From Multi-Exchange Signals to Model Building in Cosmological Correlators

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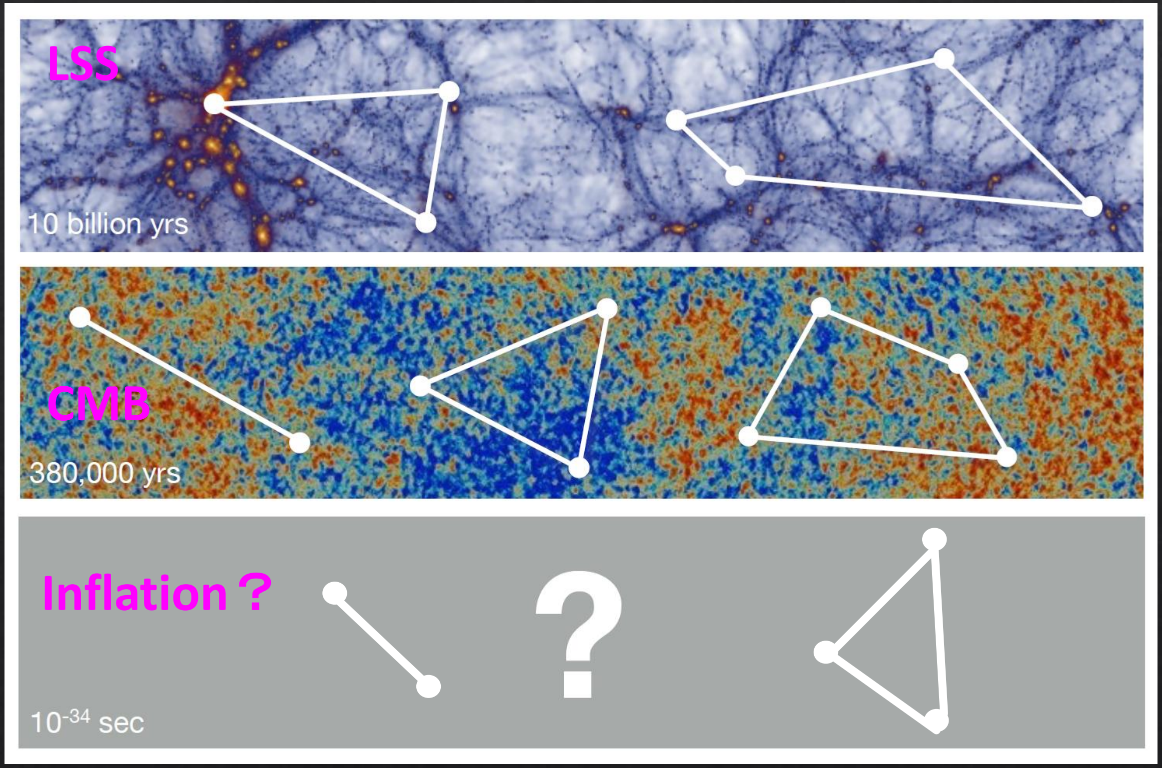
Progress of Theoretical Bootstrap
November 21, 2025

Cosmological correlators

$\delta\rho$

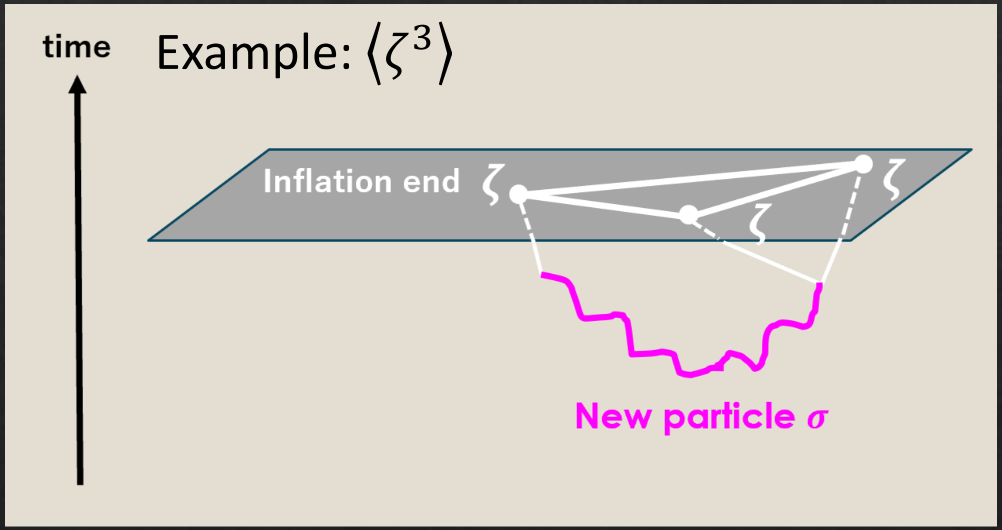
δT

ζ



Baumann, Lectures on The Cosmological Bootstrap

Correlators with massive particles



Cosmic expansion creates σ with mass $\sim H$

Correlators with massive particles

- "Cosmological Collider"

$$\langle \zeta^3 \rangle \big|_{k_L \ll k_S} \sim e^{-\pi\mu} \left(\frac{k_L}{k_s}\right)^{\frac{3}{2}} \cos\left[\mu \log\left(\frac{k_L}{k_s}\right) + \delta(\mu)\right] P_s(\cos\theta)$$

