# Infrared Divergences in de Sitter

## *Wavefunction, Density Matrix* & *Stochastic Inflation*

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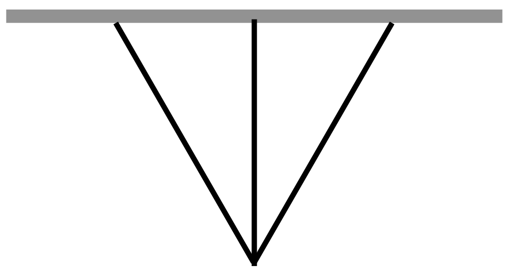
Theoretical Bootstrap Workshop @ YITP, Nov 21, 2025

# IR Divergences in de Sitter Space

**from interacting massless scalars**

Case study: massless $\Phi^3$ interaction in perturbation theory

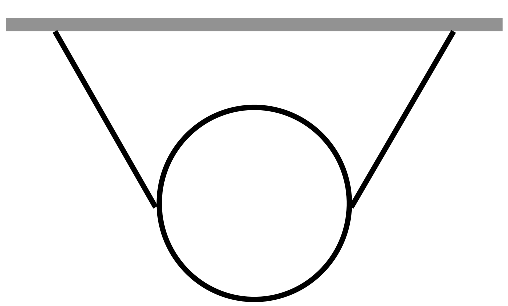


massless modes freeze after horizon-exit

cumulative interactions => **logarithmic secular growth**

$$\langle \phi_{k_1}\phi_{k_2}\phi_{k_3}\rangle' \sim i\int_{-\infty}^{\eta_0}\frac{d\eta}{\eta^4}\left[(1-ik_1\eta)(1-ik_2\eta)(1-ik_3\eta)e^{ik_T\eta}-c.c.\right]+\text{perm.}$$

$$=\frac{H^2}{2k_1^3k_2^3k_3^3}\left[(k_1^3+k_2^3+k_3^3)\left(\gamma_E-1+\log(-k_t\eta_0)\right)+4e_3-e_2k_t\right]$$



$$\langle \phi_{\mathbf{k}_1}\phi_{\mathbf{k}_2}\rangle'_{1-\text{loop}} \sim \log(kL)\log(-2k\eta_0)^2$$

**Are they really divergent? Do we need to worry about higher loops?**

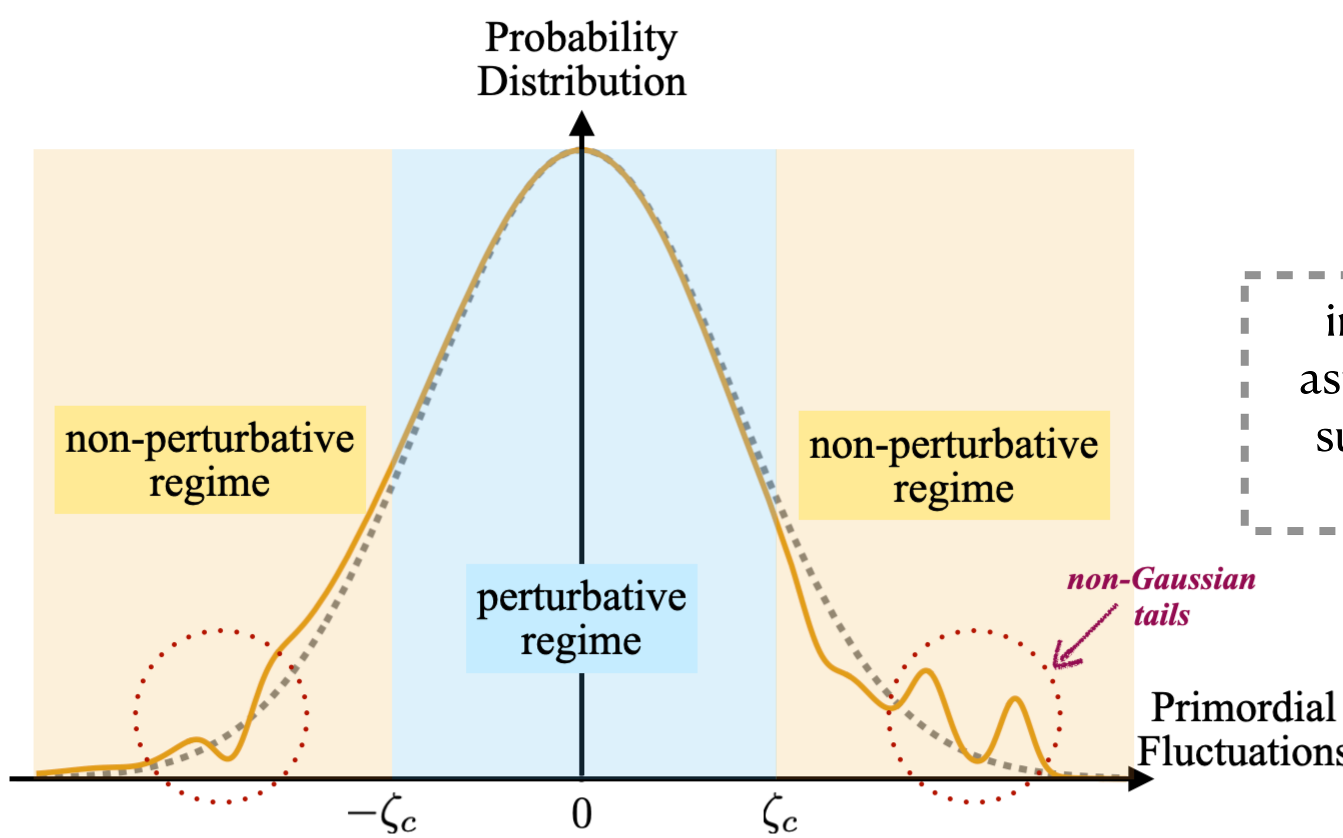
# **Motivation:** Beyond Perturbation Theory

## I. Theory

- Non-perturbative QFT in dS?
- Dynamical gravity, holography in cosmology, etc

## II. Pheno

- non-Gaussian tails of the probability distribution




important for compact astrophysical objects, e.g. supermassive BHs, early galaxies, PBHs

# Plan of the Talk

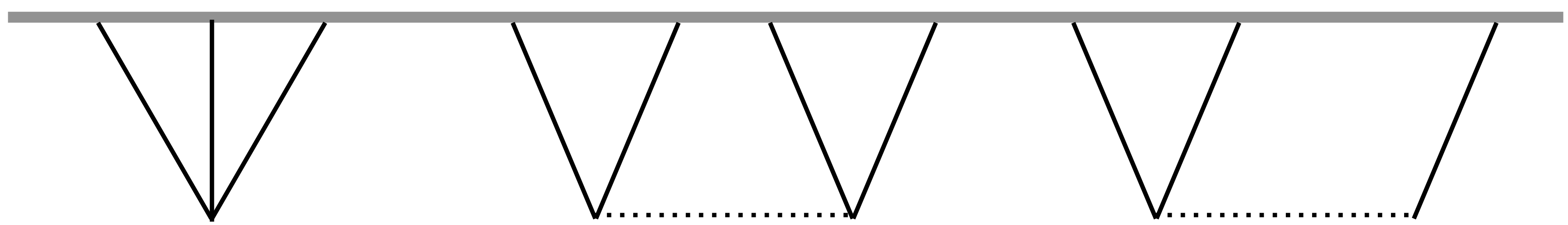
- From Wavefunction to Stochastic Formalism
  - Trees
  - Loops
  - Back to Trees

- Density Matrix & Cosmological EFTs
  - Matching to an (Open) EFT
  - $\lambda\phi^4$ as an Effective Theory in de Sitter

