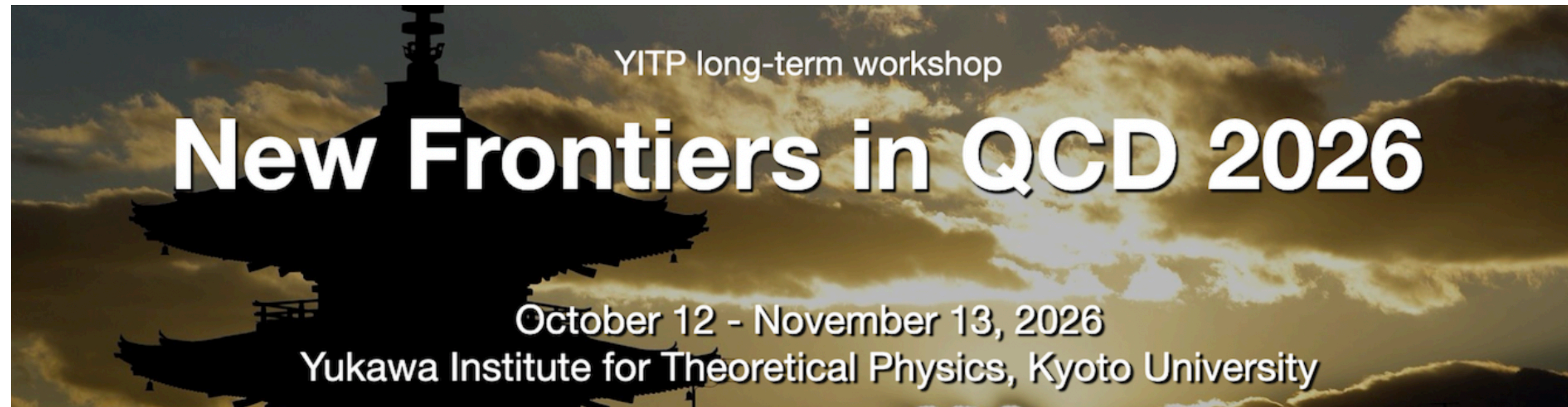
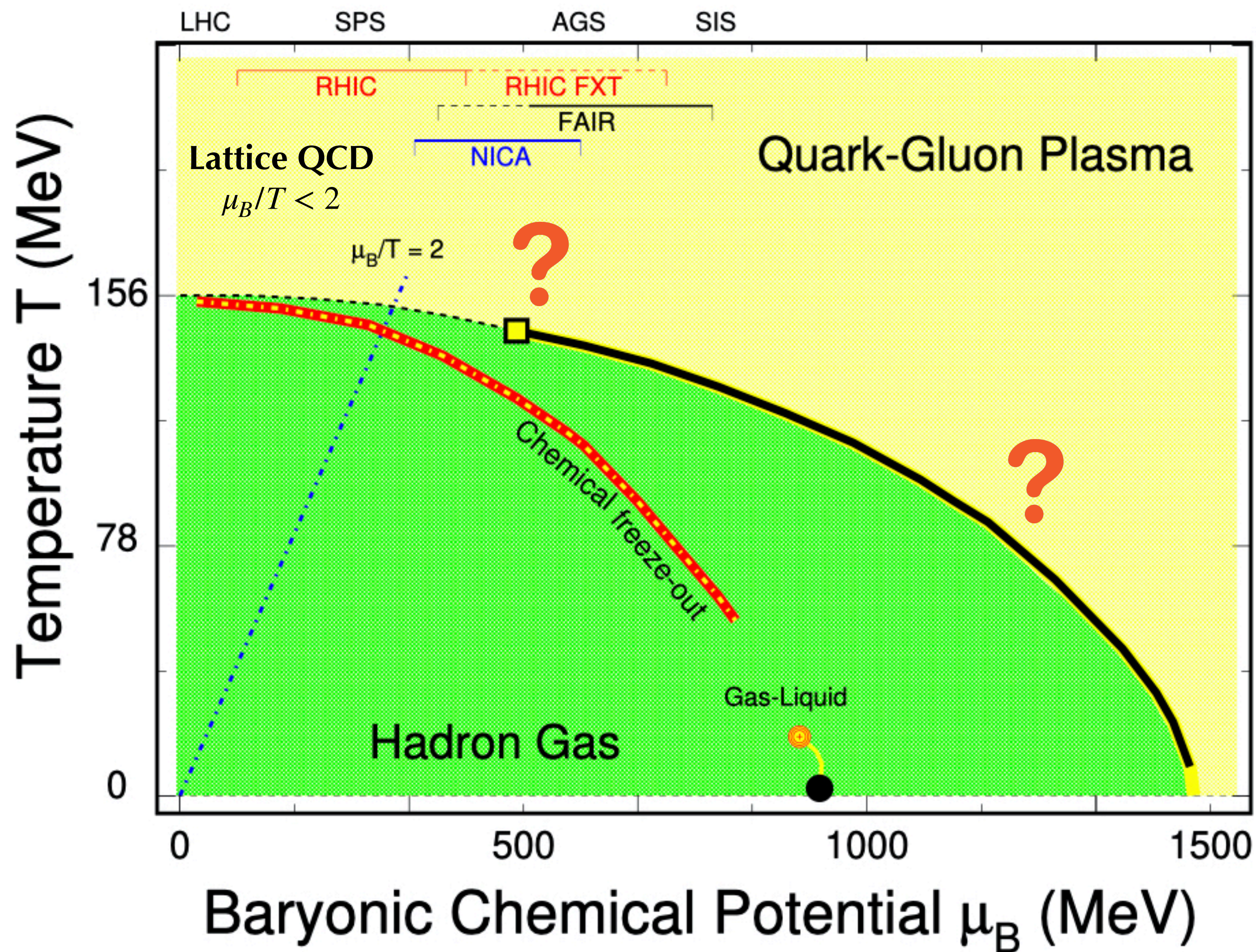


FLUCTUATION PROBES OF THE QCD PHASE STRUCTURE: RESULTS AND PROSPECTS

Ashish Pandav
Kent State University, Ohio, US

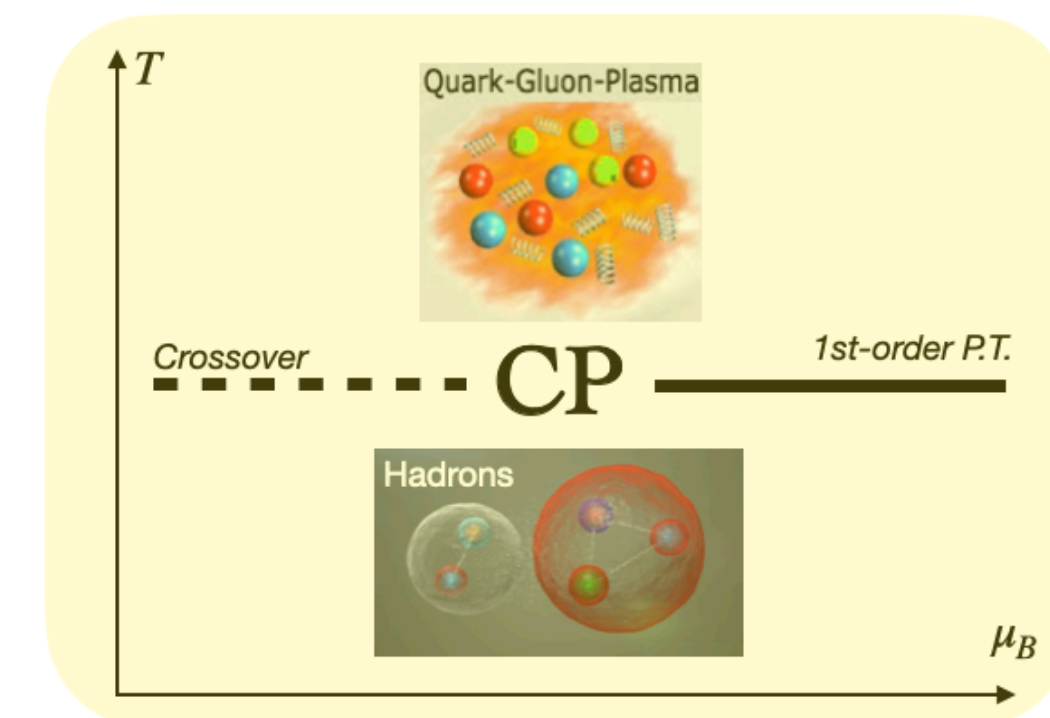


INTRODUCTION



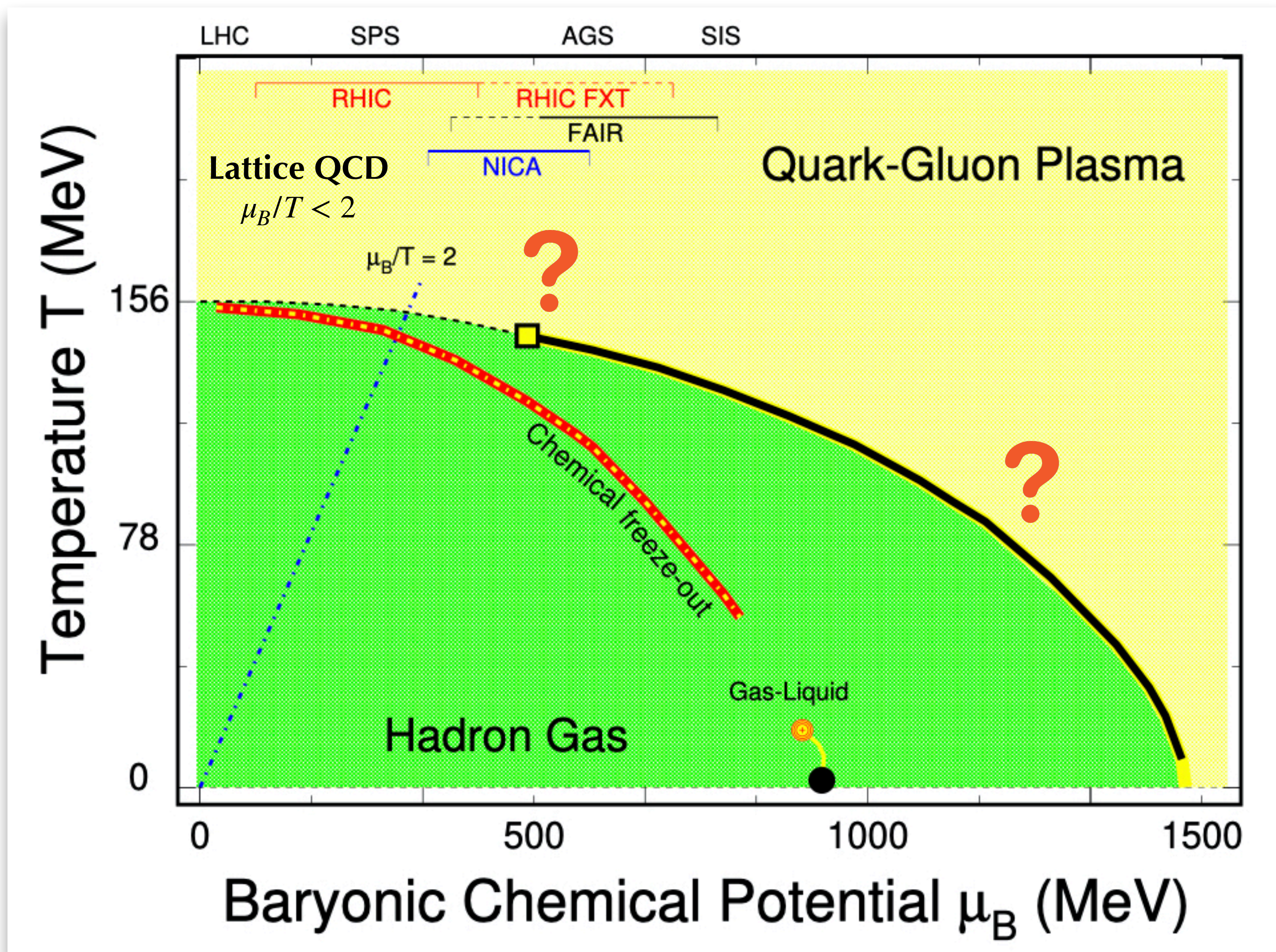
Phase diagram of strongly interacting matter:

- Hadronic & QGP phase
- Crossover at small μ_B
- Transition temperature T_c
- First-order P.T. at large μ_B
- Critical point (CP)

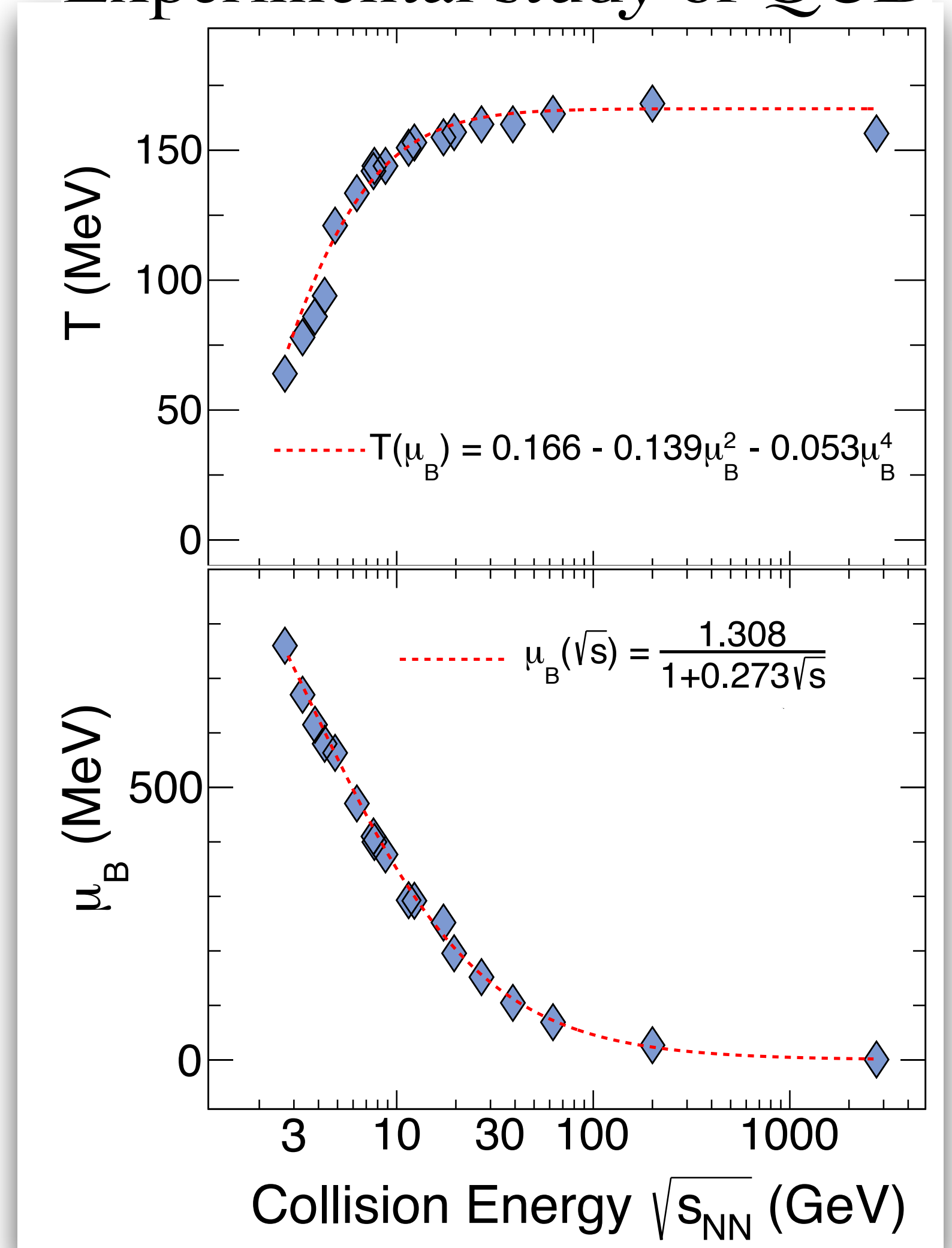


Largely conjectured, theory challenging at high μ_B , experimental exploration very important

