Bootstrapping critical 3d gauge theories

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and `arXiv:2310.08343` with N. Su

Quantum Electrodynamics in 2+1 dimensions

- Scalar QED_3 is a $U(1)$ gauge theory in 2+1 dimensions coupled to N complex scalars.

  - This theory is also sometimes called the 3d CP^{N-1} model.

- One can also add a Chern-Simons coupling with integer coefficient k, or N 2-component complex fermions

- When N (or k) is large, theory flows to CFT in IR [Appelquist, Nash, Wijewardhana '88], but less known at small N, k bc strongly coupled.

- When N small, has important condensed matter realizations!

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Condensed matter realizations

- For $N = 1$, describes the superfluid Helium phase transition.

- $N = 2$ describes the Neel-VBS phase transition [Read, Sachdev '89] , maybe realized in $SrCu_2(BO_3)_2$ [Cui et al '23] .

- $N > 1$ describe so-called deconfined quantum critical points (DQCPs), where the gauge fields are emergent [Senthil, Vishwanath, Balents, Sachdev, Fisher '04] .

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Scalar QED$_3$ definition

- QED$_3$ with N complex scalars ϕ_i:

$$\int d^3x \left[\frac{F_{\mu\nu}F^{\mu\nu}}{4e^2} + \frac{\sigma^2}{4\lambda} + |(\nabla_\mu - iA_\mu)\phi^i|^2 + (\frac{1}{4} + i\sigma)|\phi^i|^2 \right],$$

  - We have replaced the ϕ^4 term by Hubbard-stratonovich field σ.

  - One could also set $\sigma = 0$ to get the so-called tricritical theory, which has an extra relevant singlet.

- $e, \lambda \to \infty$ when we flow to IR, bc F^2 and σ^2 are irrelevant.

- Can construct operators from ϕ_i, σ, and A_μ in irreps of $SU(N)$ flavor symmetry, compute correlators at large N using Feynman diagrams [Halperin, Lubetsky, Ma '74; Kaul, Sachdev '08].

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Monopole operators

- In addition to $SU(N)$ flavor symmetry, have $U(1)_T$ symmetry: $J^\mu = \frac{1}{8\pi}\epsilon^{\mu\nu\rho}F_{\nu\rho}$ current conserved b/c $\epsilon^{\mu\nu\rho}\partial_\mu F_{\nu\rho} = 0$.

    - All fields in Lagrangian uncharged under $U(1)_T$.

- Monopole operator M_q are local operators defined as having charge q under $U(1)_T$, s.t. $\int_{S^2} F = 4\pi q$.

    - Dirac quantization condition requires $q \in \mathbb{Z}/2$.

- Lowest M_q are scalars and singlets under $SU(N)$.

    - For $k \neq 0$, M_q generically in nontrivial irreps.

    - For fermions, M_q in nontrivial irreps for all k due to zero modes.

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$N = 1$ theory

- For $N = 1$, the theory is conjectured to be dual to the critical $O(2)$ Wilson-Fisher theory [Peskin '78; Dasgupta, Halperin '81] .

- $M_q \Leftrightarrow$ lowest dimension operator made of $2q$ complex bosons ϕ.

- $O(2)$ operators computed for $q \leq 2$ at high precision from numerical bootstrap [SMC, Landry, Liu, Poland, DSD, Su, Vichi '20; Liu, Meltzer, Poland, DSD '20] .

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Wilsonian lattice simulations

- For $N \geq 1$, could study theory using standard Wilsonian lattice simulations (like lattice QCD).

- The challenge is that standard lattice does not preserve $U(1)_T$, allowing monopoles to destabilize the theory.

- Most lattice studies thus instead study $\mathbb{R}$-gauge theory, which does not have local monopole operators.

    - E.g. For $N = 1$ compute singlet $\Delta_0 = 1.508$ [Kajantie et al '04], close to $O(2)$ WF $\Delta_0 = 1.511$, check of particle-vortex duality.