*Diffusion as non-unitary effects*

# I. Trees

- Bootstrap with CFT anomalies
- Multi-field non-Gaussianities

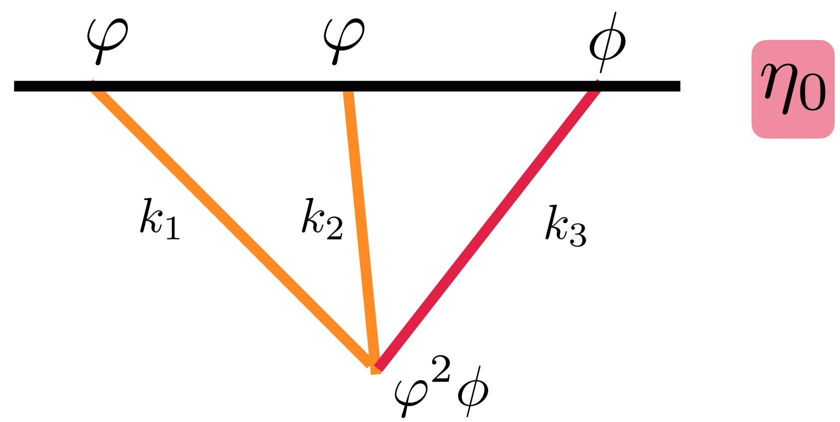


DGW, Pimentel, Achucarro 2212.14035

# I. Trees: Contact

## Three-Point Contact

$$\langle \varphi_{k_1} \varphi_{k_2} \phi_{k_3} \rangle \quad \sim$$




$$\sim I(u, \eta_0)$$

$$u \equiv \frac{k_3}{k_1 + k_2}$$

- $\phi$ is massive ⇨ **IR-finite** correlator

Arkani-Hamed, Maldacena 2015

Conformal Ward Identities

$$\left[\Delta_u - 2 + \frac{m^2}{H^2}\right] I(u) = 0$$

with $\Delta_u \equiv u^2(1-u^2)\partial_u^2 - 2u^3\partial_u$

- $\phi$ is massless ⇨ **IR-divergent** correlator

Bzowski, McFadden, Skenderis
2013, 2015, 2018
DGW, Pimentel, Achucarro 2022

*Anomalous* Conformal Ward Identities

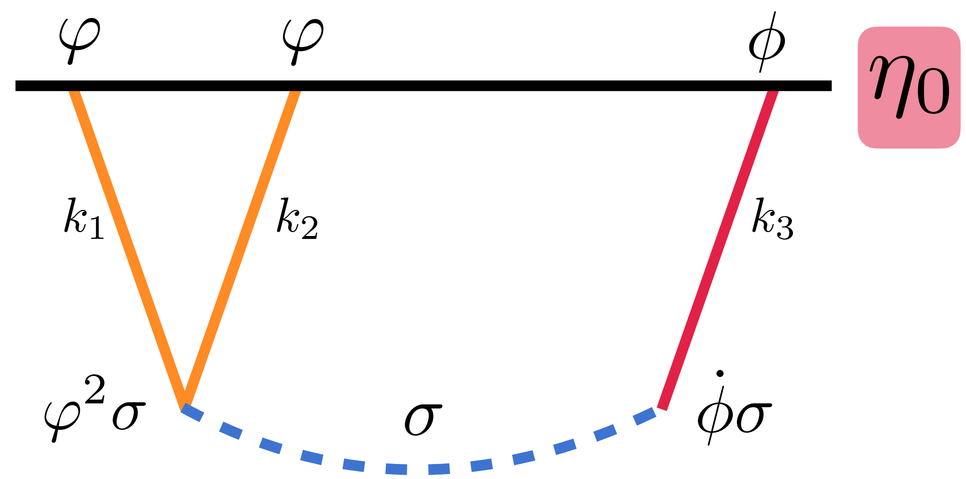
$$[\Delta_u - 2]\, I(u, \eta_0) = -\frac{6}{u}$$

an extra source term

# I. Trees: Exchange

**Three-Point Scalar Seed**

$$\langle \varphi_{k_1} \varphi_{k_2} \phi_{k_3} \rangle \sim$$




$$\sim \hat{\mathcal{I}}(u, \eta_0)$$

$$u \equiv \frac{k_3}{k_1 + k_2}$$

- $\sigma$ is massive → IR-finite correlator

Arkani-Hamed, Baumann, Lee, Pimentel 2018
Pimentel, DGW 2022

Conformal Ward Identities

$$\left(\Delta_u - 2 + \frac{m^2}{H^2}\right) \hat{\mathcal{I}} = \frac{u}{1+u}$$

with $\Delta_u \equiv u^2(1-u^2)\partial_u^2 - 2u^3\partial_u$

- $\sigma$ is massless → IR-divergent correlator

DGW, Pimentel, Achucarro 2022
Bzowski, McFadden, Skenderis 2023

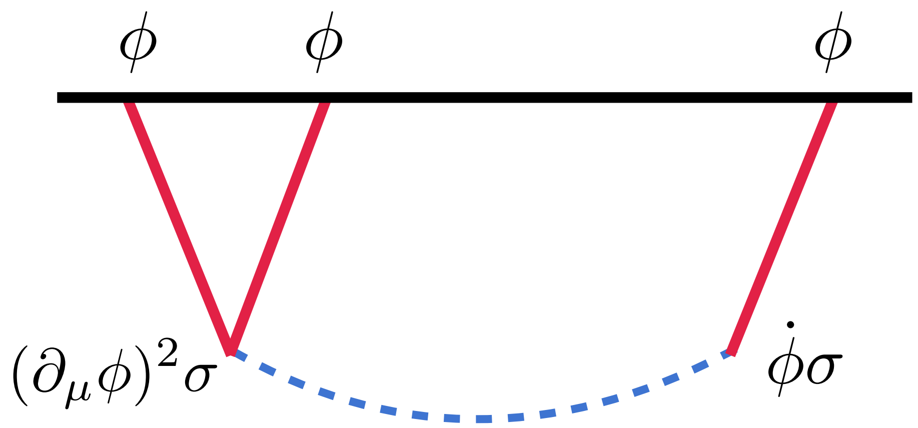
*Anomalous* Conformal Ward Identities

$$(\Delta_u - 2)\hat{\mathcal{I}}(u, \eta_0) = \frac{u}{1+u} + \frac{6}{u}\hat{\mathcal{K}}(k\eta_0)$$

an extra source term caused by the IR cutoff

# Side remark: local non-Gaussianity from multi-field inflation

DGW, Pimentel, Achucarro 2022




**massless exchange:**

the minimal setup for multi-field inflation

***Here is the real "local" shape:***

logarithmic $k_t$-pole: from the cubic vertex

$$S(k_1,k_2,k_3) \propto \frac{1}{k_1^3k_2^3k_3^3}\Big[\left(\gamma_E - 3 - \log(-k_t\eta_0)\right)(k_1^3+k_2^3+k_3^3) + k_t e_2 - 4e_3 + (k_2^3+k_3^3)\log(-2k_1\eta_0) + (k_1^3+k_3^3)\log(-2k_2\eta_0) + (k_1^3+k_2^3)\log(-2k_3\eta_0)\Big]$$

logarithmic $k_n$-pole: from the linear mixing

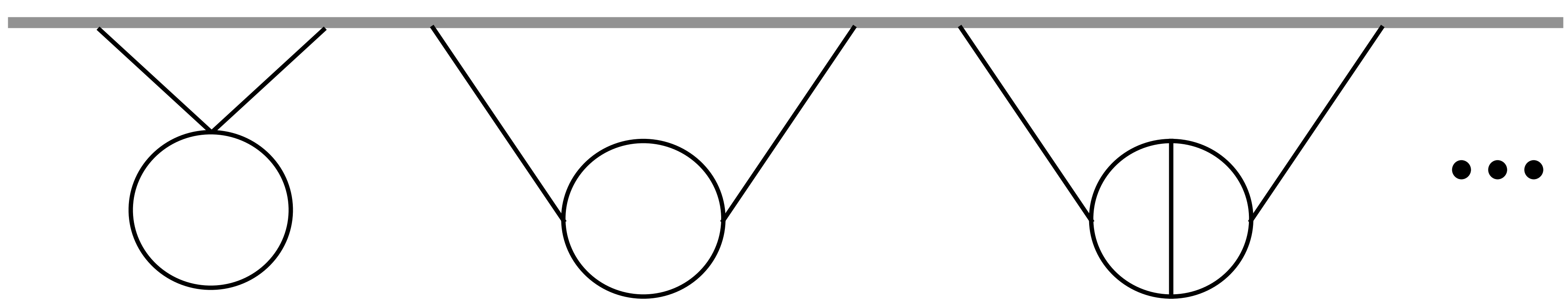
$$S^{\text{local}} = \frac{k_1^3+k_2^3+k_3^3}{k_1^3k_2^3k_3^3}$$

$$k_t \equiv k_1+k_2+k_3$$

- these poles were missed in the previous $\delta N$ formalism
- logarithmic secular growth <=> isocurvature conversion

# II. Loops

- Wavefunction: Classical v.s. Quantum
- Classical Loops and IR divergences



Cespedes, Davis, DGW 2023

# IR Divergences & the Cosmological Wavefunction

$$P = |\Psi|^2$$

**Stochastic Formalism**

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial \phi}\left(\frac{V_\phi}{3H} P\right) + \frac{H^3}{8\pi^2}\left(\frac{\partial^2 P}{\partial \phi^2}\right)$$

- Fokker-Planck eq.
- non-perturbative
- equilibrium behaviour

**Cosmological Bootstrap**

$$\Psi[\phi] = \exp\left[\sum_{n=2}^{\infty} \frac{1}{n!} \int_{\mathbf{k}_1,\ldots,\mathbf{k}_n} \psi_n(\mathbf{k}_1, \ldots, \mathbf{k}_n)\; \phi_{\mathbf{k}_1} \cdots \phi_{\mathbf{k}_n}\right]$$

- related to correlators
- perturbative
- secular growth

?

