Yang-Mills Flux Tube in AdS

Jiaxin Qiao (Kavli IPMU)
Kyoto, 2026.03.06

YITP-IAS (Kyushu Univ.) workshop
on Interfaces & Symmetry

Based on joint work with B. Gabai, V. Gorbenko **2508.08250 [hep-th]**

Related work on EST: B. Gabai, V. Gorbenko, B. Offertaler **2602.16694 [hep-th]**

WIP with: B. Gabai, V. Gorbenko, B. Offertaler, P. van Vliet (QCD);

M.Loparco, G.Mathys, J.Penedones, X.Zhao (QFT)

Outline

- Motivation
- QFT in AdS
- Yang-Mills in AdS
- Wilson loop (YM vs effective string theory)
- Non-linearly realized conformal symmetry
- Summary

Motivation

What defect?

- I am going to talk about Wilson line in gauge theories.

$$\mathcal{P}\exp\left[i\int A\right]$$

- I want to see confinement in perturbation theory.

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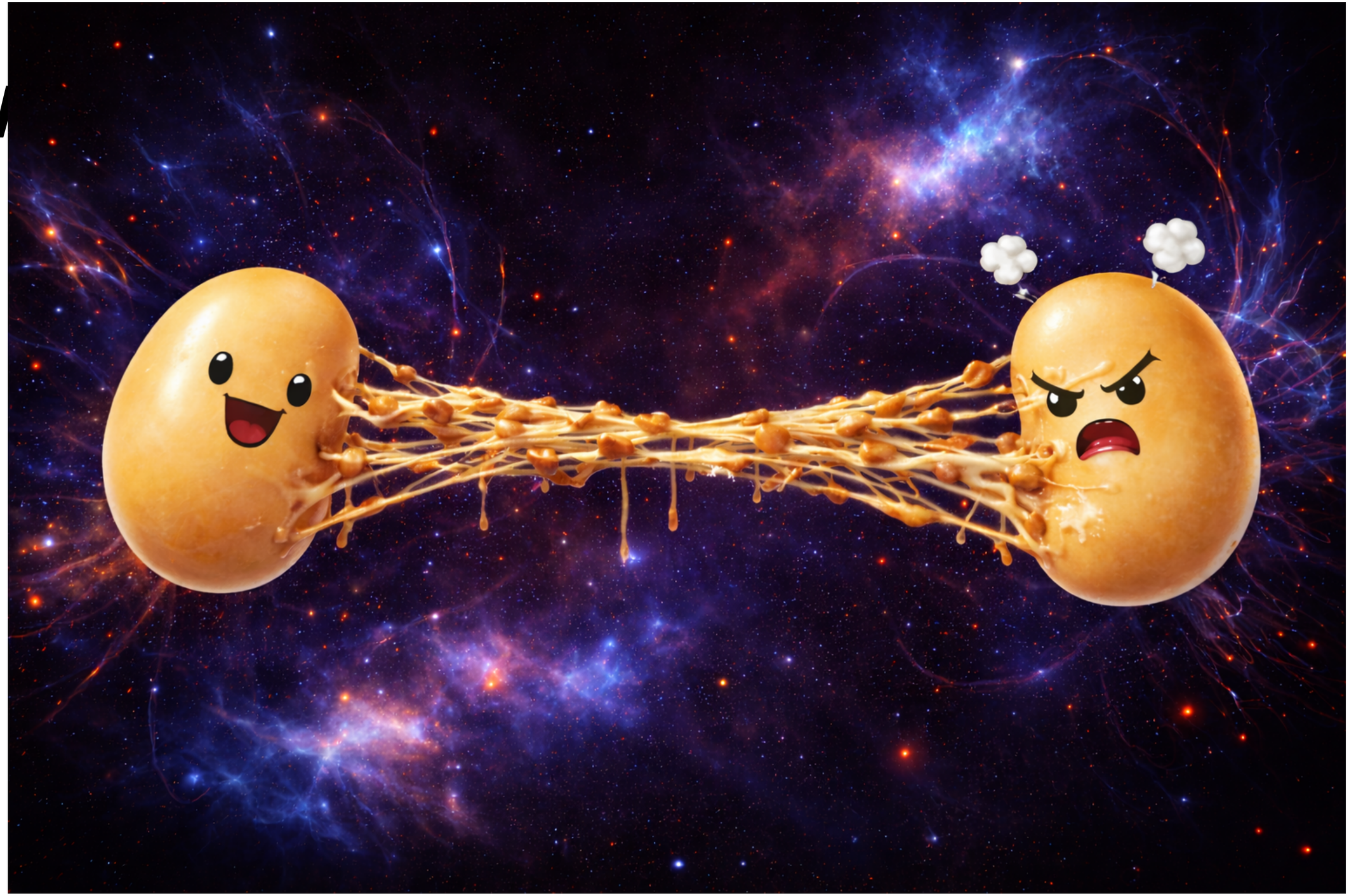

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- I also want to see conformal lines in a massive theory.

How?

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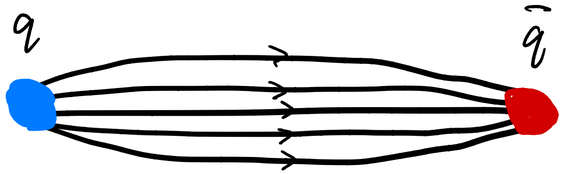
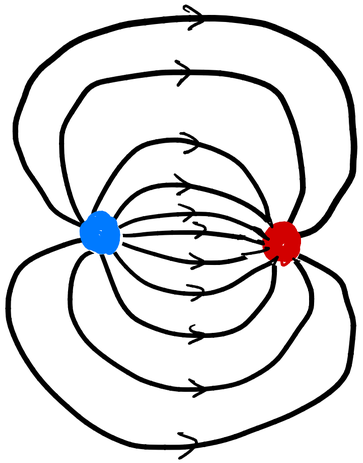
How?

- One possible way is to put the gauge theory in AdS.

QCD in flat space



Interpolation?



Effective field theory [Schwinger, Weinberg, Callan, Coleman, Wess, Zumino, Gasser, Leutwyler;…]

Effective string theory [Polchinski, Strominger ’91; Dubovsky, Flauger, Gorbenko ’12; Aharony, Komargodski ’13;…]

Lattice QCD [Wilson ’74; Creutz ’80;……]

[MILC; BMW; HPQCD; RBC; UKQCD; FLAG reviews]

Gauge theory + matter

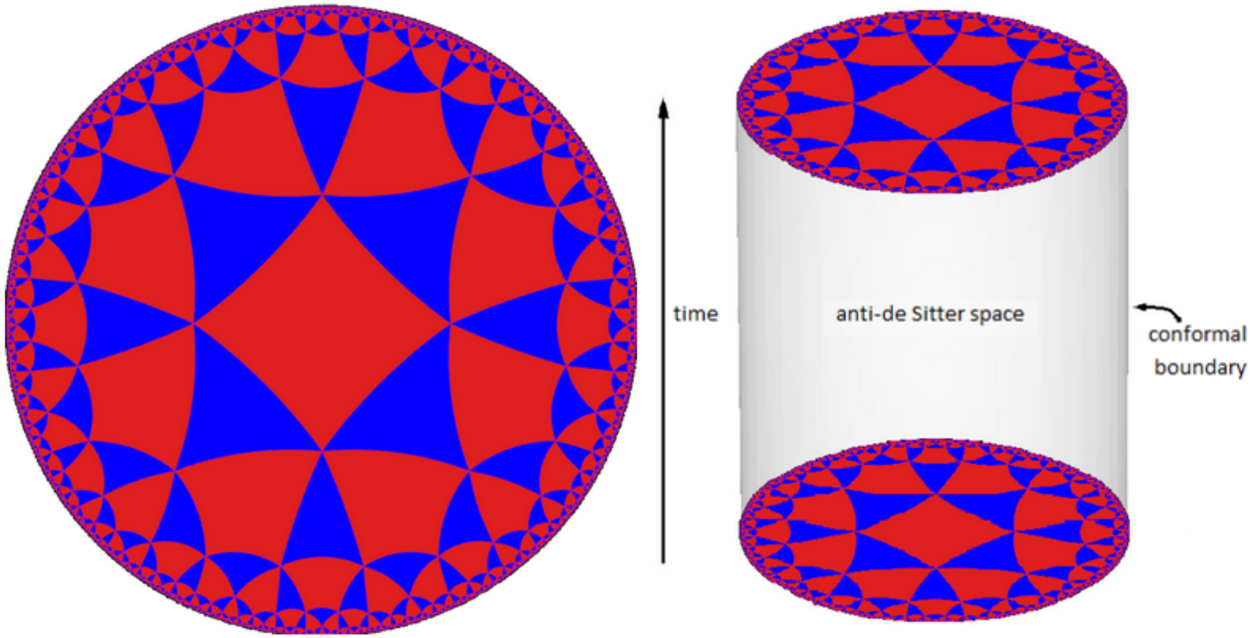
(weakly coupled)

