Towards bootstrapping de Sitter correlators

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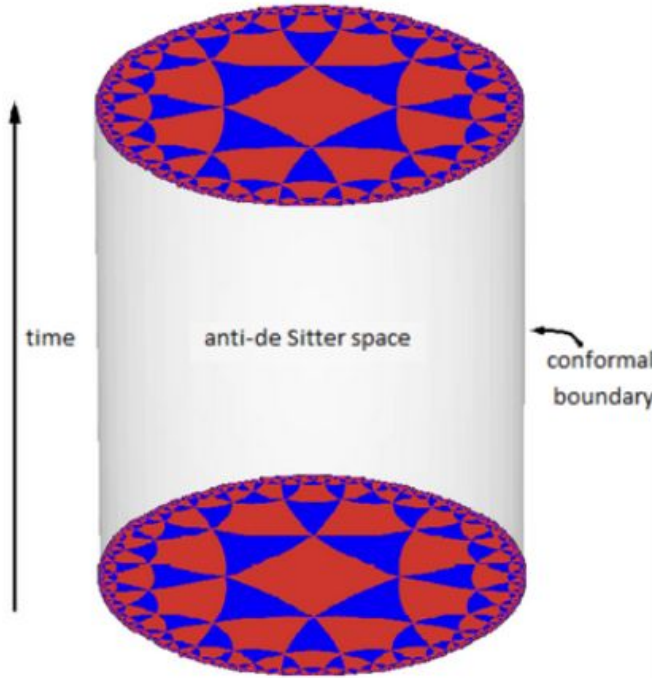
Based on arXiv: 2508.15627 (to appear in JHEP)
with Zhongjie Huang and Arthur Lipstein

Progress of theoretical bootstrap @ YITP, Kyoto University
12 November, 2025

AdS/CFT

- AdS/CFT: Relates a quantum gravity theory in AdS background to a CFT residing in its boundary. Holography duality.

  - ❑ Strongly coupled QFT from gravity.
  - ❑ Quantum gravity from QFT



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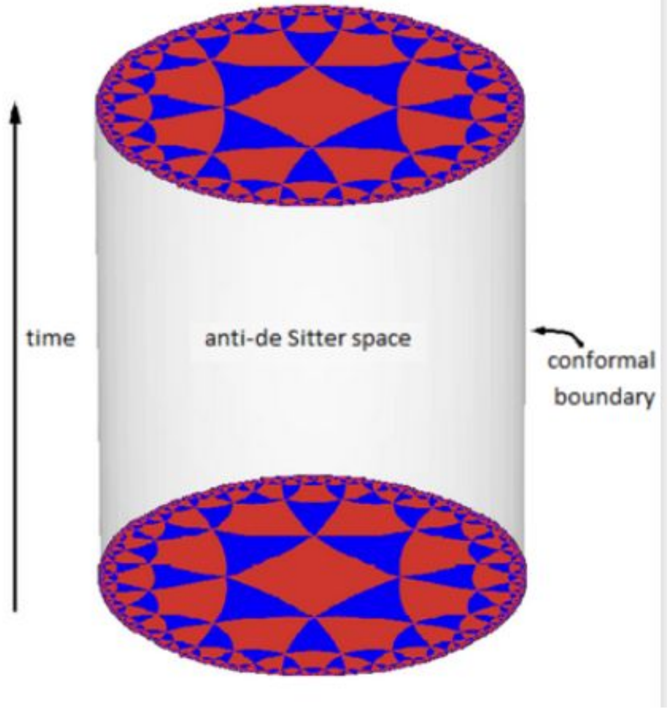
- But we do not live in AdS.



de Sitter space

- de Sitter space is a model for the evolution of accelerated Universe.
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  Difficulties

  - Absence of globally defined time-like Killing vector .
  - Definition of asymptotic region not clear.
- It is not obvious how to study QFT in dS, unlike Minkowski background.

Holography from CFT

- One of the earliest triumphs of AdS/ CFT: Heemskerk, Penedones, Polchinski, Sully

  Bulk locality follows from crossing symmetry of CFT correlators.

Holography from Conformal Field Theory

Idse Heemskerk[1], Joao Penedones[2], Joseph Polchinski[2], James Sully[1]

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Abstract

The locality of bulk physics at distances below the AdS length scale is one of the remarkable aspects of AdS/CFT duality, and one of the least tested. It requires that the AdS radius be large compared to the Planck length and the string length. In the CFT this implies a large-N expansion and a gap in the spectum of anomalous dimensions. We conjecture that the implication also runs in the other direction, so that any CFT with a large-N expansion and a large gap has a local bulk dual. For an abstract CFT we formulate the consistency conditions, most notably crossing symmetry, and show that the conjecture is true in a broad range of CFT's, to first nontrivial order in $1/N^2$: in any CFT with a gap and a large-N expansion, the four-point correlator is generated via the AdS/CFT dictionary from a local bulk interaction. We establish this result by a counting argument on each side, and also investigate various properties of some explicit solutions.

