Bootstrap Bounds on Yang-Mills in AdS

WIP with
Lorenzo Di Pietro, Stefanos Kousvos,
Marco Meineri, Marco Serone, Alessandro Vichi

Alessandro Piazza

SISSA and INFN Trieste

4 November 2025
Progress of Theoretical Bootstrap
YITP

- Confinement of Yang-Mills theory in Anti de Sitter

- Conformal Bootstrap on boundary correlators

- Non-perturbative constraints on mechanisms for confinement

1 QFT in AdS

2 Bootstrapping non-abelian currents

3 Results

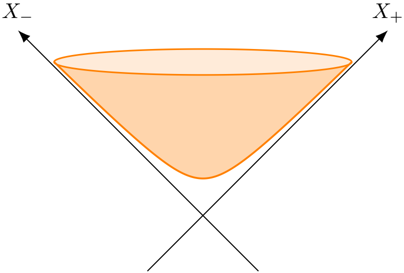
4 Conclusion

1 QFT in AdS

QFT in flat space can be very hard when IR is strongly coupled

Rigid AdS_{d+1} space is a *nice* regulator [Callan and Wilczek, 1990]

- The radius L is a natural **IR cutoff**
- Isometries $SO(d+1,1)$
  $\Rightarrow$ boundary correlators are **conformally** invariant
- **Infinite** volume at finite L
- Running of couplings $g(\mu L)$
- $L \to +\infty$ we *expect* to **recover** flat space



$$-X^+X^- + \delta_{ab}X^aX^b = -L^2$$

$$\mathrm{d}s^2 = \frac{L^2}{z^2}\left(\mathrm{d}z^2 + \delta_{ij}\mathrm{d}x^i\mathrm{d}x^j\right)$$

1 QFT in AdS: Yang-Mills

Yang-Mills in $\mathbb{R}^4$

- gauge theory G_{YM}, asymptotic freedom, dynamically generated scale Λ_{YM}
- **confinement**: mass gap + colorless asymptotic states

Yang-Mills in AdS_4 [Aharony et al., 2013]

- $L\Lambda_{\mathrm{YM}} \ll 1$: small coupling and AdS physics

  Two **boundary conditions** for the G_{YM} gauge connection $A^a_\mu(z,x)$

  - **Dirichlet**: $A^a_i(z,x) \overset{z\to 0}{\sim} z J^a_i(x) \Rightarrow G_{\mathrm{YM}}$ is **global** symmetry on the boundary
  - **Neumann**: $A^a_i(z,x) \overset{z\to 0}{\sim} a^a_i(x) \Rightarrow G_{\mathrm{YM}}$ is **gauged** on the boundary

- $L\Lambda_{\mathrm{YM}} \gg 1$: strong coupling and flat space physics

  Spectrum of asymptotic states should have a mass gap and no G_{YM} global symmetry

1 QFT in AdS: confinement scenarios

Nuemann interpolates to flat space, **Dirichlet** must disappear

[Aharony et al., 2013] [Copetti et al., 2024]

- **Decoupling**

  The boundary current J_i^a decouples from the spectrum

$$\langle J_i^a(x_1) J_j^b(x_2)\rangle = C_J(g_{\mathrm{YM}}) \frac{I_{ij}(x_1 - x_2)}{(x_1 - x_2)^4}$$

$$C_J(g_{\mathrm{YM}}) = C_J(L\Lambda_{\mathrm{YM}}) \xrightarrow{L \to L_*} 0$$

  This implies $\langle J_i^a \mathcal{O}_1 \cdots \mathcal{O}_n \rangle = 0$ in unitary CFTs