Gorbenko, Senatore 2019
Cespedes, Davis, DGW 2023

# The Field-Theoretic Wavefunction

Classical v.s. Quantum

# Field-Theoretic Wavefunction ($G_N \to 0$)

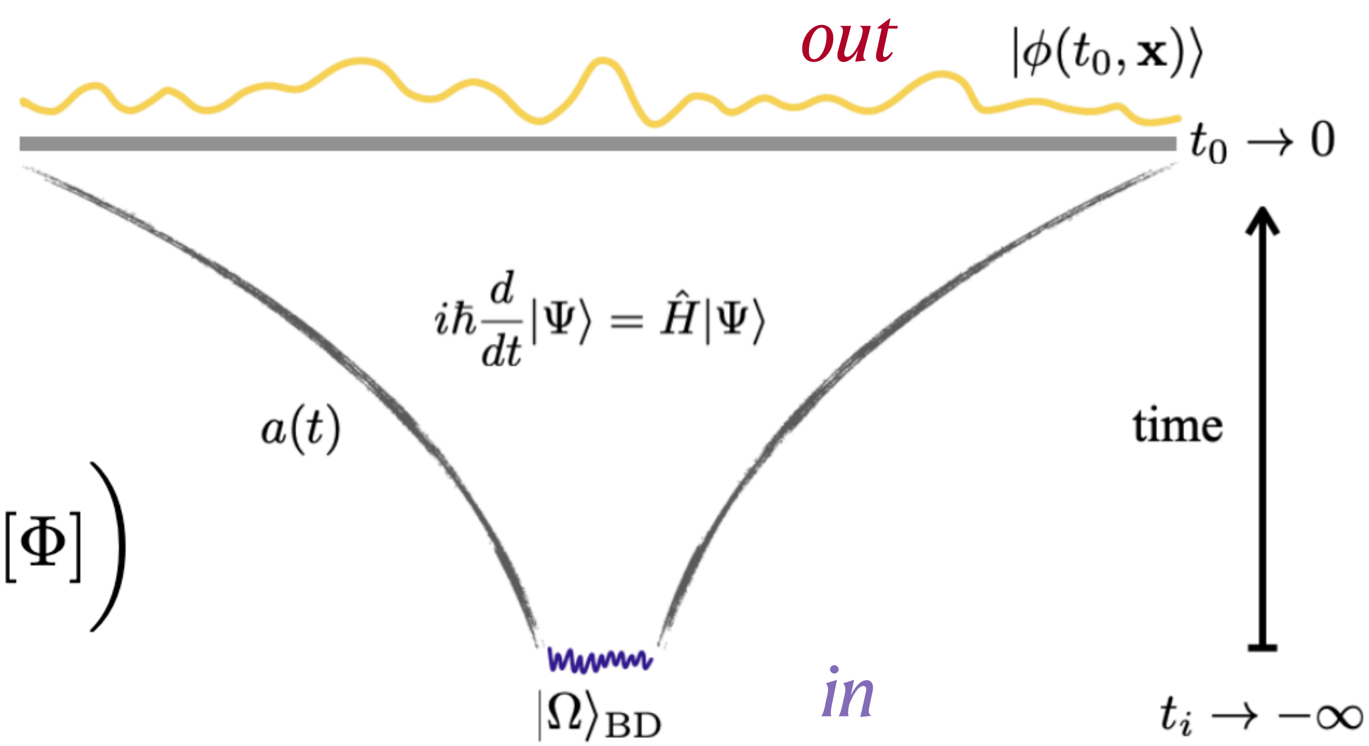
**Quantum field theory in a fixed spacetime background**

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = \frac{-d\eta^2 + d\mathbf{x}^2}{H^2\eta^2}$$

out    in

$$\Psi[\phi(\mathbf{x}), t_0] \equiv \langle \phi(\mathbf{x}) | \Omega \rangle$$

$$= \int_{\substack{\Phi(t_0)=\phi \\ \Phi(-\infty)=0}} \mathcal{D}\Phi \exp\left(\frac{i}{\hbar} S[\Phi]\right)$$




## Wavefunction Coefficients

$$\Psi[\phi] = \exp\left[\frac{1}{2}\int_{\mathbf{k}_1,\mathbf{k}_2} \underbrace{\psi_2(\mathbf{k}_1,\mathbf{k}_2)}_{\text{Free}}\, \phi_{\mathbf{k}_1}\phi_{\mathbf{k}_2} + \sum_{n=3}^{\infty}\frac{1}{n!}\int_{\mathbf{k}_1,\ldots,\mathbf{k}_n} \underbrace{\psi_n(\mathbf{k}_1,\ldots,\mathbf{k}_n)}_{\text{Interaction}}\, \phi_{\mathbf{k}_1}\cdots\phi_{\mathbf{k}_n}\right]$$

# Field-Theoretic Wavefunction: perturbation theory

## Classical Way:

- Saddle-point approximation

$$\Psi[\phi] \simeq \exp\left(\frac{i}{\hbar} S[\Phi_{\rm cl}]\right)$$

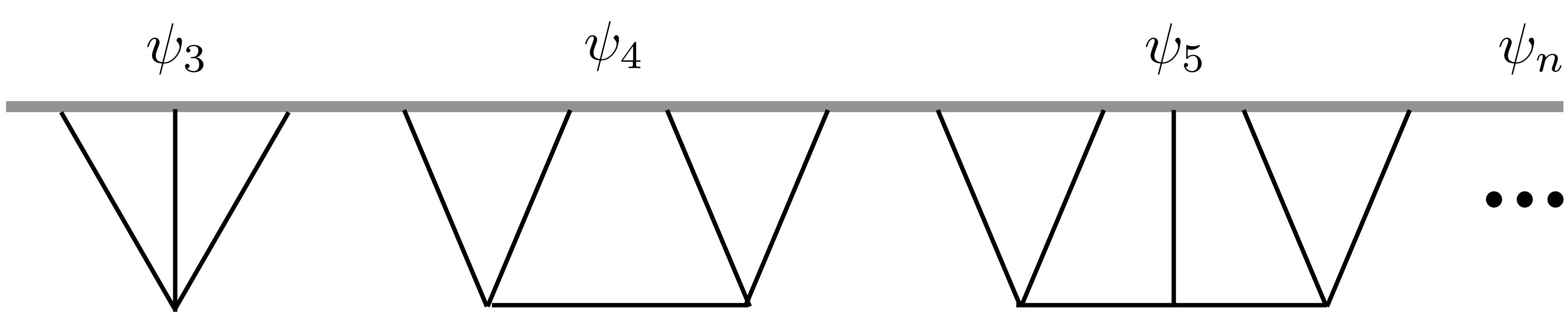
On-shell condition

bulk-to-boundary

$$(\Box - m^2)\Phi = -\frac{1}{\sqrt{-g}}\frac{\delta S_{\rm int}}{\delta \Phi} \quad \Rightarrow \quad \Phi_{\rm cl}(\eta, \mathbf{k}) = \phi_{\mathbf{k}} K(k,\eta) + \frac{i}{\hbar}\int {\rm d}\eta' G(k;\eta,\eta') \frac{\delta S_{\rm int}}{\delta \Phi_{\mathbf{k}}(\eta')}\bigg|_{\Phi=\Phi_{\rm cl}}$$

bulk-to-bulk

**<u>All the tree-level wavefunction coefficients:</u>**




$\hbar^{-1}$

(semi-)classical wavefunction coefficients

# Field-Theoretic Wavefunction: perturbation theory

Cespedes, Davis, DGW 2023

## Quantum Way:

- Functional Quantization

$$\Psi[\phi(\mathbf{x})] = \int_{\substack{\Phi(t_0)=\phi \\ \Phi(-\infty)=0}} \mathcal{D}\Phi \exp\left(\frac{i}{\hbar}S[\Phi]\right)$$

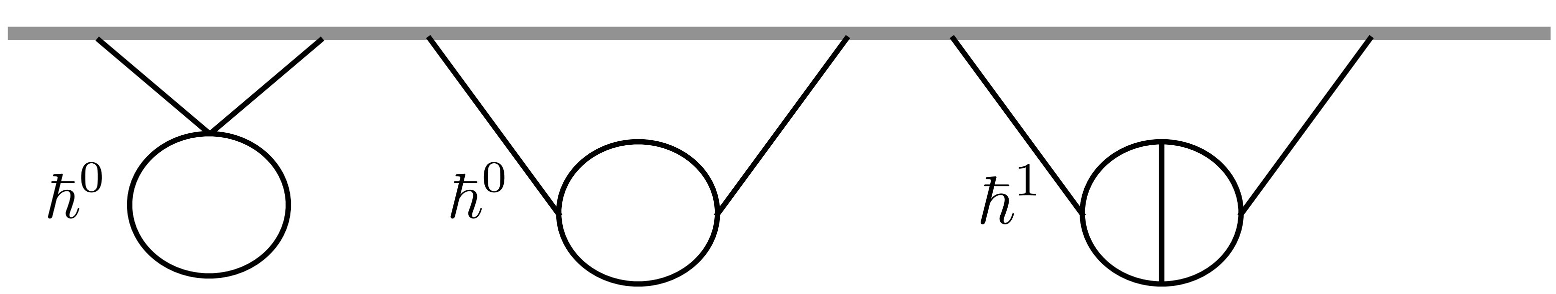
- Propagators

$$\langle \Phi_{\mathbf{k}}(\eta)\Phi_{\mathbf{k}'}(\eta')\rangle \equiv \frac{\int \mathcal{D}\Phi \Phi_{\mathbf{k}}(\eta)\Phi_{\mathbf{k}'}(\eta') \exp\left(\frac{i}{\hbar}S_0[\Phi]\right)}{\int \mathcal{D}\Phi \exp\left(\frac{i}{\hbar}S_0[\Phi]\right)} = G(k,\eta,\eta')(2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')$$

$$\langle \Pi_{\mathbf{k}}(\eta_0)\Phi_{\mathbf{k}'}(\eta)\rangle = 2i\hbar K(k,\eta)(2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')$$

**Both tree-level & loop-level wavefunction coefficients:**

$$\psi_n(\mathbf{k}_1,...,\mathbf{k}_n) \equiv \left.\frac{\delta^n\Psi[\phi]}{\delta\phi_{\mathbf{k}_1}...\delta\phi_{\mathbf{k}_n}}\right|_{\phi=0} = \frac{(i/2\hbar)^n}{\Psi[0]}\int \mathcal{D}\Phi \Pi_{\mathbf{k}_1}(\eta_0)...\Pi_{\mathbf{k}_n}(\eta_0)\exp\left(\frac{i}{\hbar}S[\Phi]\right)$$




quantum wavefunction coefficients

# From Wavefunction to Correlators

**Born rule:**

$$\langle \phi_{\mathbf{k}_1} \cdots \phi_{\mathbf{k}_n} \rangle = \frac{\displaystyle\int \mathcal{D}\phi \;\; \phi_{\mathbf{k}_1} \cdots \phi_{\mathbf{k}_n} \left|\Psi[\phi, \eta_0]\right|^2}{\displaystyle\int \mathcal{D}\phi \;\; \left|\Psi[\phi, \eta_0]\right|^2}$$