Boltzmann suppression

dilution

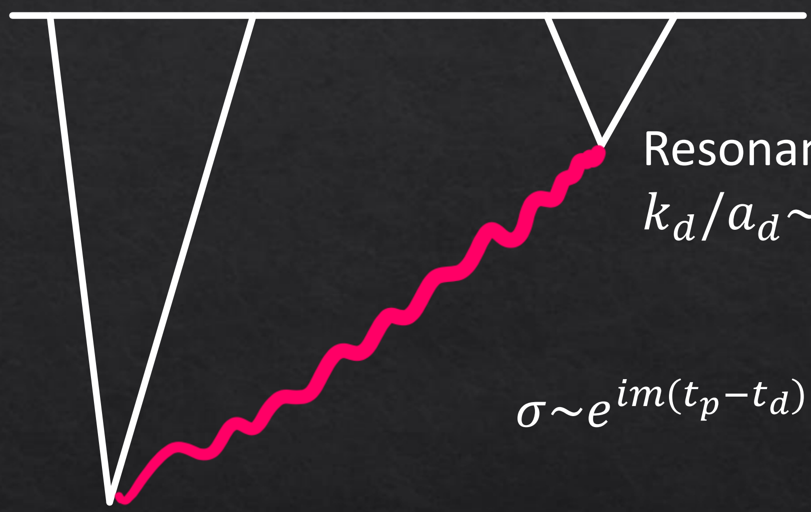
$\mu \sim m_\sigma / H$: mass

spin

Chen, Wang, '10
Baumann, Green, '12
Noumi, Yamaguchi, Yokoyama, '13
Arkani-Hamed, Maldacena, '15

- $m_\sigma \sim H_{\rm inf} \sim 10^{14}$(GeV) $\ggg\ggg 10^{4,5}$(GeV) ~ collider on earth

Why oscillation??



Resonant decay

$k_d/a_d \sim m$

$$\sigma \sim e^{im(t_p - t_d)} \sim \left(\frac{a_d}{a_p}\right)^{im/H} \sim \left(\frac{k_d}{k_p}\right)^{im/H}$$

$a = e^{Ht}$

Resonant production

$k_p/a_p \sim m$

Tong, Wang, Zhu, 2112.03448

Hubble mass??

- SUSY
  - Broken by cosmic expansion
  - Soft mass $\sim H_{\mathrm{inf}}$
- Non-minimal coupling
  - $\xi\sigma^2 R \sim 12\xi H_{\mathrm{inf}}^2 \sigma^2$

Contents

- Phenomenology (model building aspects)

  SA, A. Ghoshal and A. Strumia, 2408.07069

- Structure of correlators (multiple exchange process)

  SA, L. Pinol, F. Sano, M. Yamaguchi, Y. Zhu, 2404.09547

Phenomenology (model building aspects)

Many important works so far

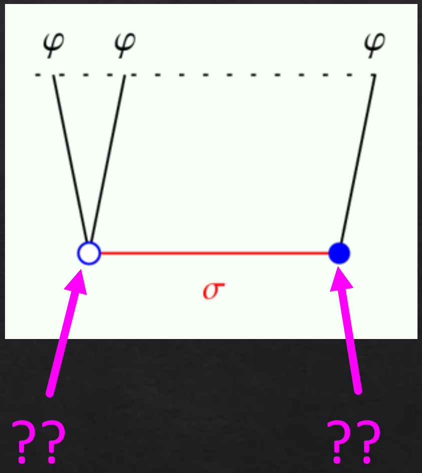
- SM background (Chen, Wang, Xianyu, 1610.06597)
- Neutrino (Chen, Wang, Xianyu, 1805.02656)
- SUSY (Baumann, Green, 1109.0292)
- Higher dimensions (Kumar, Sundrum, 1811.11200)
- GUT (Maru, Okawa, 2206.06651)
- Massive graviton (Tong, ,Xianyu 2203.06349)
- …

Questions

- Conditions for large CC signal ?

$$\langle\zeta^3\rangle\big|_{k_L\ll k_s} \sim e^{-\pi\mu}\left(\frac{k_L}{k_s}\right)^{\frac{3}{2}}\cos\left[\mu\log\left(\frac{k_L}{k_s}\right)+\delta(\mu)\right]P_s(\cos\theta)$$

??



- Important diagram?

Setup

- Target & Assumption
  - $\langle\zeta^3\rangle$ with tree scalar σ exchange
  - Inflation scenario: $\zeta\sim\varphi$ (inflaton)
- Require
  - Consistent with n_s and r (with backreaction)
  - No fine-tuning of parameters and initial conditions
- General discussion -> specific models

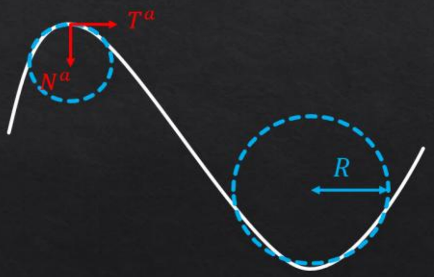
Generalities for perturbations

two scalars ϕ^a $(a = 1,2)$

$$S = \int d^4x \sqrt{|\det g|} \left[-\frac{\bar{M}_{\rm Pl}^2}{2} R + \frac{K_{ab}(\phi)}{2} (\partial_\mu \phi^a)(\partial^\mu \phi^b) - V(\phi) \right]$$

$\phi^a = \phi_0^a$ (BG) $+ \delta\phi^a$ (fluctuations)

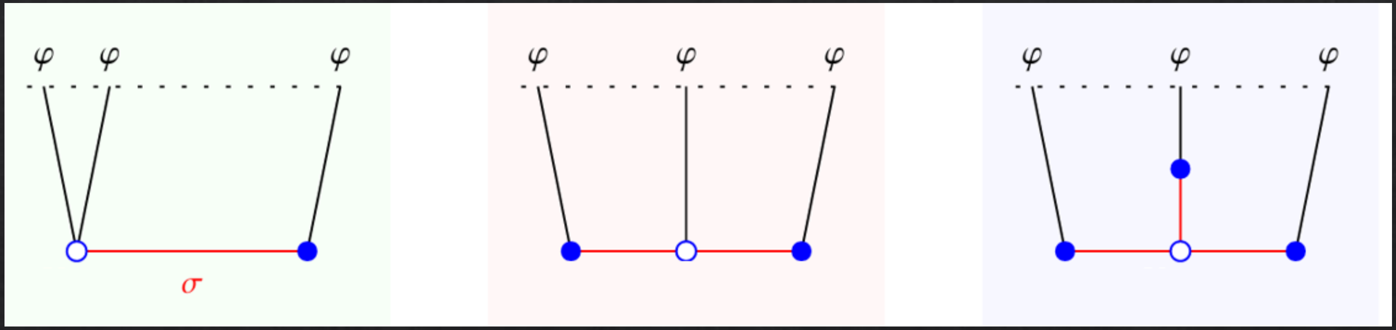
$\delta\phi^a = \varphi T^a + \sigma N^a$ $(\varphi \sim \zeta)$