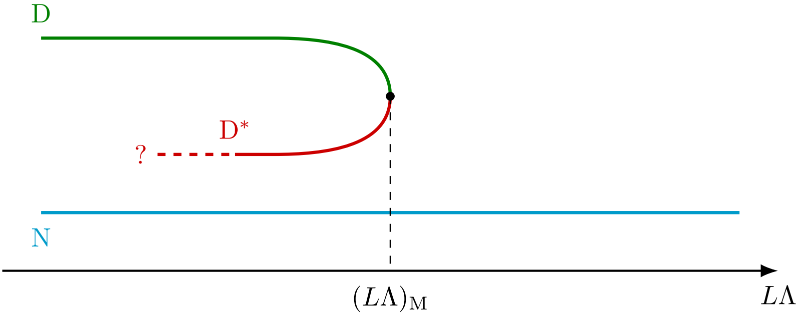
1 QFT in AdS: confinement scenarios

Nuemann interpolates to flat space, **Dirichlet** must disappear

[Aharony et al., 2013] [Copetti et al., 2024]

- **Merging**: a scalar, singlet under G_{YM}, becomes marginal

  [Kaplan et al., 2009] [Gorbenko, Rychkov, and Zan, 2018]
  [Hogervorst et al., 2021] [Lauria, Milam, and Rees, 2024]



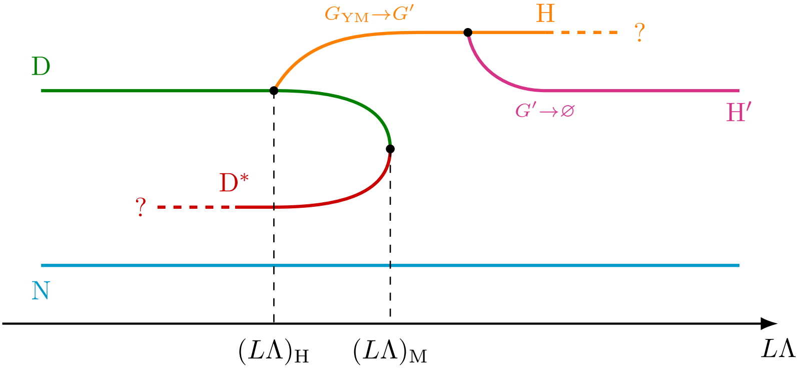
1 QFT in AdS: confinement scenarios

Nuemann interpolates to flat space, **Dirichlet** must disappear

[Aharony et al., 2013] [Copetti et al., 2024]

- **Higgsing**: a scalar, charged under $G_{\rm YM}$, becomes marginal

  We can define a new boundary condition that breaks $G_{\rm YM}$



1 QFT in AdS: confinement scenarios

Merging seems is the most “economic”

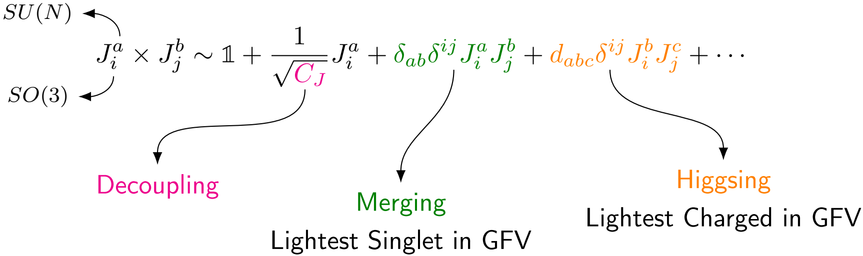
Perturbation theory is compatible with all these scenarios but favors **Merging** [Ciccone et al., 2024]

What do we learn from the bootstrap?

