



YITP long-term workshop

New Frontiers in QCD 2026

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Yukawa Institute for Theoretical Physics, Kyoto University

Initial-state and non-equilibrium effects on spin transport in nuclear collisions

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(New Frontier in QCD 2026, Sep 19, Kyoto)



Outline

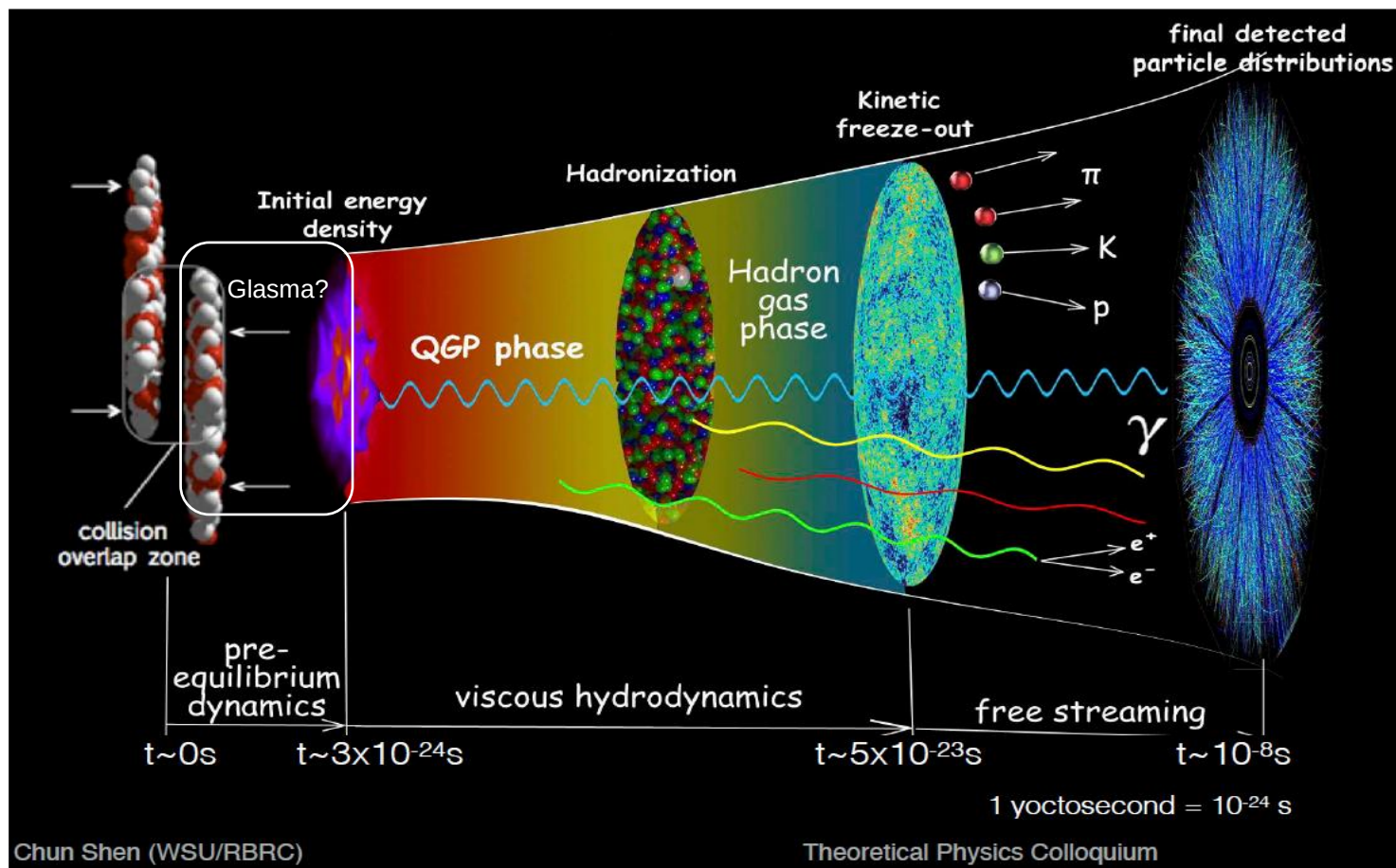
- Collective behaviors of quark-gluon plasma and spin polarization from hydrodynamic effects
- Spin transport phenomena from novel QCD effects :
 - Puzzles of spin alignment & local spin polarization
 - Chromo-electromagnetic fields (color fields) from the glasma phase & spin transport theory : quantum kinetic theory
 - Spin alignment & local spin polarization from glasma effects
 - Local spin polarization from gluon fusion (preliminary)
- Summary and outlook

References :

Y. Hidaka, S. Pu, Q. Wang, DY, PPNP 127, 103989 (2022), [arXiv:2201.07644](#)
B. Müller, DY, PRD 105, L011901 (2022), [arXiv:2110.15630](#)
A. Kumar, B. Müller, DY, PRD 108, 016020 (2023), [arXiv:2304.04181](#)
DY, PRD 110, 056005 (2025), [arXiv:2411.14822](#)
H. Sung, B. Müller, DY, [arXiv: 2507.23210](#)

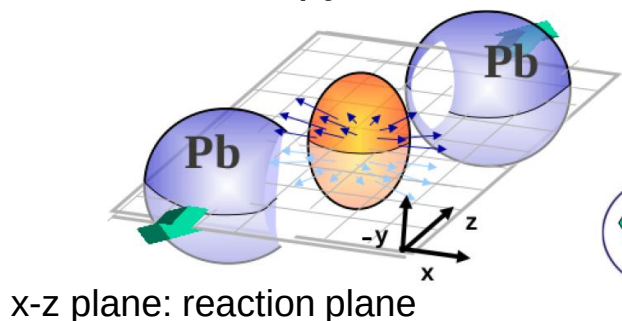
Relativistic heavy ion collisions

- Timeline for heavy ion collisions :

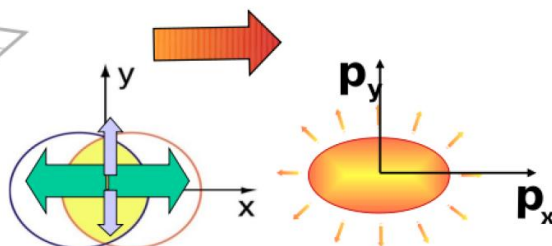


Anisotropic flow and vorticity

■ Anisotropy flow of hadrons : hydrodynamic behaviors of the QGP



Credit: Nazarova Elizaveta, 2017

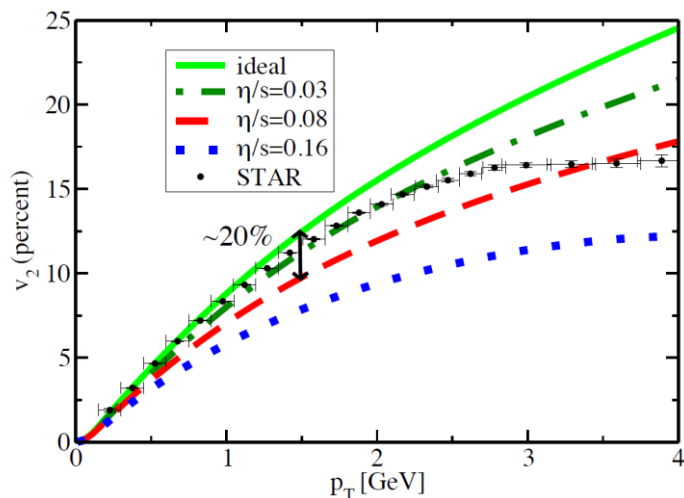


z: beam (longitudinal) dir.
x: impact parameter dir.
-y : angular-momentum dir.

collective flow :
geometric asymmetry

⇒ momentum asymmetry

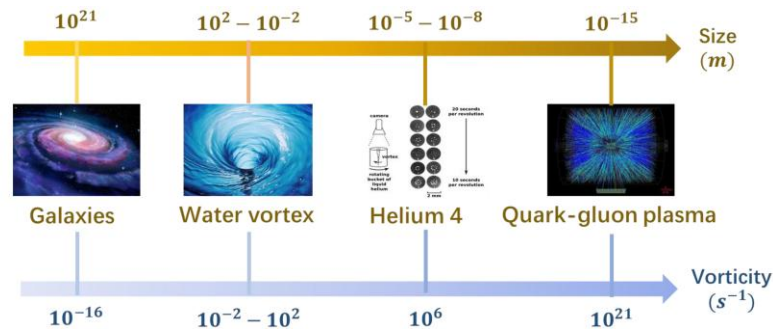
❖ Elliptic flow : $E_p \frac{dN}{d^3p} = v_0 (1 + 2v_2 \cos 2\phi_p + \dots)$ ❖ Angular momentum (AM) to vorticity :



P. Romatschke, U. Romatschke, PRL. 99 (2007) 172301

$$\mathbf{L} = \frac{1}{2} \int d^3\mathbf{r} \epsilon |\mathbf{r}|^2 (1 - \hat{\omega} \cdot \hat{\mathbf{r}}) \omega,$$

$$\omega = \nabla \times \mathbf{v} = \text{const.} \quad \epsilon : \text{energy density}$$