INTRODUCTION

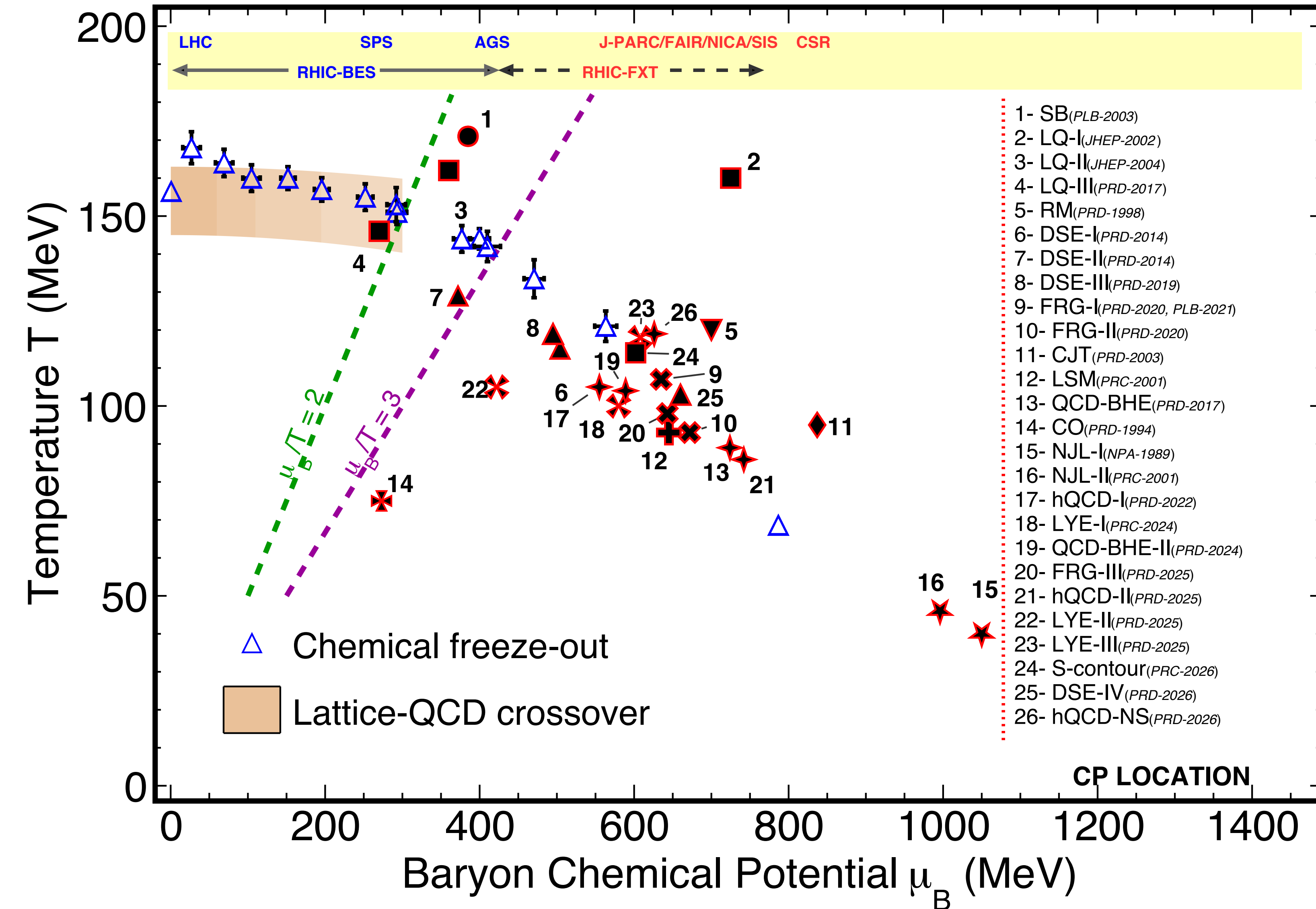


Experimental study of QCD



Beam Energy Scan to explore QCD phase structure

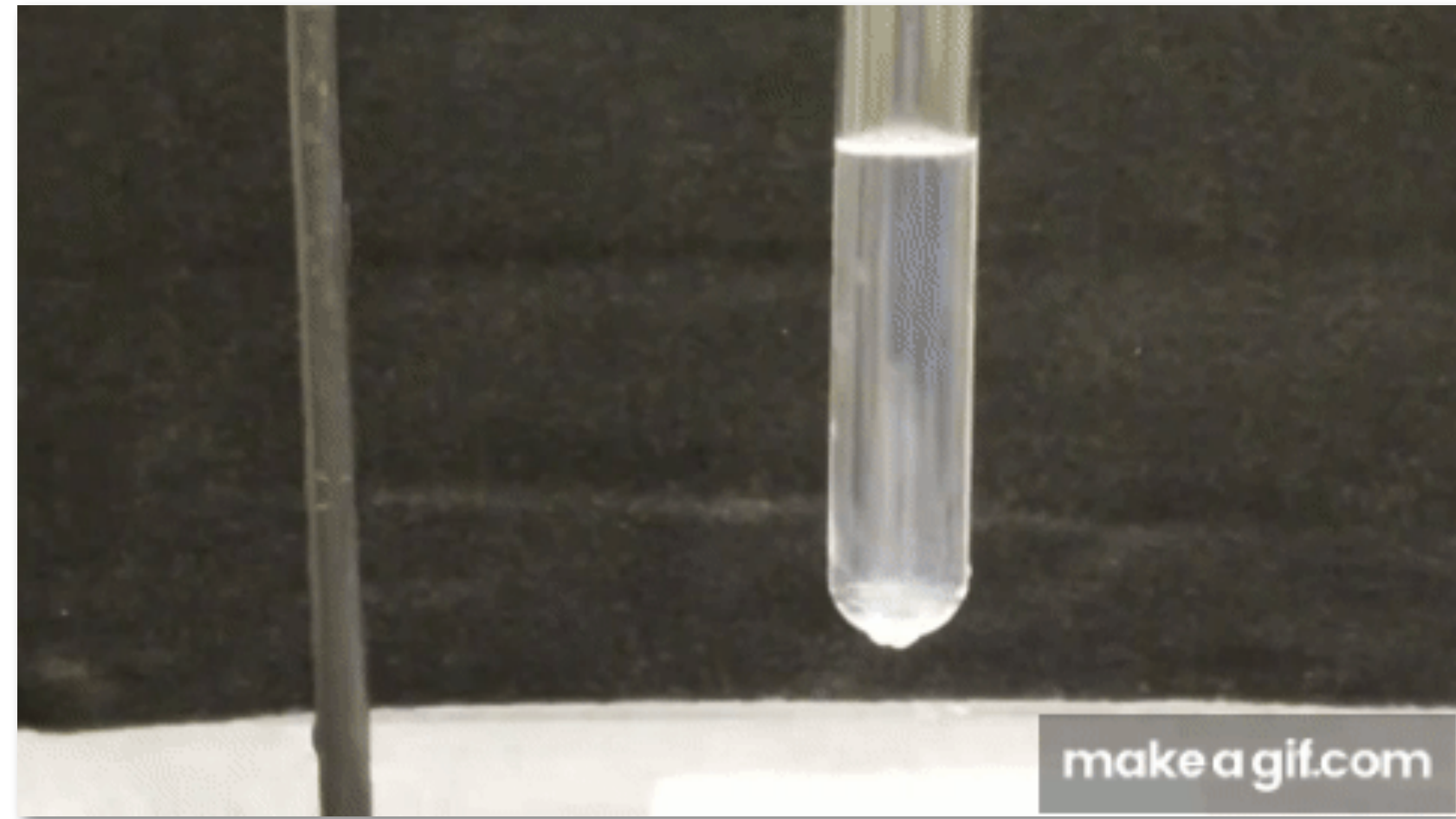
QCD CRITICAL POINT: THEORY ESTIMATES



CP locations from theory

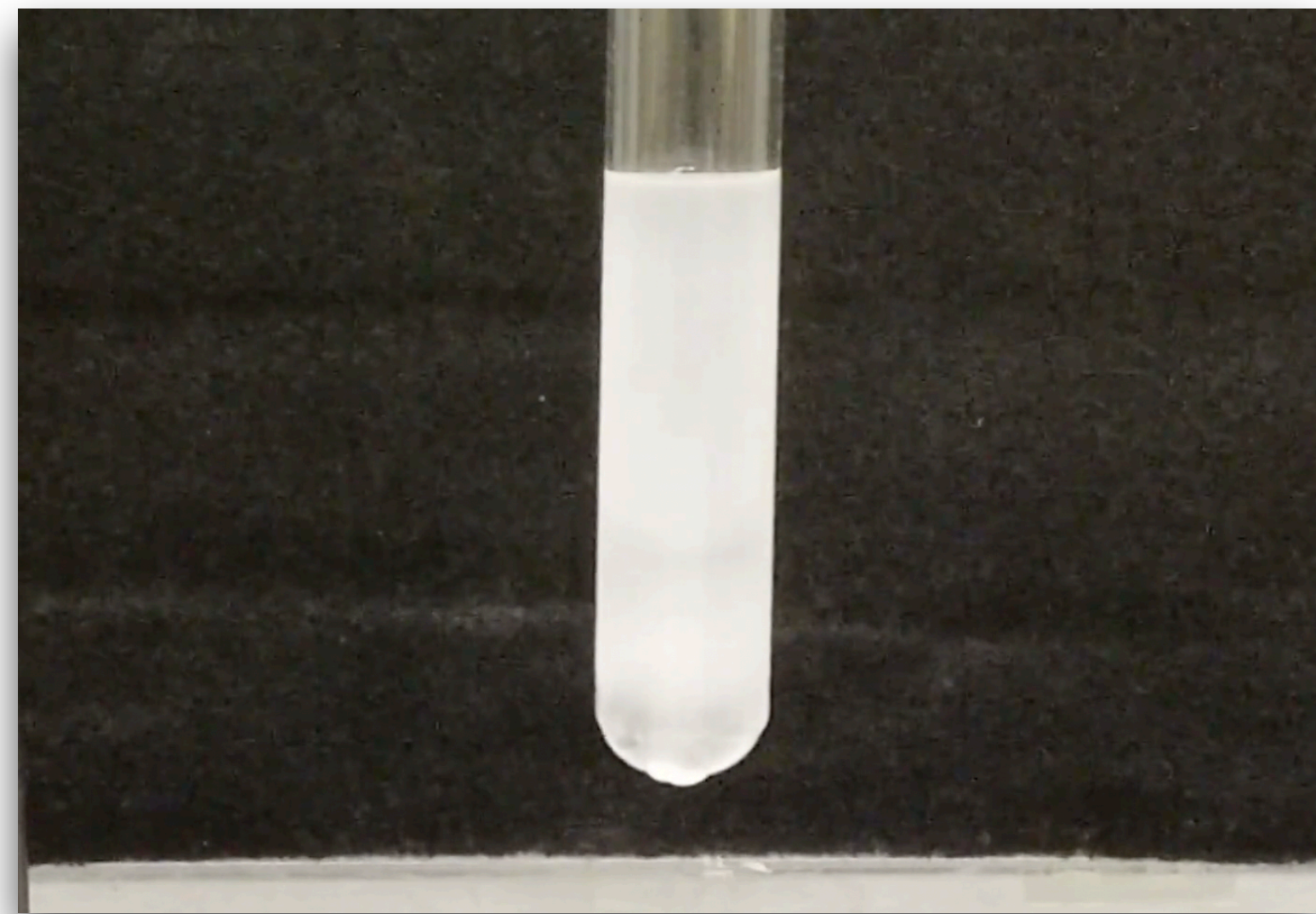
- Lattice calculations at high μ_B suffer from sign problem
- Effective models have several underlying assumptions/ approximations
- Experimental search crucial.

WHAT CAN BE GOOD OBSERVABLES?



Development of long range density fluctuations

Divergence of correlation length, thermodynamic response functions (susceptibility, compressibility etc)



Enhanced fluctuations expected near CP.

Fluctuations of particle multiplicities coupled to critical models.

How to quantify fluctuations?

Measure moments/cumulants

Critical Opalescence: CO_2 appears milky white

OBSERVABLES (SELECTED)

Observable	Definition	Comments
Particle ratio fluctuation ν_{dyn}	$\frac{\langle N_x(N_x - 1) \rangle}{\langle N_x \rangle^2} + \frac{\langle N_y(N_y - 1) \rangle}{\langle N_y \rangle^2} - 2 \frac{\langle N_x N_y \rangle}{\langle N_x \rangle \langle N_y \rangle}$	For CP search: Look for non-monotonic collision energy dependence
Momentum correlation $\langle \Delta p_{T,i} \Delta p_{T,j} \rangle$	$\frac{1}{N_{ev}} \sum_{k=1}^{N_{ev}} \frac{C_k}{N_k(N_k - 1)}$ $C_k = \sum_{i=1}^{N_k} \sum_{j=1, j \neq i}^{N_k} (p_{T,i} - \langle M(p_T) \rangle)(p_{T,j} - \langle M(p_T) \rangle)$	Look for non-monotonic collision energy dependence
Intermittency: Scaled factorial mom. $F_q(M)$	$\frac{\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i(n_i - 1) \dots (n_i - q + 1) \rangle}{\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i \rangle^q}$	Look for scaling behavior of factorial moments w.r.t bin size M
Femtoscopic correlation function $C(k^*)$	$N \frac{A(k^*)}{B(k^*)}$	Look for power law scaling of correlation function
Fluctuations/Cumulants of charged particle multiplicities C_n	$C_1 = \langle n \rangle, C_2 = \langle \delta n^2 \rangle$ $C_3 = \langle \delta n^3 \rangle$ $C_4 = \langle \delta n^4 \rangle - 3 \langle \delta n^2 \rangle^2$	Look for non-monotonic collision energy dependence

OBSERVABLES (SELECTED)

Observable	Definition	Comments
Particle ratio fluctuation ν_{dyn}	$\frac{\langle N_x(N_x - 1) \rangle}{\langle N_x \rangle^2} + \frac{\langle N_y(N_y - 1) \rangle}{\langle N_y \rangle^2} - 2 \frac{\langle N_x N_y \rangle}{\langle N_x \rangle \langle N_y \rangle}$	For CP search: Look for non-monotonic collision energy dependence
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CUMULANTS

●Cumulants: $n = \text{net-proton multiplicity in an event}$

$$C_1 = \langle n \rangle$$

$$C_2 = \langle \delta n^2 \rangle \quad * \delta n = n - \langle n \rangle$$

$$C_3 = \langle \delta n^3 \rangle$$

$$C_4 = \langle \delta n^4 \rangle - 3 \langle \delta n^2 \rangle^2$$

$$C_5 = \langle \delta n^5 \rangle - 10 \langle \delta n^3 \rangle \langle \delta n^2 \rangle$$

$$C_6 = \langle \delta n^6 \rangle - 15 \langle \delta n^4 \rangle \langle \delta n^2 \rangle - 10 \langle \delta n^3 \rangle^2 + 30 \langle \delta n^2 \rangle^3$$