**Tree-level examples:**

$$\langle \phi_{\mathbf{k}} \phi_{-\mathbf{k}} \rangle' = -\frac{1}{2\,\mathrm{Re}\psi_2'(k)}\,,$$

$$\langle \phi_{\mathbf{k}_1} \phi_{\mathbf{k}_2} \phi_{\mathbf{k}_3} \rangle' = -\frac{\mathrm{Re}\psi_3'(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)}{4\mathrm{Re}\psi_2'(k_1)\;\mathrm{Re}\psi_2'(k_2)\;\mathrm{Re}\psi_2'(k_3)}\,.$$

$$\langle \phi_{\mathbf{k}_1} \phi_{\mathbf{k}_2} \phi_{\mathbf{k}_3} \phi_{\mathbf{k}_4} \rangle' = \frac{\mathrm{Re}\psi_4'(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)}{8\prod_{a=1}^4 \mathrm{Re}\psi_2'(k_a)} - \frac{1}{8\prod_{a=1}^4 \mathrm{Re}\psi_2'(k_a)}\left[\frac{\mathrm{Re}\psi_3'(\mathbf{k}_1, \mathbf{k}_2, \mathbf{s})\,\mathrm{Re}\psi_3'(-\mathbf{s}, \mathbf{k}_3, \mathbf{k}_4)}{\mathrm{Re}\psi_2'(s)} + ..\right]$$

## Analogy to Flat Spacetime

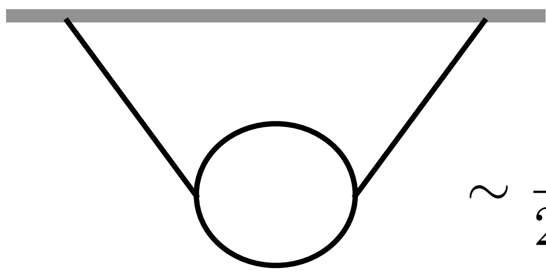
Wavefunction Coefficients ⟷ Scattering Amplitudes

Cosmological Correlators ⟷ Cross Section

# Classical Loops: leading IR logs in correlators

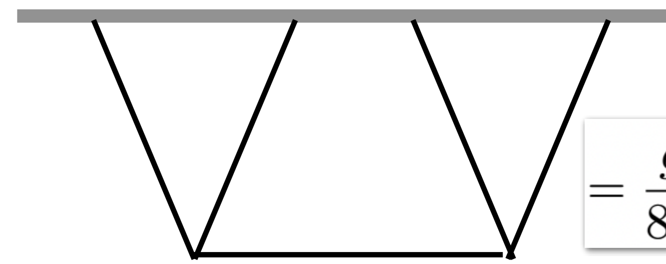
**From Wavefunction to Correlators:** One-Loop Example for $\frac{g}{3!}\Phi^3$

$\psi_2^{1-\text{loop}}$



$$\sim \frac{g^2}{2k^3}\log(-2k\eta_0)$$

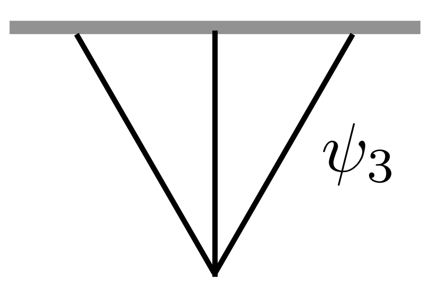
$\psi_4$



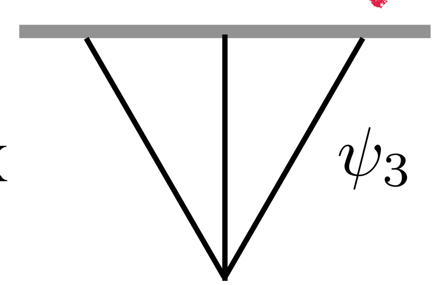
$$= \frac{g^2}{8k^3}\frac{-5}{18\pi^2}\log(kL)\log(-2k\eta_0)^2$$

$$\langle\phi_{\mathbf{k}_1}\phi_{\mathbf{k}_2}\rangle'_{1-\text{loop}} = \frac{\mathrm{Re}\,\psi'^{1-\text{loop}}_{\mathbf{k}_1\mathbf{k}_2}}{2\mathrm{Re}\,\psi'_2(k_1)\,\mathrm{Re}\,\psi'_2(k_2)} - \frac{1}{8\mathrm{Re}\,\psi'_2(k_1)\,\mathrm{Re}\,\psi'_2(k_2)}\int_{\mathbf{p}}\frac{\mathrm{Re}\,\psi'_4(\mathbf{k}_1,\mathbf{k}_2,\mathbf{p},-\mathbf{p})}{\mathrm{Re}\,\psi'_2(p)}$$
$$+\frac{1}{8\mathrm{Re}\,\psi'_2(k_1)\,\mathrm{Re}\,\psi'_2(k_2)}\int_{\mathbf{p}}\left[\frac{\mathrm{Re}\,\psi'_3(\mathbf{k}_1,\mathbf{p},-\mathbf{p}-\mathbf{k}_1)\mathrm{Re}\,\psi'_3(\mathbf{k}_2,-\mathbf{p},\mathbf{p}+\mathbf{k}_1)}{\mathrm{Re}\,\psi'_2(p)\mathrm{Re}\,\psi'_2(|\mathbf{p}+\mathbf{k}_1|)} + \frac{\mathrm{Re}\,\psi'_3(\mathbf{k}_1,\mathbf{k}_2,-\mathbf{k}_1-\mathbf{k}_2)\mathrm{Re}\,\psi'_3(\mathbf{k}_1+\mathbf{k}_2,\mathbf{p},-\mathbf{p})}{\mathrm{Re}\,\psi'_2(p)\mathrm{Re}\,\psi'_2(|\mathbf{k}_1+\mathbf{k}_2|)}\right]$$

**Classical Loops**



$\psi_3$ X



$\psi_3$

$$= \frac{g^2}{8k^3}\frac{1}{3\pi^2}\log(kL)\log(-2k\eta_0)^2$$

# IR Loops: Wavefunction is IR safe

Gorbenko, Senatore 2019

*L*-loop *n*-point wavefunction coefficient:

$$\psi_n^{L-\text{loop}} \sim \int d\eta_1 ... d\eta_m a(\eta_1)^4 ... a(\eta_m)^4 K(k_1,\eta_1)...K(k_n,\eta_m)$$
$$\int_{\mathbf{p_1},...,\mathbf{p_L}} G(p_1,\eta_a,\eta_b)...G(p_L,\eta_c,\eta_d)G(|\mathbf{p_x}+\mathbf{k_y}|,\eta_e,\eta_f)...$$

Bulk-to-bulk propagator at IR:

$$\lim_{p\to 0} G(p,\eta,\eta') = \frac{i}{6}H^2\eta^3 + \mathcal{O}(p^2) \quad \text{for} \quad \eta > \eta'$$

Momentum integration from loops:

$$\int_{\mathbf{p}} \frac{1}{p^n} = \frac{1}{(2\pi)^3}\int_{1/L}^{\Lambda} 4\pi p^{2-n} dp \xrightarrow{L\to\infty} \begin{cases} \text{IR-finite} \ , & n<3 \\ \frac{1}{2\pi^2}\log(kL) \ , & n=3 \ . \end{cases}$$

**only secular divergence from time integrals,**

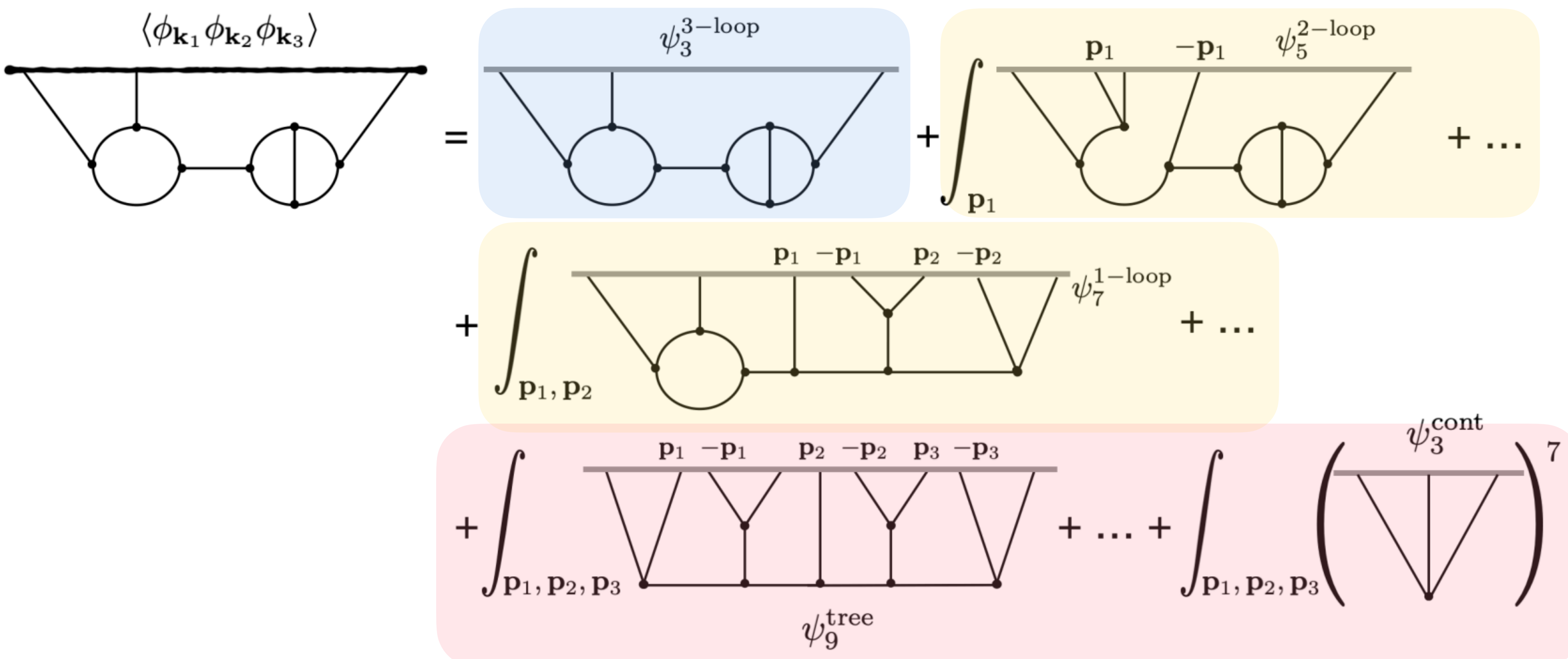
**no IR divergences from loop integrals**