Action for perturbations $\varphi(\zeta)$ and σ

Three dominant diagram

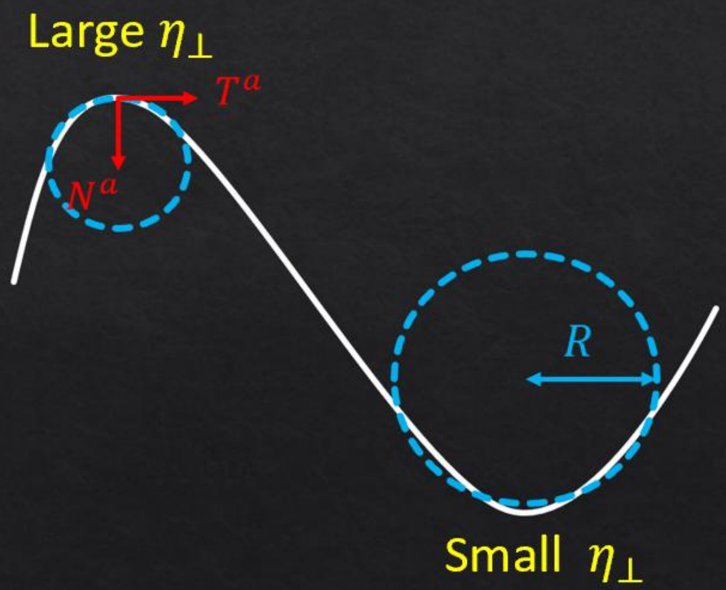
- require $m_\sigma \sim H$ w/o fine-tuning



CC signal $\propto \eta_\perp^2$ $\quad\propto \eta_\perp^2$ $\quad\propto \eta_\perp^3 \times g/H\sqrt{P_\zeta}$

Turn

- Turn rate: $\eta_\perp = \frac{d\phi}{d\mathcal{N}} \times \kappa = \sqrt{2\epsilon} M_{\mathrm{PL}} \times \kappa$
  (similar to $\omega = v/R$ in Newtonian mechanics)



Conditions for large signal

- $m_\sigma \sim H$
- Large turn rate $\eta_\perp = \sqrt{2\epsilon} M_{\mathrm{PL}} \kappa \Rightarrow$ Large $\kappa \Rightarrow$ sub-Planckian scale
- Large cubic σ^3 (triple exchange)

Check by concrete models

- ϕ (Higgs or dilaton) + R^2 (Scalaron)

  SA, A. Ghoshal and A. Strumia, 2408.07069

- Multifield α-attractor

  SA, Roest, Werth, in progress

- Axion-Saxion (Monodromy)

  SA, Otsuka, Yanagita, 2509.06739

Define $f_{\rm NL}$ in squeezed limit $k_3 \equiv k_L \ll k_{1,2} \equiv k_S$

$$S \simeq \frac{9}{10}\left[f_{\rm NL}(\nu)\left(\frac{k_L}{k_S}\right)^{1/2-\nu} + f_{\rm NL}(-\nu)\left(\frac{k_L}{k_S}\right)^{1/2+\nu}\right] \quad \text{where} \quad \nu = \sqrt{\frac{9}{4} - \frac{m_\sigma^2}{H^2}}.$$

Scalar ϕ + R^2

SA, A. Ghoshal and A. Strumia, 2408.07069

- J-frame action

$$S = \int d^4x \sqrt{|\det g|} \left[-\frac{1}{2} f(\phi) R + \frac{R^2}{6 f_0^2} + \sum_\phi \frac{(D_\mu \phi)(D^\mu \phi)}{2} - V_{\rm J}(\phi) \right]$$

  - ϕ: some scalars (specified later)
  - R^2 gives an additional scalar z (scalaron)

- Include important models

  - ϕ = Dilaton, $f = \xi \phi^2$, $V_J = \lambda(\phi)\phi^4/4$

    Dynamical generation of Planck scale Kannike et al, '15,…

  - ϕ = Higgs, $f = M_{\rm PL}^2 + \xi \phi^2$, $V_J = \lambda \phi^4/4$

    Unitarizing Higgs inflation Salvio, Mazumdar '15, Ema '17, …

Scalar ϕ + R^2

SA, A. Ghoshal and A. Strumia, 2408.07069

- J-frame action

$$S = \int d^4x \sqrt{|\det g|}\left[-\frac{1}{2}f(\phi)R + \frac{R^2}{6f_0^2} + \sum_\phi \frac{(D_\mu\phi)(D^\mu\phi)}{2} - V_{\rm J}(\phi)\right]$$

- ϕ: some scalars (specified later)
- R^2 gives an additional scalar z (scalaron)

