QCD in AdS

[2511.064752] with R Ciccone, F De Cesare, M Serone

Progress of Theoretical Bootstrap, YITP, Kyoto

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INFN

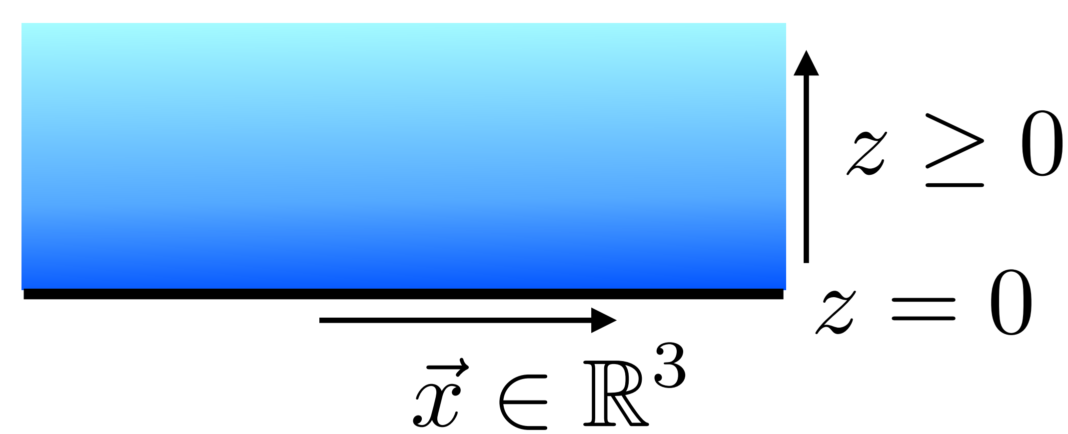
QCD: $SU(n_c)$ gauge theory
with n_f/n_s flavors of fundamental matter

$$\frac{1}{2g^2}\mathrm{tr}[F_{\mu\nu}F^{\mu\nu}] + \bar{\Psi}_I(D+M)\Psi^I \qquad I = 1, \ldots, n_f$$

or $$+\nabla_\mu \Phi_I^* \nabla^\mu \Phi^I + m^2 \Phi_I^* \Phi^I \qquad I = 1, \ldots, n_s$$

AdS: on rigid curved background of
four-dimensional Anti-de Sitter space

$$ds^2_{\mathrm{AdS}} = L^2 \frac{dz^2 + d\vec{x}^2}{z^2}$$

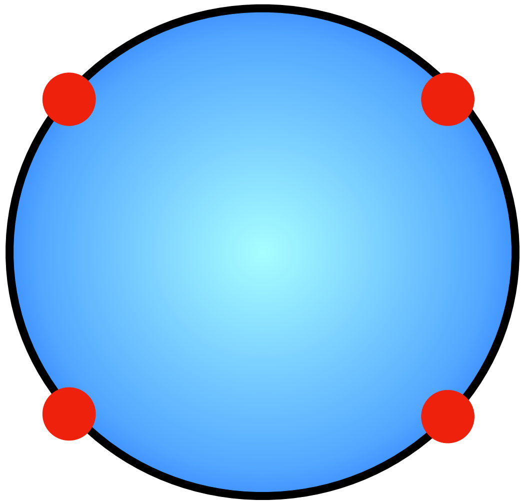


Why gauge theories in AdS? [Callan, Wilczek (1990)], …

- Dimensionless parameter $g^2(1/L)$ or $L\Lambda$, allowing perturbative expansion

- Large spacetime symmetry $SO(1,4)$

- Analogue of scattering amplitudes: local operators at boundary points



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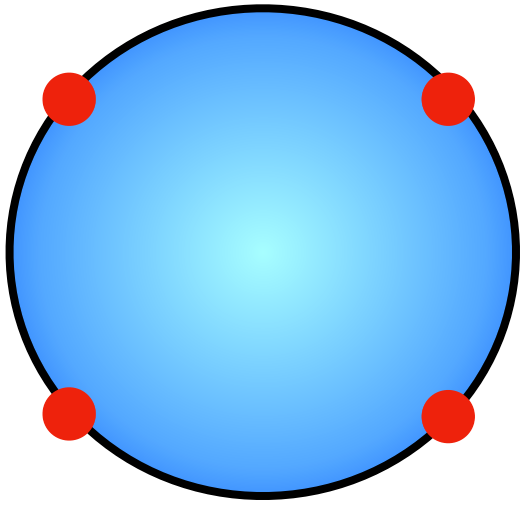

boundary CFT (bCFT): data are fixed by $L\Lambda$ but not uniquely → choice of *boundary condition*

A **CFT approach** to the dynamics of gauge theories

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Study at weak coupling:

- Possible bc’s
- Properties of the bCFTs: symmetries, MFT limit, light spectrum
- Perturbative bCFT data

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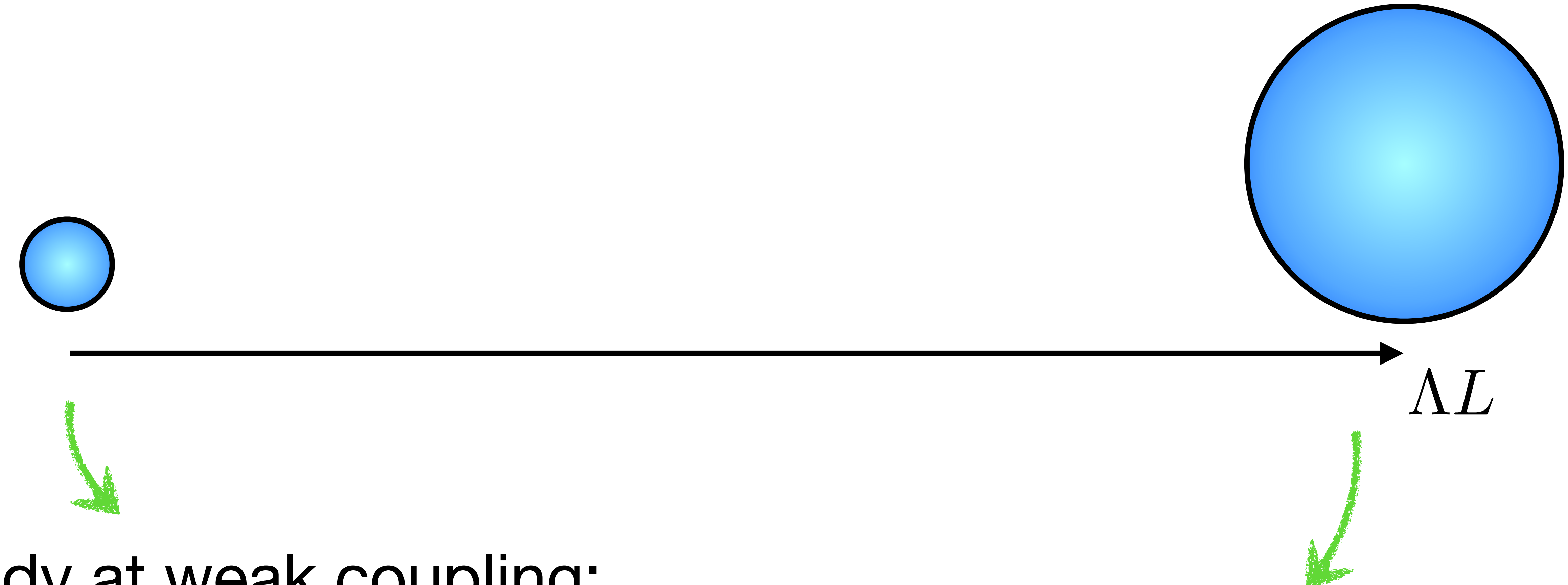

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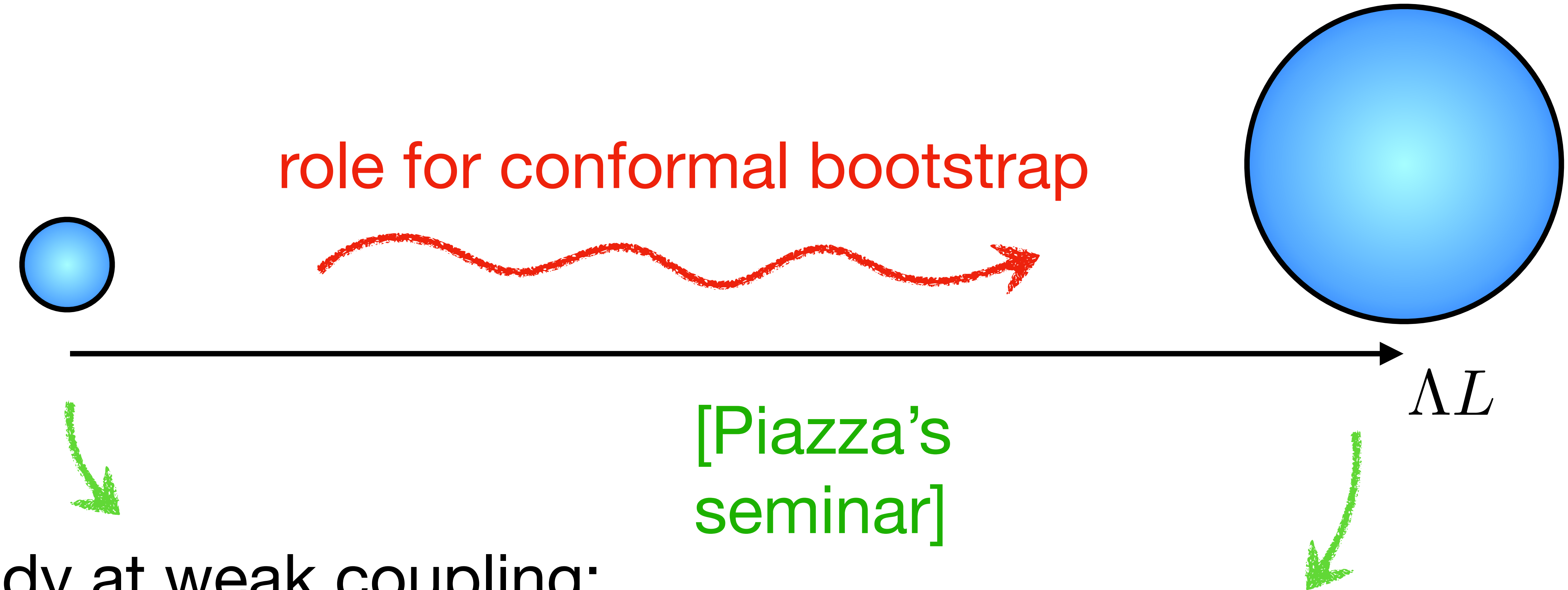

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Simplest bc's for the gauge fields at weak coupling:

<u>Dirichlet:</u> $A^a_\mu \underset{z\to 0}{\sim} z J^a_\mu + \dots$

- Continuous global symmetry:
  $SU(n_c)$ with current J^a_μ + (non-local) flavor symmetry

- bCFT at weak coupling: MFT of J^a_μ + decoupled matter

- Light spectrum: J^a_μ ($\Delta_J = 2$), $\hat{\Psi}^I_\alpha / \hat{\Phi}^I_\alpha$ ($\Delta_\Psi / \Delta_\Phi$)

  Double- & multi- trace operators in various representations of $SU(n_c)$ and flavor symmetry

Simplest bc's for the gauge fields at weak coupling:

Neumann: $$A^a_\mu \underset{z\to 0}{\sim} a^a_\mu + \dots$$

- Continuous global symmetry: only flavor, $SU(n_c)$ gauged also at the boundary

- bCFT at weak coupling: MFT of $f^a_{\mu\nu}$ + decoupled matter with restriction to $SU(n_c)$ singlet sector

- Light spectrum: $f^a_{\mu\nu}$, $\hat{\Psi}^I_\alpha/\hat{\Phi}^I_\alpha$ not gauge invariant

Gauge invariant double- & multi- trace operators in various representations of the flavor symmetry

At strong coupling?