    - Can also simulate non-local monopole operators in this theory [Karathik '18], with Δ same as local monopoles.

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Other lattice simulations

- Can simulate other theories believed to lie in same universality class as scalar QED_3.

- The JQ model [Sandvik '07] simulates the Heisenberg model Hamiltonian, with J/Q tuned to criticality:

$$H = J \sum_{\langle ij \rangle} S_i S_j - Q \sum_{ijkl} (S_i S_j S_k S_l - \text{trace}).$$

  - S_i is generator of $SU(N)$ algebra.

  - Preserves $SU(N)$ symmetry, but only $\mathbb{Z}_4 \subset U(1)_T$ on square lattice.

- Can also simulate so-called loop model, which is very similar [Nahum, Chalker, Serna, Ortuno, Somoza '15].

- Both models used to compute scaling dimensions, but with limited accuracy.

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Outline

This talk: Combine large N expansion of monopole scaling dimensions with conformal bootstrap to accurately study scalar QED3.

- Describe how to compute monopole operator scaling dimensions at large N.

- Check accuracy of large N expansion by comparison to lattice simulations, and particle-vortex duality for $N = 1$.

- Combine with bootstrap to argue that $N = 2$ theory (Neel-VBS) is tricritical, and compute CFT data.

- Combine with bootstrap to compute CFT data of $N = 3$ theory (simplest critical theory).

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- Combine with bootstrap to compute CFT data of $N = 3$ theory (simplest critical theory).

Monopole scaling dimensions

- The state-operator correspondence relates M_q on $\mathbb{R}^3$ to state on S^2 Hilbert space with $4\pi q$ magnetic flux, s.t. Δ_q given by energy on $S^2 \times \mathbb{R}$ with $4\pi q$ flux [Borokhov, Kapustin, Wu '02] .

- Consider thermal free energy $F_q \equiv \frac{-\log Z}{\beta}$ on $S^2 \times S^1_\beta$ with $4\pi q$ flux, where $\beta \equiv 1/T$ is length of S^1 [SMC, Iliesiu, Mezei, Pufu '17] .

- After integrating out matter, can compute F_q from large N saddle point.

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Leading order free energy

- From saddle we get energy (scaling dimension) and entropy:

$$F_q = N F_q^{(0)} + F_q^{(1)} + \dots, \qquad F_q^{(0)} = \Delta_q^{(0)} - \frac{1}{\beta} S_q^{(0)} + O(e^{-\beta}).$$

- Answer before zeta regularizing is:

$$\Delta^{(0)} = \sum_{j \geq q} d_j \lambda_j + \xi d_q \lambda_q,$$

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Comparison ot large *N* to *O*(2) WF

q	$\Delta_{q,0}^{(0)}$	$\Delta_{q,0}^{(1)}$	$N = 1$	$O(2)$	Error (%)
1/2	0.12459	0.38147	0.50609	0.519130434	2.5
1	0.31110	0.87452	1.1856	1.23648971	4.1
3/2	0.54407	1.4646	2.0087	2.1086(3)	4.7
2	0.81579	2.1388	2.9546	3.11535(73)	5.2
5/2	1.1214	2.8879	4.0093	4.265(6)	5.8
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- Blue values from bootstrap, red from lattice.