# Classical Loops: leading IR logs in correlators

## three-point function at three-loop order

Quantum loop

Mixture of quantum & classical loop




Classical Loops

# Classical Loops: leading IR logs in correlators

*L*-loop *n*-point correlator (with *V* vertices):

$$\langle\phi^n\rangle_{L-\text{loop}} \sim \frac{1}{(\text{Re}\psi_2)^n}\left[\text{Re}\ \psi_n^{L-\text{loop}} + \int_{\mathbf{p_1}} \frac{\text{Re}\psi_{n+2}^{(L-1)-\text{loop}}}{\text{Re}\psi_2(p_1)} + \ldots + \int_{\mathbf{p_1},\ldots,\mathbf{p_{L-1}}} \frac{\text{Re}\psi_{n+2(L-1)}^{1-\text{loop}}}{(\text{Re}\psi_2(p_1)\ldots\text{Re}\psi_2(p_{L-1}))}\right.$$
$$\left. + \int_{\mathbf{p_1},\ldots,\mathbf{p_L}} \frac{1}{\text{Re}\psi_2(p_1)\ldots\text{Re}\psi_2(p_L)}\left(\text{Re}\psi_{n+2L}^{\text{ex}} + \frac{\text{Re}\psi_{n+2L-1}^{\text{ex}}\text{Re}\ \psi_3}{\text{Re}\ \psi_2} + \ldots + \frac{(\text{Re}\ \psi_3)^V}{(\text{Re}\ \psi_2)^{3V-2L-n}}\right)\right]$$

$$\propto \lambda^V \log(kL_{\text{IR}})^L \log(-k\eta_0)^V$$

in agreement with

Baumgart, Sundrum 2019

**IR-divergent correlators are always dominated by Classical loops**

**IR divergence** ⟷ **Superhorizon physics** ⟷ **Semi-classical effect**

# III. Back To Trees

- What does the stochastic formalism resum?
- Fokker-Planck eq. from boundary RG flow

# A tempting thought

saddle-point approx.

$$\Psi[\phi] \simeq \exp\left(\frac{i}{\hbar} S[\Phi_{\rm cl}]\right)$$

Fokker-Planck equation

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial \phi}\left(\frac{V_\phi}{3H} P\right) + \frac{H^3}{8\pi^2}\left(\frac{\partial^2 P}{\partial \phi^2}\right)$$

First, screen out the short wavelength modes:

$$P[\phi_l] = \int \mathcal{D}\phi \;\; \delta\left(\phi_l - \int \frac{d^3k}{(2\pi)^3} \Omega_k \phi_k\right) |\Psi(\phi_k)|^2$$

Then, let's check the perturbative regime of $\lambda\phi^4$, where the equilibrium has not been reached.

$$\text{coupling} \times \log^2 < 1$$

- controllable playground to explicitly match two computations
- could be interesting for pheno

# Correlators from stochastic formalism

$$\langle \phi^n \rangle = \int d\phi \phi^n P(\phi) \qquad \frac{d}{dt}\langle \phi^n \rangle = \int d\phi \phi^n \frac{P(\phi,t)}{dt} = -\frac{n}{3H}\langle \phi^{n-1} V_\phi \rangle + \frac{n(n-1)H^3}{8\pi^2}\langle \phi^{n-2}\rangle$$

a set of diff. eqs.
that can be solved
perturbatively

$$\begin{cases} \frac{d}{dt}\langle \phi^2 \rangle = -\frac{1\lambda}{9H}\langle \phi^4 \rangle + \frac{H^3}{4\pi^2} \\ \frac{d}{dt}\langle \phi^4 \rangle = -\frac{2\lambda}{9H}\langle \phi^6 \rangle + \frac{3H^3}{4\pi^2}\langle \phi^2 \rangle \\ \vdots \\ \frac{d}{dt}\langle \phi^n \rangle = -\frac{n\lambda}{18H}\langle \phi^{n+2} \rangle + \frac{n(n-1)H^3}{8\pi^2}\langle \phi^{n-2} \rangle \end{cases}$$

$$\langle \phi^2 \rangle = \frac{H^2}{4\pi^2}\log a - \frac{\lambda H^4}{144\pi^4}(\log a)^3 + \frac{\lambda^2 H^6}{2880\pi^6}(\log a)^5 + \mathcal{O}(\lambda^3 (\log a)^7)$$

Free theory — one-loop — two-loop — ......

Gaussian variance

classical loops from tree-level wavefunction coefficients

quantum loops are absent

# Beyond perturbation theory

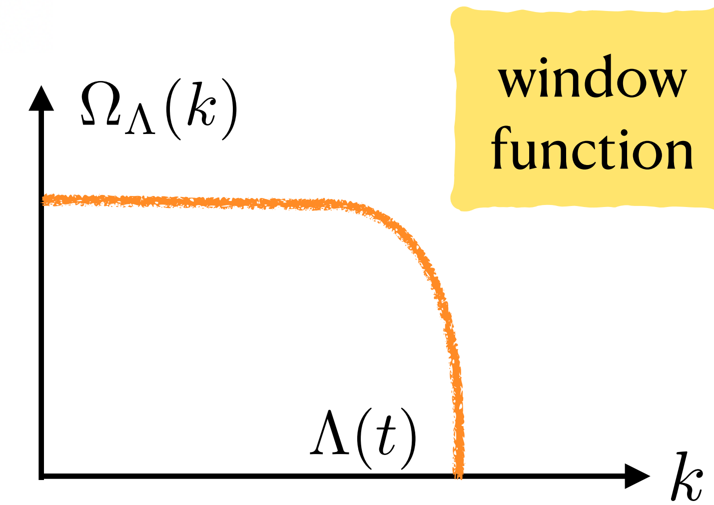
Cespedes, Davis, DGW 2023

Coarse-grained probability distribution

$$P_\Lambda[\phi, t] = |\Psi_\Lambda[\phi, t]|^2 = e^{W_0[\phi] + W_I[\phi]}$$

**free part** $W_0 = \int_{\mathbf{k}} \mathrm{Re}\ \psi_2(k)\Omega_\Lambda^{-1}(k)\ \phi_{\mathbf{k}}\phi_{-\mathbf{k}}\ ,$

**interacting part** $W_I = \sum_{n=3}^{\infty} \frac{1}{n!} \int_{\mathbf{k}_1,\ldots,\mathbf{k}_n} 2\mathrm{Re}\ \psi_n^\Lambda(\mathbf{k}_1, \ldots, \mathbf{k}_n, t)\ \phi_{\mathbf{k}_1} \cdots \phi_{\mathbf{k}_n}\ .$




$$\frac{d}{dt} P_\Lambda[\phi, t] = \frac{\partial}{\partial t} P_\Lambda[\phi, t] + \dot{\Lambda} \frac{\partial}{\partial \Lambda} P_\Lambda[\phi, t]$$

Schrodinger Eq.

RG flow?

