Computing Transmission Coefficients of CFT Interfaces via Phantom Currents

Yuma Furuta
with Yuya Kusuki (Kyushu Univ.), Toshiki Onagi (Kyoto Univ.)

Kyushu University

Based on arXiv:2511.00356 (Phys.Rev.D 113 (2026) 4)

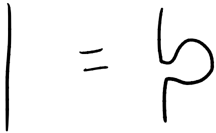
Summary

- We propose a universal mechanism to determine the transmission coefficient of a conformal interface between two CFTs.
- Key input: a spin-2 current/operator that can *emerge only after folding* the interface into a boundary problem.
- Under suitable assumptions and physical consistency requirements, we obtain a general closed-form solution for the transmission.
- The framework reproduces known explicit examples.

Topological interface

- An interface is a defect operator separating two CFTs.
- A **topological** interface commutes with the stress tensor and can be freely deformed.
- It has been investigated from long ago, mathematically and physically. (e.g. Verlinde lines)

$$[T, D] = 0 \quad \Longleftrightarrow \quad \text{topological / freely deformable interface.}$$

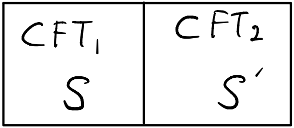


Non-topological (conformal) interface

- A general conformal interface need not commute with the stress tensor.
- Nevertheless, conformal symmetry *along the defect* is preserved via the continuity of the diagonal Virasoro:

$$T_1 - \bar{T}_1 \; = \; T_2 - \bar{T}_2 \qquad \text{at the interface.}$$

- Typical example: **RG interface**.

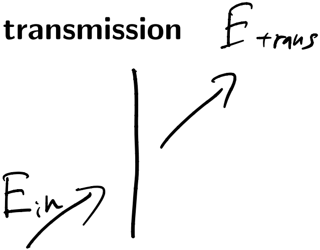


$$S' = S + \lambda \int_{x>0} dx\, \mathcal{O}(x)$$

Transmission coefficient

- Full characterization of the interface (e.g. the boundary state) is often difficult.
- Instead, measure a robust observable: the **transmission coefficient**.

$$\mathcal{T}_{\mathrm{trans}} = \frac{E_{\mathrm{trans}}}{E_{\mathrm{in}}}.$$



- It can be extracted from a stress-tensor two-point function [Meineri, Penedones, Rousset (2019)]:

$$\mathcal{T}_{\mathrm{trans}} = \frac{2\langle T_{\mathrm{L}} T_{\mathrm{R}} \rangle_I}{c_{\mathrm{L}} + c_{\mathrm{R}}}.$$

Example: free compact boson

- Massless scalar compactified on a circle of radius R (a $c = 1$ CFT).
- Consider an interface connecting radii R_1 and R_2.
- Using the $U(1)$ currents and their gluing, one can compute $\mathcal{T}_{\mathrm{trans}}$ [Bachas,de Boer, Dijkgraaf, Ooguri (2002)].

$$\begin{pmatrix} \partial\phi_{\mathrm{L}} \\ \bar{\partial}\phi_{\mathrm{R}} \end{pmatrix} = - \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix} \begin{pmatrix} \bar{\partial}\phi_{\mathrm{L}} \\ \partial\phi_{\mathrm{R}} \end{pmatrix}.$$

$$\Longrightarrow \qquad \mathcal{T}_{\mathrm{trans}} = \sin^2(2\theta).$$

How to use the gluing condition?

- Energy-Momentum tensor T_L is defined as

$$T_L =: \partial\phi_L \partial\phi_L :$$

and same for R.