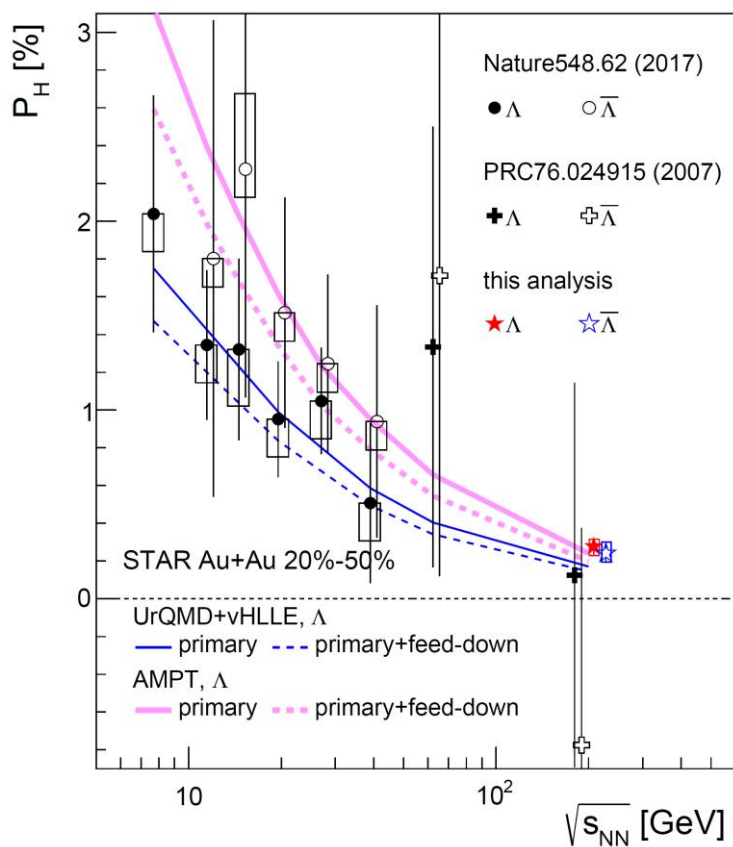
X.-G. Huang, QM 19

Global Λ polarization in HIC

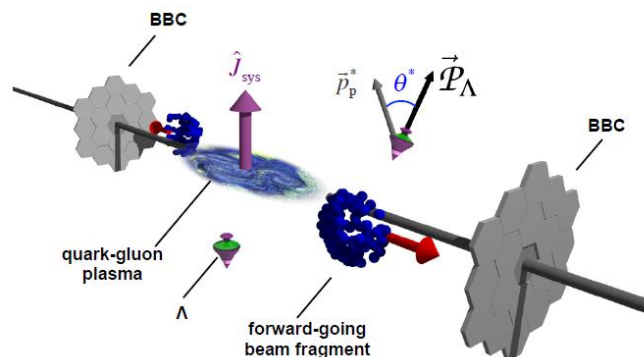
- The measurement of global polarization of Λ hyperons revealed the **spin-orbit interaction & strong vorticity** in heavy ion collisions. Z.-T. Liang & X.-N. Wang, PRL. 94, 102301 (2005)

(relativistic Barnett effect)

❖ Self-analyzing via the weak decay : $\Lambda \rightarrow p + \pi^-$



L. Adamczyk et al. (STAR), Nature 548, 62 (2017)



- ❖ Successfully described by the modified Cooper–Frye formula in **global equilibrium** : F. Becattini, et al., Ann. Phys. 338, 32 (2013)
R. Fang, et al., PRC 94, 024904 (2016)

$$\mathcal{P}^\mu = \frac{\int d\Sigma \cdot p f_p^{(0)} (1 - f_p^{(0)}) \epsilon^{\mu\nu\rho\sigma} q_\nu \omega_{\rho\sigma}}{8M_\Lambda \int d\Sigma \cdot p f_p^{(0)}},$$

$$\omega_{\rho\sigma} = \frac{1}{2} \left(\partial_\rho \left(\frac{u_\sigma}{T} \right) - \partial_\sigma \left(\frac{u_\rho}{T} \right) \right) \cdot \text{thermal vorticity}$$

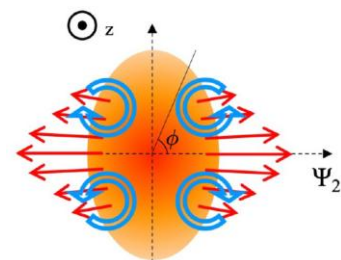
- ❖ Global pol. from (average) kinetic vorticity :

$$P_{\Lambda(\bar{\Lambda})} \approx \int_p \mathcal{P}^{-y}(p) \simeq \frac{\omega}{2T} \longrightarrow \omega = \frac{1}{2} |\nabla \times u| \sim 10^{22} s^{-1}$$

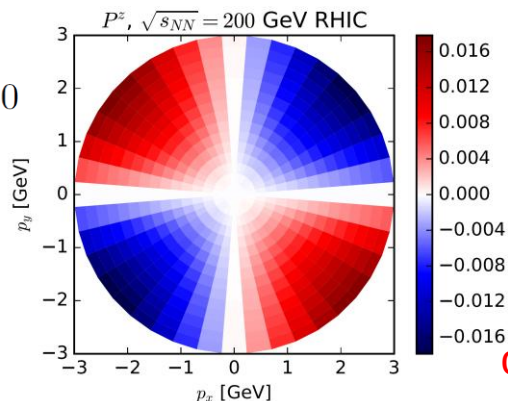
F. Becattini et al., PRC95, 054902 (2017)

Longitudinal (local) polarization

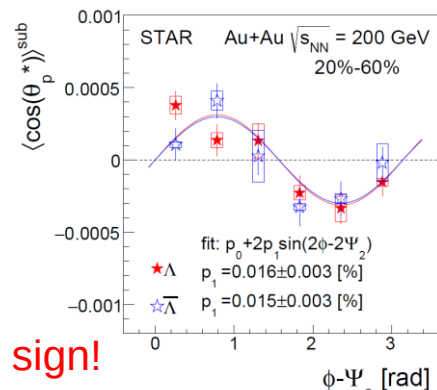
■ Longitudinal polarization along the beam direction in HIC :



F. Becattini, I. Karpenko, PRL 120, 012302 (2018).



J. Adam et al. (STAR), PRL. 123, 132301 (2019)



$P_{2,z} > 0$

$$P_{2,z} = \langle P_z \sin 2(\phi - \Psi_2) \rangle$$

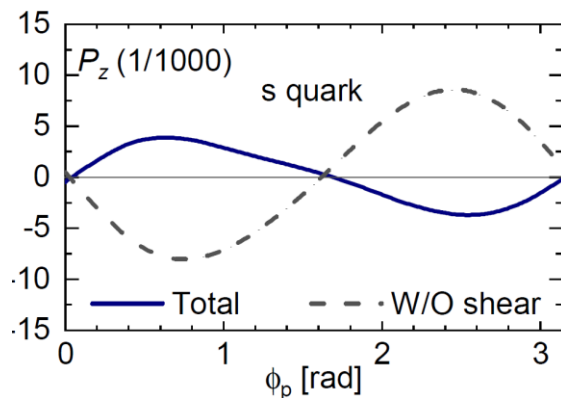
$$\approx \int_0^{2\pi} d(\phi - \Psi_2) \frac{P_z \sin 2(\phi - \Psi_2)}{2\pi}$$

v.s.

opposite sign!

❖ Thermal-shear corrections : (local equilibrium, indep. of int.)

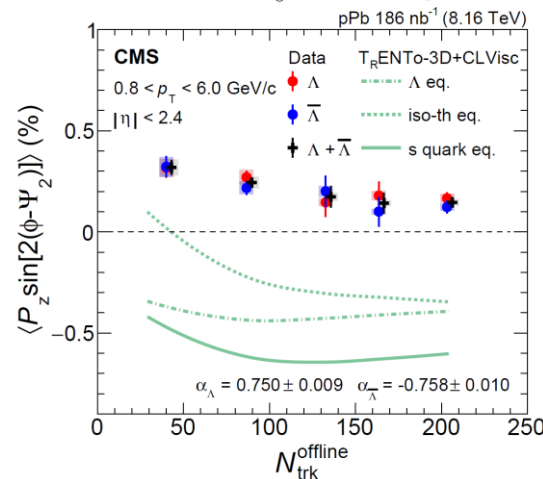
$$\mathcal{P}^\mu(p) = \frac{\int d\Sigma_x \cdot p f_p^{(0)} (1 - f_p^{(0)}) \epsilon^{\mu\nu\rho\sigma} q_\nu [\omega_{\rho\sigma} - u_\rho \pi_{\sigma\lambda} p^\lambda / (Tp \cdot u)]}{8M_\Lambda \int d\Sigma_x \cdot p f_p^{(0)}}$$



B. Fu et al., PRL 127, 142301 (2021)

See also F. Becattini et al., PRL. 127, 272302 (2021), C. Yi et al., PRC 104, 064901 (2021)

open question!