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  Derivation of bulk locality from the crossing symmetry of CFT correlators.

Calculate four-point correlator arising from local bulk interactions in AdS.

Compute the correlator by bootstrapping the dual CFT

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Bulk interactions correspond to CFT spectrum

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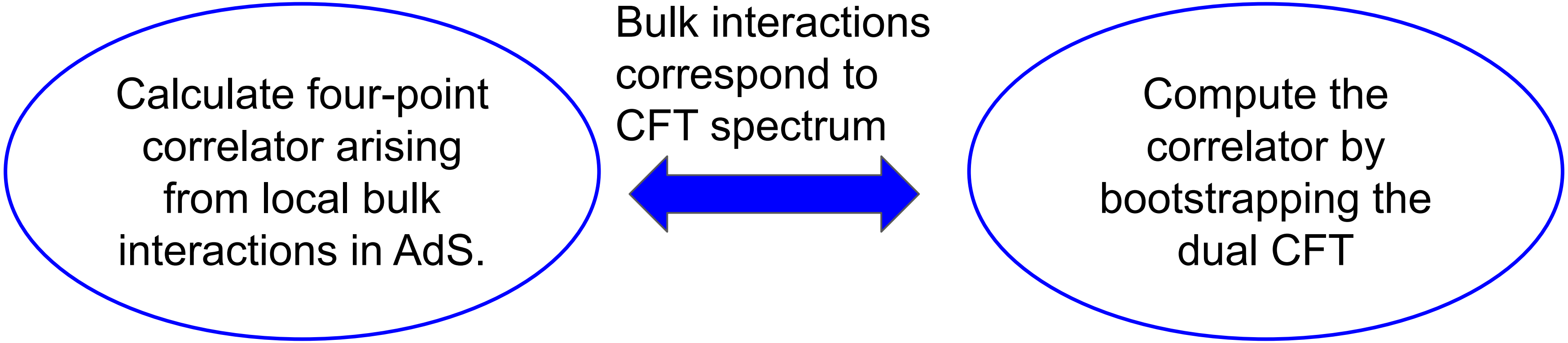

Question: Can bulk locality in dS be derived from CFT?

Key questions

- dS and Euclidean AdS are related by analytic continuation.
- Can the features of AdS/CFT dictionary lead to a dual field theory description of dS after analytic continuation?
- Are there important differences or unfamiliar consequences of dS/FT?

McFadden, Skenderis; Anninos, Hartman, Strominger, Di Pietro, Gorbenko, Komatsu ; Heckelbacher, Sachs, Skvortsov, Vanhove; Doi, Ogawa, Shinmyo, Suzuki, Takayanagi....

Questions to be addressed today

- CFT tools for dS: constraints from bootstrap?

- How to reconstruct a scalar effective field theory (EFT) in four-dimensional de Sitter space from its in-in correlators?

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Boundary perspective

- How to reconstruct a scalar effective field theory (EFT) in four-dimensional de Sitter space from its in-in correlators?

Bulk perspective

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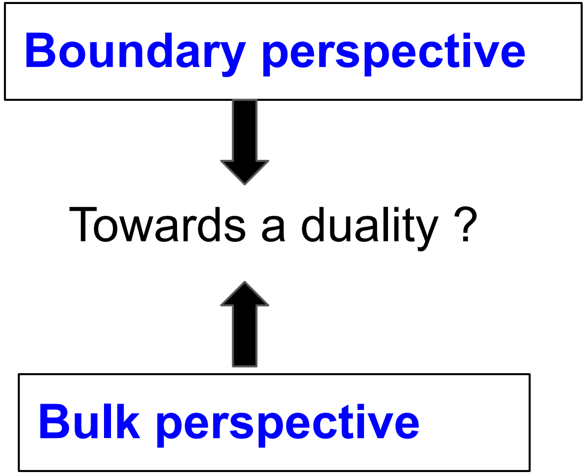

Outline

- Holography from CFT
- AdS locality from CFT in Mellin space
- Bootstrapping dS correlators
- Bulk locality in de Sitter
- Conclusion

Holography from CFT

Heemskerk, Penedones, Polchinski, Sully

CFT basics

- Consider the four point correlator in a CFT.

$$\langle\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle \sim G(U,V)$$

- Function of two cross-ratios $\quad U = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, V = \frac{x_{23}^2 x_{14}^2}{x_{13}^2 x_{24}^2}, \quad x_{ij}^2 = (x_i - x_j)^2$

- The correlator admits an expansion in conformal blocks.

$$G(U,V) = \sum_{\Delta,\ell} a_{\Delta,\ell}\, g_{\Delta,\ell}(U,V)$$

- Contains the dynamical information.

