

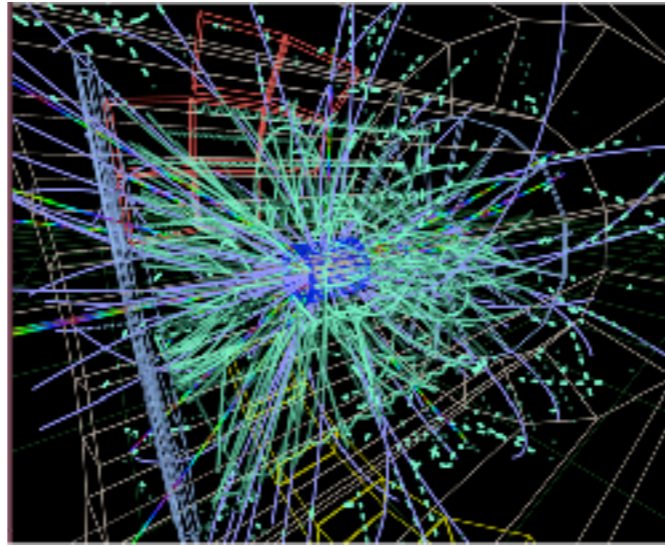
The Renaissance of Axiomatic Methods in Quantum Field Theory

Slava Rychkov

Institut des Hautes Études Scientifiques,
Bures-sur-Yvette



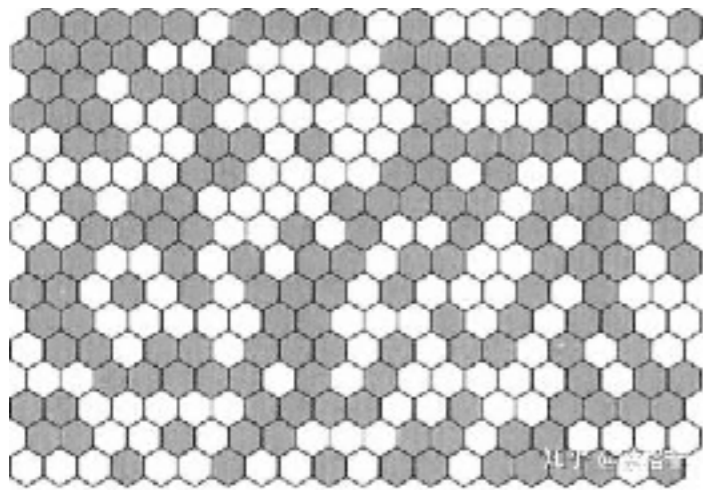
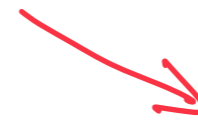
Yukawa Institute of Theoretical Physics Colloquium - 04.11.2025



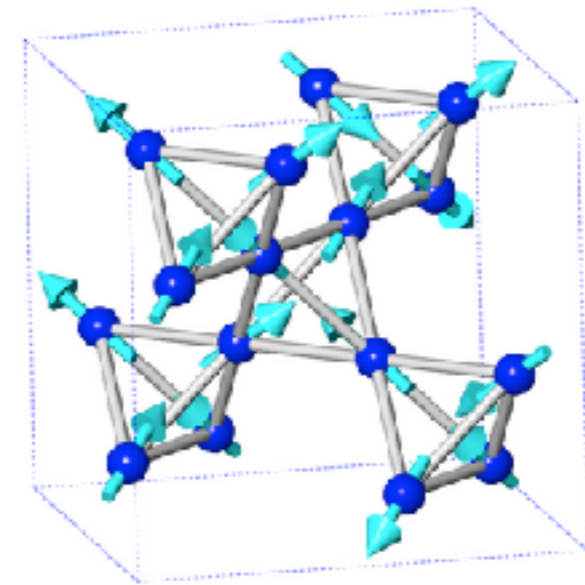
particle physics



Quantum Field Theory



statistical physics



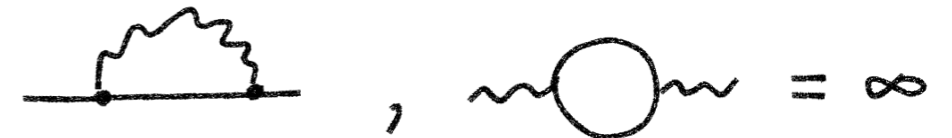
condensed matter

History of Quantum Field Theory

1927 - creation of Quantum Electrodynamics **Dirac**



1930 infinities - FIRST CRISIS **Oppenheimer**



1947-1950 Renormalization **Bethe Tomonaga Schwinger Feynman Dyson**

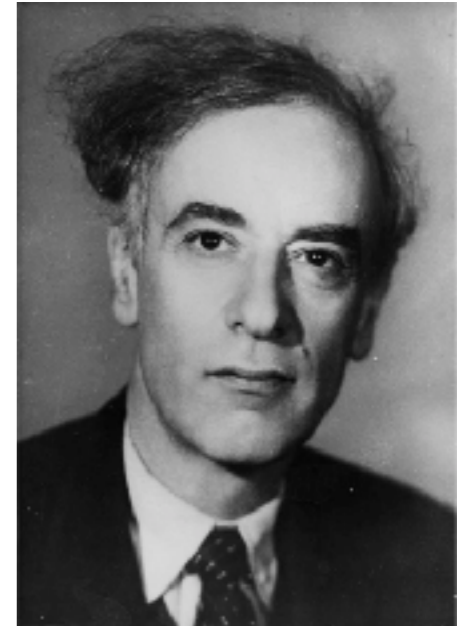


Lamb shift, Electron g-2

Some influential people declared QFT dead:

Landau (1960)

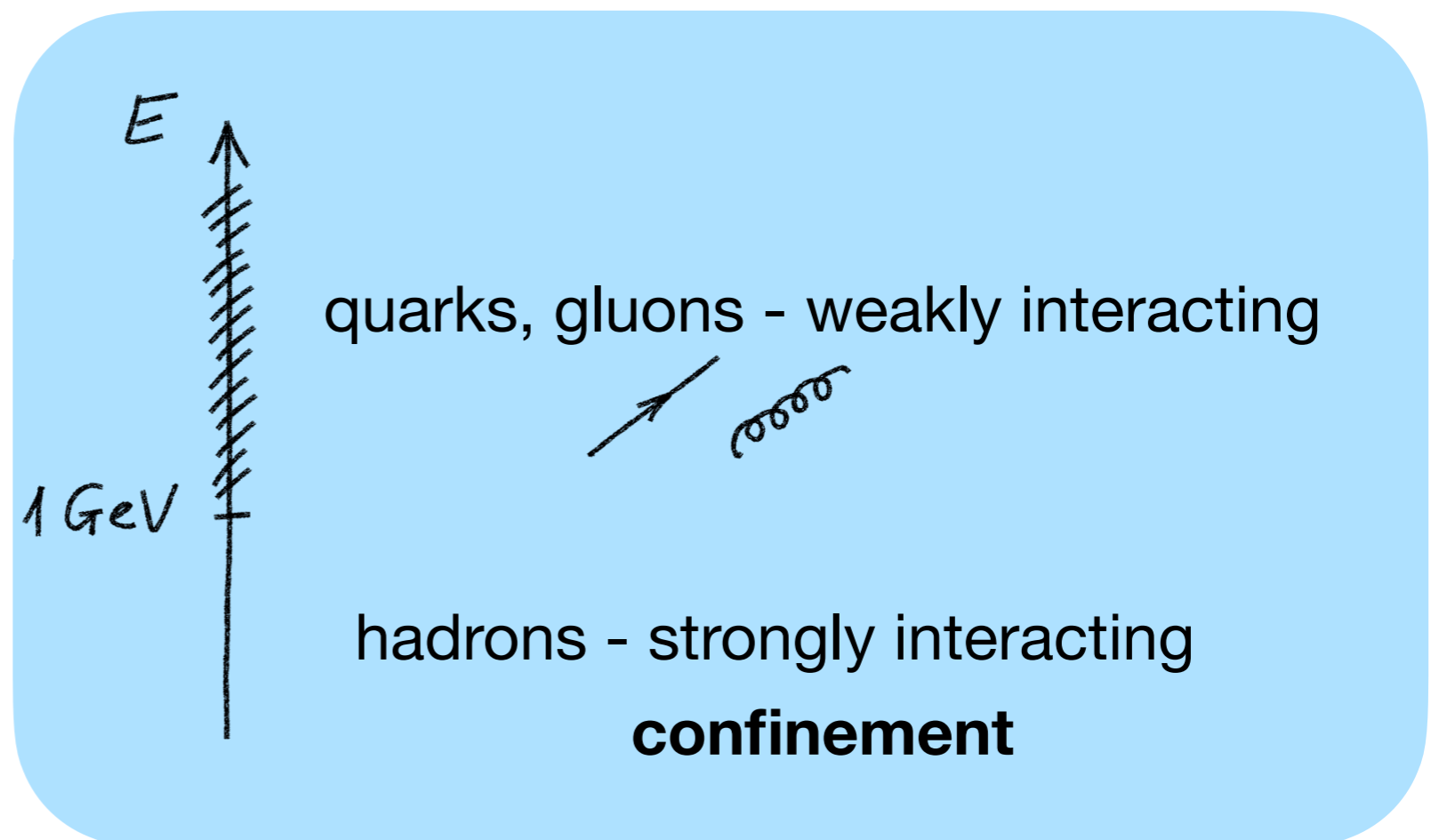
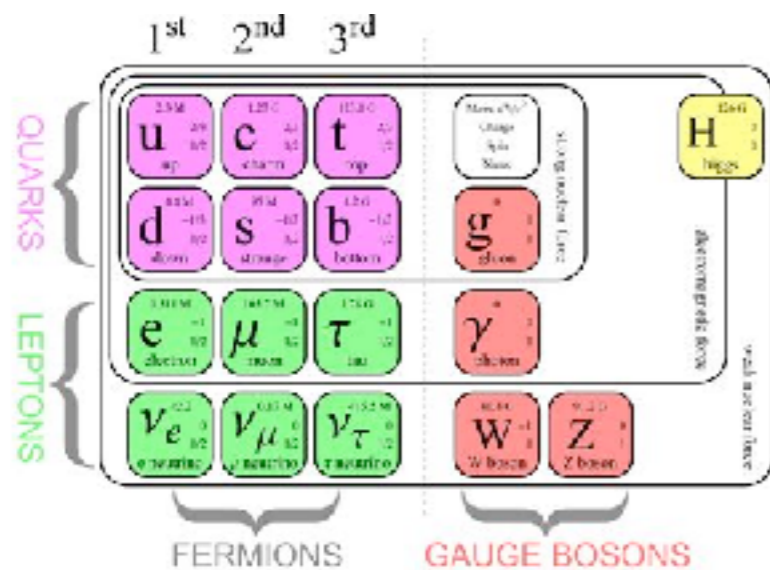
“The Hamiltonian method for strong interactions has **outlived itself** and should be **buried**, of course, with all the honors it deserves.”



