# No Shift, Sherlock

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Based on an upcoming paper with the same title with
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# Swampland and bootstrap

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- Two famous examples:

  **No free parameter conjecture** (absence of (-1)-form symmetry):

  All "parameters" in quantum gravity = VEV of some fields: $g \sim \langle \varphi \rangle$

  In string theory,

  $$g_s = e^{\langle \phi \rangle}$$

  cf. [SK, Kusuki, Meineri, Ooguri, to appear]

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- More quantitative conjectures with possible implications to BSM

  Weak gravity conjecture, infinite distance conjecture,…

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  Global symmetry in AdS QG is incompatible with entanglement wedge reconstruction (EWR).
- But….. EWR works only in the semiclassical gravity regime, expected to be modified by stringy / QG corrections.
- Can we prove no global symmetry conjecture in full QG regime by translating it to a CFT statement using AdS/CFT?

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- But proving this turns out to be difficult….

# Difficulty of proving no global symmetry

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  $\rightarrow$ difficult to distinguish it from "accidental selection rule"

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- We don’t know (yet) how to prove it in general. But there is actually one setup in which we can make progress.

Shift symmetry in AdS (or more generally, spontaneously broken symmetry)

# Shift symmetry is easier

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- Clear distinction between gauged and global symmetries.
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- Shift symmetry also changes the boundary value of $\phi$: $\phi_{\rm bdy} \to \phi_{\rm bdy} + c$
- In AdS/CFT, $\phi_{\rm bdy}$ is exactly marginal coupling constant in CFT.

$$Z_{\phi_{\rm bdy}} = \left\langle e^{-\int \phi_{\rm bdy} \mathcal{O}} \right\rangle$$

$$\phi : \text{massless} \quad \mapsto \quad \mathcal{O} : \text{marginal } (\Delta = d)$$

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  CFT which is invariant under the change of exactly marginal coupling constant $\phi_{\mathrm{bdy}} \to \phi_{\mathrm{bdy}} + c$

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* It is important to consider CFT with stress tensor. Without stress tensor, there **are** examples of trivial conformal manifold. (More on next slide)

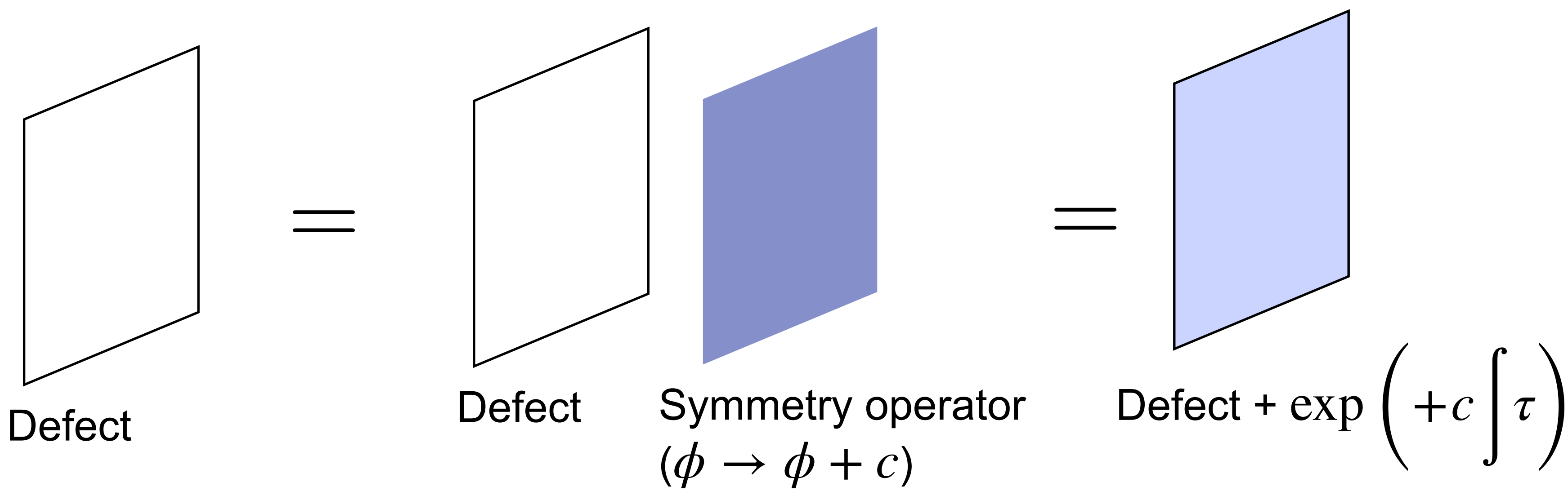
# Examples of trivial conformal mfd without $T_{\mu\nu}$

- Example 1: CFT with symmetry $G$ and conformal defect breaks it down to $H$. Drukker, Kong, Sakkas, ….