[Politzer; Gross, Wilczek ’73]

$$\left(N_c > \frac{2}{11}N_f + \frac{1}{22}N_s + \dots\right)$$

QFT in AdS

Euclidean Anti-de Space



(from Wikipedia)

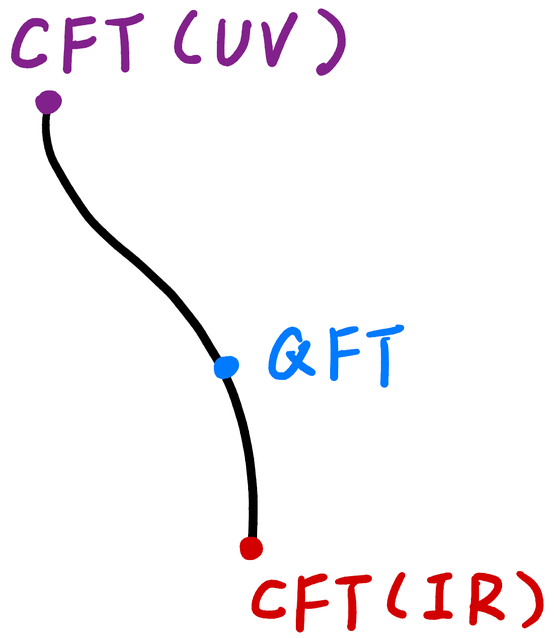
$$ds^2 = R^2 \left[\cosh^2\rho\, d\tau^2 + d\rho^2 + \sinh^2\rho\, d\Omega_{d-1}\right]$$

(AdS_{d+1})

Isometry group $O^+(1, d+1)$

QFT in AdS

AdS is a good “box” for QFTs [Callan, Wilczek 90’]



$$S_{QFT} = S_{CFT} + \lambda \int d^D x \Phi_\Delta(x), \quad [\lambda] = [E^{D-\Delta}]$$

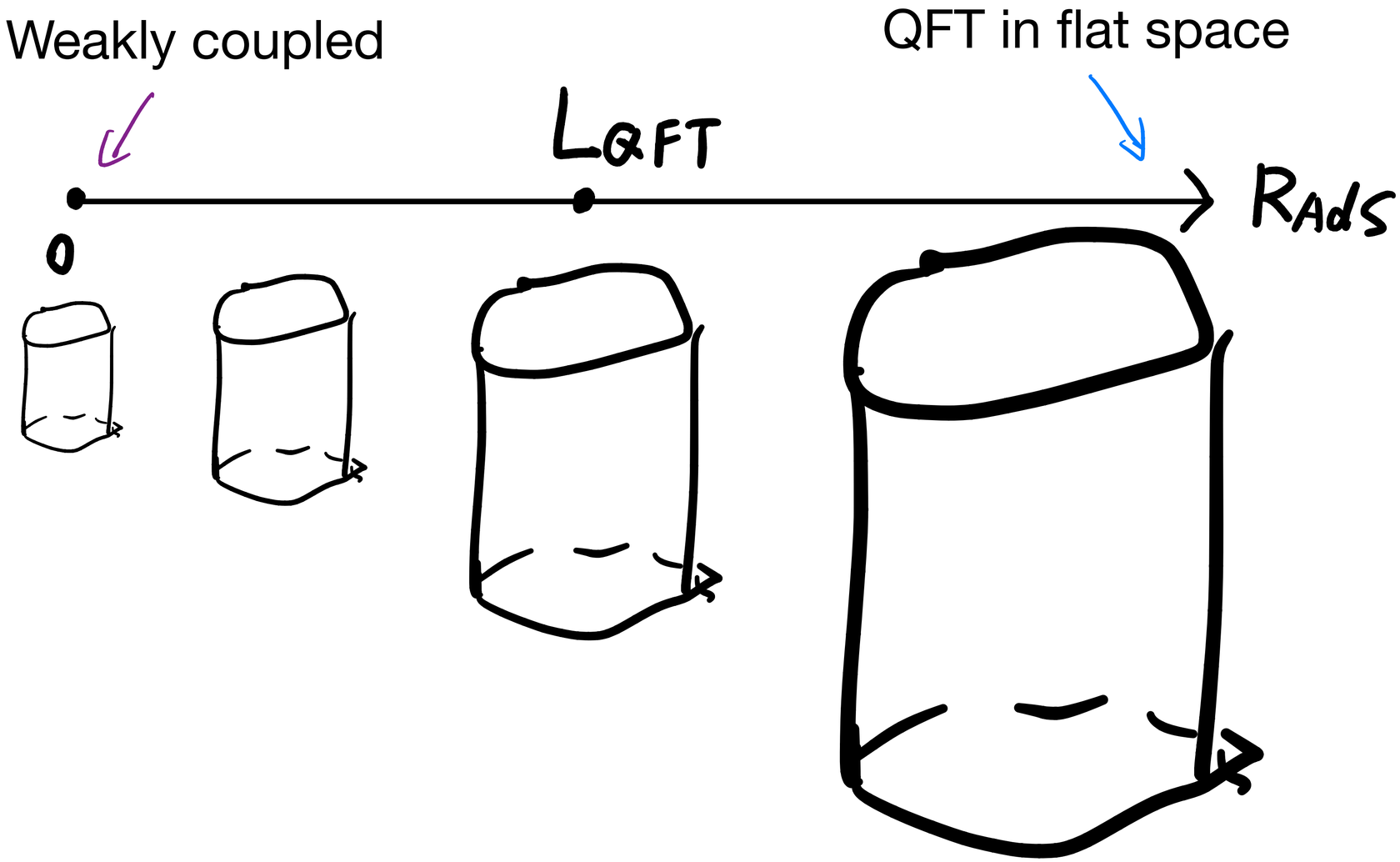
For perturbation theory, the dimensionless parameter is

$$\hat{\lambda} = \lambda \times (R_{\mathrm{AdS}})^{D-\Delta}$$

QFT in AdS

$$\hat{\lambda} = \lambda \times (R_{\rm AdS})^{D-\Delta}$$

$$(\Delta < D)$$



Yang-Mills in AdS

Yang-Mills in AdS [Aharony, Berkooz, Tong, Yankielowicz 13']