CFT data : $\underbrace{a_{\Delta,\ell}}_{\text{OPE coefficients}^2} \qquad \overbrace{\Delta}^{\text{Scaling dimension}}$

Crossing symmetry

- Expand the correlator using different OPE.

s-channel　　　　　　　　　　t-channel

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle$$

- Associativity of OPE implies these two expansions must be same.
- Statement of crossing symmetry.

Conformal bootstrap

- Functional constraint on the CFT data: Conformal bootstrap equation

$$\sum_{\Delta}\sum_{\ell=0}^{\infty} a_{\Delta,\ell}\, g_{\Delta,\ell}(U,V) = \left(\frac{U}{V}\right)^{\Delta_\phi} \sum_{\Delta}\sum_{\ell=0}^{\infty} a_{\Delta,\ell}\, g_{\Delta,\ell}(V,U)$$

Δ_ϕ : Scaling dimension of ϕ

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$$\Delta_\phi : \text{Scaling dimension of } \phi$$

- Involves a double infinite sum over scaling dimensions and spins.
- Difficult to solve the bootstrap equation directly.
- Can be solved perturbatively using an expansion parameter.

Towards solving the bootstrap equation

- Consider a CFT with only one scalar .
- This CFT contains double-trace primary operators

$$\mathcal{O}_{n,\ell} = \phi \partial_{\mu_1} \cdots \partial_{\mu_\ell} \partial^{2n} \phi \qquad n = 0, 1, 2, \cdots$$

- The CFT data can be expanded in a large parameter

$$\Delta_{n,\ell} = 2\Delta_\phi + 2n + \ell + \frac{1}{c}\gamma_{n,\ell} + \cdots$$ → anomalous dimensions

$$a_{n,\ell} = a^{(0)}_{n,\ell} + \frac{1}{c} a^{(1)}_{n,\ell} + \cdots$$ → Correction to the OPE coefficients

Towards solving the bootstrap equation

- Plug it into bootstrap equation and solve for the CFT data perturbatively in c

$$\Delta_{n,\ell} = 2\Delta_\phi + 2n + \ell + \frac{1}{c}\gamma_{n,\ell} + \cdots$$

$$a_{n,\ell} = a^{(0)}_{n,\ell} + \frac{1}{c} a^{(1)}_{n,\ell} + \cdots$$

$$\sum_{\Delta}\sum_{\ell=0}^{\infty} a_{\Delta,\ell}\, g_{\Delta,\ell}(U,V) = \left(\frac{U}{V}\right)^{\Delta_\phi} \sum_{\Delta}\sum_{\ell=0}^{\infty} a_{\Delta,\ell}\, g_{\Delta,\ell}(V,U)$$

How to reconstruct the bulk AdS dual using the CFT data?

What is the bulk perspective?

- Consider the bulk effective action of a massive scalar in AdS background

$$S = \int d^d x \sqrt{g} \left[\frac{1}{2} \left((\partial\phi)^2 + m^2\phi^2 \right) - V(\phi) \right]$$

$$ds^2 = \frac{d\vec{x}^2 + dZ^2}{Z^2}, \quad 0 \leq Z \leq \infty$$

- The dimension of boundary CFT operators is related to the mass of bulk state.

$$\Delta(\Delta - d) = m^2$$

Bulk interactions

- Bulk quartic independent interaction vertices with unfixed coefficients

$$V(\phi) = \lambda_0 \phi^4 + \lambda_4 (\partial\phi)^4 + \lambda_6 (\partial\phi)^2 (\nabla^\mu \nabla_\mu \phi)^2 + \cdots ,$$

Zero derivative — 4 derivative — 6 derivative

- We can count these vertices using the CFT solutions obtained from the bootstrap equation.

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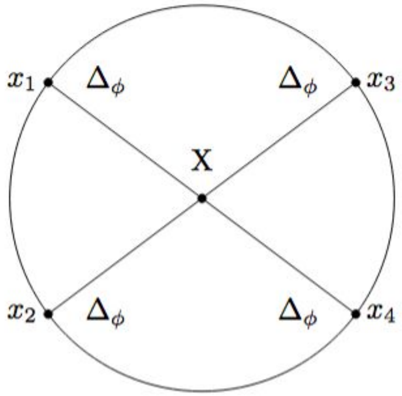
- We can count these vertices using the CFT solutions obtained from the bootstrap equation.

- Calculate the four point amplitudes of boundary CFT operators arising from these bulk interactions using Witten diagrams.

Witten diagrams for $\lambda_0 \phi^4$ interaction

- Calculate the Witten diagrams using the bulk-boundary propagators.

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4) \rangle =$$



Contact Witten diagram for quartic vertex

- Decompose these functions into conformal blocks and extract the CFT data.