**drift term** $\frac{\partial}{\partial \phi}\left(\frac{V'}{3H} P_\Lambda\right)$

**?**

# Fokker-Planck = Schrodinger + Polchinski

Cespedes, Davis, DGW 2023

The probability conservation should be unaffected by varying the artificial cutoff

$$\frac{d}{d\ln\Lambda}\int\mathcal{D}\phi\ P_\Lambda[\phi,t]=\int\mathcal{D}\phi\ \left[\frac{dW_0}{d\ln\Lambda}e^{W_I}+\frac{de^{W_I}}{d\ln\Lambda}\right]e^{W_0}=0$$

Polchinski's equation of Exact RG (modified version for *semi-classical wavefunction*)

$$e^{W_0}\frac{de^{W_I}}{d\ln\Lambda}=\frac{1}{4}\int_{\mathbf{k}}\frac{d\Omega_\Lambda}{d\ln\Lambda}\frac{1}{\mathrm{Re}\psi_2}\left[\left(\frac{\delta^2}{\delta\phi_{\mathbf{k}}\delta\phi_{-\mathbf{k}}}e^{W_I}\right)e^{W_0}-2\frac{\delta}{\delta\phi_{\mathbf{k}}}\left(e^{W_0}\frac{\delta e^{W_I}}{\delta\phi_{-\mathbf{k}}}\right)\right]$$

→ **diffusion term**

$$\dot{\Lambda}\frac{\partial}{\partial\Lambda}P_\Lambda[\phi,t]=\frac{H^3}{8\pi^2}\frac{\delta^2P_\Lambda}{\delta\phi_l^2}$$

**Fokker-Planck equation**

$$\frac{dP_\Lambda}{dt}=\frac{\partial}{\partial\phi}\left(\frac{V'}{3H}P_\Lambda\right)+\frac{H^3}{8\pi^2}\frac{\partial^2P_\Lambda}{\partial\phi^2}$$

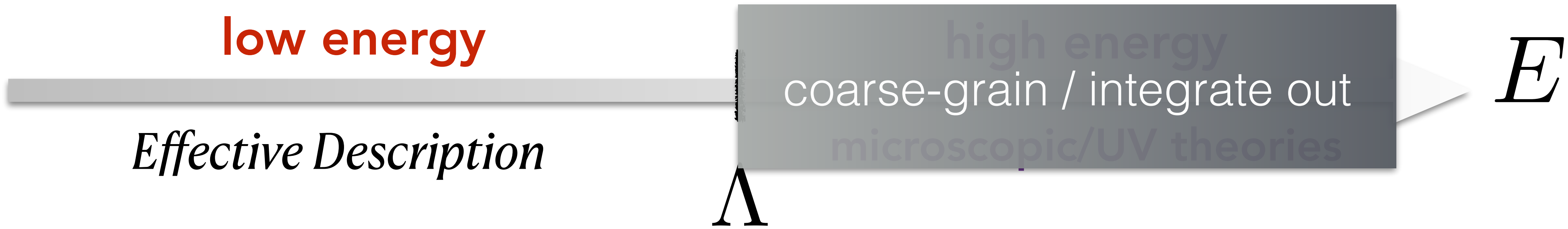
**drift: time flow** **diffusion: RG flow**

# Cosmological EFTs: $\lambda\phi^4$ *as an example*

- non-unitary effects
- non-Gaussian tails

Cespedes, Qin, DGW 2025

# Effective Field Theories from Top-Down




**integrate out heavy fields**

$$e^{iS_{\text{eff}}[\phi]} = \int \mathcal{D}\sigma e^{iS[\phi,\sigma]}$$

- decoupling with the heavy D.o.F.
- regimes of validity manifest

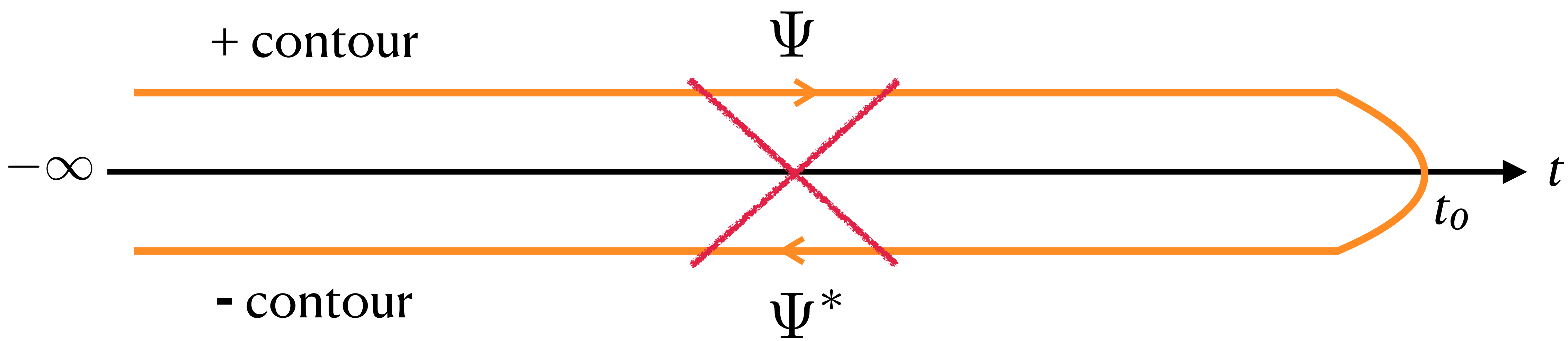
**coarse grain short modes**

$$e^{iS_\Lambda[\phi_L]} = \int \mathcal{D}\phi_H e^{iS[\phi_L,\phi_H]}$$

- Wilsonian perspective (RG flows)
- applies to stochastic formalism

***Matching*** → A *local* and *unitary* theory at low energies

# Cosmological EFTs & Density Matrix




- Single Field

$$\rho[\varphi_+,\varphi_-] = \Psi[\varphi_+]\Psi^*[\varphi_-] = \int_\Omega^{\phi_+(t_0)=\varphi_+} \mathcal{D}\phi_+ \int_\Omega^{\phi_-(t_0)=\varphi_-} \mathcal{D}\phi_- e^{iS[\phi_+]-iS[\phi_-]}$$

pure state      purity $\gamma = \mathrm{Tr}\rho^2 = 1$

- Effective Single Field

$$\rho[\varphi_+,\varphi_-] = \mathrm{Tr}_\varsigma\ \rho[\varphi_+,\varphi_-,\varsigma_+,\varsigma_-]$$

$$= \int d\varsigma \int^{\varphi_+} \mathcal{D}\phi_+ \int^{\varphi_-} \mathcal{D}\phi_- \int^{\varsigma} \mathcal{D}\sigma_+ \int^{\varsigma} \mathcal{D}\sigma_- e^{iS[\phi_+,\sigma_+]-iS[\phi_-,\sigma_-]}$$

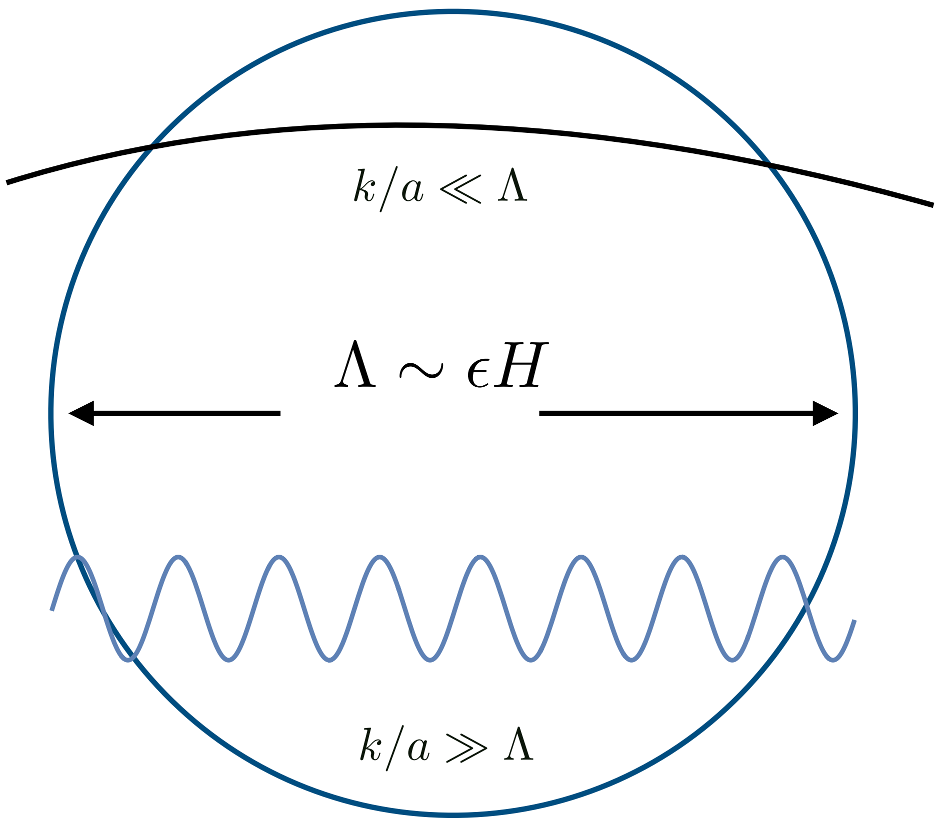
$$= \int^{\varphi_+} \mathcal{D}\phi_+ \int^{\varphi_-} \mathcal{D}\phi_- \exp\{\underbrace{iS_{\mathrm{EFT}}[\phi_+,\phi_-]}_{\text{open EFT}}\}$$

mixed state $\gamma < 1$

open EFT

# What are the possible EFTs on superhorizon scales?

- How UV physics are decoupled?
- Are the EFTs still unitary and local?
- Connection with stochastic formalism
- The regimes of validities for these EFTs;
- Beyond perturbation theory?




