

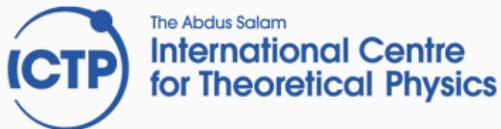
S-matrix bounds from the conformal bootstrap

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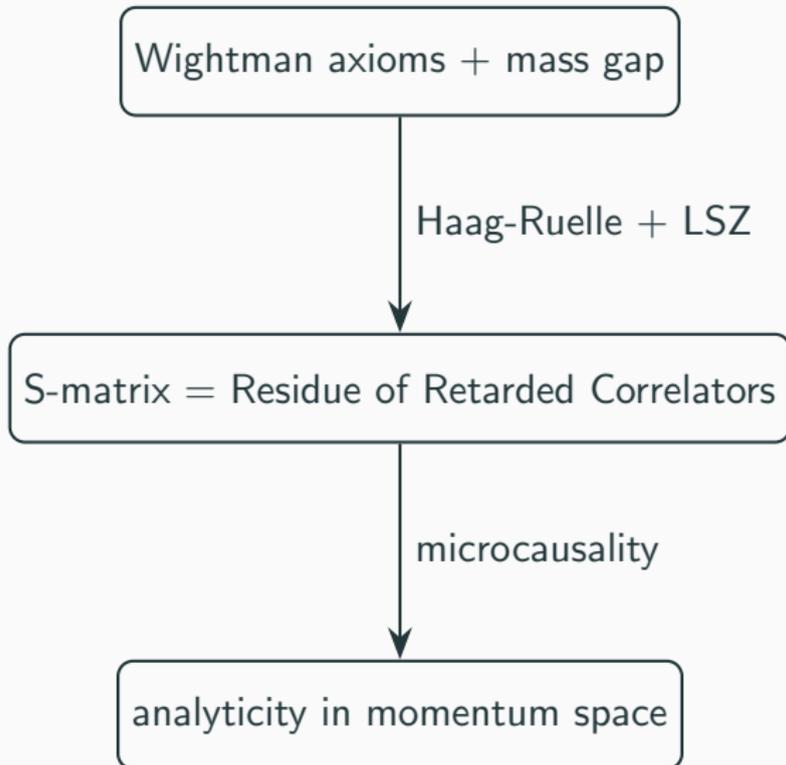
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Progress of the Theoretical Bootstrap workshop, Kyoto

W.I.P. with T. Pochart and B. van Rees



Motivation: Analyticity of S-matrices



Mismatch: Perturbative vs non-perturbative analyticity

Consider the scattering of lightest external particles in a theory

- *In perturbation theory*, scattering amplitudes are analytic in the cut s , t and u planes:

$$s, t, u \in \mathbb{C} \setminus (4m^2, \infty)$$

- *Non-perturbatively*, we have analyticity for

$$s \in \mathbb{C} \setminus (4m^2, \infty), \quad |t| < 4m^2$$

where $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$ and $u = (p_1 - p_4)^2$.

QFT in AdS

- Consider a QFT in (rigid) AdS and push operator insertions to the boundary to define boundary operators/correlation functions.
- AdS isometry acts as conformal symmetry on the boundary operators.
- Defines a conformal theory (CT) which satisfies all CFT axioms.
- Except with *no stress tensor*.

S-matrices from the flat space limit

- Mass of particles and boundary operator dimension Δ related by

$$m^2 R^2 = \Delta(\Delta - d)$$

- Thus the flat space limit \Rightarrow boundary operator dimensions $\Delta \rightarrow \infty$.
- Not a new concept - see Polchinski '99, Giddings '99, Penedones '10 ...