●Factorial cumulants:

$$\kappa_1 = C_1$$

$$\kappa_2 = -C_1 + C_2$$

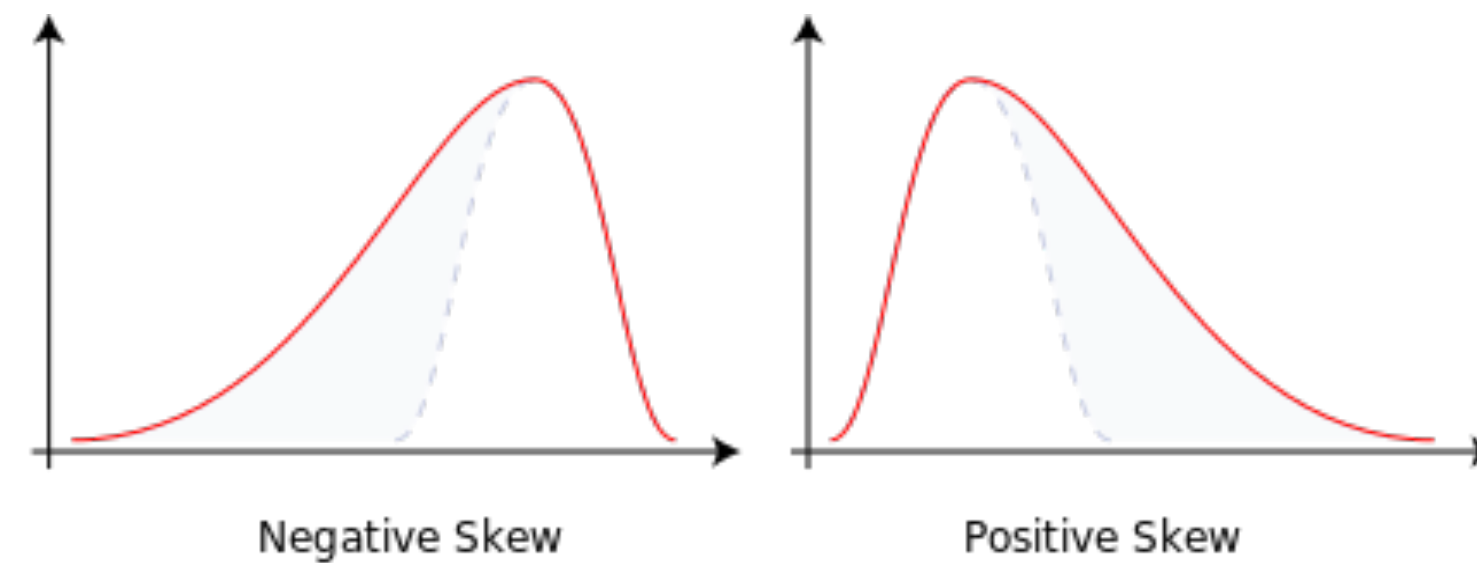
$$\kappa_3 = 2C_1 - 3C_2 + C_3$$

$$\kappa_4 = -6C_1 + 11C_2 - 6C_3 + C_4$$

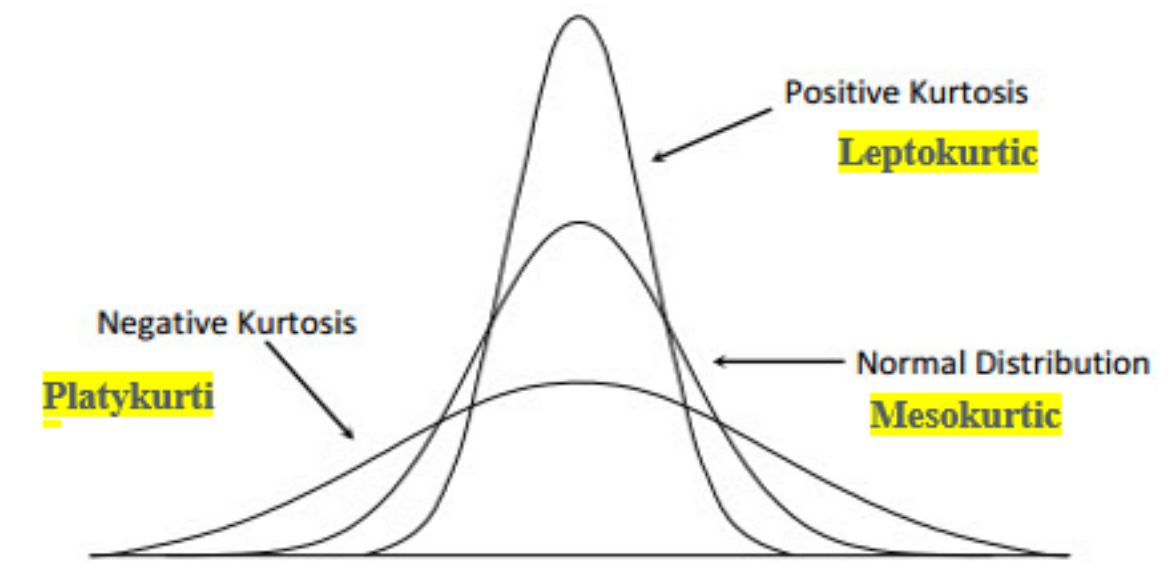
$$\kappa_5 = 24C_1 - 50C_2 + 35C_3 - 10C_4 + C_5$$

$$\kappa_6 = -120C_1 + 274C_2 - 225C_3 + 85C_4 - 15C_5 + C_6$$

Skewness: Asymmetry



Kurtosis: Peakedness



R.V. Gavai and S. Gupta, PLB696, 459(11)
S. Ejiri, F. Karsch, K. Redlich, PLB633, 275(06)
A. Bazavov et al., PRL109, 192302(12)
S. Borsanyi et al., PRL111, 062005(13)

M. A. Stephanov, PRL 107 (2011) 052301

Related to correlation length: $C_2 \sim \xi^2, C_4 \sim \xi^7$

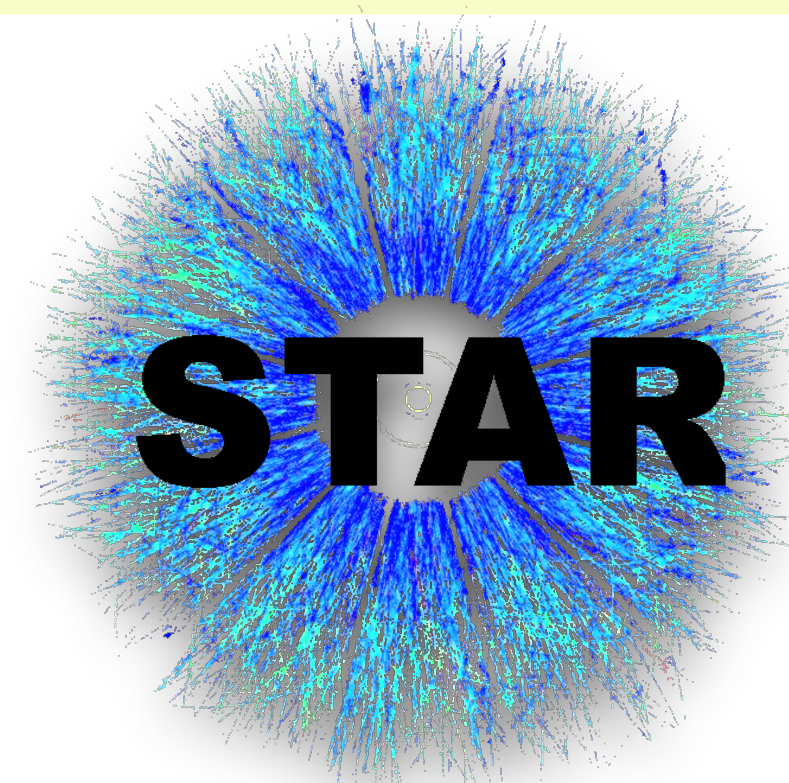
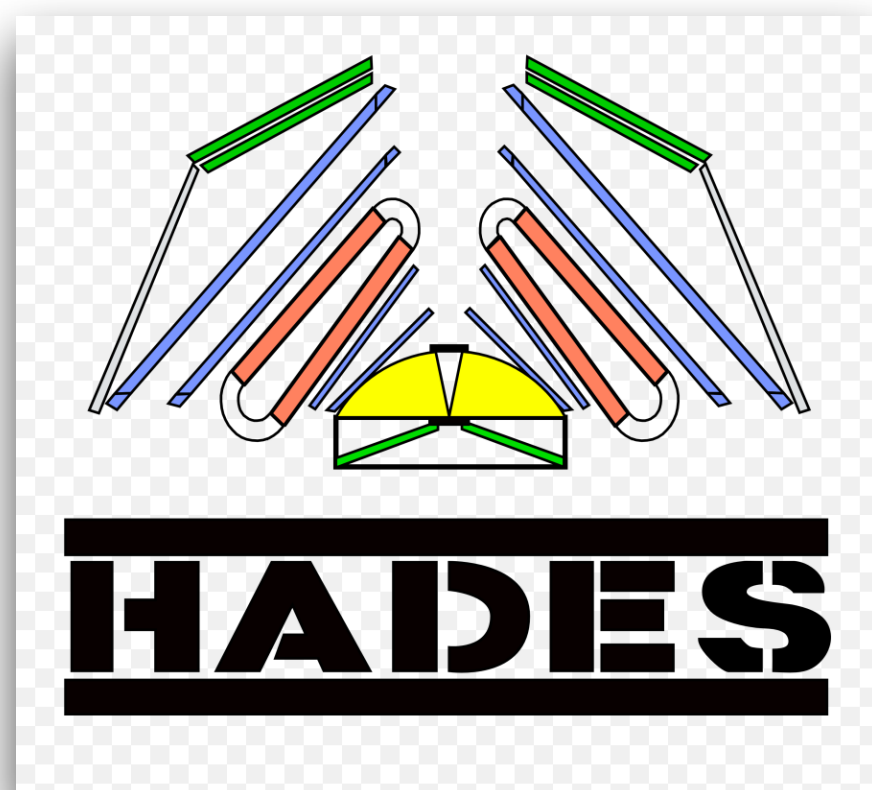
Finite size/time effects reduces ξ

Higher order $\rightarrow$ more sensitivity

Related to susceptibilities: $\frac{C_{4q}}{C_{2q}} = \frac{\chi_4^q}{\chi_2^q}, \frac{C_{6q}}{C_{2q}} = \frac{\chi_6^q}{\chi_2^q}$

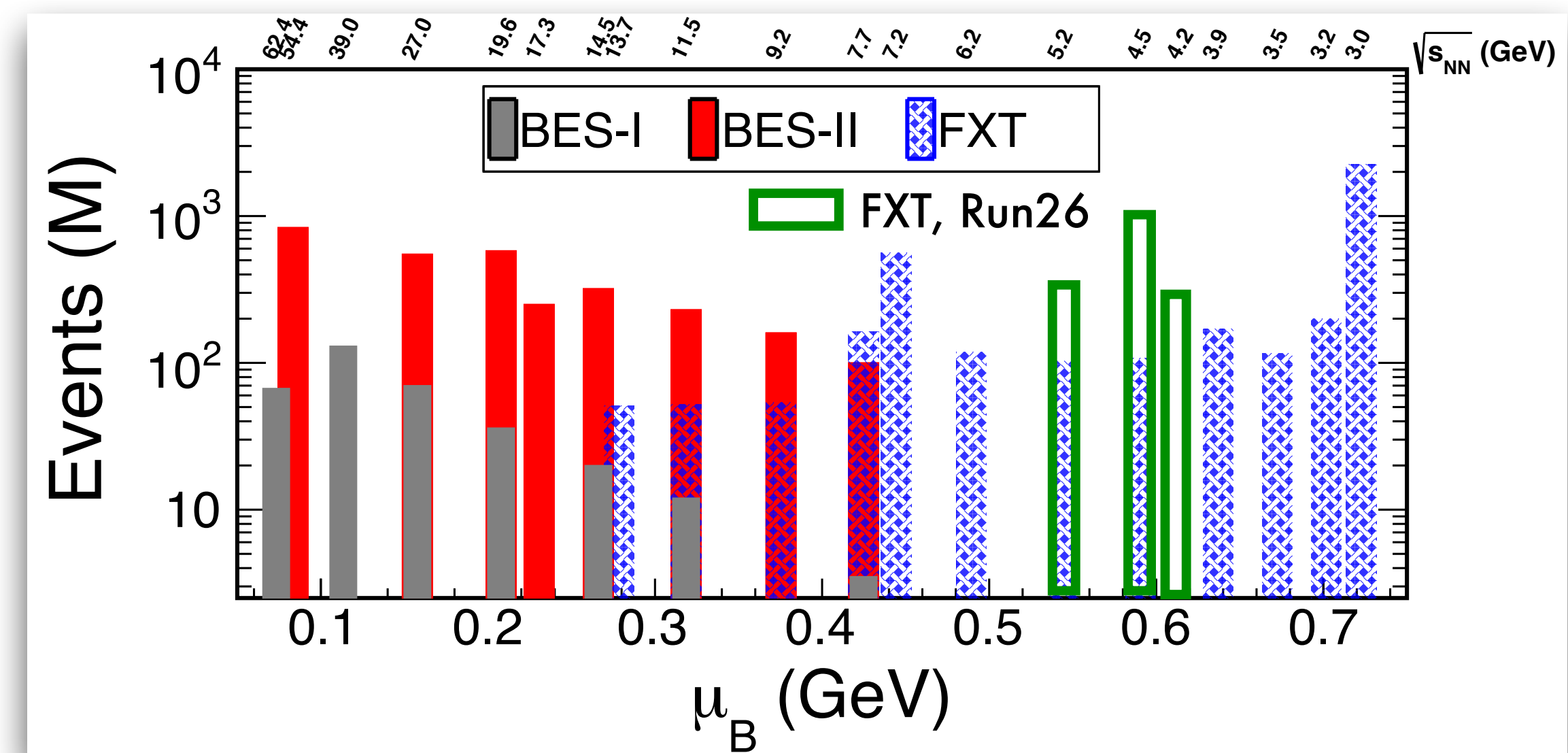
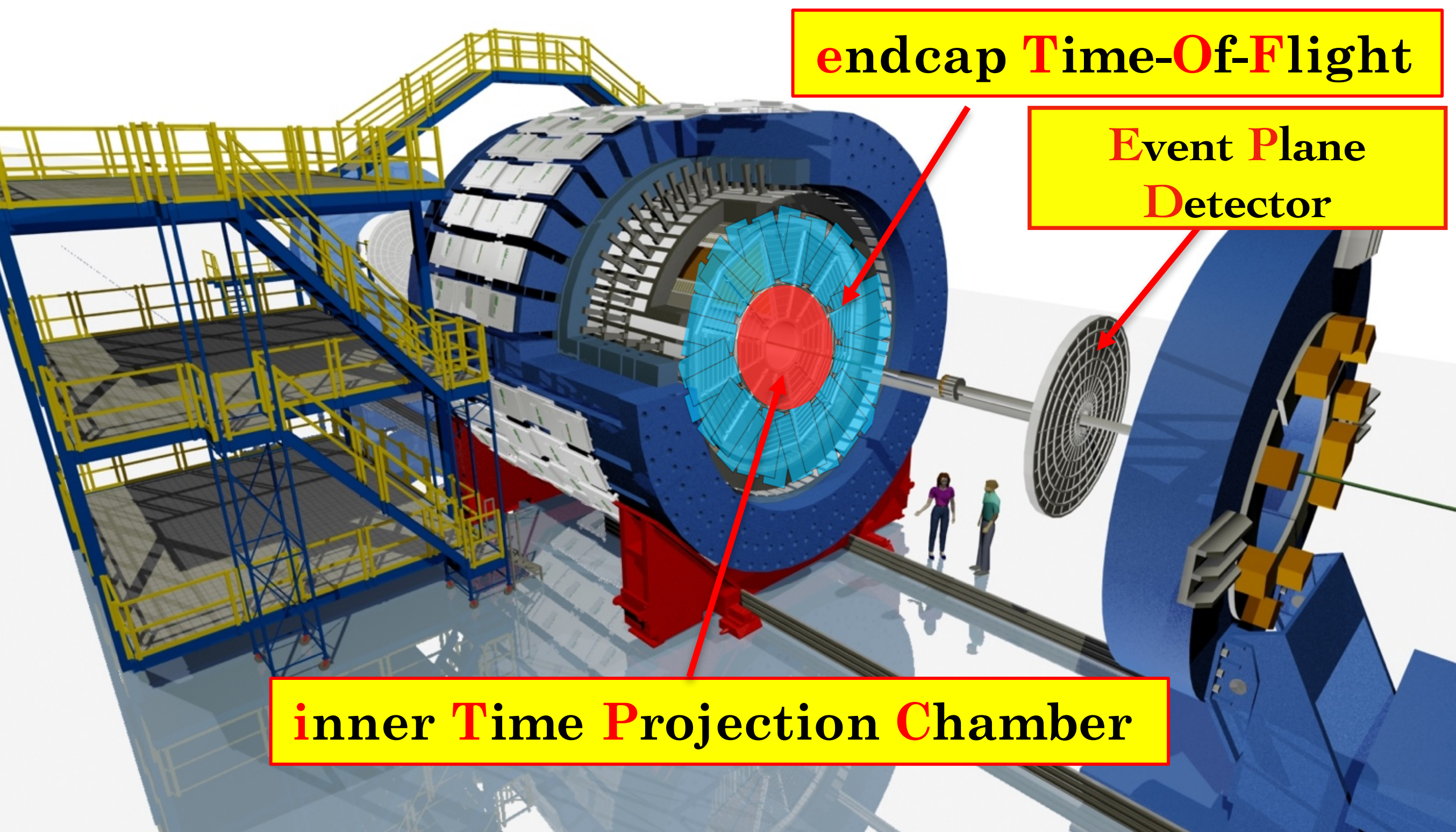
Comparison with lattice QCD,
HRG, QCD-based model calculations