→ Defect conformal manifold $G/H$ , every point on the manifold is equivalent.

$$\partial_\mu J^\mu = \underline{\tau(x_{\parallel})}\delta^{d-p}(x_\perp)$$

*Tilt operator,* $\Delta = p$, marginal



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**Example**: Maldacena Wilson loop in $\mathcal{N} = 4$ SYM

$$\mathrm{Tr}\left[\mathrm{P}\exp\left(\int dt\, iA_\mu \dot{x}^\mu + \Phi_6|\dot{x}|\right)\right]$$

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- Example 2: Free shift symmetric scalar in AdS without gravity.

→ obviously UV complete, generalized free field CFT, no stress tensor.

Or more generally SSB in AdS cf. [Carmi, Di Pietro, SK]

# Plan

1. Argument for no shift symmetry in CFT

2. Generalization to higher form

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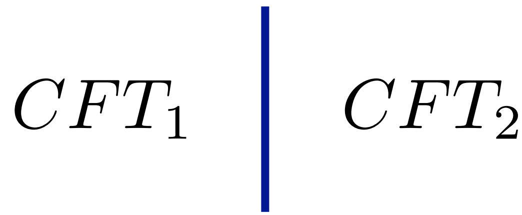
# General strategy

- A proof by contradiction in two steps:
- **Step 0**: Assume ∃CFT with a trivial conformal manifold
- **Step 1**: Use a CFT bootstrap argument to show one can construct "identity conformal interface" for any two points on a trivial conformal manifold.
- **Step 2**: Show that the existence of the identity conformal interface between nearby CFTs contradicts conformal perturbation theory

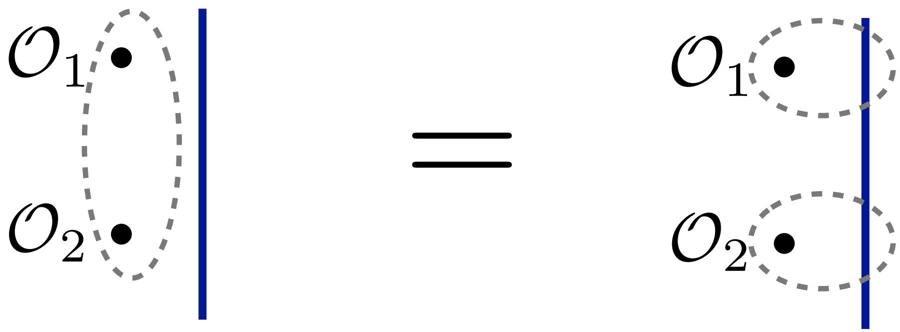
**Step 1: Trivial conformal mfd implies identity interface**

# Trivial conformal mfd implies identity interface

- Conformal interface $SO(d+1,1) \to SO(d,1)$




- Needs to satisfy various bulk-interface crossing eq

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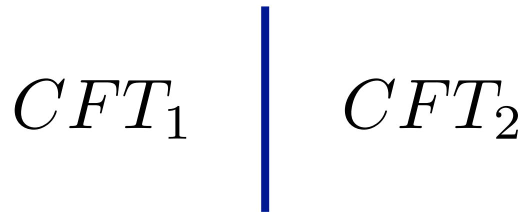


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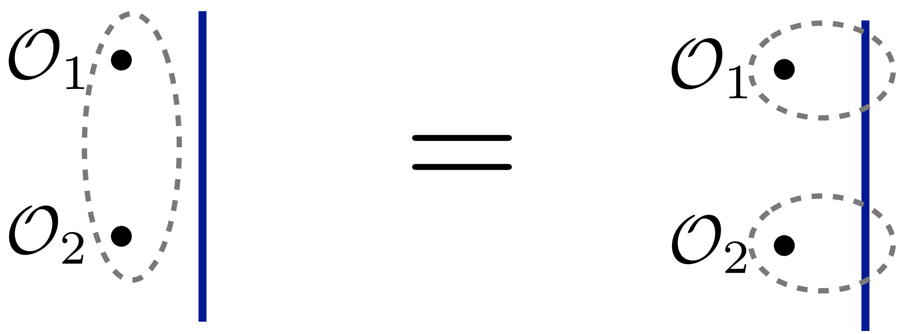


- Interface crossing eq: Bulk CFT data $\Rightarrow$ Interface CFT data
- Traditionally constructed by imposing bc to bulk fields, coupling to localized dof….
- In the modern bootstrap, sol's to crossing = conformal interface