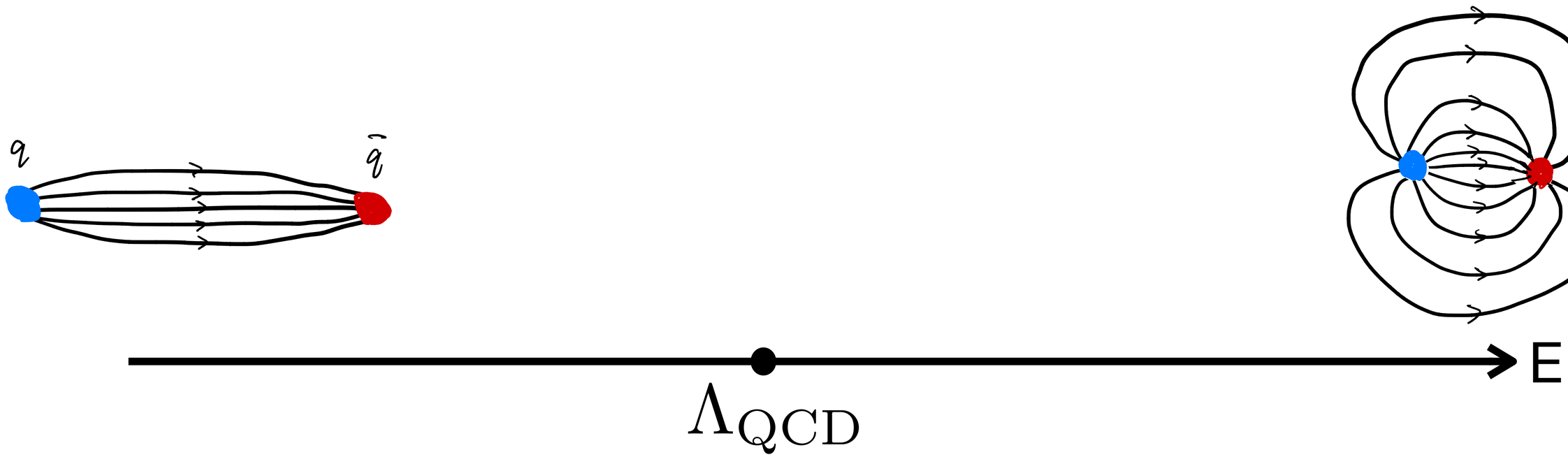
- Bulk: massive QFT (YM)

$$S_{\rm YM}[A] = \frac{1}{4R^{d-3}} \int_{{\rm AdS}_{d+1}} d^{d+1}x \sqrt{g_{\rm AdS}}\, {\rm Tr}\left(F_{\mu\nu}F^{\mu\nu}\right),$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i g_{\rm YM} R^{(3-d)/2} [A_\mu, A_\nu].$$

- Boundary condition: CFT (nonlocal)
- Weakly coupled at small AdS radius
- No dynamical gravity

What's different?



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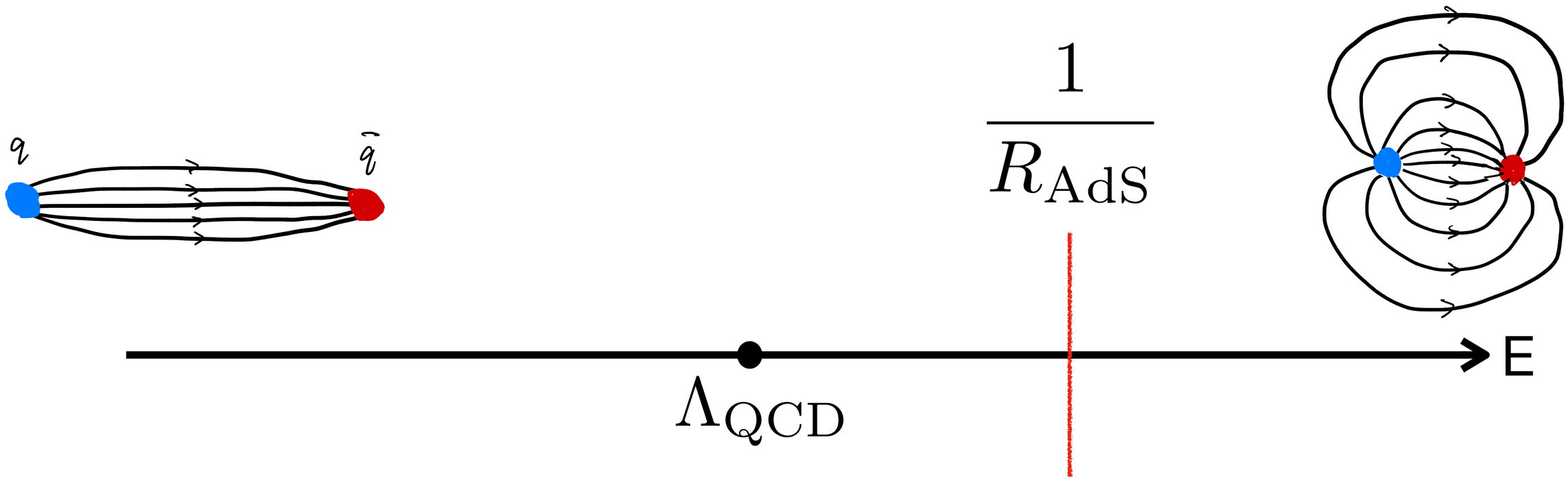
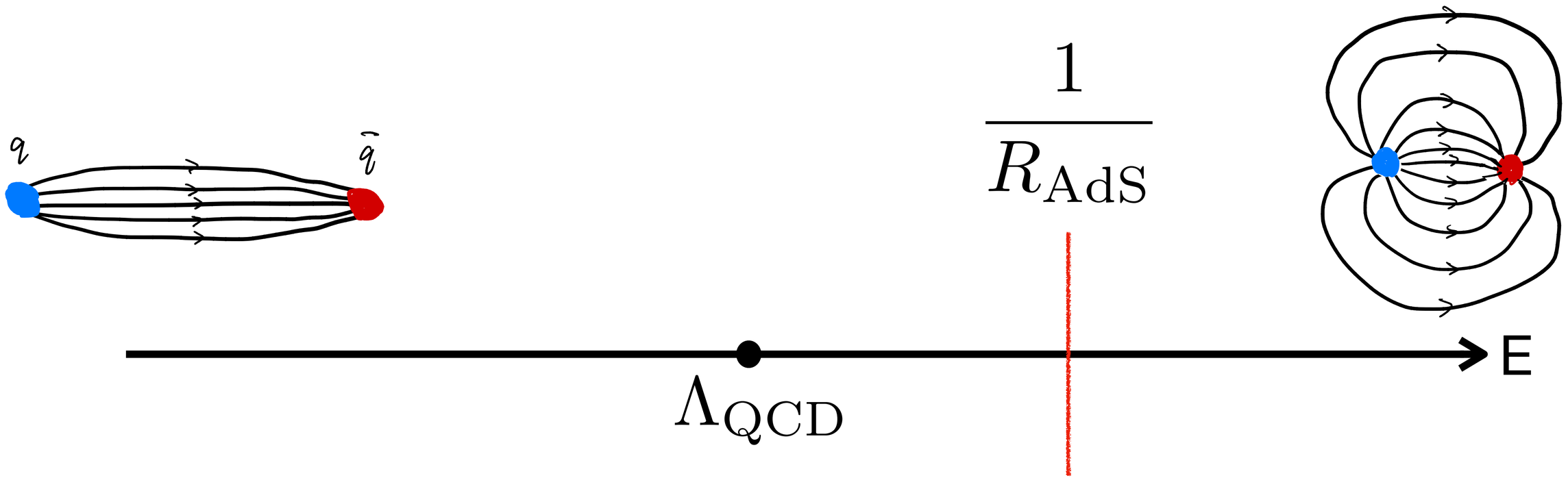


(IR cutoff)

Dirichlet boundary conditions

$$F_{ij}\Big|_{\partial} = 0 \quad (i, j \text{ parallel})$$

(No magnetic flux)