1970s Triumphant comeback of QFT

1972 Renormalizability of the Standard Model 't Hooft Veltman

1973 Asymptotic freedom, QCD Gross-Wilczek, Politzer



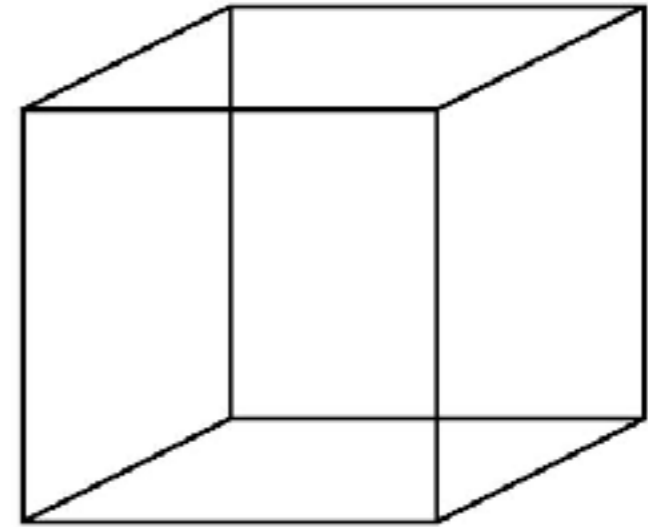
Properties of hadrons from first principles

1974 Lattice QCD [Wilson](#)

$$\frac{1}{g_{YM}^2} \int \text{Tr}(F_{\mu\nu}^2) d^4x + \int \bar{q} \not{D} q d^4x$$

QCD action

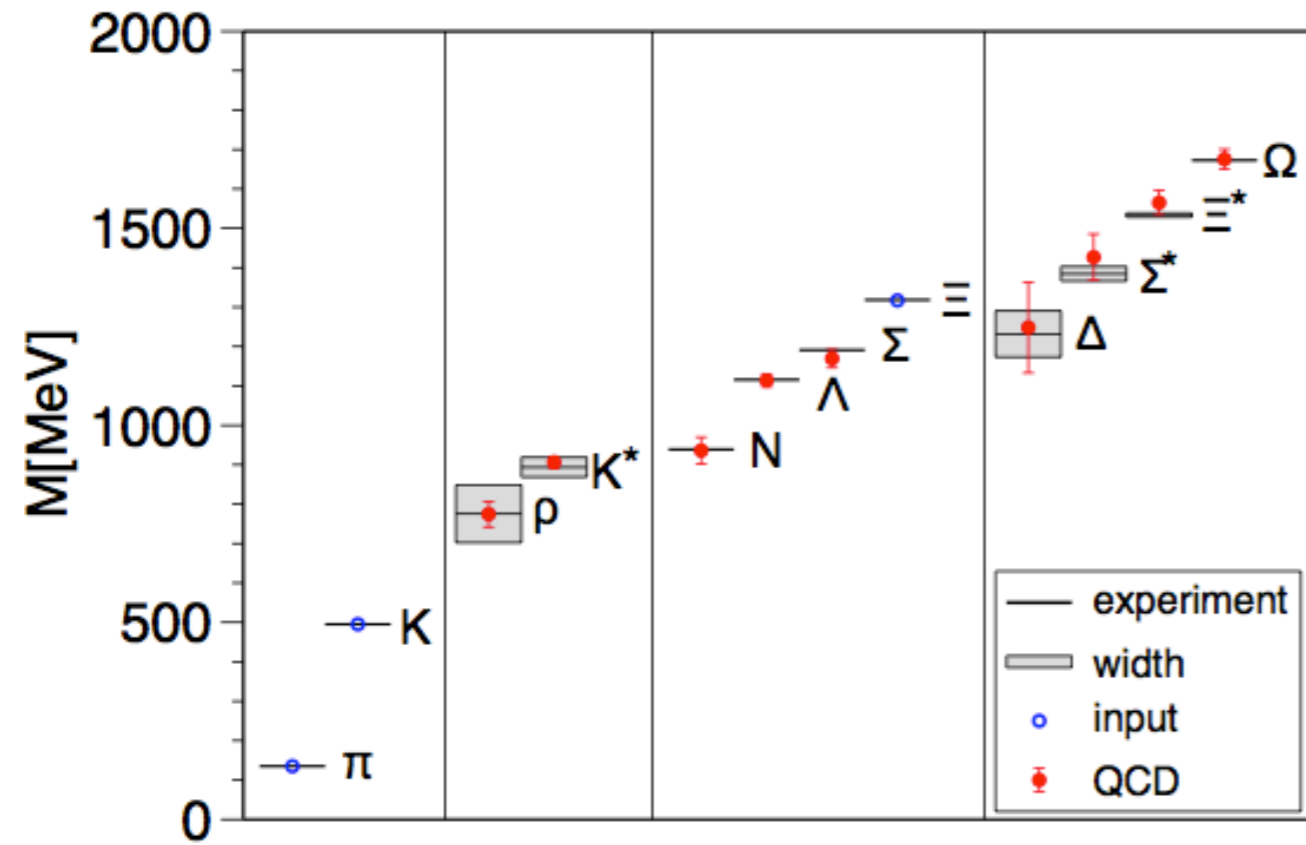
→
discretize



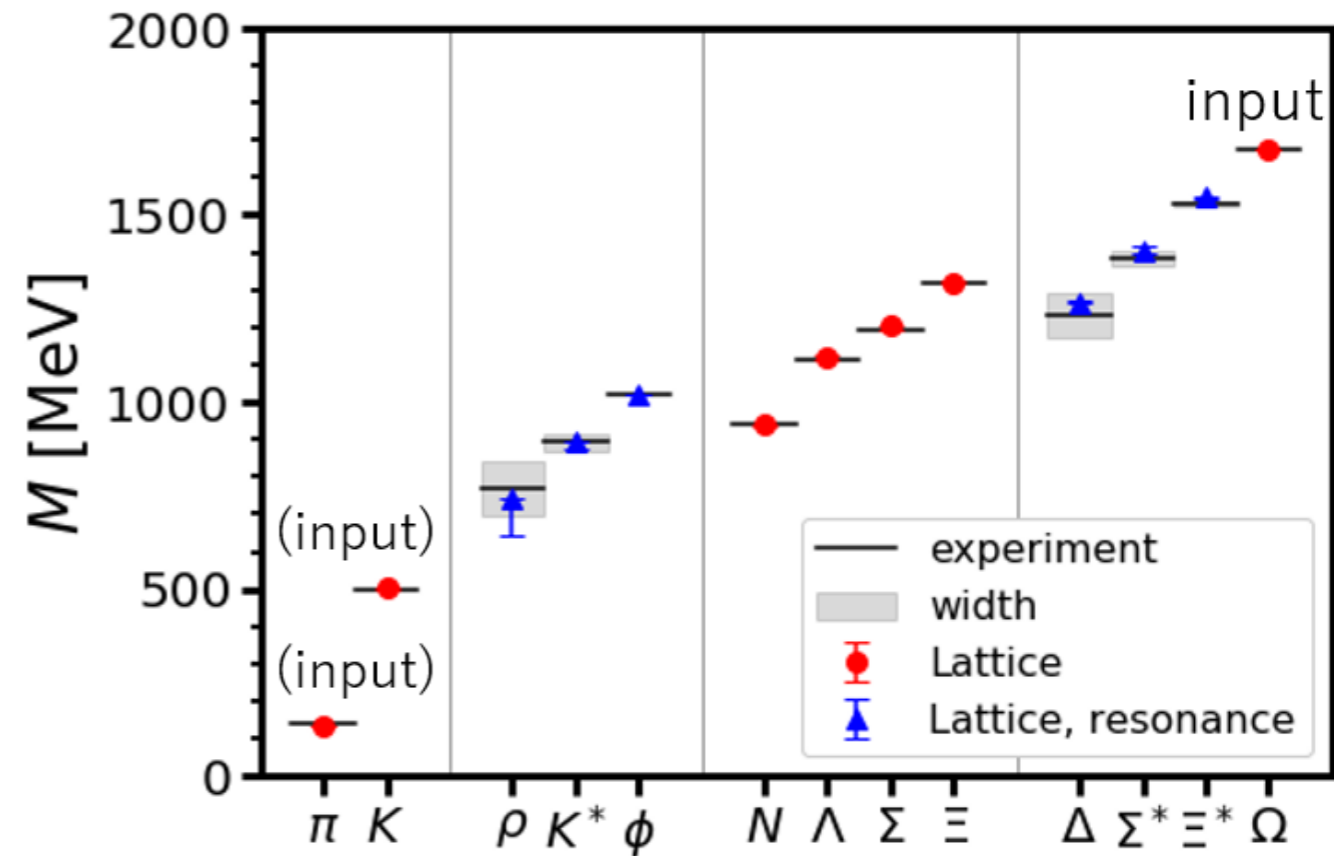
$N \times N \times N \times N$ cube in spacetime

QCD path integral = multidimensional integral over $O(N^4)$ variables

Agreement Lattice QCD / experiment



S. Dürr et al., Science 322, 1224 (2008), 0906.3599



T. Aoyama, T. M. Doi, T. Doi, E. Itou, Yan Lyu, K. Murakami and T. Sugiura [HAL QCD Collaboration], Phys.Rev. D110 (2024)094502

A question of not only aesthetics

Is there a “more theoretical” way to “compute the proton mass”?
(open problem)

Today:

encouraging results in a related problem, via “bootstrap methods”

Bootstrap = constraining by consistency conditions

In 1960s (SECOND CRISIS) people tried to impose the most general principles on QFT correlation functions and S-matrix elements, such as:

- Lorentz invariance
- Unitarity
- Causality
- Analyticity
- Crossing symmetry

Chew Mandelstam Bogoliubov
Wightman Froissart Martin
Polyakov

Bootstrap = constraining by consistency conditions

In 1960s (SECOND CRISIS) people tried to impose the most general principles on QFT correlation functions and S-matrix elements, such as:

- Lorentz invariance
- Unitarity
- Causality
- Analyticity
- Crossing symmetry

Chew Mandelstam Bogoliubov
Wightman Froissart Martin
Polyakov

Our generation picked up where they left

1. Conformal Bootstrap

Today

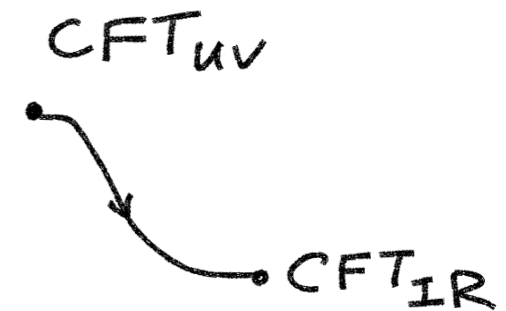
2. S-matrix Bootstrap

See weeks 4-5 of the program

Conformal Bootstrap Polyakov 1970, 1974

Applies to “**Conformal Field Theories**”, which are interesting because:

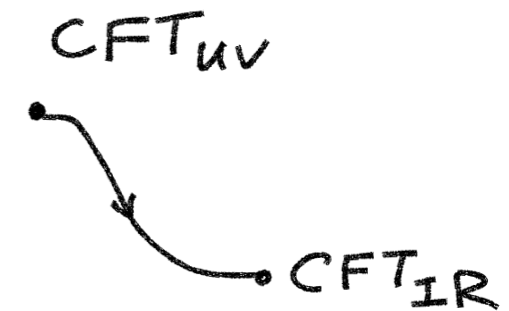
- 1) **Lampposts** in the space of all Quantum Field Theories



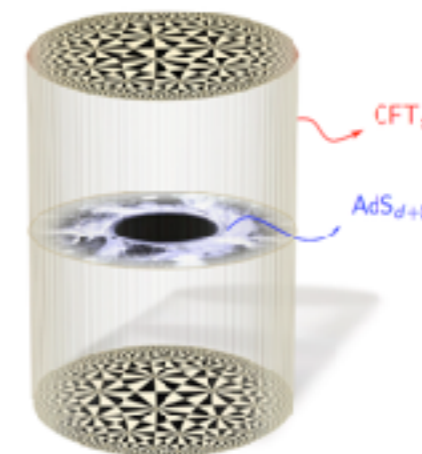
Conformal Bootstrap Polyakov 1970, 1974

Applies to “**Conformal Field Theories**”, which are interesting because:

