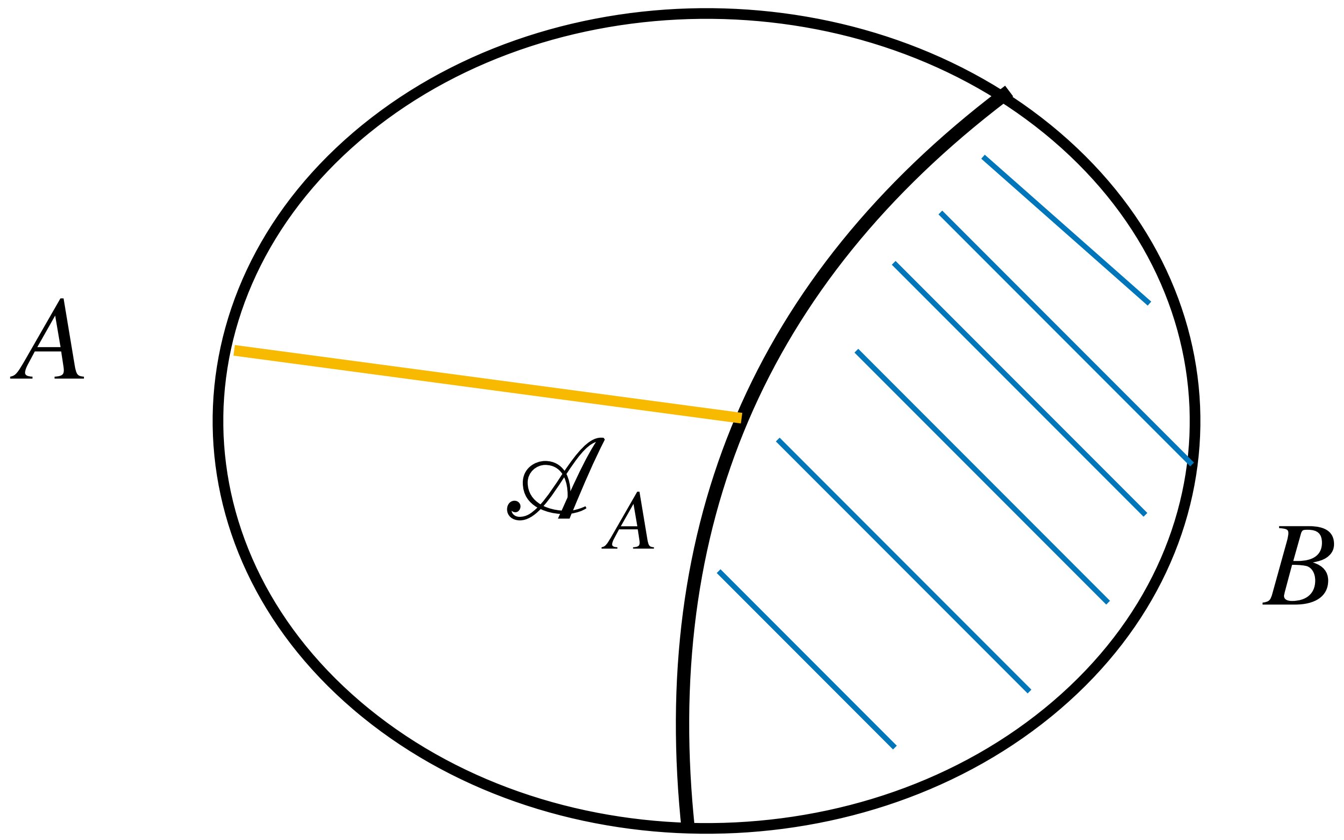




[Estes, et. al., 2014]

Background subtraction method

$$\mathcal{S} = \begin{cases} \mathcal{S}_{\text{EE}} - \frac{1}{2}\left(\mathcal{S}_{+} + \mathcal{S}_{-}\right) & \text{defect} \\ \mathcal{S}_{\text{EE}} - \frac{1}{2}\mathcal{S}_{\text{CFT}} & \text{boundary} \end{cases} \longrightarrow \mathcal{S}_{\text{defect}} = \begin{cases} D_0 + O(\epsilon) & d=2 \\ D_0 \ln \frac{R}{\epsilon} + \widetilde{D}_0 + O(\epsilon) & d=3 \\ \frac{a_1}{\epsilon} - D_0 + O(\epsilon) & d=4 \end{cases}$$

$$\mathcal{S}_{\text{boundary}} = [D_0 \rightarrow B_0]$$

$\epsilon << 1$ is a short-distance regulator which handles the infinite area of the AdS boundary.

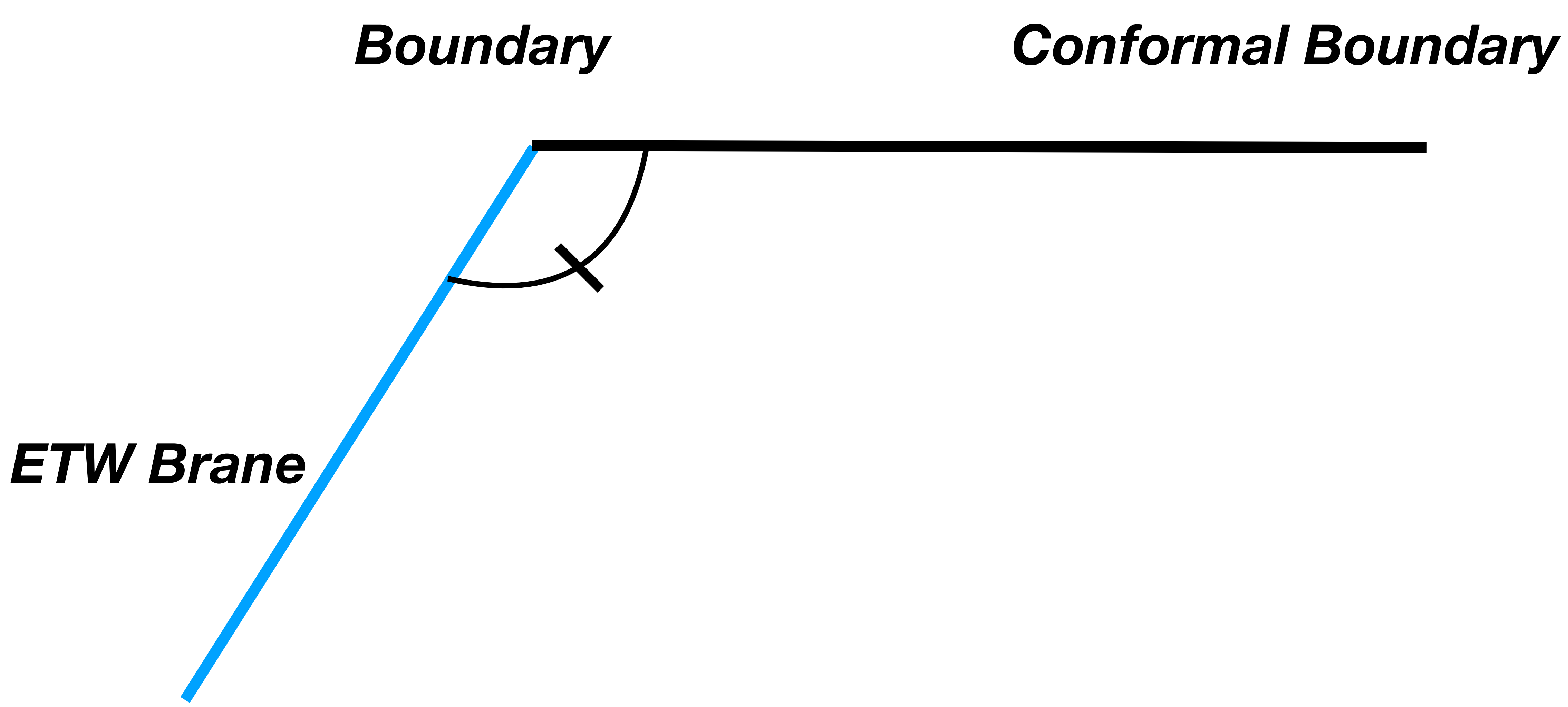
[1/5]: [3/5]

$$ds^2 = d\rho^2 + e^{A(\rho)}\left(\frac{-dt^2 + dx_\perp^2 + d\mathbf{x}^2}{x_\perp^2}\right)$$

$A(\rho)$ determined by the matter profile.

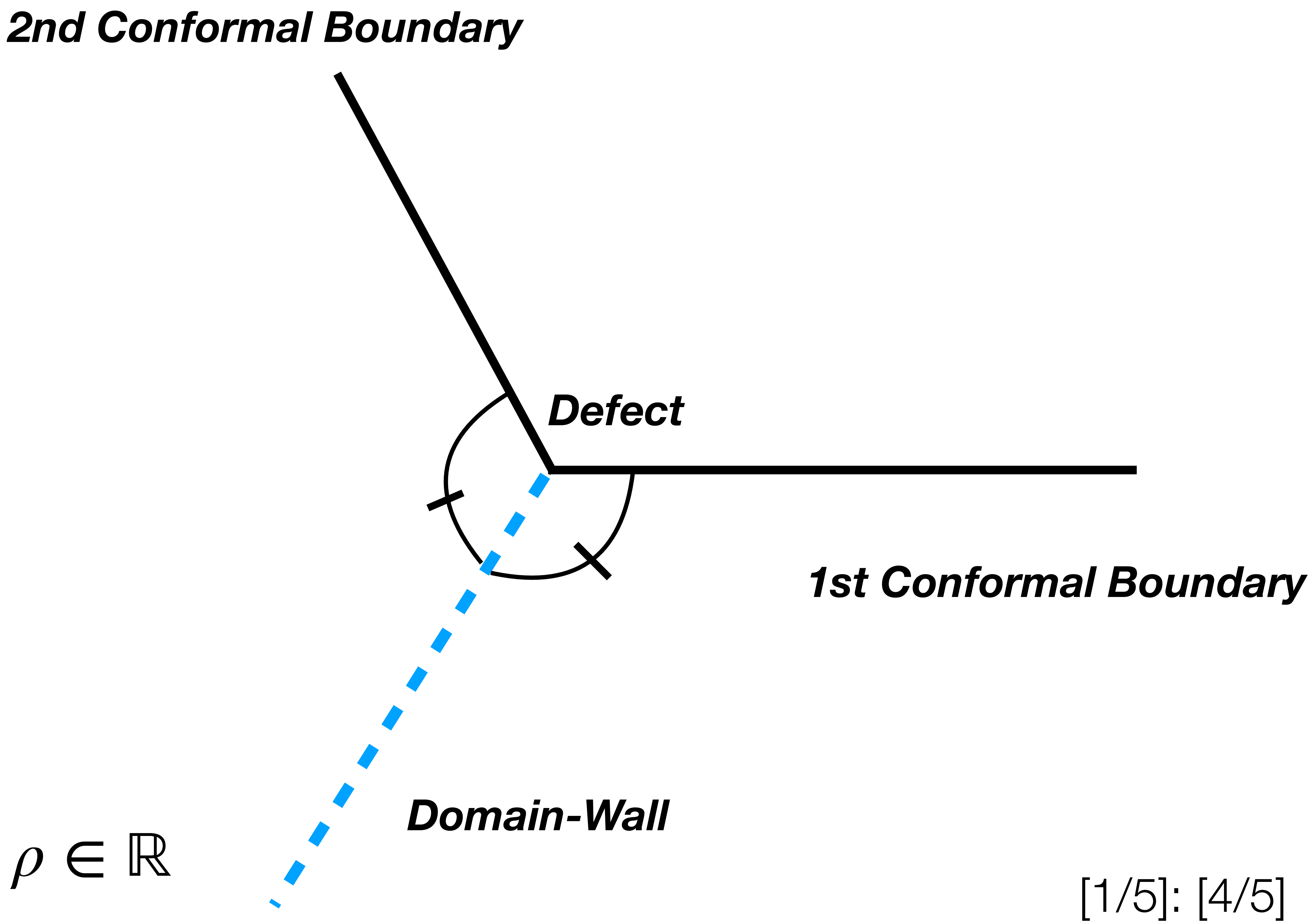
Metric is required to be asymptotic AdS via residual conformal symmetry.

Boundary Picture



$\rho \in (\rho_0, \infty)$

Defect Picture



$\rho \in \mathbb{R}$

[1/5]: [4/5]

$$\mathrm{AdS}_{d+1} = \mathrm{CFT}_d$$

ETW Brane $\longleftrightarrow$ Codimension-one boundary

Domain wall $\longleftrightarrow$ Codimension-one defect

$$\mathrm{AdS}_{d+1} = \mathrm{BCFT}_d \qquad \mathrm{AdS}_{d+1} = \mathrm{DCFT}_d$$

One can then explore notions of a modified correspondence.

In developing a modified correspondence, one may wish to work with toy models.

Going forward...

$$\mathrm{AdS}_{d+1} = \mathrm{CFT}_d$$

Bottom-up model: Toy models of AdS with an assumed holographic dual.

$$\mathrm{AdS}_{d+1} = \mathrm{BCFT}_d \qquad \mathrm{AdS}_{d+1} = \mathrm{DCFT}_d$$

Top-down construction: Superstring theory realization of a holographic theory.

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Comparing Top-Down and Bottom-up Theories

Motivation

How effective are bottom-up models at *mimicking* that of the top-down constructions?

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The parameter ϕ_b is one which is characteristic of any holographic AdS/D/BCFT theory.

Goal: We relate ϕ_b to *holographic boundary/defect entropy*, which acts as a bridge between the top-down and bottom-up theories.

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But what is ϕ_b...

…in gravity?

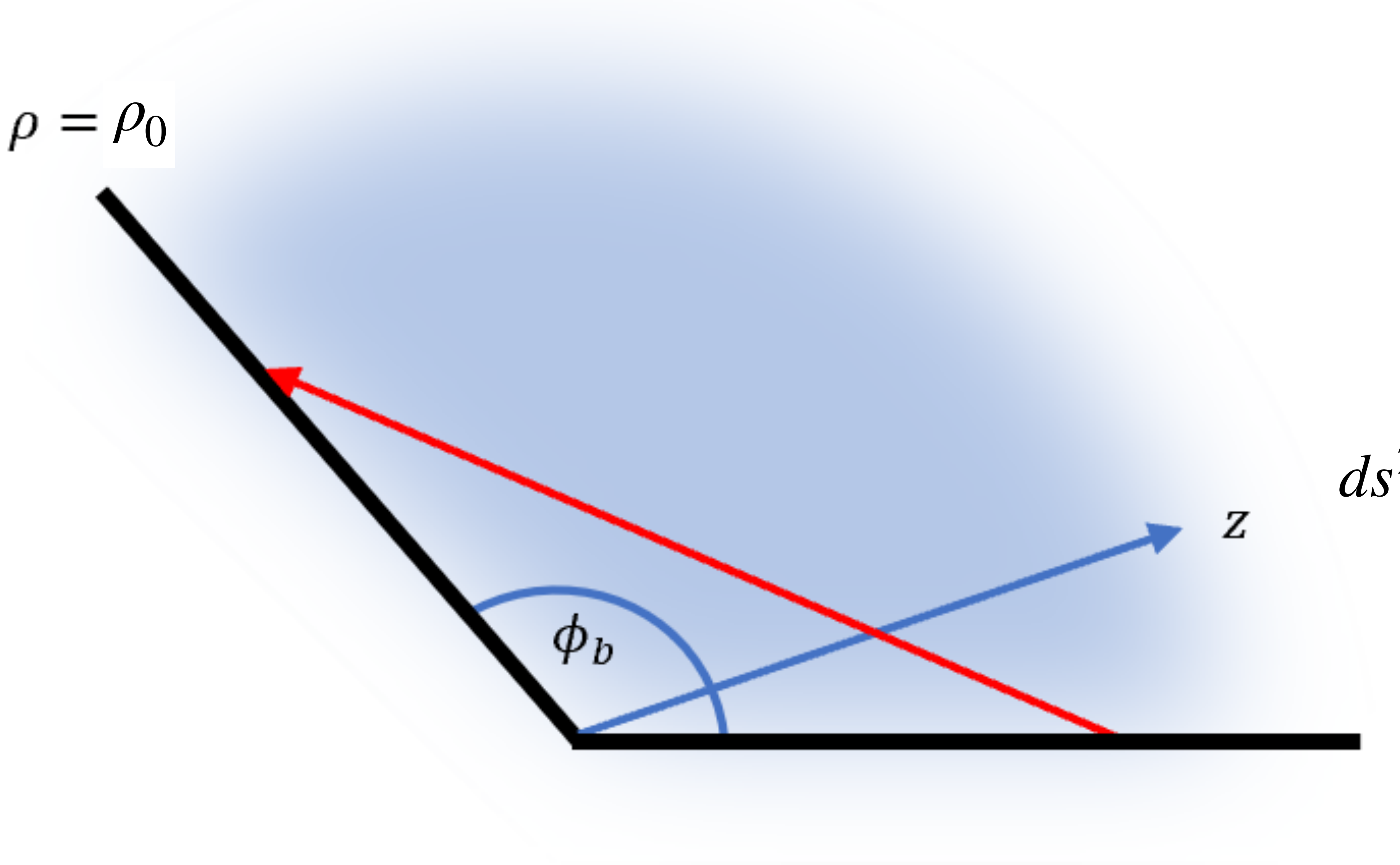
…in the B/DCFT?

What is ϕ_b in gravity?