Color confinement: asymptotic states are not charged under $SU(n_c)$

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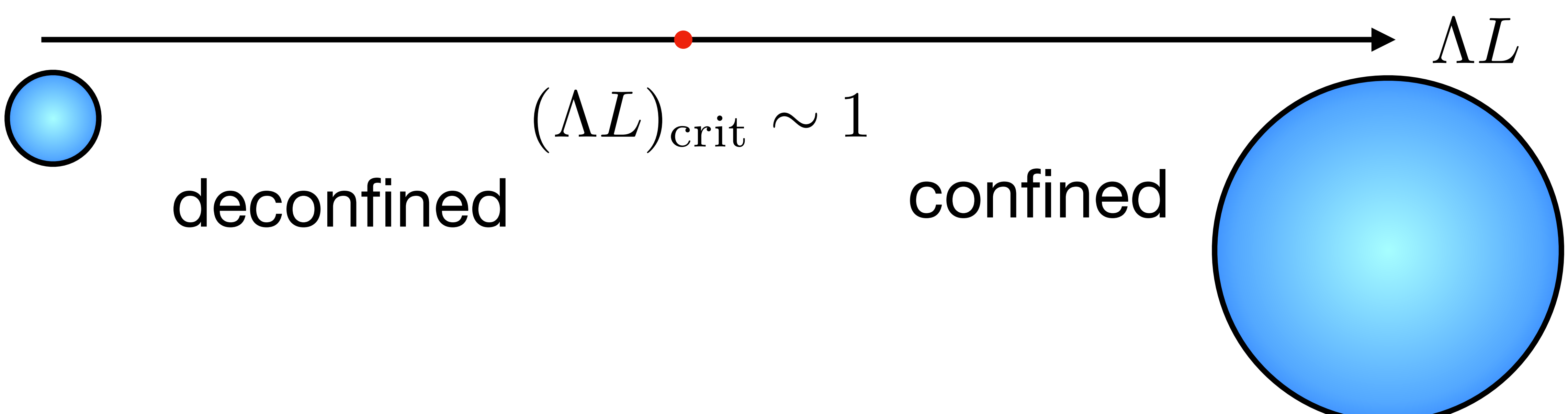
<u>**Color confinement:**</u> asymptotic states are not charged under $SU(n_c)$

→ Clash with Dirichlet bc, there must be a discontinuity

[Aharony, Berkooz, Tong, Yankielowicz (2012)]

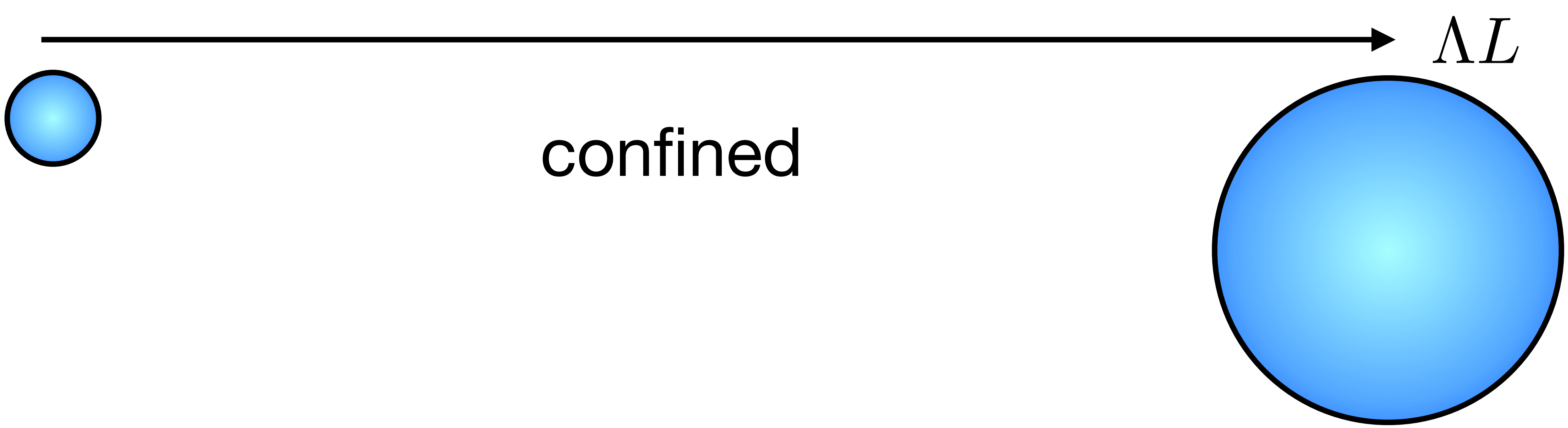


At strong coupling?

<u>Color confinement:</u> asymptotic states are not charged under $SU(n_c)$

Neumann bc satisfies the condition and can be continuous

[Aharony, Berkooz, Tong, Yankielowicz (2012)]



Observation 1:
in pure YM, also clash between **mass gap** and Dirichlet

$$\min M \leq \lim_{L\to\infty} \frac{\Delta_J}{L} = 0$$

Disappears with the inclusion of massless matter

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Disappears with the inclusion of massless matter

Observation 2:
confinement as **linear quark potential**

- Needed to tell apart confinement and higgsing
- Not immediate to import to AdS (though possible)
- Not available with dynamical fundamental flavours

Two interesting (ambitious) tasks:

(1) Show that only boundary conditions with color confinement (no $SU(n_c)$ symmetry on the boundary) are available at strong coupling / large AdS

(2) Use a continuous boundary condition to compute observables in flat space

[Gabai, Gorbenko, Qiao (2025)]

More humble (still quite ambitious) first steps:

(1') Show that Dirichlet disappears at some $(\Lambda L)_{\rm crit}$

(2') Show that Neumann is continuous

This work:

- Perturbative calculations that favor (1') and (2')

→ Generalizing and confirming similar indications found in pure Yang-Mills theory [Ciccone, De Cesare, DP, Serone (2024)]

- New phenomena due to matter

→ AdS signature of conformal window

→ Chiral symmetry breaking and relation to bc's of the fermions [Rattazzi, Redi (2009)]

How to check perturbatively for disappearance of a bc?

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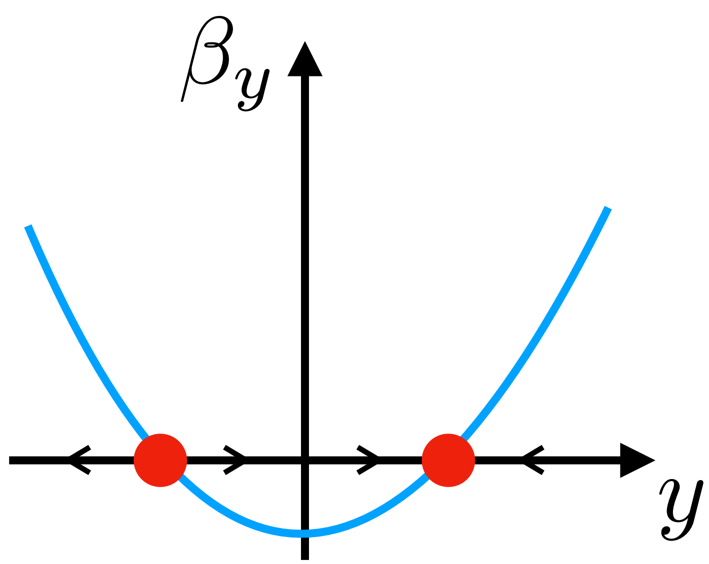
Mechanism: **Merger and Annihilation**

[Kaplan, Lee, Son, Stephanov (2009)]

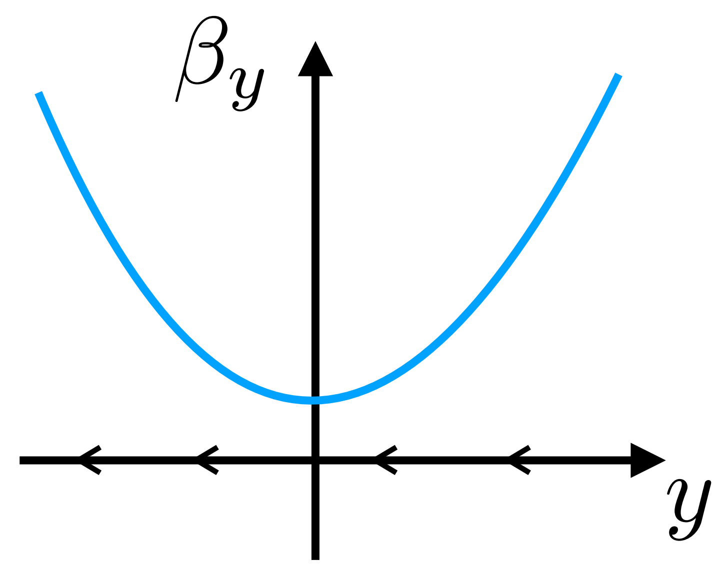
[Gorbenko, Rychkov, Zan (2018)]

A singlet scalar operator $\mathcal{O}$ becomes marginal ($\Delta = 3$) at $g^2 = g^2_{\rm crit}$ and its boundary coupling runs

$$\int_{\rm AdS_4} \frac{1}{2g^2}\,{\rm tr}[F^2] + \int_\partial y\,\mathcal{O} \;, \qquad \beta_y \sim A\left(\frac{1}{g^2_{\rm crit}} - \frac{1}{g^2}\right) + B\,y^2$$