1) **Lampposts** in the space of all Quantum Field Theories



2) Dual to models of **quantum gravity** via the AdS/CFT correspondence

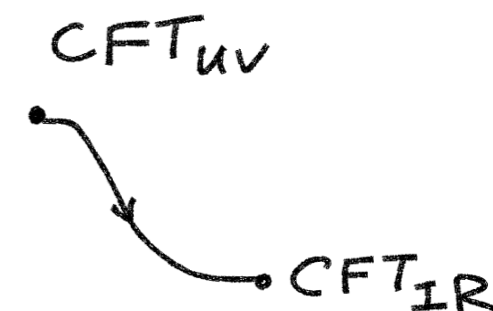


Conformal Bootstrap

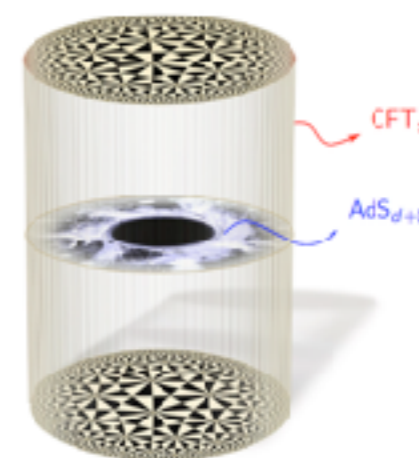
Polyakov 1970, 1974

Applies to “**Conformal Field Theories**”, which are interesting because:

1) **Lampposts** in the space of all Quantum Field Theories



2) Dual to models of **quantum gravity** via the AdS/CFT correspondence

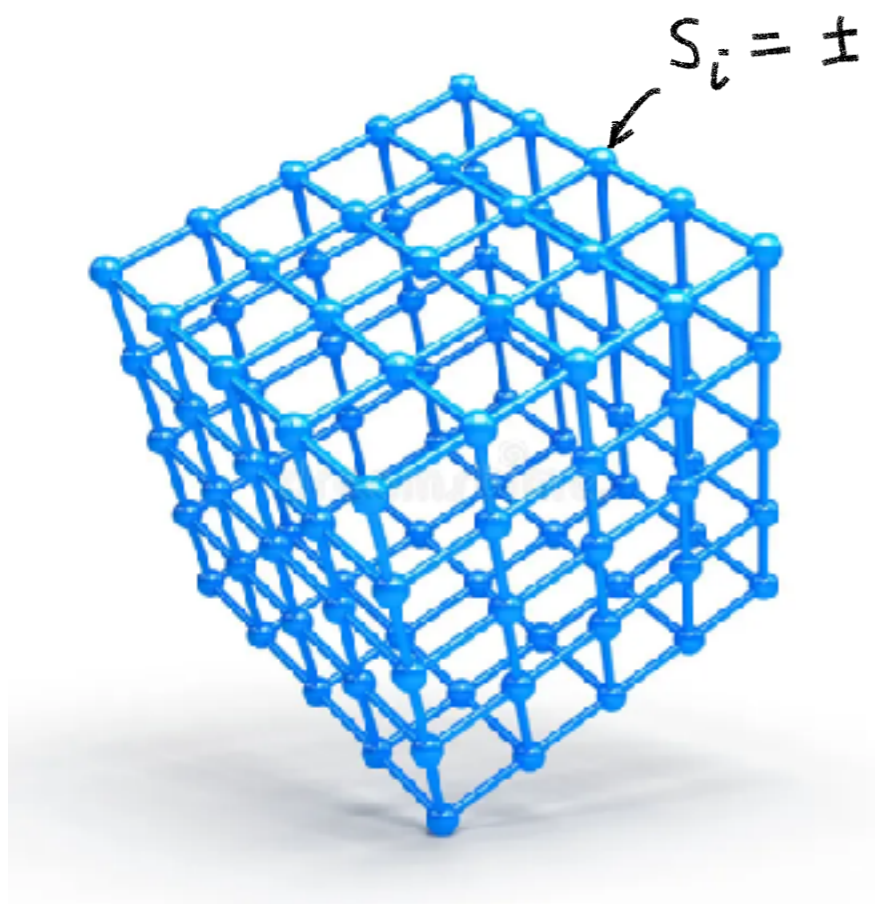


3) Describe **critical phenomena**

TODAY

in stat-phys and cond-mat - **experimental link**
(wait until the end)

Ising Model (2D or 3D)



$$E = - \sum_{\langle ij \rangle} S_i S_j$$

Energy of spin interactions

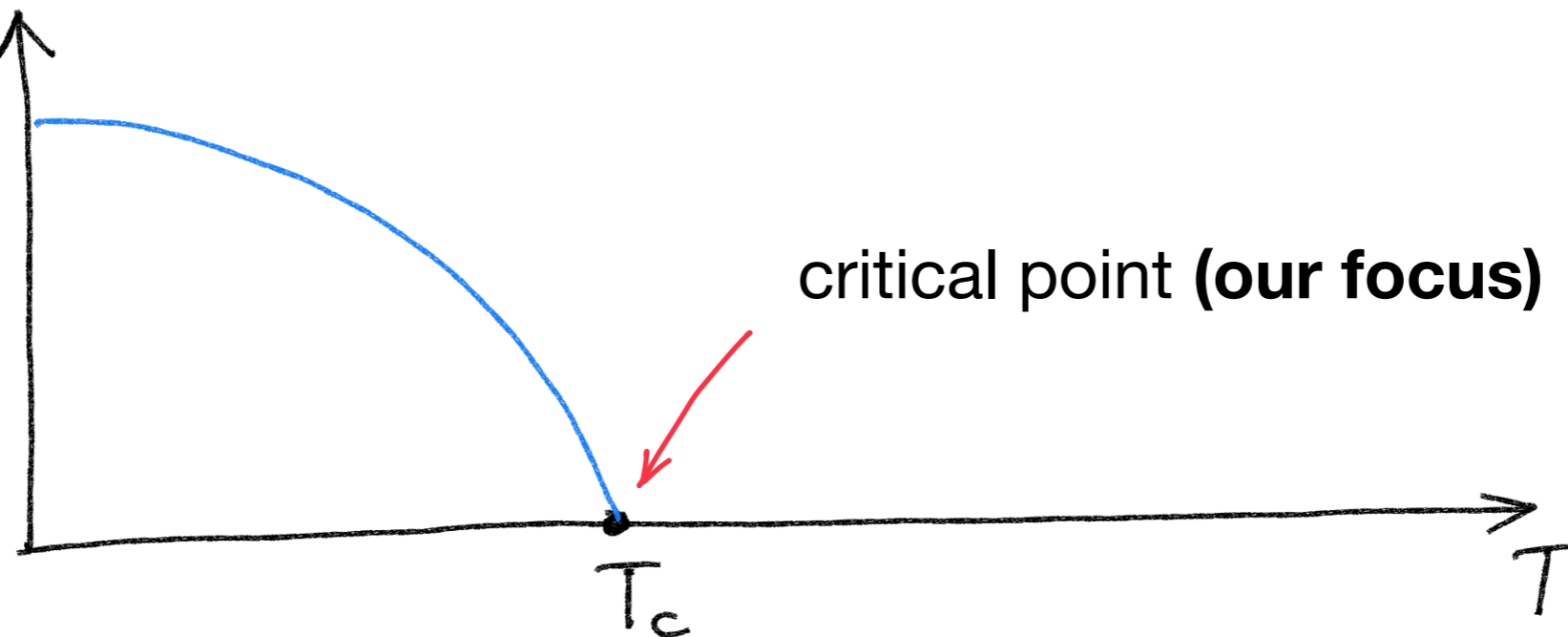
$$Z = \sum_{\{s\}} e^{-\frac{1}{T}E}$$

Thermal partition function

Phase transition in the Ising Model

spontaneous
magnetization

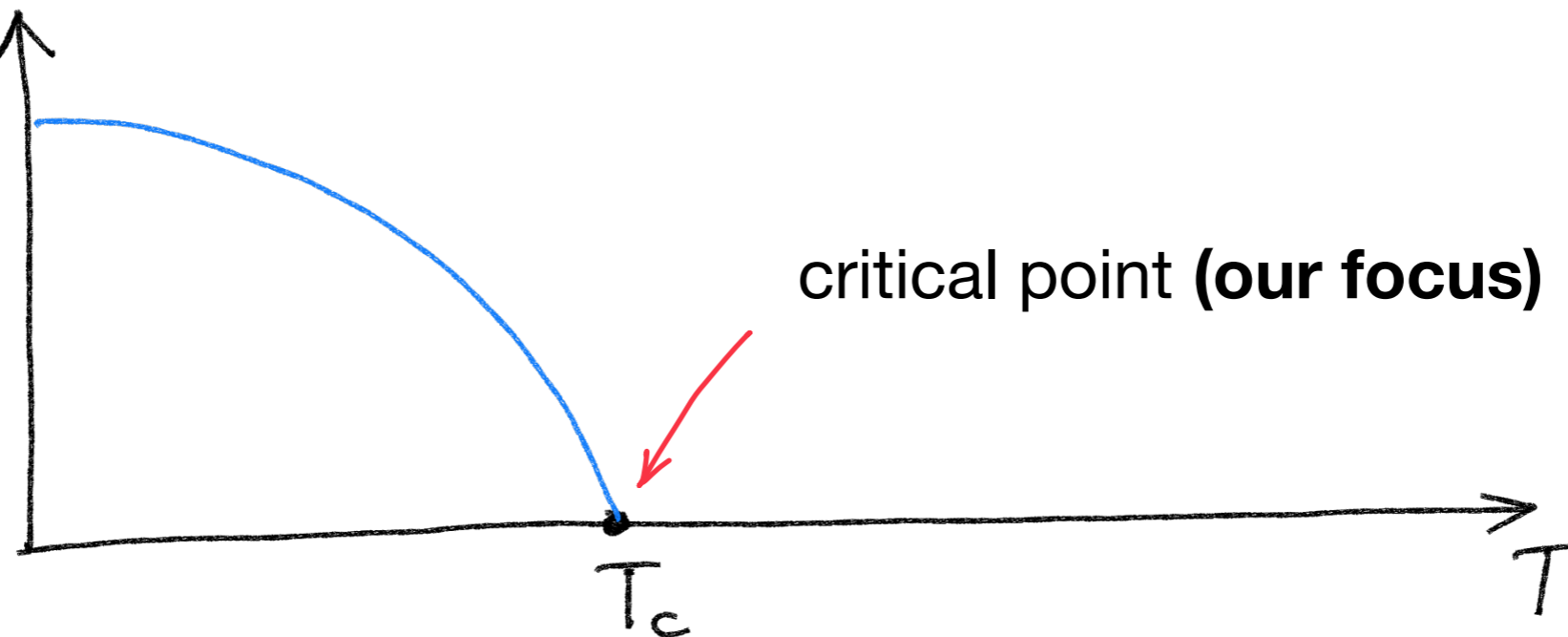
$\langle s \rangle$



Phase transition in the Ising Model

spontaneous
magnetization

$\langle s \rangle$



At $T = T_c$ correlation functions scale as powers:

$$\langle S_0 S_r \rangle \sim \frac{1}{r^{2\Delta}}$$

scaling dimension
(critical exponent)

