Bulk Reconstruction in de Sitter Space Time

Progress of Theoretical Bootstrap

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This talk is based on “Bulk Reconstruction in De Sitter Spacetime”, AG and Nirmalya Kajuri (arXiv:2405.16832)

Quick Preview

- Idea of bulk reconstruction: Representation of the bulk fields in the terms of the boundary operator.
- Originated from AdS/CFT correspondence.
- In de Sitter, boundary representation had been found only for the heavy scalar fields. Divergence arose for light scalar fields and higher spin fields.
- We obtained boundary representations for scalars of all masses as well as higher spin fields.
- Also extended results from Bunch-Davies vacuum to all α-vacua.

Contents

Outline

AdS/CFT Correspondence

- Any conformal field theory of 'd' space-time dimension is equivalent to a theory of quantum gravity in asymptotically AdS space-time in 'd+1' dimension.
- The mapping between the two theories is given by the **Extrapolate Dictionary (Banks, 1998):**

$$\lim_{z\to 0} z^{-n\Delta}\langle\phi(z,\vec{x}_1,t_1)\phi(z,\vec{x}_2,t_2)...\phi(z,\vec{x}_n,t_n)\rangle$$
$$= \langle\mathcal{O}(\vec{x}_1,t_1)\mathcal{O}(\vec{x}_2,t_2)...\mathcal{O}(\vec{x}_n,t_n)\rangle$$

$$\textbf{where } \Delta = \frac{d}{2}+\sqrt{\frac{d^2}{4}+m^2}$$

Δ is the conformal dimension.

Bulk Reconstruction

- Expressing the bulk AdS operators in terms of boundary CFT operators.
- **HKLL Method** : To find boundary representations of bulk fields by solving bulk equation of motion and using the extrapolate dictionary as the boundary condition. (Dobrev, Bena, Hamilton, Kabat, Lifschytz and Lowe)

Motivation of Bulk Reconstruction

- Extrapolate dictionary only maps bulk fields to boundary operators at the boundary.
- To complete the dictionary between bulk and boundary operator by mapping bulk fields away from the boundary to boundary operators.
- Any question of bulk physics can be translated to a question about the boundary CFT

Example: Free Scalar in AdS

- We will work in Poincare patch of AdS:

$$ds^2 = \frac{1}{z^2}(-dt^2 + d\vec{x}^2 + dz^2).$$

- For free scalar fields, Klein-Gordon equation:

$$(\Box - m^2)\phi = 0.$$

- Solution:

$$\phi(z, \vec{x}, t) = \phi_n(z, \vec{x}, t) + \phi_{nn}(z, \vec{x}, t).$$

We will discard the non-normalizable part and only consider the normalizable one.

- Expand free fields in the terms of creation and annihilation operator:

$$\phi(z,\vec{x},t) = \int d\omega d^d k \left(f_{\omega k}(z,t,\vec{x}) a_{\omega k} + f^{\dagger}_{\omega k}(z,t,\vec{x}) a^{\dagger}_{\omega k} \right).$$

$f_{\omega}(z,t,\vec{x})$ are the normalizable mode solutions of Klein-Gordon equation.

- Extrapolate dictionary is defined as:

$$\lim_{z\to 0} z^{-\Delta}\phi(z,\vec{x},t) = \mathcal{O}(\vec{x},t)$$

- Substituting the solution of $\phi(z,\vec{x},t)$ into the extrapolate dictionary and taking Fourier transform:

$$a_{\omega\vec{k}} = \int dtd^{d-1}x'\,\mathcal{O}(x',t')e^{i(\vec{k}.\vec{x}-\omega t)}f_{\vec{k}\omega}$$
$$a^{\dagger}_{\omega\vec{k}} = \int dtd^{d-1}x\,'\mathcal{O}(x',t')e^{-i(\vec{k}.\vec{x}-\omega t)}f_{\vec{k}\omega}.$$

- Substituting back to the original mode expansion, we get the boundary representation of $\phi(z,\vec{x},t)$:

$$\phi(z,\vec{x},t) = \int d^dx'dt'\,K(z,\vec{x},t;\vec{x}',t')\mathcal{O}(x',t')$$

$$K(z,x,t;x',t') = \int d^{d-1}k\,d\omega\,f_{\omega\vec{k}}(z)e^{i(\vec{k}\cdot(\vec{x}-\vec{x}')+\omega(t-t'))} + h.c.$$

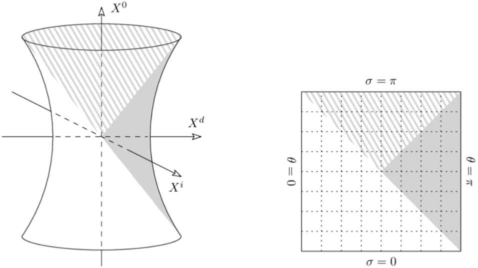
is the *smearing function.*

- This method is called the mode approach.
- There is another approach through the construction of the Green function.
- One can obtain K from the boundary limit of the Green function.