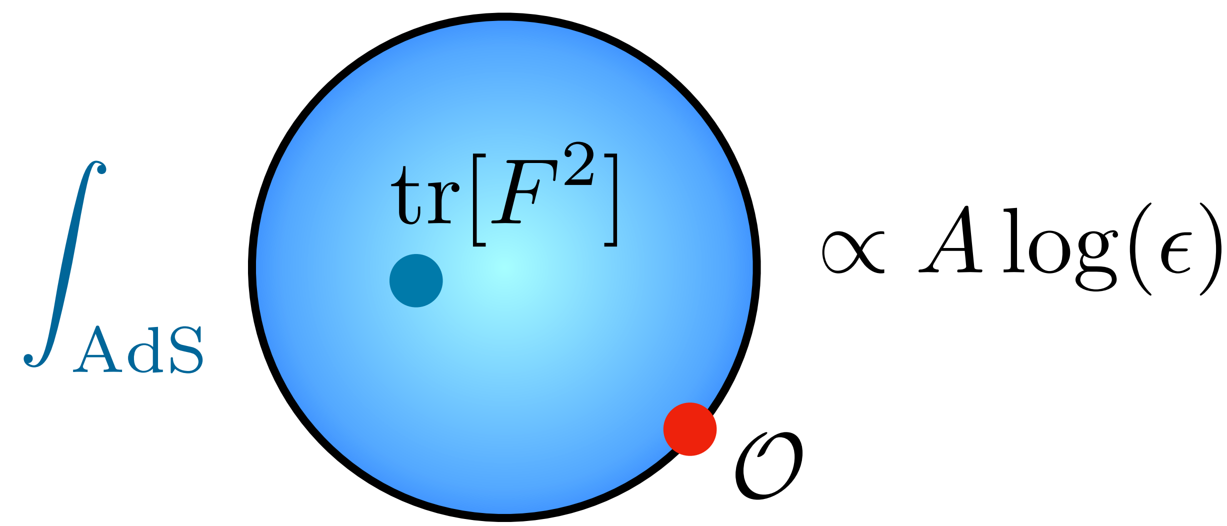


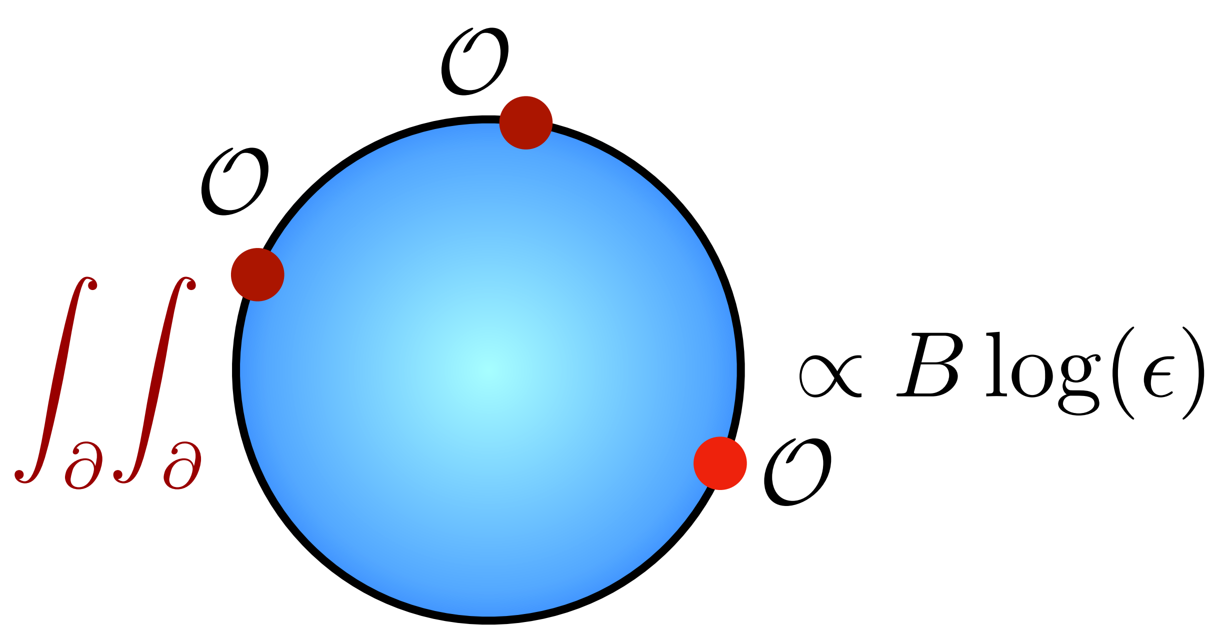
$g^2 < g^2_{\rm crit}$



$g^2 > g^2_{\rm crit}$

$$\int_{\mathrm{AdS}_4} \frac{1}{2g^2}\,\mathrm{tr}[F^2] + \int_{\partial} y\,\mathcal{O} \quad , \quad \beta_y \sim A\left(\frac{1}{g^2_{\mathrm{crit}}} - \frac{1}{g^2}\right) + B\,y^2$$





[Lauria, Milam, van Rees (2023)]

Leading candidates for $\mathcal{O}$ in perturbation theory

- From gauge fields: $\mathcal{D}_{\rm YM} = \begin{cases} J^a_\mu J^{\mu\, a} & \text{with D bc} \\ f^a_{\mu\nu} f^{\mu\nu\, a} & \text{with N bc} \end{cases}$

- From fermions: $\mathcal{D}_{\rm F} = \frac{1}{2}\widehat{\bar{\Psi}}_I \gamma^i \partial_i \widehat{\Psi}^I - \frac{1}{2}\partial_i \widehat{\bar{\Psi}}_I \gamma^i \widehat{\Psi}^I$

with bc $\Psi^I \underset{z\to 0}{\sim} z^{\frac{3}{2}}\, \widehat{\Psi}^I + \dots$, $\gamma_z \widehat{\Psi}^I = \widehat{\Psi}^I$ $\quad (M=0)$

- From scalars: $\mathcal{D}_{\rm S} = \hat{\phi}^*_I \hat{\phi}^I$

with bc $\phi^I \underset{z\to 0}{\sim} z^2\, \hat{\phi}^I + \dots \quad (m^2 = m^2_{\rm conf})$

$\mathcal{D}_{\mathrm{YM}}, \mathcal{D}_{\mathrm{F}}, \mathcal{D}_{\mathrm{S}}$ all have $\Delta = 4$ in the free limit

mixing problem

$\mathcal{D}_{\rm YM}, \mathcal{D}_{\rm F}, \mathcal{D}_{\rm S}$ all have $\Delta = 4$ in the free limit

mixing problem

Not a coincidence: for $g^2 \to 0$ the bulk theory is a sum of free CFTs.

CFT in AdS = BCFT in flat space $\mathbb{R}^3 \times \mathbb{R}_+$

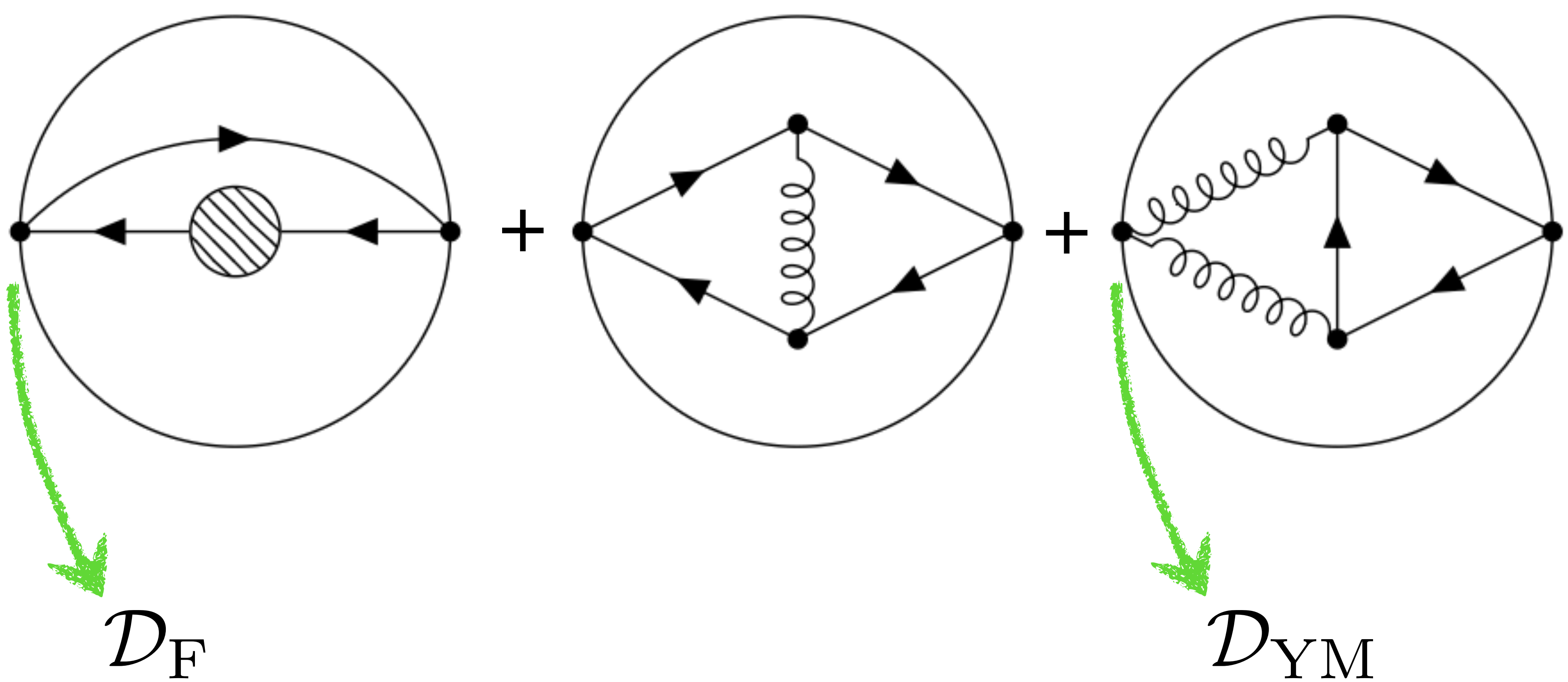
Weyl rescaling

Protected boundary operator in BCFT: **<u>displacement operator</u>**

$$T_{zz} \underset{z\to 0}{\sim} \mathcal{D} + \dots$$

$\mathcal{D}_{\rm YM}, \mathcal{D}_{\rm F}, \mathcal{D}_{\rm S}$ are the displacement operators of the free UV CFT in the bulk