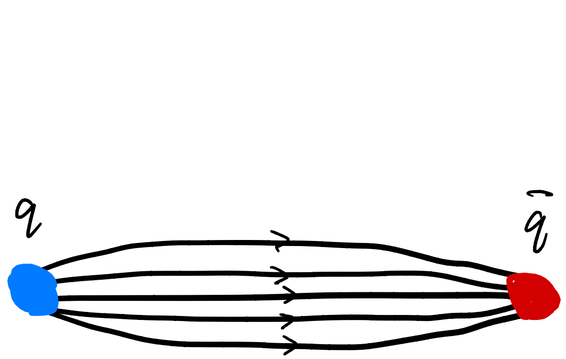
Phase transition happens. [Aharony, Berkooz, Tong, Yankielowicz 13']

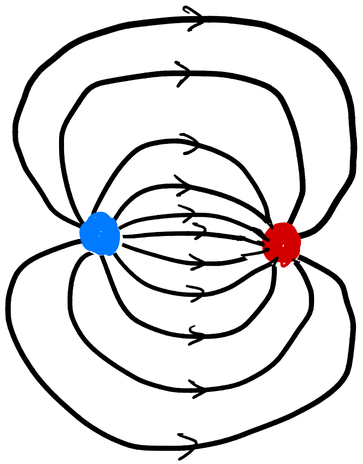
[Recently: Copetti, Di Pietro, Ji, Komatsu, Ciccone, Serone, De Sesare, Kousvos, Meineri, Piazza, …]

Neumann boundary conditions

$$F_{ni}\Big|_{\partial} = 0$$

(No electric flux)





$$\frac{1}{R_{\mathrm{AdS}}}$$

Λ_{QCD}

E

Neumann boundary conditions

$$F_{ni}\Big|_{\partial} = 0$$

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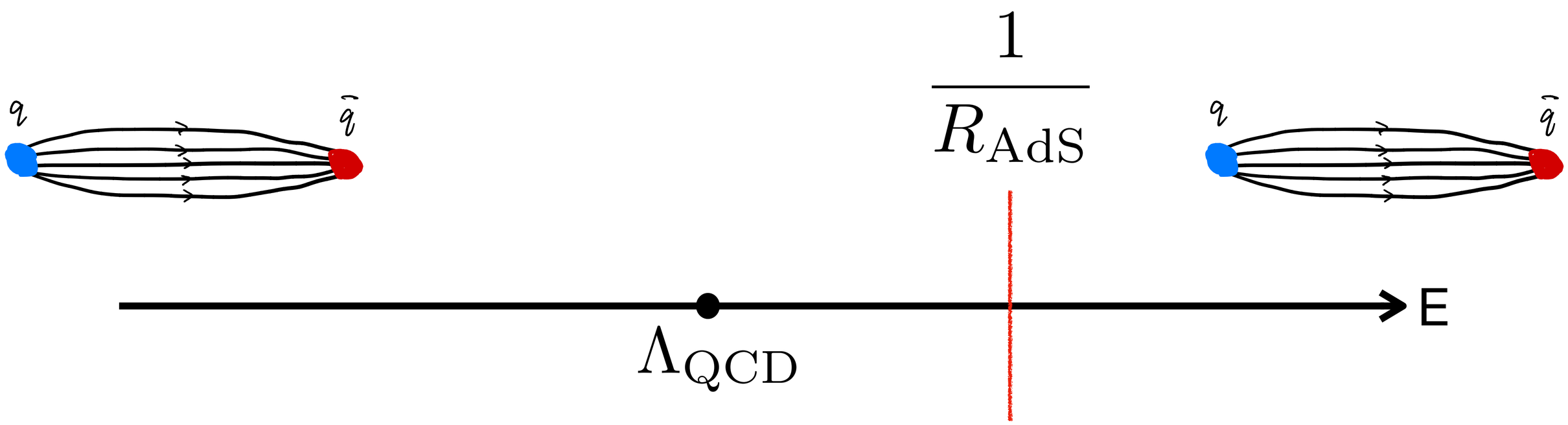


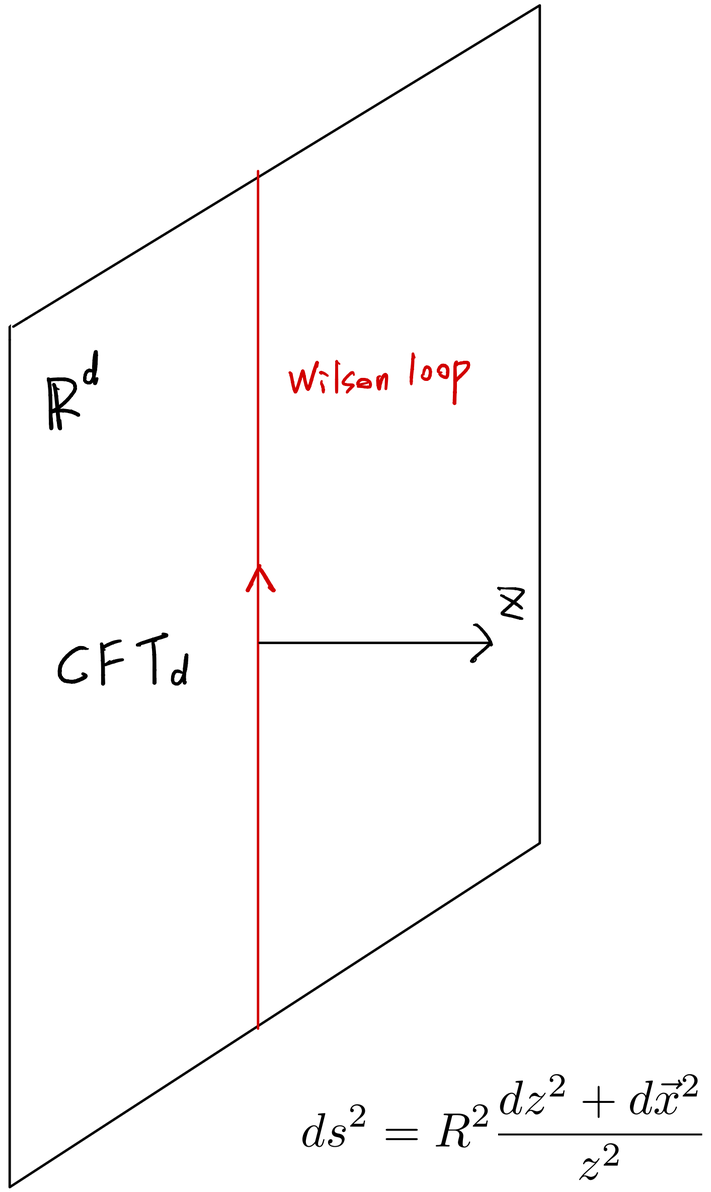
Conjecture

For pure Yang-Mills in three- and four-dimensional AdS with Neumann b.c., there is no phase transition as the AdS radius goes from 0 to ∞.

- We focus on the Neumann boundary conditions.

$$A_i(z,x) \sim a_i(x) + O(z) \quad (z \to 0)$$

- This allows us to put Wilson lines on the boundary.