- From the gluing condition, one can obtain

$$\begin{aligned} T_L &=: (\cos 2\theta \bar{\partial}\phi_L + \sin 2\theta \partial\phi_R)^2 : \\ &= \cos^2 2\theta \bar{T}_L + \sin^2 2\theta T_R + \sin 4\theta \bar{\partial}\phi_L \partial\phi_R \end{aligned}$$

- Multiplying T_R to both sides, we have

$$\mathcal{T}_{\mathrm{trans}} = \sin^2 2\theta$$

Example: Gaiotto's RG interface

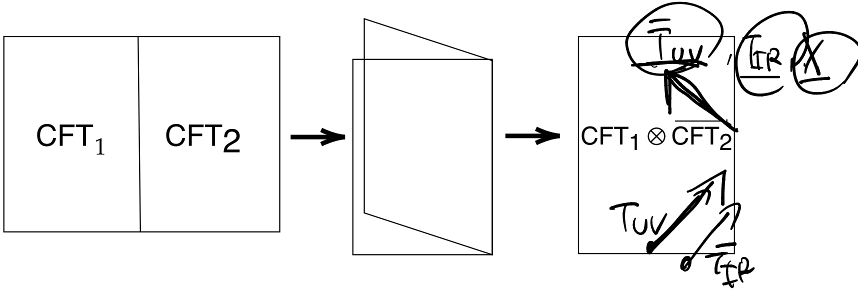
- Consider minimal models $M(p, p+1)$.
- Deform the action by the relevant operator $\phi_{1,3}$.
- The IR fixed point is again a CFT: $M(p-1, p)$ (Zamolodchikov RG flow).
- This interface was analyzed by [Gaiotto (2012)].

$$\langle T_{\rm UV}\ T_{\rm IR}\rangle = \frac{(k-1)(k+5)}{(k+2)^2}, \qquad k+2=p.$$

Folding and an emergent spin-2 current

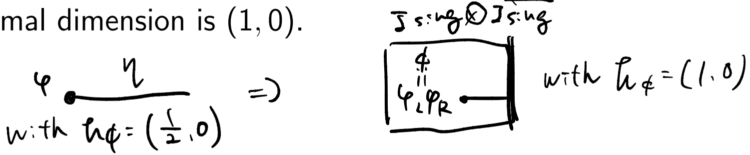
- For Zamolodchikov RG flow, folding yields a new **spin-2 "phantom current"** built from UV/IR data.
- This current appears when we act the interface to the stress tensors:

$$
\begin{cases}
T_{\rm UV} = \dfrac{3}{k(k+2)}\,\bar{T}_{\rm UV} + \dfrac{(k+1)(k+5)}{(k+2)(k+4)}\,T_{\rm IR} - \dfrac{1}{k(k+2)(k+4)}\,\bar{\phi}^{\rm UV}_{1,3}\,\phi^{\rm IR}_{3,1} \\[2ex]
\bar{T}_{\rm IR} = \dfrac{(k-1)(k+3)}{k(k+2)}\,\bar{T}_{\rm UV} + \dfrac{3}{(k+2)(k+4)}\,T_{\rm IR} + \dfrac{1}{k(k+2)(k+4)}\,\bar{\phi}^{\rm UV}_{1,3}\,\phi^{\rm IR}_{3,1}
\end{cases}
$$



More detail about the current

- Folding maps an interface problem to a boundary CFT problem.
- New currents can appear only after folding (e.g. Ising×Ising can contain a $U(1)$ current).
- Threre are some cases that we have a new current operator emerging after folding [Antinucci, Copetti, Galati, Rizi (2025)].
- One example: Ising model with a conformal defect
- Single Ising model does not have a operator with $h = (1, 0)$, but after folding along the defect, we have a current ϕ whose conformal dimension is $(1, 0)$.



More general setup

- Goal: determine $\mathcal{T}_{\mathrm{trans}}$ for a general conformal interface.
- Assumption: folding produces a spin-2 current operator of the form $X \sim \bar{\phi}_{\mathrm{L}}\phi_{\mathrm{R}}$ in the folded theory.

$$T_{\mathrm{L}} = \alpha\,\bar{T}_{\mathrm{L}} + \beta\,T_{\mathrm{R}} + \gamma\,\bar{\phi}_{\mathrm{L}}\phi_{\mathrm{R}}, \qquad \bar{T}_{\mathrm{R}} = \alpha'\,\bar{T}_{\mathrm{L}} + \beta'\,T_{\mathrm{R}} + \gamma'\,\bar{\phi}_{\mathrm{L}}\phi_{\mathrm{R}}.$$