Spin-truncated solutions

- Spins of the intermediate states:

  The interaction $\lambda_0\phi^4$ destroys and creates only two-particle states of spin 0, so the intermediate states are spin 0.

- We count the bulk interactions according to the maximum spin that they can couple to.

Spin-truncated solutions

- Spins of the intermediate states:

  The interaction $\lambda_0\phi^4$ destroys and creates only two-particle states of spin 0, so the intermediate states are spin 0.

- We count the bulk interactions according to the maximum spin that they can couple to.

- Count the CFT solutions by the maximum value of the spin for which the spectrum is non zero.

$$\Delta_{n,\ell} = 2\Delta_\phi + 2n + \ell + \frac{1}{c}\gamma_{n,\ell} + \cdots$$

$$\gamma_{n,\ell} \neq 0\,, \quad \text{for } \ell = 0, 2, \cdots L$$

Spin-truncated solutions

- These spectrum solves the spin-truncated bootstrap equation.

$$\sum_{\Delta}\sum_{\ell=0}^{L} a_{\Delta,\ell}\, g_{\Delta,\ell}(U,V) = \left(\frac{U}{V}\right)^{\Delta_\phi} \sum_{\Delta}\sum_{\ell=0}^{L} a_{\Delta,\ell}\, g_{\Delta,\ell}(V,U)$$

- Results in a recursion relation for the anomalous dimensions.

- Gives the number of solutions to crossing with maximum spin L.

Spin-truncated solutions: counting the bulk vertices

At most (L+2)(L+4)/8 solutions to crossing with maximal spin L.	Equal ⟺	Total number of bulk vertices with maximal spin L.

Strategy

- Restrict the bulk interactions with a maximum number of derivatives and make the corresponding restriction on the CFT.
- Check if the solutions are equal in number.

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Bulk interactions	Maximal spin in CFT
ϕ^4	$L=0$
$(\nabla\phi)^4,\ (\nabla\phi)^2(\nabla^\mu\nabla_\mu\phi)^2$	$L=2$

Mellin transform of CFT correlators greatly simplifies the counting!

AdS locality from CFT in Mellin space

Mellin transform of CFT correlator

- Mellin amplitude of CFT correlator

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle \sim G(U,V)$$

$$G(U,V) = \int_{-i\infty}^{i\infty} ds\ dt\ U^{\frac{s}{2}} V^{\frac{t}{2}} \mathcal{M}(s,t)\tilde{\Gamma}(s,t)$$

Product of Gamma functions

- Two Mellin variables s,t corresponding to two cross-ratios U,V.

Properties of Mellin amplitude

$$\mathcal{M}(s,t) \sim \sum_{\tau,\ell} \sum_{m=0}^{\infty} \frac{a_{\tau,\ell} \overbrace{Q_{\tau,\ell,m}(t)}^{\text{Mack polynomial}}}{\underbrace{s-\tau-2m}_{\text{pole}}}, \quad \tau = \Delta - \ell : \text{twist}$$

Penedones; Costa, Goncalves, Penedones

- Mellin amplitude is meromorphic functions of s,t .
- Poles correspond to scaling dimensions (twist) of exchange operators.
- Residues are related to OPE coefficients.
- Mack polynomial is a polynomial in t of degree ℓ

Mellin amplitude of spin-truncated solution

- Crossing symmetry in terms of Mellin amplitude

$$\mathcal{M}(s,t) = \mathcal{M}(t,s)$$

Mellin amplitude of spin-truncated solution

- Crossing symmetry in terms of Mellin amplitude

$$\boxed{\mathcal{M}(s,t) = \mathcal{M}(t,s)}$$

- We require the spin of the operators are truncated. $\ell \leq L$

$$\mathcal{M}(s,t) \sim \sum_{\tau} \sum_{\ell=0}^{L} \sum_{m=0}^{\infty} \frac{a_{\tau,\ell}}{s-\tau-2m} \overbrace{\mathcal{Q}_{\tau,\ell,m}(t)}^{\text{Mack polynomial}} \ .$$

degree: t^{ℓ}

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s-channel:
Polynomial in t with degree $\ell \leq L$

t-channel:
Polynomial in s with degree $\ell \leq L$

⬇

$$\mathcal{M}(s,t) = \text{symmetric polynomial in } s,t \text{ with degree} \leq L$$

Implication of spin-truncated solution

- The s-channel pole corresponds to the exchanged operator.
- Expanding the pole results in

$$\frac{1}{s-\tau-2m}$$
$$\sim \frac{1}{\tau+2m}\left(1+\frac{s}{\tau+2m}+\frac{s^2}{(\tau+2m)^2}+\cdots+\frac{s^L}{(\tau+2m)^L}+\frac{s^{L+1}}{(\tau+2m)^{L+1}}+\cdots\right)$$