Diagrammatic calculation of mixing, e.g. $\mathcal{D}_{\rm YM}/\mathcal{D}_{\rm F}$



+ diagrams involving only the gauge field, already computed for pure YM theory [Ciccone, De Cesare, DP, Serone (2024)]

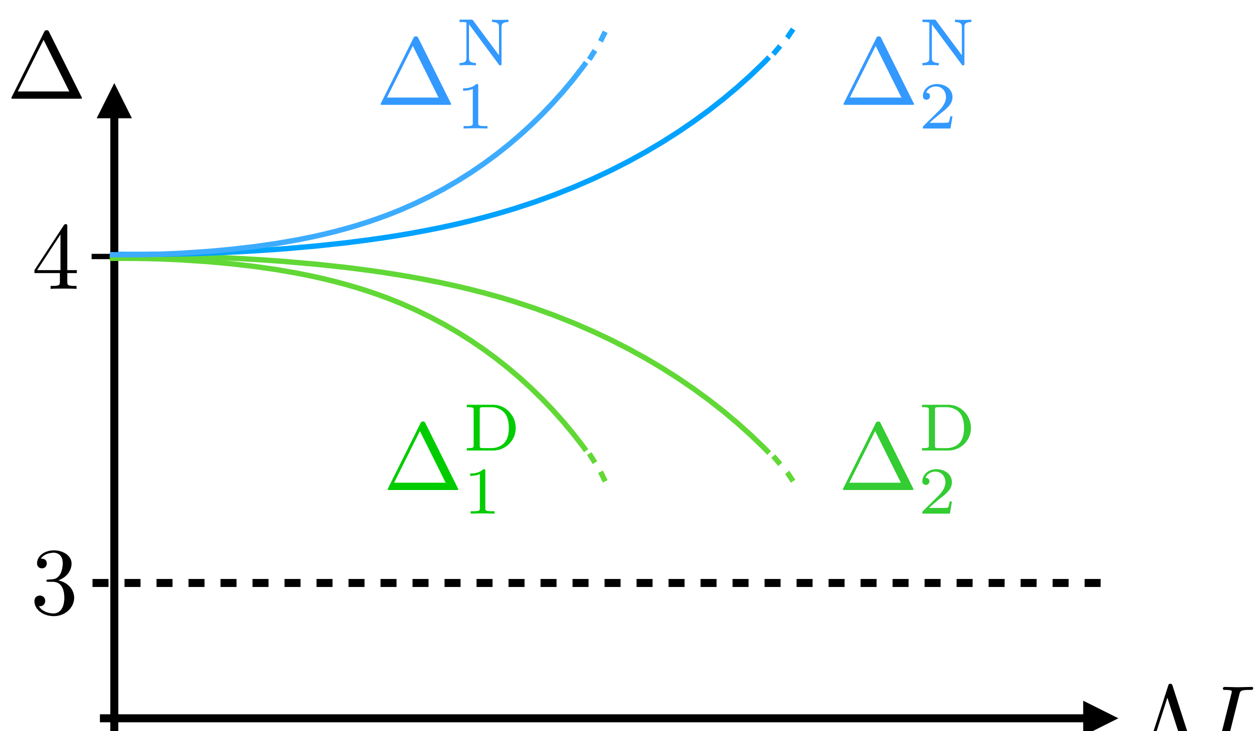
Alternative technique: <u>Broken conformal Ward Identity</u>

Result

Eigenvalues of the mixing matrix, fermionic matter

$$\gamma^{\mathrm{D}}_{1,2} = -\gamma^{\mathrm{N}}_{1,2}$$

$$= \frac{-(15n_c^2 - 4) \pm \sqrt{(15n_c^2-4)^2 - 32\,(n_c^2-1)\,n_c\left(\frac{11n_c}{2} - n_f\right)}}{48\pi^2 n_c}$$

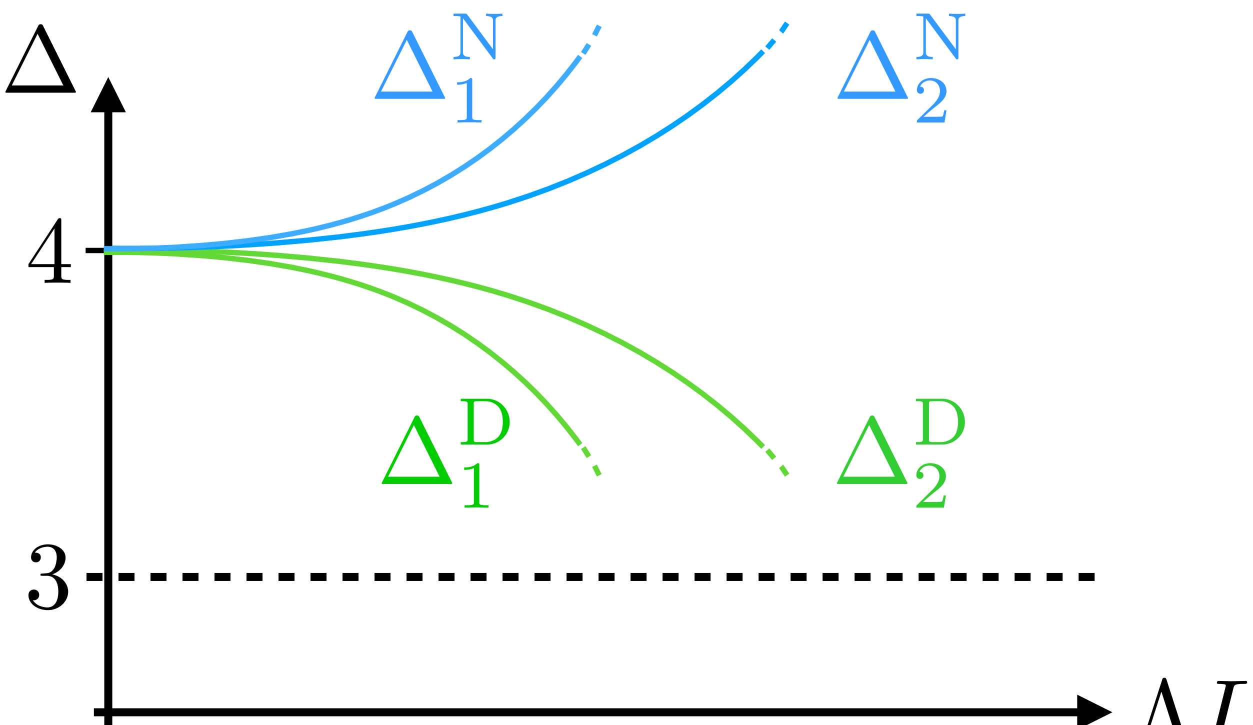


Result

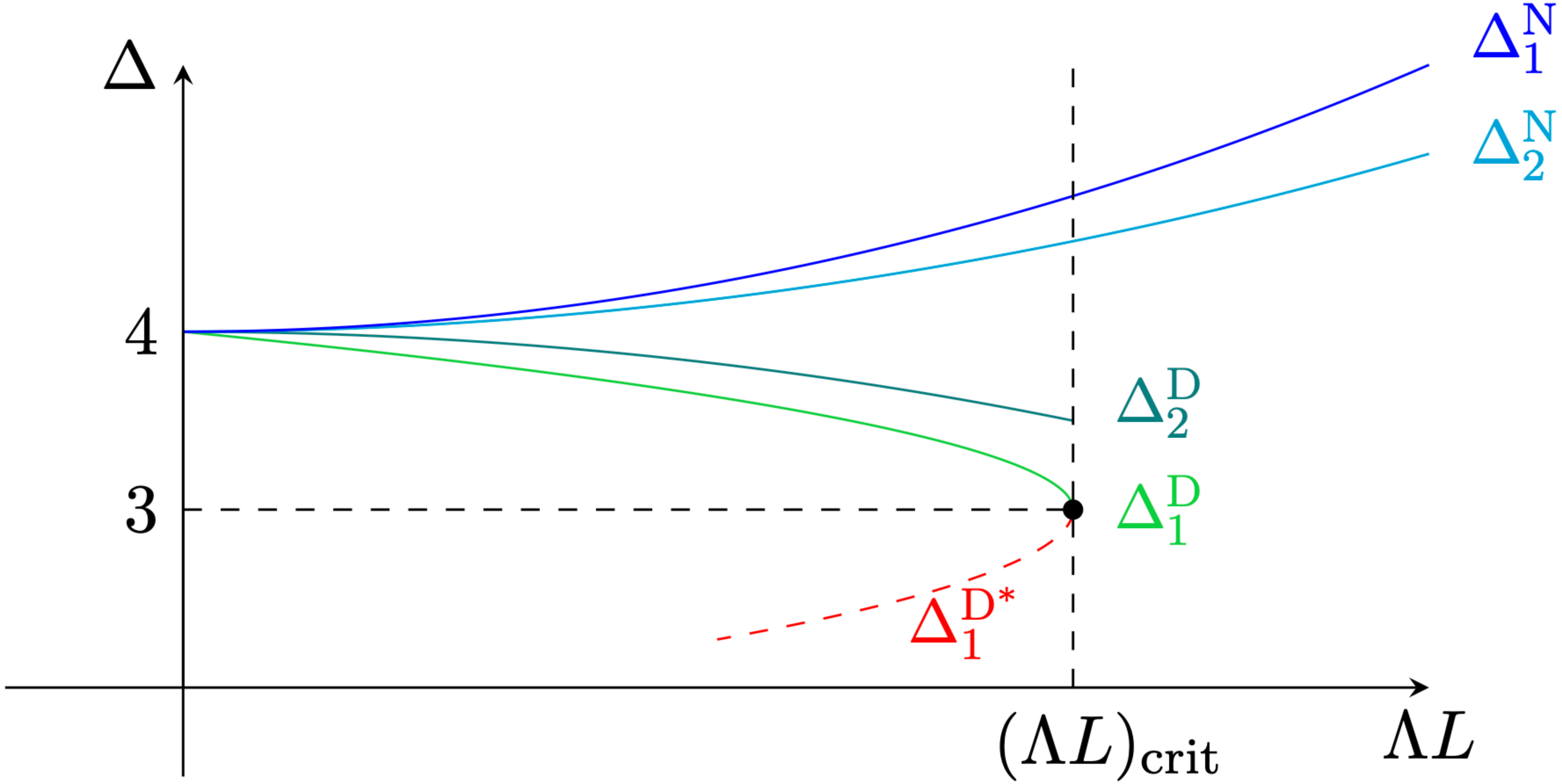
Eigenvalues of the mixing matrix, scalar matter

$$\gamma_{1,2}^{\rm D} = -\gamma_{1,2}^{\rm N}$$

$$= \frac{-(14n_c^2-3) \pm \sqrt{(14n_c^2-3)^2 - 6\,(n_c^2-1)\,n_c(22n_c-n_s)}}{48\pi^2 n_c}$$