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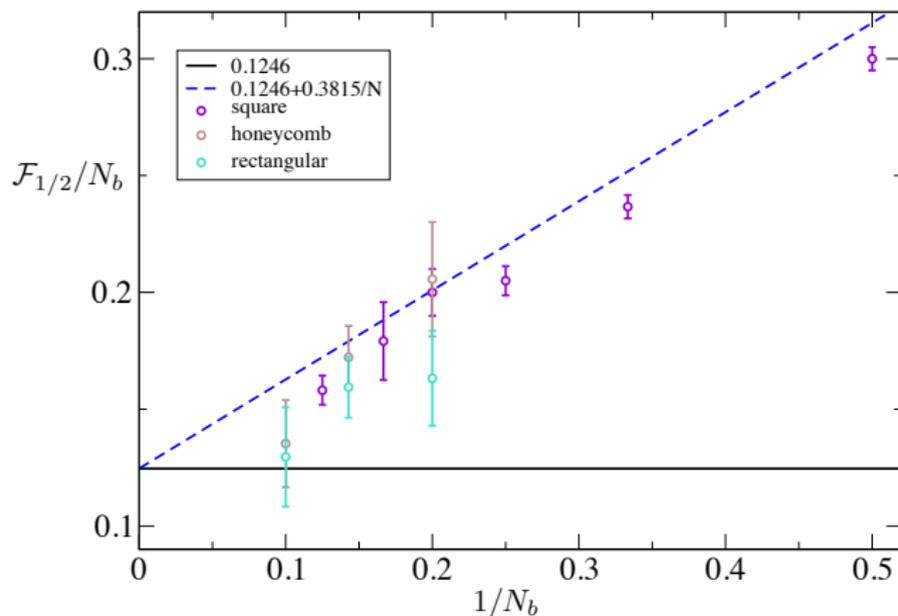| q	$\Delta_{q,0}^{(0)}$	$\Delta_{q,0}^{(1)}$	$N = 1$	$O(2)$	Error (%)
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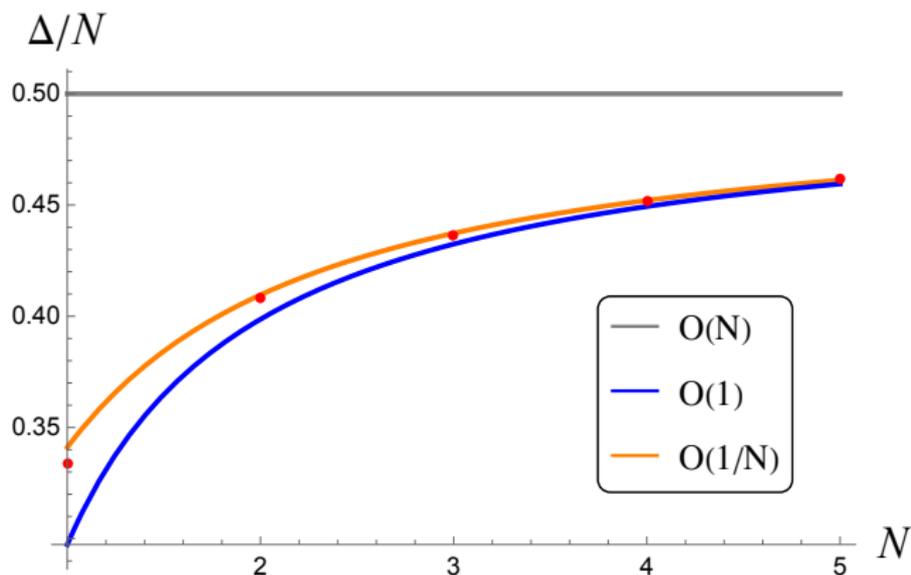
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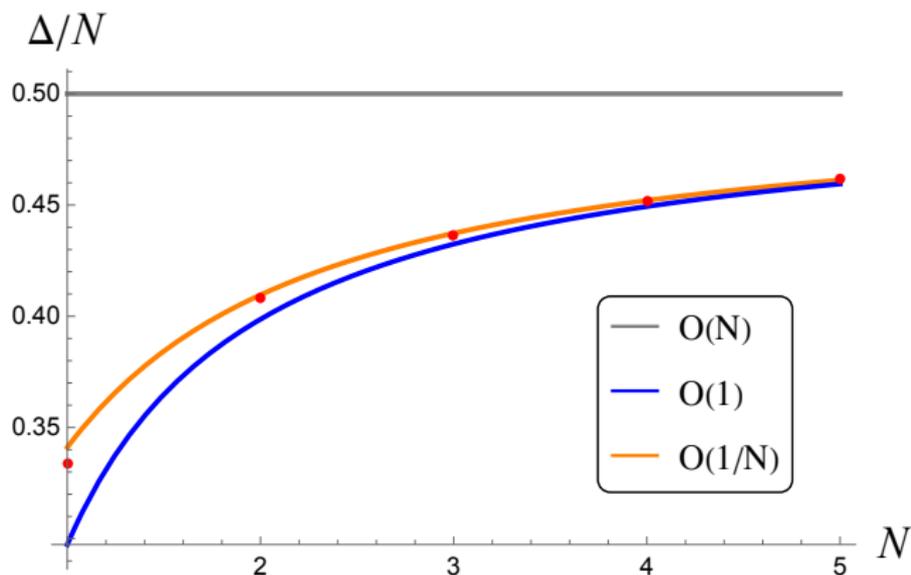
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- Bootstrap takes as input just the symmetries of the theory, the irreps of operators, and assumptions about the spectrum.