[Reeves, et. al., 2021]

[Harvey, Jensen, Uzu, 2023]

A geometric argument (using a boundary).



$$ds^2 = d\rho^2 + e^{2A(\rho)}\left(\frac{-dt^2 + d\mathbf{x}^2 + dz^2}{z^2}\right) \longrightarrow ds^2 = \frac{e^{2A(\rho)}}{z^2}\left(-dt^2 + d\mathbf{x}^2 + dz^2 + z^2 d\phi^2\right)$$

$$d\phi = -\frac{d\rho}{e^A}$$

$$\phi_b = \int_{\rho_0}^{\infty} d\rho\, e^{-A(\rho)}$$

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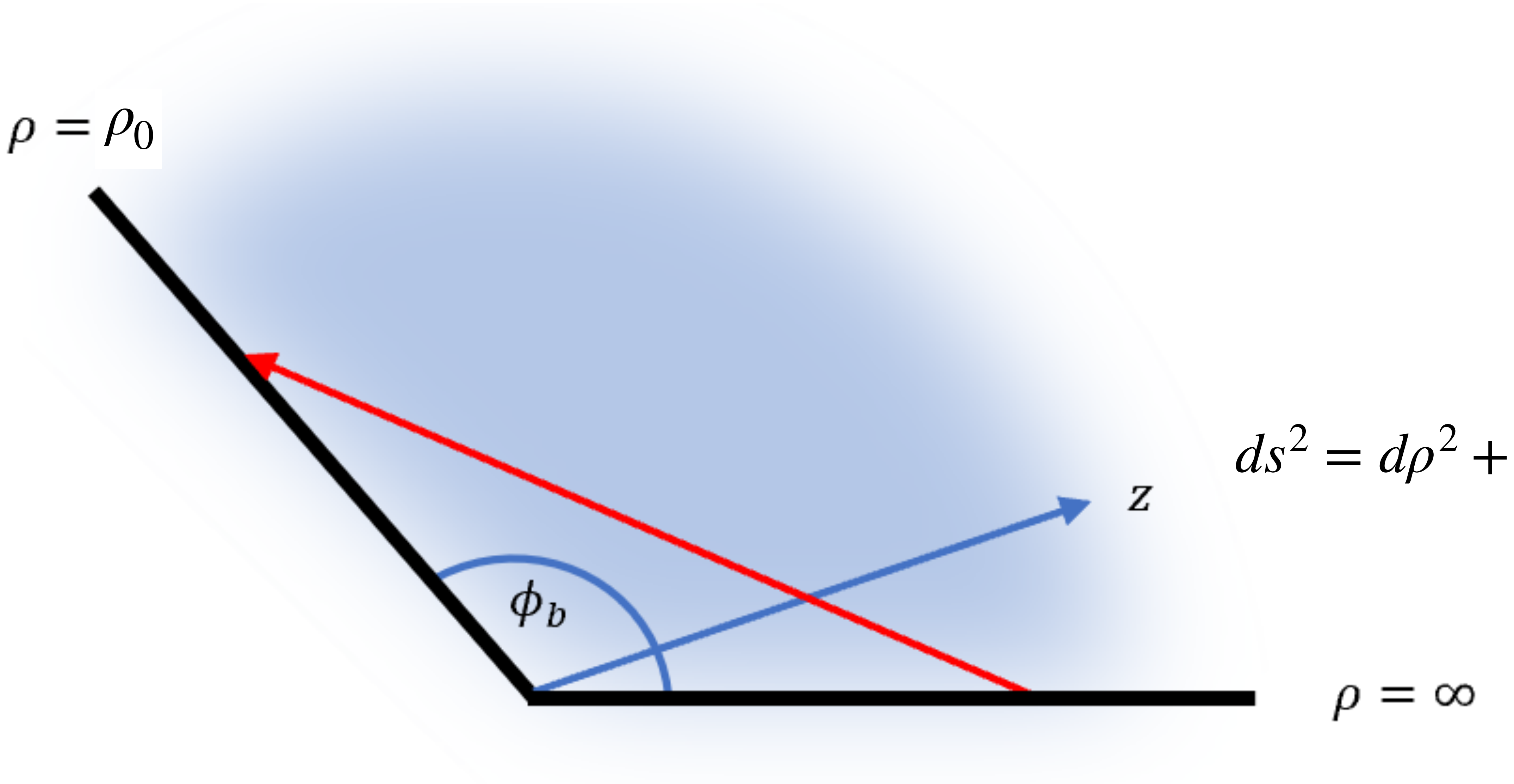

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Clearly, a null ray can never travel to the boundary if $\phi_b \geq \pi$

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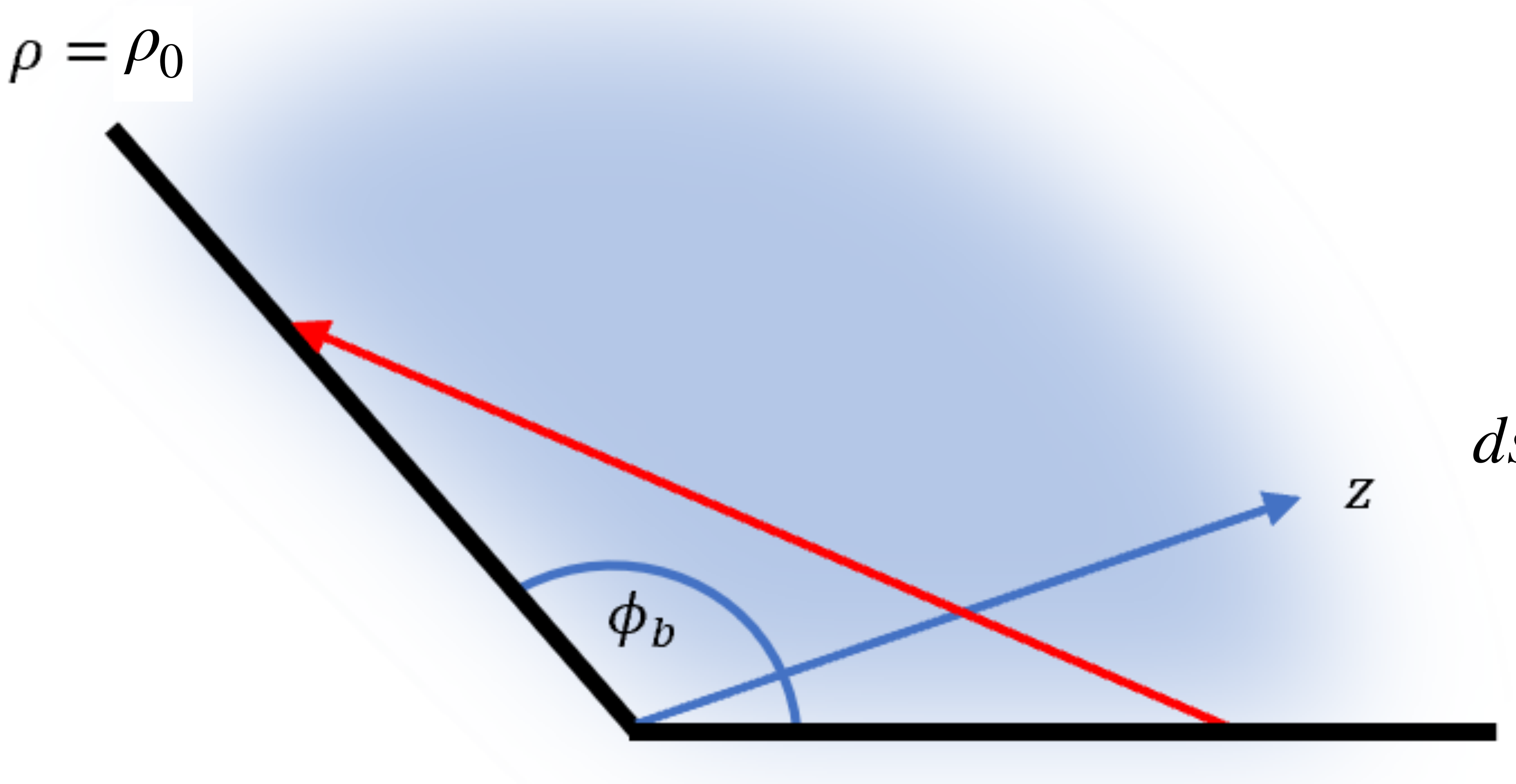

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Global Picture (at boundary)

$$ds^2_{\text{global}} = \frac{-d\tau^2 + d\theta^2 + \sin^2(\theta) d\Omega^2_{d-2}}{\cos^2(\theta)}$$

Geodesic in (τ, ϕ)-plane

$$\phi_b = \Delta\tau$$

$$ds^2 = d\rho^2 + e^{2A(\rho)} ds^2_{\text{global}} \longrightarrow ds^2 = \frac{e^{2A}}{\cos^2(\theta)}\left(-d\tau^2 + d\theta^2 + \sin^2(\theta) d\Omega^2_{d-2} + \cos^2(\theta) d\phi^2\right)$$

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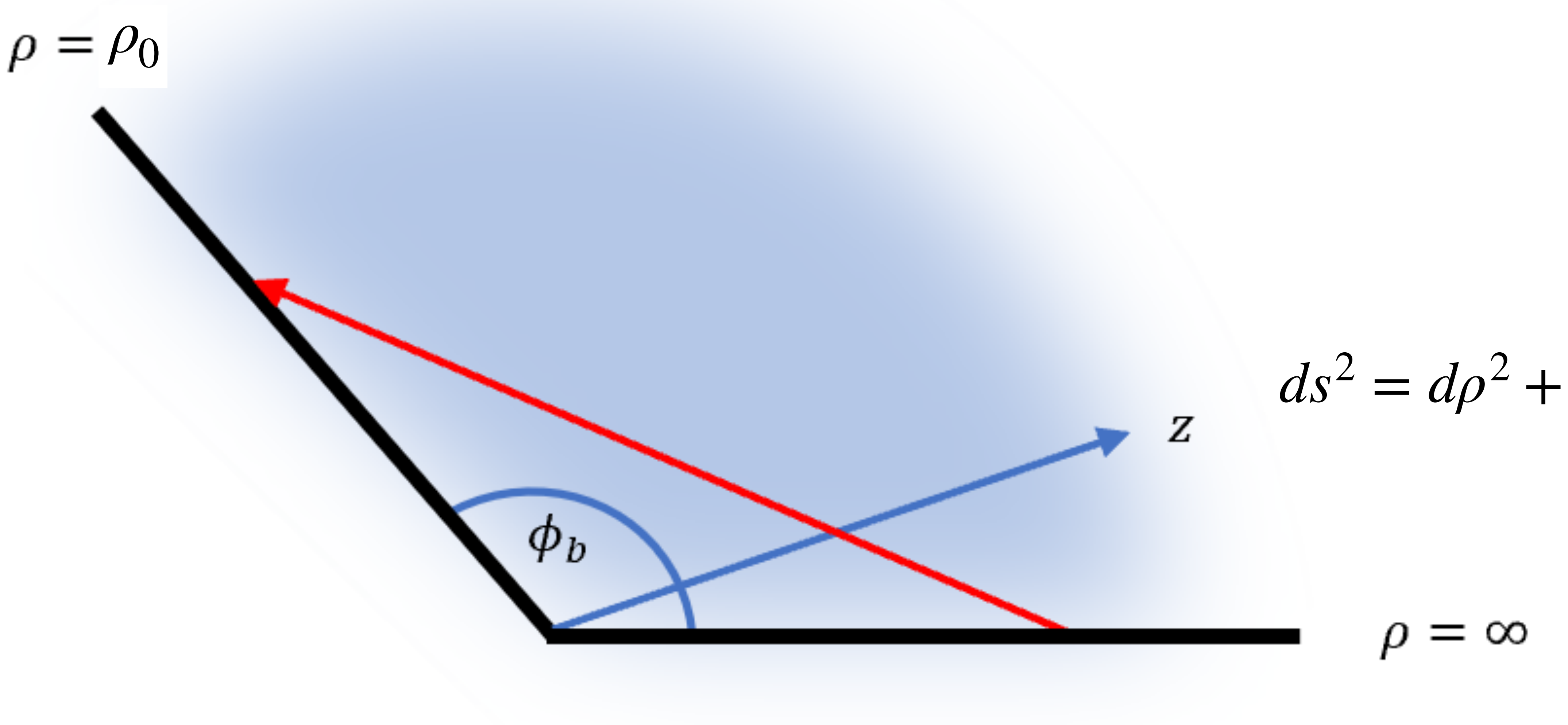

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Global Picture (at boundary)

These geodesics always exist $\quad ds^2_{\text{global}} = \dfrac{-d\tau^2 + d\theta^2 + \sin^2(\theta)d\Omega^2_{d-2}}{\cos^2(\theta)}$

If path is along poles

$$ds^2_{\text{global}} = -d\tau^2 + d\theta^2 \longrightarrow \Delta\tau = \pi$$

$$\tau = \pi \iff t \to \infty$$

What is ϕ_b in gravity?

[Reeves, et. al., 2021]

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Since $\tau = \pi \iff t \to \infty$, ϕ_b is therefore interpreted as a *global* **light-crossing time.**

$dz^2 + z^2 d\phi^2)$

$d\phi = - \frac{\ }{e^A}$

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Since $\tau = \pi \iff t \to \infty$, ϕ_b is therefore interpreted as a *global* **light-crossing time.**

Boundary Case: Travel time from the conformal boundary to the ETW brane.

$$\phi_b = \int_{\rho_0}^{\infty} d\rho \, e^{-A(\rho)}$$

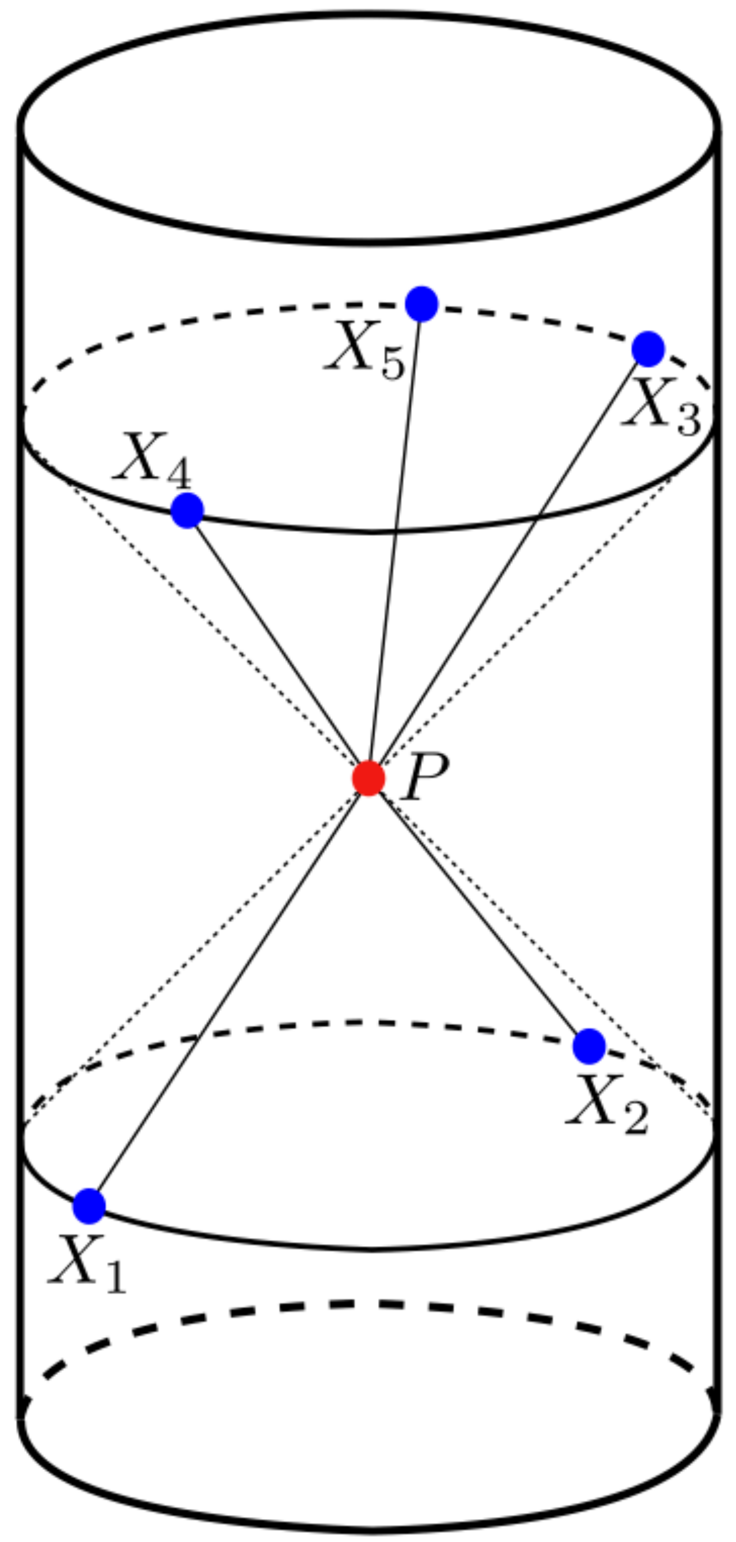
Domain-Wall / Interface Case: Travel time from the conformal boundary across a domain-wall towards a second conformal boundary.

$$\phi_b = \Delta\tau$$

Bulk Locality: What is ϕ_b in the D/BCFT?

Lorentzian correlation functions from a CFT have certain singularities in perturbation theory when certain operator insertions connect to null-ray diagrams in the bulk.

[Maldacena, Simmons-Duffin, Zhiboedov, 2015]



These singularities can assist in bulk reconstruction by providing light-cone structure, and thus the causal geometry.

Hence, these specific singularities in the CFT are said be probe sof bulk locality.

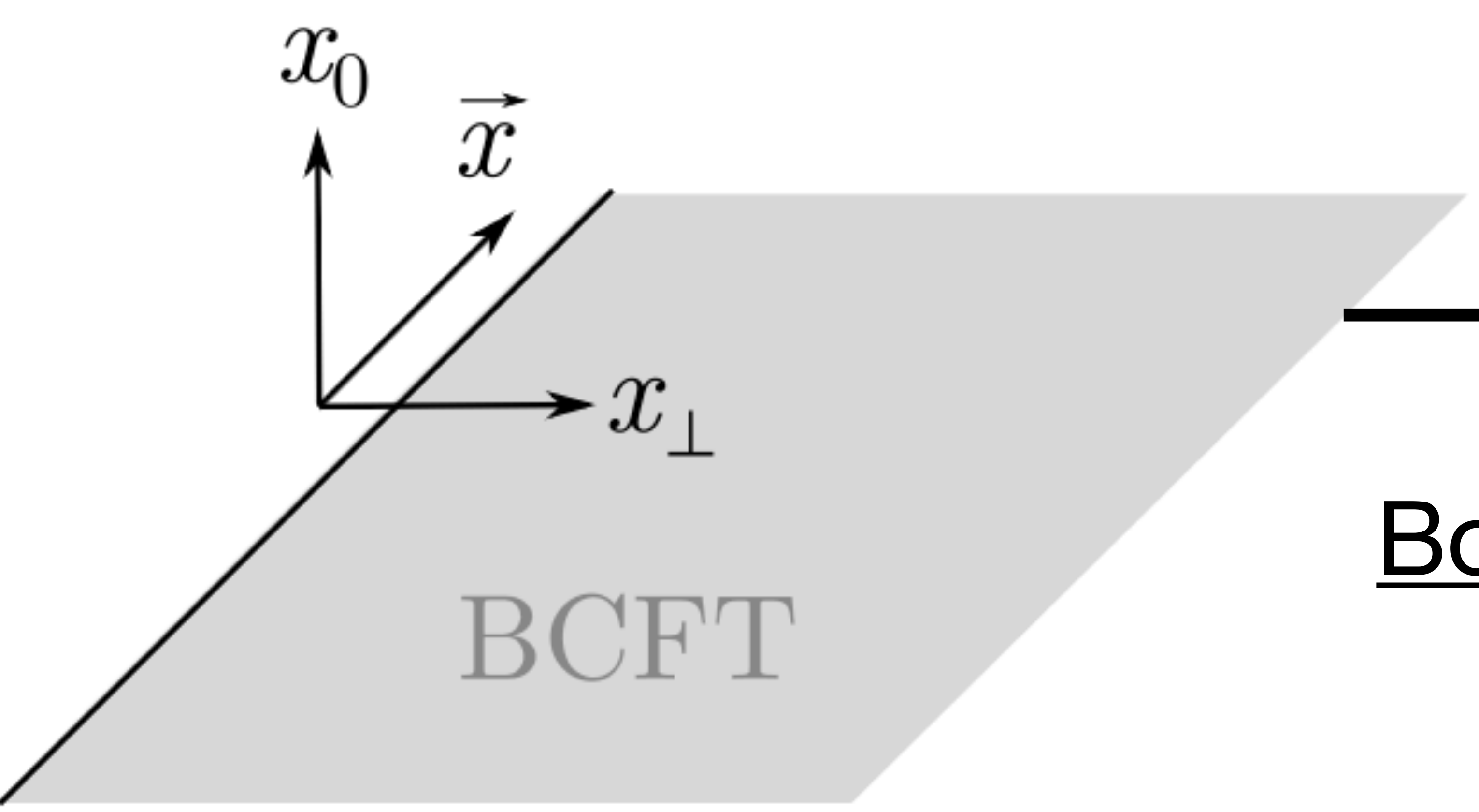
Note: it is expected that these singularities smooth out at the string scale.