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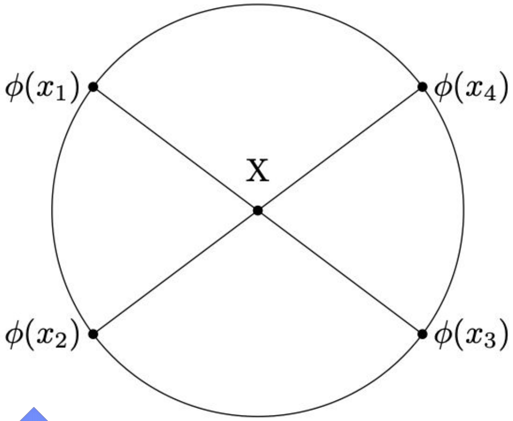
$$=\frac{1}{\tau+2m}\left(\underbrace{1+\frac{s}{\tau+2m}+\frac{s^2}{(\tau+2m)^2}+\cdots+\frac{s^L}{(\tau+2m)^L}}_{\text{allowed}}+\overbrace{\frac{s^{L+1}}{(\tau+2m)^{L+1}}+\cdots}^{\text{forbidden}}\right)$$

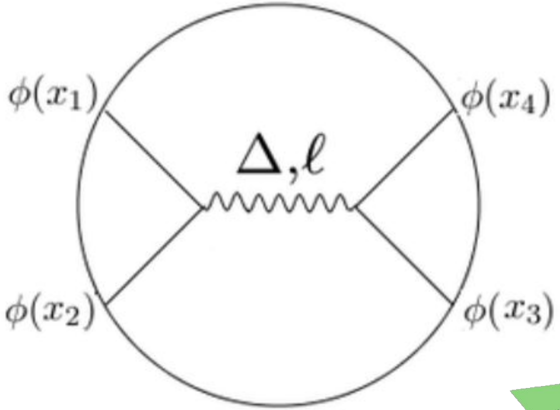
- Exchange diagrams are excluded. Only contact diagrams are allowed.

Bulk perspective

- Contact Witten diagrams are polynomials in Mellin space.



Allowed



Forbidden

First few spin-truncated solutions

- $L = 0 : \mathcal{M}(s,t) = a_0$

- $L = 2 : \mathcal{M}(s,t) = c_0 + c_1(s^2 + t^2 + u^2) + c_2 stu$

- Note: Correlators of identical scalars, no odd spin operators in OPE.

$$\boxed{L = 1, 3, \cdots \ \text{forbidden}}$$

- Contact Witten diagrams with independent bulk vertices reproduce these solutions obtained from bootstrap.

dS to AdS

Sleight, Toronna ; Di Pietro, Gorbenko, Komatsu; Heckelbacher, Sachs, Skvortsov, Vanhove

dS and EAdS geometry

- dS4 and EAdS4 are realised as a 5-dimensional hypersurface satisfying

$$-(X^0)^2 + \sum_{i=1}^{3}(X^i)^2 + (X^4)^2 = \mp R^2.$$

-1 for EAdS, +1 for dS

EAdS in Poincare patch

$$X^A = \left(\frac{1+\vec{x}^2+Z^2}{2Z}, \frac{x^i}{Z}, \frac{1-\vec{x}^2-Z^2}{2Z}\right)$$

dS in Poincare patch

$$X^A = \left(\frac{1+\vec{x}^2-\eta^2}{2\eta}, \frac{x^i}{\eta}, \frac{1-\vec{x}^2+\eta^2}{2\eta}\right)$$

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EAdS in Poincare patch

$$ds^2 = \frac{d\vec{x}^2 + dZ^2}{Z^2}, \quad 0 \leq Z \leq \infty$$

dS in Poincare patch

$$ds^2 = \frac{d\vec{x}^2 - d\eta^2}{\eta^2}, \quad -\infty < \eta < 0$$

$$(Z, R) \leftrightarrow (i\eta, -iR)$$

Set $R = 1$

dS and EAdS geometry

$$ds^2 = \frac{d\vec{x}^2 - d\eta^2}{\eta^2}, \quad -\infty < \eta < 0$$

$$ds^2 = \frac{d\vec{x}^2 + dZ^2}{Z^2}, \quad 0 \leq Z \leq \infty$$

Late time in dS

$$\eta = 0 \Leftrightarrow Z = 0$$

Boundary of AdS

Question: Can we understand the late time dS correlators from CFT?

Observables in dS

- Compute the correlation function of operators at a fixed time.

$$\langle Q(t) \rangle$$

- In-in formalism to compute the correlators using Schwinger-Keldysh method.