- Consider the four point function of identical scalar operators in the CT

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4) \rangle = \frac{\mathcal{G}(s, t, u)}{x_{12}^{2\Delta_\phi} x_{34}^{2\Delta_\phi}}$$

where

$$s = 4 \left(\frac{1 - \sqrt{\rho\bar{\rho}}}{1 + \sqrt{\rho\bar{\rho}}} \right)^2, \quad t = 4 \left(\frac{\sqrt{\rho} + \sqrt{\bar{\rho}}}{1 + \sqrt{\rho\bar{\rho}}} \right)^2, \quad u = 4 - s - t$$

and

$$\rho = \frac{1 - \sqrt{1 - z}}{1 + \sqrt{1 - z}}, \quad \bar{\rho} = \frac{1 - \sqrt{1 - \bar{z}}}{1 + \sqrt{1 - \bar{z}}}$$

- KPvRZ '20 conjectured that if one further writes

$$\mathcal{G}(s, t, u) = \mathcal{G}_{\text{gff}}(s, t, u) + \mathcal{G}_c(s, t, u) T(s, t, u)$$

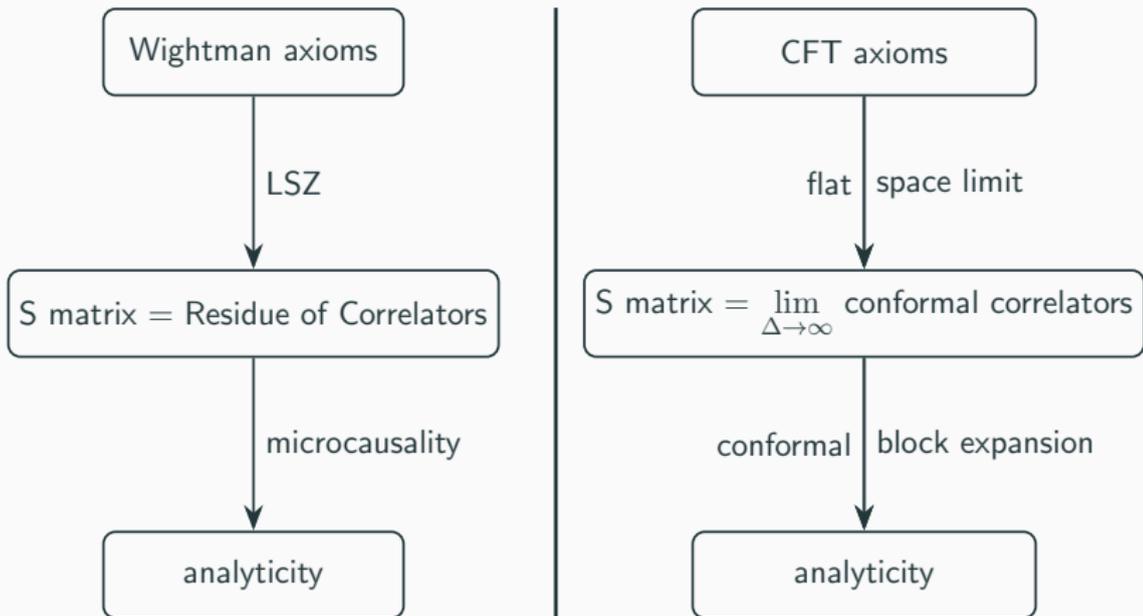
where \mathcal{G}_{gff} is the disconnected correlator and \mathcal{G}_c is a contact Witten diagram in AdS,

- Then the scattering amplitude $M(s, t, u)$ is given by:

$$\lim_{\Delta_\phi \rightarrow \infty} T(s, t, u) = M(s, t, u)$$

where now the variables on the right hand side are the usual Mandelstam variables.

QFT in AdS instead of LSZ



Non-perturbative test: Numerical conformal bootstrap

- For the numerics, specialize to AdS_3/CT_2 .
- To emphasize: no stress tensor and no Virasoro symmetry, only global conformal symmetry.
- Bound the value of the correlation function $\mathcal{G}(z, \bar{z})$ at a point following Paulos, Zheng '21

The bootstrap algorithm of Paulos, Zheng '21

$$\sum_{\mathcal{O}} \text{Crossing Diagram} = \sum_{\mathcal{O}} \text{Non-crossing Diagram}$$

- Consider the crossing equation

$$F_{0,0}(z, \bar{z}) + \sum_{\Delta, \ell} C_{\Delta, \ell}^2 F_{\Delta, \ell}(z, \bar{z}) = 0$$

here $F_{\Delta, \ell}$ is the crossing block:

$$F_{\Delta, \ell}(z, \bar{z}) \equiv ((1-z)(1-\bar{z}))^{\Delta_\phi} G_{\Delta, \ell}(z, \bar{z}) - (z\bar{z})^{\Delta_\phi} G_{\Delta, \ell}(1-z, 1-\bar{z})$$

- The algorithm: Find two functionals α_{\pm} such that

$$\alpha_{\pm}(\Delta, \ell) \geq \pm G_{\Delta, \ell} \left(\frac{1}{2}, \frac{1}{2} \right)$$

where $\alpha(\Delta, \ell) \equiv \alpha(F_{\Delta, \ell}(z, \bar{z}))$.