What is ϕ_b in the D/BCFT?

A BCFT 2-point function is similar to a CFT 4-point function: $\langle \mathcal{O}(x)\mathcal{O}(y)\rangle \propto \frac{1}{4x_\perp y_\perp}\mathcal{G}(\xi)$

In which we have the cross-ratio

[Reeves, et. al., 2021]



$$\xi = \frac{(x-y)^2}{4x_\perp y_\perp} \longrightarrow \xi = \frac{(1-r)^2}{4r} \qquad \xi \in (0,\infty) \to r \in (0,1)$$

Boundary conformal block:

$$g^b_{\hat{\Delta}} = (4r)^{\hat{\Delta}}{}_2F_1\left(\hat{\Delta}, \frac{d-1}{2}; \hat{\Delta} - \frac{d}{2} + \frac{3}{2}; r^2\right)$$

Expected bulk point singularities

$$r = 0, \pm 1, \infty$$

If there are others, this must occur upon summing the blocks

What is ϕ_b in the D/BCFT?

[Reeves, et. al., 2021]

Note:* ρ *is* r *from the previous slide

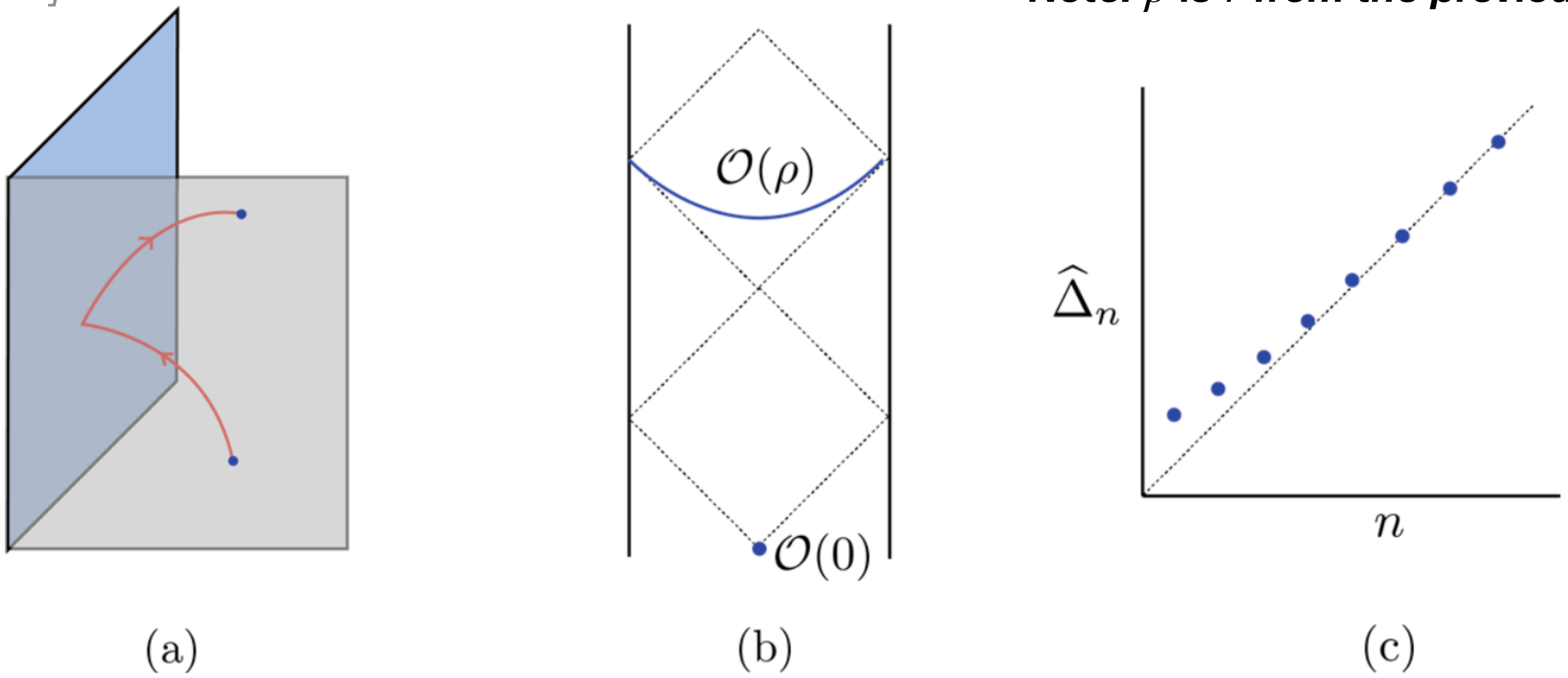


Figure 1: (a) A light ray leaving the boundary and returning to the boundary at a later time (in this example reflecting off an ETW brane); (b) The bulk causal structure then implies new 'bulk brane' singularities in the BCFT to the future of a BCFT operator; (c) The bulk brane singularities require a careful alignment of operator dimensions appearing on the boundary of the BCFT.

What is ϕ_b in the D/BCFT?

[Reeves, et. al., 2021]

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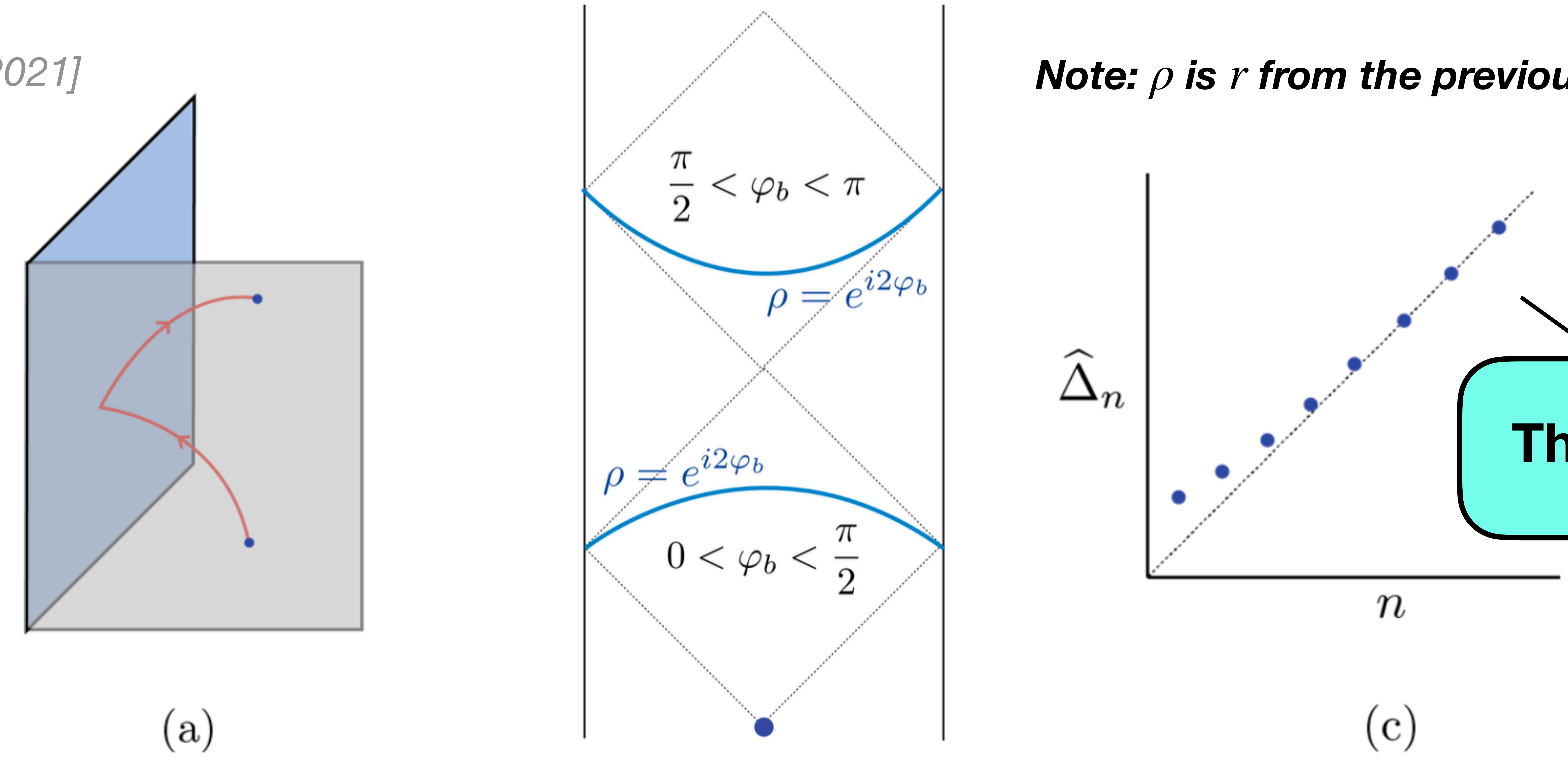

This does happen!

[Reeves, et. Al., 2021] Predicts new singularities with ***finite causal depth*** at $r = e^{2i\phi_b}$.

"Finite" is defined by a single ϕ_b, not $2\phi_b$

What is ϕ_b in the D/BCFT?

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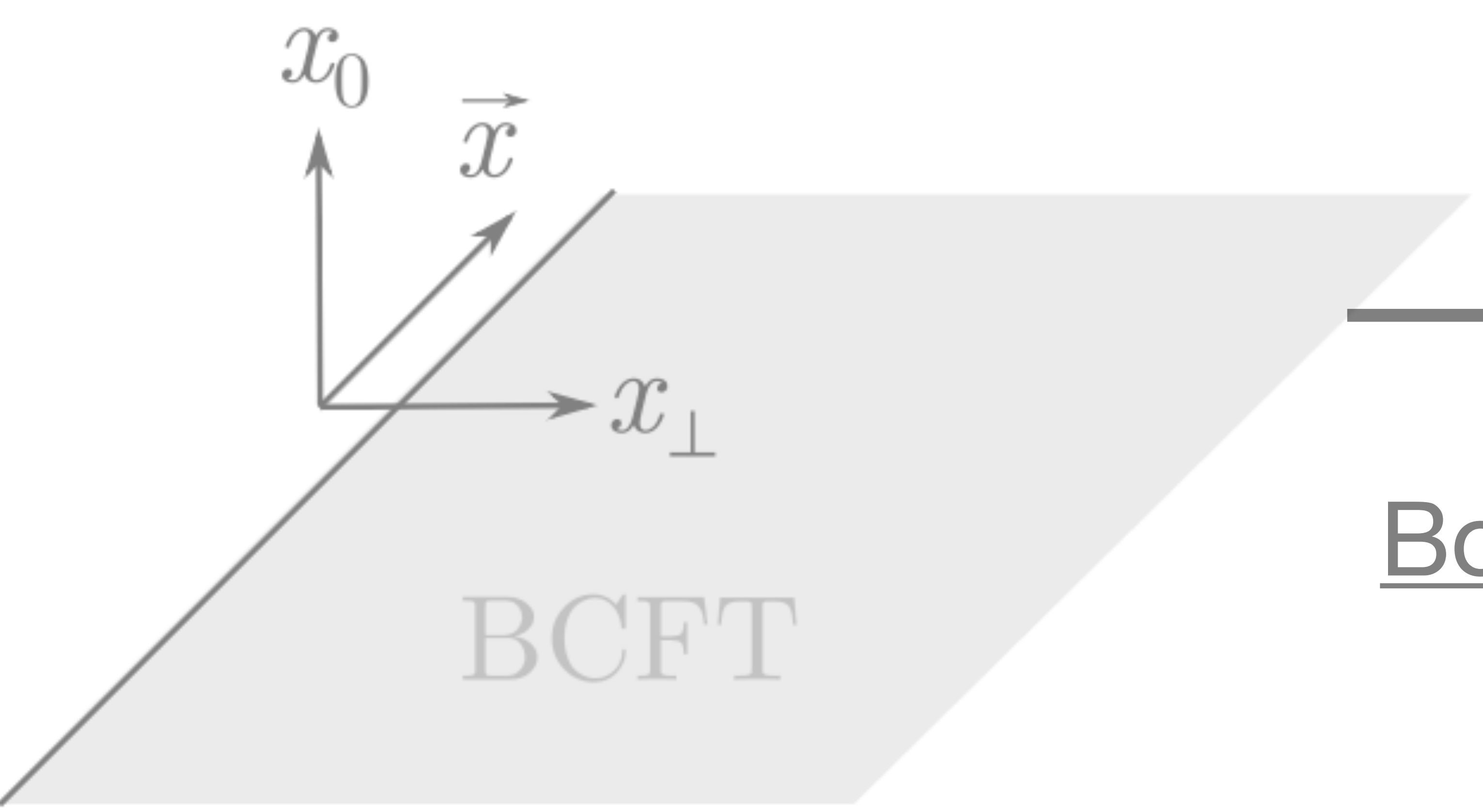

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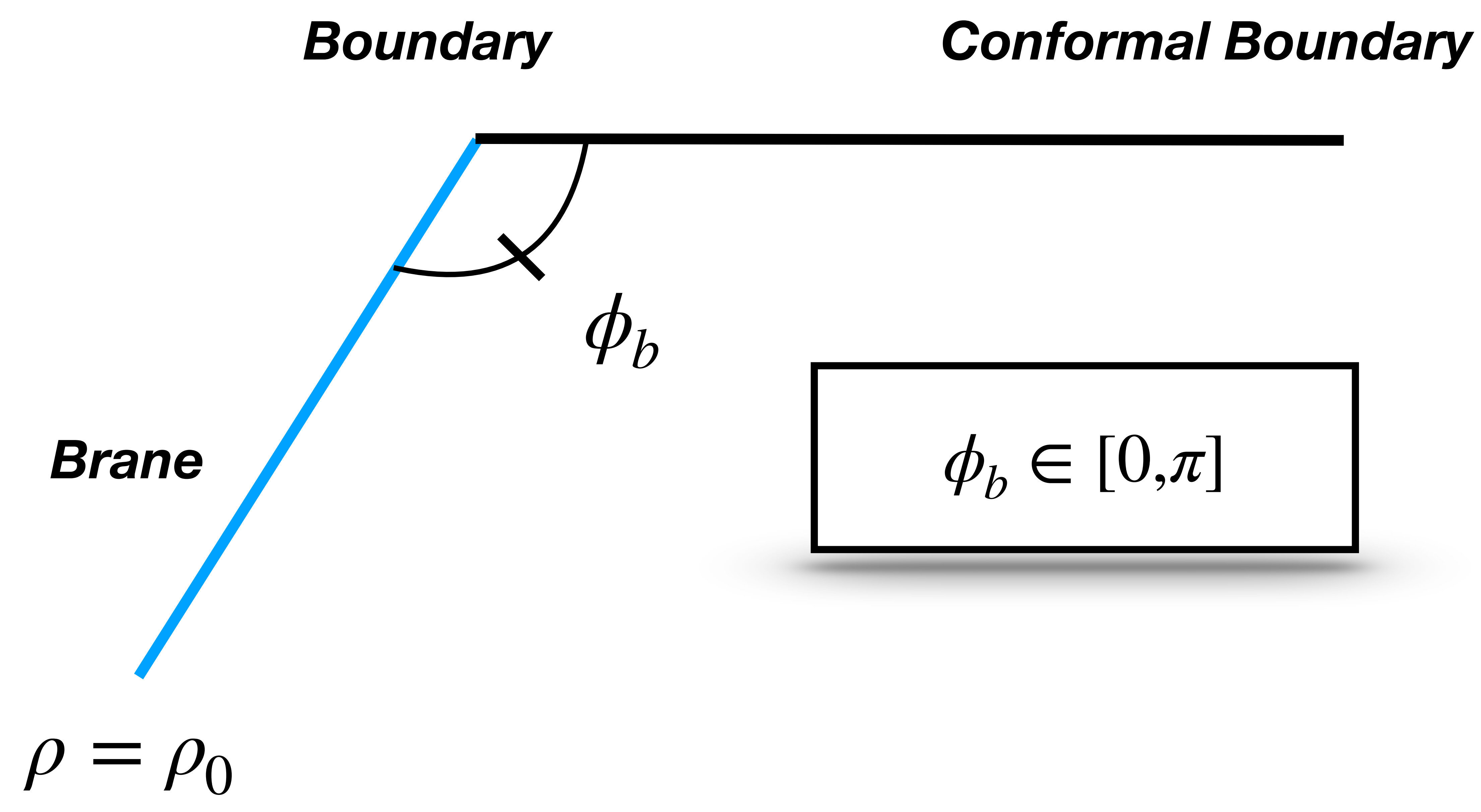
Expected bulk point singularities