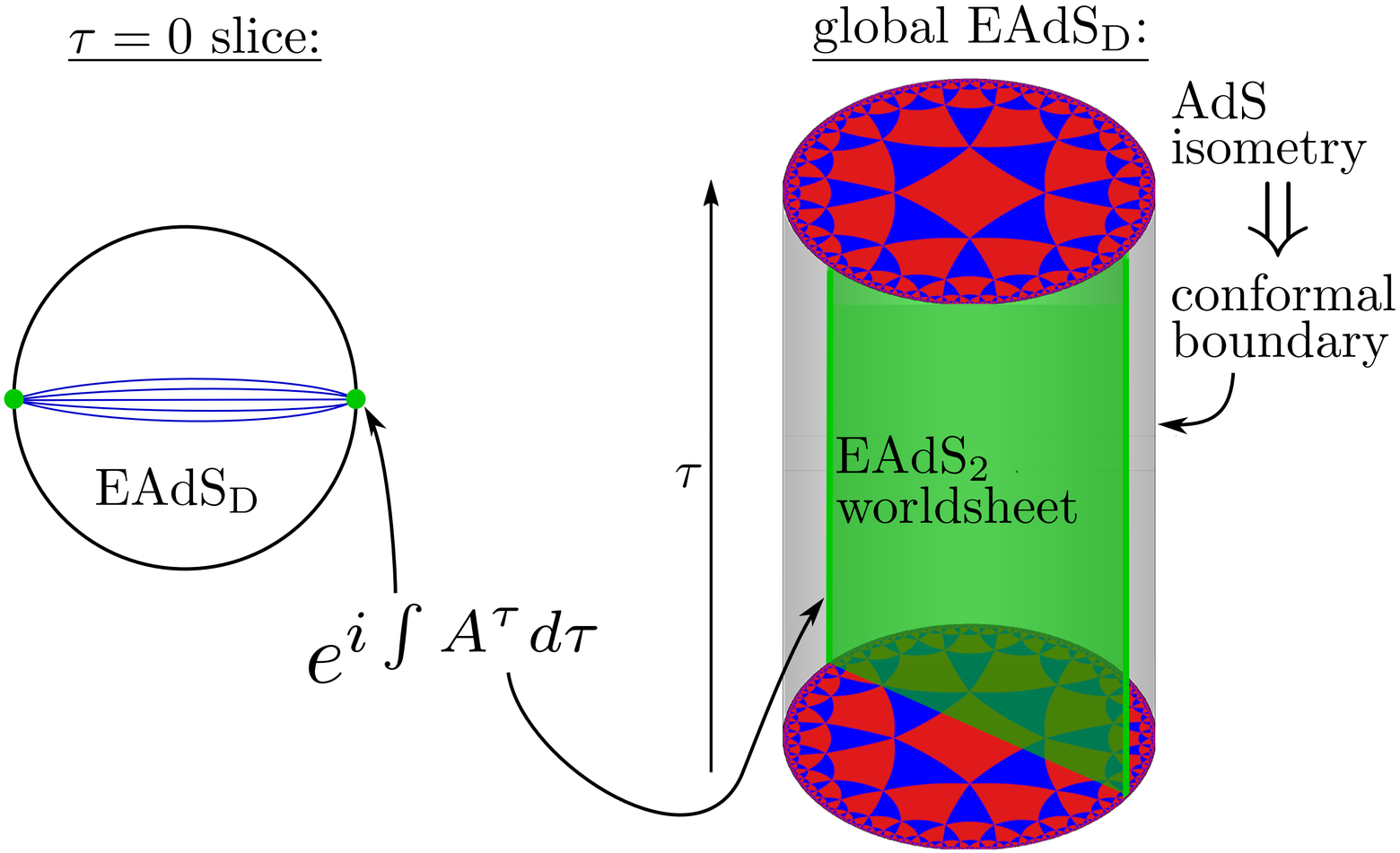


$$ds^2 = R^2 \frac{dz^2 + d\vec{x}^2}{z^2}$$

Wilson loop

Wilson loop

- Put a Wilson loop on the boundary



Wilson loop

- Conformal line defect for the boundary CFT
- Conformal symmetry breaking (superficial)

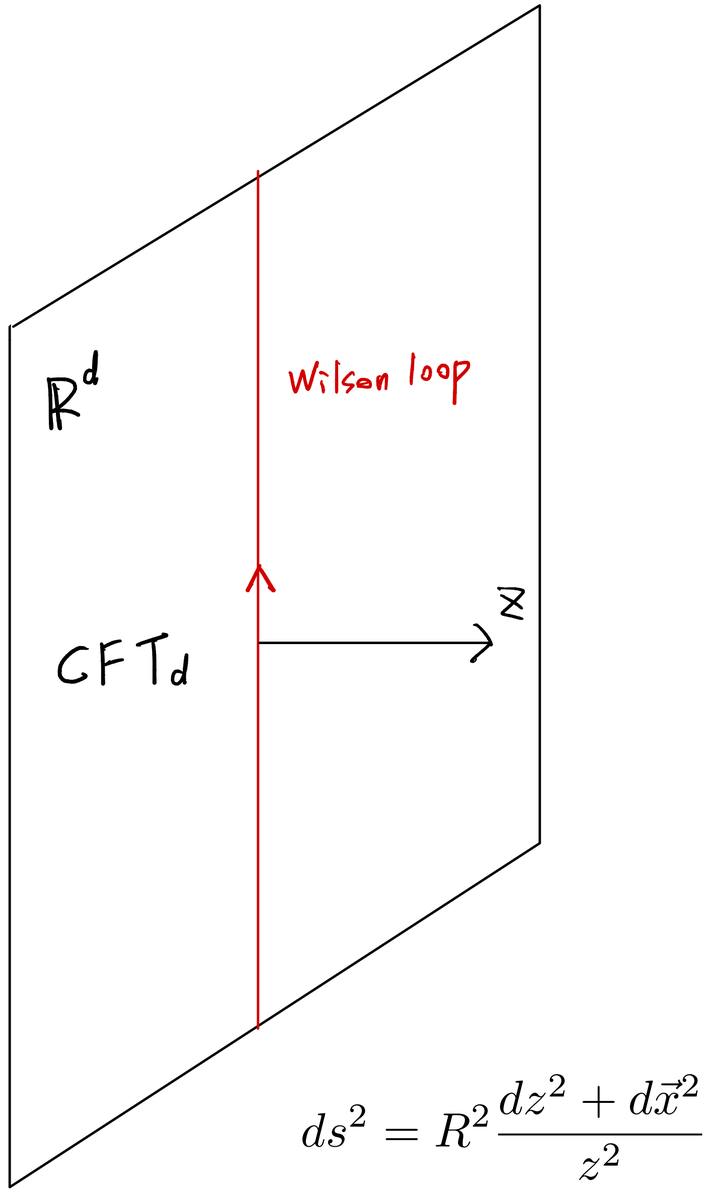
$$O(1,d+1) \longrightarrow O(1,2) \times SO(d-1)$$

- Displacement operator

$$\delta W = \int_{-\infty}^{+\infty} d\tau \, \mathrm{Tr} \left(\mathcal{P} \left(igF_{\tau i}(\tau) \, \delta x^i(\tau) \right) e^{-ig \int_\gamma A} \right)$$

$$\Rightarrow \quad \mathbb{D}_i \quad \text{“=”} \quad igF_{\tau i}$$

- Protected dimension $\Delta_{\mathbb{D}_i} = 2$



Wilson loop - operator content (3D)