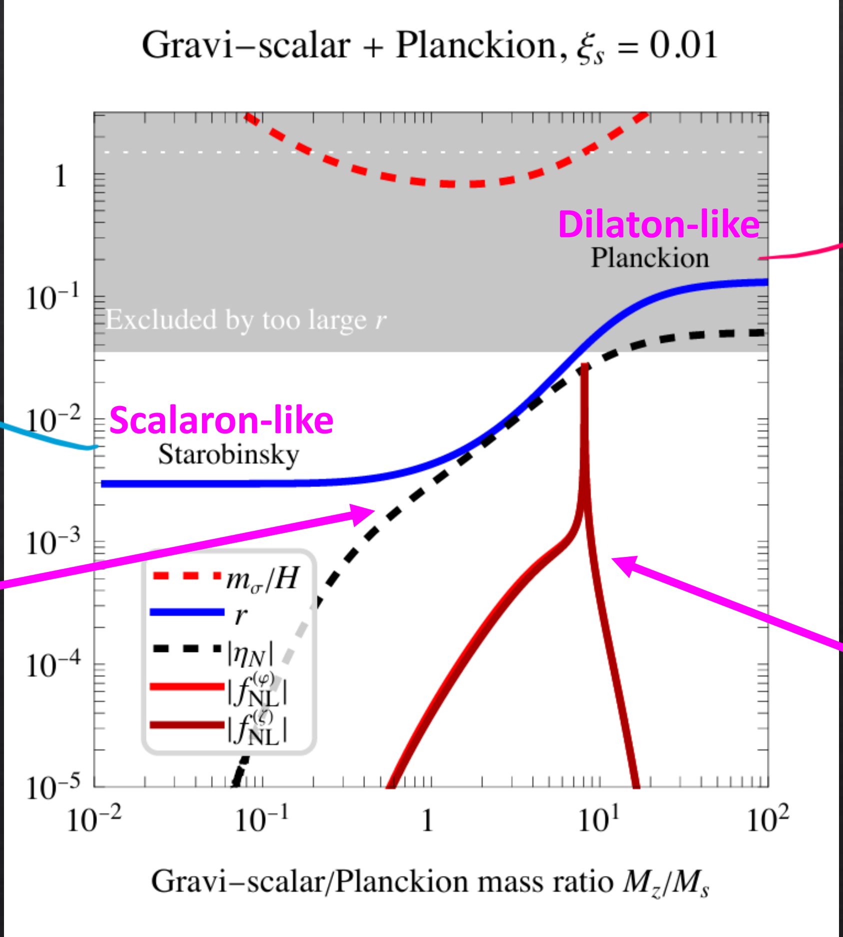
- E-frame action

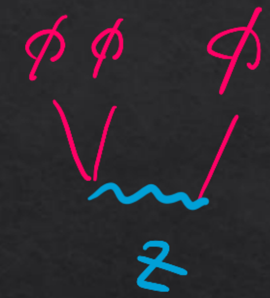
$$S = \int d^4x \sqrt{|\det g|}\left[-\frac{\bar{M}_{\rm Pl}^2}{2}R + \frac{6\bar{M}_{\rm Pl}^2}{z^2}\frac{(\partial_\mu z)^2 + \sum_\phi (D_\mu\phi)^2}{2} - V(\phi, z)\right]$$

$$V = \left(\frac{6\bar{M}_{\rm Pl}^2}{z^2}\right)^2\left[V_{\rm J}(\phi) + \frac{3}{8}f_0^2\left(f + \xi_z z^2\right)^2\right].$$

$$\xi_z = -1/6,$$

CC signal from dilaton (Higgs)+R^2





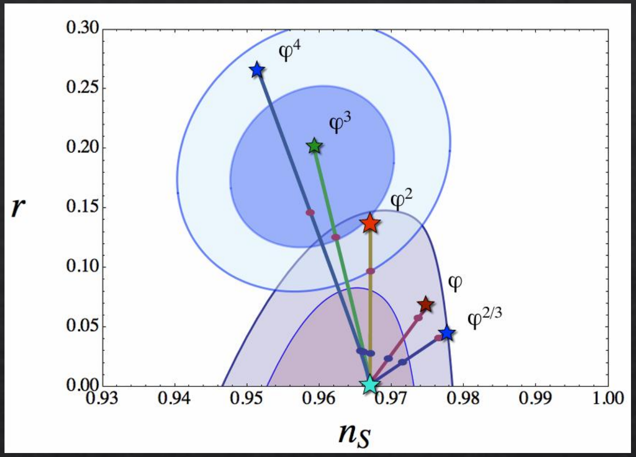
$\kappa \propto 1/M_{\rm Pl} \Rightarrow \eta_\perp \propto \sqrt{\epsilon}$

CC signal$\sim \mathcal{O}(\epsilon)$

Multifield α-attractor

- α-attractor model Kallosh, Linde, Roest, 1311.0472, 1405.3646

$$-\frac{\alpha(\partial\phi)^2}{(1-\phi^2/6)^2} - V(\phi)$$



- Two field generalization of α-attractor (α, $R_m \equiv m_\chi^2/m_\phi^2$)

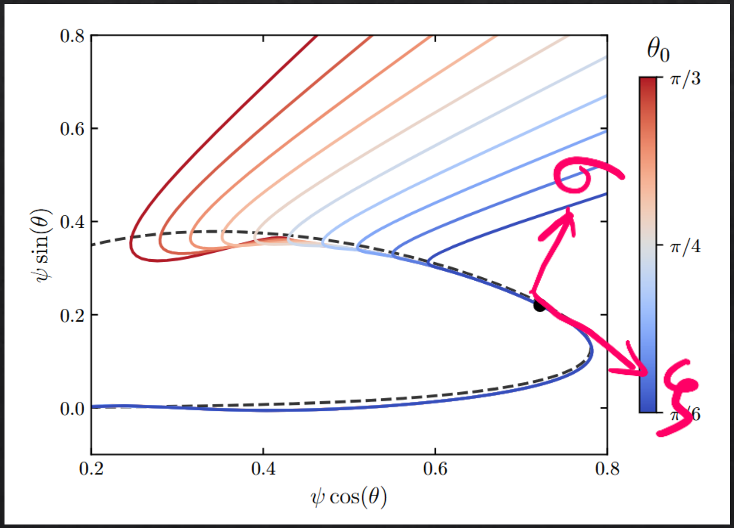
$$G_{IJ} = \frac{6\alpha}{(1-\phi^2-\chi^2)^2}\delta_{IJ}\,.$$

$$V(\phi,\chi) = \frac{\alpha}{2}\left(m_\phi^2\phi^2 + m_\chi^2\chi^2\right)\,,$$