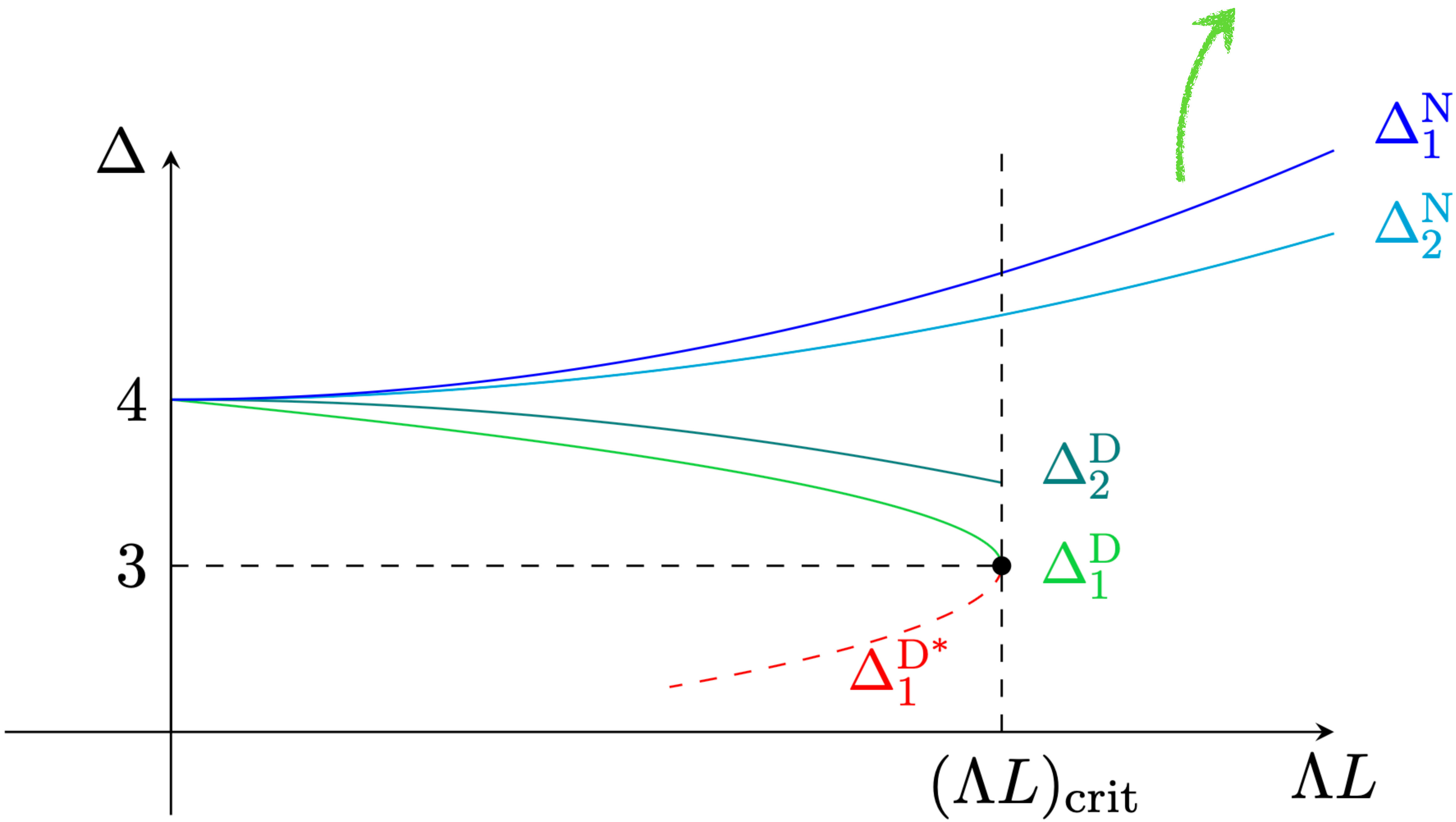


Proposed extrapolation

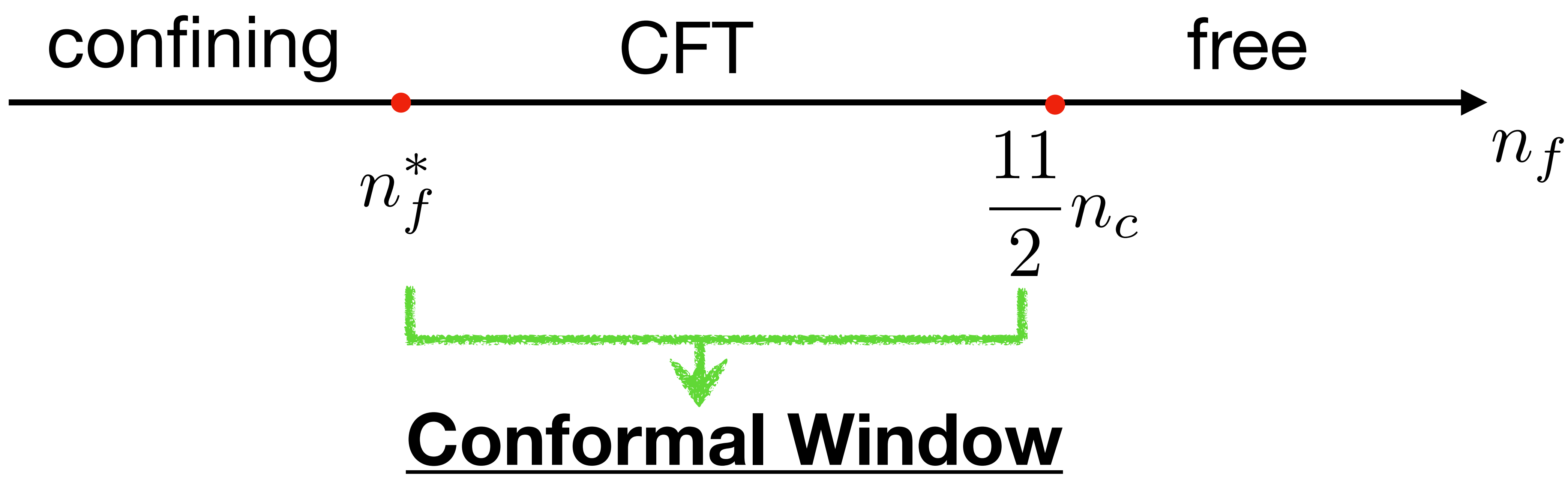


Proposed extrapolation

asymptotically:
$\propto ML$



With matter we can also have different IR behaviors in the bulk:



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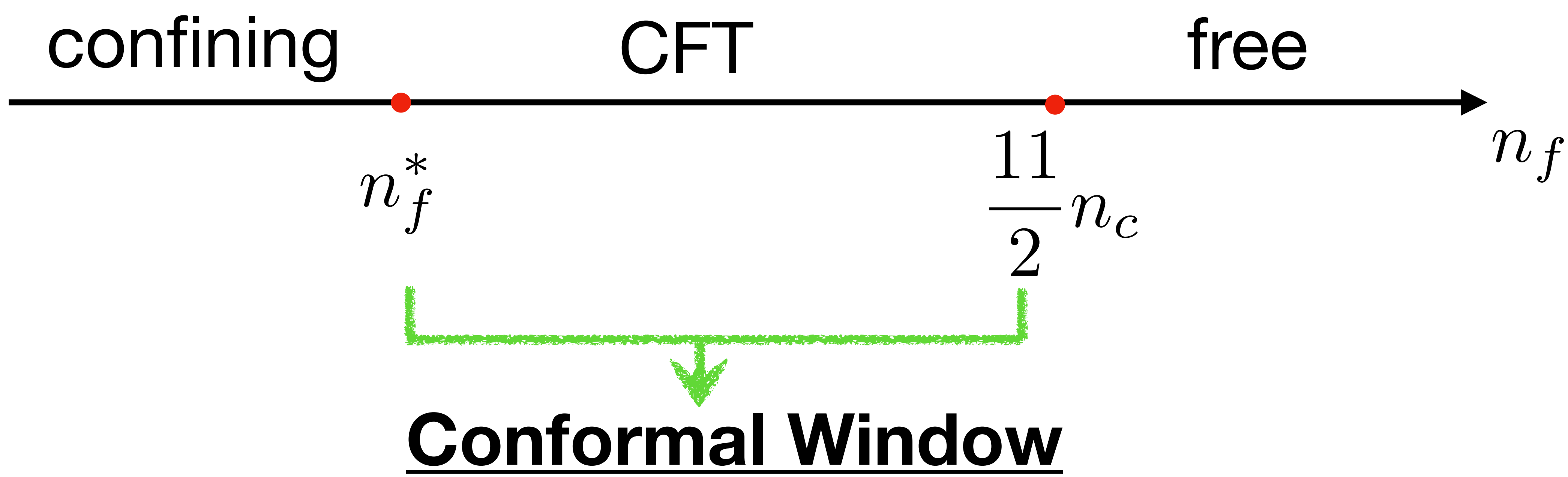
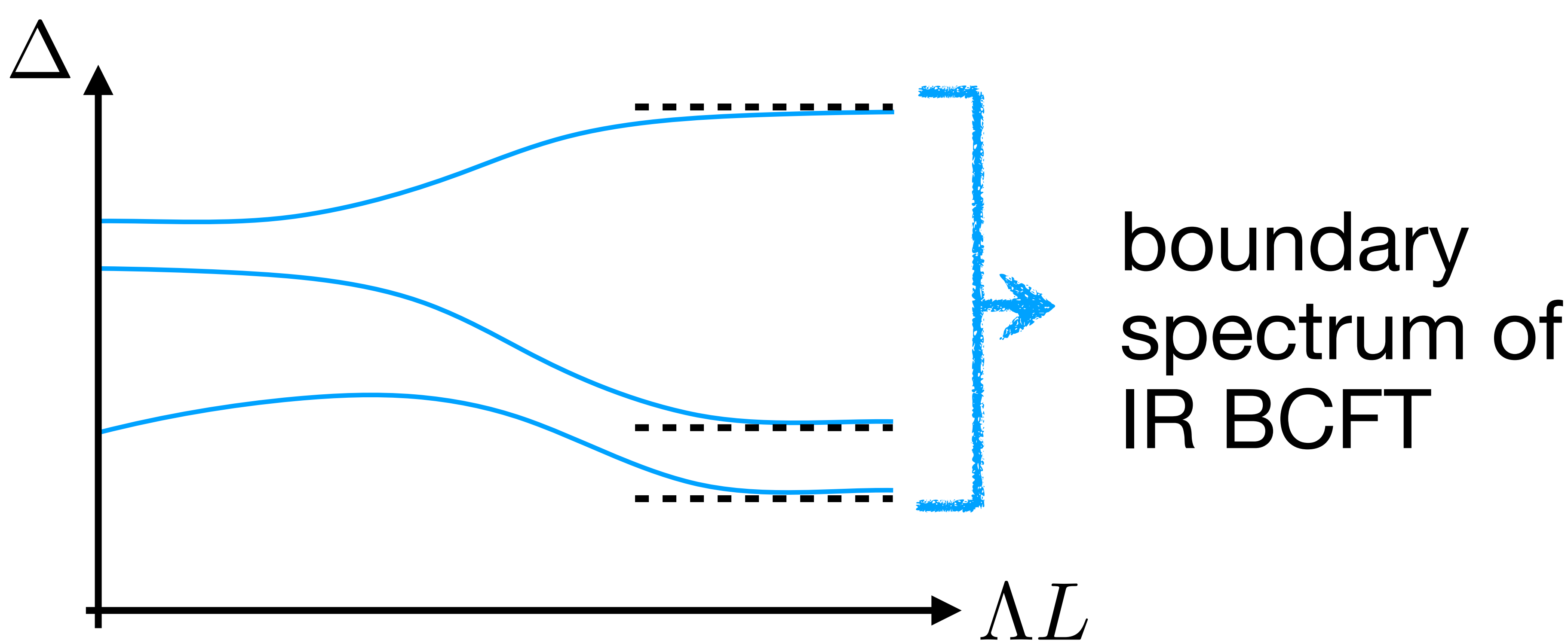