$$\Delta = \frac{1}{8} \quad (D=2)$$

$$\Delta \approx 0.518 \quad (D=3)$$

Scale invariance

4 samples of random spin distribution at criticality, at different scales

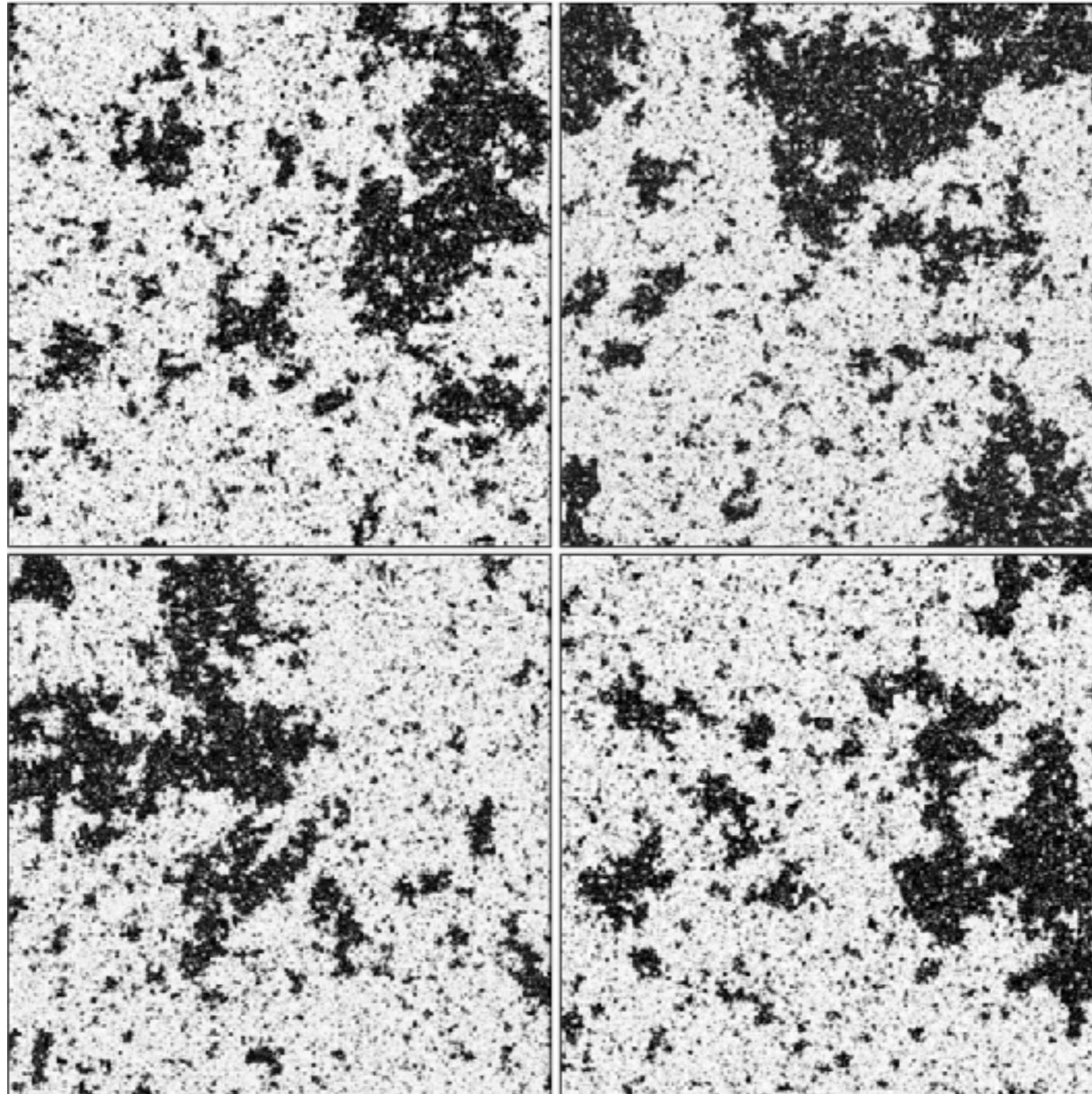


Fig: Douglas Ashton, youtube

Scale invariance

4 samples of random spin distribution at criticality, at different scales

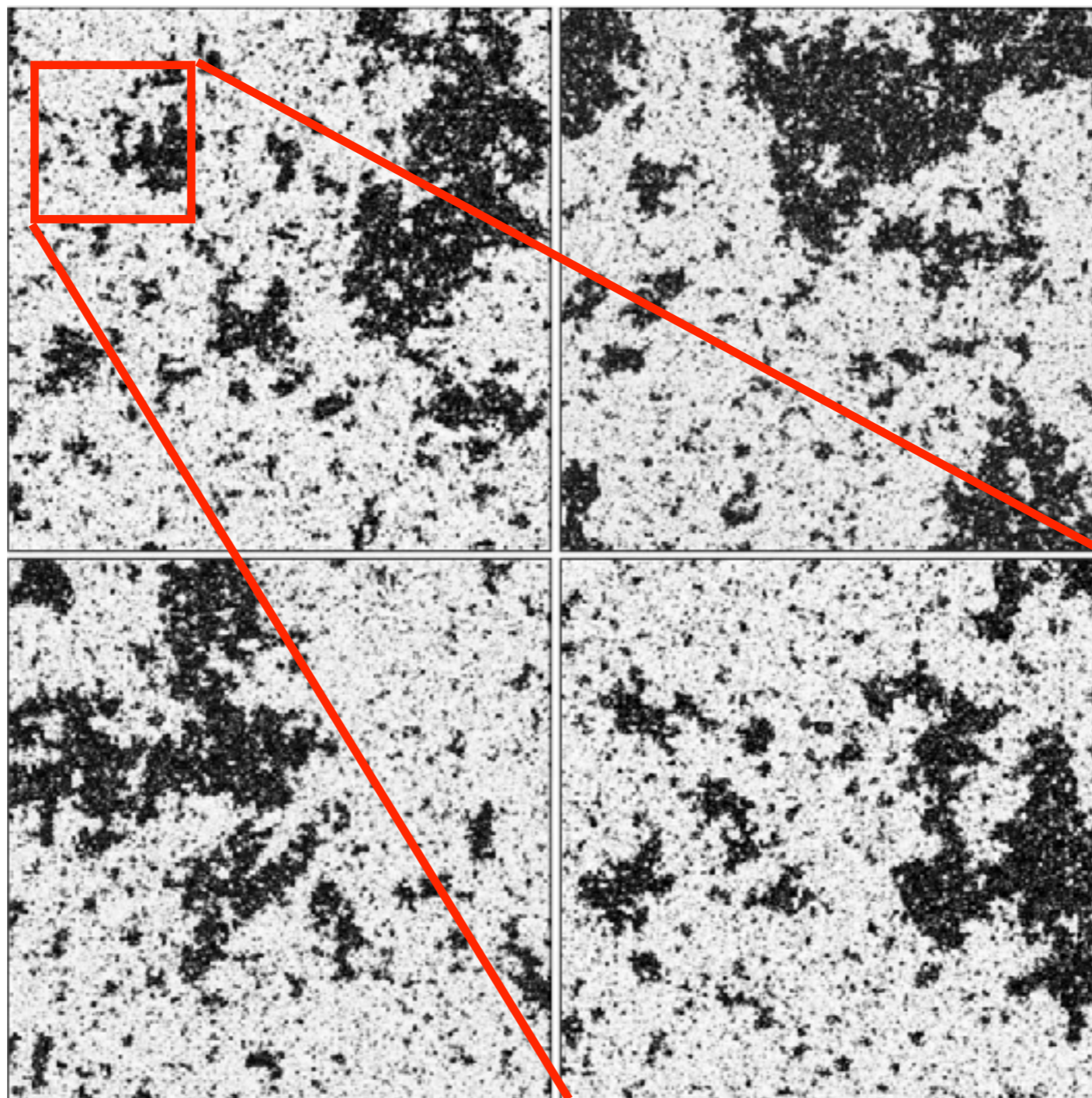


Fig: Douglas Ashton, youtube

Conformal invariance

Scale transformations act as $x^M \rightarrow \lambda x^M$

Conformal transformations act nonlinearly

(D=2) $z \rightarrow \frac{az + b}{cz + d} \quad (z \in \mathbb{C})$

- Map circles to circles (or straight lines)
- Preserve intersection angles

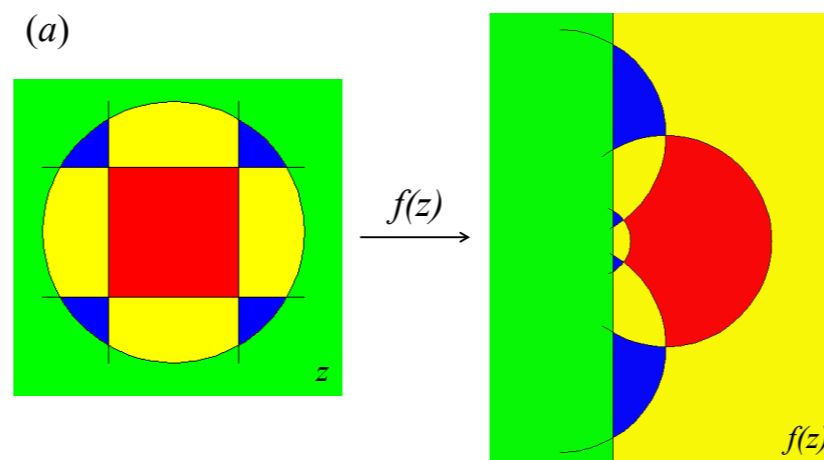


Fig: Ali Shaberi, 1504.02898

Generalize to D=3 - finite-dimensional Lie group $SO(4,2)$

Statistics of Ising spins at criticality is conformally invariant

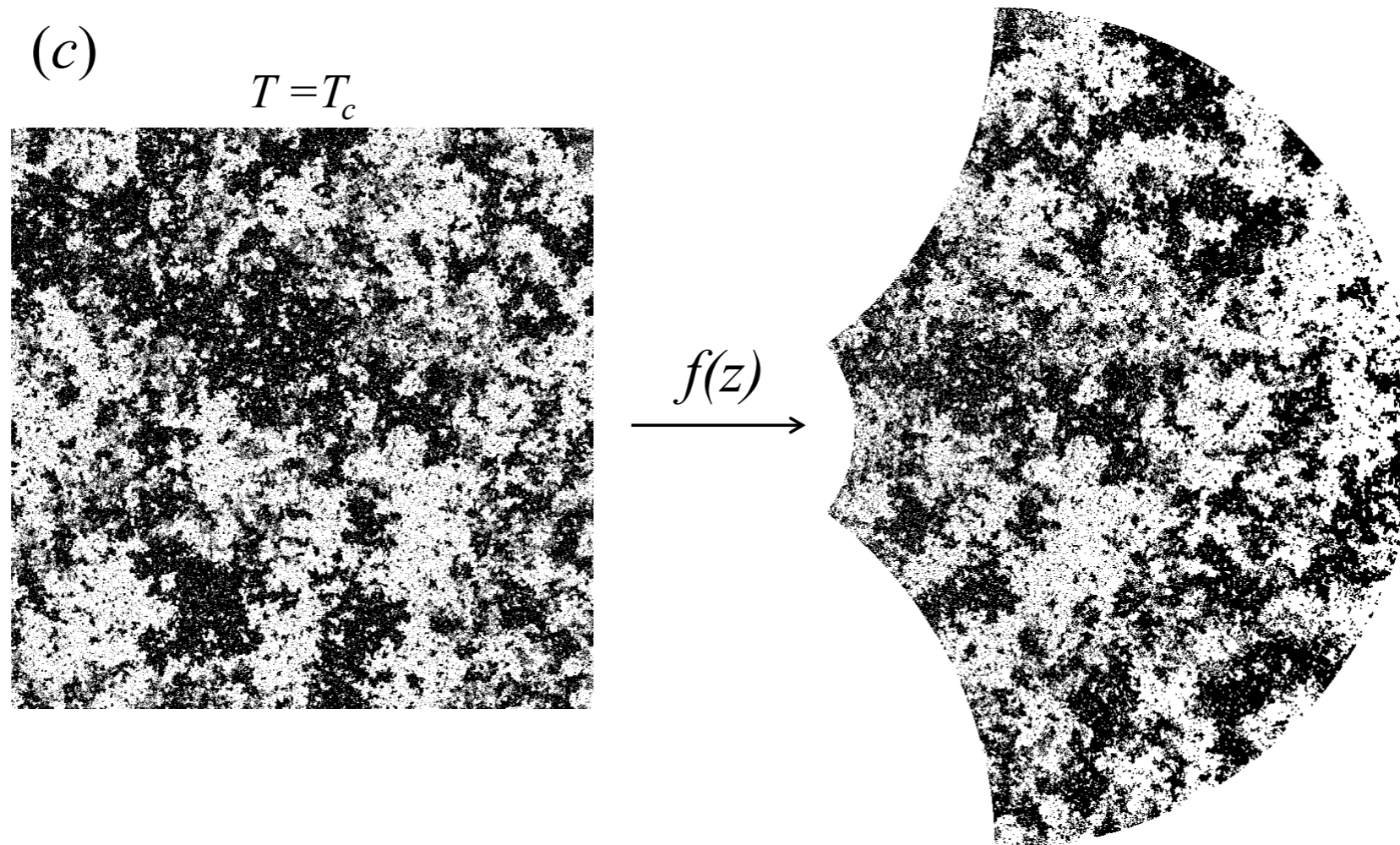


Fig: Ali Shaberi, 1504.02898

Statistics of Ising spins at criticality is conformally invariant

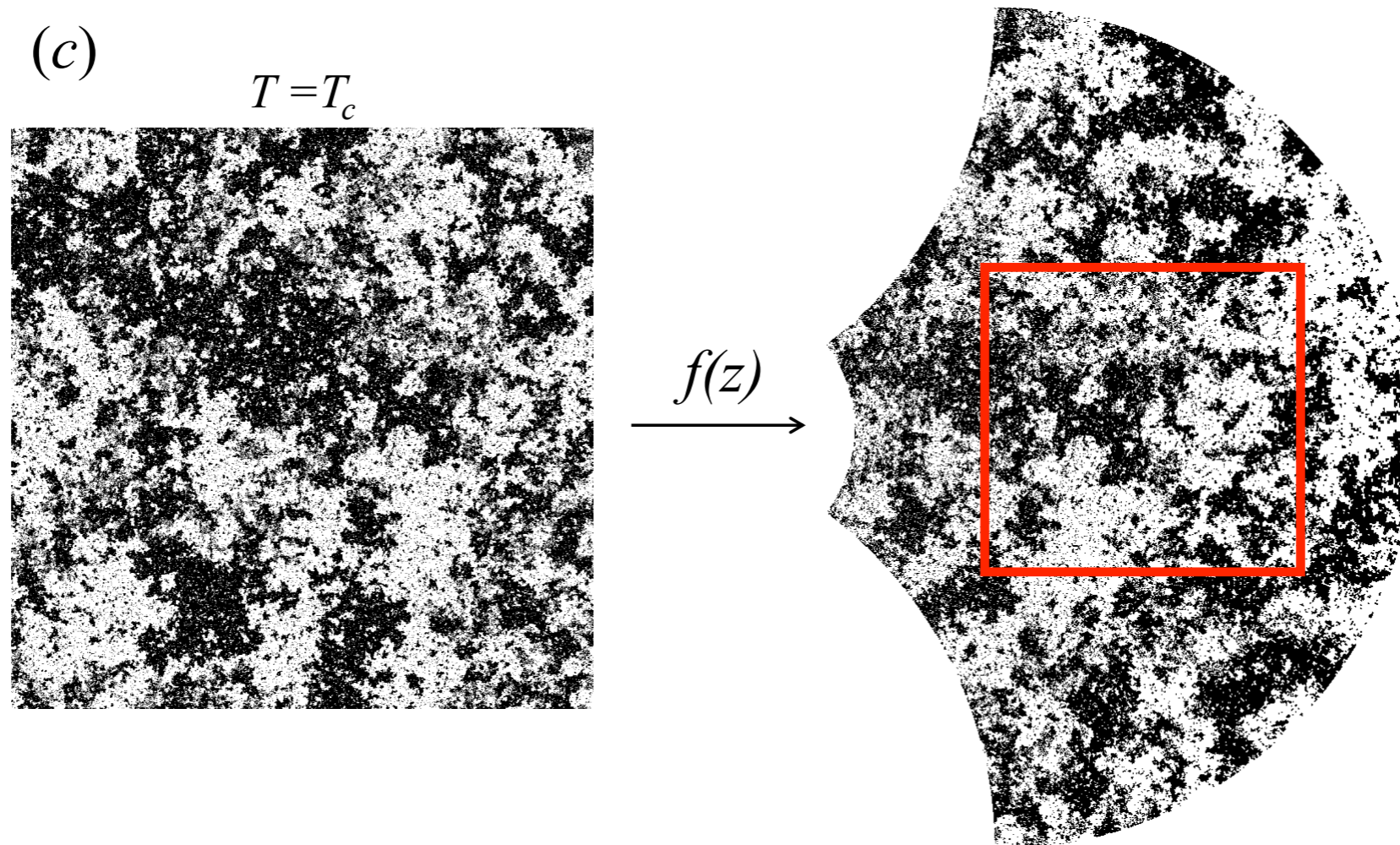


Fig: Ali Shaberi, 1504.02898

Field Theory

Wilson-Fisher 1972

$$S = \int d^d x [(\partial\phi)^2 + m^2\phi^2 + \lambda\phi^4]$$

- scale invariance in IR
- fundamental field ϕ
- composite fields
 $\phi^2, \phi^3, (\partial\phi)^2, \dots$
- scaling dimension
= classical + anomalous

$$\Delta = \Delta_{cl} + \gamma$$

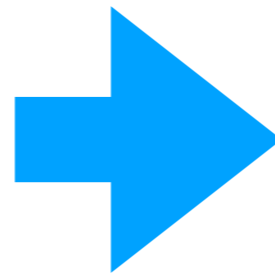
- conformal invariance not used

Field Theory

Wilson-Fisher 1972

$$S = \int d^d x [(\partial\phi)^2 + m^2\phi^2 + \lambda\phi^4]$$