CMS, PRL 135 (2025) 132301

C. Yi et al., PRC 111 (2025) 044901

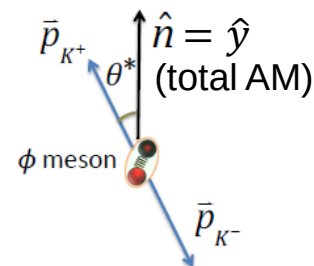
new effects needed

Spin alignment of vector mesons

- Angular dep. of the decay particle w.r.t the spin quantization axis :

$$\frac{dN}{d \cos \theta^*} \propto [1 - \rho_{00} + \cos^2 \theta^* (3\rho_{00} - 1)]$$

$$\rho_{00} = \frac{1 - \langle \mathcal{P}_q^y \mathcal{P}_{\bar{q}}^y \rangle}{3 + \langle \mathcal{P}_q^y \mathcal{P}_{\bar{q}}^y \rangle}$$

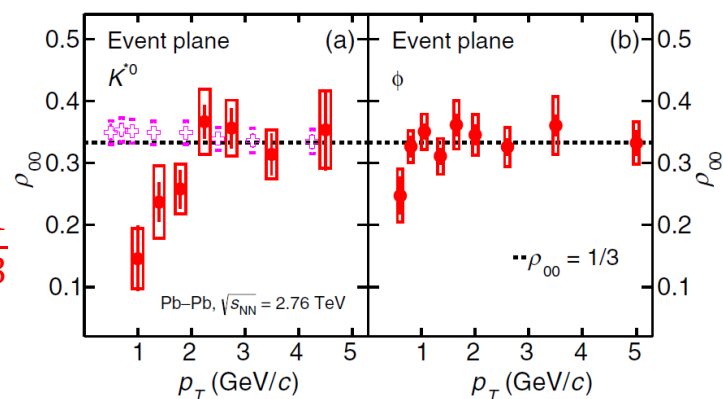


Z.-T. Liang and X.-N. Wang, PLB 629, 20 (2005)

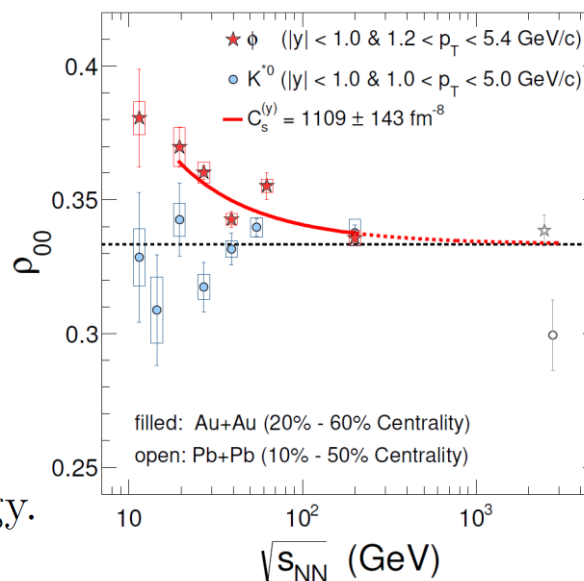
$\rho_{00} \neq 1/3$: spin correlation

LHC:

$$\rho_{00} < \frac{1}{3}$$



S. Acharya et al. (ALICE), PRL.125, 012301 (2020)



RHIC:

$$\rho_{00} > \frac{1}{3}$$

- Spin alignment puzzle : the deviation from 1/3 is **unexpectedly large**.

e.g. $\rho_{00} \approx \frac{1}{3} - \left(\frac{\omega}{T}\right)^2$, $\frac{\omega}{T} \sim 0.1\%$ at LHC energy.

➡ **Beyond hydro**: opportunities to probe microscopic int. in QCD matter

- Electromagnetic fields can polarize the spin. How about gluon fields?

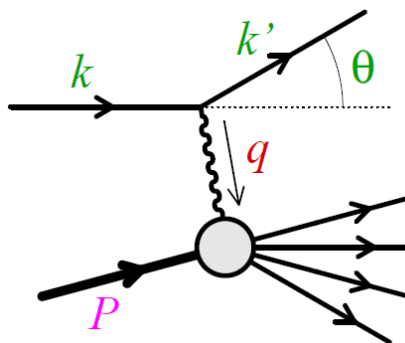
$$\mathcal{P} \propto \mathbf{B} - \mathbf{u} \times \mathbf{E}$$

magnetic pol. spin Hall effect with momentum anisotropy

Saturated gluons in high-energy collisions

❖ Could we generate the chromo-electromagnetic (color) fields in nuclear collisions?

■ Deep inelastic scattering :

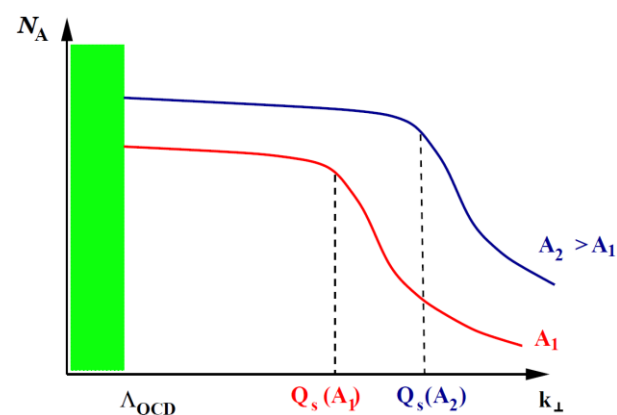
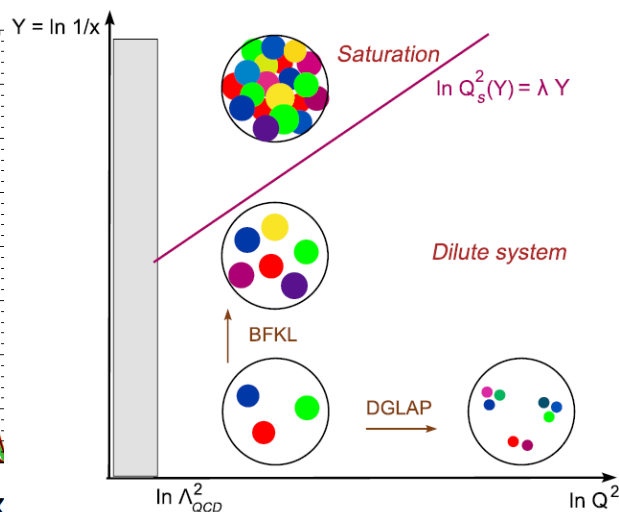
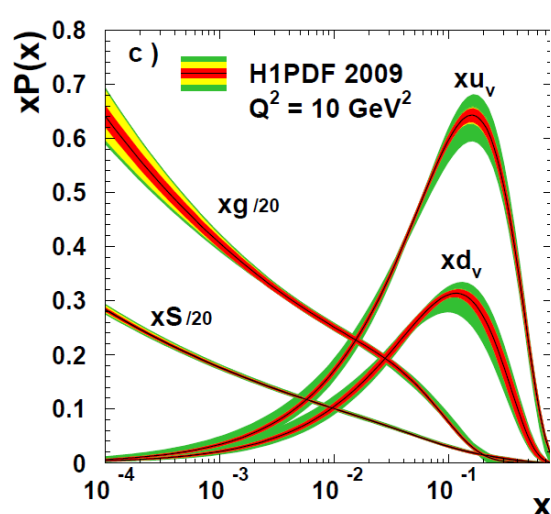


$$Q^2 = -q^2$$

$$\text{Bjorken scaling : } x \approx \frac{Q^2}{s}$$

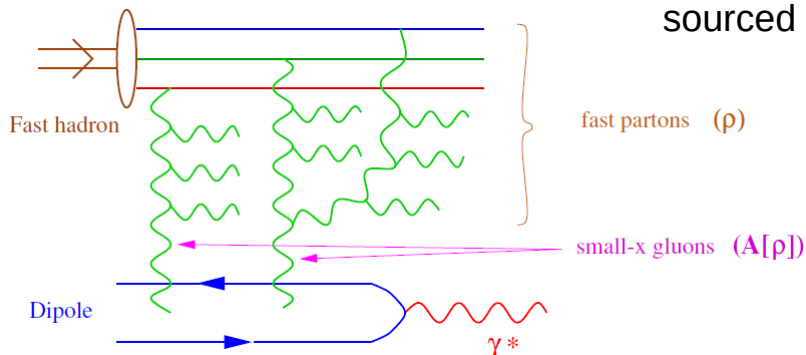
■ Saturated gluons at small x : splitting + recombination

review: F. Gelis et al.,
Ann.Rev.Nucl.Part.Sci.60:463-489,2010



Chromo-electromagnetic fields in HIC

- **Color glass condensate (CGC)** : EFT describing highly populated soft gluons ($E < Q_s$) sourced by hard partons



L. McLerran & R. Venugopalan, PRD 49, 2233 (1994), PRD 49, 3352 (1994).

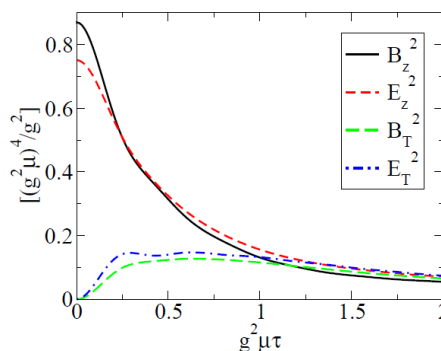
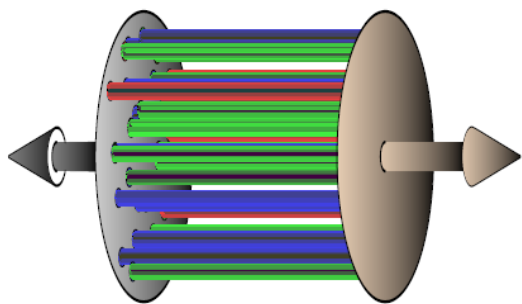
$$D_\mu \boxed{F^{\mu\nu,a}} = \delta^{\nu+} \delta(x^-) \boxed{\rho^a(x_\perp)}$$

soft gluons

hard partons

Reviews : F. Gelis et al., Ann.Rev.Nucl.Part.Sci.60:463-489,2010
J. Berges et al., Rev. Mod. Phys. 93 (2021) 3, 035003

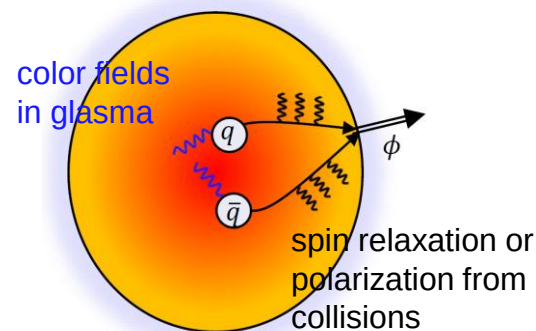
- **Glasma** in HIC : color flux tubes (at the top RHIC and LHC energies)



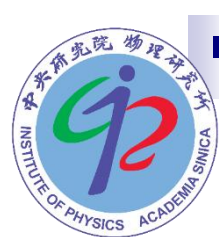
T. Lappi, PLB 643 (2006) 11-16

Spin alignment from glasma fields?