In AdS: interacting BCFT at large ΛL



The bulk conformal symmetry implies the existence of a displacement operator: protected dimension $\Delta = 4$

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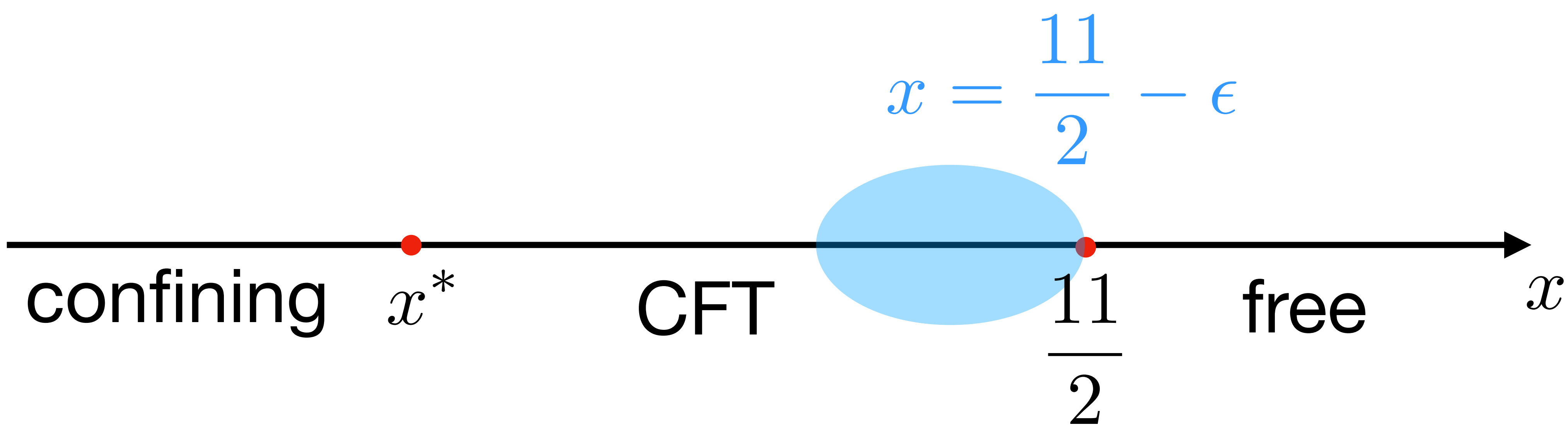
We can verify in the **Banks-Zaks expansion**



$$n_c \to \infty\,, \quad n_f \to \infty\,, \quad \text{with} \quad \lambda \equiv \frac{g^2 n_c}{16\pi^2} \quad x \equiv \frac{n_f}{n_c}$$

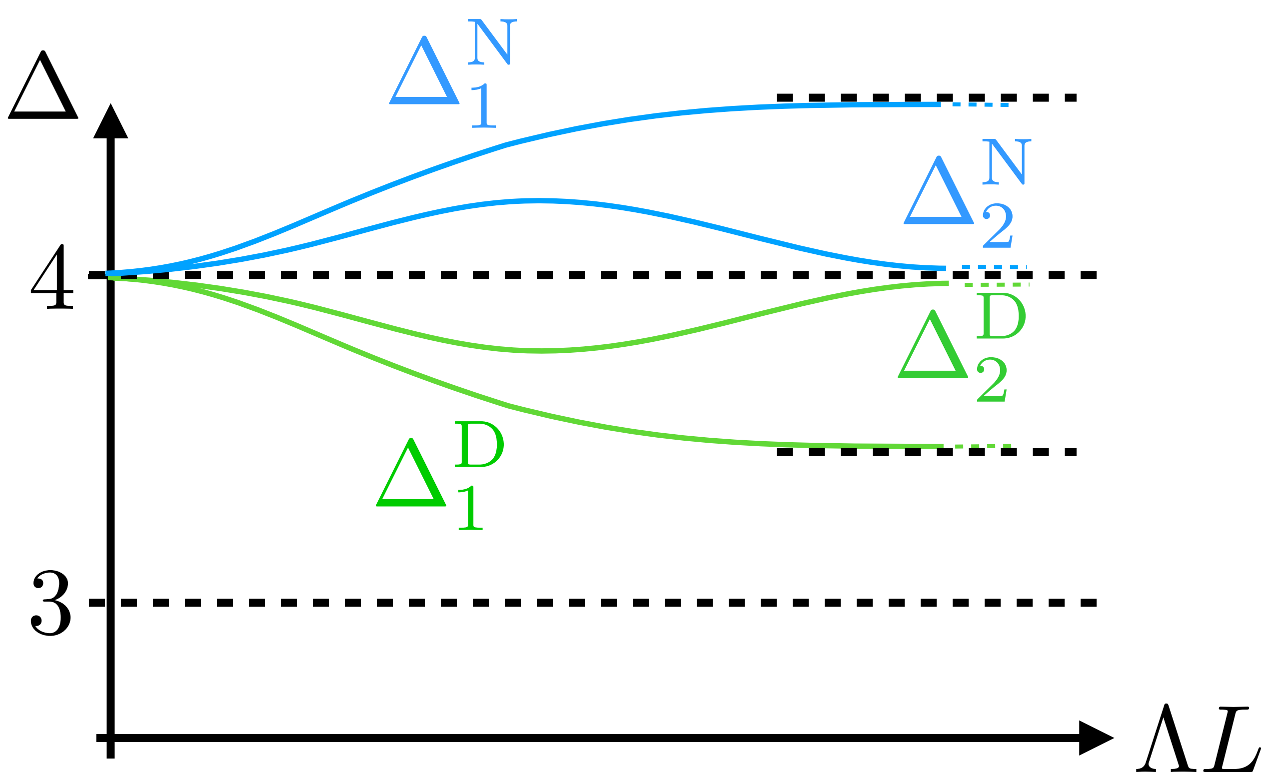
$$\beta^{\text{Ven}} = \frac{4\lambda^2}{3}\left(x - \frac{11}{2}\right) + \frac{2\lambda^3}{3}(13x - 34) + \mathcal{O}(\lambda^4)$$

Fixed point: $$\lambda_{\text{BZ}} = \frac{4}{75}\epsilon + \mathcal{O}(\epsilon^2)$$

Plugging in the previous formulas for the anomalous dimensions

$$\left(\Delta_1^{\mathrm{D/N}}\right)_{\mathrm{BZ}} = 4 \mp \frac{8}{15}\epsilon + \mathcal{O}(\epsilon^2)$$ → unprotected bdry scalar of BCFT

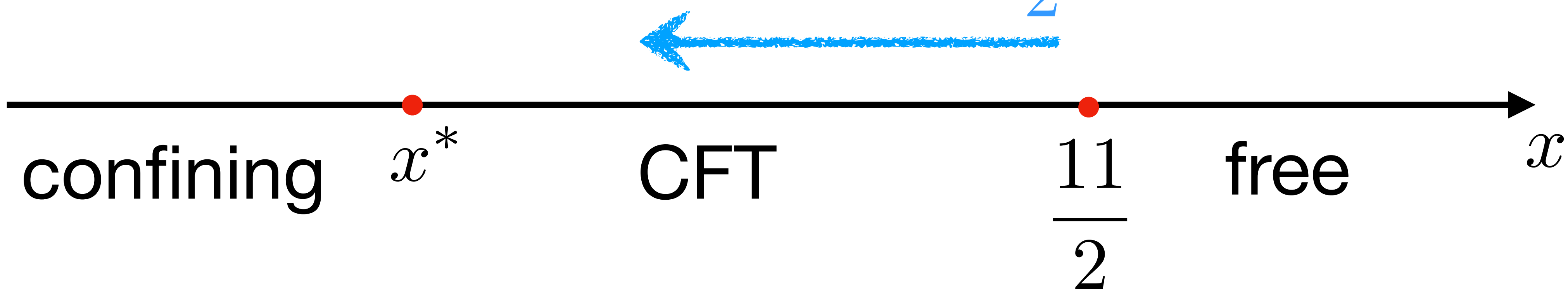
$$\left(\Delta_2^{\mathrm{D/N}}\right)_{\mathrm{BZ}} = 4 + \mathcal{O}(\epsilon^2)$$ → displacement ✓



Δ_{BZ} are the boundary dimensions at large ΛL

$$(\Delta_1^{\mathrm{D}})_{\mathrm{BZ}} = 4 - \frac{8}{15}\epsilon + \mathcal{O}(\epsilon^2)$$

$$x = \frac{11}{2} - \epsilon$$



Extrapolating the one-loop answer:

$$(\Delta_1^{\mathrm{D}})_{\mathrm{BZ}} = 3 \quad \text{for} \quad x = \bar{x} \approx 3.6$$

Similarly, for $n_c = 3$ the IR boundary scaling dimension crosses marginality for $n_f = \bar{n}_f \approx 11$