Outline

Description of dS

- de Sitter space is the maximally symmetric vacuum solution of Einstein's field equations with a positive cosmological constant Λ.



- The global chart of de sitter spacetime has conformal boundaries at timelike infinity in the past and future both.

- In this paper, we will work with the flat slicing which covers the shaded region and has only one boundary(future boundary).
- Metric of flat slicing :

$$ds^2 = \frac{R^2}{\eta^2}(-d\eta^2 + dX^2)$$

  R is the radius of de sitter space.
  Here, $\eta \to 0$ define the boundary.
- The flat slicing of de sitter space-time describes an inflationary universe, so ideally suitable for cosmological applications.

de Sitter vs anti-de Sitter

1. Positive curvature

2. dS has two spacelike boundaries in the infinite past and in the infinite future

3. KG equation of dS has two normalizable solutions.

1. Negative curvature

2. AdS has only one conformal boundary, that is timelike.

3. AdS has only one normalizable solution

Outline

dS/CFT correspondence

- There is a long standing proposal that there could be a conformal field theory (CFT) that is dual to the quantum gravity in de Sitter spacetime.(Strominger, Maldacena, Susskind, Balasubramanian, Anninos, Hartman..)
- It's still under investigation.
- If there is indeed such a correspondence between dS and CFT living at the $\eta \to 0$ boundary then, our construction will be useful to translate bulk physics to the boundary CFT.

The Cosmological Bootstrap Program

- CMB observations probe correlation functions of bulk fields at the end of inflation i.e at $\eta \to 0$.
- The Cosmological Bootstrap help us to derive this correlators purely from a boundary perspective using principles like conformal symmetry, unitarity etc
- If we fix boundary correlators by bootstrap, one can use our smearing function to find correlation function at other times.

Outline

Review of previous work

- Bulk reconstruction in de Sitter spacetime has been done only for heavy scalar fields ($(m/H)^2 > d^2/4$) using the Green function method (Xiao, PRD, 2014)
- This strategy only applies to scalar fields belonging to the principal series. The validity of the results beyond this range was unclear.
- Divergences arise for fields whose dual operator has conformal dimension zero(gravition) or negative integer(high spin fields).
- Only Bunch-Davies vacuum was considered.

Preview of our work

- In our work, we use the mode sum approach (reviewed earlier).
- We constructed the smearing function for fields of all masses and spins.
- We find the origin of the the divergences that appeared earlier.
- We also extend the construction of boundary representations to arbitrary α-vacua.

Solution of field equation of motion

- We will follow the same method as explained for AdS space.
- We consider scalar fields in d+1 spacetime dimension dS spacetime,
- dS metric in flat spacetime:

$$ds^2 = \frac{-d\eta^2 + dx^i dx_i}{\eta^2} \qquad (i = 1, ...d)$$

- Two linearly independent solutions are obtain as,

$$\phi(\eta, x) = c_1\, \phi_{\Delta}(\eta, x) + c_2\, \phi_{d-\Delta}(\eta, x)$$

Boundary fall-off.

$$\lim_{\eta\to 0} \phi_\Delta(\eta, x) \sim \eta^\Delta \mathcal{O}_\Delta(x)$$
$$\lim_{\eta\to 0} \phi_{d-\Delta}(\eta, x) \sim \eta^{d-\Delta} \mathcal{O}_{d-\Delta}(x).$$

□

- Conformal dimension for dS

$$\Delta = \frac{d}{2} + \sqrt{\frac{d^2}{4} - m^2}$$

- Unlike AdS, both these modes are normalizable.

- Unlike AdS/CFT, a bulk field in dS/CFT has two dual operators in the boundary $\mathcal{O}_\Delta, \mathcal{O}_{d-\Delta}$.
- The boundary representation of a bulk field in de Sitter will be:

$$\phi(\eta, x) = c_1 \phi_\Delta(\eta, x) + c_2 \phi_{d-\Delta}(\eta, x)$$

where:

$$\phi_\Delta(\eta, x) = \int K_\Delta\left(\eta, x; x'\right) \mathcal{O}_\Delta\left(x'\right) d^d x'$$

$$\phi_{d-\Delta}(\eta, x) = \int K_{d-\Delta}\left(\eta, x; x'\right) \mathcal{O}_{d-\Delta}\left(x'\right) d^d x'$$

- Hence we have to find two smearing functions K_Δ and $K_{d-\Delta}$.
- c_1, c_2 to be determinded from boundary conditions.