- Bootstrap was first applied to fermionic QED$_3$ with $k = 0$, where lowest monopoles transform under both $SU(N) \times U(1)$.

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- So main focus is $N = 4$ theory, where $M_{1/2}$ in vector of $SO(6) \cong SU(4)$, M_1 in rank 2, etc.

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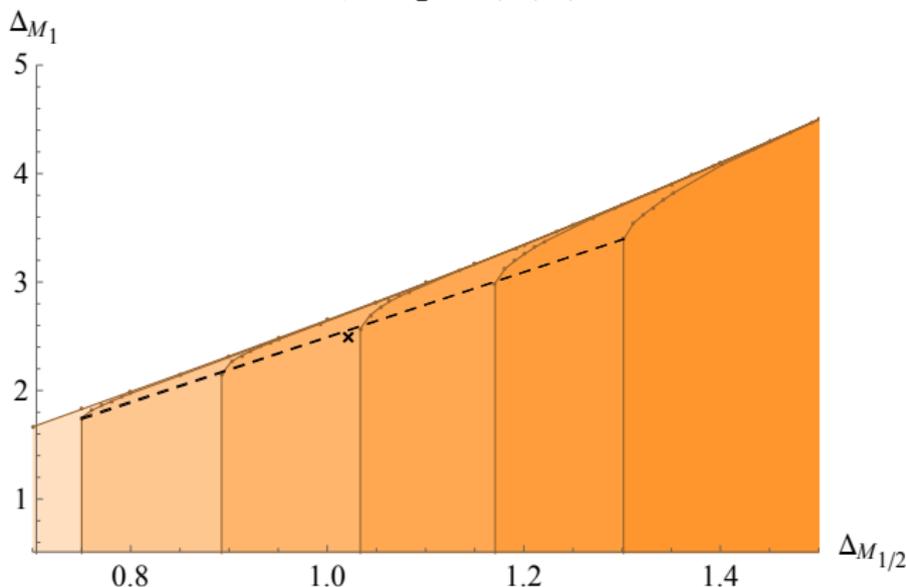
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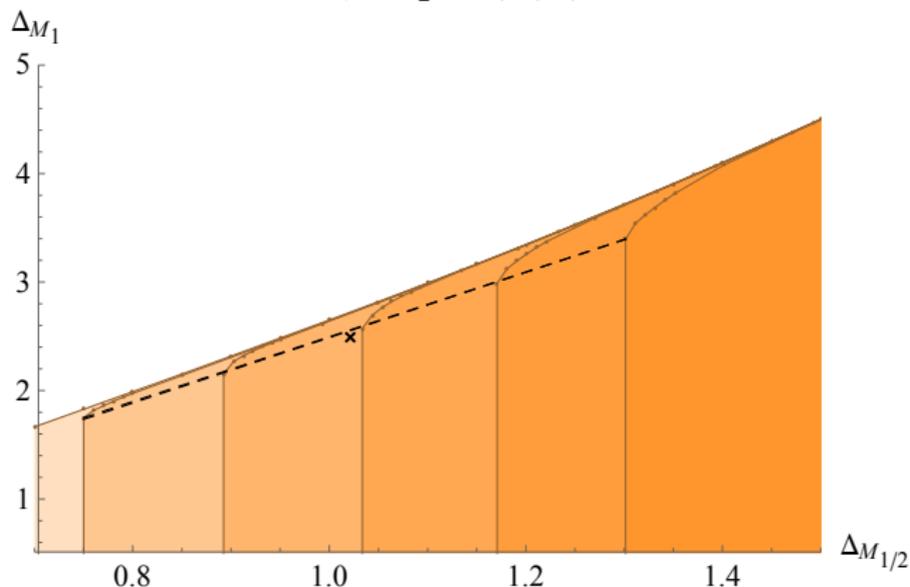