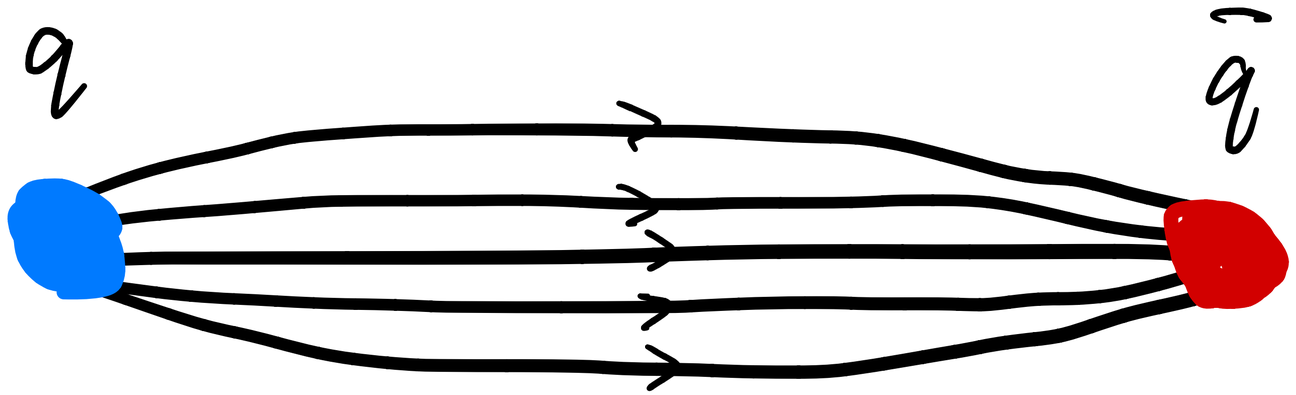
Operator	Dimension	R	CT	KK mode notation
$\mathbb{D} = F_{\tau 1}$	2	-	+	$a^{(1)}$
$\partial_1 F_{\tau 1}$	3	+	+	$a^{(2)}$
$\partial_1^2 F_{\tau 1}$	4	-	+	$a^{(3)}$
$F_{\tau 1} F_{\tau 1}$	4	+	+	$a^{(1)} a^{(1)}$
$\partial_1^3 F_{\tau 1}$	5	+	+	$a^{(4)}$
$[\partial_1 F_{\tau 1} F_{\tau 1}]^- = \partial_1 F_{\tau 1} F_{\tau 1} - F_{\tau 1} \partial_1 F_{\tau 1}$	5	-	-	$a^{(2)} a^{(1)} - a^{(1)} a^{(2)}$
$[\partial_1 F_{\tau 1} F_{\tau 1}]^+ = \partial_1 F_{\tau 1} F_{\tau 1} + F_{\tau 1} \partial_1 F_{\tau 1}$	5	-	+	$a^{(2)} a^{(1)} + a^{(1)} a^{(2)}$
$[F_{\tau 1} F_{\tau 1}]_1 = \partial_\tau F_{\tau 1} F_{\tau 1} - F_{\tau 1} \partial_\tau F_{\tau 1}$	5	+	+	$\partial_\tau a^{(1)} a^{(1)} - a^{(1)} \partial_\tau a^{(1)}$

Table 1: The lowest lying defect operators that can be inserted on the Wilson line.

(Low-lying operators)

Effective string theory

Wilson loop (large radius)



For a large Wilson loop, the **long-range dynamics** of the flux tube is described by the effective string theory.

[Luscher ’81; Polchinski, Strominger ’91; Luscher, Weisz ’04; Dubovsky, Flanger, Gorbenko ’12, Aharony, Komargodski ’13,…]

(EST) [Cuomo, Dubovsky, Hernandez-Chifflet, Monin, Zare ’24; Athenodorou, Dubovsky, Luo, Tepper ’24 x2;…]

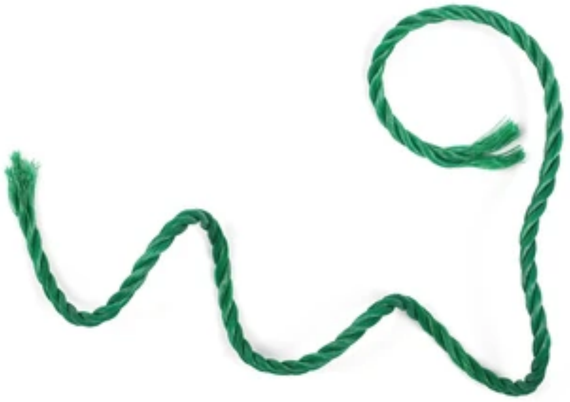
(S-matrix) [Miro, Guerrieri, Hebbar, Penedones, Vieira ’19; Miro, Guerrieri ’21; Gaikwad, Gorbenko, Guerrieri ’24; Guerrieri, Homrich, Vieira ’24;…]

Wilson loop (large radius)

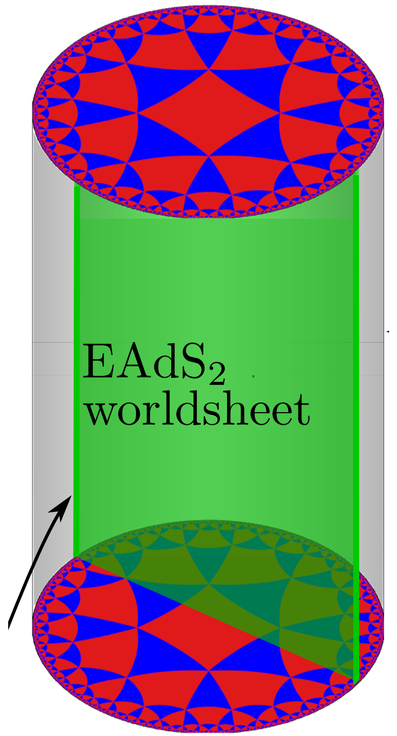
- Bulk: effective string theory (EST)

$$S_{\mathrm{EST}} = \frac{1}{\ell_s^2} \int d^2\sigma \sqrt{\det\left(g_{\mu\nu}(X)\,\partial_\alpha X^\mu \partial_\beta X^\nu\right)} + \text{non-universal terms}$$

(Nambu-Goto action)



embed



Wilson loop (large radius)

- Bulk: effective string theory (EST) [Nambu ’70; Goto ’71; Polchinski, Strominger ’91; Dubovsky et al ’12, …]

$$S_{\mathrm{EST}} = \frac{1}{\ell_s^2} \int d^2\sigma \sqrt{\det\left(g_{\mu\nu}(X)\, \partial_\alpha X^\mu \partial_\beta X^\nu\right)} + \text{non-universal terms}$$

(Nambu-Goto action)

- Dirichlet boundary conditions (static quarks)
- Weakly coupled at large AdS radius

Wilson loop (large radius)

- Weakly coupled at large AdS radius

  [Gabai, Gorbenko, Offertaler **2602.16694]**

Nambu-Goto

$$S^{(3\mathrm{D})}_{\mathrm{EST}}[z] = \int d^2\sigma\sqrt{h}\Big[\lambda^2 + \frac{1}{2}\partial_\alpha z\partial^\alpha z + z^2 - \frac{1}{8\lambda^2}(\partial_\alpha z\partial^\alpha z)^2 + \frac{1}{3\lambda^2}z^4$$
$$+ \frac{\kappa}{\lambda^6}\left((\nabla_\alpha\nabla_\beta z\nabla^\alpha\nabla^\beta z)^2 - 4(\partial_\alpha z\partial^\alpha z)^2 + \frac{20}{3}z^4 + O(z^6)\right) + O(z^6\lambda^{-4}, z^4\lambda^{-8})\Big]$$

curvature term

$$\left(\lambda = \frac{R_{\mathrm{AdS}}}{\ell_s}\right)$$

- Very similar story in flat space (e.g. Ising interface)

Wilson loop (large radius)

- Weakly coupled at large AdS radius

  [Gabai, Gorbenko, Offertaler **2602.16694]**

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$$\left(\lambda = \frac{R_{\mathrm{AdS}}}{\ell_s}\right)$$

- Mass=2 (in AdS units) $\Rightarrow \quad \Delta_z = 2 \quad (\mathrm{Dirichlet\ boundary\ conditions})$
- Very similar story in flat space (e.g. Ising interface)

Wilson loop (large radius)