- Since

$$\mathcal{G} \left(\frac{1}{2}, \frac{1}{2} \right) = 1 + \sum_{\Delta, \ell} C_{\Delta, \ell}^2 G_{\Delta, \ell} \left(\frac{1}{2}, \frac{1}{2} \right)$$

- we have the upper and lower bounds

$$1 + \alpha_{-}(0, 0) \leq \mathcal{G} \left(\frac{1}{2}, \frac{1}{2} \right) \leq 1 - \alpha_{+}(0, 0)$$

where we used the fact that OPE coefficients $C_{\Delta, \ell}^2 \geq 0$.

- Use derivatives acting at $z = \bar{z} = \frac{1}{2}$ as basis for the functionals.
- In other words, we write

$$\alpha(\Delta, \ell) = \sum_{m+n \leq \Lambda} \alpha_{m,n} \partial_z^m \partial_{\bar{z}}^n F_{\Delta, \ell}(z, \bar{z})|_{z=\bar{z}=1/2}$$

- Λ controls the size of our basis and it is expected that as $\Lambda \rightarrow \infty$ we scan over all possible functionals.
- Note that in AdS_2/CT_1 these functionals can be constructed analytically as was done by Cordova, He, Paulos '22.

Gap assumptions

- Since $mR \sim \Delta \Rightarrow$ twist gap scales like Δ_ϕ .
- We set twist gap to be $\sqrt{3}\Delta_\phi$.
- In the flat space limit \Rightarrow all other external states have mass greater than $\sqrt{3}$.
- So the earlier equation becomes

$$\alpha_{\pm}(\Delta, \ell) \geq \pm G_{\Delta, \ell} \left(\frac{1}{2}, \frac{1}{2} \right)$$

for $\Delta - \ell \geq \sqrt{3}\Delta_\phi$.

- At the point $z = \bar{z} = \frac{1}{2}$ i.e. $s = 2, t = 0$, we have

$$\mathcal{G}_c \sim \frac{1}{4^{\Delta_\phi} \sqrt{\Delta_\phi}}$$

Since

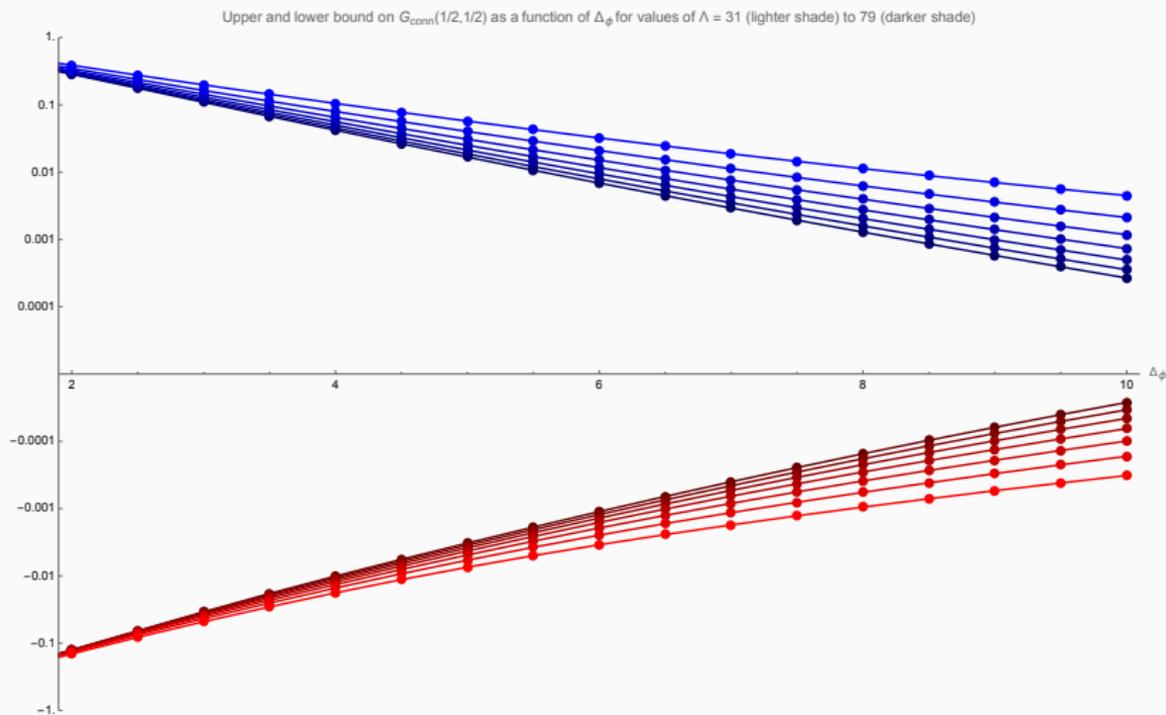
$$T \sim \frac{\mathcal{G} - \mathcal{G}_{\text{gff}}}{\mathcal{G}_c}$$