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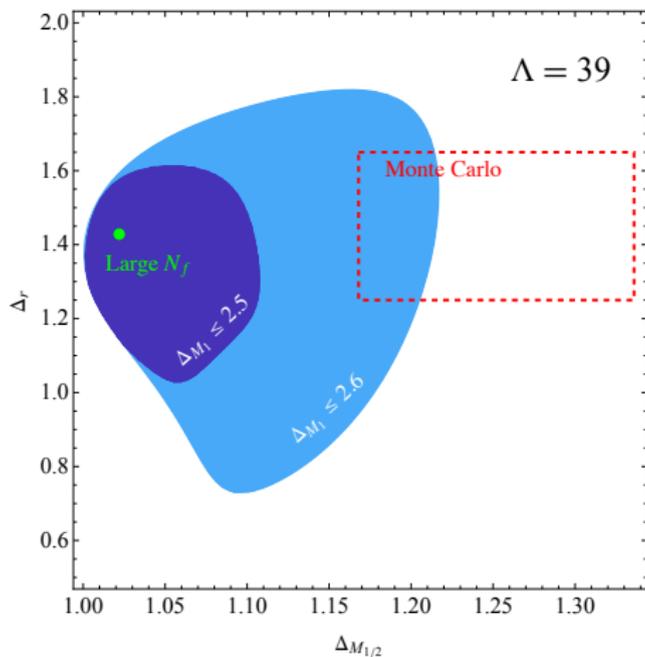- With assumptions on gaps of many different operators, get island.
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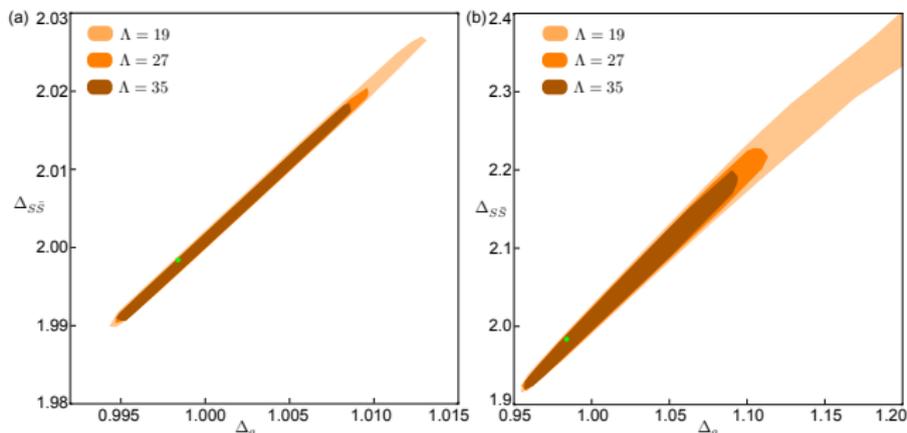
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- Many other theories with relevant adjoint of $SU(N)$ symmetry, e.g. QCD$_3$, Chern-Simons matter theories, tricritical QED$_3$.