Weinberg

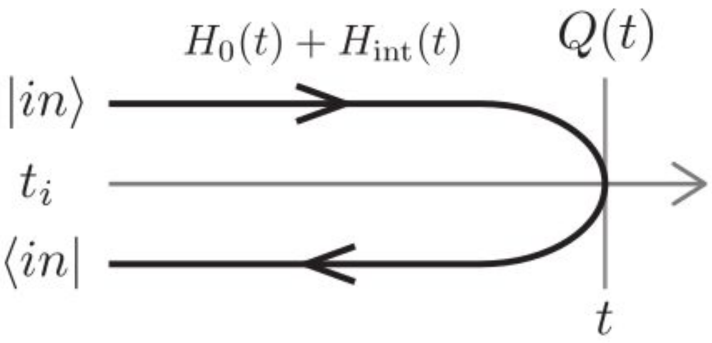

Fig. from 1807.03098, Baumann

de Sitter correlators from EAdS

- Observables in dS : in-in correlator with respect to Bunch-Davies vacuum.

- In-in correlators can be computed in terms of Witten diagrams in Euclidean AdS.

  Sleight, Toronna ; Di Pietro, Gorbenko, Komatsu; Heckelbacher, Sachs, Skvortsov, Vanhove

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  Sleight, Toronna ; Di Pietro, Gorbenko, Komatsu; Heckelbacher, Sachs, Skvortsov, Vanhove
- The correlation functions in de Sitter can be mapped to Euclidean AdS.
- Write down the Lagrangian in EAdS which computes dS correlators.

dS to EAdS Lagrangian

- Start with a general effective action of a scalar field in dS.

$$S_{dS} = \int d^4x \sqrt{g} \left[\frac{1}{2} \left((\partial\phi)^2 + m^2\phi^2 \right) - V(\phi) \right]$$

dS to EAdS Lagrangian

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$$S_{dS} = \int d^4x\sqrt{g}\left[\frac{1}{2}\left((\partial\phi)^2 + m^2\phi^2\right) - V(\phi)\right]$$

- Map this dS action to an action in Euclidean AdS of two scalar fields.

$$S_{EAdS} = \int \frac{dZd^3x}{Z^4}\left[-\frac{1}{2}\left((\partial\phi_+)^2 + m^2\phi_+^2\right) + \frac{1}{2}\left((\partial\phi_-)^2 + m^2\phi_-^2\right) - V(\phi_+,\phi_-)\right]$$

- EAdS action contains non-trivial interaction between two scalars.

Auxiliary action for EAdS

$$S_{EAdS} = \int \frac{dZ\,d^3x}{Z^4}\left[-\frac{1}{2}\left((\partial\phi_+)^2 + m^2\phi_+^2\right) + \frac{1}{2}\left((\partial\phi_-)^2 + m^2\phi_-^2\right) - V(\phi_+,\phi_-)\right]$$

- For simplicity, we restrict to conformally coupled scalars $\Rightarrow$ $\Delta_+ = 2, \;\; \Delta_- = 1$

Auxiliary action for EAdS

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- For simplicity, we restrict to conformally coupled scalars $\Rightarrow \Delta_+ = 2, \quad \Delta_- = 1$

ϕ^4 interaction in dS results in this term in EAdS

$$V(\phi_+, \phi_-) = \underbrace{\alpha(\phi_+^4 - 6\phi_+^2\phi_-^2 + \phi_-^4)}_{\text{0- derivative interactions}} + \text{higher derivative terms}$$

Goal: Can we reconstruct this EAdS action using CFT tools?

CFT perspective of dS correlators

CFT perspective

- Consider a CFT with two CFT operators

$$\mathcal{O}_+, \mathcal{O}_- \qquad \Delta_+ = 2 \text{ and } \Delta_- = 1$$

Three types of correlators

$$\langle ++++\rangle$$
$$\langle ----\rangle$$
$$\langle ++--\rangle$$

Corresponding Mellin amplitudes

$$\mathcal{M}_{++++}$$
$$\mathcal{M}_{----}$$
$$\mathcal{M}_{++--}$$

What is the solution of spin-truncated bootstrap equation for these correlators?

CFT perspective

- OPE of non identical scalars admit odd spin solutions.

$$\mathcal{O}_+ \times \mathcal{O}_+ \sim \text{even spin}$$
$$\mathcal{O}_- \times \mathcal{O}_- \sim \text{even spin}$$
$$\mathcal{O}_+ \times \mathcal{O}_- \sim \text{odd spin}$$

- Crossing symmetry of 4-point CFT correlators admits more solutions than in the case of a single bulk scalar field.