Retarded/Advanced basis

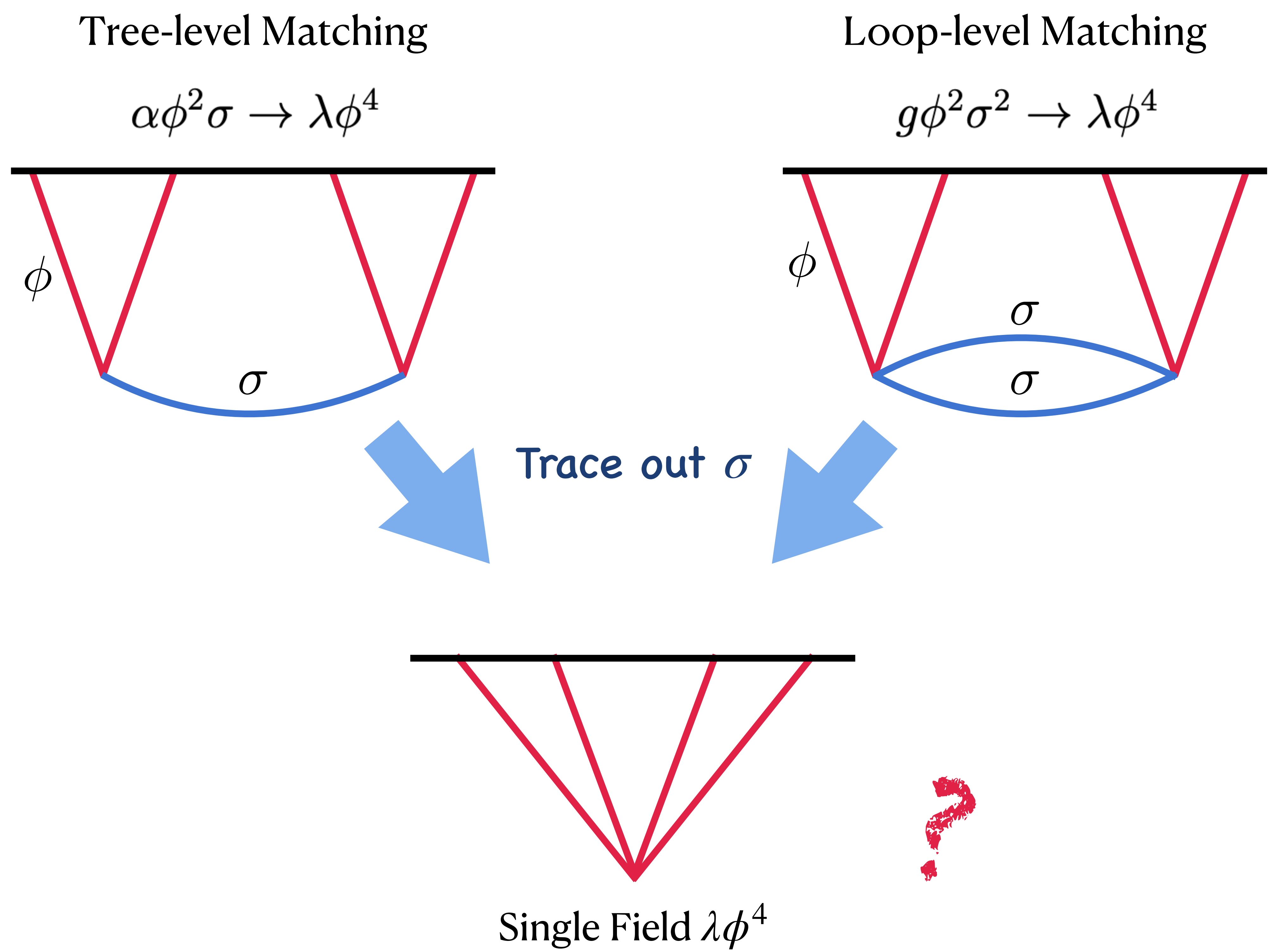
$$\phi_r = \frac{1}{2}(\phi_+ + \phi_-) \ , \qquad \phi_a = \phi_+ - \phi_-$$

Single Field $\lambda\phi^4$

$$S[\phi_a, \phi_r] = \int d^4x a^3 \left[ -\partial_\mu \phi_a \partial^\mu \phi_r - \lambda \phi_a \phi_r^3 - \frac{\lambda}{4} \phi_a^3 \phi_r \right]$$

# UV realisations of $\lambda\phi^4$

Cespedes, Qin, DGW 2025



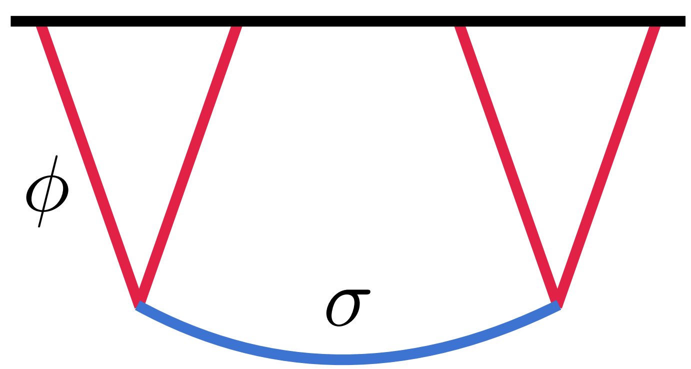
# Matching at Tree Level

$$\alpha\phi^2\sigma \to \lambda\phi^4$$

Cespedes, Qin, DGW 2025

$$\langle \varphi_{\boldsymbol{k}_1}\varphi_{\boldsymbol{k}_2}\varphi_{\boldsymbol{k}_3}\varphi_{\boldsymbol{k}_4}\rangle' = -\frac{\alpha^2 H^4}{8m^2\prod k_i^3}\sum k_i^3 \log(-k_T\eta_0)$$

IR log




$$+\ \frac{\alpha^2 H^4}{m^2}\mathcal{B}(k_i)$$

rational polynomial terms (derivative EFT expansion)

$$+\ \frac{\alpha^2 H^6}{m^4} e^{-\pi m/H}\,{}_2F_1$$

particle production

*m*>>*H* $\qquad \lambda_{\rm eff} = -\frac{1}{2}\frac{\alpha^2}{m^2}$

local and unitary $\lambda\phi^4$

$\gamma \to 1$

*m*<<*H* $\qquad \lambda_{\rm eff} = -\frac{\alpha^2 H^2}{2m^4}$

$$S_{\rm EFT} = -\int d^3x \int dt\sqrt{-g}\left[\partial_\mu\phi_a\partial^\mu\phi_r + \lambda_{\rm eff}\phi_a\phi_r^3 + \bar{\lambda}_{\rm eff}\phi_a^3\phi_r\right]$$
$$+\, i\int d^3x\int d^3y\int dt a^3(t)\ \phi_a^2(\boldsymbol{x},t)\lambda_{\rm NL}(|\boldsymbol{x}-\boldsymbol{y}|)\phi_r^2(\boldsymbol{y},t)$$