Assuming s quarks are produced by e.g., **Schwinger mechanism** from chromo-E fields
e.g., H. Taya, PRD 96, 014033 (2017)



- (1) intrinsic saturation scale $Q_s \gg \omega$
- (2) fluctuating (minor effect on global Λ pol.)
- (3) intrinsic anisotropy (need not be along $\hat{n}$)



Wigner functions & kinetic theory

- Dynamical polarization of quarks in a medium : phase space info is needed
- **Wigner functions and kinetic theory** : Review (w/o spin) : J. Blaizot, E. Iancu, Phys.Rept. 359, 355-528 (2002)
Review (with spin) : Y. Hidaka, S. Pu, Q. Wang, DY, PPNP 127, 103989 (2022)

$$S^<(p, x) = \int d^4Y e^{\frac{i p \cdot y}{\hbar}} \langle \bar{\psi}(x_2) U(x_2, x_1) \psi(x_1) \rangle, \quad x = \frac{x_1 + x_2}{2}, \quad y = x_1 - x_2.$$

- Kadanoff-Baym (KB) equation with the $\hbar$ expansion : ($\partial/p \ll 1$)

$$(\not{D} - m)S^< + \gamma^\mu i \frac{\hbar}{2} \nabla_\mu S^< = \frac{i\hbar}{2} \left(\underbrace{\Sigma^< \star S^>}_{\text{gain}} - \underbrace{\Sigma^> \star S^<}_{\text{loss}} \right)$$

$$\nabla_\mu = \Delta_\mu + \mathcal{O}(\hbar^2),$$

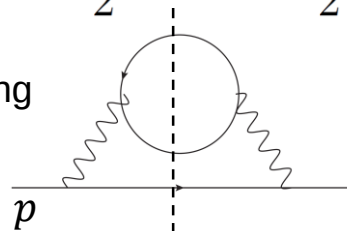
$$\Delta_\mu = \partial_\mu + e F_{\nu\mu} \partial_p^\nu,$$

$$\Pi^\mu = p^\mu + \mathcal{O}(\hbar^2),$$

$$A \star B = AB + \frac{i\hbar}{2} [A, B]_{\text{P.B.}} + \mathcal{O}(\hbar^2),$$

$$[A, B]_{\text{P.B.}} = (\partial_p^\mu A)(\partial_\mu B) - (\partial_\mu A)(\partial_p^\mu B).$$

e.g., 2-2 scattering



- Clifford-algebra decomposition :

$$S^{\lessgtr} = \mathcal{S}^{\lessgtr} + i \mathcal{P}^{\lessgtr} \gamma^5 + \mathcal{V}_\mu^{\lessgtr} \gamma^\mu + \mathcal{A}_\mu^{\lessgtr} \gamma^5 \gamma^\mu + \frac{\mathcal{S}_{\mu\nu}^{\lessgtr}}{2} \sigma^{\mu\nu}, \quad \Sigma^{\lessgtr} = \Sigma_S^{\lessgtr} + i \Sigma_P^{\lessgtr} \gamma^5 + \Sigma_V^{\lessgtr} \gamma^\mu + \Sigma_A^{\lessgtr} \gamma^5 \gamma^\mu + \frac{\Sigma_T^{\lessgtr}}{2} \sigma^{\mu\nu}.$$

- ❖ Vector/axial-vector components :

$$\left. \begin{aligned} \mathcal{V}^\mu(p, X) &= \frac{1}{4} \text{tr} (\gamma^\mu S^<(p, X)), \\ \mathcal{A}^\mu(p, X) &= \frac{1}{4} \text{tr} (\gamma^\mu \gamma^5 S^<(p, X)) \end{aligned} \right\} \mathcal{P}^\mu(\mathbf{p}) = \frac{\int d\Sigma \cdot p \mathcal{A}^\mu(\mathbf{p}, X)}{2m \int d\Sigma \cdot \mathcal{V}(\mathbf{p}, X)} \quad \mathcal{A}^\mu(\mathbf{p}, x) = \int \frac{dp_0}{2\pi} \mathcal{A}^\mu(p, x)$$



Quantum kinetic theory for spin transport

- KB eq. $\rightarrow$ master eqs. : constraints, onshell conditions, kinetic equations

- Leading-order Wigner functions (from master eqs.) :

➤ Vector WF: $\mathcal{V}^\mu(\mathbf{p}, x) = \frac{p^\mu}{2\epsilon_{\mathbf{p}}} f_V(p, x)|_{p_0=\epsilon_{\mathbf{p}}=\sqrt{|\mathbf{p}|^2+m^2}} + \boxed{\mathcal{O}(\partial/p)} \hbar \text{ expansion}$

➤ Axial WF: $\mathcal{A}^\mu(\mathbf{p}, x) = \frac{1}{2\epsilon_{\mathbf{p}}} \left[\tilde{a}^\mu - \underbrace{\frac{\hbar}{2} e \tilde{F}^{\mu\nu} \left(\partial_{p\nu} f_V - \frac{\epsilon_{\mathbf{p}}}{2} \partial_{p_\perp\nu} (f_V/\epsilon_{\mathbf{p}}) \right)}_{\mathcal{P} \propto B - \mathbf{u} \times \mathbf{E}} \right]_{p_0=\epsilon_{\mathbf{p}}=\sqrt{|\mathbf{p}|^2+m^2}}.$

($\tilde{a}^\mu(p, x)$: effective spin four vector) $\mathcal{P} \propto B - \mathbf{u} \times \mathbf{E}$ magnetic pol. & spin Hall effect

- Axial kinetic theory : scalar/axial-vector kinetic eqs. (SKE/AKE)

➤ SKE : $\mathbf{p} \cdot \Delta f_V = \mathcal{C}[f_V], \quad \Delta_\mu = \partial_\mu + e F_{\nu\mu} \partial_p^\nu.$

K. Hattori, Y. Hidaka, DY, PRD 100, 096011 (2019)

DY, K. Hattori, Y. Hidaka, JHEP 20, 070 (2020)

standard Boltzmann (Vlasov) eq.

➤ AKE : $\mathbf{p} \cdot \Delta \tilde{a}^\mu + e F^{\nu\mu} \tilde{a}_\nu - \frac{\hbar}{2} \epsilon^{\mu\nu\rho\sigma} p_\rho (\partial_\sigma e F_{\beta\nu}) \partial_p^\beta f_V = q_\nu \widehat{\Sigma_V^\nu} \tilde{a}^\mu + \dots + \frac{\hbar}{2} \epsilon^{\mu\nu\rho\sigma} q_\nu (\Delta_\sigma \widehat{\Sigma_{V\rho}}) f_V,$

entangled f_V & $\tilde{a}^\mu$

$\widehat{AB} = A^ - A^>B^<.$

- ❖ Generalization to include color dof. : $\mathcal{A}^\mu = \mathcal{A}^{s\mu} \mathbf{I} + \mathcal{A}^{a\mu} t^a$

DY, JHEP 06, 140 (2022)

B. Müller, DY, PRD 105, L011901 (2022)

➤ (color-octet) axial WF : $\mathcal{A}^{ai}(\mathbf{p}, x) \approx -\frac{\hbar g}{4\epsilon_{\mathbf{p}}} e^{\frac{-\Delta t_{QGP}}{\tau_R^0}} \left[B^{ai}(t_i) - \frac{\epsilon^{ijk} p_j E_k^a(t_i)}{\epsilon_{\mathbf{p}}} \right] \partial_{\epsilon_{\mathbf{p}}} f_V^s(\epsilon_{\mathbf{p}}, t_i)$

(no contribution to Λ pol.)

spin relaxation in QGP

spin polarization from glasma fields



Spin alignment from glasma

■ Spin alignment from quark coalescence :