CFT perspective

- Odd-spin solutions to truncated crossing equations are allowed, which correspond to bulk interactions of the form

$$V(\phi_+, \phi_-) = \partial_\mu \phi_+ \partial^\mu \phi_+ \phi_-^2$$

- Amplitude is proportional to s : Spin 1 interaction

Counting the solutions

$$\mathcal{M}_{----} = (\text{degree} \leq L \text{ symmetric polynomial in } s, t\ ),$$
$$\mathcal{M}_{++++} = (\text{degree} \leq L \text{ symmetric polynomial in } s, t),$$
$$\mathcal{M}_{++--} = (\text{degree} \leq L \text{ symmetric polynomial in } t, u\ )$$

$$\text{Number of monomials } s^m (tu)^n \text{ satisfying } m + n \leq L \text{ and } 2n \leq L$$

- Number of independent crossing-symmetric monomials in s,t,u gives the total number of solutions to crossing upto maximum spin L.

$$\frac{\left(\left\lfloor \frac{L}{2} \right\rfloor + 1\right)\left(\left\lfloor \frac{L}{2} \right\rfloor + 2L + 6\right)}{2}.$$

$\lfloor x \rfloor$: Floor function

Crossing symmetry cannot reconstruct the bulk

- We count the independent bulk interaction vertices in EAdS Lagrangian upto spin L (modulo EOM and integration by parts)

$$\frac{\left(\lfloor \frac{L}{2} \rfloor + 1\right)\left(\lfloor \frac{L}{2} \rfloor + 2L + 6\right)}{2} \Rightarrow$$

This number is greater than the number of free coefficients in the EAdS action upto spin L!

- Crossing cannot fully determine the EAdS action.

- Need to impose additional constraints.

Mixing of operators in the OPE

- The following double trace operators appear in the conformal block expansion of the correlators

$$\mathcal{O}_-\Box^n\partial^\ell\mathcal{O}_-,\quad \mathcal{O}_+\Box^n\partial^\ell\mathcal{O}_+,\quad \mathcal{O}_+\Box^n\partial^\ell\mathcal{O}_-.\qquad \Delta_+=2 \text{ and } \Delta_-=1$$

- Degenerate operators: Same classical dimension

$$\mathcal{O}_+\Box^n\partial^\ell\mathcal{O}_+ \text{ and } \mathcal{O}_-\Box^{n+1}\partial^\ell\mathcal{O}_- \qquad \Delta=4+2n+\ell$$

- They mix together due to quantum corrections and we need to unmix them.

Aprile, Drummond, Heslop, Paul

Expanding the CFT data

We expand the CFT data in central charge

$$\mathcal{M} = \mathcal{M}^{(0)} + \frac{1}{c}\mathcal{M}^{(1)} + \cdots\,,$$

$$\tau = 2\Delta_\phi + 2n + \frac{1}{c}\gamma^{(1)}_{n,\ell} + \cdots\,, \qquad \tau = \Delta - \ell$$

$$a_{\tau,\ell} = a^{(0)}_{n,\ell} + \frac{1}{c}a^{(1)}_{n,\ell} + \cdots\,,$$

$\langle a^{(0)}\gamma^{(1)}\rangle_{n,\ell}$: Averaged anomalous dimension at $O(1/c)$

Question: How do we compute these data?

Dispersive sum rule

- We have a dispersion relation in Mellin space to obtain the CFT data. Based on Cauchy's residue theorem.
- We have the following sum rule to recursively solve for the averaged anomalous dimensions.

Related to Mack polynomial

$$\frac{1}{N!}\partial_s^N\left(\frac{\mathcal{M}^{(1)}(s,t)}{2\Delta_\phi+2k-s}\right)=\sum_\ell\sum_{n=0}^{k}\frac{\langle a^{(0)}\gamma^{(1)}\rangle_{n,\ell}\,f(\tau_0,\ell,k-n,t)}{2(2\Delta_\phi+2k-s)^{N+1}}.$$

$N, k:$ Arbitrarily chosen integers

Sample result: CFT data for spin 0

- Can be solved recursively giving the anomalous dimensions

$$\langle (a^{(0)}\gamma^{(1)})_{n,0}\rangle_{----} = \frac{\sqrt{\pi}\, 4^{-2n+1}\Gamma(2n+1)}{(2n+1)\Gamma\left(2n+\frac{1}{2}\right)}\, A_0,$$
$$\langle (a^{(0)}\gamma^{(1)})_{n,0}\rangle_{++++} = \frac{\sqrt{\pi}\, 4^{-2n-1}(n+1)^2\Gamma(2n+4)}{3\Gamma\left(2n+\frac{5}{2}\right)}\, B_0,$$
$$\langle (a^{(0)}\gamma^{(1)})_{n,0}\rangle_{++--} = \frac{\sqrt{\pi}\, 4^{-2n-1}(n+1)\Gamma(2n+3)}{\Gamma\left(2n+\frac{5}{2}\right)}\, C_0.$$

A_0, B_0, C_0: unfixed constants

Two more constraints

- Denote the anomalous dimensions after unmixing as $\gamma_{n,l}^{\text{mixed},\pm}$
- We impose one of the anomalous dimensions is zero after unmixing

$$\boxed{\gamma_{n,l}^{\text{mixed},-} = 0.}$$

- Denote the anomalous dimensions of $\mathcal{O}_+ \Box^n \partial^\ell \mathcal{O}_-$ ➡ $\gamma_{n,l}^{\text{pure}}$
- Impose the constraint

$$\boxed{\gamma_{n,l}^{\text{mixed},+} = \gamma_{n,l}^{\text{pure}}}$$

Two more constraints

- What is the origin of these additional constraints?
- These additional constraints eliminate the odd-spin interactions and fix the coefficients of the bulk interactions.
- Encodes the information that there is only one underlying scalar field in the original effective action in de Sitter.