- Bulk: effective string theory (EST)

$$S_{\rm EST} = \frac{1}{\ell_s^2}\int d^2\sigma \sqrt{\det\left(g_{\mu\nu}(X)\,\partial_\alpha X^\mu \partial_\beta X^\nu\right)} + \text{non-universal terms}$$

- AdS isometry = global symmetry of EST

$$X^M(\sigma) \to \Lambda^M{}_N\, X^N(\sigma)$$

- Goldstone modes $\quad O(1,d+1) \longrightarrow O(1,2)\times SO(d-1)$

$$\text{SSB} \;\Rightarrow\; (d-1)\ \text{Goldstone modes in}\ \text{AdS}_2$$

- Protected dimension of boundary primaries $\quad \Delta = 2$

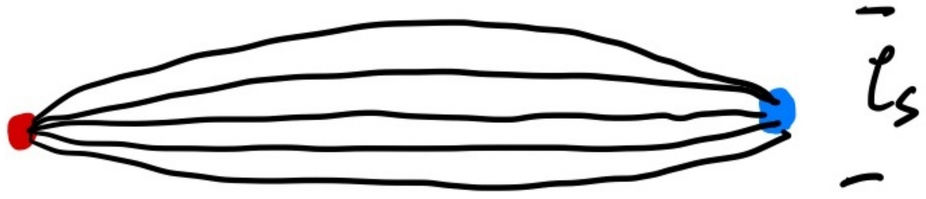
Wilson loop - summary

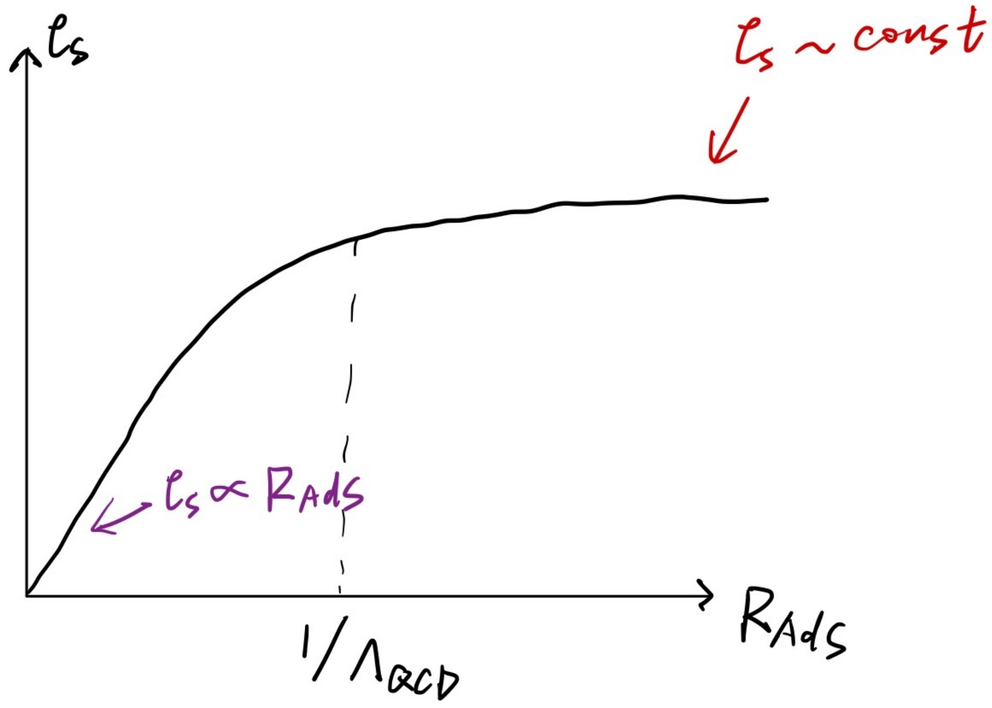
- Boundary CFT: conformal line defect implies d-1 displacement operators
- They are single gluon states at zero AdS radius
- Bulk EST: the spontaneously broken AdS isometry implies d-1 operators
- They correspond to Goldstone modes
- Same protected dimension, same degeneracy

Therefore, as the AdS radius goes from zero to infinity, the lightest single gluon states interpolates to Goldstone modes in the EST.

Some comments

- In flat space, the effective string description breaks down at short distance.
- At small AdS radius, the flux tube may still be squeezed around the minimal surface since the gravity is strong.





Non-linearly realized conformal symmetry

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- For general CFT with conformal defects
- Symmetry breaking by a conformal line defect ?

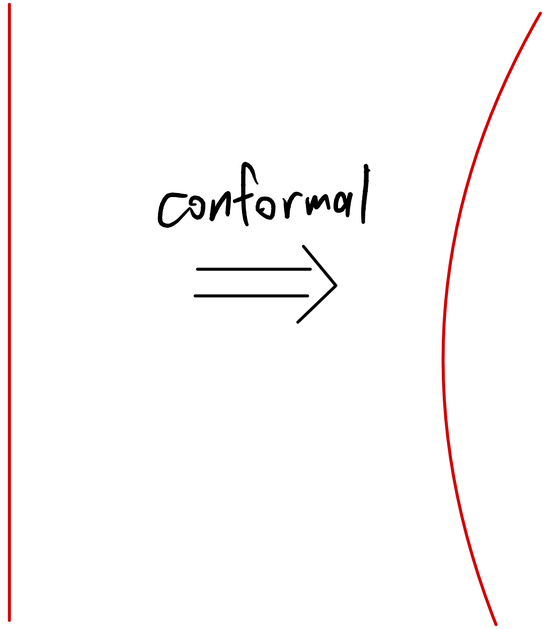
$$O^+(1,d+1) \longrightarrow O^+(1,2) \times SO(d-1)$$

Non-linearly realized conformal symmetry

- For general CFT with conformal defects
- Symmetry breaking by a conformal line defect ?

$$O^+(1,d+1) \longrightarrow O^+(1,2) \times SO(d-1)$$

- Actually the symmetry is not broken !



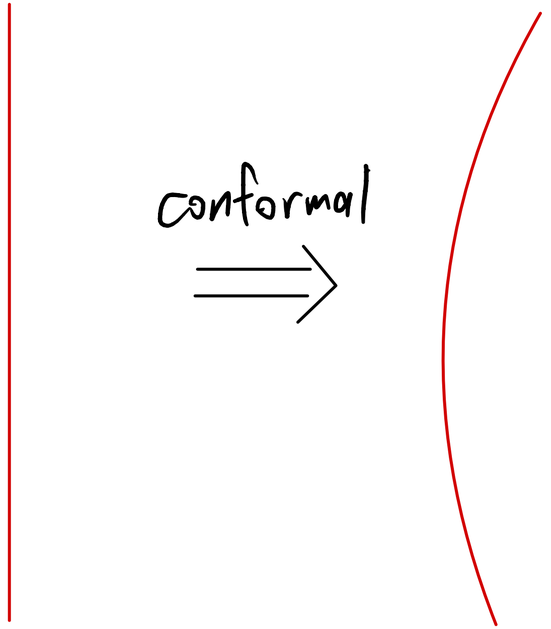
How do we write down the Ward identities for the “broken symmetry”,

without changing the shape of the defect?

Non-linearly realized conformal symmetry

- There are two ways to change the shape of the defect
- A: conformal map
- B: using displacement operator

$$\delta_v \mathcal{L} = \int_{-\infty}^{\infty} dt\, \mathbb{D}_i(\tau) v^i(\tau) \mathcal{L} \quad \text{(infinitesimal)}$$



- The agreement between A and B leads to constraints
- In practice, the infinitesimal version is more useful.