Previous Results

Smearing functions obtained for the principal series ($m^2 > \frac{d^2}{4}$) for Bunch-Davies vacuum: (Xiao, 2014)

$$K_{\Delta}(\eta, x; x') = \frac{\Gamma\left(\Delta - \frac{d}{2} + 1\right)}{\Gamma(\Delta - d + 1)} \left(\frac{\eta^2 - |\Delta x|^2}{\eta}\right)^{\Delta - d} \theta(\eta - |\Delta x|)$$

$$K_{d-\Delta}\left(\eta, x; x'\right) = \frac{\Gamma(\frac{d}{2} - \Delta + 1)}{\Gamma(1 - \Delta)} \left(\frac{\eta^2 - |\Delta x|^2}{\eta}\right)^{-\Delta} \theta(\eta - |\Delta x|)$$

Bulk reconstruction for scalar of all masses

The solutions of the KG equation:

For non-integer Δ:

$$f_k(\eta) = b_1 \eta^{d/2} J_{\Delta - d/2}(k\eta) + b_2 \eta^{d/2} J_{d/2-\Delta}(k\eta).$$

where $J_\mu(\eta)$ is the Bessel function of first kind. □

For integer Δ:

$$f_k(\eta) = c_1 \eta^{d/2} J_{\Delta - d/2}(k\eta) + c_2 \eta^{d/2} Y_{\Delta-d/2}(k\eta)$$

$Y_\mu(\eta)$ is the Bessel function of the second kind. □

Analysis of Smearing Function

For the above solution integral form of smearing function is,

$$K_\Delta\left(\eta, x; x'\right) = \frac{\Gamma(\Delta - d/2 + 1)2^\Delta}{|\Delta x|^{d/2-1}} \int_0^\infty \eta^{d/2} k^{d-\Delta} J_{\Delta-d/2}\left(k\eta\right) J_{\frac{d}{2}-1}(k|\Delta x|) dk$$

- This is a Weber-Schafheitlin type integral.
- It has analytic expression for some parameters and distributional for the others.
- This is the origin of the divergences that we mentioned earlier.

Analysis of Smearing Function

Smearing function for ϕ_Δ:

$$K_\Delta(\eta, x; x') = \frac{\Gamma\left(\Delta - \frac{d}{2} + 1\right) 2^{d/2}}{\Gamma(\Delta - d + 1)} \left(\frac{\eta^2 - |\Delta x|^2}{\eta}\right)^{\Delta - d}$$

□

- Smearing function has analytic solution for principal series of all masses when d=1.
- Light mass has analytic solution for mass range $d - 1 \leq m^2 \leq \frac{d^2}{4}$ in space dimension $d \geq 2$.
- For other range its distributional.

Analysis of Smearing Function for $\phi_{d-\Delta}$

Non-integer Δ

- We will follow up with two different cases for two Δ values. Integral form of smearing function is:

$$K_{d-\Delta}\left(\eta, x; x'\right) = \frac{\Gamma(d/2-\Delta+1)2^{d-\Delta}}{|\Delta x|^{d/2-1}} \int d^d k\, \eta^{d/2} k^{\Delta} J_{d/2-\Delta}(k\eta) J_{d/2-1}(k|\Delta x|)$$

- This is the same type of integral we encountered earlier.

For $\phi_{\Delta-d}$,**Non-integer Δ.**

$$K_{d-\Delta}\left(\eta, x; x'\right) = \frac{\Gamma(d/2-\Delta+1)}{\Gamma(1-\Delta)}\left(\frac{\eta^2-|\Delta x|^2}{\eta}\right)^{-\Delta}$$

□

- For principal series has analytic solution for d=1.
- It is distributional for other cases.

For $\phi_{d-\Delta}$,**For integer Δ.**

$$K_{d-\Delta}(\eta, x; x') = -\frac{(4\pi)^{\frac{d}{2}}\Gamma(\Delta)}{\Gamma(\Delta - d/2)} \sin\left(\frac{\pi}{2}(d-1)\right)\left(\frac{\eta^2 - |\Delta x|^2}{\eta}\right)^{-\Delta}$$

□

- For odd d dimensions, $\sin\left(\frac{\pi}{2}(d-1)\right) = 0$.
- For odd dimensional space-times, the functional form of the smearing function matches with the non-integer case

Choices of Vacua

- Solutions of KG equation are:

$$\phi(\eta, x) = c_1\, \phi_\Delta(\eta, x) + c_2\, \phi_{d-\Delta}(\eta, x)$$

- The constants above can be fixed with different choice of initial conditions which correspond to different choice of vacua.
- Usual choice of vacuum approaching Minkowski vacuum at early times gives Bunch Davies vacuum
- This choice is not unique. There is a class of states which are invariant under dS isometries called α-vacua.