To get a finite amplitude, we need

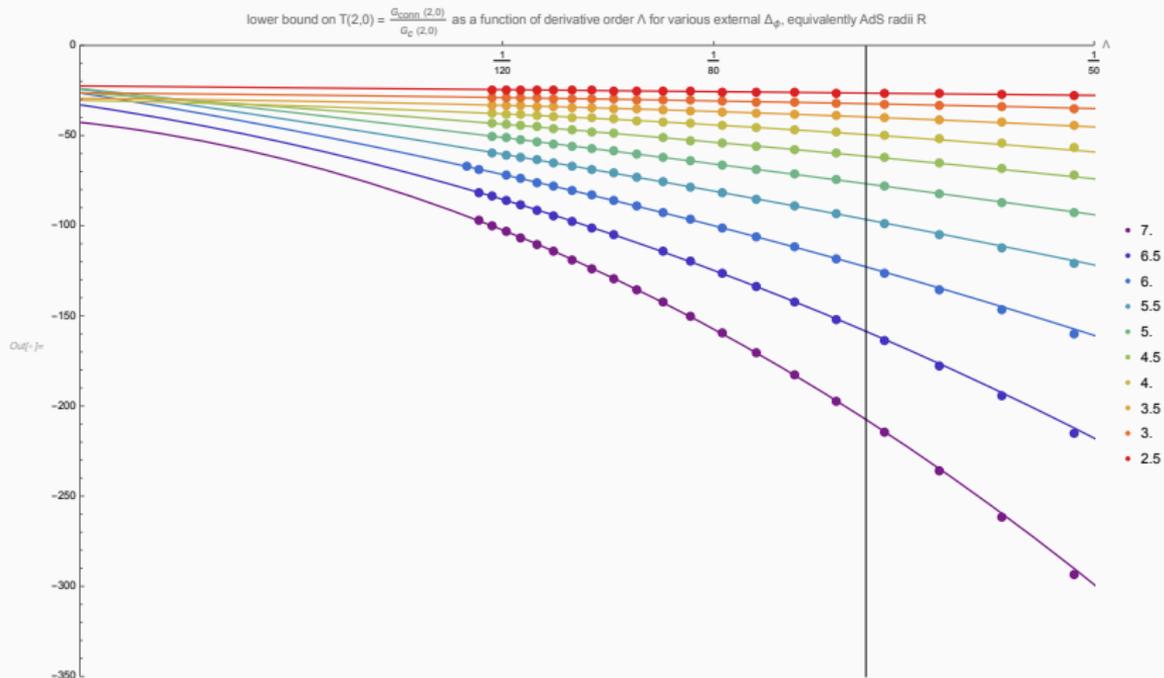
$$|\mathcal{G} - \mathcal{G}_{\text{gff}}| \sim \frac{1}{4^{\Delta_\phi} \sqrt{\Delta_\phi}}$$

- Any correlator must look like the GFF correlator in this limit!