- Various gap assumptions are imposed to exclude these theories, based on conjectures of how spectrum should differ.

- With these assumptions, get islands at large N consistent with large N Δ's of non-monopoles, but no islands at small N.

Bootstrap for large *N* [He, Rong, Su '21]

- With assumptions on gaps of many different operators, get island.
- Consistent with both large *N* of adjoint Δ_a and $\Delta_{S\bar{S}}$ of other irrep in OPE of adjoints (made from four ϕ's).

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$N = 2$ scalar QED$_3$

- The theory (Neel-VBS) explicitly has $SU(2) \times U(1)_T$ symmetry, and is assumed to have just one relevant singlet.

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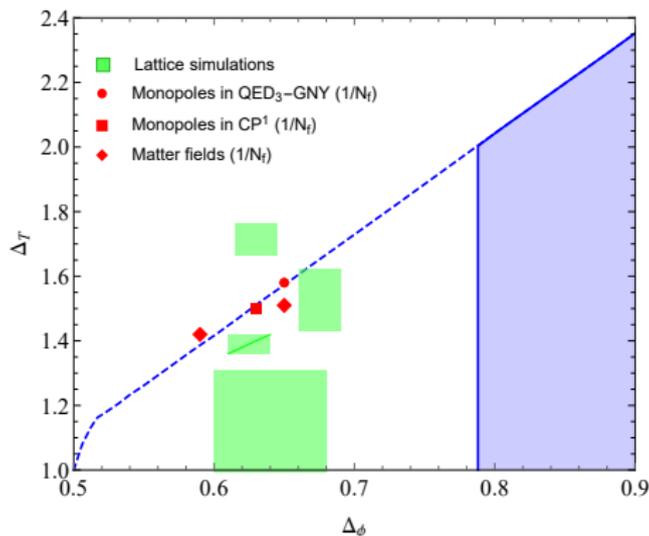 - As change coupling, see singlet Δ_s goes from above to below 3.

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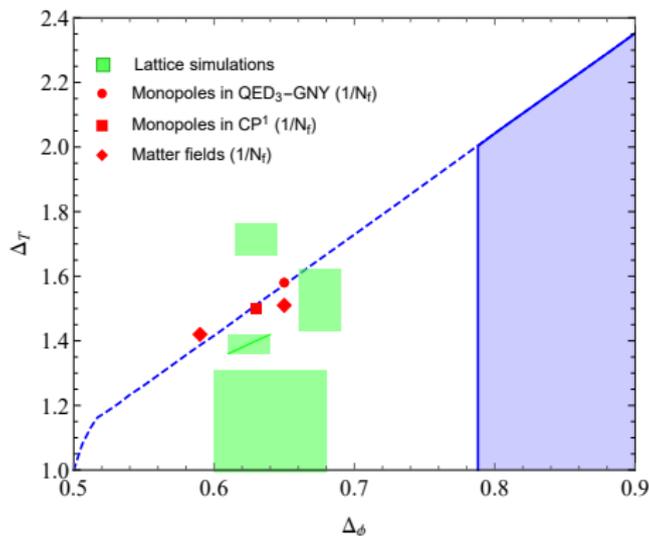
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- Bootstrap correlators of $SO(5)$ singlet s, vector v, rank-2 t, which gives access also to rank-3 t_3 and rank-4 t_4.

- Assume one relevant s (so two relevant $SU(2) \times U(1)_T$ singlets), one relevant v, t, t_3, everything else irrelevant.

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- 29 crossing equations, use Skydive [Liu, DSD, Su, Rees '23] to get allowed region in space of $\{\Delta_v, \Delta_s, \Delta_t, \Delta_{t_3}, \frac{\lambda_{sss}}{\lambda_{vvt}}, \frac{\lambda_{tts}}{\lambda_{vvt}}, \frac{\lambda_{vvs}}{\lambda_{vvt}}, \frac{\lambda_{ttt}}{\lambda_{vvt}}\}$.