Bunch-Davies and α vacua

- Solution corresponding to Bunch-Davies vacuum:

$$f_k^{BD}(\eta) = \eta^{d/2} H^{(2)}_{\Delta - d/2}(k\eta)$$

where, $H^{(2)}_{\nu}$ is the Hankel function of second kind.

Bunch-Davies and α vacua

- Solution corresponding to Bunch-Davies vacuum:

$$f_k^{BD}(\eta) = \eta^{d/2} H^{(2)}_{\Delta - d/2}(k\eta)$$

where, $H^{(2)}_\nu$ is the Hankel function of second kind.

- Solution corresponding to α- vacua:

$$f_k^{\alpha}(\eta) = (\cosh\alpha) f_k^{BD}(\eta) + \sinh\alpha \left(f_k^{BD}(\eta) \right)^* .$$

Boundary Representation for Bunch-Davies vacuum

- Using

$$H_\nu^{(2)}(x) = J_\nu(x) - iY_\nu(x)$$

we have $c_1 = 1, c_2 = -i$. This gives us:

Boundary Representation for Bunch-Davies vacuum

- Using

$$H_\nu^{(2)}(x) = J_\nu(x) - iY_\nu(x)$$

we have $c_1 = 1, c_2 = -i$. This gives us:

Bunch-Davies Boundary Representaiton.

$$\phi(\eta, x) = \int d^d x' K_\Delta(\eta, x; x') \mathcal{O}_\Delta(x') - i \int d^d x' K_{d-\Delta}(\eta, x; x') \mathcal{O}_{d-\Delta}(x')$$

□

Boundary Representation for α vacuum

- The constants are:

$$c_1 = \cosh\alpha + \sinh\alpha$$
$$c_2 = i\,(\sinh\alpha - \cosh\alpha)$$

Hence we get:

α-vacua Boundary Representation.

$$\phi(\eta, x) = (\cosh\alpha + \sinh\alpha)\int d^d x' K_\Delta(\eta, x; x')\mathcal{O}_\Delta(x') + i\,(\sinh\alpha - \cosh\alpha)\int d^d x' K_{d-\Delta}(\eta, x; x')\mathcal{O}_{d-\Delta}(x')$$

□

Outline

Spin-1 Case

For spin-1, Source free Maxwell equation reduce to the K.G. equation of scalar of mass $m^2 = d - 1$,

- Using holographic gauge condition
  $\mathrm{A}_\eta(\eta, x) = 0.$
- Spin-1 fields can be recast as a of scalar fields of equal mass
  $\phi(\eta, x) = \eta A(\eta, x).$

Smearing function For $K_\Delta(\eta, x)$:

$$K_\Delta(\eta, x; x') = \frac{\Gamma\left(\Delta - \frac{d}{2} + 1\right) 2^{d/2}}{\Gamma(\Delta - d + 1)} \left(\frac{\eta^2 - |\Delta x|^2}{\eta}\right)^{\Delta - d}$$

□

Smearing function for $K_{d-\Delta}$.

$$K_{d-\Delta}(\eta, x; x') = -\frac{(4\pi)^{\frac{d}{2}}\Gamma(\Delta)}{\Gamma(\Delta - d/2)} \sin\left(\frac{\pi}{2}(d-1)\right) \left(\frac{\eta^2 - |\Delta x|^2}{\eta}\right)^{-\Delta}$$

□

General integer spin

- Obtain the K.G. equation of motion with mass of scalar field $m^2 = (2-s)(s+d-2)$.
- Using Holographic gauge condition obtain as

  $\mathrm{F}_{\eta\ldots\eta} = F_{\mu_1\eta\ldots\eta} = \cdots = F_{\mu_1..\mu_{s-1}\eta} = 0.$

  $F_{A_1\ldots A_s}$ symmetric tensor of rank s.
- Higher spin fields can be written as a multiplet of scalar fieldsof equal mass

  $\phi_{\mu_1..\mu_s} = \eta^s F_{\mu_1..\mu_s}$

General spin smearing function

- $K_{\Delta}(\eta, x)$ will be analytic and obtain the same form as free scalar (similar type as shown for the spin-1 cases).
- $K_{d-\Delta}(\eta, x)$ also give a analytic solution same as scalar fields (here also the story will be same).

Outline

Summary

- Earlier work obtained the boundary representation for principal series scalars but ran into divergence issues with other mass range and higher spin fields.
- We obtained boundary representations for scalars of all mass and higher spin fields.
- Depending on the values of the parameters Δ and the dimension, smearing function become analytic or distributional.
- We also extended the initial condition from Bunch-Davies to general α-vacua.

Thank You !
Questions or comments