*diffusion*

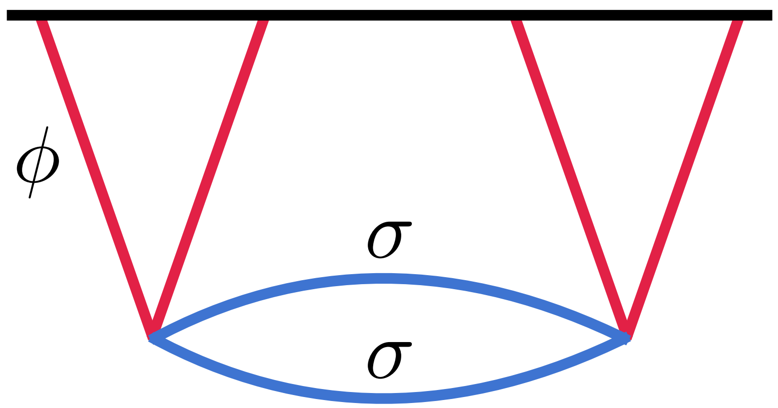
$\gamma \ll 1$

**potential unbounded from below => no non-perturbative analysis**

# Matching at Loop Level

Cespedes, Qin, DGW 2025

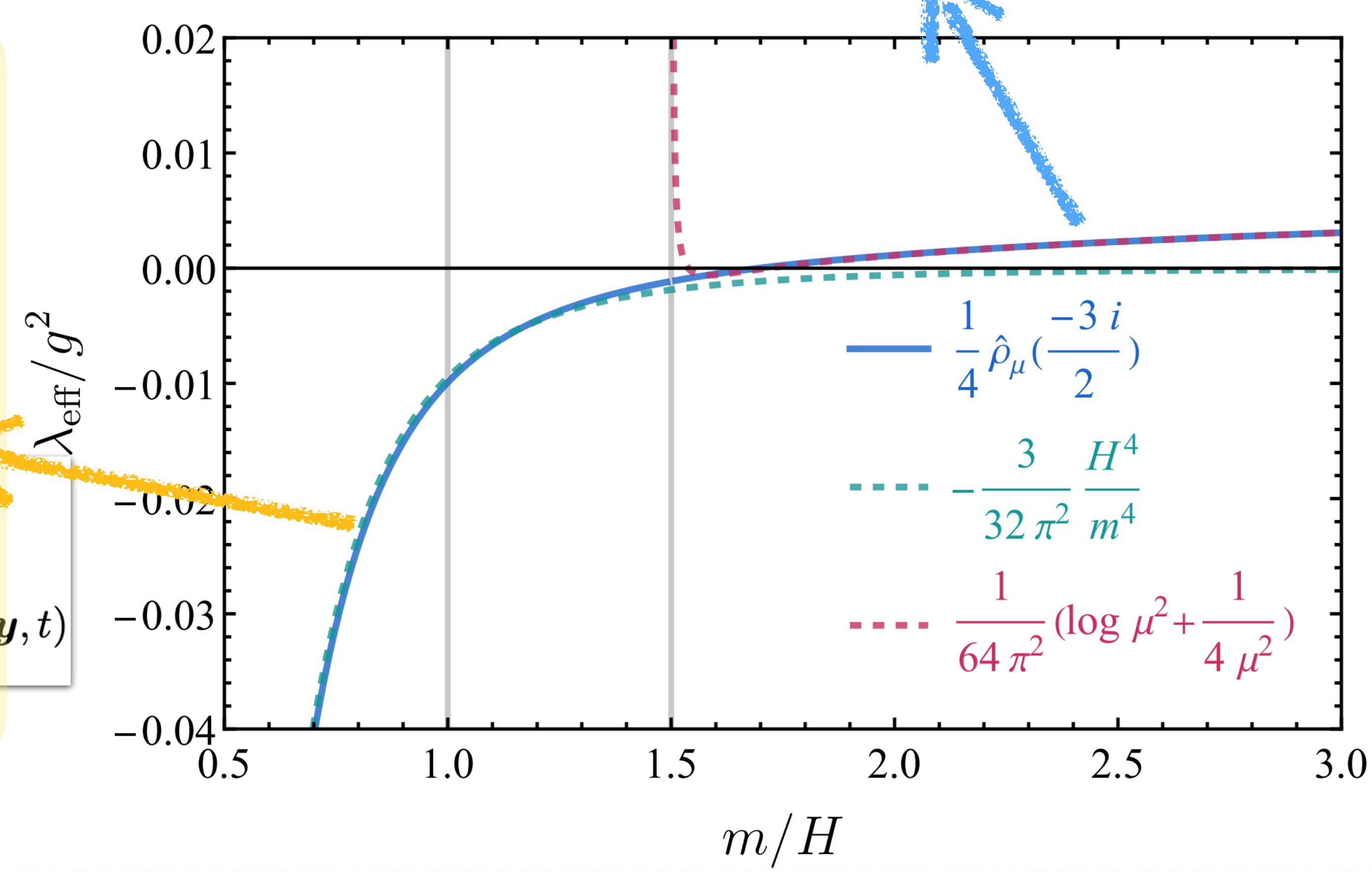
$$g\phi^2\sigma^2 \to \lambda\phi^4$$




$m$>>$H$ $\lambda_{\text{eff}} > 0$

local and unitary $\lambda\phi^4$

$\gamma \to 1$




$m$<<$H$

$$\lambda_{\text{eff}} = -\frac{3}{32\pi^2}\frac{g^2H^4}{m^2}$$

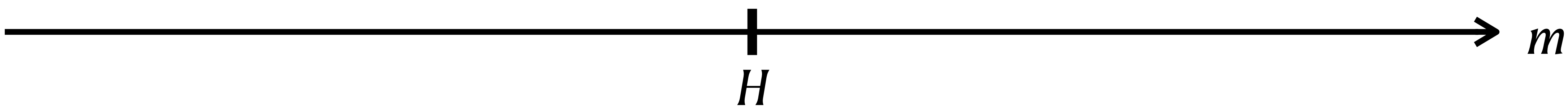
$$S_{\text{EFT}} = -\int d^3x \int dt\sqrt{-g}\,\left[\partial_\mu\phi_a\partial^\mu\phi_r + \lambda_{\text{eff}}\phi_a\phi_r^3 + \bar{\lambda}_{\text{eff}}\phi_a^3\phi_r\right]$$
$$+\, i\int d^3x\int d^3y\int dt a^3(t)\ \phi_a^2(\boldsymbol{x},t)\lambda_{\text{NL}}(|\boldsymbol{x}-\boldsymbol{y}|)\phi_r^2(\boldsymbol{y},t)$$

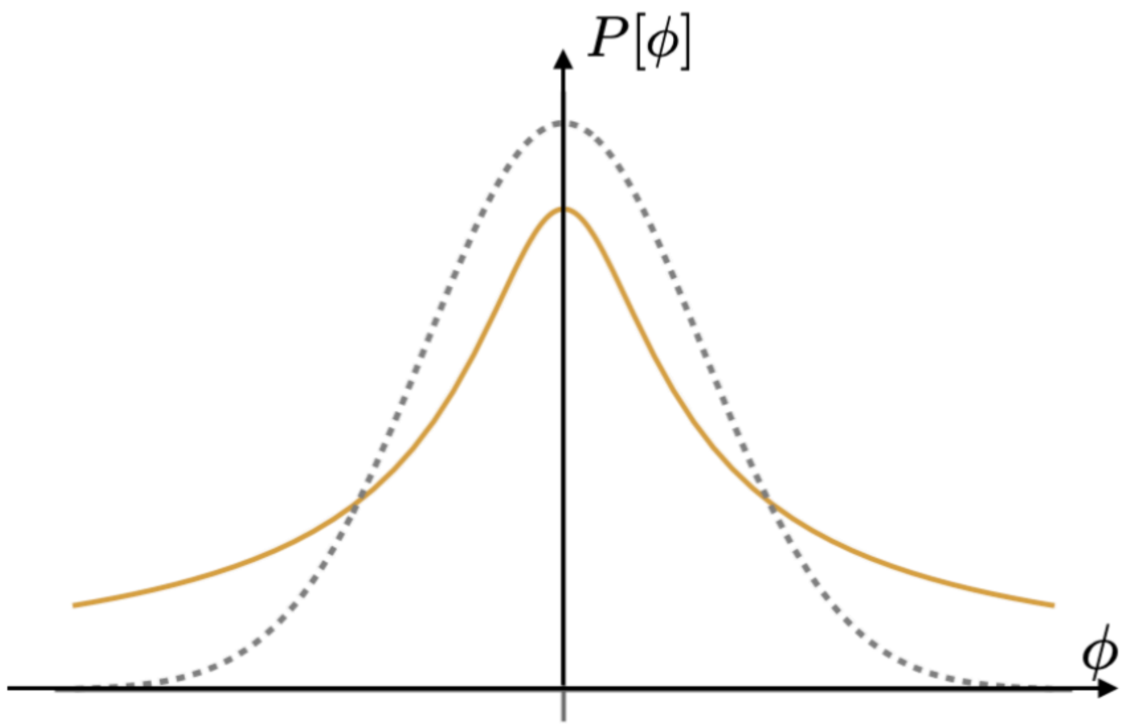
# Beyond Perturbation Theory

$$g\phi^2\sigma^2 \to \lambda\phi^4$$

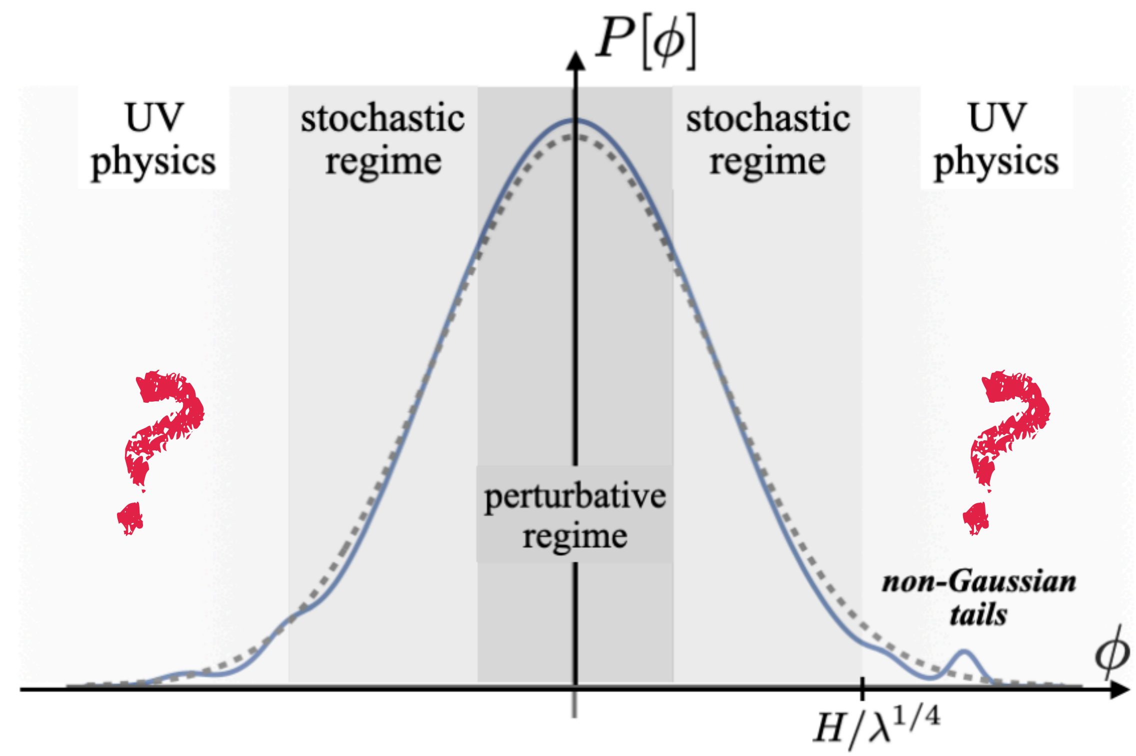
$$\lambda_{\text{eff}} = -\frac{3}{32\pi^2}\frac{g^2 H^4}{m^2} < 0$$

$$\lambda_{\text{eff}} > 0$$




$$P[\phi] \propto \left(\frac{H^2}{2m^2 + g\phi^2}\right)^{1/2}$$




$$P[\phi] \propto e^{-\frac{8\pi^2\lambda_{\text{eff}}}{3}\frac{\phi^4}{H^4}}$$

# Take Home Messages

$$P = |\Psi|^2$$

- Stochastic formalism is a resummation of classical loops;
- Fokker—Planck = Schrodinger + Polchinski;
- Non-unitary effects (diffusion, decoherence, etc) can become important in cosmological EFTs;
- Non-Gaussian tails may reveal UV physics beyond low-energy EFTs.