# Trivial conformal mfd implies identity interface

- On trivial conformal manifold, $CFT_1$ and $CFT_2$ share the same bulk CFT data by assumption.
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- 1-to-1 correspondence between interfaces within $CFT_1$ and interfaces connecting $CFT_1$ and $CFT_2$

$$CFT_1 \;\Big|\; CFT_1 \quad \simeq \quad CFT_1 \;\Big|\; CFT_2$$

- In particular, it implies the “identity interface” between $CFT_{1,2}$ that corresponds to inserting nothing in $CFT_1$

$$CFT_1 \;\vdots\; CFT_1 \quad \simeq \quad CFT_1 \;\Big|\; CFT_2$$

## **Step 2**: **Identity interface contradicts conformal perturbation theory**

# Identity interface is incompatible

- Let's analyze the "identity interface" between nearby CFTs.


$$CFT_1 + \delta\phi \int_{x_\perp > 0} \mathcal{O}$$

- In addition, it may involve deformation localized at the interface

e.g. $$\delta\rho \int_{x_\perp = 0} \mathcal{O}' \qquad \Delta_{\mathcal{O}'} = d - 1$$

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- No other deformation away from the interface is allowed since it will violate the stress tensor conservation in the bulk.

$$\partial_\mu T^{\mu\nu} \neq 0$$

# Identity interface is incompatible


$$CFT_1 + \delta\phi \int_{x_\perp > 0} \mathcal{O} \left( + \delta\rho \int_{x_\perp = 0} \mathcal{O}' \right)$$

- Half-space deformation turns on one-point function of $\mathcal{O}$

$$\langle \mathcal{O}(x) \rangle_I = \delta\phi \int_{y_\perp > 0} d^d y \langle \mathcal{O}(x) \mathcal{O}(y) \rangle_I = \frac{\pi^{d/2} \delta\phi}{2^d \Gamma(d+1)} \frac{1}{|x_\perp|^d}$$

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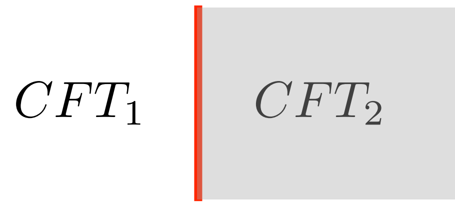


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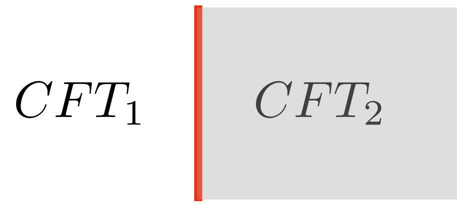


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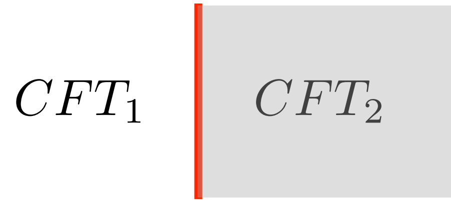
- One can argue that other possible deformations at the interface (including non-local def) cannot resolve the issue.

Thus, the identity interface is incompatible with conformal perturbation

# Sketch of more systematic argument




- One can make the argument slightly more systematic using displacement operator $D$.

$$\partial_\mu T^{\mu\perp} = D(x_{||})\delta^{d-1}(x_\perp)$$

- In identity interface, $D = 0$.

- Half-space deformation gives $D \propto \mathcal{O}$.

- Any other interface-localized deformation induces other operator for $D$.

# Recap

- A proof by contradiction in two steps:

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# Generalization to higher form symmetry

- 1-form symmetries of Maxwell theory.

$$dF = 0\,, \quad d * F = 0$$

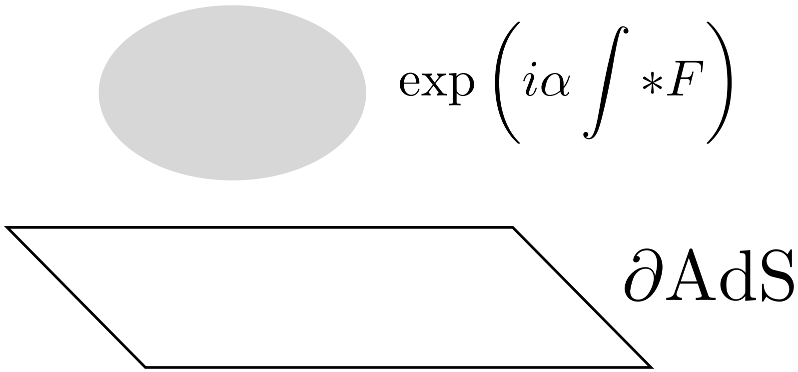
- In QG, they are expected to be violated by the presence of charged particles.