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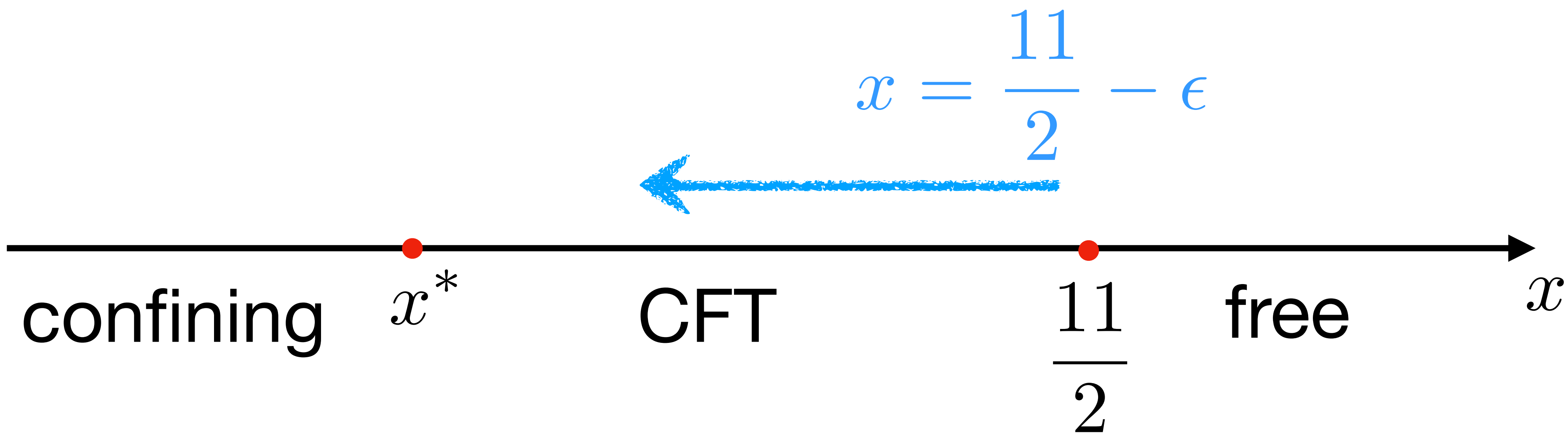

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Close to estimates of x^* and n_f^* in the literature!

[see DP, Serone (2020) and references therein]

End of conformal window **detected from the boundary**?

Possible connection: continuity of boundary data at large ΛL as a function of x

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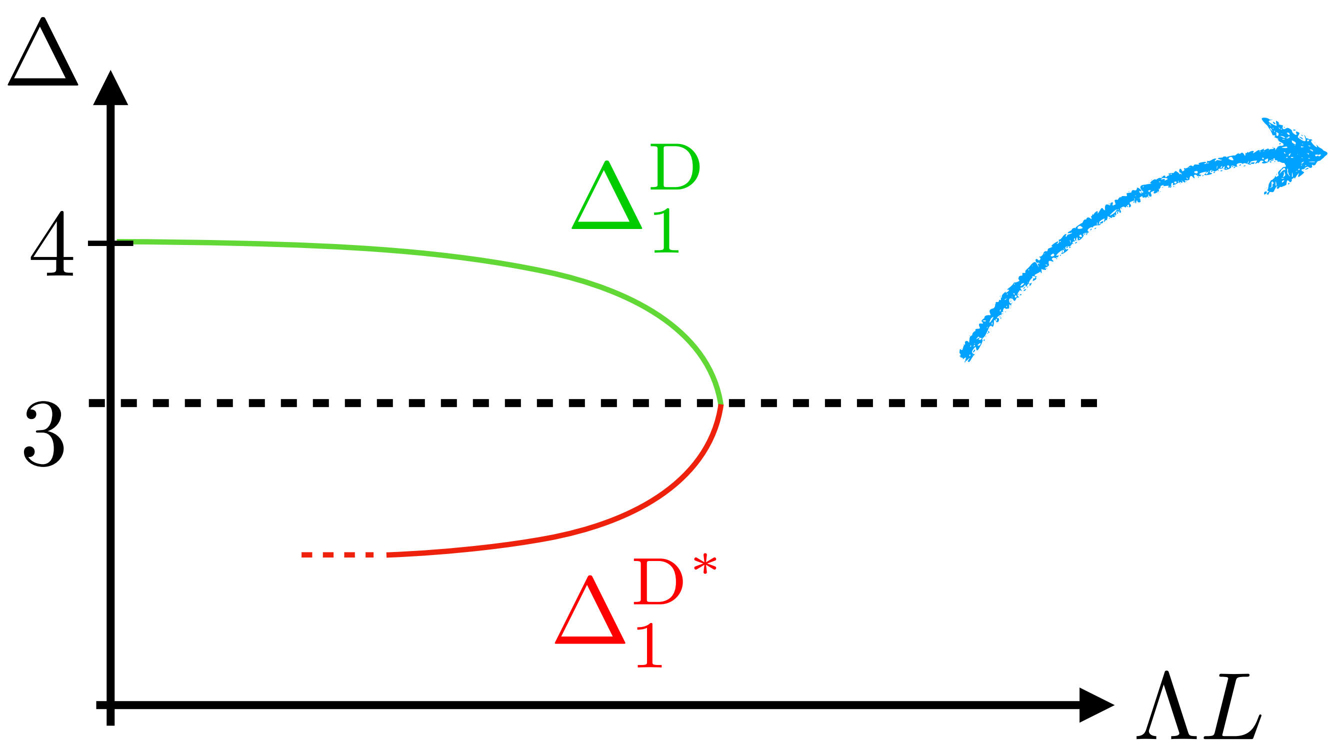
As soon as $x < x^*$: confining phase



At large ΛL the D bc only exists as complex CFT

For this to remain true as we transition to $x > x^*$:

$$x^* \leq \bar{x}$$

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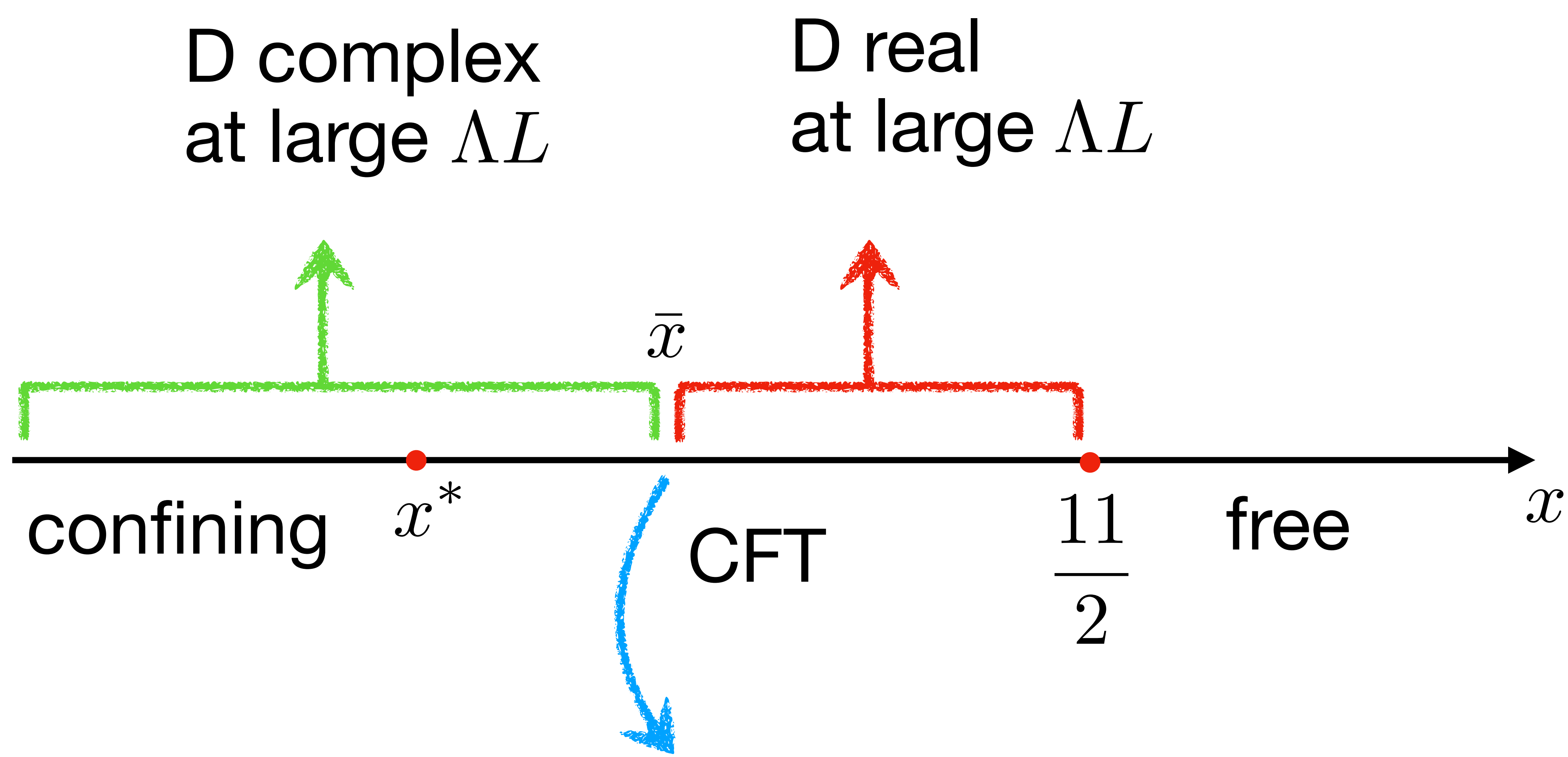
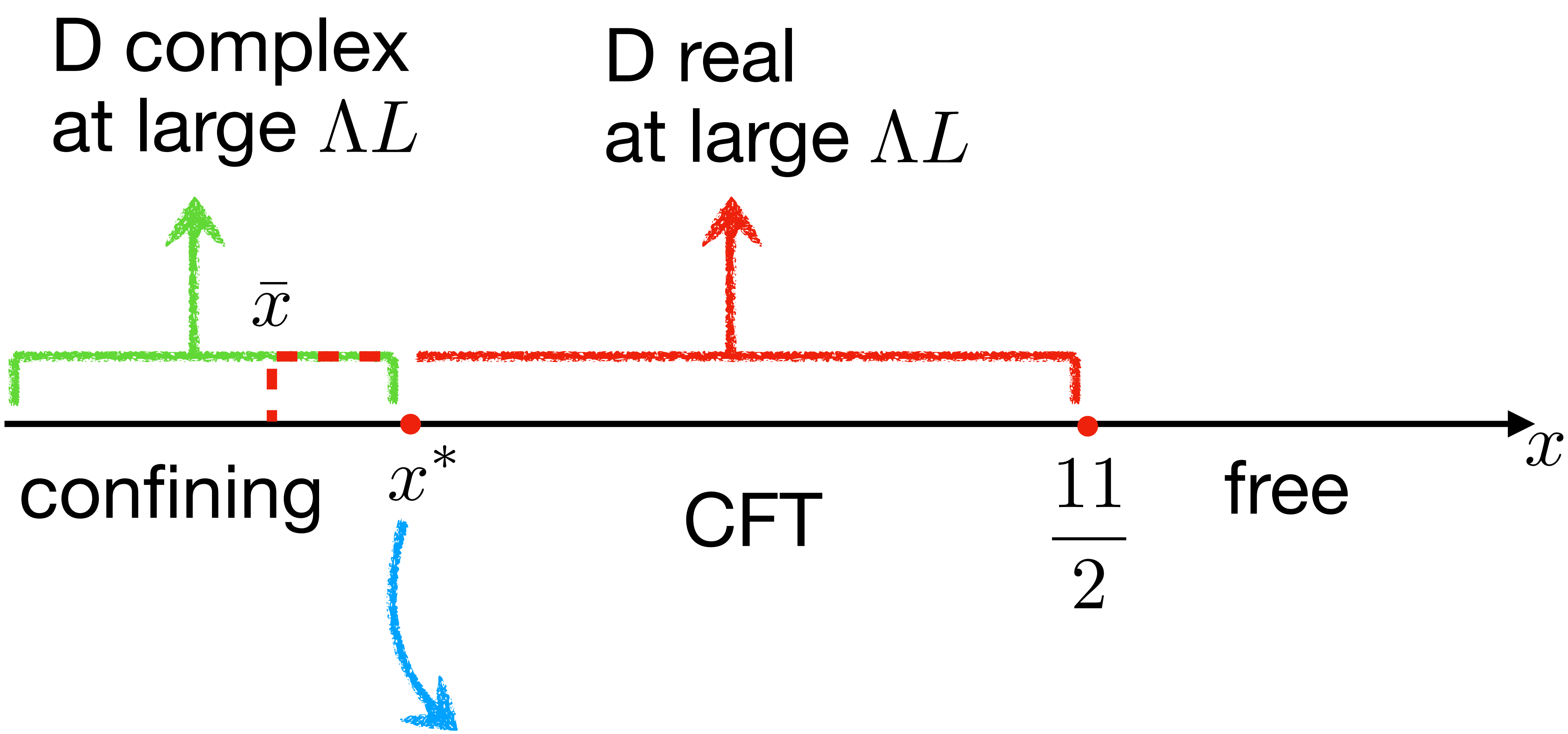