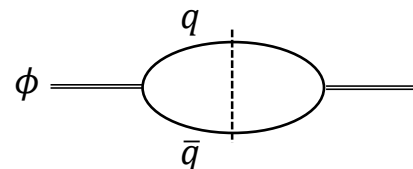
KE for vector mesons : $q \cdot \partial f_\lambda^\phi \approx \epsilon_\mu^*(\lambda, q) \epsilon_\nu(\lambda, q) \mathcal{C}_{\text{coal}}^{\mu\nu}(q, x)$
 $(\rho_{\lambda\lambda} \propto f_\lambda^\phi, \lambda = 0, \pm 1)$

quark-meson int. :

$$\mathcal{L}_{\text{int}} = g_\phi \bar{\psi} \gamma^\mu V_\mu \psi$$

$$\chi_B^{ij} = \langle B^{ai}(0, x + \mathbf{k} t^{\text{th}}/p_+) B^{aj}(0, x - \mathbf{k} t^{\text{th}}/p_-) \rangle_{|\mathbf{k}|=\tilde{k}},$$

$$\Rightarrow \rho_{00}(q) = \frac{\int d\Sigma_X \cdot q f_0^\phi(q, X)}{\int d\Sigma_X \cdot q (f_0^\phi(q, X) + f_{+1}^\phi(q, X) + f_{-1}^\phi(q, X))} \neq \frac{1}{3}$$



■ Correlation of initial glasma fields :

$$\chi_B^{zz} = \chi_E^{zz} = \chi_{\text{GBW}} \equiv \frac{Q_s^4(N_c^2 - 1)}{2g^2 N_c} \left(\frac{1 - e^{-Q_s^2 \tilde{k}^2 (t^{\text{th}} \lambda_m)^2 / 4}}{Q_s^2 \tilde{k}^2 (t^{\text{th}} \lambda_m)^2 / 4} \right)^2 \xrightarrow{\tilde{k} \rightarrow 0} \frac{Q_s^4(N_c^2 - 1)}{2g^2 N_c}$$

T. Lappi, S. Schlichting, PRD 97, 034034 (2018)

K. J. Golec-Biernat, M. Wusthoff, PRD 59, 014017 (1998)

■ Initial quark distribution function : $f_V^s(\epsilon_p, t_i) = (e^{\epsilon_p/Q_s} + 1)^{-1}$

❖ Considering ϕ mesons with $Q_s \sim 2 \text{ GeV}$ & $m_{q/\bar{q}} \sim 0.5 \text{ GeV}$

❖ Heavy-quark approx. & pQCD : $\tau_R^o \approx \left(\frac{g^2 C_2(F) m_D^2 T}{6\pi m^2} \ln g \right)^{-1} \approx 5 \text{ fm}/c$

M. Hongo et al., JHEP 08, 263 (2022)

Phenomenological estimation : S. Banerjee et al., PRC 111, 064912 (2025)

(model dependent)

Transverse spin alignment spectra

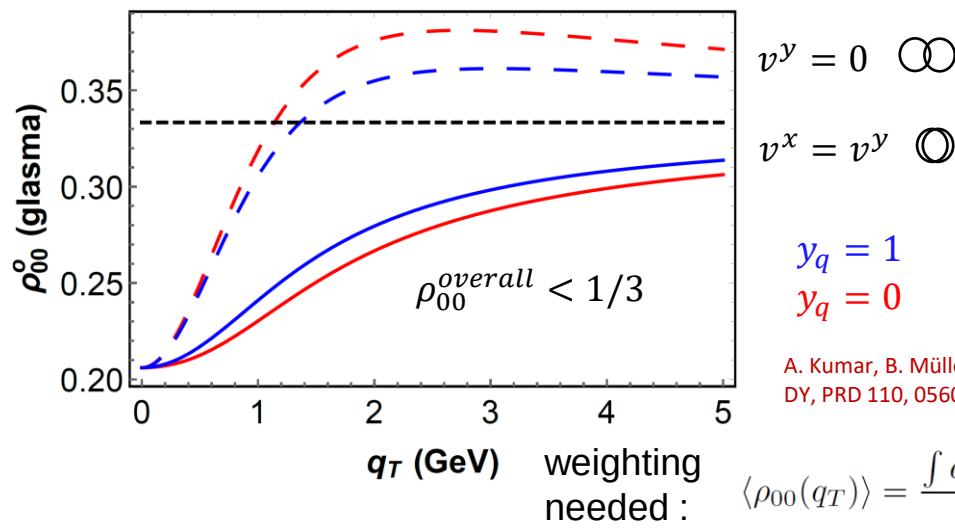
- Boost the color fields to the lab frame to retrieve momentum dependence

$$B_r^{ai} = \gamma(B^{ai} + \epsilon^{ijk} v_j E_k^a) - (\gamma - 1) \mathbf{B}^a \cdot \hat{\mathbf{v}} \hat{v}^i, \quad v^i = q^i / \sqrt{|\mathbf{q}|^2 + M^2} \text{ and } \hat{v}^i = v^i / |\mathbf{v}|.$$

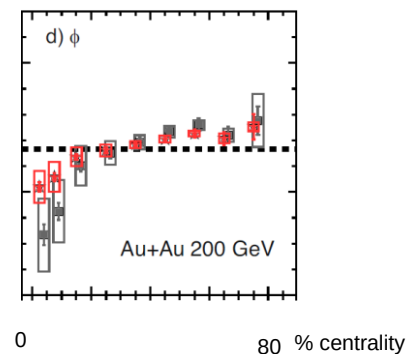
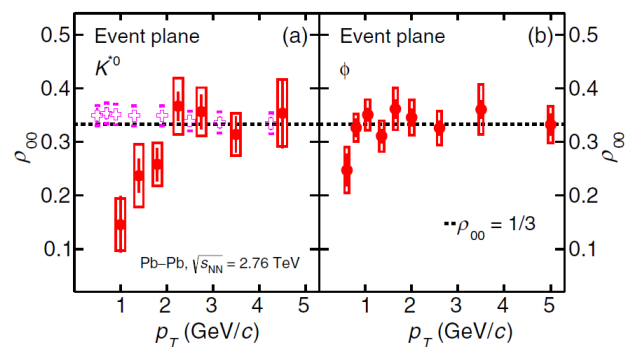
- ❖ Two sources of anisotropy :
longitudinal fields v.s. momentum anisotropy

- Out-of-plane spin alignment (ϕ mesons) :

- ❖ Initial-state effect (glasma) : **high-energy**



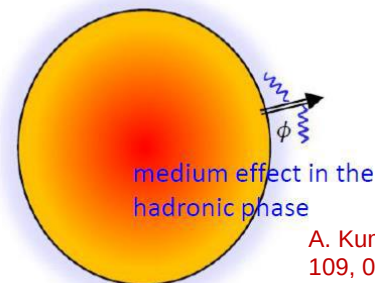
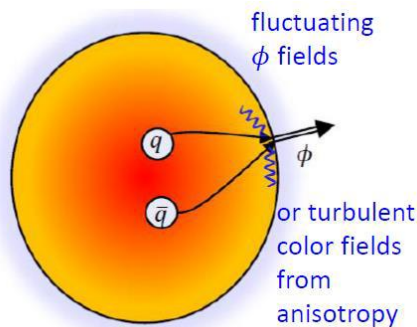
A. Kumar, B. Müller, DY, PRD 108, 016020 (2023)
DY, PRD 110, 056005 (2025)



Color fields in the QGP phase

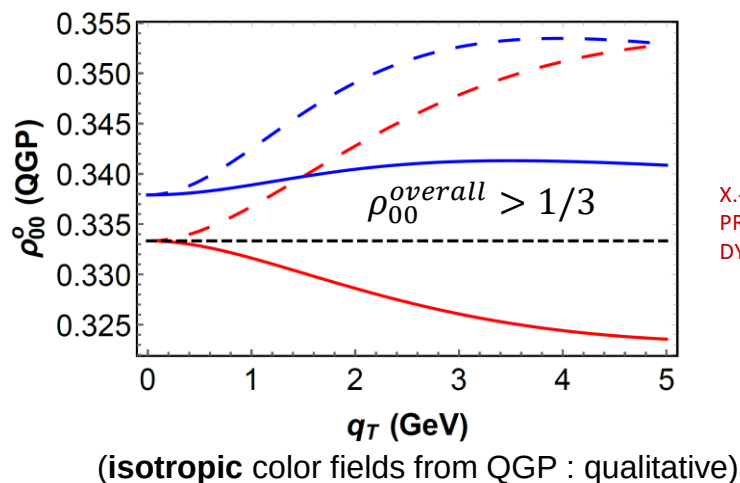
■ Late-time effects :

X.-L. Sheng et al., PRD 109, 036004, (2024)
PRL 131, 042304 (2023)
B. Müller, DY, PRD 105, L011901 (2022)
DY, JHEP 06, 140 (2022)

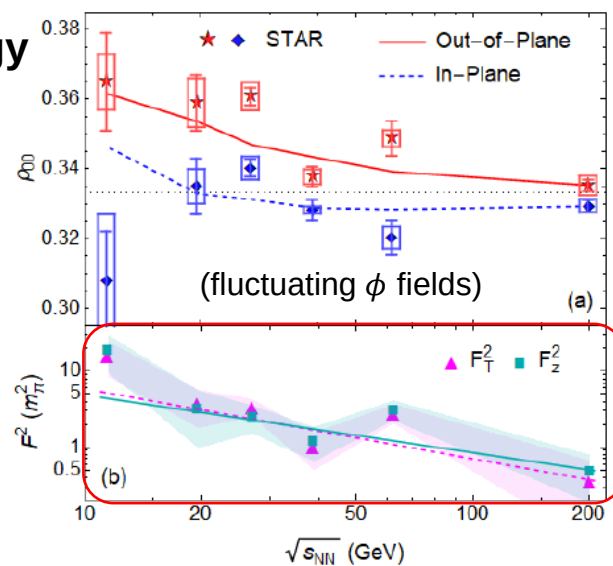


A. Kumar, Philipp Gubler, DY, PRD 109, 054038 (2024)
D. Wagner, N. Weickgenannt, E. Speranza, PRR 5, 013187 (2023)
F. Li, S. Liu, arXiv:2206.11890