$$r = 0, \pm 1, \infty$$

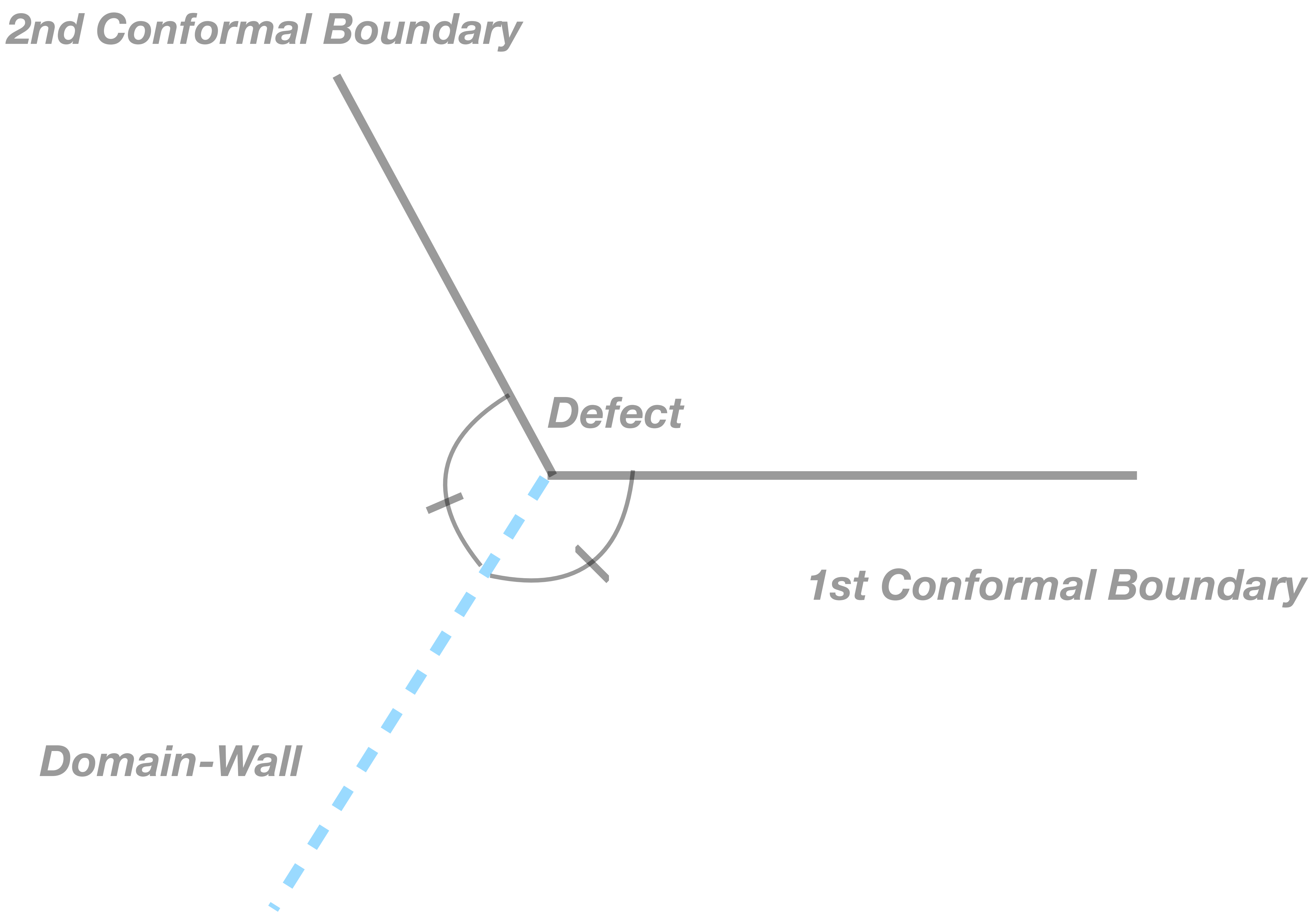
After summing the blocks: $\langle \mathcal{O}(x)\mathcal{O}(y)\rangle \propto \frac{1}{\left(r - e^{2i\phi_b}\right)^{1-a}}$

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle \propto \frac{1}{\left(r - e^{2i\phi_b}\right)^{1-a}} \longleftrightarrow$$ **Geometric interpretation** of $2\phi_b$?



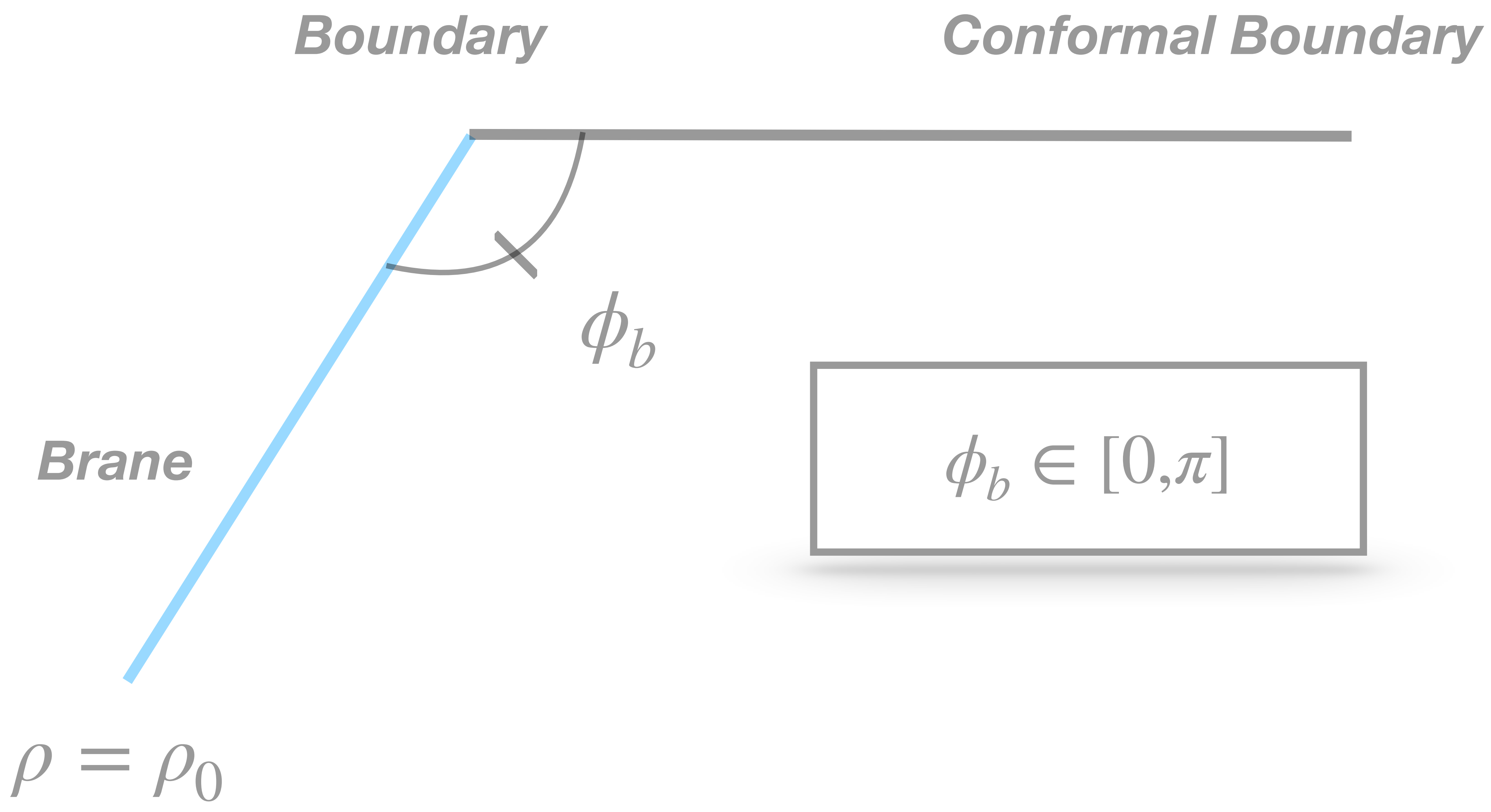
$\phi_b \in [0,\pi]$

To ETW brane *and* back.



$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle \propto \frac{1}{\left(r - e^{i\phi_b}\right)^{1-a}}$$ ⟷ **Geometric interpretation** of ϕ_b

Notation: *the singularity is located at ϕ_b for this scenario, but ϕ_b is twice that of the boundary case.*



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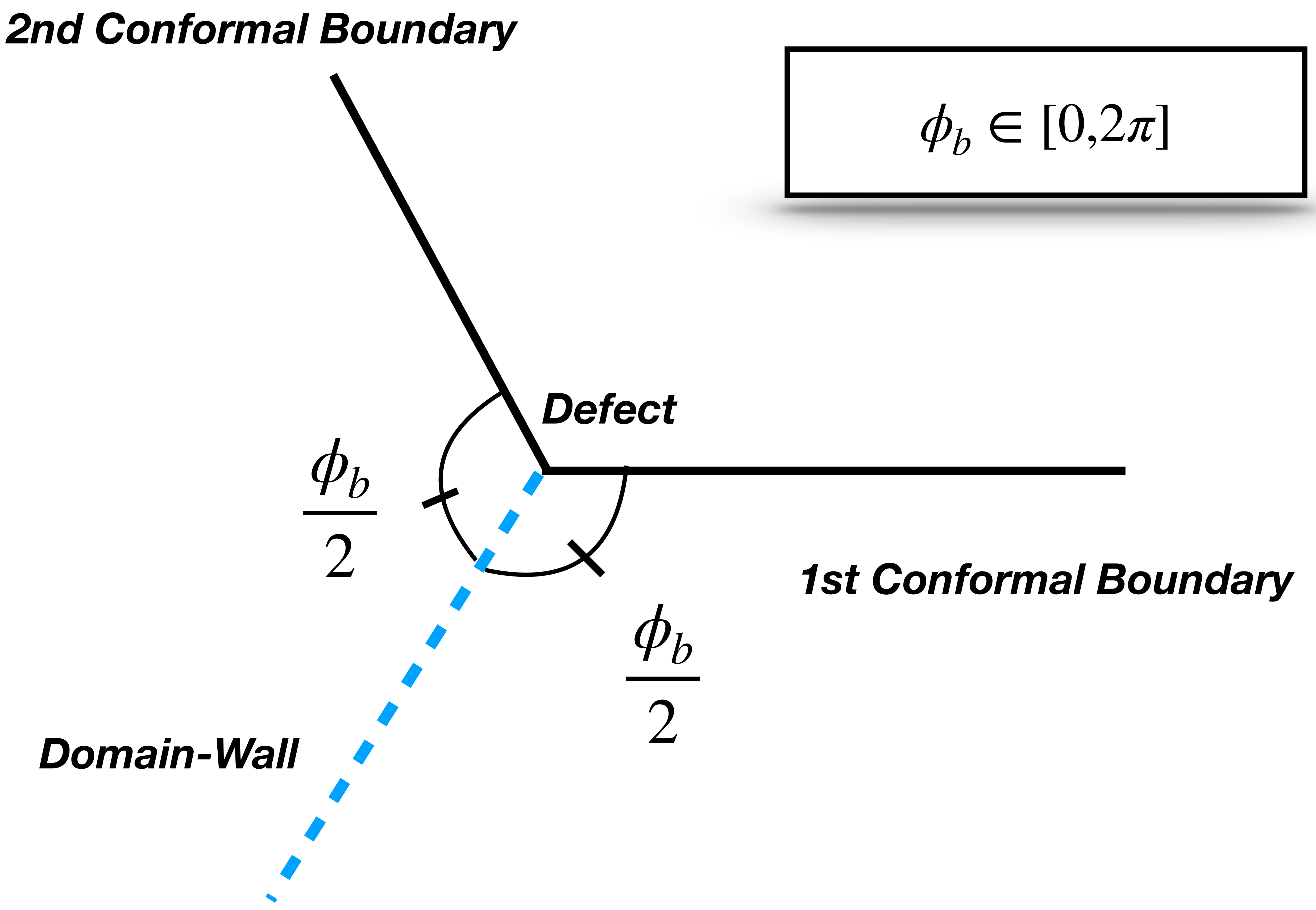

Across domain-wall to 2nd conformal boundary.

Main idea: Comparing Theories

<u>Bottom-up:</u> ϕ_b can be directly related to the <u>ETW brane/domain-wall location</u> $\rho = \rho_0$, (which itself can be related to an <u>effective tension</u>), and therefore a presumed **<u>holographic defect/boundary entropy</u>** can be constructed.

<u>Top-down:</u> ϕ_b can be expressed in terms of <u>string parameters</u>, and therefore **<u>holographic boundary/defect entropy</u>**, accordingly.

<u>Procedure:</u>

Calculate ϕ_b for our top-down and bottom-up theories, and then relate those to the dual <u>holographic boundary/defect entropy</u> for a 1-1 comparison.

Note: We strictly computed ϕ_b from geometry.

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The Boundary Bottom-up Model: Causal Structure

[Takayanagi, 2011]

$$S = \frac{1}{16\pi G_N}\int_N d^{d+1}\sqrt{-g}(R + d(d-1)) + \frac{1}{8\pi G_N}\int_Q d^d x\sqrt{-h}(K - T) + \dots$$

Conformal boundary (M) and other terms.

Bulk manifold

Boundary manifold of the ETW brane

Constant brane tension term

$$T = -(d-1)\tanh(\rho_0)$$

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$$T = -(d-1)\tanh(\rho_0)$$

$$ds^2 = d\rho^2 + e^{2A(\rho)}\left(\frac{-dt^2 + d\mathbf{x}^2 + dz^2}{z^2}\right)$$

Empty AdS ⟶ $A(\rho) = \ln(\cosh(\rho))$

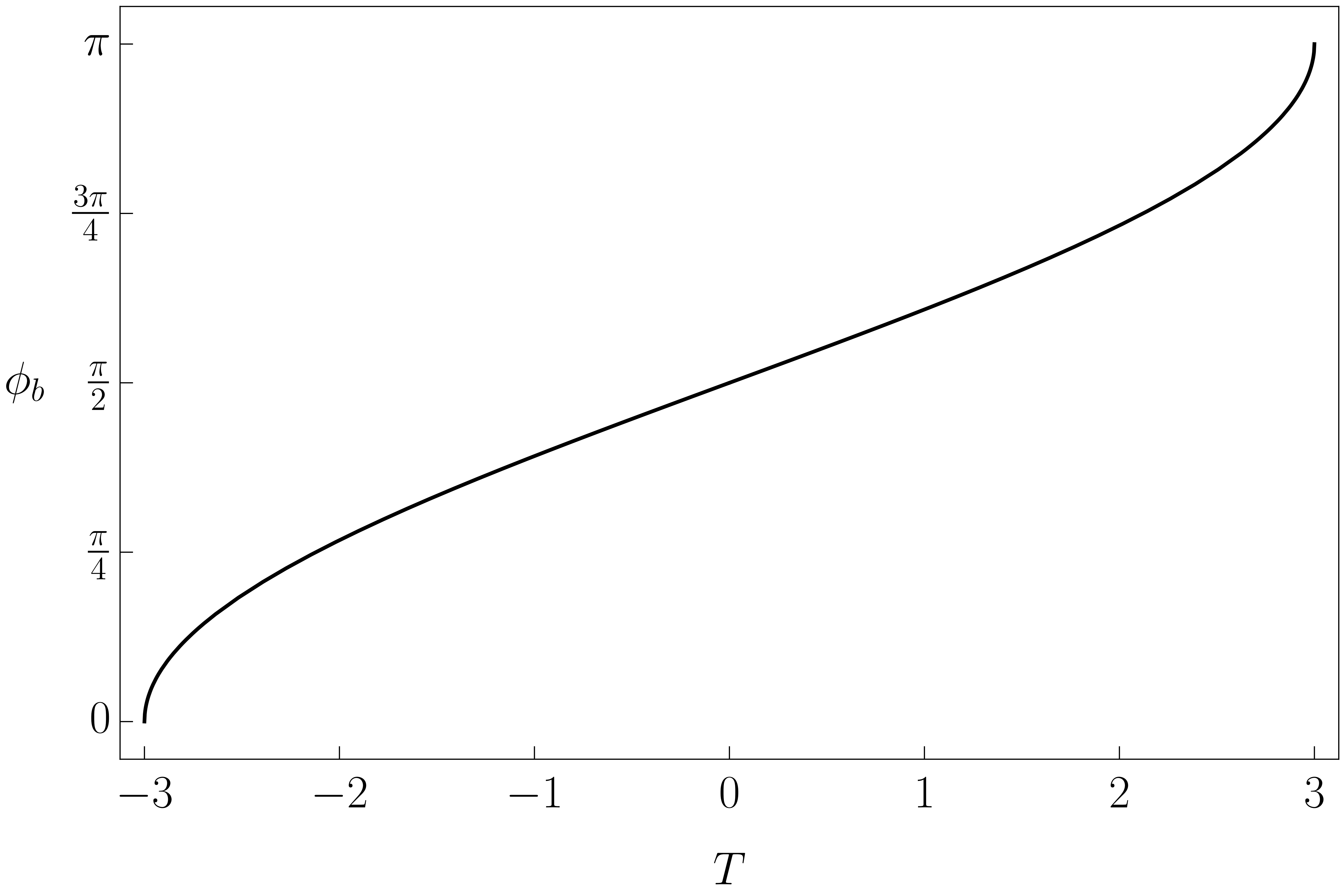
No different from truly empty AdS

$$\phi_b = \int_{\rho_0}^{\infty} e^{-A} d\rho = \frac{\pi}{2} - 2\tan^{-1}\left(\tanh\left(\frac{\rho_0}{2}\right)\right) \longrightarrow 0 \le \phi_b \le \pi$$

For different $A(\rho)$, see the Appendix A of [Harvey, Jensen, Uzu, 2025], but the above bounds do not change!