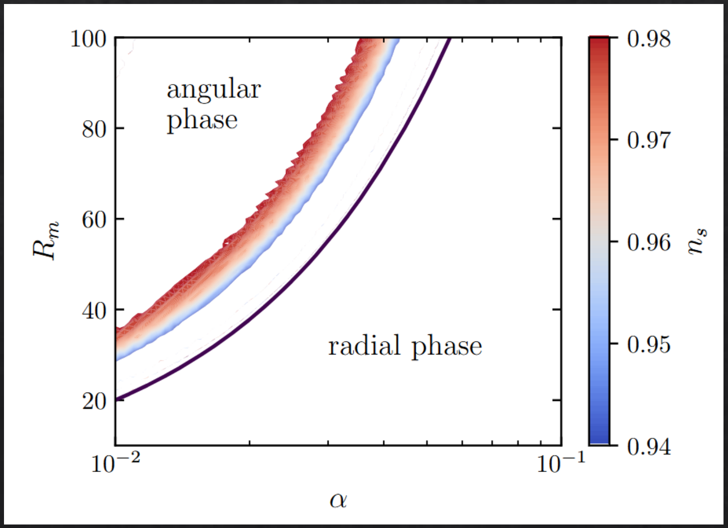
Multifield α-attractor

- $\phi = r\cos\theta, \chi = r\sin\theta$
  - Angular (θ) inflation for $\alpha \ll 1$ Christodoulidis, Roest, Sfakianakis, 1803.09841

Trajectory



Parameter space



CC signal

- Large turn rate: $\eta_\perp \sim 1$ for $\alpha \sim \epsilon \ll 1$

$$\eta_\perp^2 \simeq \frac{4\epsilon}{3\alpha} \quad \Rightarrow \kappa \propto 1/(\sqrt{\alpha} M_{\rm Pl})$$

- σ-mass: $$m_{\sigma,\rm eff}^2 = \eta_\perp^2 H^2$$ $m_{\sigma,\rm eff} \sim H$ for $\eta_\perp \sim 1$

- Large CC signal $\sim \epsilon/\alpha$ by a new scale $\sim \sqrt{\alpha} M_{\rm Pl}$
- However, non-perturbative in interesting range $\Rightarrow$ numerical method

SA, Roest, Werth, in progress

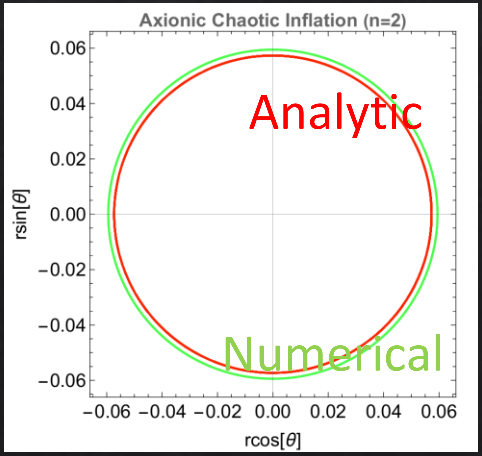
Axion-Saxion (Monodromy)

SA, Otsuka, Yanagita, 2509.06739

- Axion (θ), Saxion (r)
- Flat target space $G_{IJ} = \mathrm{diag}\,(1, r^2)$
- $$V(r,\theta) = \alpha\theta^n + \frac{M^2}{2}(r-R)^2\,,$$

➢ Turn rate: $\eta_\perp^2 \simeq \frac{2\epsilon M_{\rm Pl}^2}{r^2}$ $\Rightarrow \kappa \propto 1/r$

➢ Large CC signal$\sim\epsilon \mathrm{M_{pl}}^2/r^2$ by a new scale $\sim r$

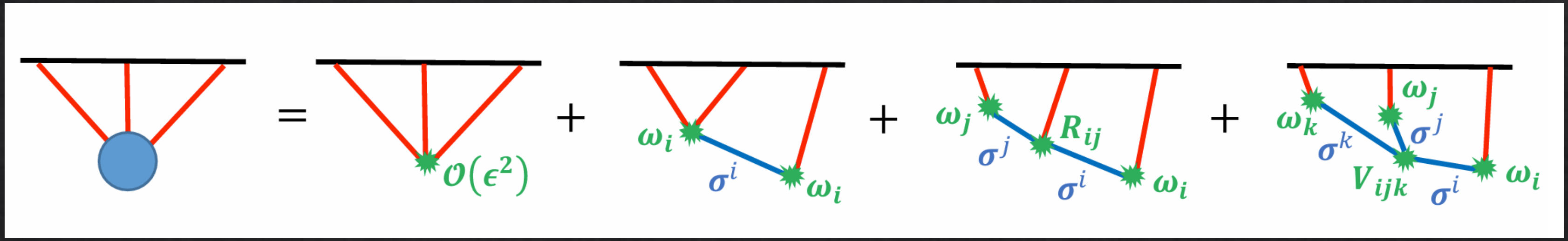


Summary

$\phi + R^2$	Multi α-attractor	Monodromy
$\kappa \sim \frac{1}{\mathrm{M_{pl}}}$	$\kappa \sim \frac{1}{\sqrt{\alpha}\mathrm{M_{pl}}}$	$\kappa \sim \frac{1}{r}$
CC signal$\sim\epsilon$	CC signal$\sim\epsilon/\alpha$	CC signal$\sim\epsilon\frac{\mathrm{M_{pl}^2}}{r^2}$

Many other fun models to try!

Multiple Exchange



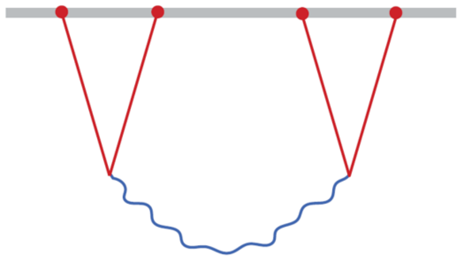
- Multiple-exchange diagrams can become equally important.
- Need the full analytic structure to get the net size.
- Rich phenomenology (specific CC signal)

Challenges

- **Challenges**
  - Mode functions of massive particles
  - Nested time integral

- Structure of correlators
  e.g., $\langle \zeta^4 \rangle$ with tree level single exchange

$F =$



$$\sim -g^2 \int \frac{\mathrm{d}\eta}{\eta^2}\frac{\mathrm{d}\eta'}{\eta'^2}\, e^{i(k_1+k_2)\eta} e^{i(k_3+k_4)\eta'}\, G(|\mathbf{k}_1+\mathbf{k}_2|, \eta, \eta')\,.$$