- scale invariance in IR
- fundamental field ϕ
- composite fields
 $\phi^2, \phi^3, (\partial\phi)^2, \dots$
- scaling dimension
= classical + anomalous
 $\Delta = \Delta_{cl} + \gamma$
- conformal invariance not used



Conformal Field Theory

- scale and conformal invariance at all distances
- all fields on equal footing
 A_1, A_2, A_3, \dots
 $\Delta_1 < \Delta_2 < \Delta_3 < \dots$
- scaling dimensions are not divided into classical + anomalous

Conformal invariance constrains correlators Polyakov 1970

3pt functions

$$\langle A_1(x_1)A_2(x_2)A_3(x_3) \rangle = \frac{f_{123}}{|x_1 - x_2|^{\Delta_1 + \Delta_2 - \Delta_3} |x_1 - x_3|^{\Delta_1 + \Delta_3 - \Delta_2} |x_2 - x_3|^{\Delta_2 + \Delta_3 - \Delta_1}}$$

OPE coefficients

scaling dimensions

4pt functions

$$\langle A(x_1)A(x_2)A(x_3)A(x_4) \rangle = \frac{g(u, v)}{|x_1 - x_2|^{2\Delta_A} |x_3 - x_4|^{2\Delta_A}}$$

u, v: conformal cross-ratios

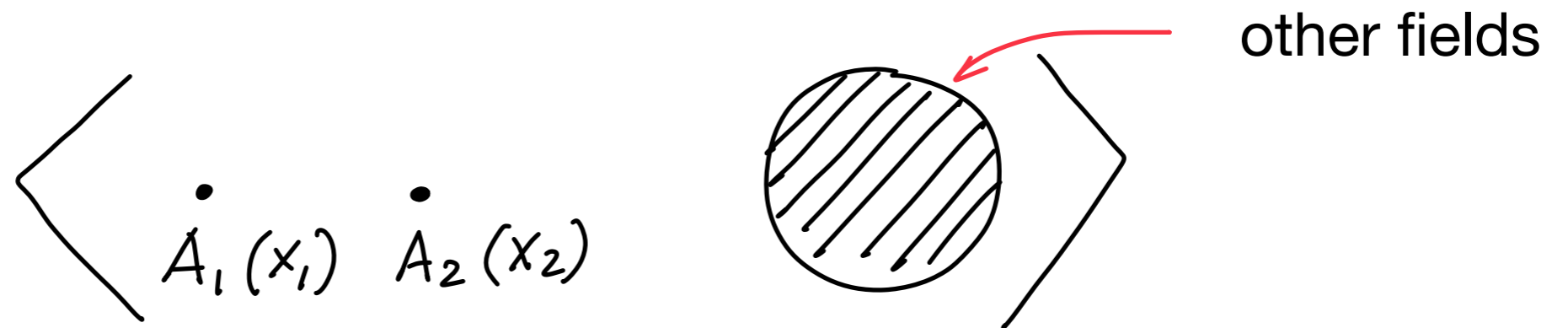
$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}$$
$$v = u|_{1 \leftrightarrow 3}$$

Key point:

$g(u, v)$ can be computed if Δ_i and f_{ijk} are known

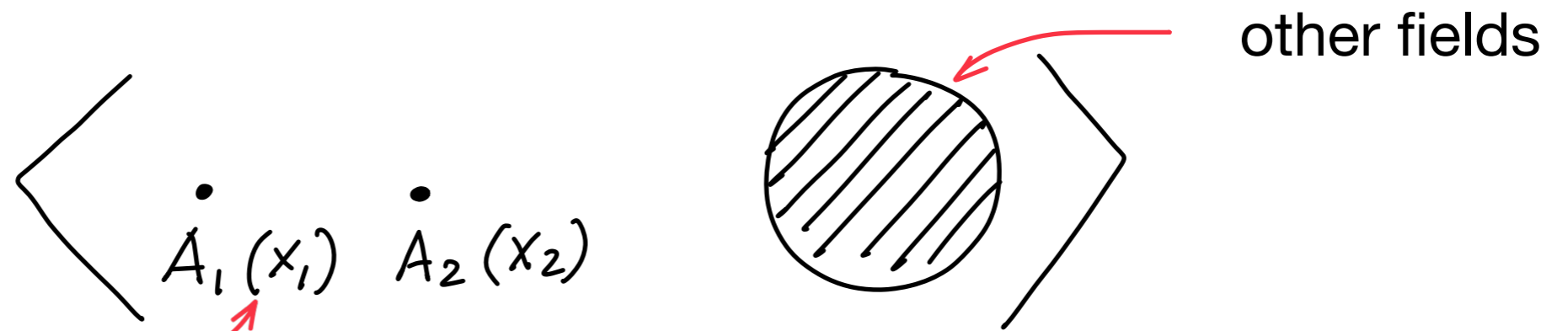
Operator Product Expansion

Wilson Kadanoff Polyakov Ferrara-Gatto-Grillo



Operator Product Expansion

Wilson Kadanoff Polyakov Ferrara-Gatto-Grillo

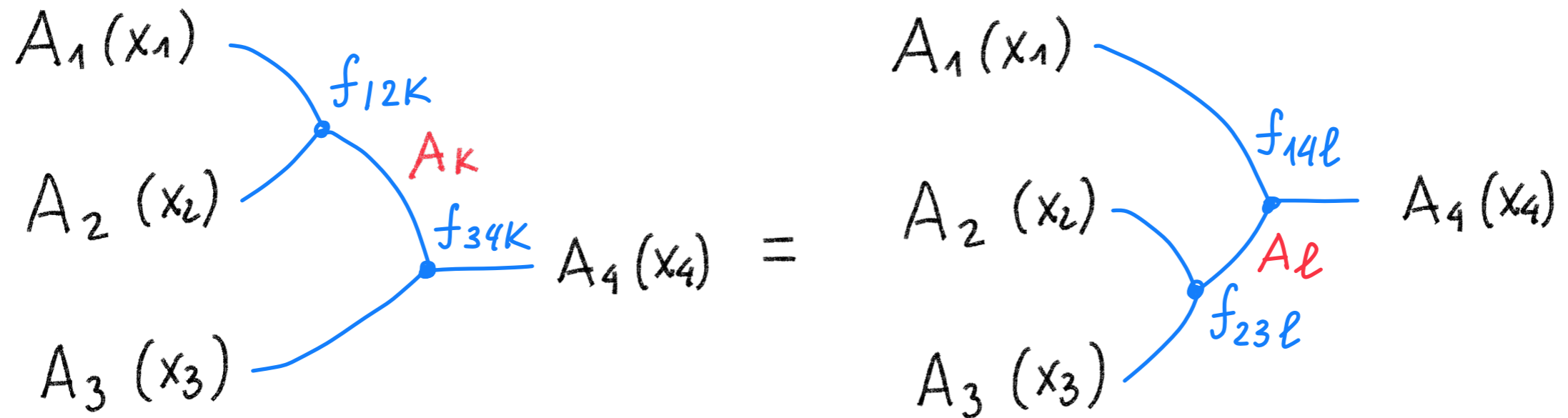


$$A_1(x_1)A_2(x_2) = \frac{1}{|x_1 - x_2|^{\Delta_1 + \Delta_2}} \sum_{k=1}^{\infty} f_{12k} |x_1 - x_2|^{\Delta_k} \left[A_k\left(\frac{x_1 + x_2}{2}\right) + \dots \right]$$