The Boundary Bottom-up Model: Causal Structure

Example: $d = 4$



$$0 \leq \phi_b \leq \pi$$

This also implies:

$$\tanh\left(\rho_0\right) = \cos\left(\phi_b\right)$$

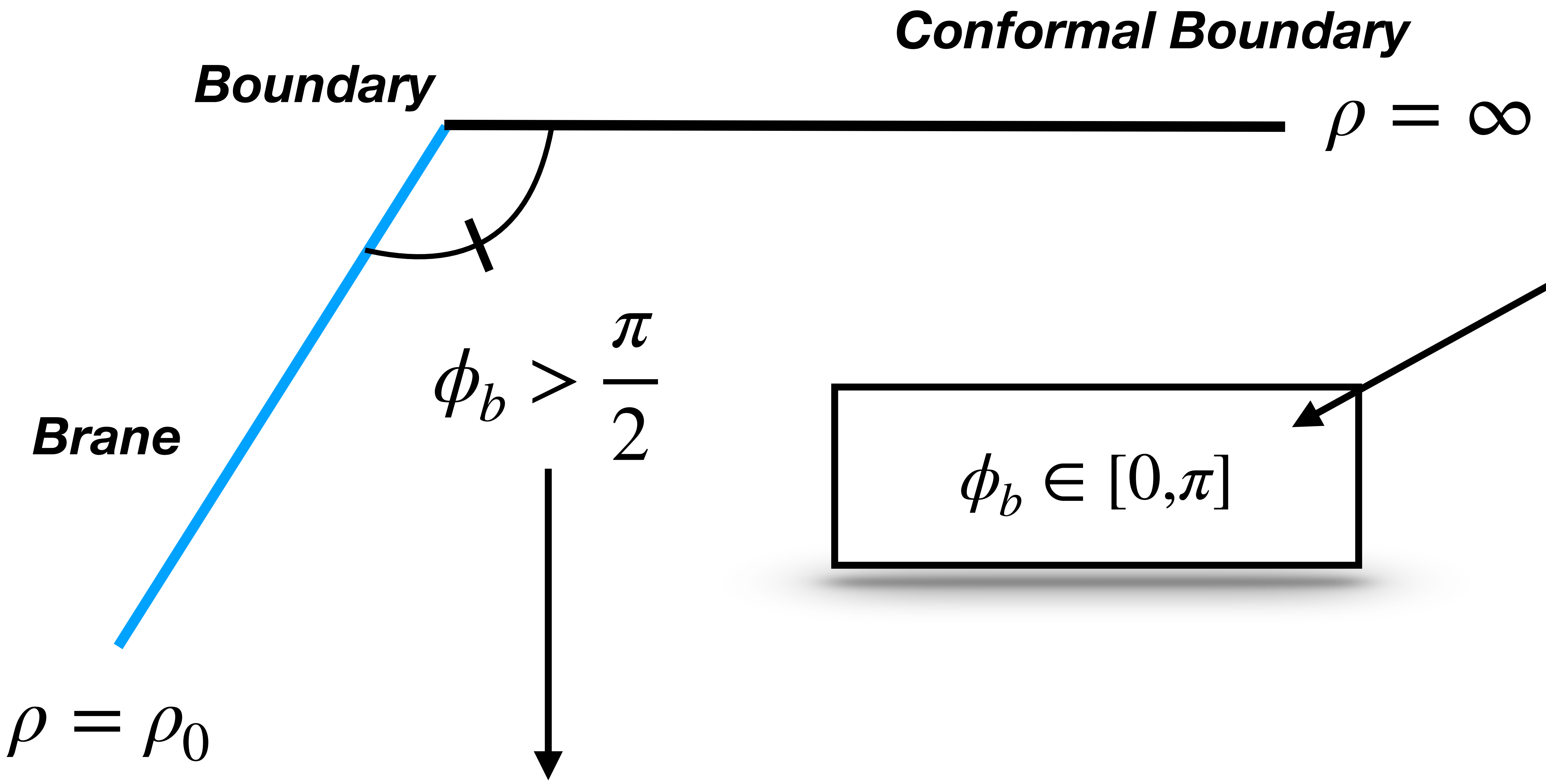
$$T = -(d-1)\cos\left(\phi_b\right)$$

$$\phi_b < \frac{\pi}{2} \longleftrightarrow T < 0$$

$$\phi_b > \frac{\pi}{2} \longleftrightarrow T > 0$$

[3/5]: [2/4]

The Boundary Bottom-up Model: Causal Structure



$\phi_b \in [0,\pi]$

$T > 0$

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The Defect Bottom-up Model: Causal Structure

[Karch, Randall, 2001]

The Karch-Randall braneworld is constructed similarly, except now it's two AdS cutouts glued together across a common interface.

$$0 \leq \phi_b \leq 2\pi$$

This also implies:

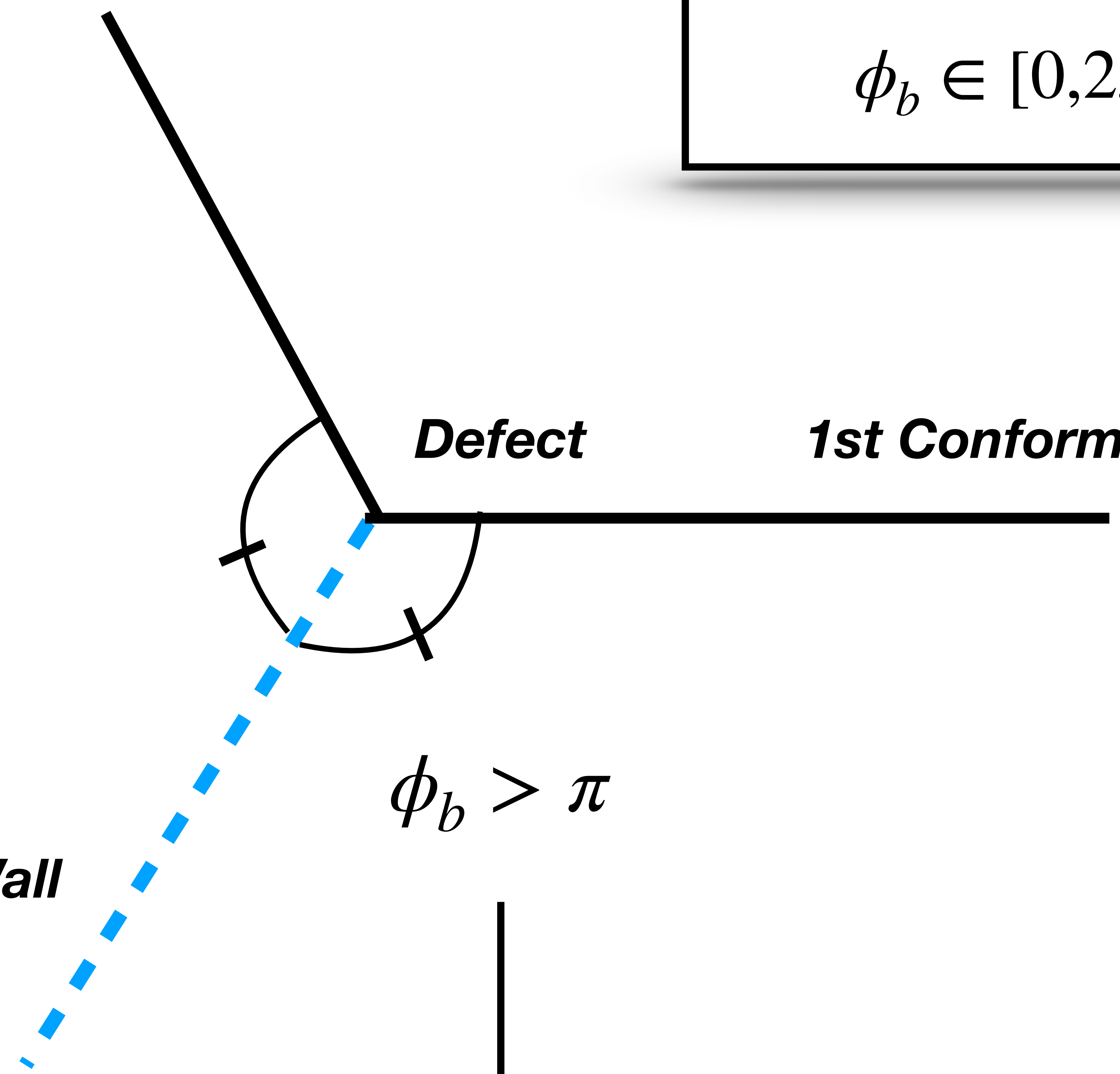
$$\tanh\left(\rho_0\right) = \cos\left(\frac{\phi_b}{2}\right)$$

$$T = -2(d-1)\cos\left(\frac{\phi_b}{2}\right)$$

$$\phi_b < \pi \longleftrightarrow T < 0$$

$$\phi_b > \pi \longleftrightarrow T > 0$$

The Defect Bottom-up Model: Causal Structure



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↓

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This also implies:

$$\tanh\left(\rho_0\right) = \cos\left(\frac{\phi_b}{2}\right)$$

$$T = -2(d-1)\cos\left(\frac{\phi_b}{2}\right)$$

$$\phi_b < \pi \longleftrightarrow T < 0$$

$$\phi_b > \pi \longleftrightarrow T > 0$$

The Defect Bottom-up Model: Causal Structure

2nd Conformal Boundary

$\phi_b \in [0,2\pi]$

It will turn out that the entropy for the KR models is twice that of the ETW brane model for the same ρ_0. I will therefore just focus on the former.

$T > 0$

$$0 \leq \phi_b \leq 2\pi$$

This also implies:

$$\tanh\left(\rho_0\right) = \cos\left(\frac{\phi_b}{2}\right)$$

$$T = -2(d-1)\cos\left(\frac{\phi_b}{2}\right)$$

$$\phi_b < \pi \longleftrightarrow T < 0$$

$$\phi_b > \pi \longleftrightarrow T > 0$$

Boundary Entropy for ETW brane model

[Kobayashi et al., 2019]

$$B_0 = \frac{1}{4G_N}\frac{\pi^{(d-1)/2}}{\Gamma\left(\frac{d-1}{2}\right)}\cos(\phi_b)\,{}_2F_1\left(\frac{1}{2},\frac{d}{2},\frac{3}{2},\cos^2\phi_b\right)\times\begin{cases}1\,, & d \text{ even}\,,\\ \frac{2}{\pi}\,, & d \text{ odd}\,.\end{cases}$$

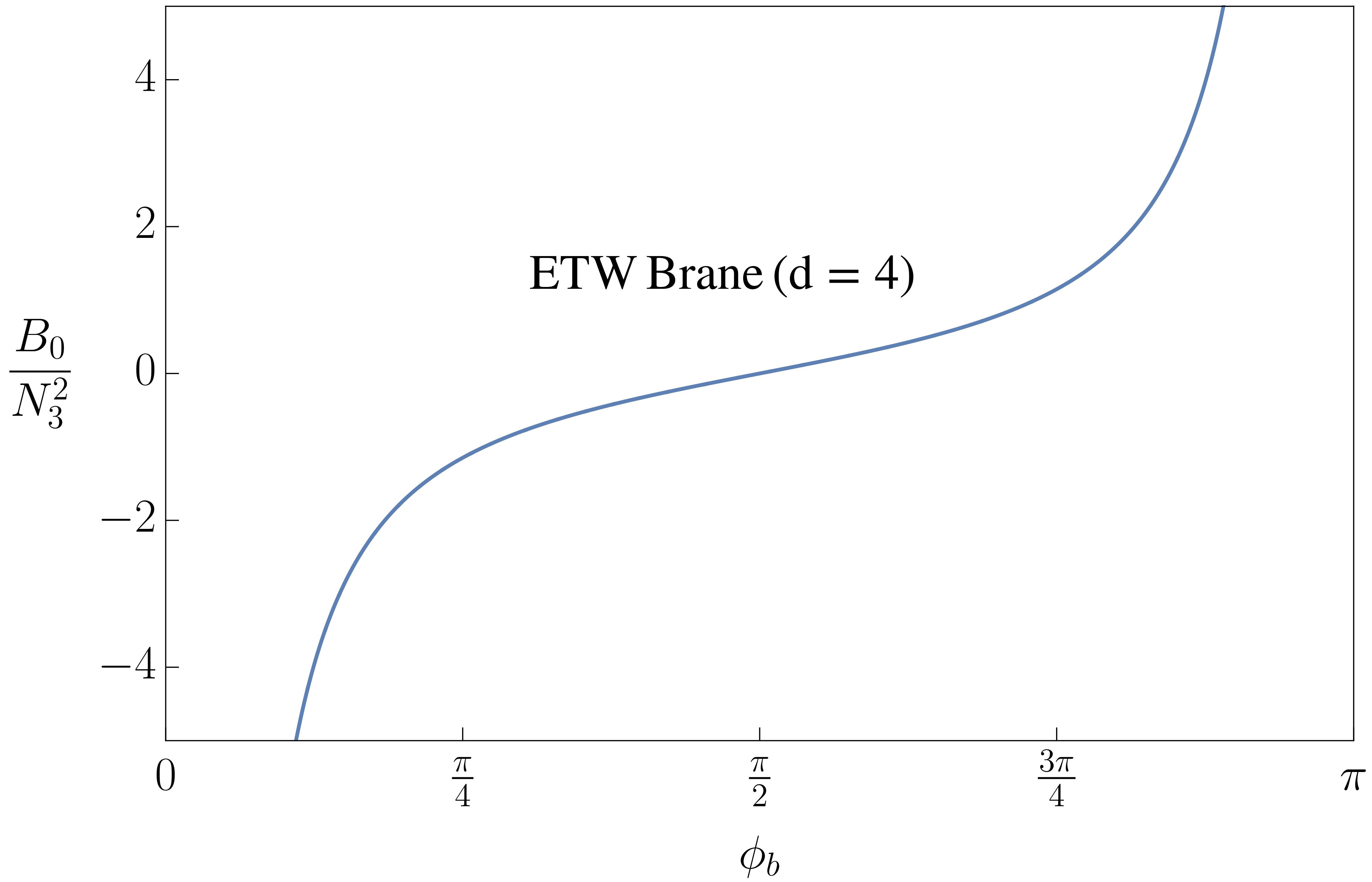
$D_0 = 2B_0$ *for the same* ρ_0

Example: $d = 4$

$$\mathcal{S}_{\text{bdy}} = \frac{a_1}{\epsilon} - B_0 + O(\epsilon)$$

Negative sign implicit in plot below

Example: $d = 4$



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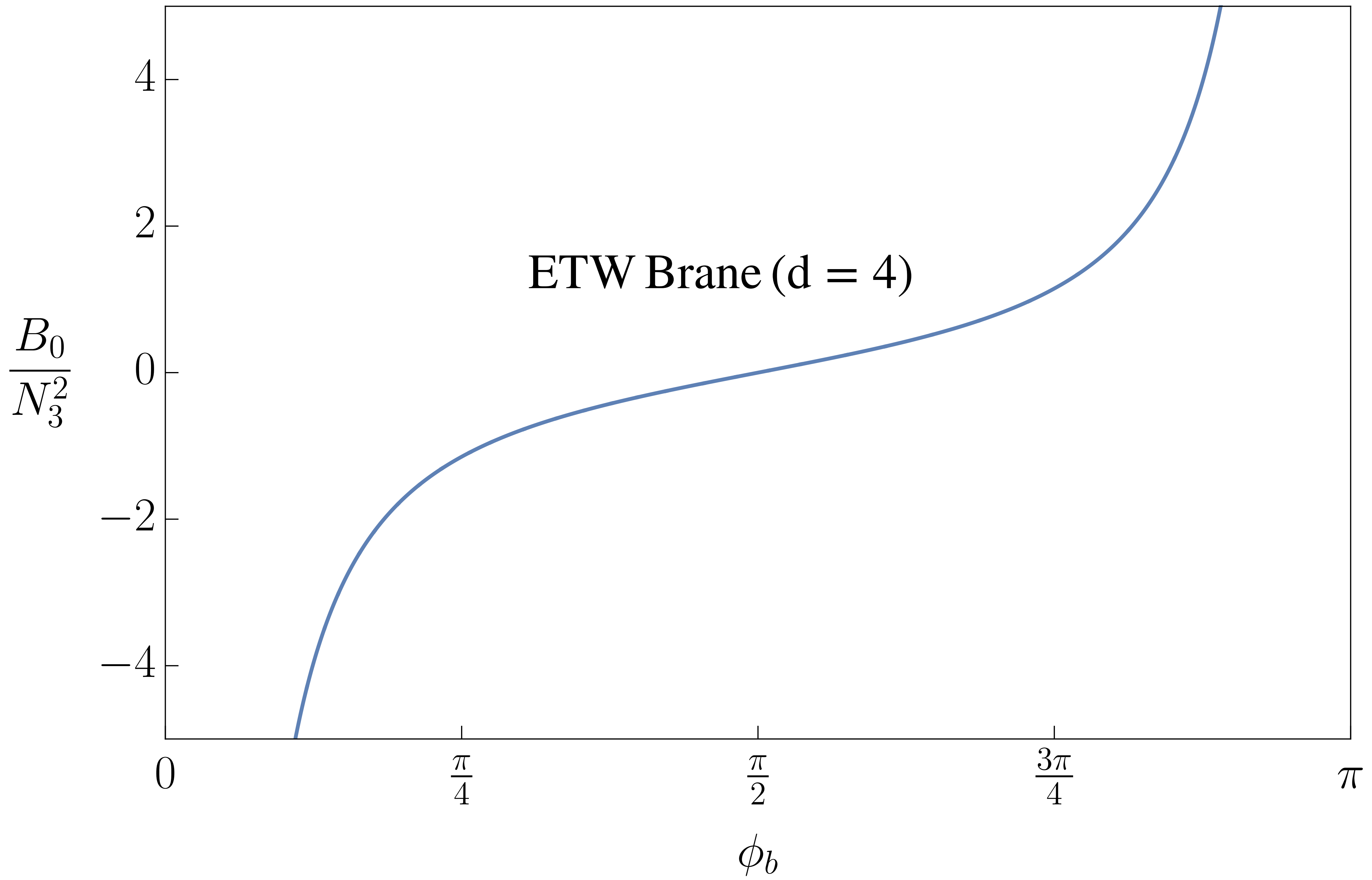

Universal entropy B_0 is monotonically increasing with ϕ_b, but is bounded from above.

$B_0 < 0 \to B_0 > 0$ identically to tension T.

All of the above is also the case for D_0.

Overview

- **Intro:** AdS/B/DCFT & Holographic Entanglement Entropy [1/5]: [5/5]
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List of Theories & Current Status

Holographic Theory	**Dual dimension** d	ϕ_b	D_0/B_0
non-SUSY Janus	4		
SYM SUSY Janus	4		
D3/D5 Intersection	4		
D1/D5 SUSY Janus	2		
M-Theory Janus	3		
Karch-Randall Braneworld	To-match		
D3/D5 BCFT	4		
M2/M5 BCFT	4		
ETW Brane Toy Model	To-match		

List of Theories & Current Status

[Estes et al., 2014]
[Chiodaroli, Gutperle, Hung, 2010]

Holographic Theory	Dual dimension d	ϕ_b	D_0/B_0
non-SUSY Janus	4		✓
SYM SUSY Janus	4		✓
D3/D5 Intersection	4		✓
D1/D5 SUSY Janus	2	*DCFT*	✓
M-Theory Janus	3		✓
Karch-Randall Braneworld	To-match		✓
D3/D5 BCFT	4		✓
M2/M5 BCFT	4		
ETW Brane Toy Model	To-match	✓	✓

Top-Down: Obtaining ϕ_b

1. Start with the geometry from our list of top-down constructions:

$$ds^2 = f_1^2 ds^2_{\mathrm{AdS_d}} + (\mathrm{otherterms})_{\mathrm{D-d}}$$

This is to try and minimize the causal depth due to the constraints of the bottom-up models.