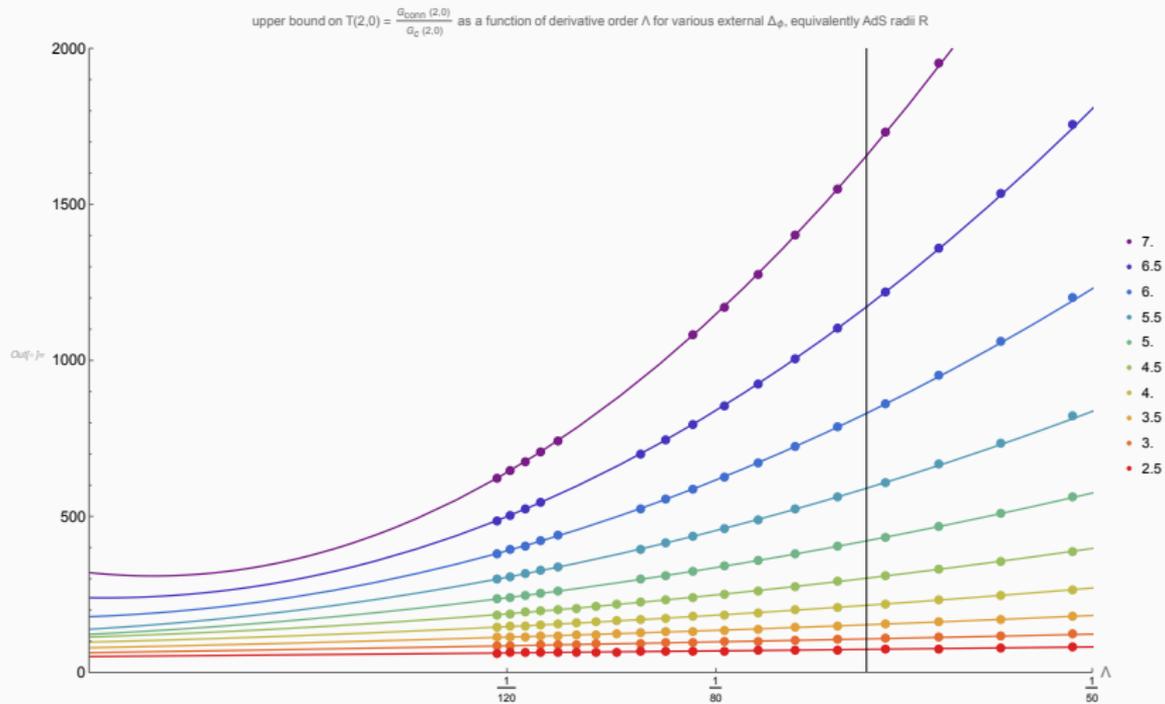
0th order check: Connected correlator



S matrix bounds: lower



S matrix bounds: upper



Conclusion

- It seems possible to get S-matrix bounds from conformal bootstrap albeit with a lot of effort.
- In progress: bounds on the derivative of the S-matrix at $s = 2, t = 0$.
- For the future: massive particles - analyticity domains, anomalous thresholds?
- For the future: massless particles?

Auxiliary slides

We augment standard numerical bootstrap in two ways:

- Large spin and large Δ positivity of the functionals as an extra constraint which reduces the number of spin constraints that we need to put in.
- The usual idea is that due to the convergence of the conformal block expansion, we need to only impose the condition

$$\alpha_{\pm}(\Delta, l) \geq \pm G_{\Delta, l} \left(\frac{1}{2}, \frac{1}{2} \right)$$

upto some finite (but large) l_{max} .

- Experimentally we were finding that this $l_{max} \sim 2\Lambda$ without imposing the aforementioned large spin constraints.
- Imposing the large spin and large energy constraints reduces this to $l_{max} \sim \Lambda$.
- (Technical) We also factor out the large Δ behaviour of the conformal blocks before which gives a better behaved polynomial approximation to the conformal blocks at large Δ . This allows us to use polynomials of lower order than usual.