EXPERIMENTS



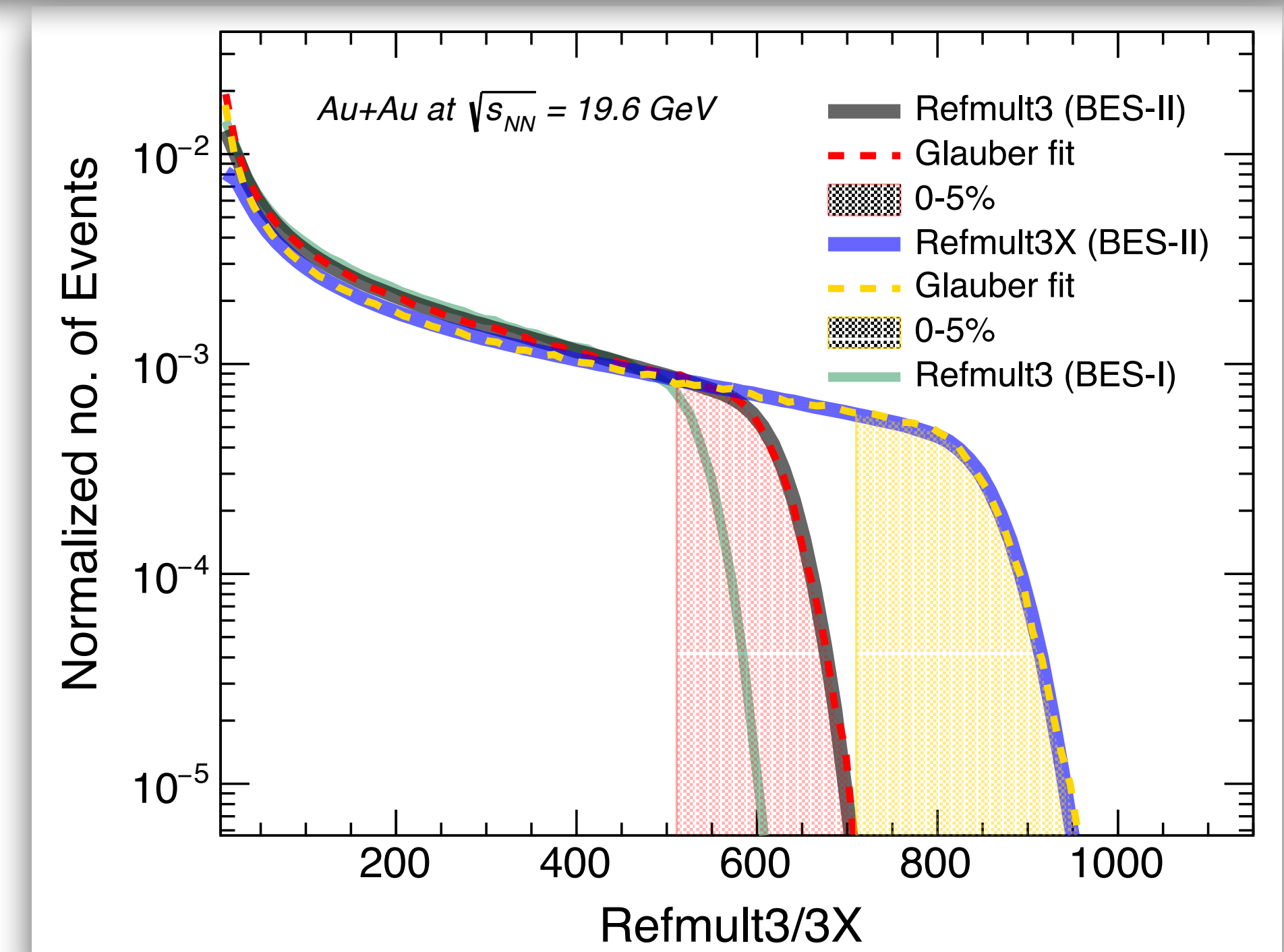
Experiment	Facility	Mode	Colliding energy ($\sqrt{s_{NN}}$)	Systems* <i>*Not all are listed</i>
HADES	SIS18	FXT	2.32 - 2.7 GeV	Au+Au, Ag+Ag, C+C, p+p
NA61/ SHINE	SPS	FXT	5.1 - 17.3 GeV	Pb+Pb, Be+Be, Ar+Sc, p+p
STAR	RHIC	COL/ FXT	3 - 200 GeV	Au+Au, U+U, Zr+Zr, Ru+Ru, Cu+Cu, d+Au, He3+Au, p+Au, p+p
ALICE	LHC	COL	2.76 - 13 TeV	Pb+Pb, Xe+Xe, p+Pb, p+p

BEAM ENERGY SCAN II AT STAR

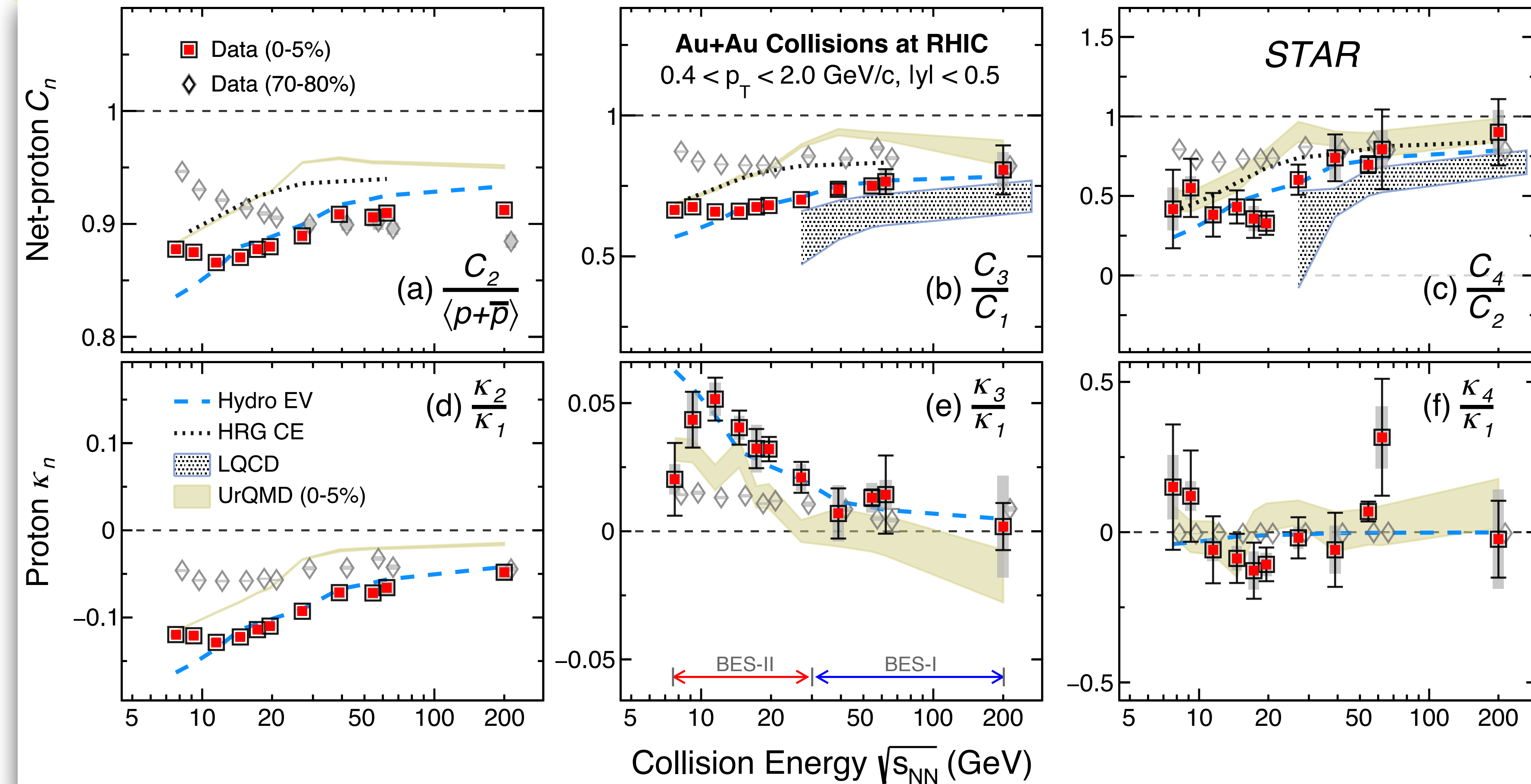


Precision measurements with high event statistics and important detector upgrades

Access to wide range of QCD phase diagram
 $3 \leq \sqrt{s_{NN}} \text{ (GeV)} \leq 200 \rightarrow 750 \geq \mu_B \text{ (MeV)} \geq 25$



RESULTS: DATA VS NON-CP MODELS



Non-CP models:

-Hydrodynamical model with excluded volume effects

-Hadronic transport model UrQMD

-Thermal model HRG

All models include exact baryon number conservation

STAR: PRL 135, 142301 (2025)

HRG CE: P. B. Munzinger et al, NPA 1008, 122141 (2021)

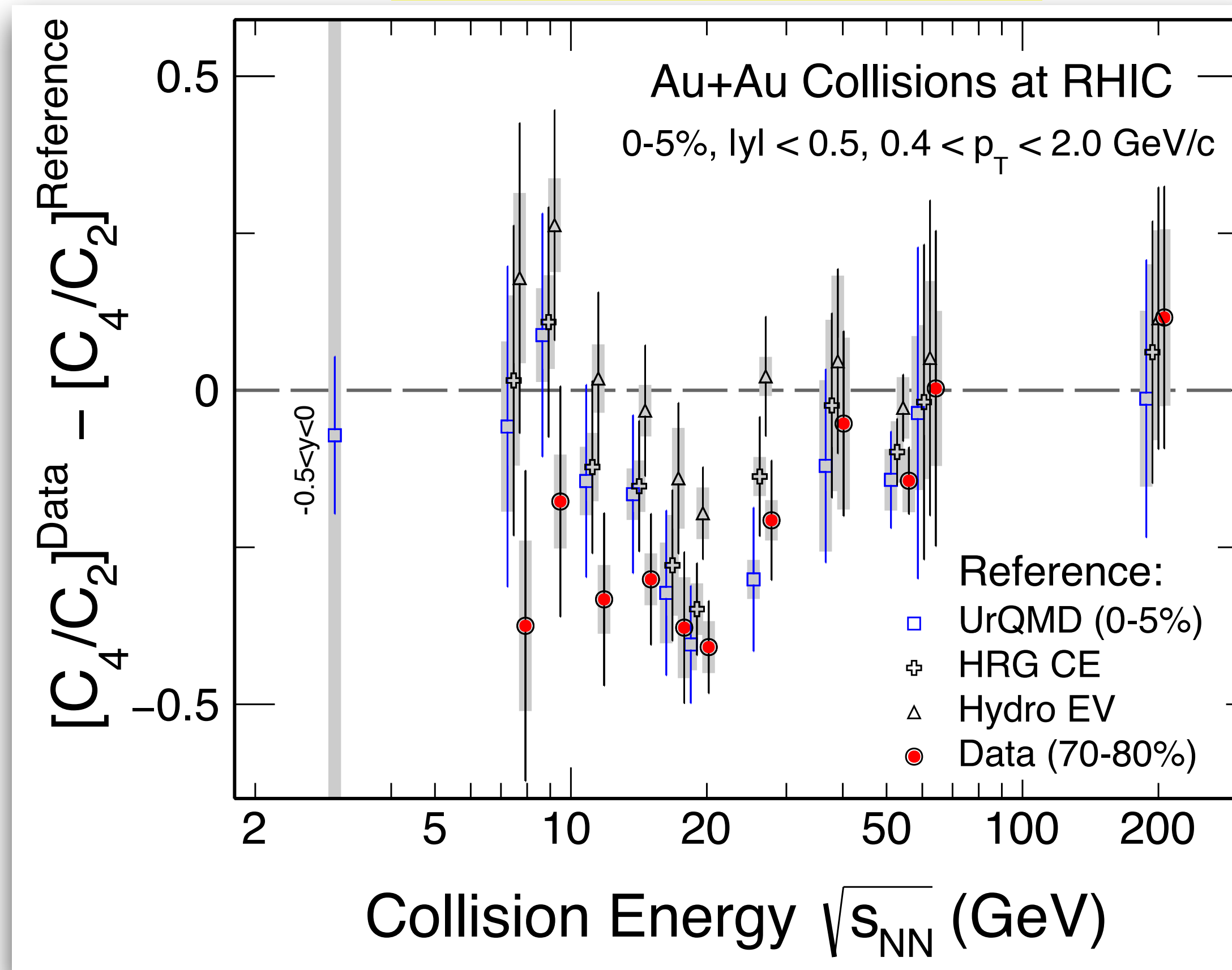
Hydro EV: V. Vovchenko et al, PRC 105, 014904 (2022)

LQCD: D. Bollweg et al, PRD 110, 054519 (2024)