2. Find the shortest geodesic path from the conformal boundary to the gravitating boundary (or domain-wall), and find the normal-coordinate (*called ρ here*).

$$ds^2 = f_1^2 ds^2_{\mathrm{AdS_d}} + f_2^2 d\rho^2 + (\mathrm{otherterms})_{\mathrm{D-d-1}}$$

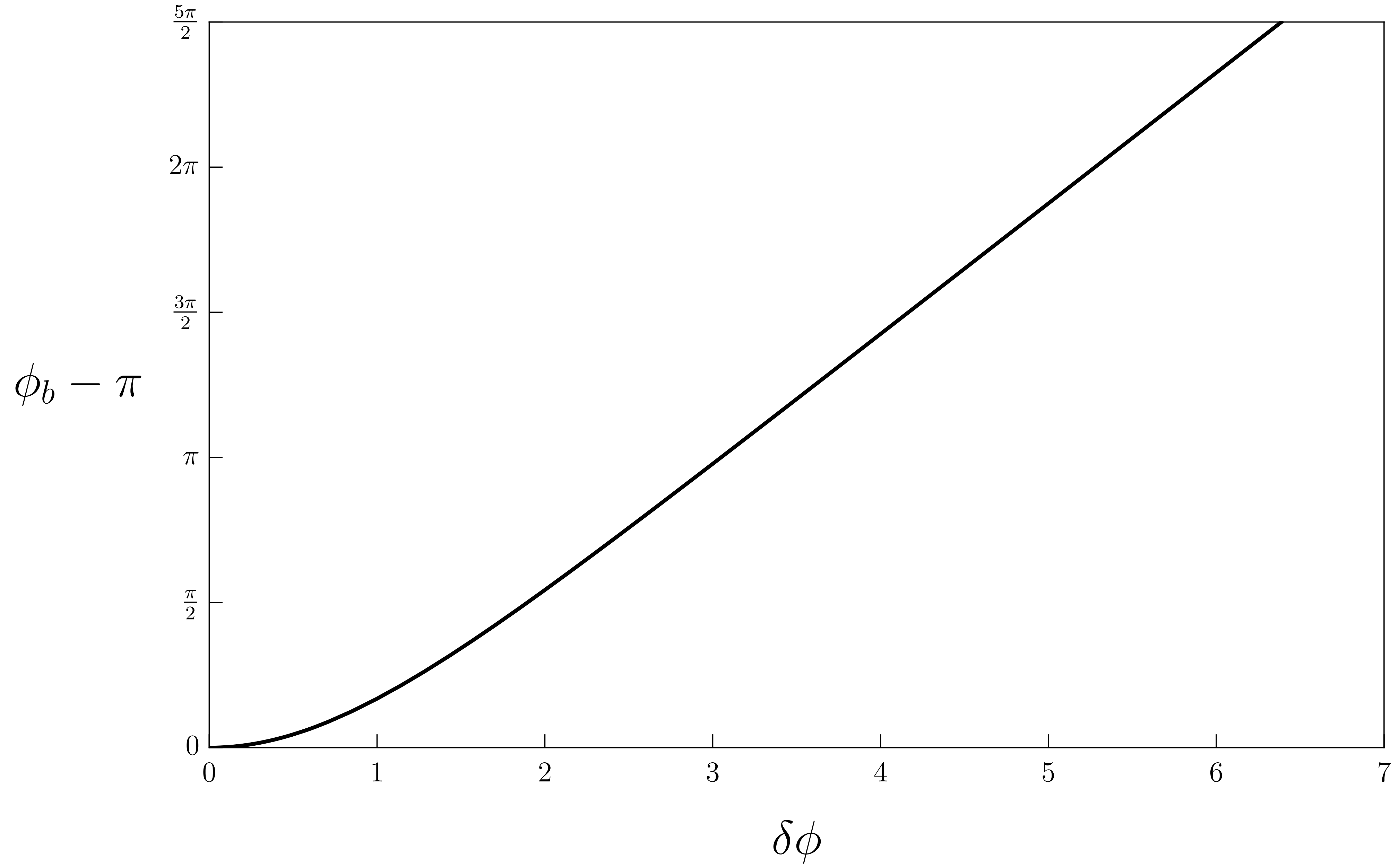
Some of these may have vanished.

3. Extract the light-crossing integrand:

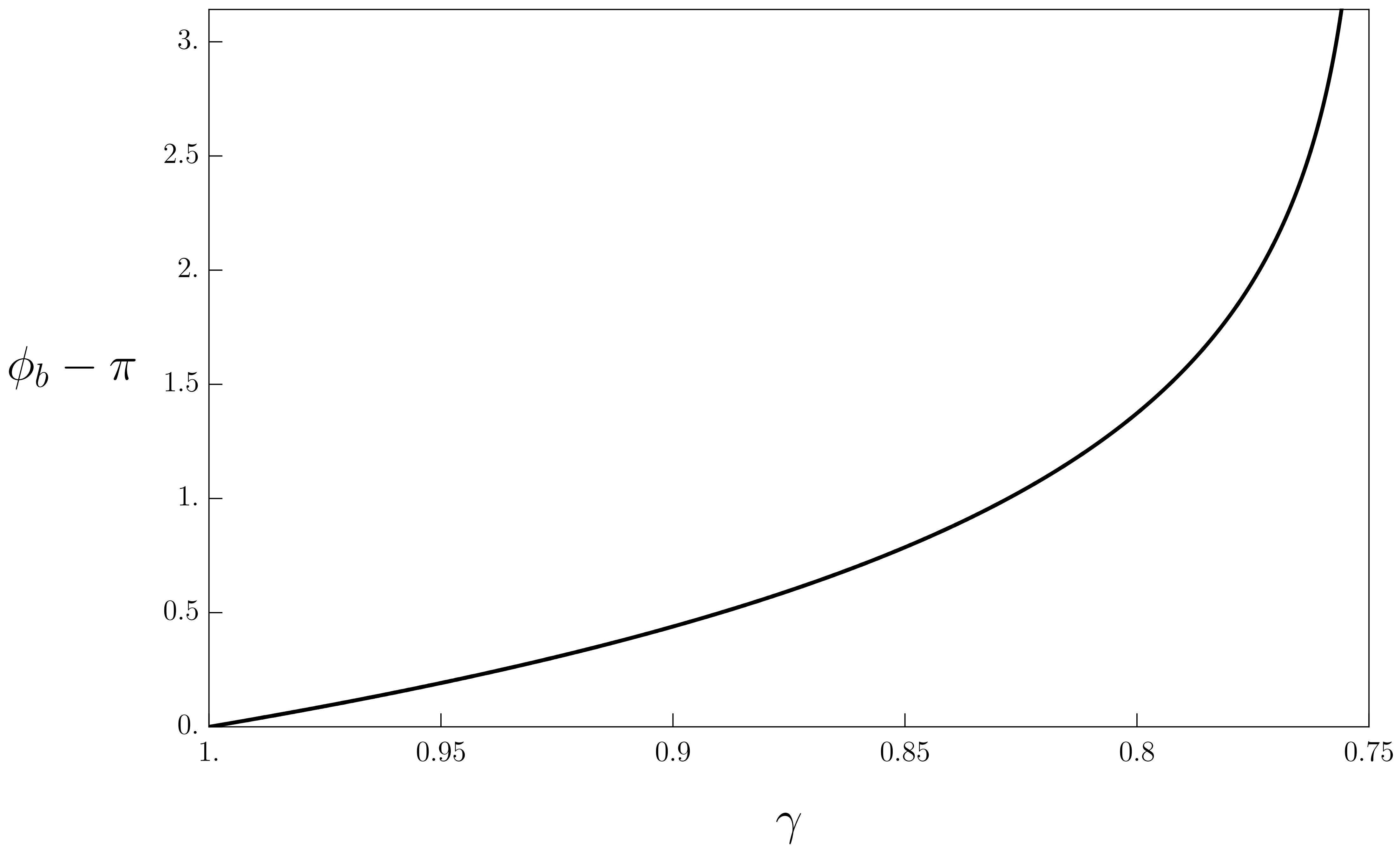
$$e^{-A(\rho)} d\rho = \frac{f_2}{f_1} d\rho$$

and then integrate to obtain ϕ_b.

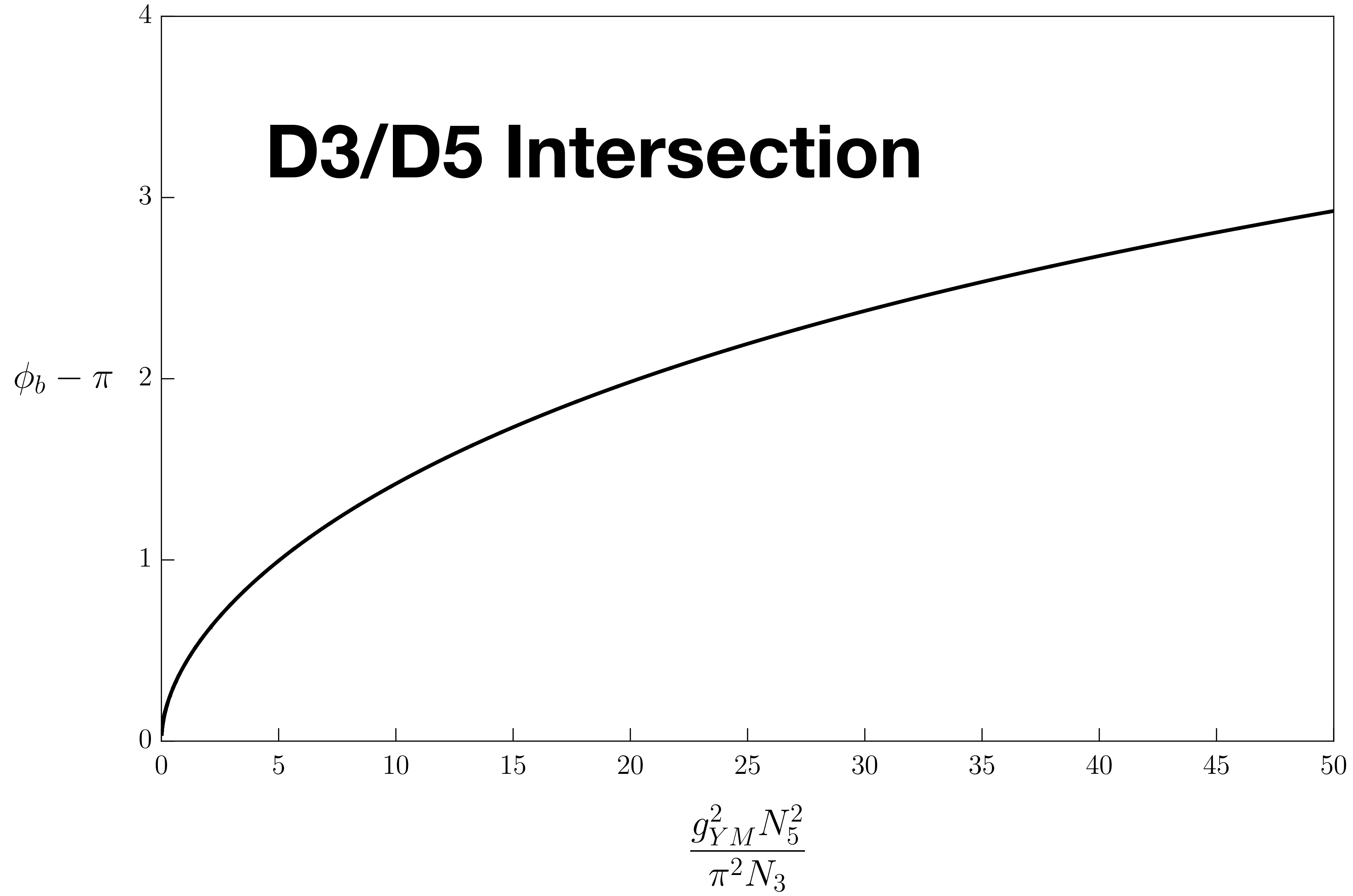
SYM SUSY Janus



non-SUSY Janus



D3/D5 Intersection

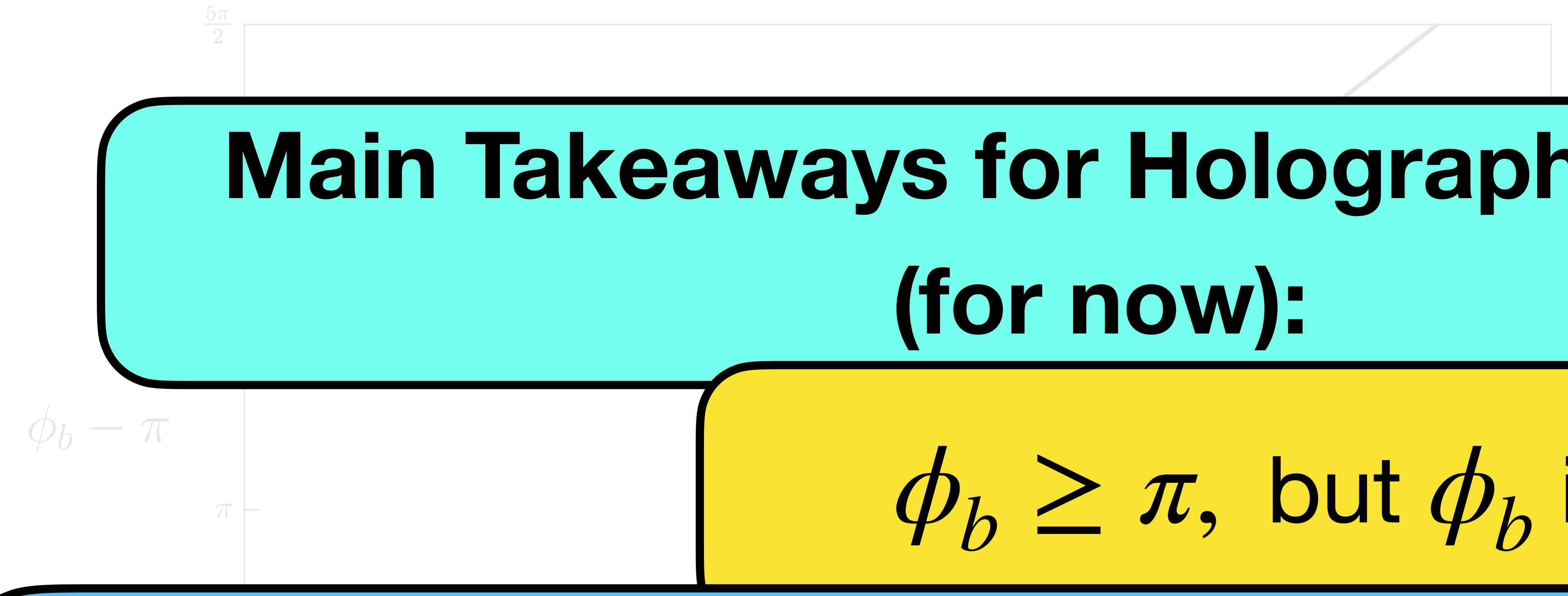


Agrees with DCFT Result: [M. Chiodaroli, J. Estes, and Y. Korovin, 2017], [Reeves, et. al., 2021]

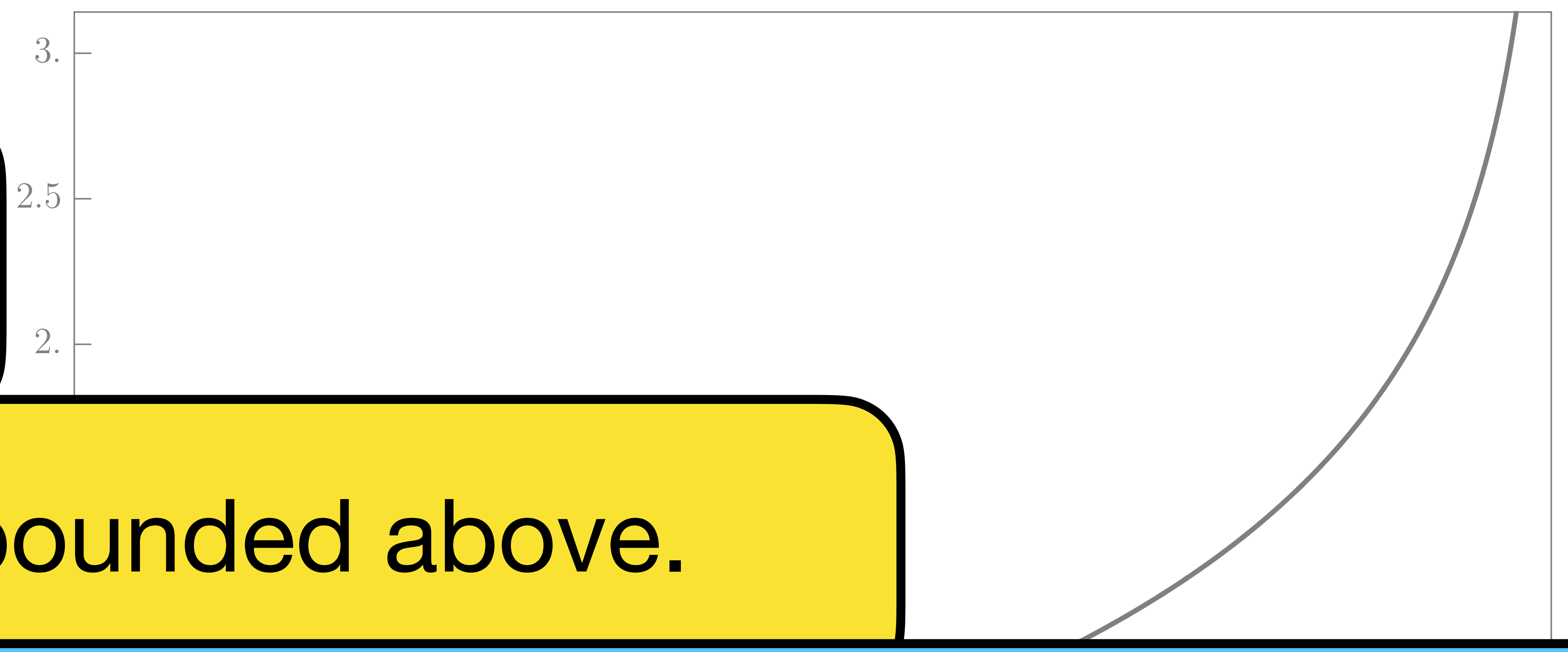
D1/D5 Intersection $\phi_b = \kappa\pi, \quad \kappa \geq 1$

M-Theory Janus $\phi_b = \sqrt{1+\lambda^2}\,\pi, \quad \lambda \in \mathbb{R}^+$