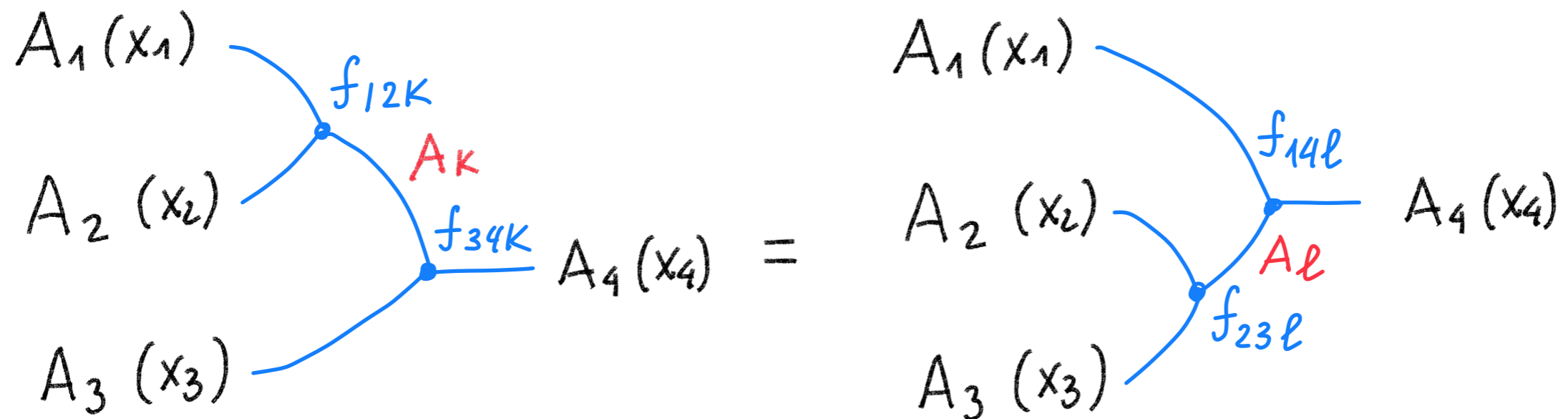
OPE coefficients scaling dimensions

- operator statement, to be used inside correlators
- convergent at finite distances (to the next field insertion)

OPE associativity



OPE associativity



Gives quadratic equation on OPE coefficients:

$$\sum_{k=1}^{\infty} f_{12k} f_{34k} G_k(x_1, \dots, x_4) = \sum_{l=1}^{\infty} f_{14l} f_{23l} G_l(x_1, \dots, x_4)$$

explicitly known special functions
("conformal blocks")

History of conformal bootstrap

- Basically all so far is in [Polyakov 1970, 1974](#) (checks but no applications)

Non-Hamiltonian approach to conformal quantum field theory

A. M. Polyakov

L. D. Landau Theoretical Physics Institute, USSR Academy of Sciences

(Submitted July 9, 1973)

Zh. Eksp. Teor. Fiz. 66, 23–42 (January 1974)

- 1984: [Polyakov, Belavin, Zamolodchikov](#) apply this in $D=2$, solve 2D Ising CFT

Need special 2D tricks (Virasoro, the number of fields finite)

=> much subsequent work in $D=2$,

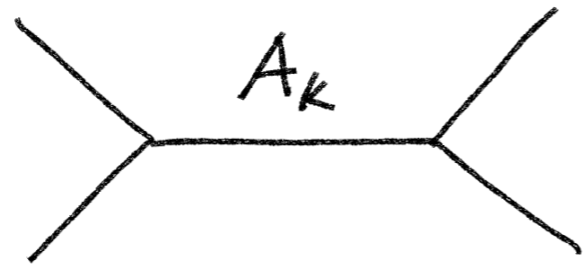


but widespread belief that this won't work in $D>2$

Renaissance of conformal bootstrap

Rattazzi, SR, Tonni, Vichi 2008

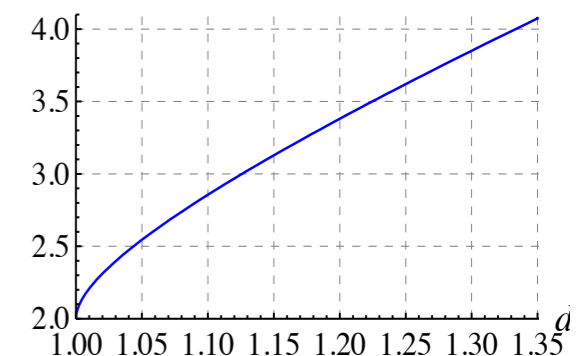
Key observations:



Number of exchanged fields A_k is ∞ but:

- **decoupling:** the fields of high scaling dimension give **exponentially small** contributions
- **positivity:** those contributions often have **definite sign**

=> Rigorous bounds on scaling dimensions of low-lying fields



Conformal bootstrap for 3D Ising CFT

El-Showk, Paulos, Poland, SR, Simmons-Duffin, Vichi 2012, 2014

Kos, Poland, Simmons-Duffin 2014, 2016

Chang, Dommers, Erramilli, Homrich, Kravchuk, Liu, Mitchell, Poland, Simmons-Duffin 2024

Two main fields of 3D Ising CFT are:

σ	\mathbb{Z}_2 -odd	(renormalized versions of ϕ, ϕ^2 in WF)
ε	\mathbb{Z}_2 -even	

Conformal bootstrap for 3D Ising CFT

El-Showk, Paulos, Poland, SR, Simmons-Duffin, Vichi 2012, 2014

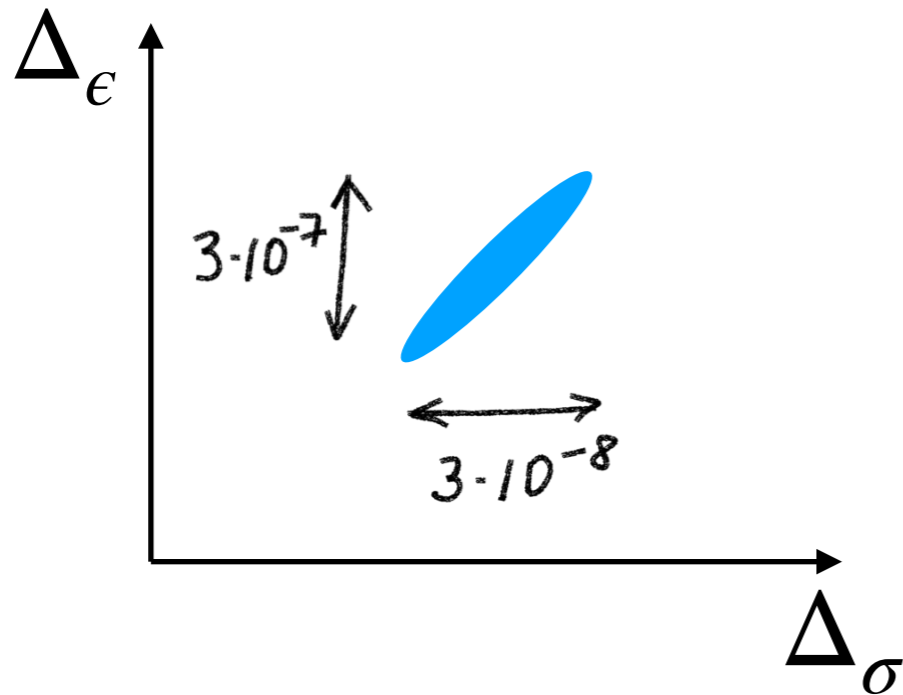
Kos, Poland, Simmons-Duffin 2014, 2016

Chang, Dommers, Erramilli, Homrich, Kravchuk, Liu, Mitchell, Poland, Simmons-Duffin 2024

Two main fields of 3D Ising CFT are:

$$\begin{array}{ll} \sigma & \mathbb{Z}_2\text{-odd} \\ \varepsilon & \mathbb{Z}_2\text{-even} \end{array} \quad (\text{renormalized versions of } \phi, \phi^2 \text{ in WF})$$

Bootstrap => their scaling dimensions must live in a small region:



Conformal bootstrap for 3D Ising CFT

El-Showk, Paulos, Poland, SR, Simmons-Duffin, Vichi 2012, 2014

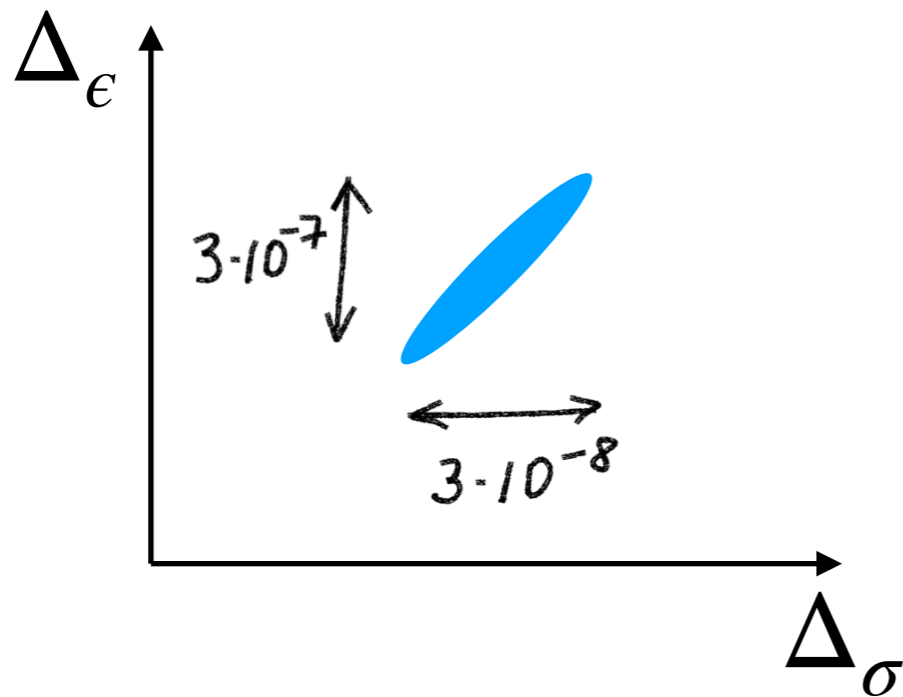
Kos, Poland, Simmons-Duffin 2014, 2016

Chang, Dommers, Erramilli, Homrich, Kravchuk, Liu, Mitchell, Poland, Simmons-Duffin 2024