Related to charge completeness conjecture

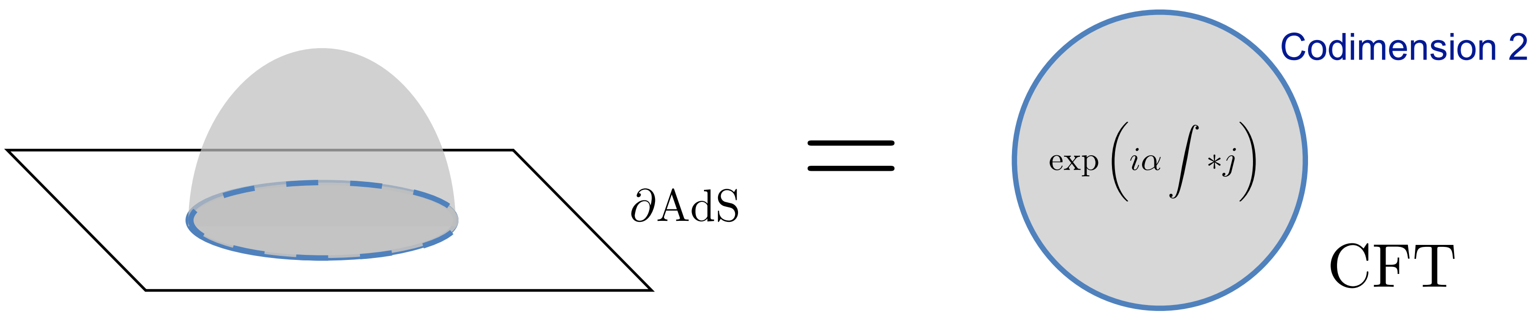
- Can we show the absence of 1-form symmetries of Maxwell theory in AdS QG?

# Generalization to higher form symmetry

- If there was (electric) 1-form symmetry, one can construct the codim-2 topological interface in AdS.




- By pushing them to boundary, one gets a monodromy defect (co-dimension 2 + codimension 1)

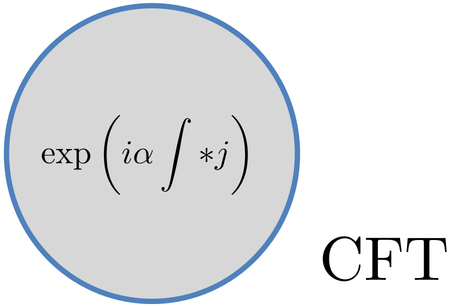



- 1-form symmetry in the bulk predicts that the monodromy defect is topological.

# CFT counterpart of no 1-form symmetry in AdS

**CFT "No 1-form" conjecture**

*In CFT with stress tensor, monodromy defects cannot be topological.*



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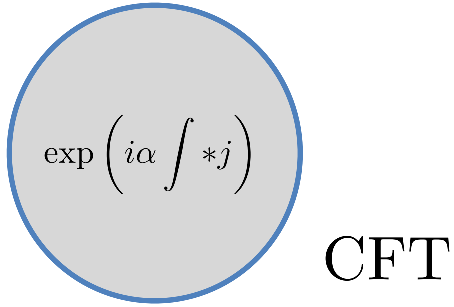


- Working in $\alpha \ll 1$ regime, one can show that the monodromy defect turns on one-point function of $j_\mu$ .

$$\langle j_\mu(x) \int *j\rangle \neq 0$$

Contradiction to topologicalness.

- One can also argue that any local deformation at the defect (edge of the circle) cannot remedy this.

Thus, the topological monodromy defect cannot arise in CFT.

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# Conclusion

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- CFT-based argument for no shift symmetry in AdS QG.
  - Works beyond the semi-classical gravity regime.
  - But works only for shift symmetry + spontaneously broken symmetries.

- Our argument is based on proof by contradiction. No quantitative prediction on the amount of violation of symmetry.

  $\Rightarrow$ Bootstrap bound on the amount of violation?

- Monodromy defects seem important for studying Swampland conjectures in AdS.

  $\Rightarrow$ A systematic bootstrap analysis of monodromy defect?
  Application to weak gravity conjecture?

- Non-spontaneously broken symmetries?

- Other related swampland conjectures? Infinite distance conjecture? No free parameter conjecture? [SK, Kusuki, Meineri, Ooguri, to appear]

# Caveat

- Strictly speaking, the argument for higher-form only shows the absence of topological symmetry operator in AdS.

- In QFT, symmetry = topological operators

[Gaiotto, Kapustin, Seiberg, Willet 14]

- It was argued recently that there is an obstruction for constructing topological symmetry operators in gravitational EFT.

[Bah, Jefferson, Roumpedakis, Waddleton 24] [Calvo, Mignosa, Rodriguez-Gomez 25]

$$e^{\alpha \int *j} = 1 + \alpha \int *j + \frac{\alpha^2}{2} \underbrace{\int *j \int *j}_{\text{Need to regularize}} + \cdots$$

↓

We need to introduce a thickness / regulator $\delta$, and later take $\delta \to 0$ limit

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Gravity backreaction becomes strong in the $\delta \to 0$ limit

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The argument does not exclude having other notions of symmetry like Ward identity etc.