SYM SUSY Janus



non-SUSY Janus

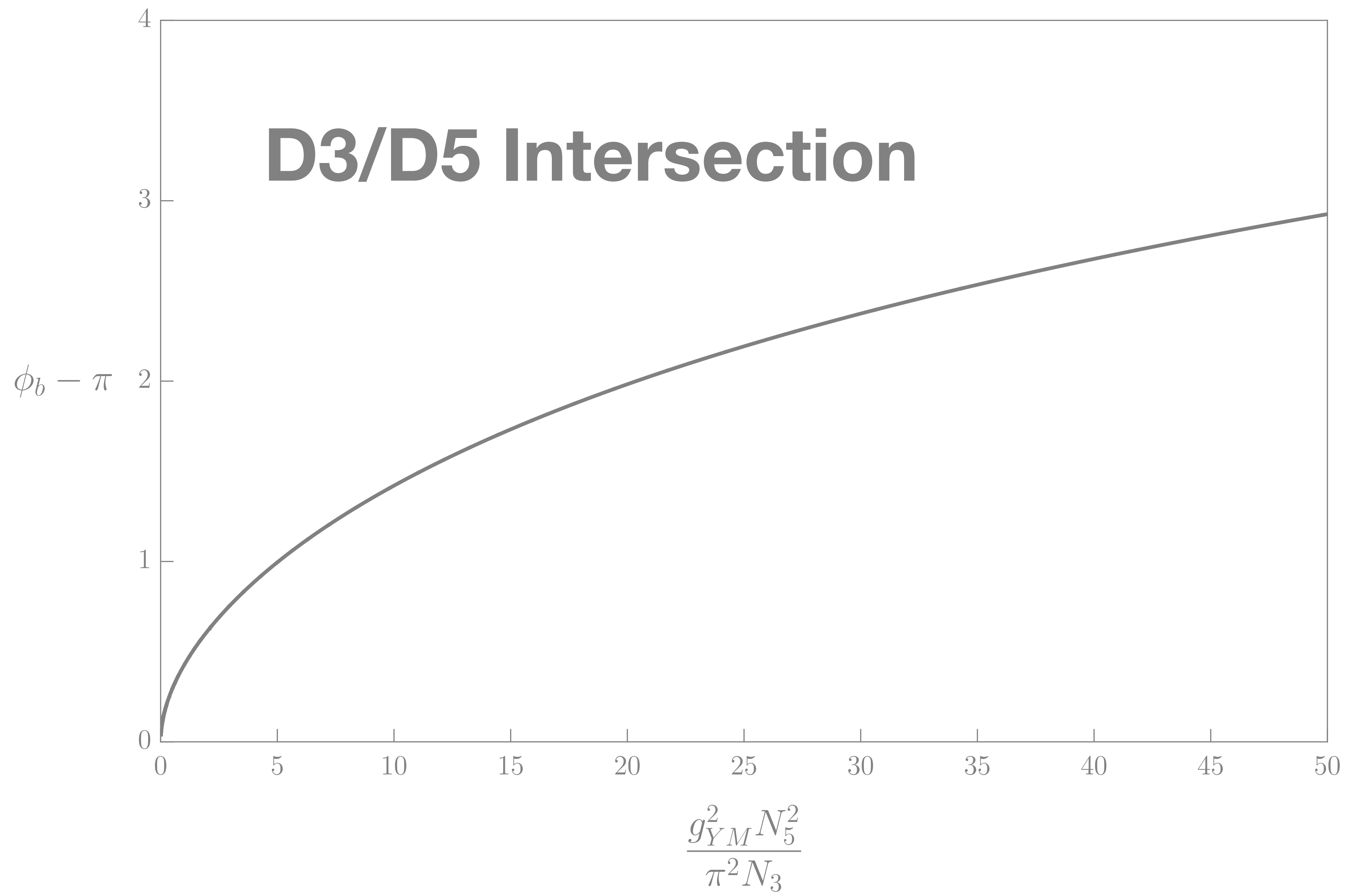


Main Takeaways for Holographic <u>DCFTs</u> (for now):

$\phi_b \geq \pi$, but ϕ_b is *mostly* unbounded above.

Non-SUSY Janus is bounded via $0.75 \leq \gamma \leq 1$, where the lower-bound corresponds to a linear dilation solution.

[D. Bak, M. Gutperle, S. Hirano, 2003], [E. D'Hoker, J. Estes, and M. Gutperle, 2006]



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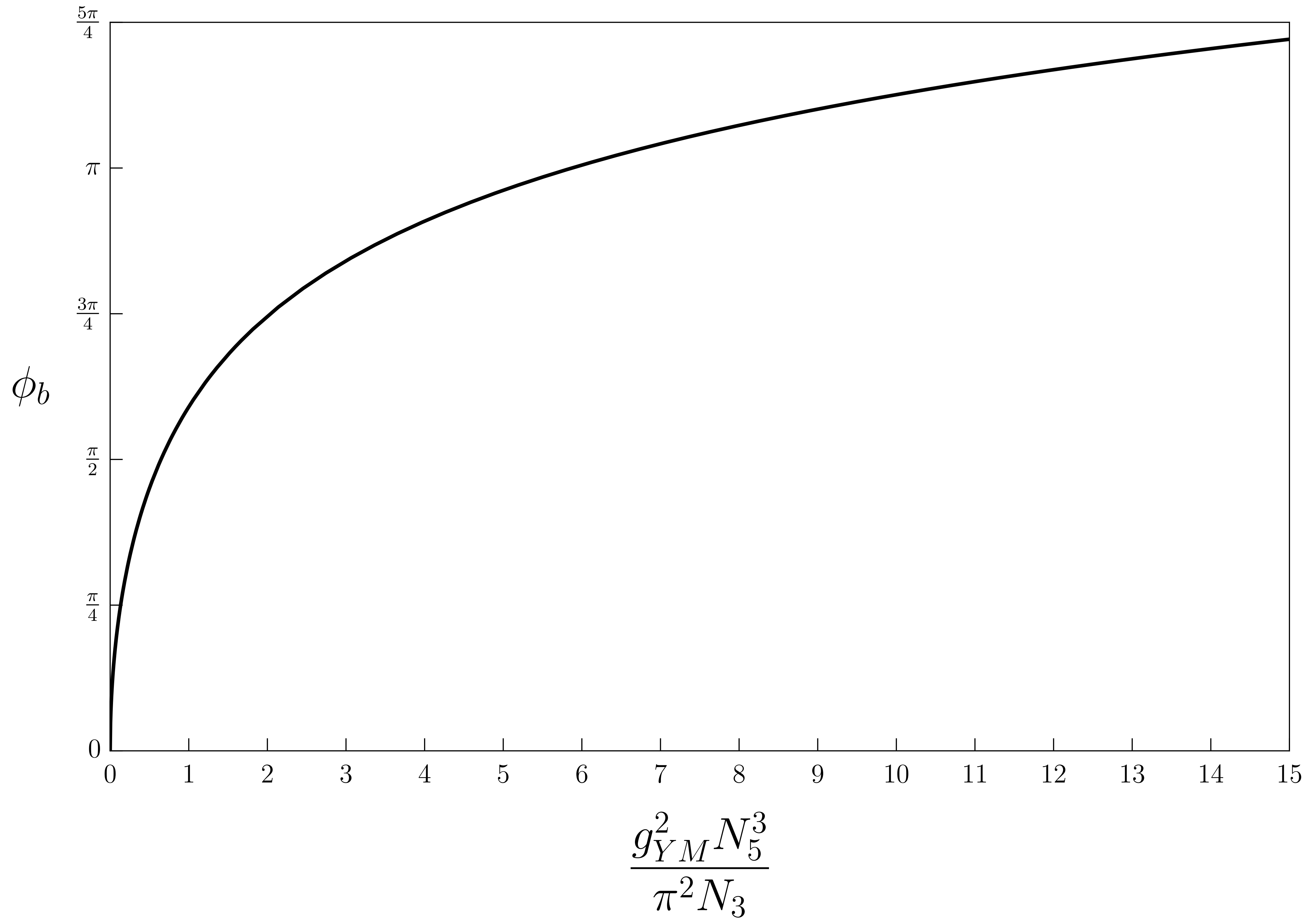
Top-down $\phi_b \geq \pi \not\Leftrightarrow T < 0$ for **Bottom-up**.

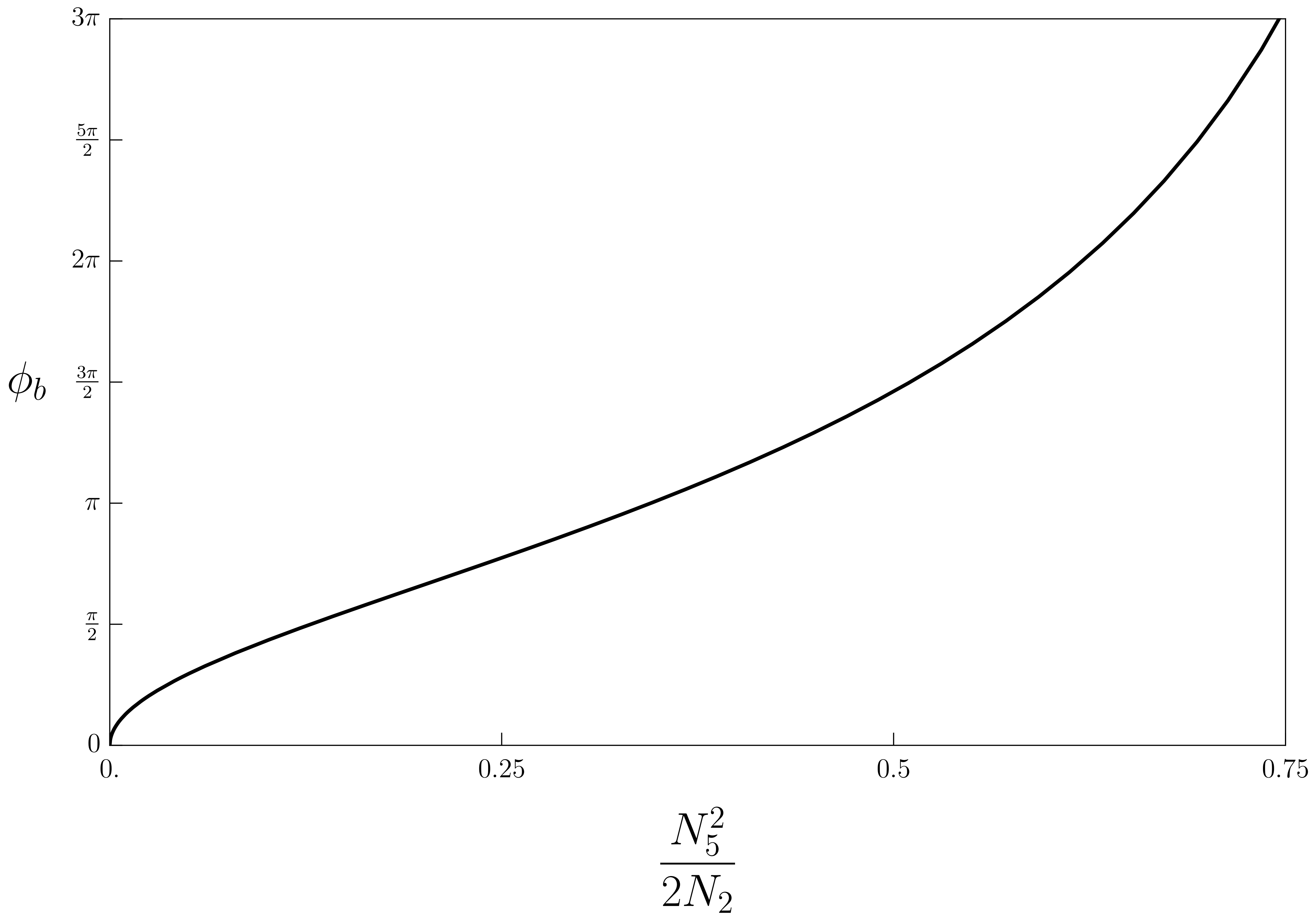
$\therefore$ A bottom-up KR model can only hope to mimic the top-down theories when $T > 0$.

Even then, only *up to* $\phi_b = 2\pi \Rightarrow T = 2(d-1)$

D3/D5 BCFT



M2/M5 BCFT

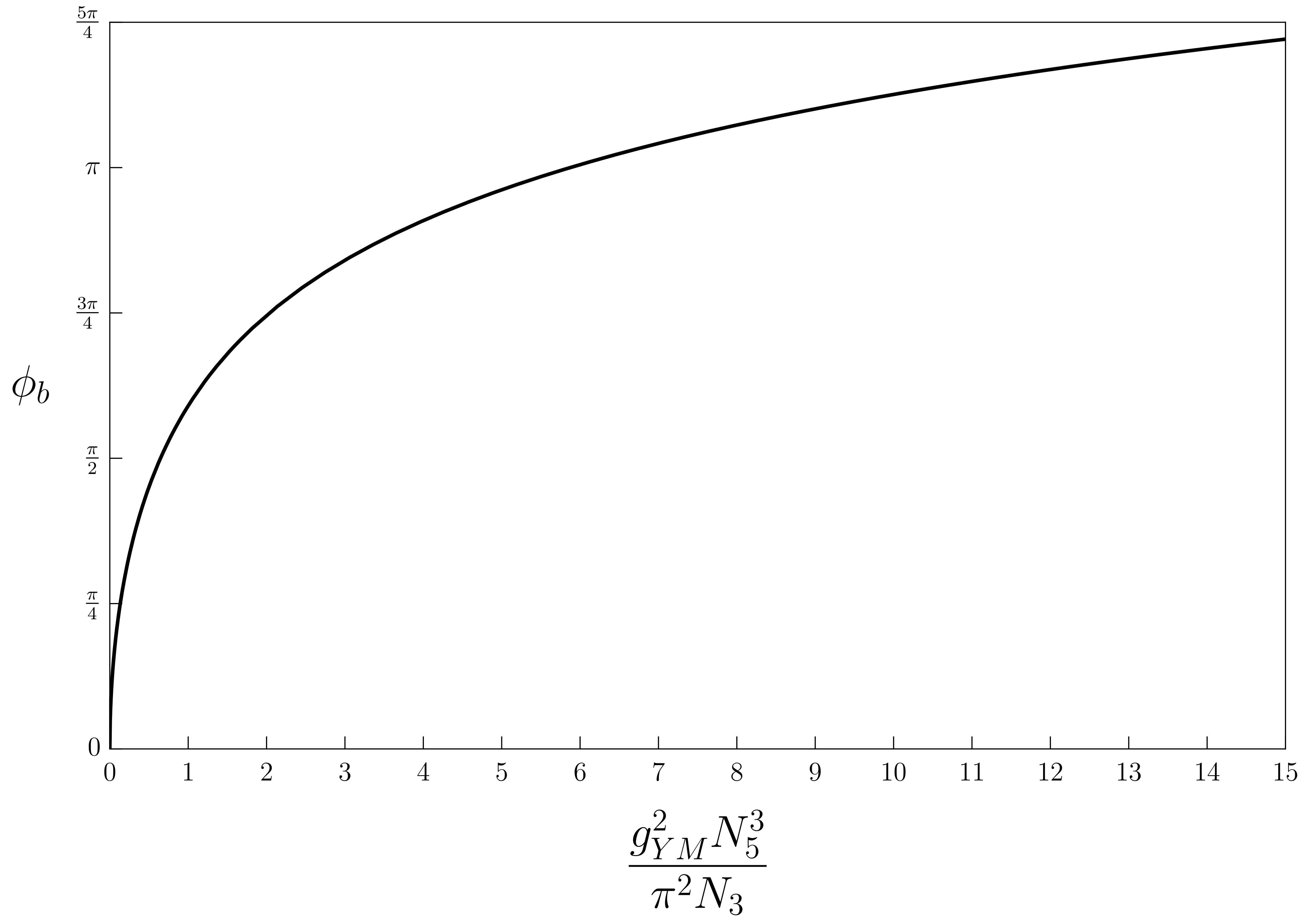


Main Takeaways for Holographic BCFTs (for now):

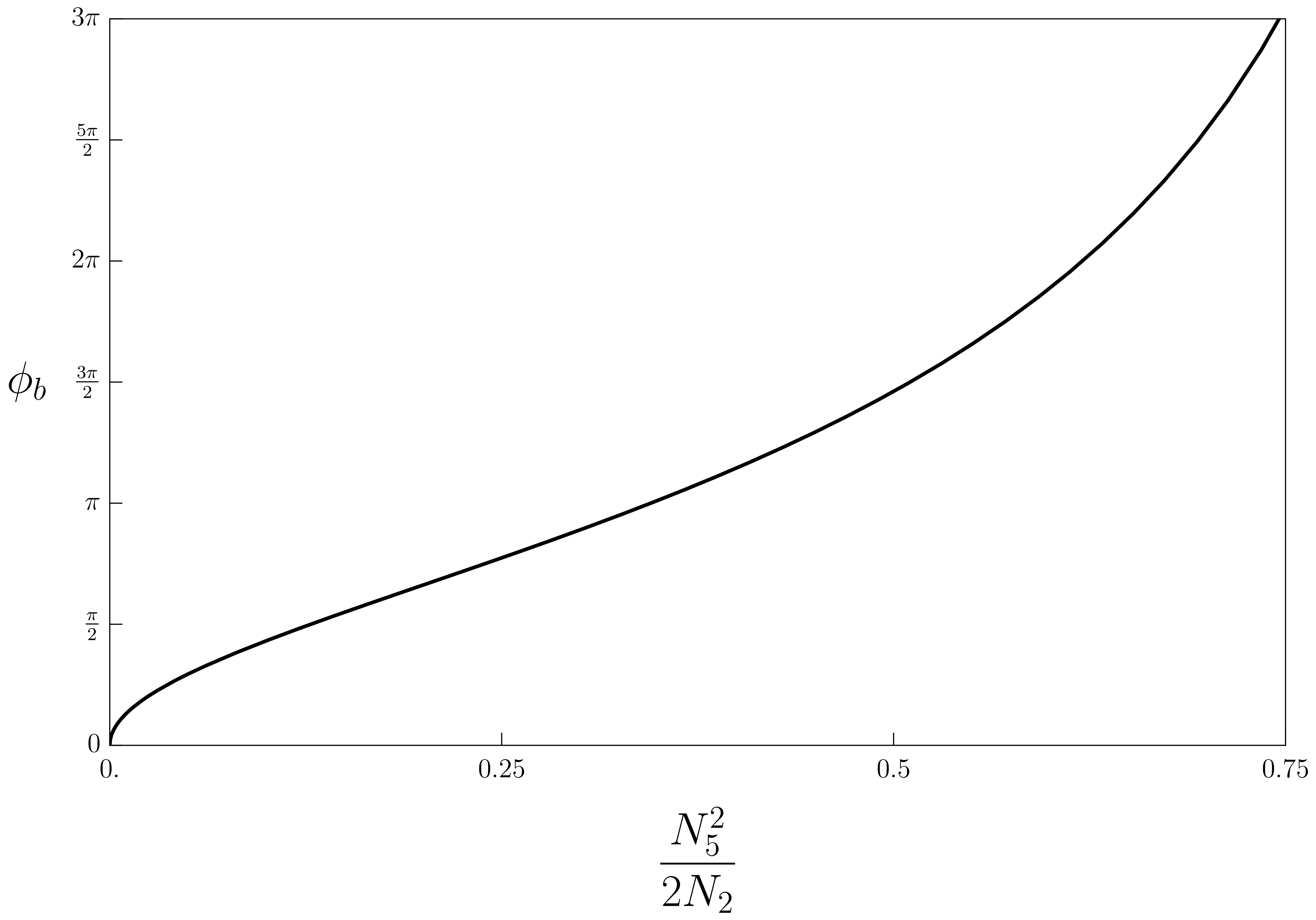
$\phi_b \geq 0$, but ϕ_b is also unbounded above.