Two main fields of 3D Ising CFT are:

$$\begin{array}{ll} \sigma & \mathbb{Z}_2\text{-odd} \\ \varepsilon & \mathbb{Z}_2\text{-even} \end{array} \quad (\text{renormalized versions of } \phi, \phi^2 \text{ in WF})$$

Bootstrap => their scaling dimensions must live in a small region:



$$\Delta_\sigma = 0.518148806(24)$$

$$\Delta_\varepsilon = 1.41262528(29)$$



critical exponents $\alpha, \beta, \gamma, \delta, \nu, \eta$
with $\sim 10^{-7}$ accuracy

More than 100 are known, discovered from the OPEs $\sigma \times \sigma$, $\sigma \times \epsilon$, $\epsilon \times \epsilon$

\mathcal{O}	\mathbb{Z}_2	ℓ	Δ	$\tau = \Delta - \ell$	$f_{\sigma\sigma\mathcal{O}}$	$f_{\epsilon\epsilon\mathcal{O}}$
ϵ	+	0	1.412625(10)	1.412625(10)	1.0518537(41)	1.532435(19)
ϵ'	+	0	3.82968(23)	3.82968(23)	0.053012(55)	1.5360(16)
	+	0	6.8956(43)	6.8956(43)	0.0007338(31)	0.1279(17)
	+	0	7.2535(51)	7.2535(51)	0.000162(12)	0.1874(31)
$T_{\mu\nu}$	+	2	3	1	0.32613776(45)	0.8891471(40)
$T'_{\mu\nu}$	+	2	5.50915(44)	3.50915(44)	0.0105745(42)	0.69023(49)
	+	2	7.0758(58)	5.0758(58)	0.0004773(62)	0.21882(73)
$C_{\mu\nu\rho\sigma}$	+	4	5.022665(28)	1.022665(28)	0.069076(43)	0.24792(20)
	+	4	6.42065(64)	2.42065(64)	0.0019552(12)	-0.110247(54)
	+	4	7.38568(28)	3.38568(28)	0.00237745(44)	0.22975(10)
	+	6	7.028488(16)	1.028488(16)	0.0157416(41)	0.066136(36)

\mathcal{O}	\mathbb{Z}_2	ℓ	Δ	$\tau = \Delta - \ell$	$f_{\sigma\epsilon\mathcal{O}}$	-
σ	-	0	0.5181489(10)	0.5181489(10)	1.0518537(41)	
σ'	-	0	5.2906(11)	5.2906(11)	0.057235(20)	
	-	2	4.180305(18)	2.180305(18)	0.38915941(81)	
	-	2	6.9873(53)	4.9873(53)	0.017413(73)	
	-	3	4.63804(88)	1.63804(88)	0.1385(34)	
	-	4	6.112674(19)	2.112674(19)	0.1077052(16)	
	-	5	6.709778(27)	1.709778(27)	0.04191549(88)	

\mathbb{Z}_2	ℓ	Δ	$\tau = \Delta - \ell$	$f_{\sigma\sigma\mathcal{O}}$	$f_{\epsilon\epsilon\mathcal{O}}$
+	2	3	1	0.32613776(45)	0.8891471(40)
+	4	5.022665(28)	1.022665(28)	0.069076(43)	0.24792(20)
+	6	7.028488(16)	1.028488(16)	0.0157416(41)	0.066136(36)
+	8	9.031023(30)	1.031023(30)	0.0036850(54)	0.017318(30)
+	10	11.0324141(99)	1.0324141(99)	0.00087562(13)	0.0044811(15)
+	12	13.033286(12)	1.033286(12)	0.000209920(37)	0.00115174(59)
+	14	15.033838(15)	1.033838(15)	0.000050650(99)	0.00029484(56)
+	16	17.034258(34)	1.034258(34)	0.000012280(18)	0.00007517(18)
+	18	19.034564(12)	1.034564(12)	$2.98935(46) \cdot 10^{-6}$	$0.0000191408(89)$
+	20	21.0347884(84)	1.0347884(84)	$7.2954(10) \cdot 10^{-7}$	$4.8632(23) \cdot 10^{-6}$
+	22	23.034983(11)	1.034983(11)	$1.78412(27) \cdot 10^{-7}$	$1.23201(72) \cdot 10^{-6}$
+	24	25.035122(11)	1.035122(11)	$4.37261(60) \cdot 10^{-8}$	$3.1223(15) \cdot 10^{-7}$
+	26	27.035249(11)	1.035249(11)	$1.07287(18) \cdot 10^{-8}$	$7.8948(42) \cdot 10^{-8}$
+	28	29.035344(19)	1.035344(19)	$2.6409(19) \cdot 10^{-9}$	$1.9992(23) \cdot 10^{-8}$
+	30	31.035452(16)	1.035452(16)	$6.447(24) \cdot 10^{-10}$	$5.003(20) \cdot 10^{-9}$
+	32	33.035473(28)	1.035473(28)	$1.640(25) \cdot 10^{-10}$	$1.308(21) \cdot 10^{-9}$
+	34	35.035632(67)	1.035632(67)	$3.58(22) \cdot 10^{-11}$	$2.90(19) \cdot 10^{-10}$
+	36	37.035610(41)	1.035610(41)	$1.15(13) \cdot 10^{-11}$	$9.6(11) \cdot 10^{-11}$
+	38	39.035638(58)	1.035638(58)	$2.26(71) \cdot 10^{-12}$	$1.93(60) \cdot 10^{-11}$
+	40	41.03564(13)	1.03564(13)	$7.3(15) \cdot 10^{-13}$	$6.3(13) \cdot 10^{-12}$

\mathbb{Z}_2	ℓ	Δ	$\tau = \Delta - \ell$	$f_{\sigma\sigma\mathcal{O}}$	$f_{\epsilon\epsilon\mathcal{O}}$
+	4	6.42065(64)	2.42065(64)	0.0019552(12)	-0.110247(54)
+	6	8.4957(75)	2.4957(75)	0.000472(49)	-0.0431(48)
+	8	10.562(12)	2.562(12)	0.0001084(69)	-0.0139(11)
+	10	12.5659(57)	2.5659(57)	0.00002598(39)	-0.004437(62)
+	12	14.633(21)	2.633(21)	$6.10(33) \cdot 10^{-6}$	-0.001224(60)
+	14	16.6174(75)	2.6174(75)	$1.417(34) \cdot 10^{-6}$	-0.0003791(54)
+	16	18.678(24)	2.678(24)	$3.547(59) \cdot 10^{-7}$	-0.0000972(64)
+	18	20.654(22)	2.654(22)	$7.99(90) \cdot 10^{-8}$	-0.0000284(26)
+	20	22.651(27)	2.651(27)	$1.83(13) \cdot 10^{-8}$	$-7.58(47) \cdot 10^{-6}$
+	22	24.671(18)	2.671(18)	$4.55(72) \cdot 10^{-9}$	$-2.09(19) \cdot 10^{-6}$
+	24	26.681(20)	2.681(20)	$1.168(29) \cdot 10^{-9}$	$-5.67(17) \cdot 10^{-7}$
+	26	28.706(24)	2.706(24)	$2.81(17) \cdot 10^{-10}$	$-1.49(11) \cdot 10^{-7}$
+	28	30.6923(81)	2.6923(81)	$6.69(36) \cdot 10^{-11}$	$-4.162(88) \cdot 10^{-8}$
+	30	32.702(11)	2.702(11)	$1.62(16) \cdot 10^{-11}$	$-1.066(59) \cdot 10^{-8}$
+	32	34.718(17)	2.718(17)	$4.15(42) \cdot 10^{-12}$	$-2.83(18) \cdot 10^{-9}$
+	34	36.717(16)	2.717(16)	$9.44(77) \cdot 10^{-13}$	$-7.33(59) \cdot 10^{-10}$
+	36	38.697(17)	2.697(17)	$2.40(39) \cdot 10^{-13}$	$-2.12(34) \cdot 10^{-10}$
+	38	40.701(19)	2.701(19)	$5.4(17) \cdot 10^{-14}$	$-5.2(15) \cdot 10^{-11}$
+	40	42.726(18)	2.726(18)	$1.59(49) \cdot 10^{-14}$	$-1.55(48) \cdot 10^{-11}$
+	42	44.729(15)	2.729(15)	$4.2(12) \cdot 10^{-15}$	$-4.4(11) \cdot 10^{-12}$

\mathbb{Z}_2	ℓ	Δ	$\tau = \Delta - \ell$	$f_{\sigma\sigma\mathcal{O}}$	$f_{\epsilon\epsilon\mathcal{O}}$
+	0	3.82968(23)	3.82968(23)	0.053012(55)	1.5360(16)
+	2	5.50915(44)	3.50915(44)	0.0105745(42)	0.69023(49)
+	4	7.38568(28)	3.38568(28)	0.00237745(44)	0.22975(10)
+	6	9.32032(34)	3.32032(34)	0.00055657(42)	0.06949(11)
+	8	11.2751(24)	3.2751(24)	0.00013251(91)	0.01980(15)
+	10	13.2410(10)	3.2410(10)	0.00003234(15)	0.005459(39)
+	12	15.2301(64)	3.2301(64)	$7.64(14) \cdot 10^{-6}$	0.001538(22)
+	14	17.1944(55)	3.1944(55)	$1.930(46) \cdot 10^{-6}$	0.000386(14)
+	16	19.1950(62)	3.1950(62)	$4.568(72) \cdot 10^{-7}$	0.0001107(16)
+	18	21.1720(23)	3.1720(23)	$1.153(27) \cdot 10^{-7}$	0.00002798(33)
+	20	23.167(10)	3.167(10)	$2.74(11) \cdot 10^{-8}$	$7.45(52) \cdot 10^{-6}$
+	22	25.163(10)	3.163(10)	$6.88(22) \cdot 10^{-9}$	$1.937(51) \cdot 10^{-6}$
+	24	27.1491(82)	3.1491(82)	$1.716(45) \cdot 10^{-9}$	$4.92(42) \cdot 10^{-7}$
+	26	29.1460(53)	3.1460(53)	$4.183(78) \cdot 10^{-10}$	$1.347(62) \cdot 10^{-7}$
+	28	31.1306(52)	3.1306(52)	$1.056(50) \cdot 10^{-10}$	$3.35(10) \cdot 10^{-8}$
+	30	33.126(12)	3.126(12)	$2.54(10) \cdot 10^{-11}$	$8.35(42) \cdot 10^{-9}$
+	32	35.1299(77)	3.1299(77)	$6.71(17) \cdot 10^{-12}$	$2.36(13) \cdot 10^{-9}$
+	34	37.1174(64)	3.1174(64)	$1.39(14) \cdot 10^{-12}$	$4.87(48) \cdot 10^{-10}$
+	36	39.1079(78)	3.1079(78)	$4.84(56) \cdot 10^{-13}$	$1.70(17) \cdot 10^{-10}$
+	38	41.101(29)	3.101(29)	$8.4(28) \cdot 10^{-14}$	$2.5(11) \cdot 10^{-11}$
+	40	43.102(18)	3.102(18)	$2.63(64) \cdot 10^{-14}$	$9.0(26) \cdot 10^{-12}$
+	42	45.116(27)	3.116(27)	$7.9(22) \cdot 10^{-15}$	$3.42(95) \cdot 10^{-12}$

Proliferation like for hadrons - but in this case we have a theory

Summary so far

- 3D Ising critical point is a **CFT**.