❖ Final-state effect (strong-force fields in QGP) : low-energy



X.-L. Sheng et al., PRD 109, 036004, (2024)
PRL 131, 042304 (2023)
DY, PRD 110, 056005 (2025)



□ Collision-energy dependence may come from initial v.s. final state effects



Flow induced polarization from color fields

- Lambda spin polarization from background color fields?

strange-equilibrium scenario : $\mathcal{P}^\mu(\mathbf{p}) = \frac{\text{tr}_c \left[\int d\Sigma \cdot \mathbf{p} \mathcal{A}^\mu(\mathbf{p}, x) \right]}{2M_\Lambda \text{tr}_c \left[\int d\Sigma \cdot \mathcal{V}(\mathbf{p}, x) \right]}$ color-singlet WFs of s quarks

- For just the non-dynamical contribution ($\tilde{a}^{s\mu} = 0$) :

$$\mathcal{A}^{s\mu}(\mathbf{p}, x) = \left[\frac{\hbar g^2}{8p_0 N_c} \int_{t_i}^{x_0} dx'_0 \langle \tilde{F}^{a\mu\nu}(x) F_{\alpha\beta}^a(x') \rangle \mathcal{D}_{p\nu} \left(p^\beta \partial_p^\alpha f_V^s(p, x') \right) \right]_c \Big|_{p_0=\epsilon_p},$$

$$\xrightarrow{\partial_p^\alpha f_V^s \propto u^\alpha} \mathcal{A}^{s\mu} \neq 0 \quad \left\{ \begin{array}{l} u^i = 0: \langle E^{ai}(x) B^{aj}(x') \rangle \neq 0 \quad (\text{difficult}) \\ u^i \neq 0: \langle B^{ai}(x) B^{aj}(x') \rangle \neq 0 \text{ or } \langle E^{ai}(x) E^{aj}(x') \rangle \neq 0. \end{array} \right.$$

B. Müller, DY, PRD 105, L011901 (2022)

- An intuitive argument :

$$\mathbf{F}^a = g(\mathbf{E}^a + \mathbf{u} \times \mathbf{B}^a), \quad \mathcal{P}^a = g(\mathbf{B}^a - \mathbf{u} \times \mathbf{E}^a).$$

chromo-Lorentz force

chromo-magnetic polarization & spin Hall effect

H. Sung, B. Müller, DY, arXiv: 2507.23210

- Construction a p - even axial-vector :

$$\mathcal{P} \sim \langle (\mathbf{p} \cdot \mathbf{F}^a) \mathcal{P}^a \rangle \quad \text{flow-induced (local) polarization}$$

$$\sim (\mathbf{p} \times \mathbf{u}) (\langle \mathbf{B}^a \cdot \mathbf{B}^a \rangle + \langle \mathbf{E}^a \cdot \mathbf{E}^a \rangle)$$

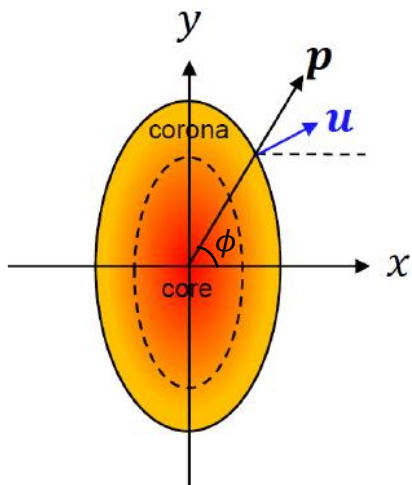
Local Λ polarization (focused on pA)

■ Longitudinal polarization from the **corona of glasma** (early hadronization) :

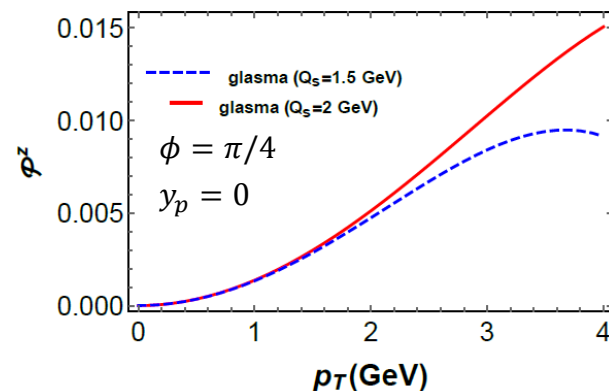
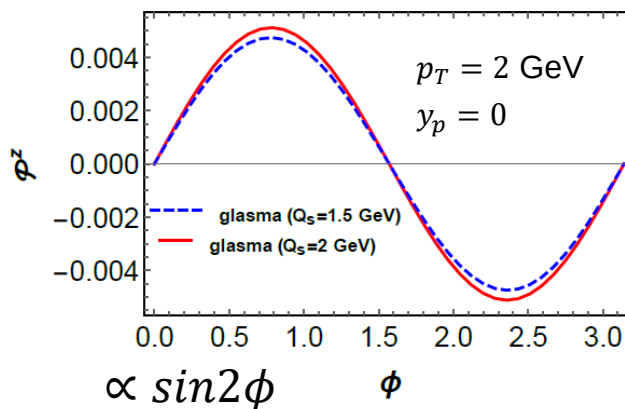
H. Sung, B. Müller, DY, arXiv: 2507.23210

$$p^z \approx \frac{\Delta t}{8M_\Lambda N_c \epsilon_p} \int d\Sigma \cdot p (p^x u^y - p^y u^x) \quad \mathbf{p} \times \mathbf{u} \text{ structure}$$

$$\times \langle B_z^a(x) B_z^a(x) \rangle (u^0 \partial_{p \cdot u} - \epsilon_p^{-1}) \partial_{p \cdot u} f_V^s, \quad f_V^s = e^{-\frac{\pi (p \cdot u)^2}{|gE^a|}}.$$



weak initial anisotropic flow
from pressure gradient

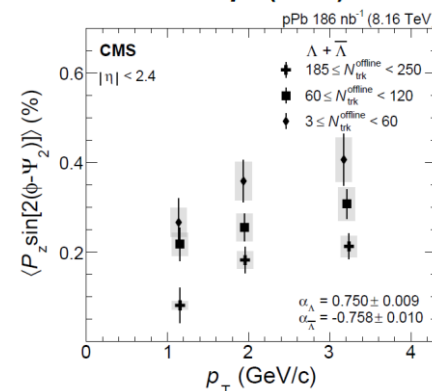


- ✓ Consistent sign & comparable order of magnitude with observations? sensitive to $\int d\Sigma \cdot p$ & $|p \cdot u|$!
- ✓ Combination with the core contribution :

For smaller systems & high p_T

For larger systems & low p_T

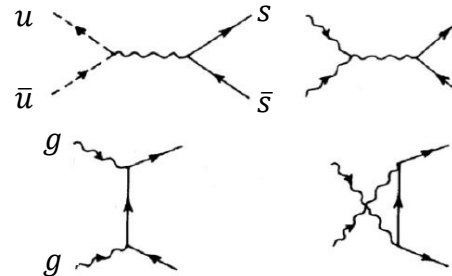
$$\diamond \text{ Full pol. : } \mathcal{P}^z = \frac{N_{\text{GL}} \mathcal{P}_{\text{GL}}^z + N_{\text{QGP}} \mathcal{P}_{\text{QGP}}^z}{N_{\text{GL}} + N_{\text{QGP}}}$$



Strangeness production in early times

■ Strangeness enhancement :

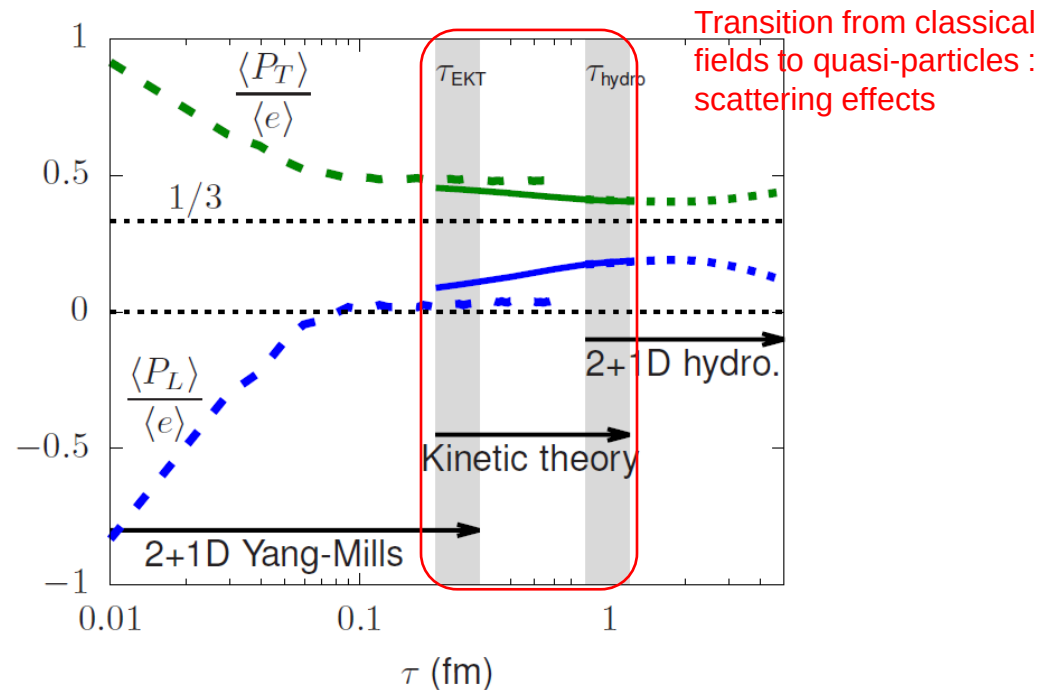
J. Rafelski, Phys. Rept. 88, 331 (1982)
J. Rafelski & B. Müller, PRL 48, 1066 (1982)



(+ QKT for s polarization from gluon fusion?)