[4/5]: [4/4]

D3/D5 BCFT



M2/M5 BCFT



Main Takeaways for Holographic <u>BCFTs</u> (for now):

$\phi_b \geq 0$, but ϕ_b is also unbounded above.

Top-down $\phi_b \geq 0 \Leftrightarrow T < 0$ for **Bottom-up** *is possible.*

Toy model still limited up to $\phi_b = \pi \Rightarrow T = (d-1)$

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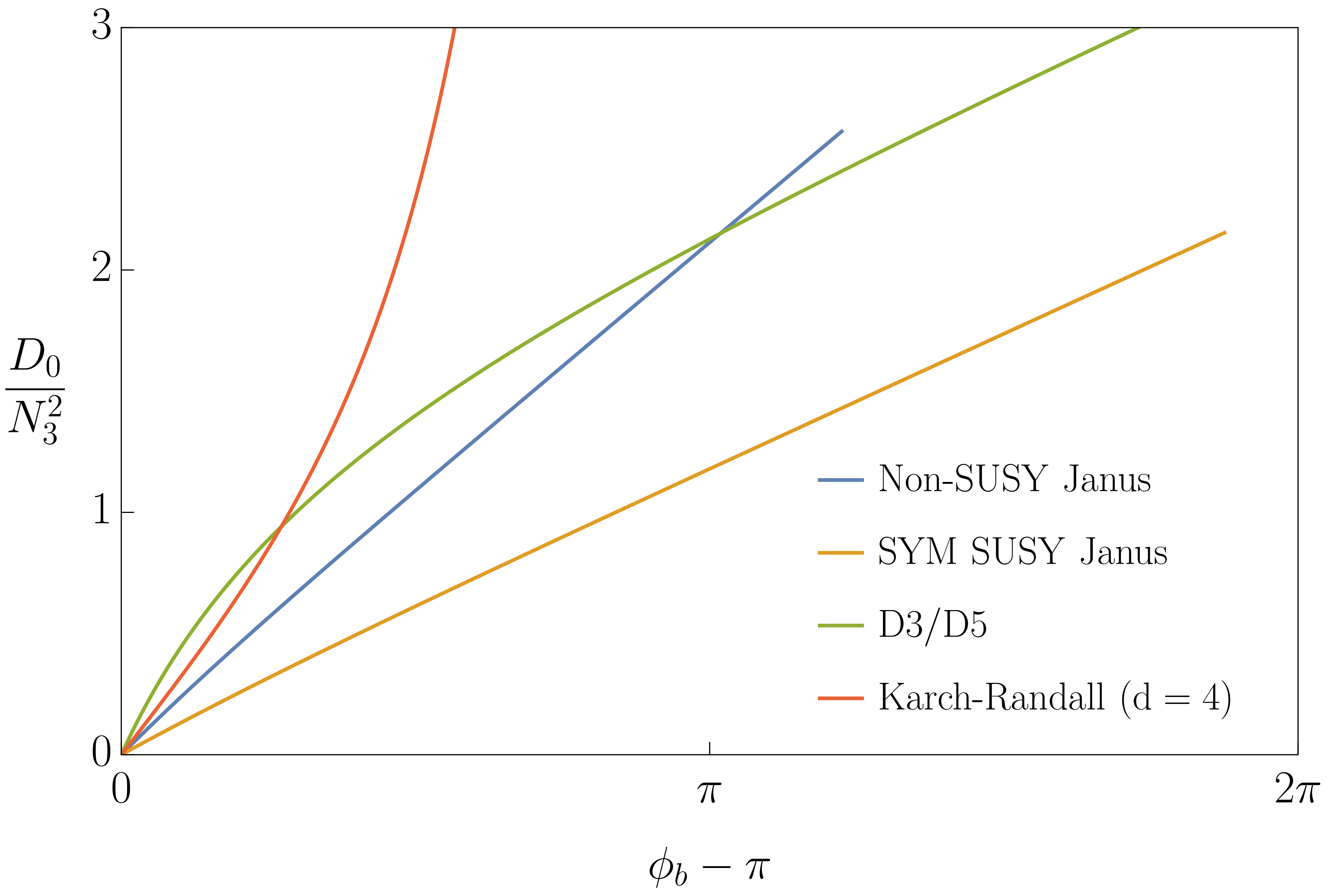
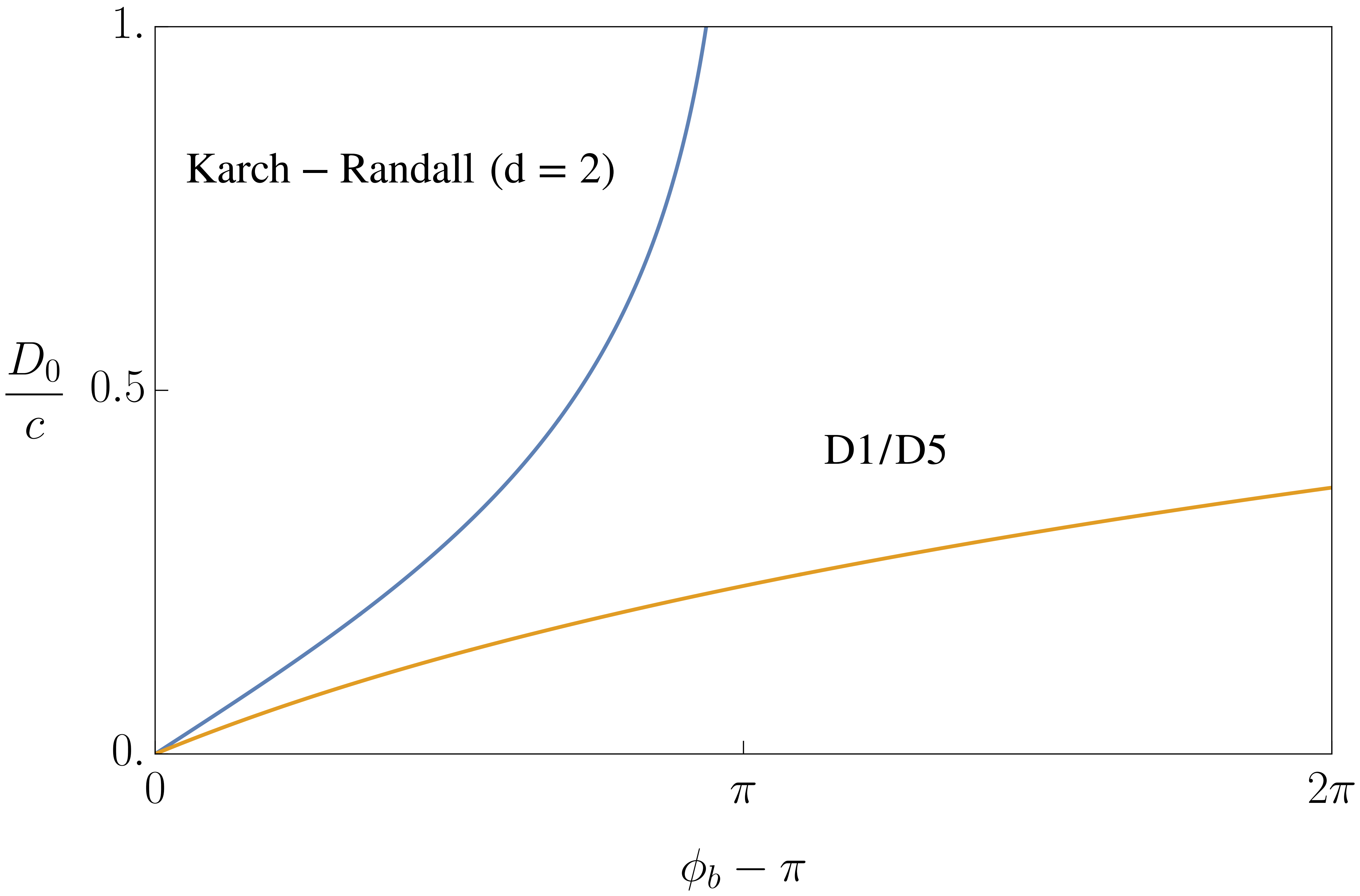
All Comparison Results: Defects

M-theory Janus: $D_0 = 0$





Book keeping:

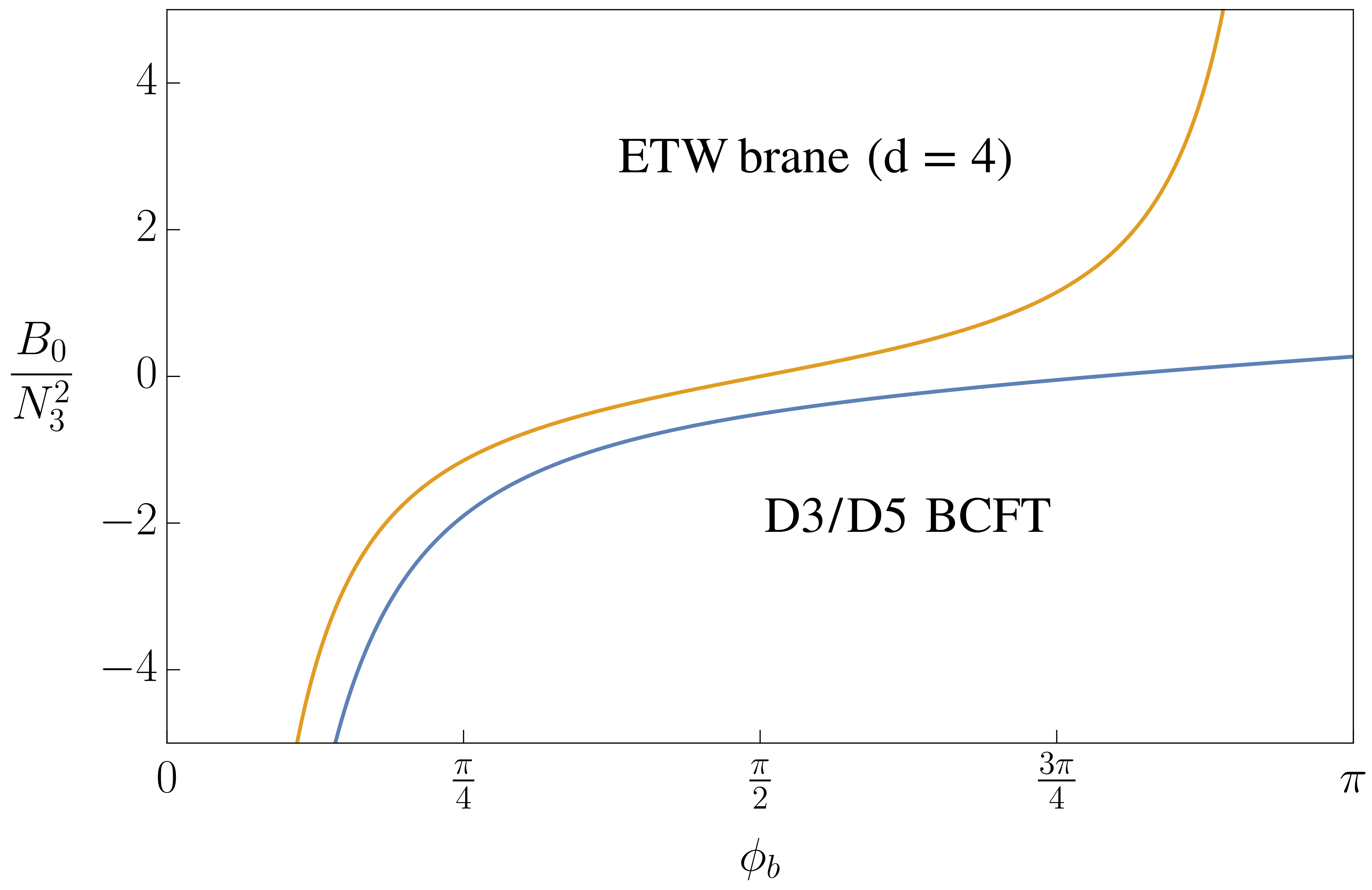
$d = 4,3,2$

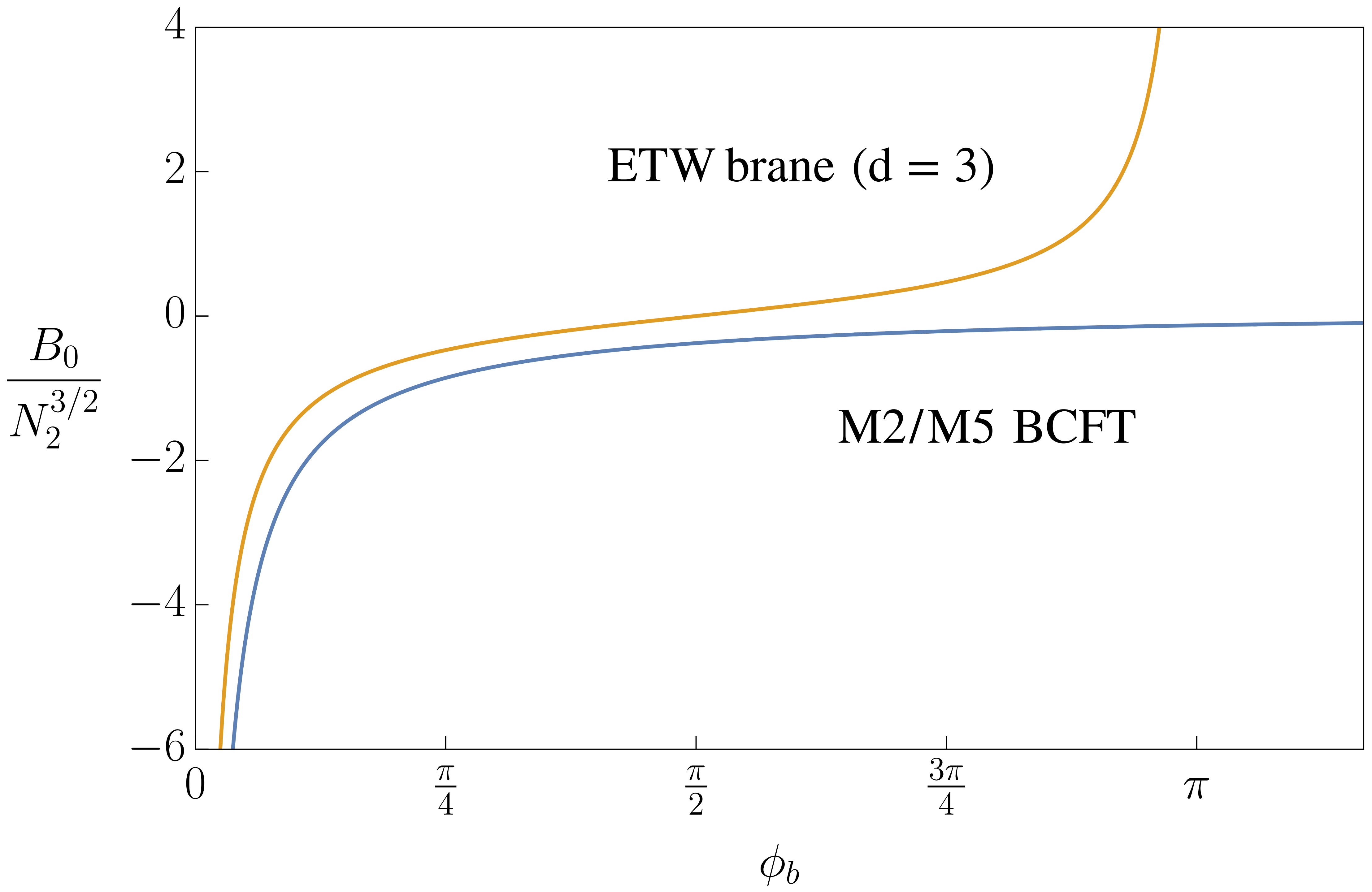
$$\frac{L^8 \mathrm{vol}(\mathbb{S}^5)}{16\pi G_{10}} = \frac{L^8 \pi^3}{(2\pi)^7 l_p^8} = \frac{L^3}{16\pi G_5} \Rightarrow \frac{L^3}{G_5} = \frac{2}{\pi} N_3^2$$

$$\frac{L^2 (2L)^7 \mathrm{vol}(\mathbb{S}^7)}{16\pi G_{11}} = \frac{L^9}{6\pi^4 l_p^9} = \frac{L^2}{16\pi G_4} \Rightarrow \frac{L^2}{G_4} = \frac{2^{3/2}}{3} N_2^{3/2}$$

$$c = \frac{12\pi^2 L^2}{G_6} = \frac{3L}{2G_3}$$

All Comparison Results: Boundaries





Regime of small ϕ_b (negative tension for the bottom-up model) shows close agreement.

$$\frac{B_0}{N_3^2} = \begin{cases} -\frac{1}{2\phi_b^2} + \frac{1}{2}\ln(\phi_b) + \mathcal{O}(\phi_b^0) & \text{Bottom-up}\,, \\ -\frac{\pi^2}{12\phi_b^2} + \frac{1}{2}\ln(\phi_b) + \mathcal{O}(\phi_b^0) & \text{D3/D5} \end{cases}$$

$$\frac{B_0}{N_2^{3/2}} = \begin{cases} -\frac{2}{3\sqrt{2}\phi_b} + \mathcal{O}(\phi_b) & \text{Bottom-up} \\ -\frac{1}{\sqrt{2}\phi_b} + \mathcal{O}(\phi_b) & \text{M2/M5} \end{cases}$$

[5/5]: [2/3]

Summary

Bottom-up Models:

1. ϕ_b is bounded from above, and therefore cannot mimic top-down constructions above this regime.

$$\phi_b \in \begin{cases} [0,\pi] & \text{boundary} \\ [0,2\pi] & \text{defect} \end{cases}$$

2. Mid-point of each bound (going up) changes the sign of the brane tension $T < 0 \rightarrow T > 0$.

Top-Down Defects: $\phi_b \geq \pi$, and *most* are unbounded from above.

Non-SUSY bounded by $0.75 \leq \gamma \leq 1$.

Importantly: No negative tension analogue.

Top-Down Boundaries:

1. $\phi_b \geq 0$, and unbounded from above.

2. Good agreement with the bottom-up model when $T < 0$.

M2/M5 BCFT: B_0 monotonically increases with ϕ_b just like the rest.

$$S_{d=3,\mathrm{BCFT}} \sim B_0 \ln\left(\frac{R}{\epsilon}\right) \sim \frac{b}{3}\ln\left(\frac{R}{\epsilon}\right) \qquad \textit{with} \qquad b_{UV} - b_{IR} \geq 0$$

D3/D5 BCFT

For sufficiently small N_5, the geometry becomes AdS_5 cut-off by an effective ETW brane.

Predicted by: [Coccia, Uhlemann, 2022]

Thank You!