✗	**Decoupling**	$C_J \to 0$
☺	**Merging**	$\Delta_{\text{singlet}} \to 3$
☹	**Higgsing**	$\Delta_{\text{charged}} \to 3$

2 Bootstrapping non-abelian currents

- At $g_{\mathrm{YM}} = 0$, boundary CFT_3 of Yang-Mills in AdS_4 is a **Generalized Free Vector** theory (GFV): $\langle JJ \rangle$ + Wick contractions
- As $g_{\mathrm{YM}} \neq 0$ we have a **continuous** deformation of GFV spectrum $\Rightarrow$ 1 parameter family of CFT data
- For concreteness we take $G_{\mathrm{YM}} = SU(N)$, $N = 2, 3$
- CFT data involved in the scenarios is accessible in the OPE



$SU(N)$

$$J_i^a \times J_j^b \sim \mathbb{1} + \frac{1}{\sqrt{C_J}} J_i^a + \delta_{ab}\delta^{ij} J_i^a J_j^b + d_{abc}\delta^{ij} J_i^b J_j^c + \cdots$$

$SO(3)$

Decoupling

Merging

Lightest Singlet in GFV

Higgsing

Lightest Charged in GFV

2 Bootstrapping non-abelian currents

Abelian [Dymarsky et al., 2019], Non Abelian [He et al., 2024]

- $SO(4,1)_0$, **parity**, $SU(N)$ and **charge conjugation**
- Multiple conformal tensor structures
- OPE

$$J_i^a(x) \times J_j^b(0) \sim \sum_{\mathcal{O}} \sum_I \lambda^{(I)}_{JJ\mathcal{O}} C^{(I)}_{\mathcal{O}}(x,\partial)\mathcal{O}_{\Delta,\ell,p,\mathbf{r}}(0) \quad \mathbf{r} \in \mathbf{adj} \otimes \mathbf{adj}$$

- Conformal block decomposition

$$\langle J_i^a J_j^b J_k^c J_l^d \rangle = \sum_K \mathbb{T}^{(K)}_{ijkl} \sum_{\mathcal{O}_{\Delta,\ell,p,\mathbf{r}}} T^{abcd}_{\mathbf{r}} \sum_{I,J} \lambda^{(I)}_{JJ\mathcal{O}} \lambda^{(J)}_{JJ\mathcal{O}} g^{(K,I,J)}_{\Delta,\ell}(z,\bar{z})$$

- Crossing equation

$$\sum_{\mathcal{O}_{\Delta,\ell,p,\mathbf{r}}} \lambda^{\mathsf{T}}_{JJ\mathcal{O}} \cdot \vec{\mathcal{V}}_{\Delta,\ell,p,\mathbf{r}} \cdot \lambda_{JJ\mathcal{O}} = 0 \qquad \vec{\mathcal{V}} \text{ vectors of matrices}$$

3 Results: bound on C_J

$$\vec{\mathcal{V}}_{\mathbb{1}} + \frac{1}{C_J}\theta \cdot \vec{\mathcal{V}}_J \cdot \theta + \sum_{\mathcal{O}} \lambda_{JJ\mathcal{O}}^{\mathsf{T}} \cdot \vec{\mathcal{V}}_{\mathcal{O}} \cdot \lambda_{JJ\mathcal{O}} = 0 \qquad \theta = \begin{pmatrix} 3 - 5\alpha_{JJJ} \\ -\alpha_{JJJ} \end{pmatrix}$$

$$p = \max \alpha(\vec{\mathcal{V}}_{\mathbb{1}})$$

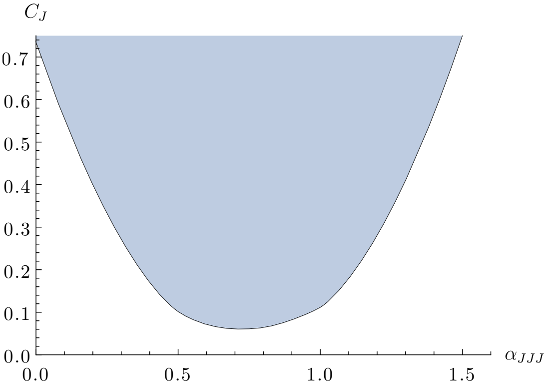
$$\alpha(\theta \cdot \vec{\mathcal{V}}_J \cdot \theta) = 1$$