■ Thermalization (hydrodynamization) of the QCD matter :

A. Kurkela, et al. PRL 122 (2019) 12, 122302



Local spin polarization from anisotropic flow

- Non-equilibrium phase in early times ($f_V \ll 1$) :

$$p \cdot \partial f_V \approx p_\mu \Sigma_V^{<\mu} + m \Sigma_S^{<} = \Gamma^{<}, \quad (\text{gain term dominates})$$

N. Benoit, DY, preliminary

$$p \cdot \partial \tilde{a}_\mu \approx -\frac{\hbar}{2} \tilde{F}^{<\mu\nu} p_\nu, \quad \tilde{F}^{\leq\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}^{\leq}, \quad F_{\alpha\beta}^{\leq} = \partial_{[\alpha} \Sigma_{V\beta]}^{\leq}.$$

$$\longrightarrow \mathcal{P}^\mu(\mathbf{p}, x) \approx \frac{\tilde{a}^\mu(\mathbf{p}, x)}{2M_\Lambda f_V(\mathbf{p}, x)}, \quad \tilde{a}^\mu(\mathbf{p}, x) = \frac{\hbar}{2\epsilon_{\mathbf{p}}} \int_{t_i}^{x_0} dx'_0 p_\nu \tilde{F}^{<\mu\nu} \left(x'_0, \mathbf{x} - \frac{\mathbf{p}}{\epsilon_{\mathbf{p}}} (x_0 - x'_0) \right),$$

(instantaneously hadronized)

In the corona region

$$f_V(\mathbf{p}, x) = \frac{1}{2\epsilon_{\mathbf{p}}} \int_{t_i}^{x_0} dx'_0 \Gamma^{<} \left(x'_0, \mathbf{x} - \frac{\mathbf{p}}{\epsilon_{\mathbf{p}}} (x_0 - x'_0) \right).$$

- Initial anisotropic flow of gluons ($n_g(q, x) = n_g(q \cdot u)$) :

$$p_\nu \tilde{F}^{<\mu\nu} = \frac{2g^4}{(2\pi)^4} \epsilon^{\mu\nu\alpha\beta} p_\nu \left[\rho_t u_\beta \hat{p}_\perp^\rho \partial_\alpha u_\rho + \rho_s \partial_\alpha u_\beta \right]$$

(sym or antisym) (antisym)

$$\longrightarrow \tilde{a}^i(\mathbf{p}, x) \approx \frac{\hbar \alpha_s^2}{\pi^2 \epsilon_{\mathbf{p}}} \rho_s(\mathbf{p}) \epsilon^{ijk} p^k \Delta u^j, \quad \longrightarrow \mathcal{P}^i(\mathbf{p}, x) \approx \frac{\rho_s(\mathbf{p}) \epsilon^{ijk} p^k (\partial_0 u^j)}{16\pi^4 M_\Lambda \epsilon_{\mathbf{p}} \rho_V(\mathbf{p})}$$

(time-dep & weak flow) $\Delta u^j = u^j(x_0) - u^j(t_i).$ $(x_0 - t_i \rightarrow 0)$

$$\mathcal{P}(\mathbf{p}) \propto \dot{\mathbf{u}} \times \mathbf{p}$$

for small systems?

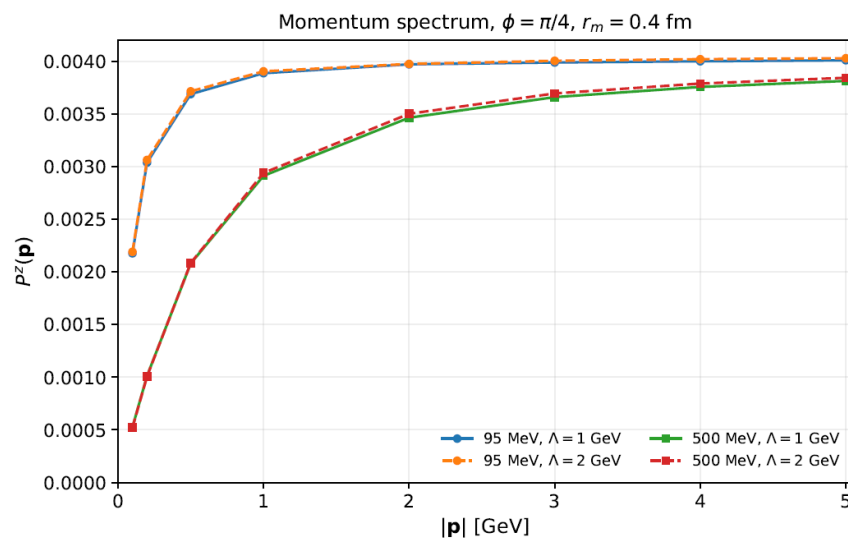
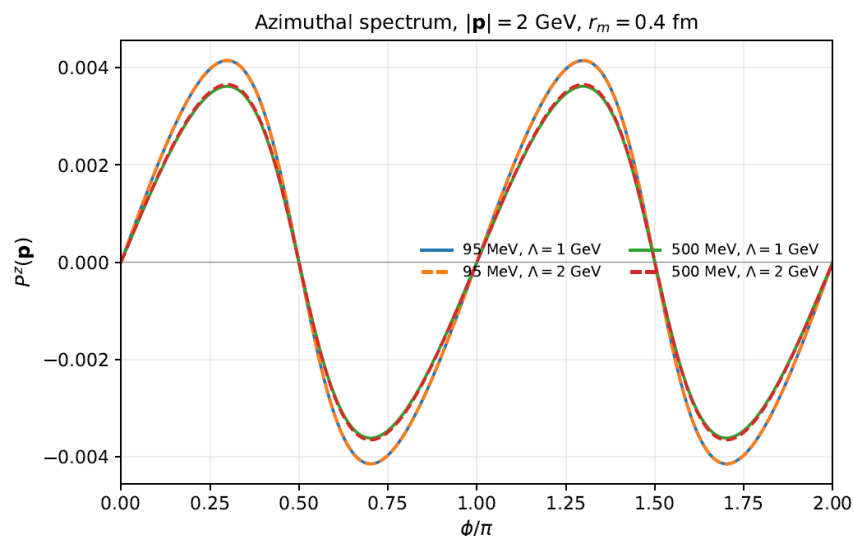
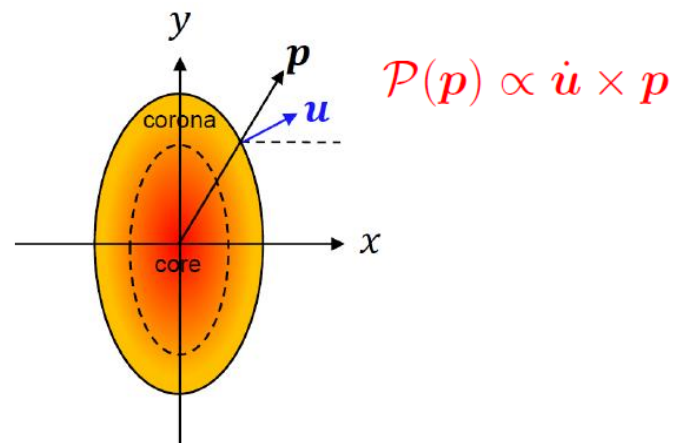
Numerical estimate

■ Longitudinal polarization:

$$\mathcal{P}^z(\mathbf{p}, x) \approx \frac{\rho_s(|\mathbf{p}|)}{16\pi^4 M_\Lambda \rho_V(|\mathbf{p}|) \epsilon_p} (p^y (\partial_0 u^x) - p^x (\partial_0 u^y))$$

$$\mathcal{P}^z(\mathbf{p}, t) = \frac{1}{2\pi} \int_0^{2\pi} d\Phi \mathcal{P}^z(\mathbf{p}, x) \Theta(\mathbf{p} \cdot \mathbf{u})|_{r=r_m, \tau=t}$$

N. Benoit, DY, preliminary





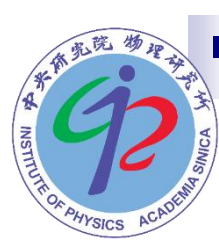
Summary & outlook

❑ Conclusions :

- ✓ Spin polarization phenomena for hadrons (**Λ polarization & vector-meson spin alignment**) provide a new opportunity to study QCD matter (glasma & QGP) in nuclear collisions.
- ✓ **QKT** is a useful tool to study **non-equilibrium** spin transport.
- ✓ **Overpopulated initial-state gluons** could play a significant role on local spin polarization and spin alignment on top of the “hydro” contributions.
- ✓ The **size** and **energy** dependence of collision systems may be helpful to disentangle the competing effects.