CFT results : spin 0

Mellin amplitudes determined in terms of a single parameter (3 unfixed constants-2 constraints)

$$\mathcal{M}_{----} = \alpha,$$
$$\mathcal{M}_{++++} = 3\alpha,$$
$$\mathcal{M}_{++--} = -\,\alpha.$$

Bulk computations: Witten diagrams

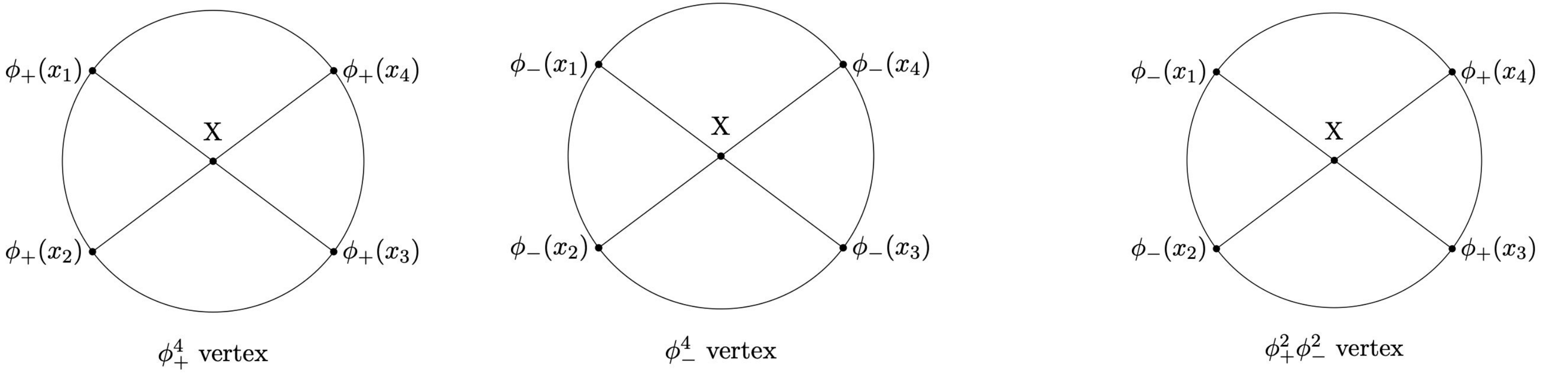
$$V(\phi_+, \phi_-) = \lambda_1 \phi_+^4 + \lambda_2 \phi_-^4 + \lambda_3 \phi_+^2 \phi_-^2$$

- Start with the independent bulk interaction vertices with 3 unfixed parameters.

- Compute the correlators using contact Witten diagrams.

Bulk computations: Witten diagrams

$$V(\phi_+,\phi_-) = \lambda_1\phi_+^4 + \lambda_2\phi_-^4 + \lambda_3\phi_+^2\phi_-^2$$



- Take the Mellin transform of the corresponding Witten diagrams.
- Compare with Mellin amplitude obtained from CFT.

Results : spin 0

Mellin amplitudes determined in terms of a single parameter (3 unfixed constants-2 constraints)

$$\mathcal{M}_{----} = \alpha,$$
$$\mathcal{M}_{++++} = 3\alpha,$$
$$\mathcal{M}_{++--} = -\alpha.$$

These amplitudes arise from the following bulk interaction vertices

$$V(\phi_+, \phi_-) = \alpha(\phi_+^4 - 6\phi_+^2\phi_-^2 + \phi_-^4)$$

Reproduces the EAdS action coming from dS.

Summary

- Bulk locality in dS is derived from boundary correlators.
- dS correlators generated from EAdS Lagrangian.
- Crossing symmetry allows for many terms in EAdS Lagrangian, but only a subset of them can arise upon rotation from dS.
- Bulk action is reconstructed using crossing and additional constraints.

What's next?

- We have considered conformally coupled scalars. Can this be done for massless scalars?

  In progress with Z. Huang and A. Lipstein

- Implications of additional constraints?

- Tree-level exchange or loop diagrams?

- Generalisation to gravitational theories in bulk?

Thank you!