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	Δ_v	Δ_t	Δ_{t_3}	Δ_{t_4}	Δ_s
Bootstrap '23	0.630*	1.519	2.598	3.884	2.359
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- Our prediction verified by lattice study [Sandvik et al '24] !

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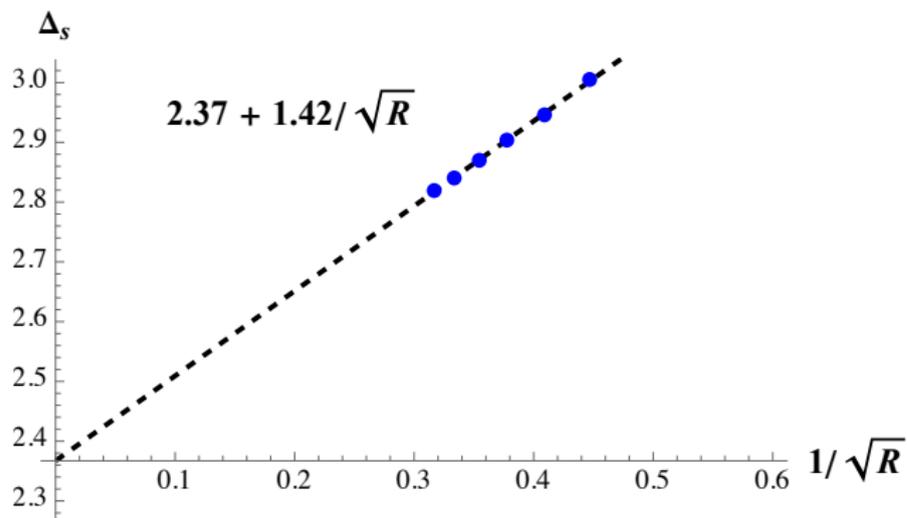
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- Fix $\Delta_v = .755$ from large N for $\Delta_{1/2}$, and minimize Δ_0 to look at boundary of allowed region and read off $\Delta_0, \Delta_1, \Delta_{3/2}, \Delta_2$.

$N = 3$ scalar QED$_3$

- Since $N = 2$ scalar QED3 tricritical, simplest DQCP is $N = 3$.

- Some lattice studies say its critical [Lou, Sandvik, Kawashima '09] , others say only critical for $N > 4$ [Bonati, Pelissetto, Vicari '20] .

- We [SMC, Piazza, Reehorst, Su TBA] bootstrap correlators of singlet, $q = 1/2, 1$ monopoles, which also gives access to $q = 3/2, 2$.

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Our bootstrap results

	$\Delta_{1/2}$	Δ_1	$\Delta_{3/2}$	Δ_2	Δ_0
Bootstrap	0.755^*	1.840(1)	3.173(4)	4.65(9)	1.61(1)
Large N	0.755	1.81	3.10	4.59	–
Lattice	0.71(4)	–	–	–	1.46(7)

- We input one value $\Delta_{1/2}$, to get predictions for 4 values Δ_1, $\Delta_{3/2}$, Δ_2, Δ_0 that all match large N and lattice!

- Cannot yet access $SU(3)$ sector, bc trivial mixing with $U(1)$ sector

- Results for $N > 3$ not as good, prob require bigger gap above Δ_0.

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Large charge effective theory

- Effective theory for operators with $q \gg 0$ and $\ell \ll \sqrt{q}$ for ANY CFT with $U(1)$ symmetry [Hellerman, Orlando, Reffert, Watanabe '15] :

$$\Delta_{q,\ell} = c_{\frac{3}{2}} q^{\frac{3}{2}} + c_{\frac{1}{2}} q^{\frac{1}{2}} - 0.0937 + \sqrt{\frac{\ell(\ell+1)}{2}} + O(q^{-\frac{1}{2}}),$$

  - $O(1)$ terms are universal, but c_i terms are theory dependent.