$$\alpha(\vec{\mathcal{V}}_{\mathcal{O}}) \succeq 0$$

$$\Rightarrow \; C_J \geq -\frac{1}{p}$$

C_J is **strictly** positive

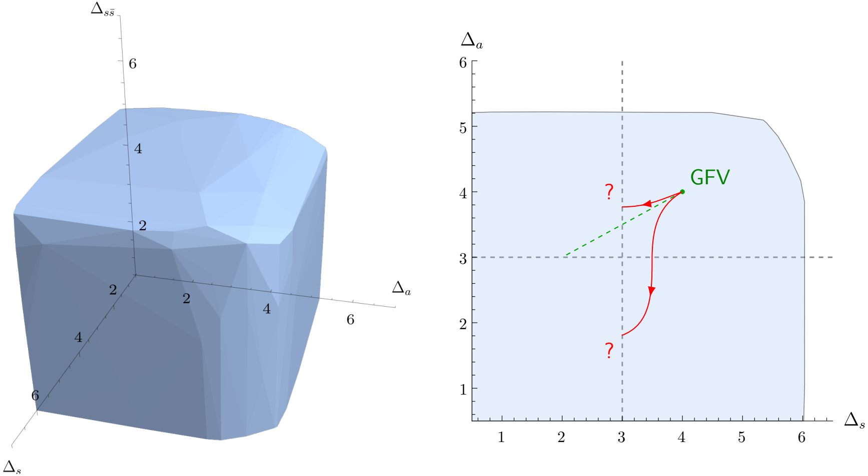
Decoupling ✗



$[SU(2),\ \Lambda = 23]$

3 Results: bound on dimensions

Without assumptions we can only put upper bounds



$[SU(3),\, \Lambda = 19]$

3 Results: hunting the RG flow in AdS

- One parameter (g_{YM}) family of CFT data where $g_{\mathrm{YM}} = 0$ is GFV
- Assume a spectrum which is a **parametric** deformation of GFV

$$\Delta_{\ell,p,\mathbf{r}} \in \left[\Delta_{\mathrm{GFV}} - \delta/\ell, \Delta_{\mathrm{GFV}} + \delta/\ell\right] \cup \left[\Delta'_{\mathrm{GFV}} - \delta, +\infty\right)$$

(first operator is $[J,J]_{n,\ell}$)

- Explore the allowed region of lowest lying scalars $\mathcal{O}$ as a function of δ

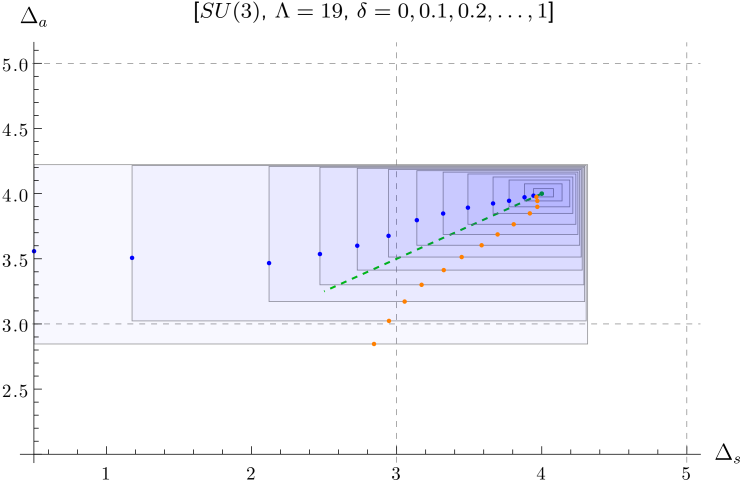
$$\Delta_{0,+,\mathbf{r}} \in \{\Delta_{\mathcal{O}}\} \cup \left[\Delta'_{\mathrm{GFV}} - \delta, +\infty\right)$$

- Compute the spectrum of primaries in GFV (characters)
- Search in $(\Delta_s, \Delta_a, \Delta_{s\bar{s}}, \alpha_{JJJ})$ with Navigator [Reehorst et al., 2021]
- For which δ: $\min \Delta_s < 3$? $\min \Delta_a < 3$? $\min \Delta_{s\bar{s}} < 3$?