continuous transition at $\left(\Delta_1^{\mathrm{D}}\right)_{\mathrm{BZ}} = 3$

$x^* > \bar{x}$:



dicontinuous transition from real to complex at large ΛL ✗

Chiral Symmetry Breaking

AdS version of SSB: symmetry broken by the bc

Even at weak coupling, bc cannot preserve full

$$SU(n_f)_L \times SU(n_f)_R \times U(1)_B$$

[Rattazzi, Redi (2009)]

Can be seen as a consequence of

't Hooft anomaly $\simeq$ no bc preserve the symmetry

Boundary conditions at $z = \epsilon$

$$B_I{}^J \gamma_z \Psi_J(\epsilon, \vec{x}) = \Psi_I(\epsilon, \vec{x})$$
$$\bar{\Psi}^I(\epsilon, \vec{x}) \gamma_z B_I{}^J = -\bar{\Psi}^J(\epsilon, \vec{x})$$

$(B_I{}^J B_J{}^K = \delta_I{}^K)$

with boundary action

$$S_\partial = \frac{1}{2} \int_{z=\epsilon} d^d\vec{x}\, \epsilon^{-d} \bar{\Psi}^I B_I{}^J \Psi_J$$

$B_I{}^J$: flavor-matrix parameter of the bc

Near-boundary expansion for massless fermions

$$\Psi_I(z, \vec{x}) \underset{z \to 0}{\sim} z^{\frac{d}{2}} \widehat{\Psi}_I(\vec{x})$$
$$\bar{\Psi}^I(z, \vec{x}) \underset{z \to 0}{\sim} z^{\frac{d}{2}} \widehat{\bar{\Psi}}^I(\vec{x})$$

with

$$B_I{}^J \gamma_z \widehat{\Psi}_J(\vec{x}) = \widehat{\Psi}_I(\vec{x})$$
$$\widehat{\bar{\Psi}}^I(\vec{x}) \gamma_z B_I{}^J = -\widehat{\bar{\Psi}}^J(\vec{x})$$

Spontaneous breaking of $SU(n_f)_L \times SU(n_f)_R$ to the subgroup commuting with ${B_I}^J$

$$\langle \bar{\Psi}_\alpha^J \Psi_I^\alpha \rangle = -{B_I}^J \frac{n_c}{4\pi^2 L^3} (1 + \# g^2 + \# g^4 + \dots)$$

aligned one-point function already at tree-level

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aligned one-point function already at tree-level

AdS analogue of pions: "tilt" operators

$$J^A_z \underset{z\to 0}{\sim} z^{d-1}\widehat{\bar{\Psi}}^I [B, T^A]_I{}^J \widehat{\Psi}_J$$

$$J^{5\,A}_z \underset{z\to 0}{\sim} z^{d-1}\widehat{\bar{\Psi}}^I \{B, T^A\}_I{}^J \gamma^5 \widehat{\Psi}_J$$

marginal operators that move between vacua

Many possible breaking patterns of $SU(n_f)_L \times SU(n_f)_R$ for small AdS, given by choices of ${B_I}^J$

However only one pattern survives in flat space

$${B_I}^J = {\delta_I}^J$$ [Vafa, Witten (1984)]

Many possible breaking patterns of $SU(n_f)_L \times SU(n_f)_R$ for small AdS, given by choices of ${B_I}^J$

However only one pattern survives in flat space

$${B_I}^J = {\delta_I}^J \qquad \text{[Vafa, Witten (1984)]}$$

N for gauge fields, no confinement/deconfinement discontinuity

other fermionic bc should all disappear

Questions for the future

- Connection between perturbative AdS condensate and non-perturbative flat space one?

- Disappearance of “wrong” fermionic bc’s?

- Pion amplitudes from tilt correlators?

- Axial ABJ anomaly?

- Conformal window in 3d QCD/QED?

Thank You

Broken Conformal Ward Identity

UV: direct sum of decoupled CFTs $T^i_{\mu\nu}$

Displacement operators $T^i_{zz} \underset{z\to 0}{\sim} C_{T^i\mathcal{D}_i}\, z^{d-1}\mathcal{D}_i$

$$\langle T^i_{AB}(X)\mathcal{D}_j(P)\rangle = \delta_{ij} C_{T^i\mathcal{D}_i} \frac{H_{AB}(X,P)}{(-2P\cdot X)^{d+1}}$$

$$H_{AB} = \frac{G_{AC}(X)G_{BD}(X)P^C P^D}{(-2P\cdot X)^2} - \frac{G_{AB}(X)}{4(d+1)}$$

$$G_{AB}(X) = \eta_{AB} + X_A X_B$$

Fixed by conservation and tracelessness

Deformation that couples the CFT's

$$S = \sum_{i=1}^{n} S_{\mathrm{CFT}_i} + S_{\mathrm{int}}\,, \qquad S_{\mathrm{int}} = \sum_{p=1}^{m} \lambda_p \int dx\, O_p(x)$$

Mixing $\quad \mathcal{D}_i = Q(\boldsymbol{\lambda})_{ij} \overline{\mathcal{D}}_j$

$$\langle \overline{\mathcal{D}}_i(\vec{x}_1) \overline{\mathcal{D}}_j(\vec{x}_2) \rangle = \frac{\delta_{ij}}{(\vec{x}_{12}^{\,2})^{\Delta_i(\boldsymbol{\lambda})}}\,, \qquad \Delta_i(\boldsymbol{\lambda}) = d + 1 + \gamma_i(\boldsymbol{\lambda})$$

Removing traceless constraint:

$$\langle T_{AB}(X) \overline{\mathcal{D}}_i(P) \rangle$$

$$= \frac{C_{T\overline{\mathcal{D}}_i}(\boldsymbol{\lambda})}{(-2P \cdot X)^{\Delta_i(\boldsymbol{\lambda})}} \left(H_{AB}(X,P) - \frac{(\Delta_i(\boldsymbol{\lambda}) - d - 1)d}{4(d+1)\Delta_i(\boldsymbol{\lambda})} G_{AB}(X) \right)$$

Broken conformal symmetry in the bulk

$$G^{AB}T_{AB}(X) = \sum_p \beta_{\lambda_p}(\boldsymbol{\lambda}) O_p(X)$$

Plugging this operator equation in $\langle T\mathcal{D}\rangle$

$$-\frac{d(\Delta_i(\boldsymbol{\lambda}) - d - 1)}{4\Delta_i(\boldsymbol{\lambda})} C_{T\overline{\mathcal{D}}_i}(\boldsymbol{\lambda}) = \sum_{p=1}^{m} \beta_{\lambda_p}(\boldsymbol{\lambda}) C_{O_p\overline{\mathcal{D}}_i}(\boldsymbol{\lambda})$$

where $\langle O_p(X)\overline{\mathcal{D}}_i(P)\rangle = \dfrac{C_{O_p\overline{\mathcal{D}}_i}(\boldsymbol{\lambda})}{(-2P\cdot X)^{\Delta_i(\boldsymbol{\lambda})}}$

one coupling, leading order: $-\dfrac{d}{4(d+1)}\gamma_i C_{T\overline{\mathcal{D}}_i} = \beta_0 C_{O\overline{\mathcal{D}}_i}$

We do not know a priori the basis $\overline{\mathcal{D}}_i$

Arbitrary basis:

$$\langle \mathcal{D}_i(\vec{x}_1)\mathcal{D}_j(\vec{x}_2)\rangle$$

$$= \frac{1}{(\vec{x}_{12}^{\,2})^{d+1}}\Big(\delta_{ij} + \lambda M_{ij} - \lambda\Gamma_{ij}\log(\vec{x}_{12}^{\,2}) + \mathcal{O}(\lambda^2)\Big)$$

eigenvalues γ_i

$$\Rightarrow \quad -\frac{d}{4(d+1)}\Gamma_{ij}C_{T\mathcal{D}_j} = \beta_0 C_{O\mathcal{D}_i}$$

tree level

In QCD: 2 constraints for 3 entries. However one entry coincides with pure YM.