- The coefficient β controls transmission:

$$\mathcal{T}_{\mathrm{trans}} = \frac{c_{\mathrm{R}}\,\beta}{c_{\mathrm{L}} + c_{\mathrm{R}}}.$$

Physical requirements

1. **Diagonal Virasoro is preserved**:

$$T_{\mathrm{L}} + \bar{T}_{\mathrm{R}} = \bar{T}_{\mathrm{L}} + T_{\mathrm{R}}.$$

2. **Central charge matching** (normalization consistency):

$$c_{\mathrm{L}} = \alpha^2 c_{\mathrm{L}} + \beta^2 c_{\mathrm{R}} + 2\gamma^2 N_X, \qquad c_{\mathrm{R}} = {\alpha'}^2 c_{\mathrm{L}} + {\beta'}^2 c_{\mathrm{R}} + 2{\gamma'}^2 N_X.$$

3. **Cluster decomposition**:

$$T_{\mathrm{L}}\, \bar{T}_{\mathrm{R}} \sim 0 \quad \text{(no long-range correlations across the boundary).}$$

Here N_X is the (model-dependent) normalization of the folded spin-2 operator X.

Solving the constraints

Solving the three conditions, for the non-trivial case $\gamma \neq 0$, $c_R h_R - c_L h_L \neq 0$ the system reduces to:

$$c_{\rm L} = \alpha\, c_{\rm L} + \beta\, c_{\rm R},$$

$$c_{\rm L}\, h_{\rm R}\, \alpha(1-\alpha) = c_{\rm R}\, h_{\rm L}\, \beta(1-\beta),$$

where $h_{\rm L}, h_{\rm R}$ are the conformal weights associated with $\phi_{\rm L}, \phi_{\rm R}$.

- This yields a general solution for (α, β):

$$\alpha = \frac{h_L(c_R - c_L)}{c_R h_R - c_L h_L},$$

$$\beta = \frac{c_L(h_R - h_L)}{c_R h_R - c_L h_L}$$

- The case $\gamma = 0$ corresponds to "trivial" solutions (topological or factorizing interfaces).

Particular solutions

- In particular, if $\gamma \neq 0$, $c_{\rm L} h_{\rm R} = c_{\rm R} h_{\rm L}$, and $c_{\rm L} \neq c_{\rm R}$, we have

$$(1 - \alpha)(c_{\rm L} - c_{\rm R}) = 0,$$

  which leads us to $\alpha = 1, \beta = 0$. Threrefore this case is topological.

- If $\gamma \neq 0$, $c_{\rm L} h_{\rm R} = c_{\rm R} h_{\rm L}$, and $c_{\rm L} = c_{\rm R}$, we have a one parameter solutions:

$$\alpha = \cos^2 x, \beta = \sin^2 x.$$

Consistency with known examples

- **Free boson:** one can parameterize $\beta = \sin^2 x$. With $c_{\mathrm{L}} = c_{\mathrm{R}} = 1$,

$$\mathcal{T}_{\mathrm{trans}} = \sin^2 x,$$

consistent with $\mathcal{T}_{\mathrm{trans}} = \sin^2(2\theta)$.

- **Gaiotto RG interface:** This corresponds to the case $c_{\mathrm{L}} h_R \neq c_{\mathrm{R}} h_L$. Solving in that setup reproduces

$$\langle T_{\mathrm{L}} T_{\mathrm{R}} \rangle_{I} = \frac{(k-1)(k+5)}{(k+2)^2}.$$

Summary

- We proposed a universal mechanism to determine $\mathcal{T}_{\text{trans}}$ for CFT interfaces.
- The derivation uses a spin-2 operator ("phantom current") that emerges upon folding.
- With reasonable assumptions and physical requirements, one can solve for $\mathcal{T}_{\text{trans}}$ in closed form.
- The result matches known explicit examples.

Future directions

- Explore alternative assumptions (e.g. multiple independent phantom currents).
- Use the resulting $\mathcal{T}_{\mathrm{trans}}$ to constrain fusion of interfaces.
- Formulate the existence/structure of the emergent spin-2 current in a more mathematical language.