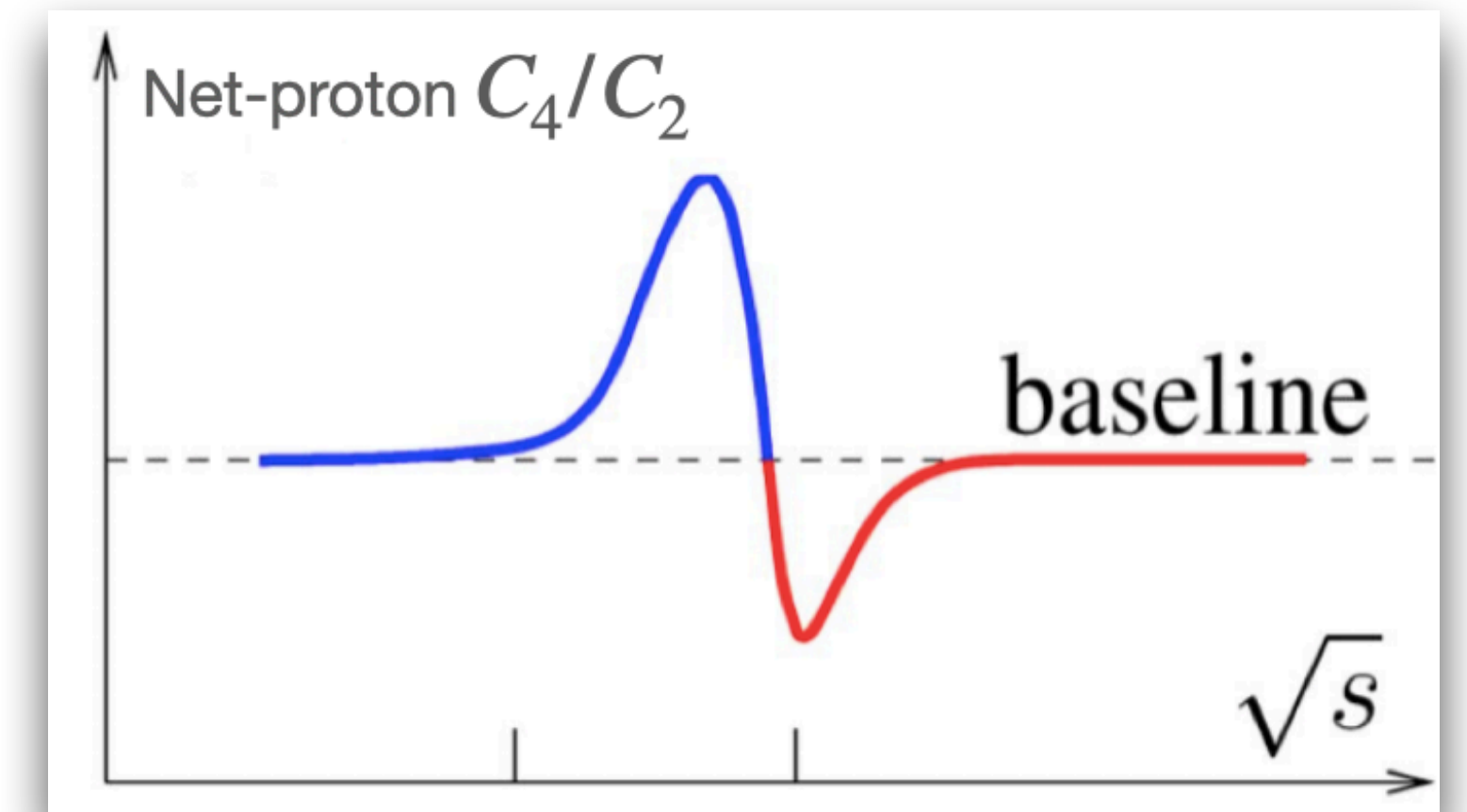
- Deviation between data and non-CP models seen in low collisions energies
 - Change of trends in $C_2 / \langle p + \bar{p} \rangle$ and κ_2 / κ_1 around $\sqrt{s_{NN}} \sim 10$ GeV. An opposite trend seen for κ_3 / κ_1 .
 - A dip in C_4 / C_2 (0-5%) at ~ 20 GeV compared to models
- All non-CP models are monotonic, do not capture the full energy dependence trends.

RESULTS: DATA VS NON-CP MODELS

C_4/C_2 : Data vs. model



M. A. Stephanov, PRL 107 (2011) 052301

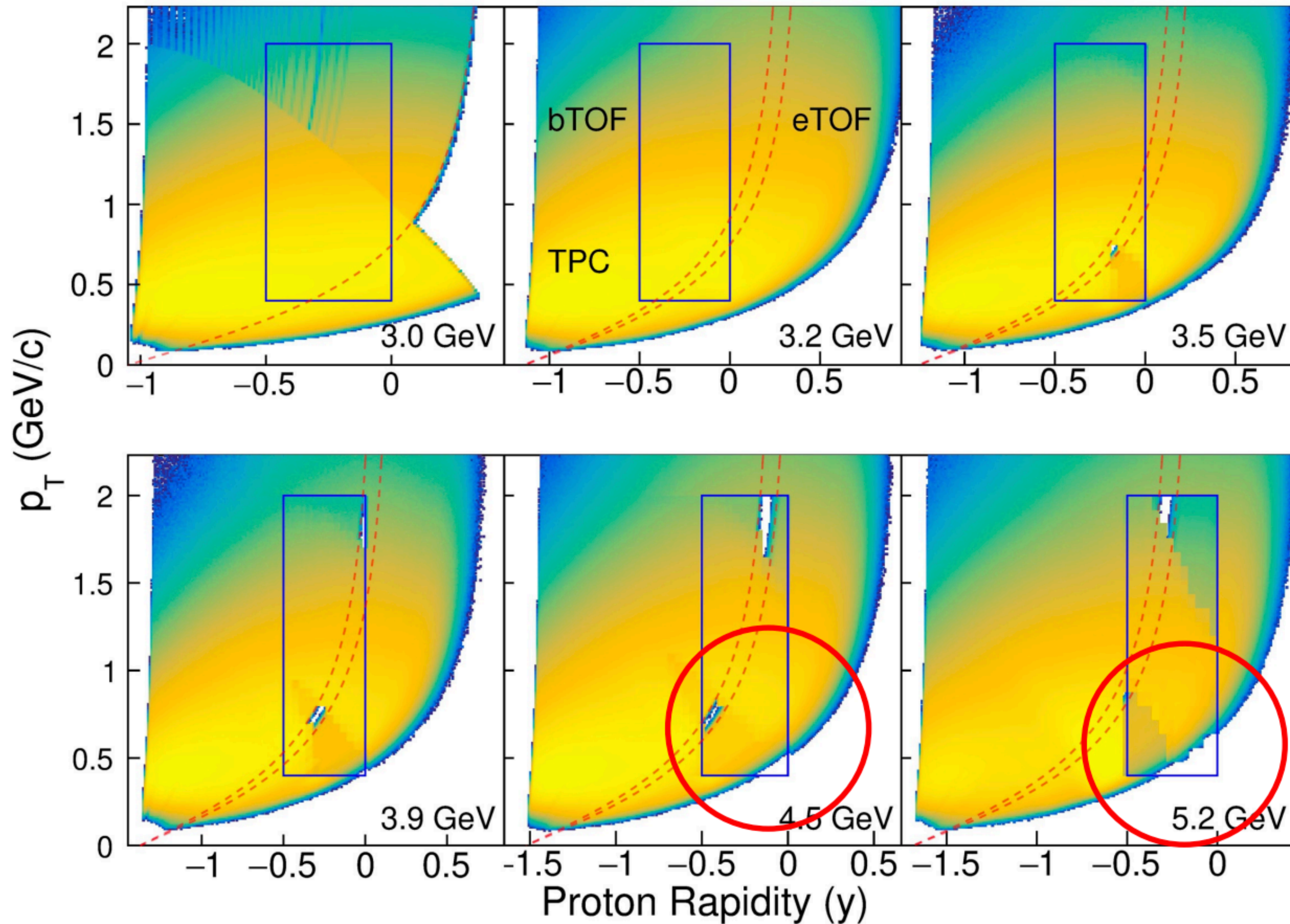


Qualitative features near CP:

- A minimum/negative deviation
- A maximum/positive deviation
- Consistency with baseline away from CP

- A minimum in C_4/C_2 w.r.t. non-CP models and peripheral data at $2-5\sigma$ level around 20 GeV
- Consistency with non-CP baseline for data at $\sqrt{s_{NN}} \geq 27$ GeV
- Dynamic baseline with a CP needed for quantitative assessment of data

EXTENSION TO LOWER ENERGIES: STAR FXT



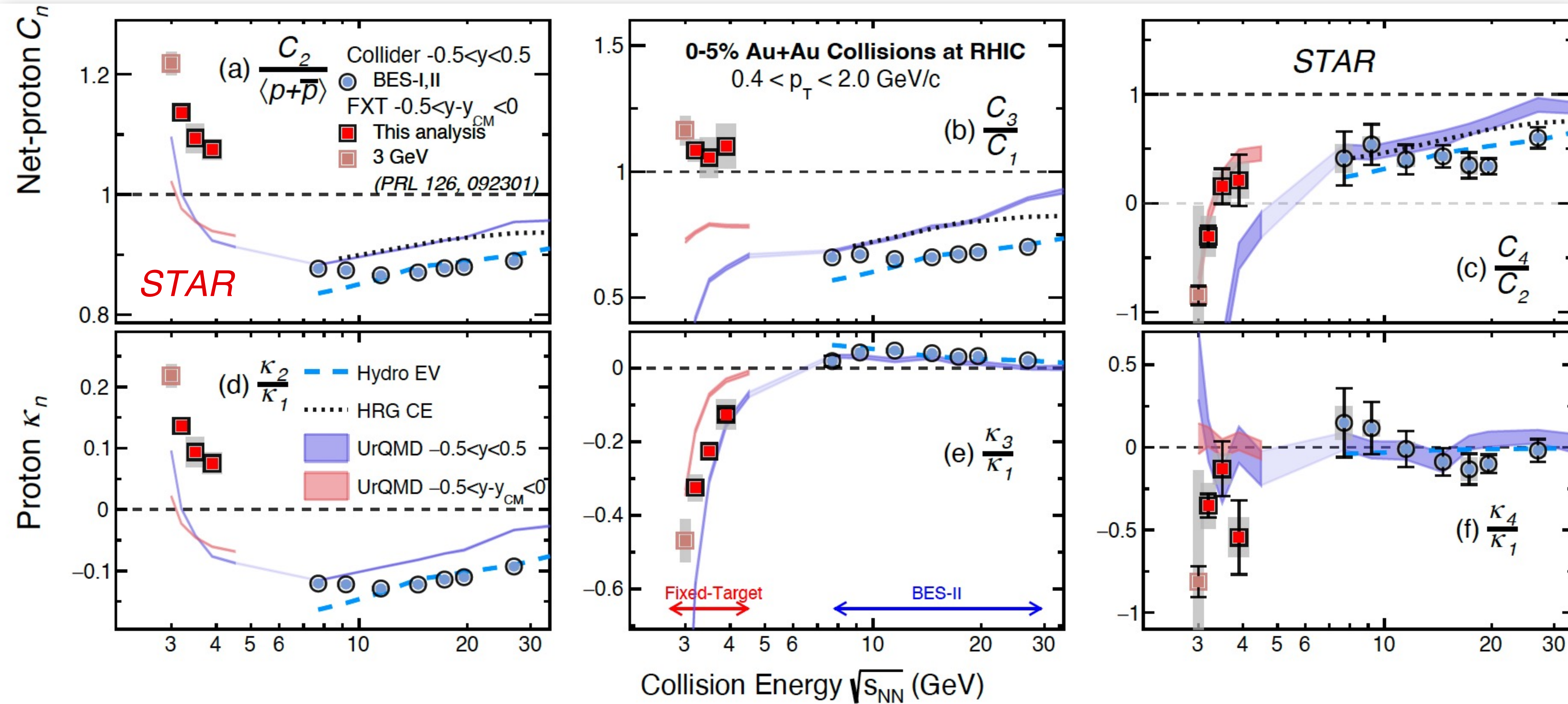
STAR FXT Collisions:

$\sqrt{s_{NN}} = 3, 3.2, 3.5, 3.9, 4.5, 5.2$ GeV

Mid-rapidity acceptance: $-0.5 < y < 0$

Uniform acceptance challenging
for higher energies

EXTENSION TO LOWER ENERGIES: STAR FXT



Results from $\sqrt{s_{NN}} = 3\text{--}3.9$ GeV:

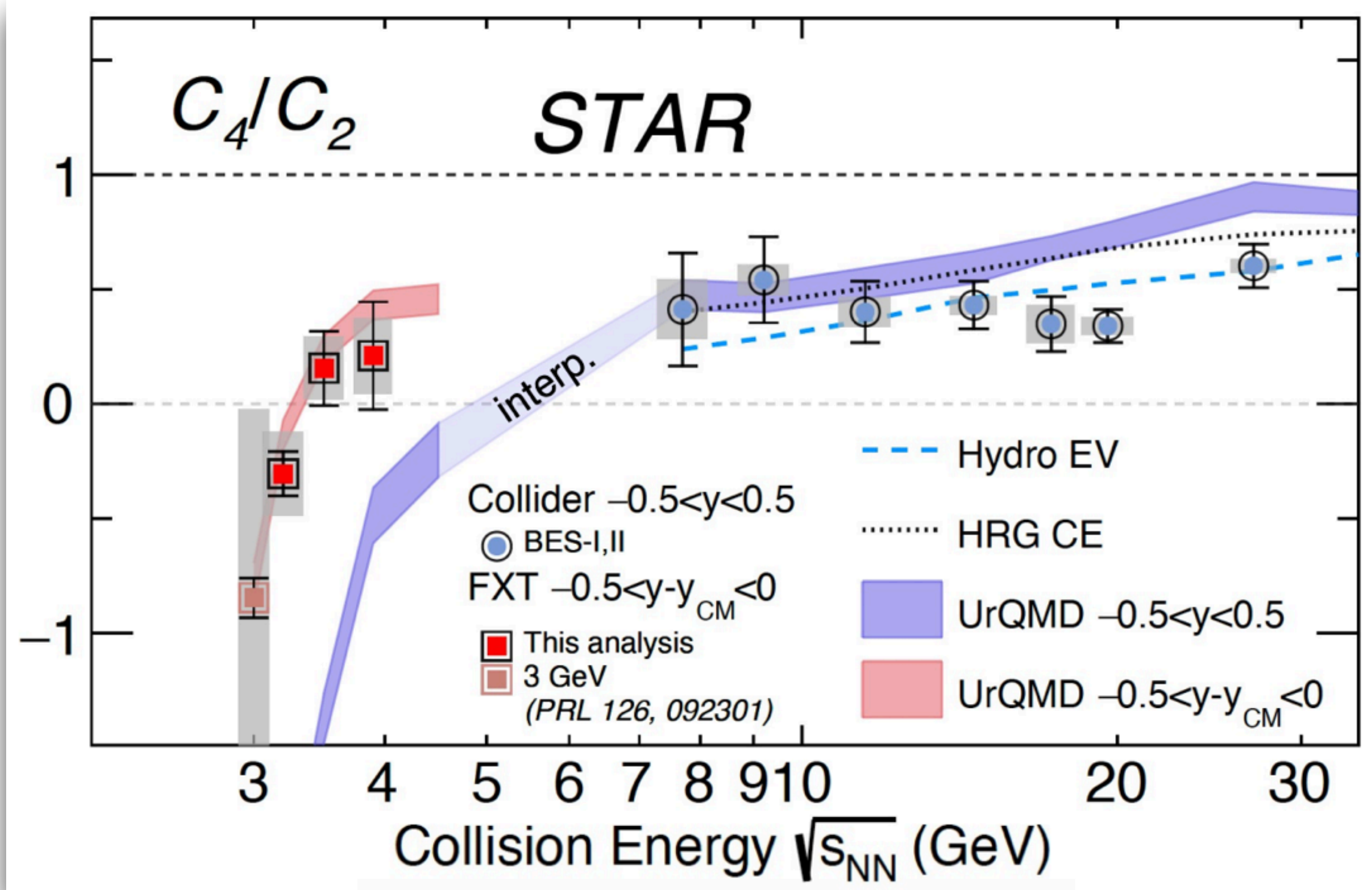
2nd and 3rd order ratios show change in trends with decreasing $\sqrt{s_{NN}}$, deviate from UrQMD