↑ External line for inflaton

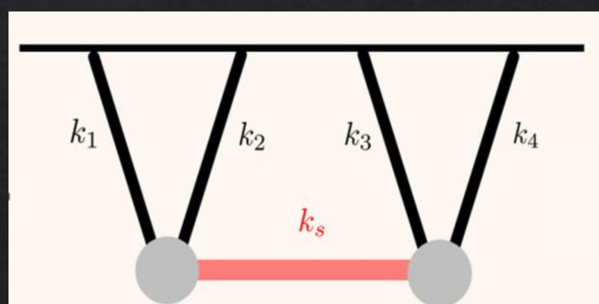
↑ Propagator of σ (complicated function)

Baumann, Lectures on The Cosmological Bootstrap

Differential equation technique

Arkani-Hamed, Maldacena, 1503.08043
Arkani-Hamed, Baumann, Lee, Pimentel, 1811.00024

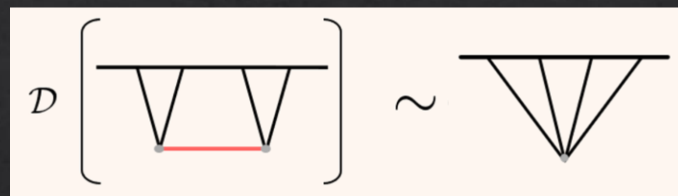
- Correlators satisfy differential equations w.r.t momentum
  $u \equiv k_s/(k_1+k_2)$ & $v \equiv k_s/(k_3+k_4)$



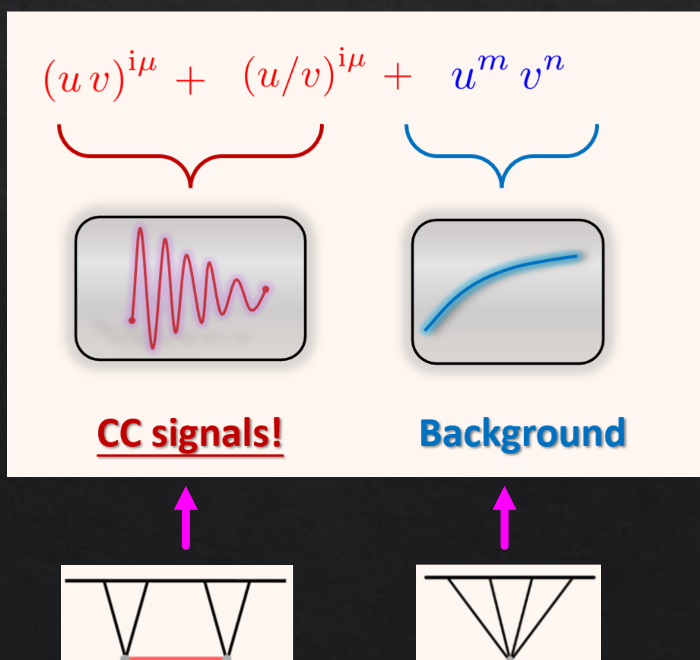
$$\left(\Delta_u + M^2\right) F = uv/(u+v)$$
$$\left(\Delta_v + M^2\right) F = uv/(u+v)$$

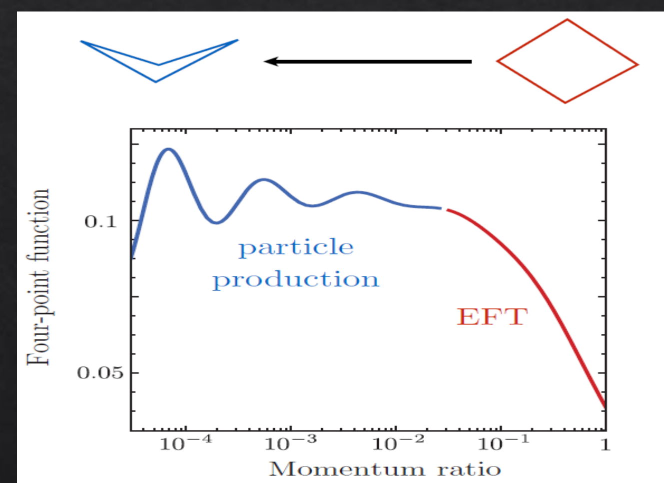
$$\Delta_u \equiv u^2(1-u^2)\partial_u^2 - 2u^3\partial_u \,,$$

Solutions



+ B.C.