3 Results: hunting the RG flow in AdS

Parametric gaps: $\Delta \in \left[\Delta_{\mathrm{GFV}} - \delta/\ell, \Delta_{\mathrm{GFV}} + \delta/\ell\right] \cup \left[\Delta'_{\mathrm{GFV}} - \delta, +\infty\right)$

$[SU(3),\ \Lambda = 19,\ \delta = 0, 0.1, 0.2, \ldots, 1]$

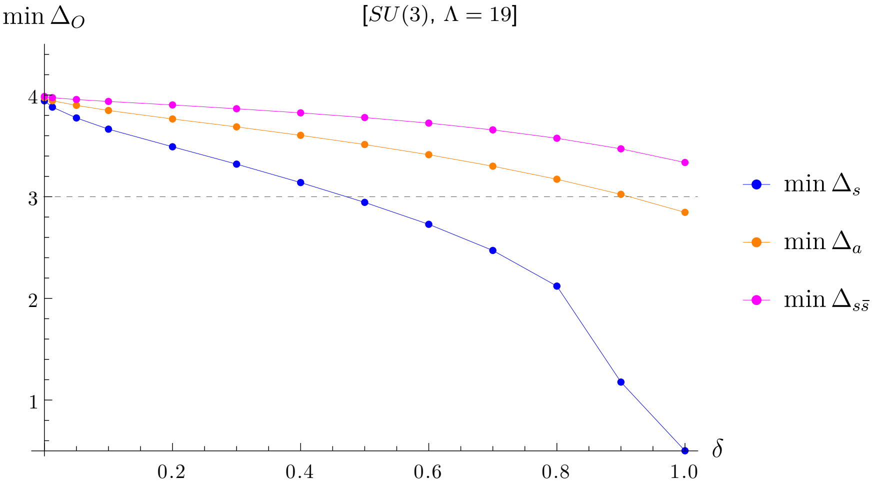


☺ Merging

☹ Higgsing

3 Results: hunting the RG flow in AdS

Parametric gaps: $\Delta \in \left[\Delta_{\text{GFV}} - \delta/\ell, \Delta_{\text{GFV}} + \delta/\ell\right] \cup \left[\Delta'_{\text{GFV}} - \delta, +\infty\right)$



☺ Merging ☹ Higgsing

4 Conclusion

- We considered the problem of **confinement** of pure Yang-Mills from the point of view of **QFT in AdS**
- Compatibility with **flat space** implies that Dirichlet must disappear
- The **non-abelian currents bootstrap** can place bounds on scenarios
    - Rule out Decoupling
    - Evidence for Merging vs Higgsing
- Open questions and future
    - What is **Dirichlet***?
    - Is the flow saturating $\min \Delta_s$?
    - Bootstrapping the RG flow?

4 Conclusion

- We considered the problem of **confinement** of pure Yang-Mills from the point of view of **QFT in AdS**
- Compatibility with **flat space** implies that Dirichlet must disappear
- The **non-abelian currents bootstrap** can place bounds on scenarios
    - Rule out Decoupling
    - Evidence for Merging vs Higgsing
- Open questions and future
    - What is **Dirichlet***?
    - Is the flow saturating $\min \Delta_s$?
    - Bootstrapping the RG flow?

Thank you!