Z. Sweger (STAR) at QM2025

EXTENSION TO LOWER ENERGIES: STAR FXT

C_4/C_2 : Data vs. model

Z. Sweger (STAR) at QM2025

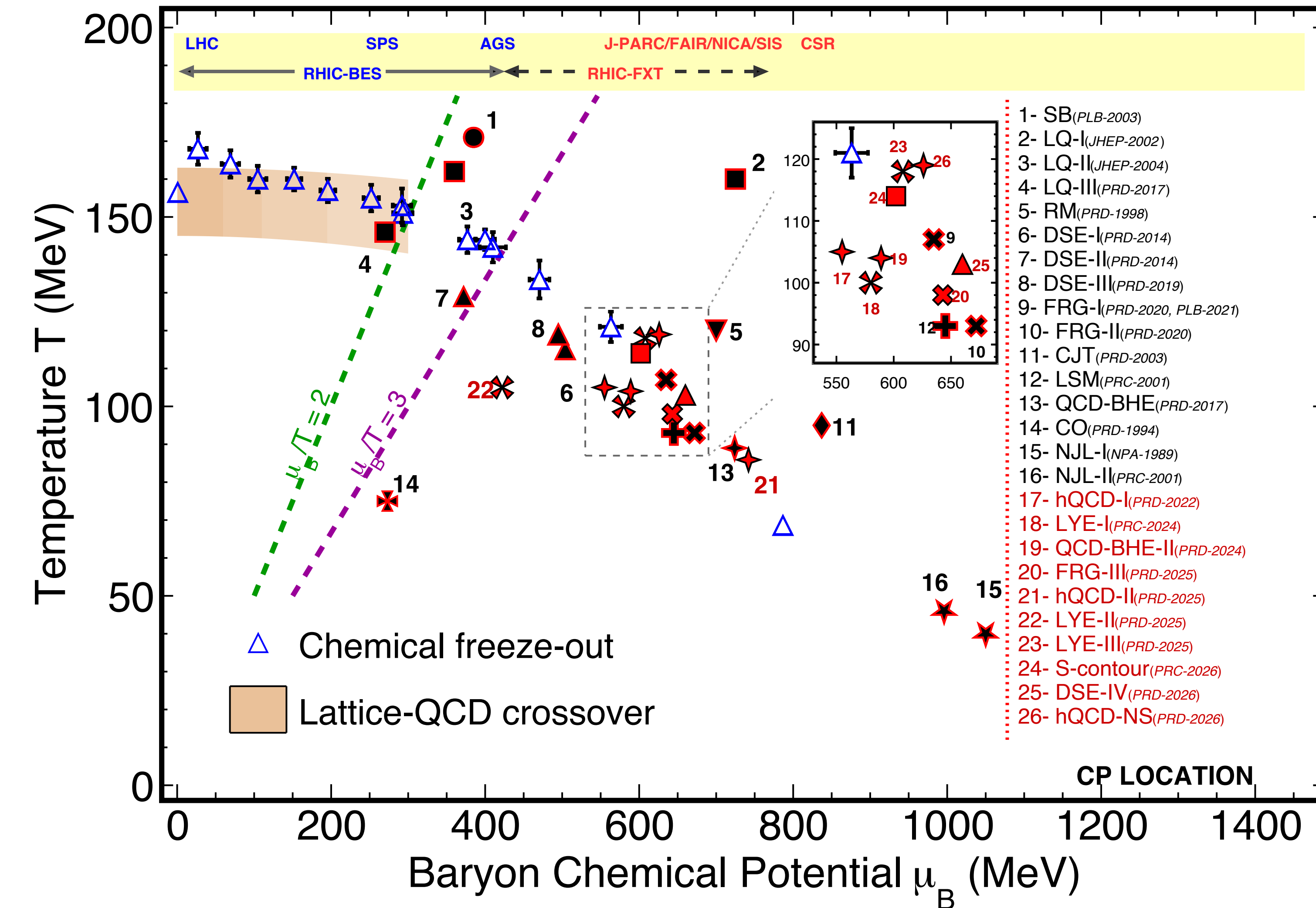


Results from $\sqrt{s_{NN}} = 3\text{--}3.9$ GeV FXT

- C_4/C_2 data consistent with non-CP baseline UrQMD
- FXT data on 4.5 GeV, 5.2 GeV awaited!

RECENT DEVELOPMENTS

Complementary motivations for precision exploration of the $\sqrt{s_{NN}} = 4 - 5$ GeV region



Recent CP estimates from diverse theory calculations

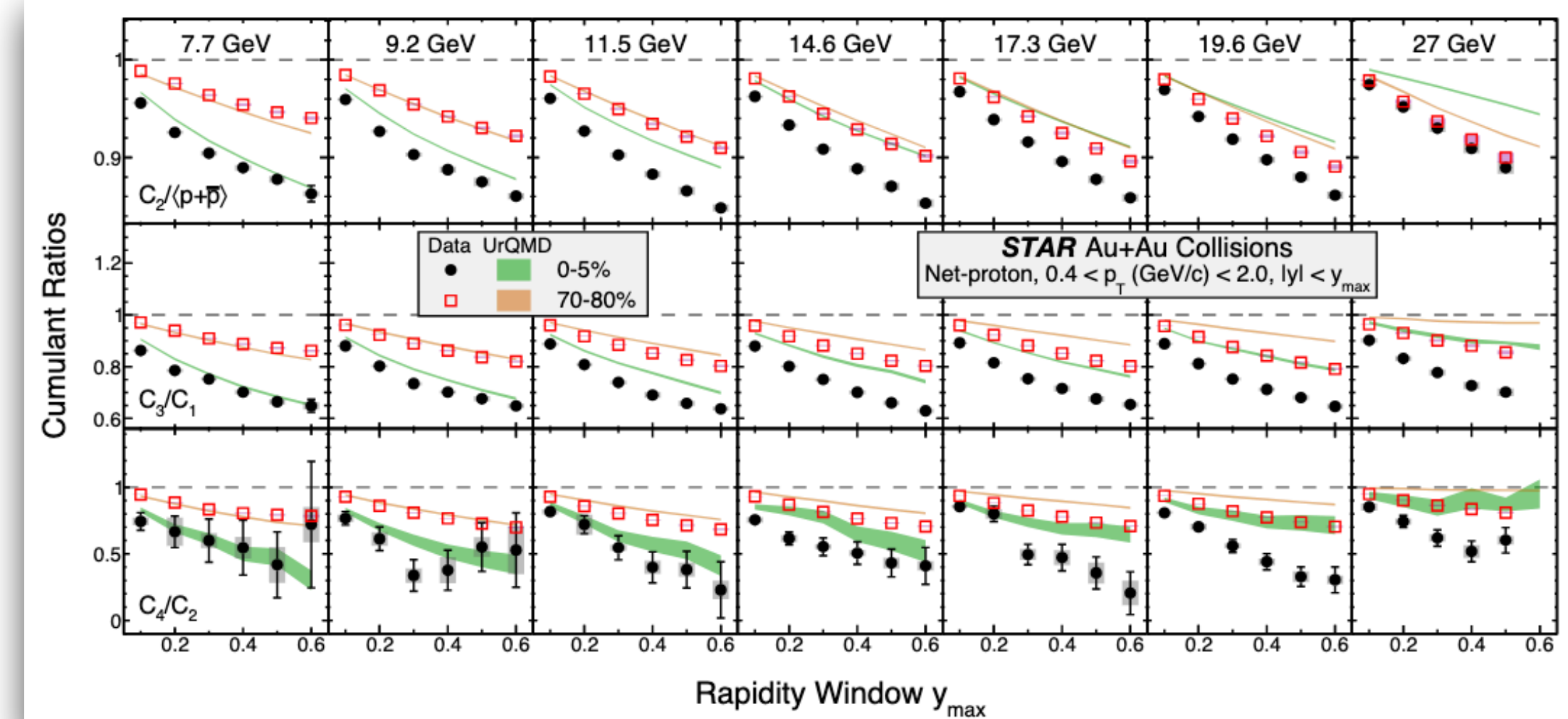
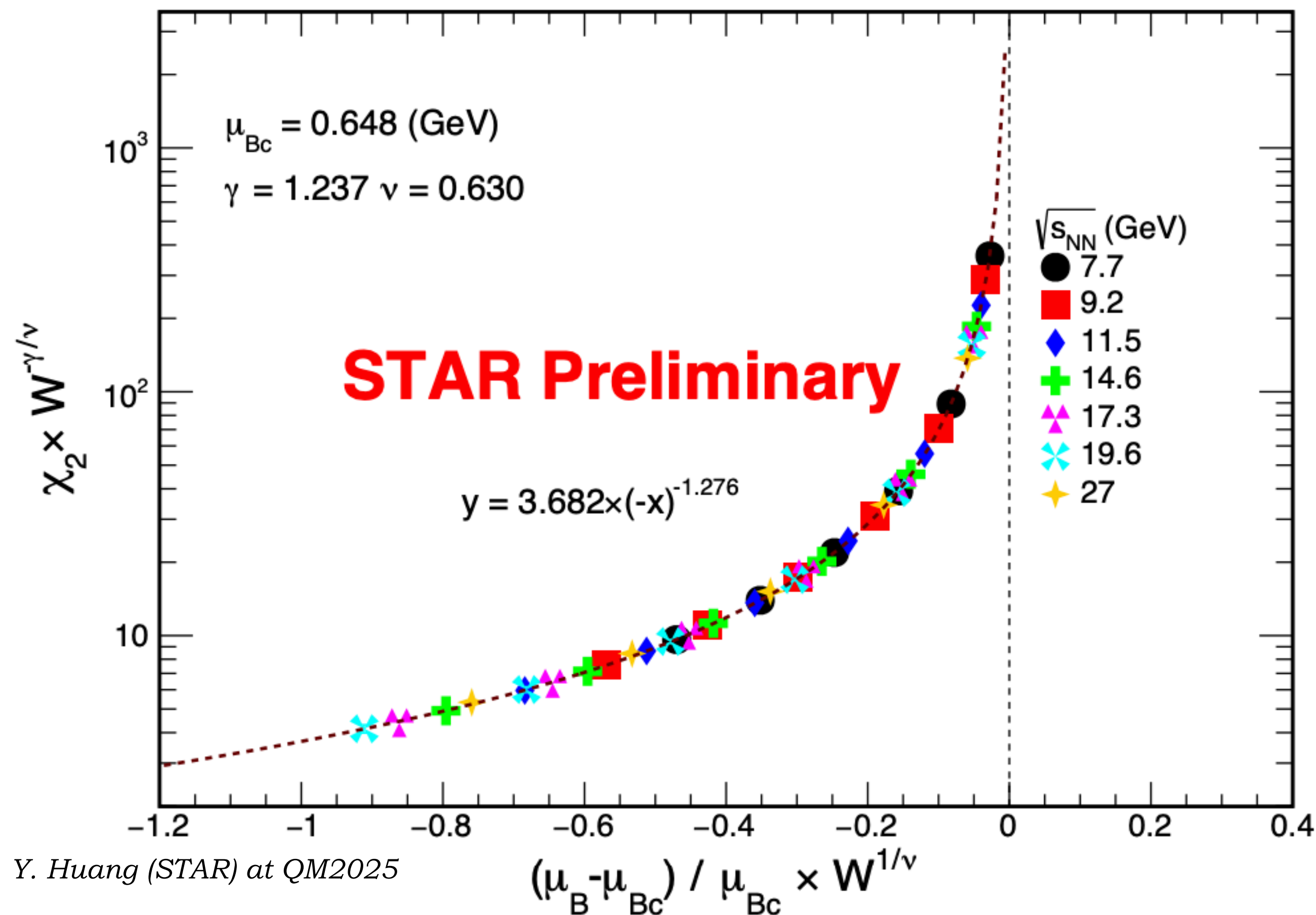
Different approaches, different extrapolations and assumptions at large μ_B

$$\mu_{BCP} \sim 550 - 650 \text{ MeV}$$

$$T_{CP} \sim 90 - 120 \text{ MeV}$$

Corresponds to $\sqrt{s_{NN}} \sim 4 - 5$ GeV in collision energy

RECENT DEVELOPMENTS



A.Sorensen and P.Sorensen: *arXiv:2405.10278*

$$\chi(W, t) = W^{\gamma/\nu} \Phi(t W^{1/\nu}) \quad t = \frac{\mu_B - \mu_{Bc}}{\mu_{Bc}}$$

$$\text{Susceptibility: } \chi_2(W, \mu_{fo}) = \frac{C_2(W, \mu_{fo})}{T_{fo}^3 W dV_{fo}/dy}$$

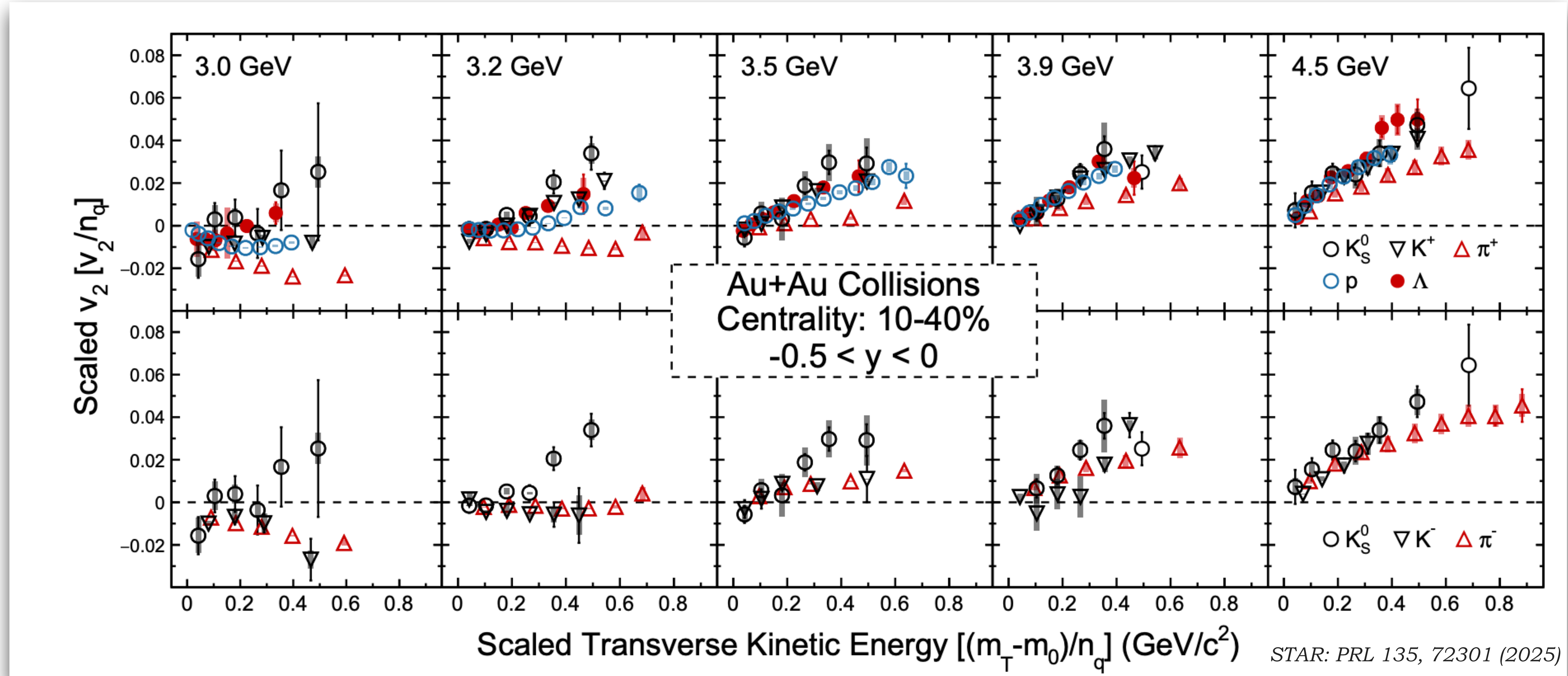
Rapidity window size: W

Freeze out parameters^{[1][2]}: $T, \mu, dV/dy$

-Finite-size scaling study using BES-II collider data leads to extraction of critical parameter $\mu_{Bc} = 648_{-61}^{+6}$ MeV, which corresponds to $\sqrt{s_{NN}} \sim 4 - 4.5$ GeV