It has infinitely many fields, scaling dimensions Δ_i and OPE coefficients f_{ijk}

Summary so far

- 3D Ising critical point is a **CFT**.
It has infinitely many fields, scaling dimensions Δ_i and OPE coefficients f_{ijk}
- **Conformal bootstrap** finds this “CFT data” by imposing OPE associativity

Summary so far

- 3D Ising critical point is a **CFT**.
It has infinitely many fields, scaling dimensions Δ_i and OPE coefficients f_{ijk}
- **Conformal bootstrap** finds this “CFT data” by imposing OPE associativity
- Not limited to 3D Ising - dozens of other CFTs have been studied (e.g. Heisenberg and XY)
(some interesting puzzles solved where RG was not precise enough)

Reviews: SR, Poland, Vichi - RMP 2018
Poland, Simmons-Duffin 2022
Ning Su, SR - RMP 2024

Summary so far

- 3D Ising critical point is a **CFT**.
It has infinitely many fields, scaling dimensions Δ_i and OPE coefficients f_{ijk}
- **Conformal bootstrap** finds this “CFT data” by imposing OPE associativity
- Not limited to 3D Ising - dozens of other CFTs have been studied (e.g. Heisenberg and XY)
(some interesting puzzles solved where RG was not precise enough)

Reviews: SR, Poland, Vichi - RMP 2018
Poland, Simmons-Duffin 2022
Ning Su, SR - RMP 2024

Promising research direction:

Adapt methods of conformal bootstrap to massive theories (S-matrix bootstrap)

Review: Kruczenski, Penedones, van Rees 2022

weeks 4,5 of the program

Experimental tests of conformal invariance

- **Polyakov** (1970): conformal invariance is a symmetry of critical phenomena
- *Bootstrap is based solely on conformal invariance, gives critical exponents in agreement with experiments, RG and Monte Carlo*
=> as theorists we are convinced of conformal invariance
- In Monte Carlo one has access to full statistics of configurations, can check conformal invariance [Cosme, Lopes, Penedones 2015](#)

Experimental tests of conformal invariance

- **Polyakov** (1970): conformal invariance is a symmetry of critical phenomena
 - *Bootstrap is based solely on conformal invariance, gives critical exponents in agreement with experiments, RG and Monte Carlo*
=> as theorists we are convinced of conformal invariance
 - In Monte Carlo one has access to full statistics of configurations, can check conformal invariance [Cosme, Lopes, Penedones 2015](#)
- BUT:** in classic experiments one measured only two point functions, so direct experimental tests of only scale invariance

Experimental tests of conformal invariance

- **Polyakov** (1970): conformal invariance is a symmetry of critical phenomena
- *Bootstrap is based solely on conformal invariance, gives critical exponents in agreement with experiments, RG and Monte Carlo*
=> as theorists we are convinced of conformal invariance
- In Monte Carlo one has access to full statistics of configurations, can check conformal invariance [Cosme, Lopes, Penedones 2015](#)

BUT: in classic experiments one measured only two point functions, so direct experimental tests of only scale invariance

Some experimental possibilities to test conformal invariance

- 1) two-point functions in presence of a boundary [with Alessandro Podo](#)
- 2) cold atom experiments [with Junchen Rong and experimentalists in Paris](#)

3) Dream test of conformal invariance

Percolation is conformally invariant in 2D and 3D (it's a log-CFT)

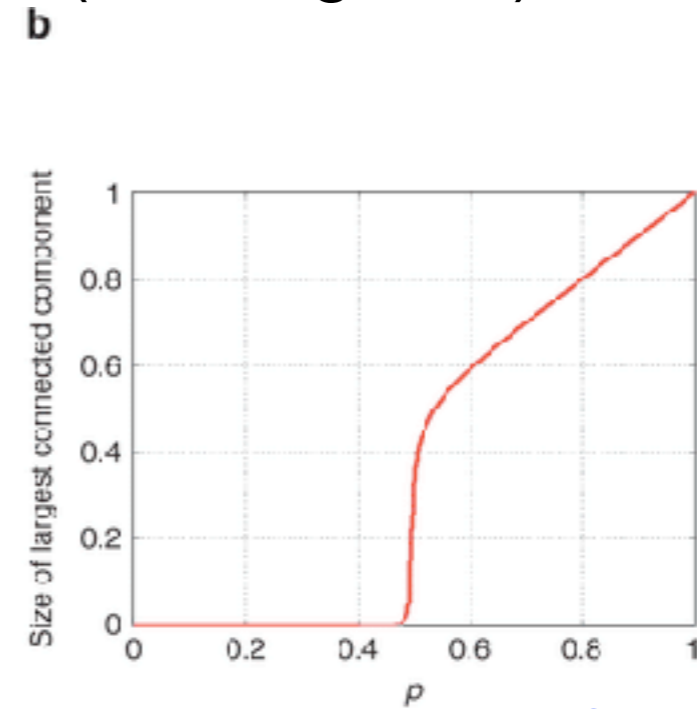
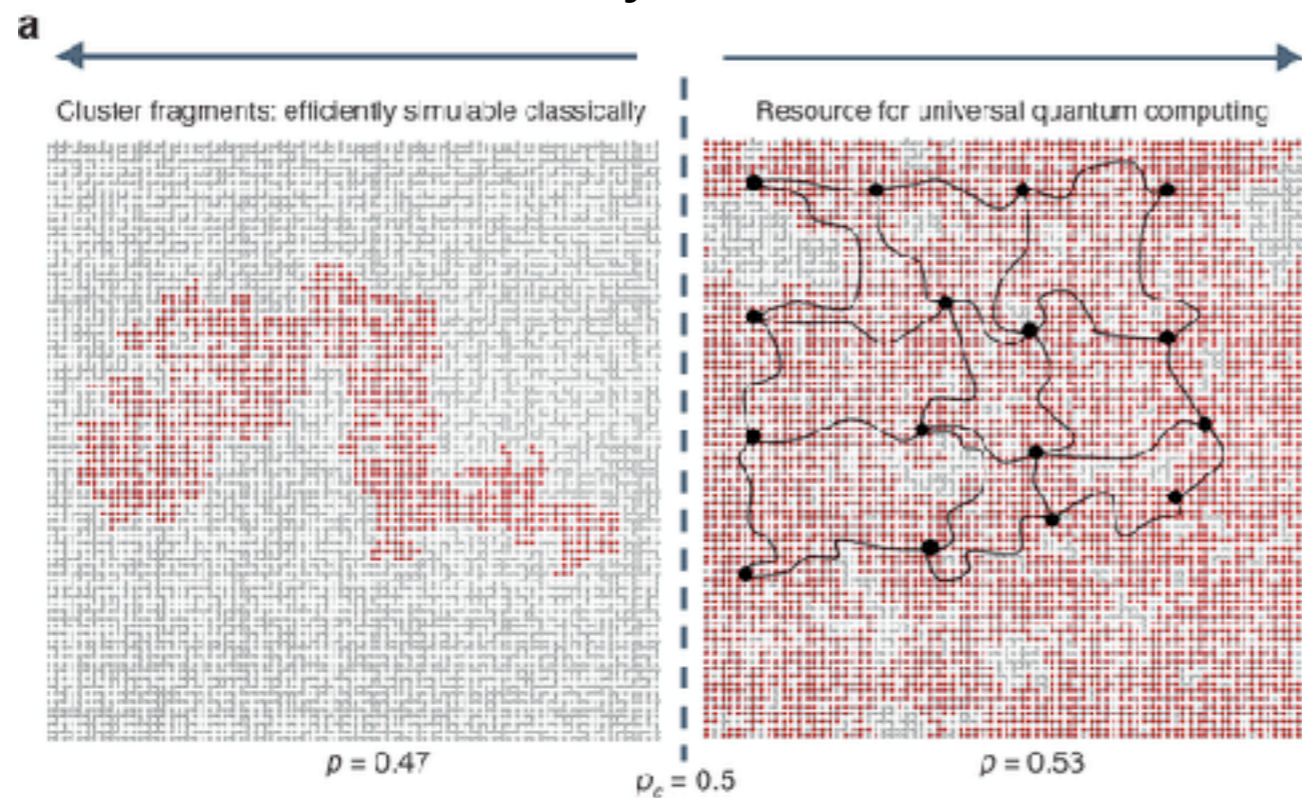


figure: Pant et al, Nature Comm., 2019

3) Dream test of conformal invariance

Percolation is conformally invariant in 2D and 3D (it's a log-CFT)

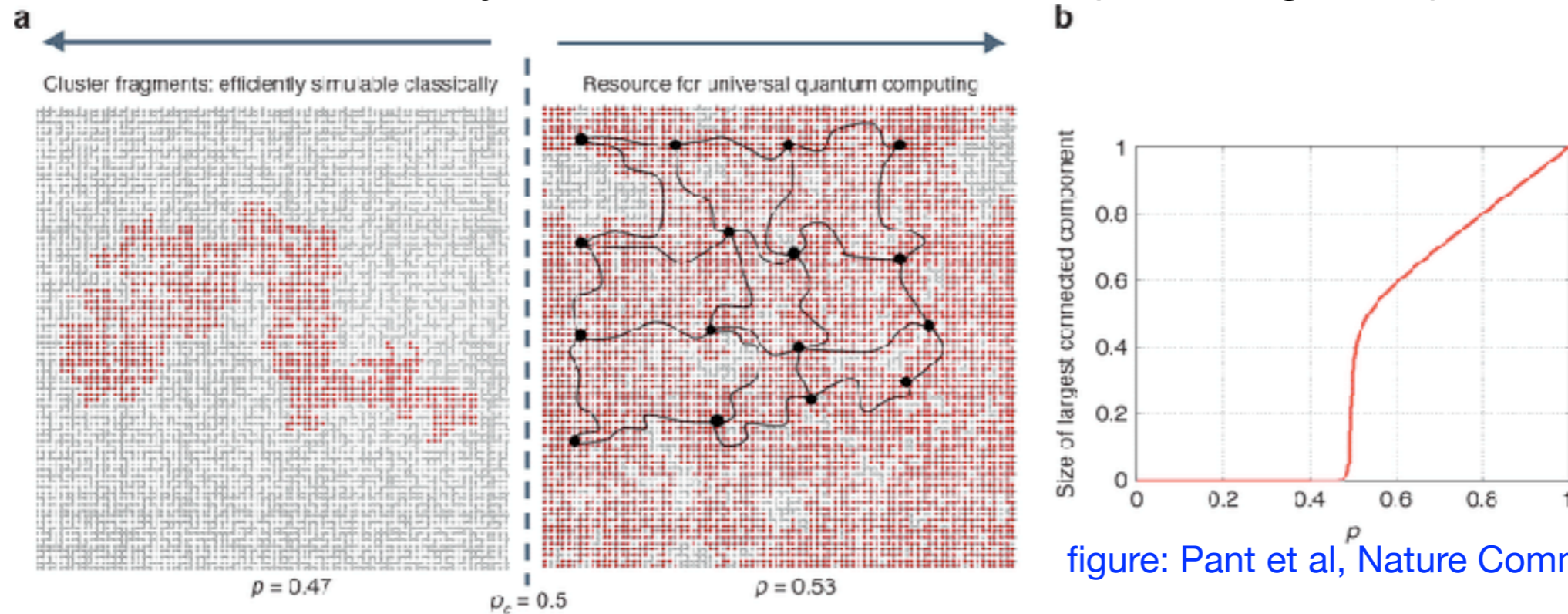


figure: Pant et al, Nature Comm., 2019

Percolation plays a role in coffee making



figure: <https://fooddrinklife.com>

Let's test conformal invariance with an espresso machine!