Supplemental material

5 Symmetries of YM in AdS

$$G = G_{\text{space}} \times G_{\text{global}} = \Big(SO(4,1)_0 \rtimes \mathbb{Z}_2^{\mathcal{P}} \Big) \times \Big(PSU(N) \rtimes \mathbb{Z}_2^{\mathcal{C}} \Big)$$

- **conformal**: boundary correlators of QFT in AdS

  Representations: (Δ, ℓ)

- **parity**: $A_\mu^a(z,x) \xrightarrow{\mathcal{P}} \mathcal{P}_\mu^\nu A_\mu^a(z,-x)$, $\mathcal{P}_\mu^\nu = \mathrm{diag}(1,-1,-1,-1)$

  Commutes with with D, L_i, anti-commutes with P_i, K_i

  Representations: $(\Delta, \ell, p = \pm)$, p "intrinsic" parity

- $\boldsymbol{SU(N)}$: global version of G_{YM}

  Local operators are blind to the center ⇒ $PSU(N) \cong SU(N)/\mathbb{Z}_N$

  Representations: $\lambda = (\lambda_1, \ldots, \lambda_{N-1})$ Dynkin labels (↔ Young diagrams)

- **charge conjugation**: $A_\mu(z,x) \xrightarrow{\mathcal{C}} A_\mu(z,x)^* = \big(\mathcal{C}^{ab} A_\mu^b(z,x)\big) T^a$

  Representations: $\lambda_c \cong \lambda \oplus \lambda^*$ or $\lambda_\pm$

6 Characters and GFV spectrum

Representations: $R = (\Delta, \ell, p = \pm, \lambda_{(c,\pm)})$

Characters: $q, x, \vec{z}$ fugacities, $g_{\mathcal{P}} \in \{\mathbb{1}, \mathcal{P}\}$, $g_{\mathcal{C}} \in \{\mathbb{1}, \mathcal{C}\}$

$$\begin{aligned}\chi_R(q, x, \vec{z}, g_{\mathcal{P}}, g_{\mathcal{C}}) &= \mathrm{tr}_R\Big[q^D x^L z_1^{H_1} \cdots z_{N-1}^{H_{N-1}} g_{\mathcal{P}} g_{\mathcal{C}}\Big] \\ &= \mathrm{tr}_{(\Delta,\ell,p)}\Big[q^D x^L g_{\mathcal{P}}\Big]\ \mathrm{tr}_{\lambda_{(c,\pm)}}\Big[z^H g_{\mathcal{C}}\Big]\end{aligned}$$

Twining characters from [Fuchs, Schellekens, and Schweigert, 1996]

Generating function of symmetric products of $J = (2, 1, +, (1, 1)_-)$ $(N = 3)$

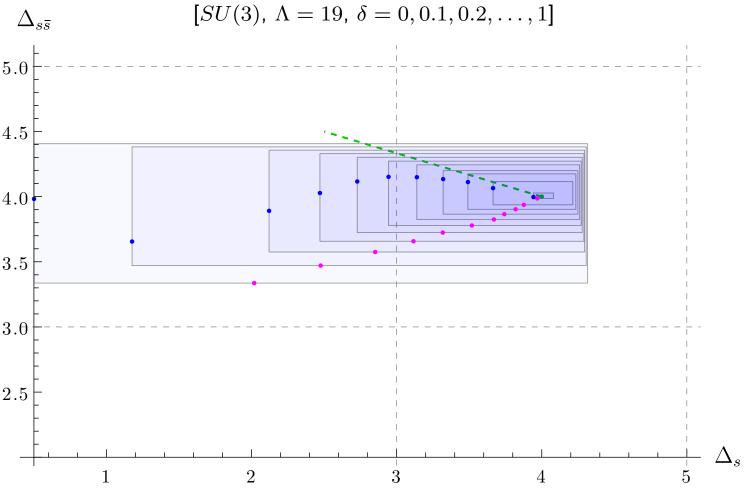
$$Z_J(\eta, g) = \exp\left(\sum_{n=0}^{+\infty} \frac{\eta^n}{n} \chi_J(g^n)\right) \qquad\qquad g = q^D x^L z^H g_{\mathcal{P}} g_{\mathcal{C}}$$