RECENT DEVELOPMENTS

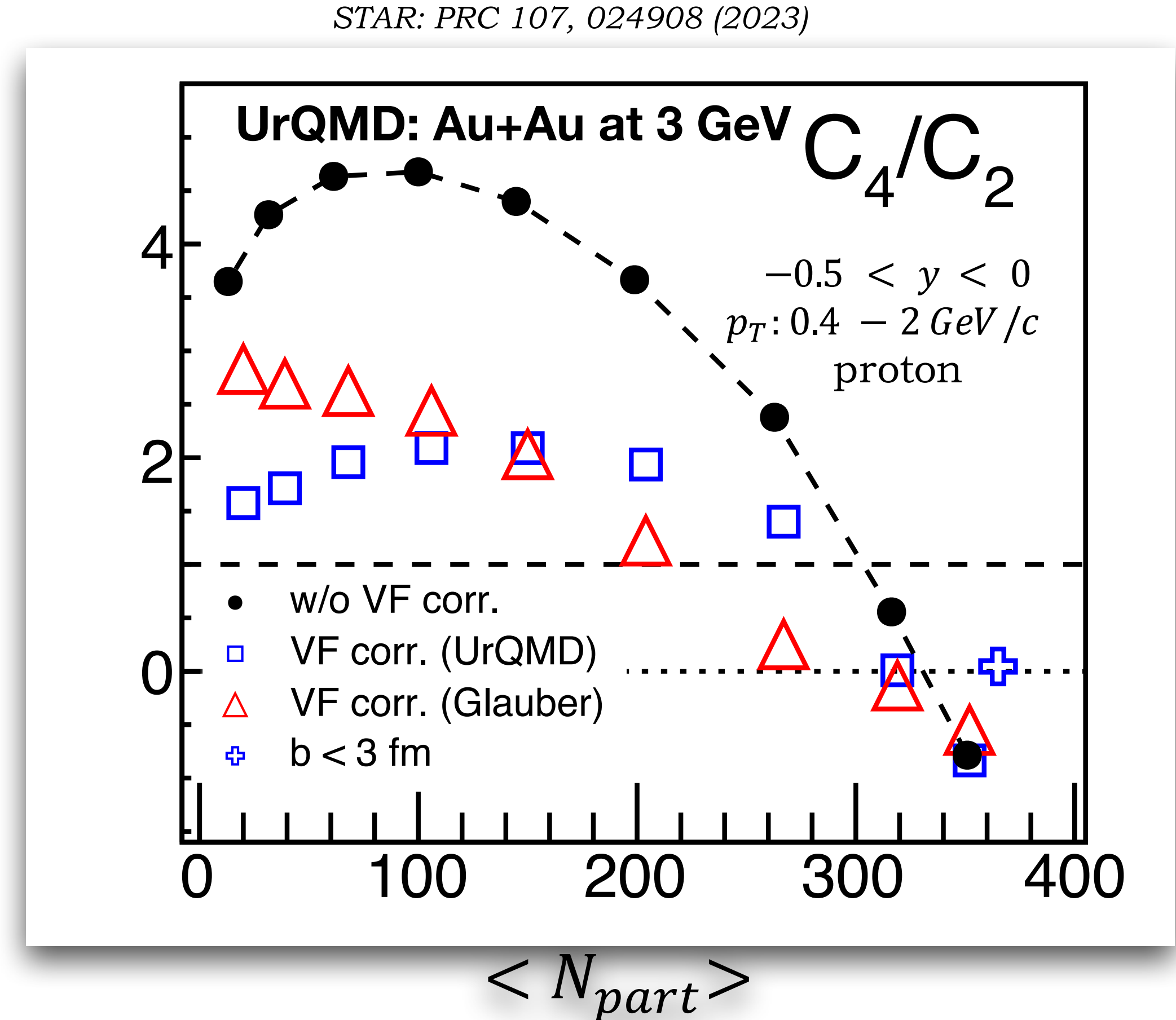
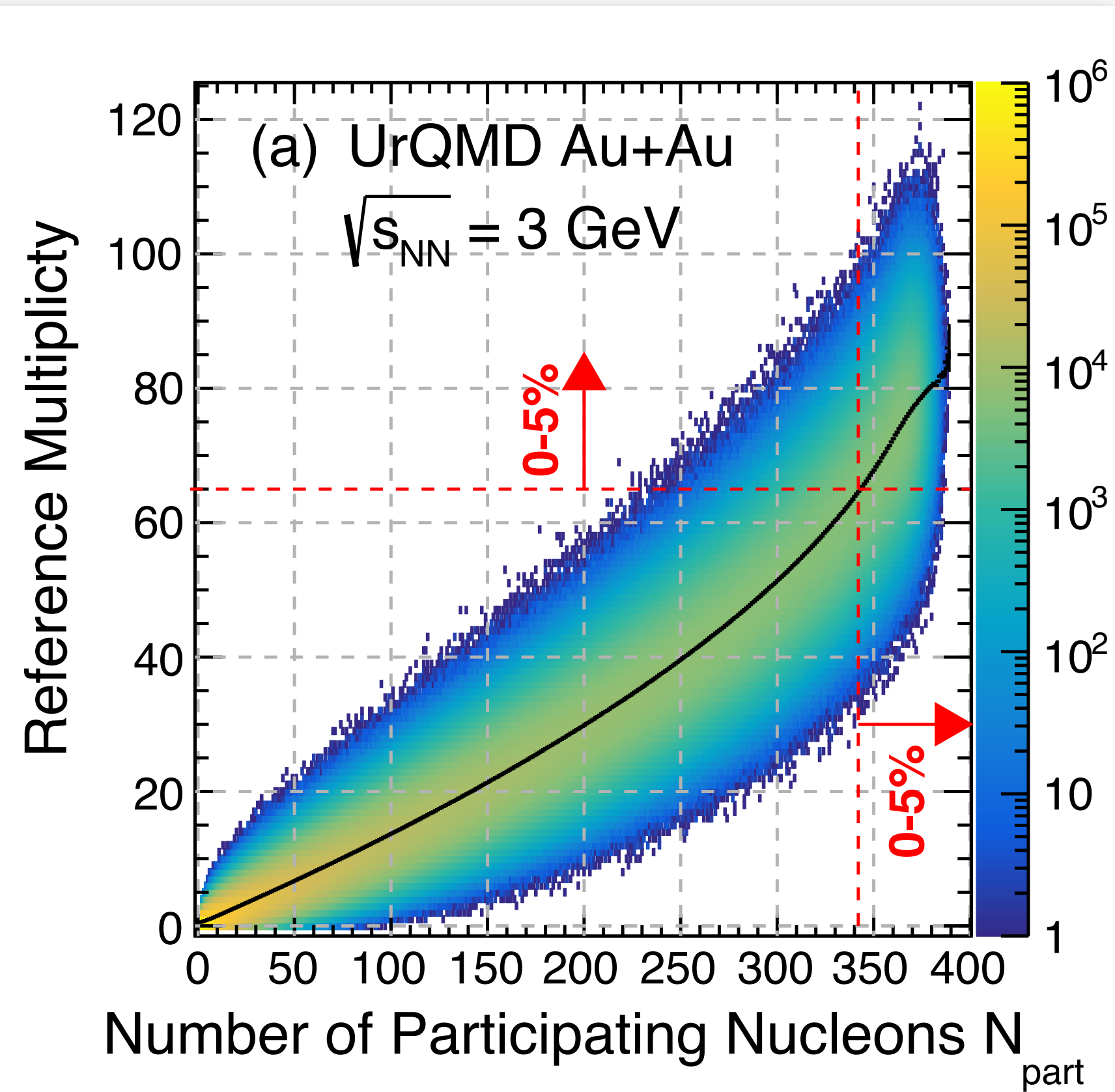
Elliptic flow measurements at FXT energies



NCQ scaling restoration at $\sqrt{s_{NN}} \sim 4.5$ GeV - possible QGP formation

CHALLENGES AND PROGRESS

Initial Volume Fluctuations

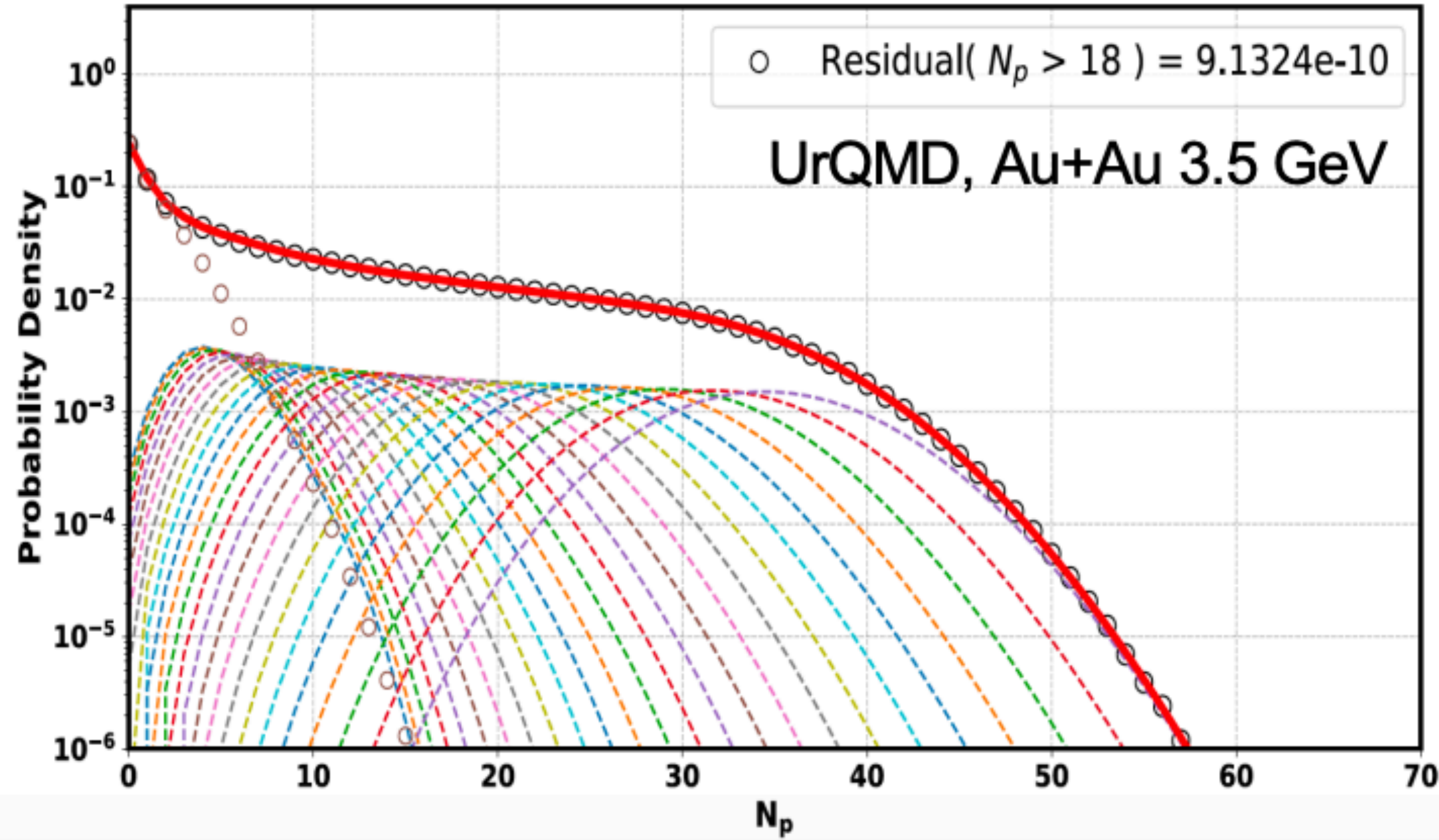


FXT energies while interesting region for CP search, are also experimentally challenging
Lower collision energy (STAR FXT) $\rightarrow$ lower charged particle multiplicity $\rightarrow$ poorer centrality resolution $\rightarrow$ larger initial volume fluctuations

CHALLENGES AND PROGRESS

A new method:

ZH. Wang and XF. Luo, PLB 871 (2025) 139984



Bayesian fit to the *total* proton distribution

- Sum of several sub-distributions, one per volume class
- Each sub-distribution: Edgeworth expansion with its own $C_1 \dots C_4$ as parameters

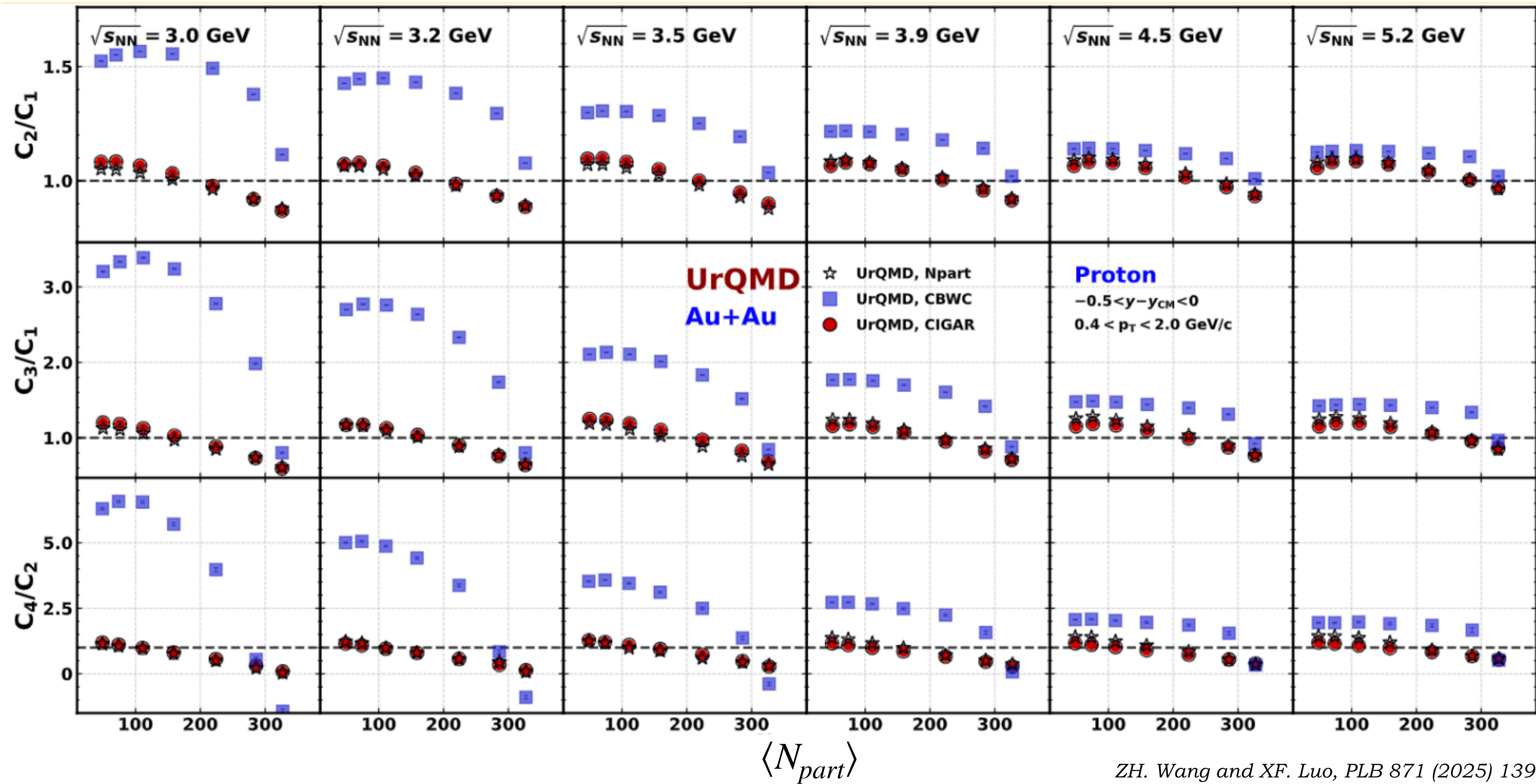
$$P(N_p) = \sum_V w_V P(N_p | V)$$

Edgeworth expansion :

$$\sigma p(\sigma x) = Z(x) \left\{ 1 + \sum_{s=1}^{\infty} \left[\sigma^s \sum_{\{k_m\}} H_{e_{s+2r}}(x) \prod_{m=1}^s \frac{1}{k_m!} \left(\frac{S_m + 2}{m + 2} \right)^{k_m} \right] \right\}$$

where Chebyshev-Hermite polynomials H_{e_n} is defined as: $H_{e_n}(x) = (-1)^n \exp(x^2/2) \frac{d^n}{dx^n} \exp(-x^2/2)$