  - Formula does not apply for $\ell = 1$, bc its descendent state.

- From large N expansion for scalar QED3 for $\ell = 0$ [de la Fuente '18] :

$$c_{\frac{3}{2}} = 0.4983 , \qquad c_{\frac{1}{2}} = 0.3449 .$$

- For critical $O(2)$, can fix $c_{\frac{3}{2}} = .337 , c_{\frac{1}{2}} = .266$ from lattice data for $\ell = 0$ operators [Banerjee, Chandrasekharan, Orlando '18] .

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Bootstrap vs. large charge for scalar QED3

Match for $\ell \leq q$, better than expected $\ell \ll \sqrt{q}$.

Bootstrap	$\ell = 0$	$\ell = 1$	$\ell = 2$	$\ell = 3$	$\ell = 4$
$q = 0$	1.61(1)	2	3.76(2)	4.328(2)	5.0(1)
$q = 1$	0.755*	3.328(4)	3.21(1)	4.1(1)	5.42(3)
$q = 2$	1.840(1)	4.83(3)	3.634(6)	5.64(9)	5.6(1)
$q = 3$	3.173(4)	6.0(1)	4.893(4)	5.6(1)	6.82(2)
$q = 4$	4.65(9)	–	6.1(2)	–	7.87(7)

Large charge	$\ell = 0$	$\ell = 1$	$\ell = 2$	$\ell = 3$	$\ell = 4$
$q = 1$	0.750	–	2.482	3.199	3.912
$q = 2$	1.8035	–	3.536	4.253	4.966
$q = 3$	3.093	–	4.825	5.543	6.256
$q = 4$	4.583	–	6.315	7.032	7.745

Bootstrap vs. large charge for critical $O(2)$

Bootstrap results from [SMC, Landry, Liu, Poland, DSD, Su, Vichi '19; Liu, Meltzer, Poland, DSD '20] :

Bootstrap	$\ell = 0$	$\ell = 1$	$\ell = 2$	$\ell = 3$	$\ell = 4$
$q = 1$	0.519	2.950	3.650	4.615	5.700
$q = 2$	1.236	5.800	3.015	5.766	5.030
$q = 3$	2.106	2.078	3.884	4.582	5.852
$q = 4$	3.115	–	4.893	–	6.730

Large charge	$\ell = 0$	$\ell = 1$	$\ell = 2$	$\ell = 3$	$\ell = 4$
$q = 1$	0.509	–	2.241	2.958	3.672
$q = 2$	1.236	–	2.968	3.685	4.399
$q = 3$	2.118	–	3.851	4.568	5.281
$q = 4$	3.135	–	4.867	5.584	6.297

Conclusion

- Large N expansion of monopole scaling dimensions Δ_q accurate even for small N (unlike non-monopoles).

    - Checked by comparison to particle-vortex for $N = 1$, and lattice for $N > 1$.

- Conformal bootstrap of $N = 2$ theory assuming $SO(5)$ symmetry and relevant singlet saturated by large N for Δ_q and gives prediction $\Delta_s \approx 2.36$.

    - Recently matched by lattice calculation and fuzzy sphere!

- Conformal bootstrap for $U(1)$ sector of $N = 3$ theory is saturated by large N for Δ_q.

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Future directions

- Large N calculation of monopoles to $O(1/N)$.

- Include $SU(N)$ sector in bootstrap for $N > 2$ theories.

- Impose large charge relations, maybe get islands?

- Simulate scalar QED_3 with Villain lattice [Sulejmanpasic, Gattringer '19], which correctly preserves $U(1)_T$.

- Try to bootstrap other 3d gauge theories, e.g. QCD_3, Chern-simons matter, etc.

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Bootstrap 2026

See everyone next year in London, August 3-14!