Expand and read off the spectrum (with degeneracies)

$$Z_J(\eta, g) = \sum_{R,n} d_{R,n} \eta^n \chi_R(g) = 1 + \eta\, \chi_J + \eta^2\, \chi_{(4,0,+,(0,0)_+)} + \cdots$$

7 Hunting the RG flow in AdS

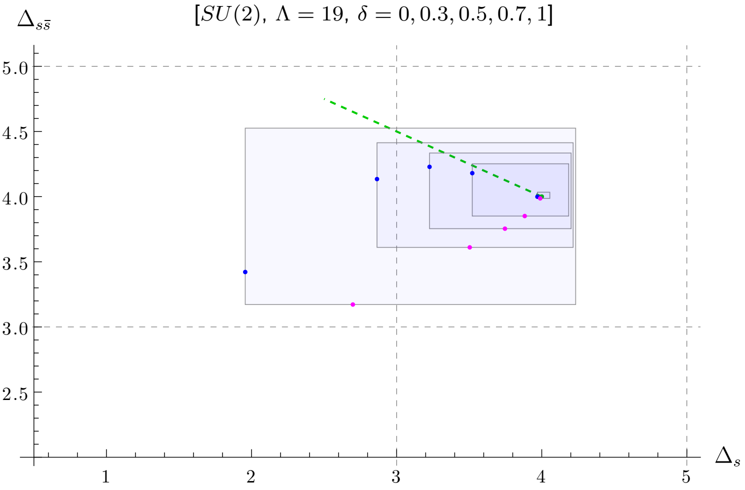
Parametric gaps: $\Delta \in \left[\Delta_{\mathrm{GFV}} - \delta/\ell, \Delta_{\mathrm{GFV}} + \delta/\ell\right] \cup \left[\Delta'_{\mathrm{GFV}} - \delta, +\infty\right)$



☺ Merging ☹ Higgsing

7 Hunting the RG flow in AdS

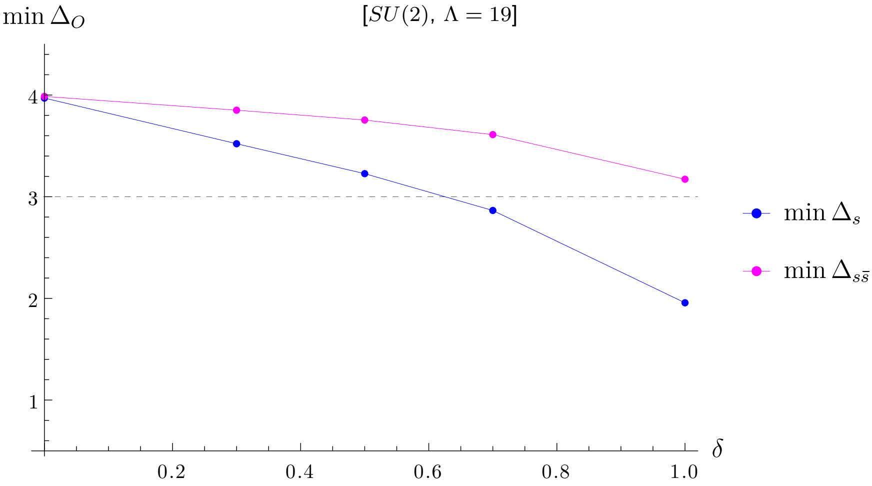
Parametric gaps: $\Delta \in \left[\Delta_{\mathrm{GFV}} - \delta/\ell, \Delta_{\mathrm{GFV}} + \delta/\ell\right] \cup \left[\Delta'_{\mathrm{GFV}} - \delta, +\infty\right)$



☺ Merging ☹ Higgsing

7 Hunting the RG flow in AdS

Parametric gaps: $\Delta \in \left[\Delta_{\mathrm{GFV}} - \delta/\ell, \Delta_{\mathrm{GFV}} + \delta/\ell\right] \cup \left[\Delta'_{\mathrm{GFV}} - \delta, +\infty\right)$



☺ Merging

☹ Higgsing