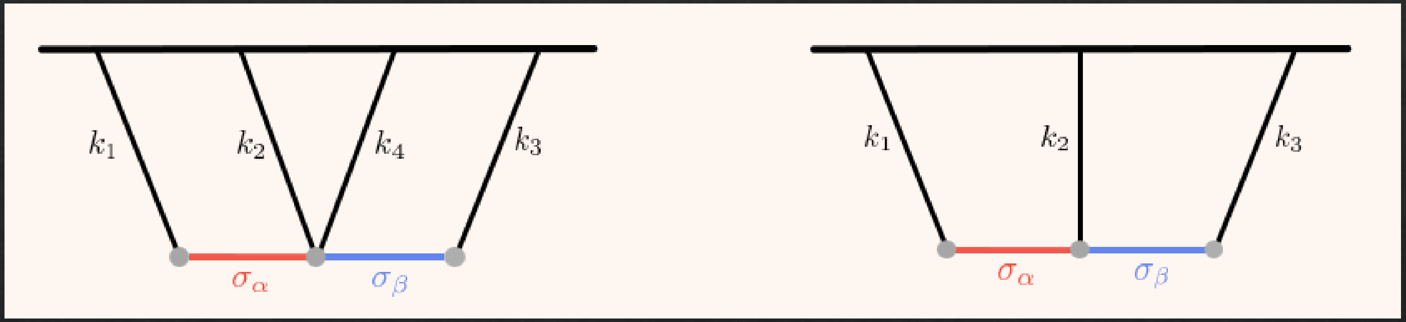




Baumann, Lectures on The Cosmological Bootstrap

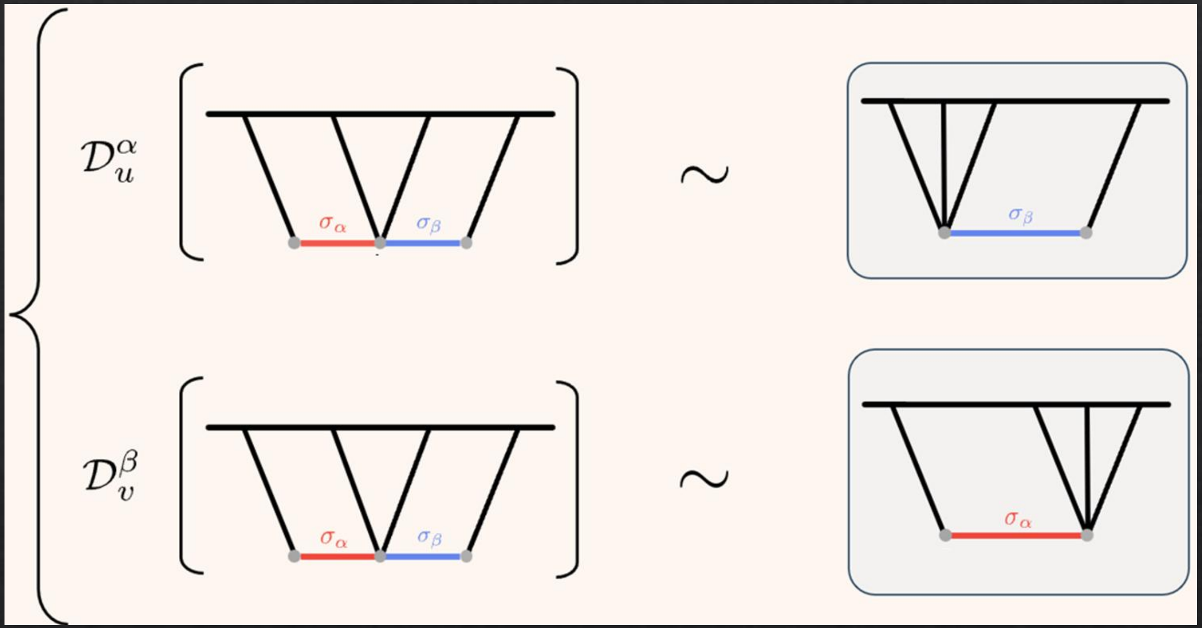
Double exchange

SA, L. Pinol, F. Sano, M. Yamaguchi, Y. Zhu, 2404.09547



$$F = F(u, v) \text{ with } u \equiv \frac{k_1}{k_2+k_4},\ v \equiv \frac{k_3}{k_2+k_4}$$

Differential equations



(u-v mixed) differential operators

$$\mathcal{D}_u^\alpha \equiv u^2\partial_u^2 - 2u\partial_u + \mu_\alpha^2 + \frac{9}{4} - u^2\left(u\partial_u + v\partial_v\right)\left(u\partial_u + v\partial_v - 1\right)$$

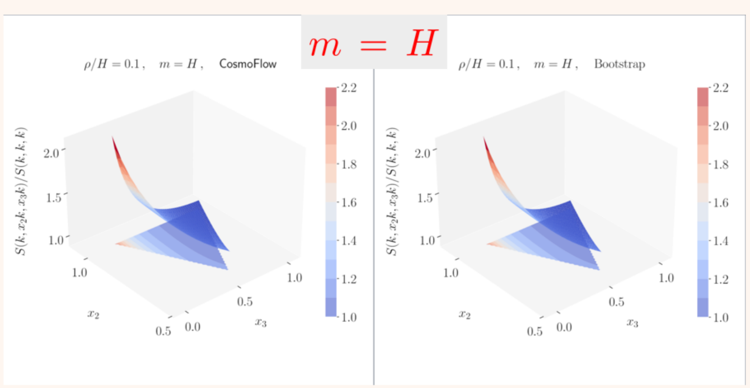
$$\mathcal{D}_v^\beta = \mathcal{D}_u^\alpha(u \leftrightarrow v, \alpha \leftrightarrow \beta)$$

Result

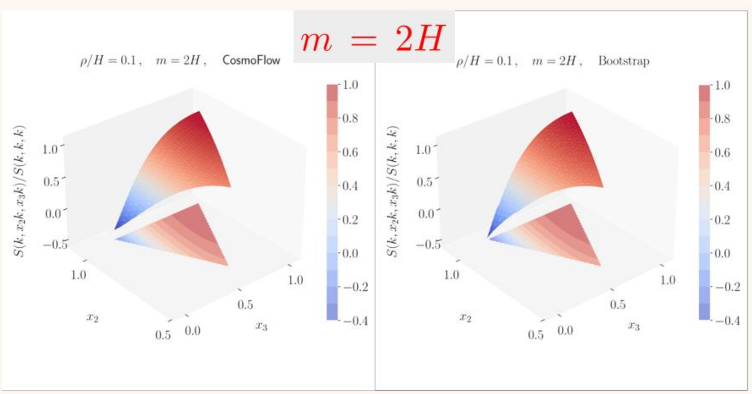
- Analytic solutions (based on Appell series $F_4(u^2, v^2)$)

- valid in any kinematic configuration
  (subtle in taking 3pt limit $k_4 \to 0$)