[Gabai, Sever, Zhong ’24], [Girault, Paulos, van Vliet ’25] [Drukker, Kong, Kravchuk ’25]

Non-linearly realized conformal symmetry

- Define

$$\mathcal{F}^{\mathcal{O}\mathbb{D}\mathbb{D}\mathcal{O}}(t) \equiv \frac{\langle \mathcal{O}(\tau_1)\,\mathbb{D}(\tau_2)\,\mathbb{D}(\tau_3)\,\mathcal{O}(\tau_4)\rangle_W}{\langle \mathbb{D}(\tau_2)\mathbb{D}(\tau_3)\rangle_W\,\langle \mathcal{O}(\tau_1)\mathcal{O}(\tau_4)\rangle_W} - [\mathrm{GFF}]\,,$$

$$\mathcal{F}^{\mathbb{D}\mathcal{O}\mathbb{D}\mathcal{O}}(t) \equiv \frac{1}{(1+t)^4}\frac{\langle \mathbb{D}(\tau_1)\,\mathcal{O}(\tau_2)\,\mathbb{D}(\tau_3)\,\mathcal{O}(\tau_4)\rangle_W}{\langle \mathbb{D}(\tau_1)\mathbb{D}(\tau_3)\rangle_W\,\langle \mathcal{O}(\tau_2)\mathcal{O}(\tau_4)\rangle_W} - [\mathrm{GFF}]\,,$$

$$t \equiv \frac{\tau_{21}\,\tau_{43}}{\tau_{41}\,\tau_{32}}\,.$$

- Homogeneous constraints

$$\int_0^\infty dt\,\big[\,t^2\,\mathcal{F}^{\mathbb{D}\mathcal{O}\mathbb{D}\mathcal{O}}(t) + \big(1+2t+2t^2\big)\mathcal{F}^{\mathcal{O}\mathbb{D}\mathbb{D}\mathcal{O}}(t)\big] = 0\,,$$

$$\int_0^\infty dt\,\big[\,-t\,\mathcal{F}^{\mathbb{D}\mathcal{O}\mathbb{D}\mathcal{O}}(t) + 2(t+t^2)\mathcal{F}^{\mathcal{O}\mathbb{D}\mathbb{D}\mathcal{O}}(t)\big] = 0\,.$$

- Inhomogeneous constraints

$$\int_0^\infty dt\,\big[(t^2-1)\log t\,\mathcal{F}^{\mathbb{D}\mathcal{O}\mathbb{D}\mathcal{O}}(t) - 2(1+2t)\log\left(\frac{t}{1+t}\right)\mathcal{F}^{\mathcal{O}\mathbb{D}\mathbb{D}\mathcal{O}}(t)\big] = \frac{2\Delta_{\mathcal{O}}}{C_{\mathbb{D}}}\,.$$

Non-linearly realized conformal symmetry

- These constraints are satisfied by other CFTs.
- Free lunch: one-loop anomalous dimensions [Gabai, JQ, Gorbenko 25']

$$C_{1,1,2}C_{n,n,2} = \frac{5\left(n^2+n+3\right)}{72n(2n-1)\pi}\lambda + O(\lambda^2)\ ,$$

$$C^2_{1,n-1,n} = \frac{\left(n^2+1\right)^2}{4n(2n-1)\left(n^2-1\right)\pi}\lambda + O(\lambda)\ ,$$

(Tree-level OPE coefficients)

$$\gamma_n = \frac{1}{2\pi}\left(\frac{2}{n+1} + 2(\psi(n)+\gamma) - 1\right)\lambda + O(\lambda^2)\ ,$$

(1-loop anomalous dim)

$$\left(\lambda = g^2_{\rm YM}\, N_c\, R^{1/2}_{\rm AdS}\right)$$

Non-linearly realized conformal symmetry

- Free lunch: one-loop anomalous dimensions
    - Observation 1: Not satisfied in the free theory

Non-linearly realized conformal symmetry

- Free lunch: one-loop anomalous dimensions
    - Observation 1: Not satisfied in the free theory
    - Observation 2: anomalous dimensions and some OPE coefficients

$$\gamma_i \sim \lambda^2 \quad C_{ijk} \sim \lambda$$

Non-linearly realized conformal symmetry

- Free lunch: one-loop anomalous dimensions
    - Observation 1: Not satisfied in the free theory
    - Observation 2: anomalous dimensions and some OPE coefficients

$$\gamma_i \sim \lambda^2 \quad C_{ijk} \sim \lambda$$

- Observation 3: $\int_0^a x^{\Delta-n-1} dx = \frac{1}{\Delta - n} + \dots$
- Enhancement $\quad \text{constraint} \quad \supset \quad \sum \frac{C_{ijk}^2}{\gamma_i} \to \text{const} \quad \text{as} \quad \lambda \to 0$

Non-linearly realized conformal symmetry

- These constraints are satisfied by other CFTs.
- Free lunch: one-loop anomalous dimensions

$$C_{1,1,2}C_{n,n,2} = \frac{5\left(n^2+n+3\right)}{72n(2n-1)\pi}\lambda + O(\lambda^2)\ ,$$

$$C^2_{1,n-1,n} = \frac{\left(n^2+1\right)^2}{4n(2n-1)\left(n^2-1\right)\pi}\lambda + O(\lambda)\ ,$$

(Tree-level OPE coefficients)

$$\gamma_n = \frac{1}{2\pi}\left(\frac{2}{n+1} + 2(\psi(n)+\gamma) - 1\right)\lambda + O(\lambda^2)\ ,$$

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Verified in Feynman diagram calculation

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(1-loop anomalous dim)

Todo

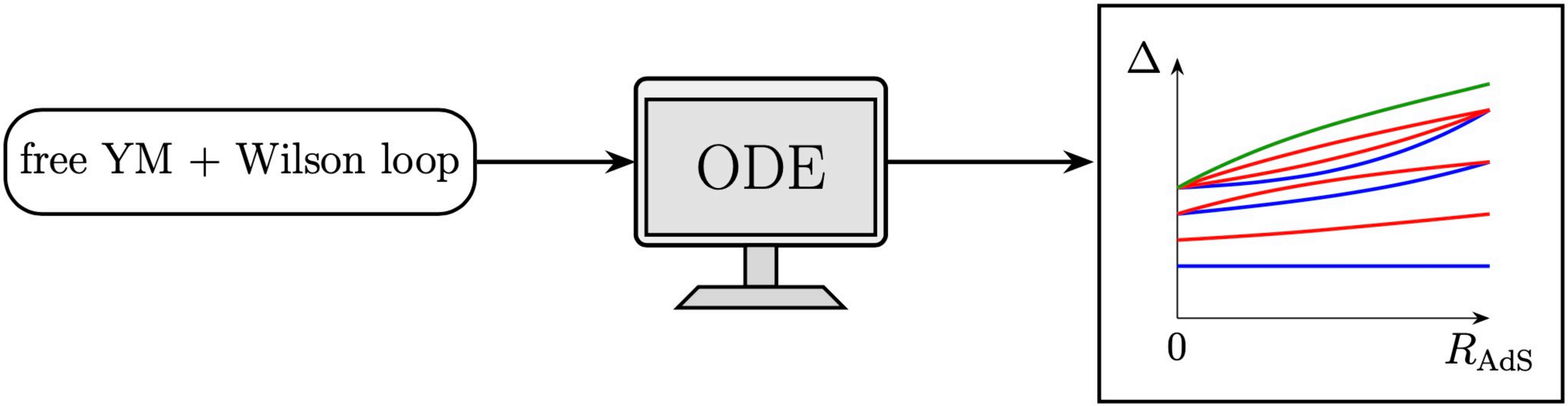
Summary

- AdS is a good box for gauge theory and also other QFTs
- Wilson loop background: YM vs EST, displacement vs NGB
- Non-linearly realized conformal symmetry

Outlook

- More consistency checks in perturbation theory
- More non-perturbative arguments based on symmetries
- Numerical tools (conformal bootstrap, ODE, …)

Dream goal



ありがとうございます!