CHALLENGES AND PROGRESS



ZH. Wang and XF. Luo, PLB 871 (2025) 139984

Results from multiplicity based centrality and N_{part} based centrality method disagree at low energy
 New method seen to effectively removes volume fluctuation effects

CHALLENGES AND PROGRESS

Observables with reduced sensitivity to volume fluctuations

$$\nu_{dyn}[A, B] = \frac{\langle N_A(N_A - 1) \rangle}{\langle N_A \rangle^2} + \frac{\langle N_B(N_B - 1) \rangle}{\langle N_B \rangle^2} - 2 \frac{\langle N_A N_B \rangle}{\langle N_A \rangle \langle N_B \rangle}$$

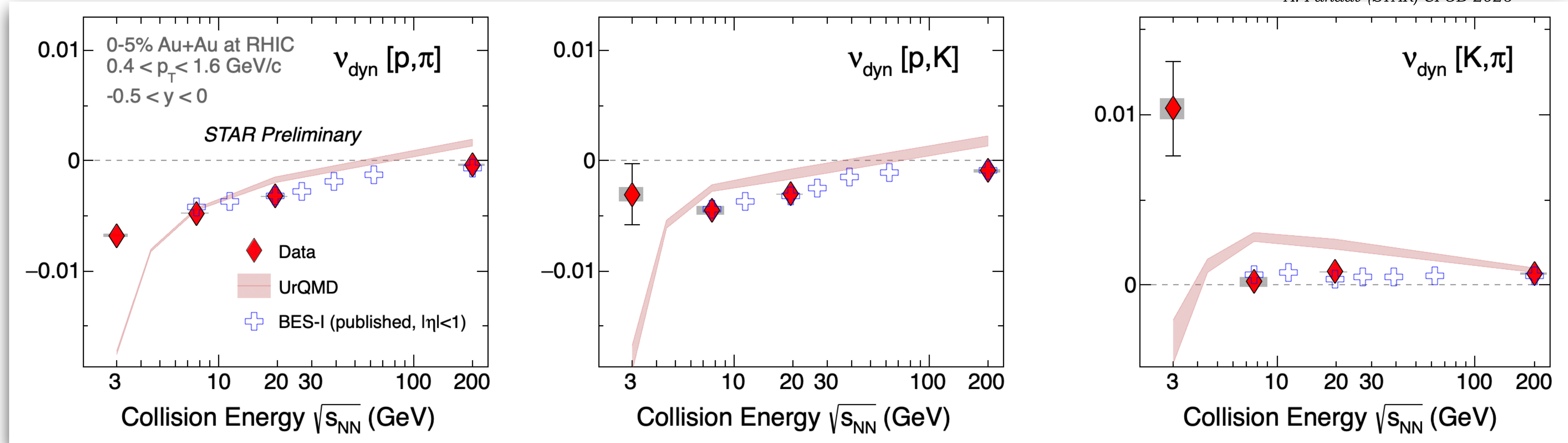
Insensitive to Volume effects in the wounded nucleon model picture

$$\nu_{dyn}[p, \pi] = \nu_{dyn}[p + \bar{p}, \pi^+ + \pi^-]$$

$$\nu_{dyn}[p, K] = \nu_{dyn}[p + \bar{p}, K^+ + K^-]$$

$$\nu_{dyn}[K, \pi] = \nu_{dyn}[K^+ + K^-, \pi^+ + \pi^-]$$

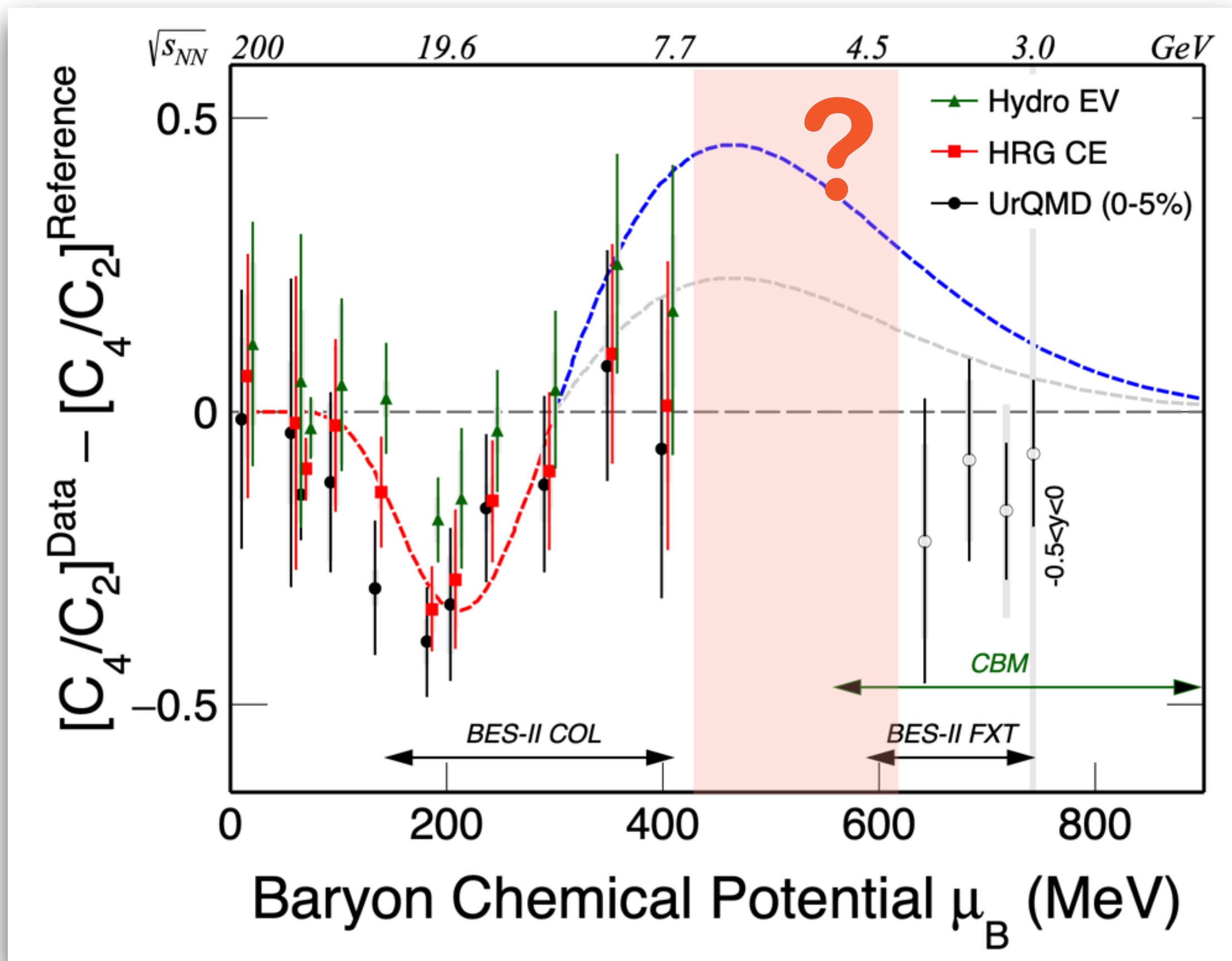
A. Pandav (STAR) CPOD 2026



- Data show a monotonically decreasing trend for $\nu_{dyn}[p, \pi]$ and $\nu_{dyn}[p, K]$. UrQMD follows but large differences compared to data seen at 3 GeV. $\nu_{dyn}[K, \pi]$ show opposite trend.

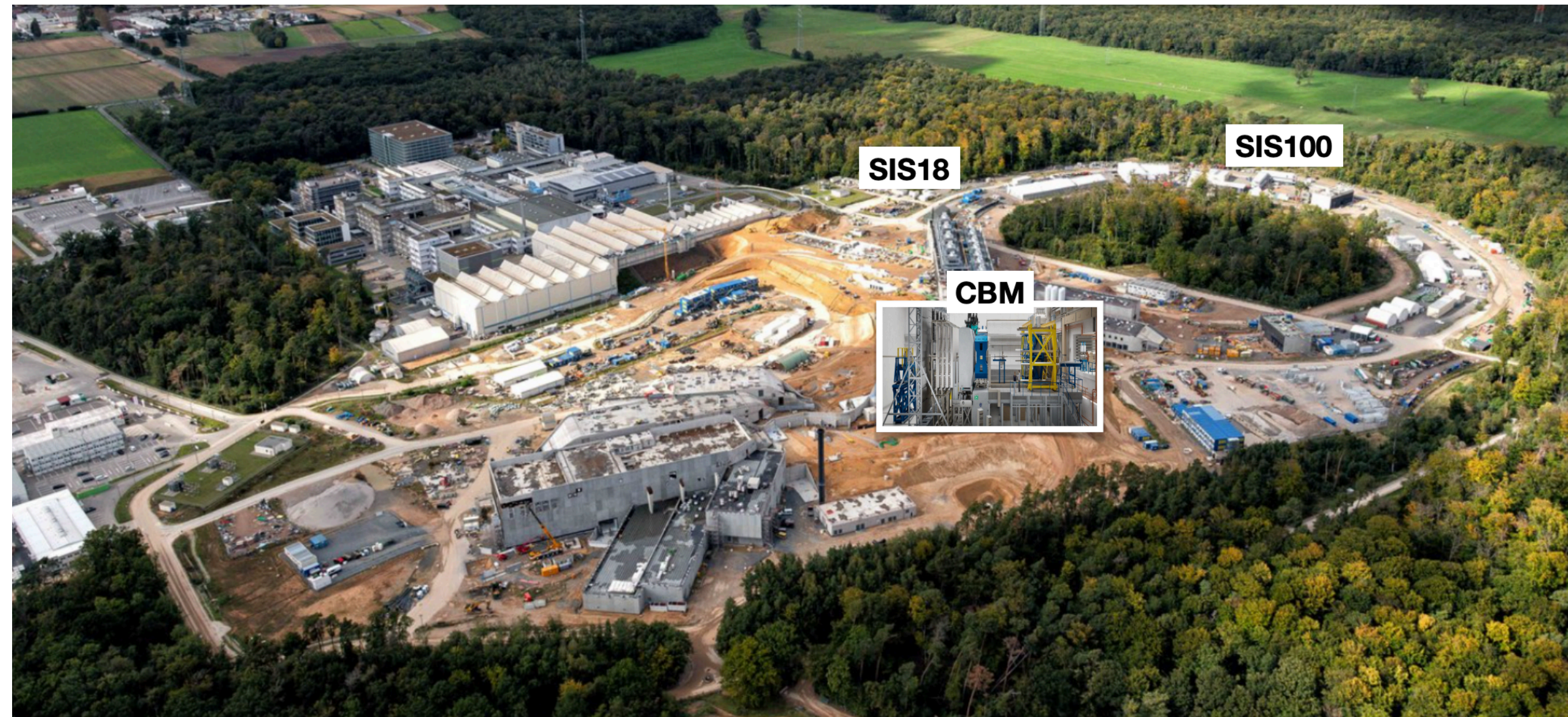
CHALLENGES AND PROGRESS

Bridging the $\sqrt{s_{NN}}$ gap from 3.9 - 7.7 GeV

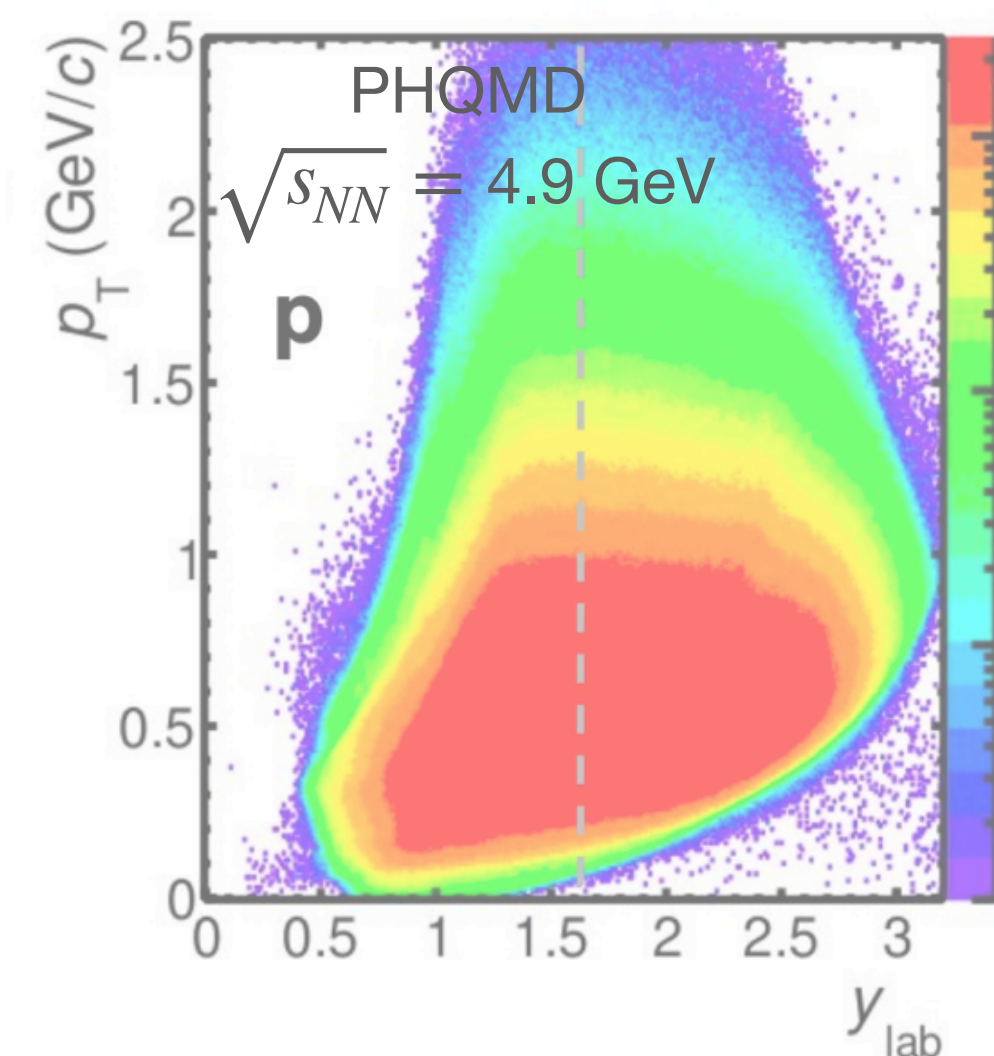


STAR: PRL 135, 142301 (2025)

High statistics data at $\sqrt{s_{NN}} = 4.2, 4.5, 5.2$ GeV collected by STAR (300-1000M) in its last run (2026)



CBM (projected 2028):
Precision data at
 $\sqrt{s_{NN}} = 2.7 - 4.9$ GeV with
mid-rapidity coverage



SUMMARY AND OUTLOOK

○ Era of precision measurement of fluctuations:

- STAR C_4/C_2 shows minimum around $\sqrt{s_{NN}} \sim 20$ GeV w.r.t. non-CP models at $2 - 5\sigma$ level.
- STAR FXT C_4/C_2 data from $\sqrt{s_{NN}} = 3 - 3.9$ GeV consistent with non-CP model UrQMD
- Lower order cumulant ratios also show large deviations from models at low collision energies.

○ Challenges and progress:

- Volume Fluctuation effect: New analysis frameworks have been developed to suppress these effects, currently being applied to experimental data
- $\sqrt{s_{NN}} \sim 4 - 5$ GeV region interesting for CP search. STAR collected 300-1000 Million events at 4.2, 4.5, and 5.2 GeV. CBM to take data over $\sqrt{s_{NN}} = 2.7 - 4.9$ GeV, with mid rapidity coverage for protons.

○ Theoretical assessment of data:

- Need for dynamical model calculations with a critical point to make quantitative assessment of data. Efforts underway.

ACKNOWLEDGEMENTS

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*STAR Collaboration, RNC Berkeley group, EHEP group at Kent, Theory Colleagues. Organizers for the opportunity. **Thank you all.***

THANK YOU FOR YOUR ATTENTION.