- vs Numeric

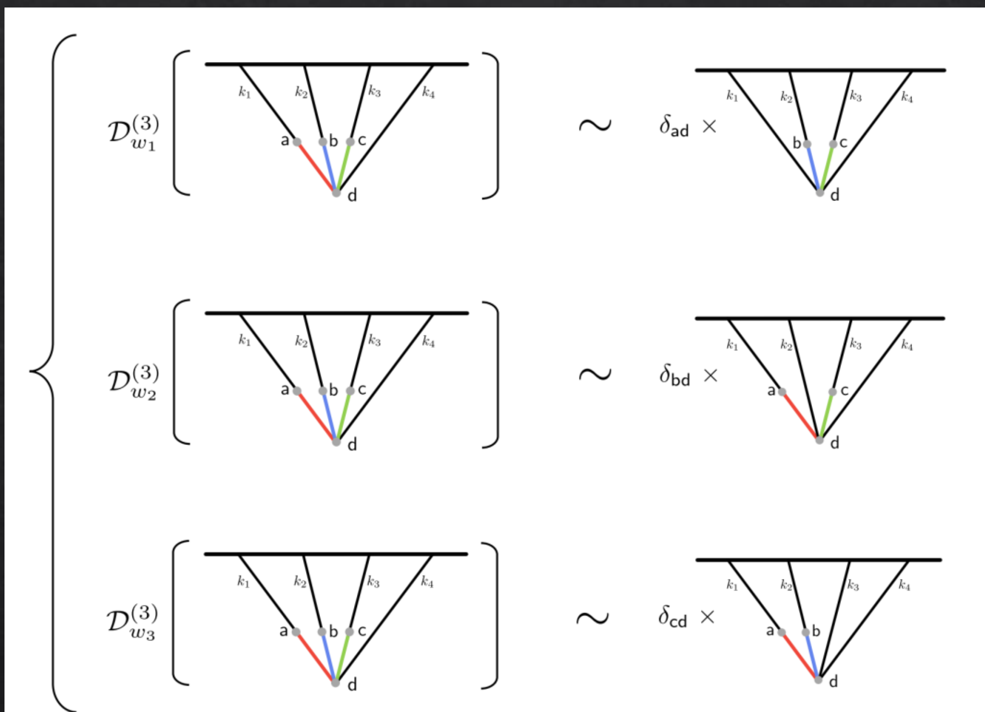


$\sim$ Local



$\sim$ equilateral

Toward triple exchange



$$w_i \equiv \frac{k_i}{k_4}\,, \quad (i=1,2,3)$$

$$\mathcal{D}^{(3)}_{w_i} \equiv w_i^2\partial^2_{w_i} - 2w_i\partial_{w_i} + \mu_i^2 + \frac{9}{4} - w_i^2\left(\sum_{j=1}^{3} w_j\partial_{w_j} + p_4 + 2\right)\left(\sum_{j=1}^{3} w_j\partial_{w_j} + p_4 + 1\right)$$

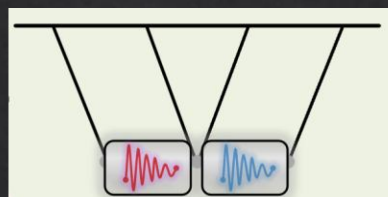
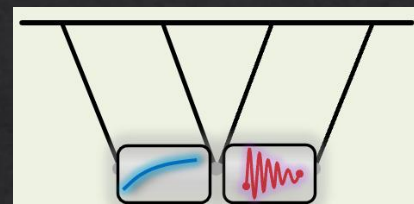
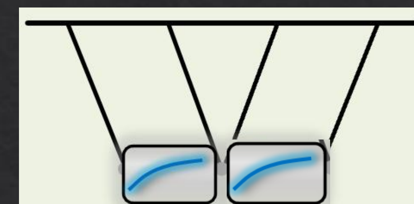
Homogenous solutions:

$$\mathcal{I}^{p_1p_2p_3p_4}_{\rm hom} \sim \sum_{\mathsf{abc}=\pm} w_1^{\frac{3}{2}-\mathrm{i}\mathsf{a}\mu_1} w_2^{\frac{3}{2}-\mathrm{i}\mathsf{b}\mu_2} w_3^{\frac{3}{2}-\mathrm{i}\mathsf{c}\mu_3} \times F_C\left[\begin{matrix}\frac{2p_4+11-2\mathrm{i}(\mathsf{a}\mu_1+\mathsf{b}\mu_2+\mathsf{c}\mu_3)}{4}, \frac{2p_4+13-2\mathrm{i}(\mathsf{a}\mu_1+\mathsf{b}\mu_2+\mathsf{c}\mu_3)}{4} \\ 1-\mathrm{i}\mathsf{a}\mu_1, 1-\mathrm{i}\mathsf{b}\mu_2, 1-\mathrm{i}\mathsf{c}\mu_3\end{matrix}\middle| w_1^2, w_2^2, w_3^2\right]$$

Lauricella's hypergeometric function

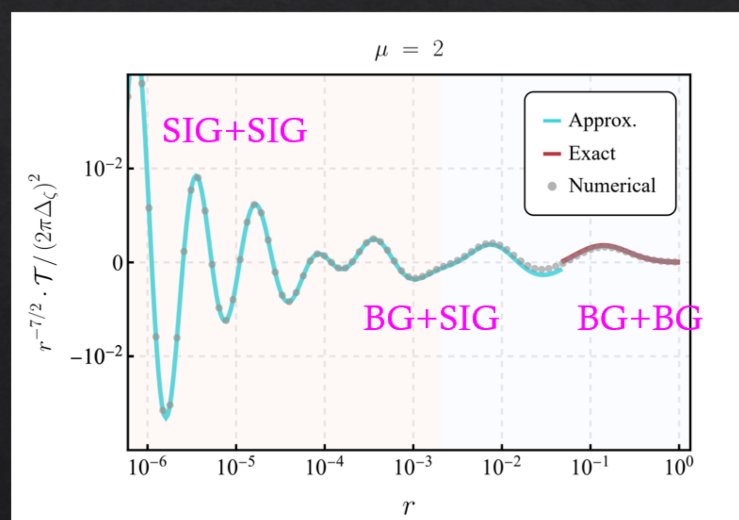
See Liu, Xianyu, 2412.07843 for more generalization

Phenomenological implication

$F =$  $+$  $+$ 

SIG+SIG BG+SIG BG+BG

Specific oscillation signal (change of frequency $\mu \to 2\mu$) which cannot be mimicked by single exchange



Summary

- Cosmological Correlators can contain high energy information (e.g., mass, spin, couplings of heavy fields = cosmological collider)

- Model building aspects
  - Need big turn rate for large signal ⇒ $\eta_{\perp} = \sqrt{2\epsilon}M_{\mathrm{PL}} \times \kappa$ ⇒ Large κ
  - Three models : ϕ + R^2, Multifield α-attractor, Monodromy
  - Many other fun models to try!

- Multiple exchange
  - Differential equation technique
  - Full analytic expressions of double exchange

Thank you!!