❑ Outlook :

- Gluon fusion with anisotropic flow may also lead to local spin polarization of strange quarks. ➡ interplay btw **strangeness enhancement & local pol. of Λ ?** (preliminary)
- More sophisticated modeling & simulations are needed.
- (Collective) spin transport for heavy quarks : more sensitive to the early-state (e.g., glasma) effects.

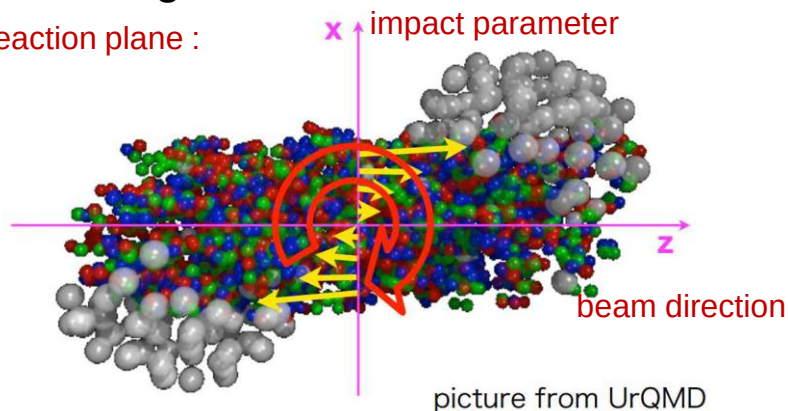


Thank you!

Subatomic swirls

■ Strong vortical fields in HIC :

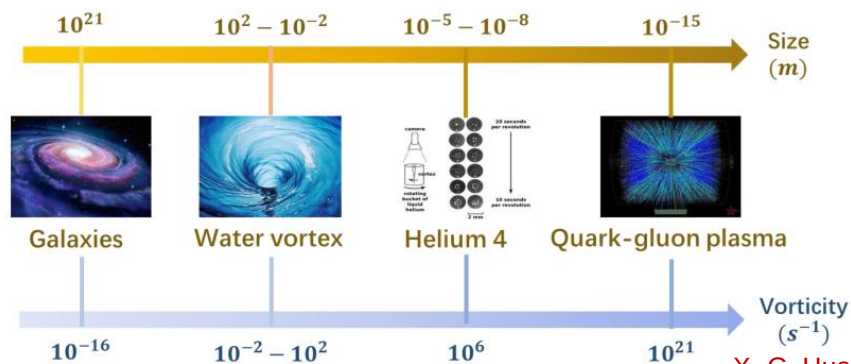
Reaction plane :



❖ Angular momentum (AM) to vorticity :

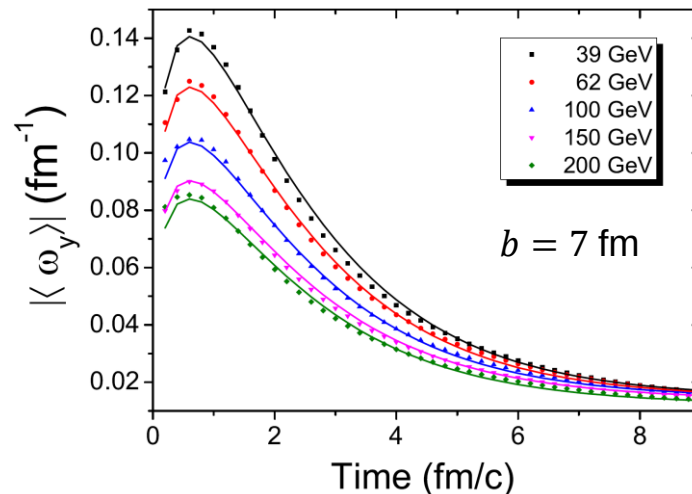
$$\mathbf{L} = \frac{1}{2} \int d^3\mathbf{r} \epsilon |\mathbf{r}|^2 (1 - \hat{\omega} \cdot \hat{\mathbf{r}}) \boldsymbol{\omega},$$

$$\boldsymbol{\omega} = \nabla \times \mathbf{v} = \text{const.} \quad \epsilon : \text{energy density}$$

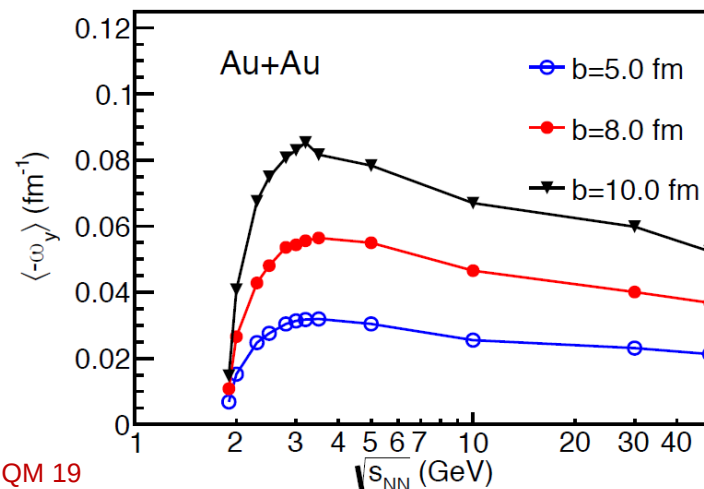


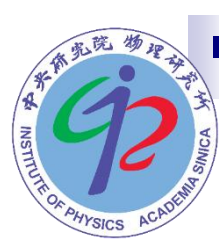
X.-G. Huang, QM 19

Y. Jiang, Z.-W. Lin, J. Liao, PRC 94, 044910 (2016)
see also W.-T. Deng and X.-G. Huang, PRC 93, 064907 (2016)



X.-G. Deng et al., PRC101.064908 (2020)





Spin polarization near local equilibrium

■ Polarization spectrum in local equilibrium :

local equilibrium, indep. of int.

$$\mathcal{A}_{\text{eq}}^{\mu}(\mathbf{p}, x) = \left(\mathcal{J}_{\text{thermal}}^{\mu} + \mathcal{J}_{\text{shear}}^{\mu} + \mathcal{J}_{\text{accT}}^{\mu} + \mathcal{J}_{\text{chemical}}^{\mu} + \mathcal{J}_{\text{EB}}^{\mu} \right)_{p_0 = \epsilon_{\mathbf{p}}}$$

(+ nonequilibrium corrections, collisions dep.)

Y. Hidaka, S. Pu, DY, PRD 97, 016004 (2018)
S. Liu, Y. Yin, JHEP 07, 188 (2021)
F. Becattini, et al., PLB 820, 136519 (2021)
C. Yi, S. Pu, DY, PRC 104, 064901(2021)

$$\left\{ \begin{array}{l} \mathcal{J}_{\text{thermal}}^{\mu} = \frac{a}{2} \epsilon^{\mu\nu\alpha\beta} p_{\nu} \omega_{\alpha\beta}, \quad \Rightarrow \quad J_5^{\mu} = \sigma_5 \omega^{\mu} \\ \mathcal{J}_{\text{shear}}^{\mu} = \frac{-a}{(u \cdot p)T} \epsilon^{\mu\nu\alpha\beta} p_{\alpha} u_{\beta} p^{\sigma} \pi_{\sigma\nu}, \quad \pi_{\mu\nu} = \partial_{\mu} u_{\nu} + \partial_{\nu} u_{\mu} - u_{\mu} (u \cdot \partial) u_{\nu}, \\ \mathcal{J}_{\text{accT}}^{\mu} = \frac{-a}{2T} \epsilon^{\mu\nu\alpha\beta} p_{\nu} u_{\alpha} (D u_{\beta} - \frac{1}{T} \partial_{\beta} T), \\ \mathcal{J}_{\text{chemical}}^{\mu} = \frac{a}{(u \cdot p)} \epsilon^{\mu\nu\alpha\beta} p_{\alpha} u_{\beta} \partial_{\nu} \frac{\mu}{T}, \\ \mathcal{J}_{\text{EB}}^{\mu} = \frac{a}{(u \cdot p)T} \epsilon^{\mu\nu\alpha\beta} p_{\alpha} u_{\beta} E_{\nu} + a \frac{B^{\mu}}{T}, \quad \Rightarrow \quad J_5^{\mu} = \sigma_5 B^{\mu} \end{array} \right.$$

kinetic vorticity :

$$\omega^{\mu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} u_{\nu} \partial_{\alpha} u_{\beta}$$

$$a = \frac{\hbar}{2\epsilon_{\mathbf{p}}} \text{sign}(u \cdot p) f_V^{(0)} (1 - f_V^{(0)}),$$

$$f_V^{(0)} = \frac{1}{e^{p \cdot u / T} + 1}.$$

$$\text{global pol. : } \mathcal{P}^{\mu} \sim \int_{Vp} d^3 p \mathcal{P}^{\mu}(p) \sim J_5^{\mu}$$



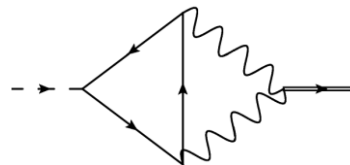
Axial chiral vortical effect (aCVE)

■ Radioactive correction on aCVE (massless):

S. Golkar, D. T. Son, JHEP 02, 169 (2012).

D.-F. Hou, H. Liu, H.-c. Ren, PRD 86, 121703 (2012).

$$J_5^\mu = N_c N_f \frac{T^2}{6} \left(1 + \frac{g^2(N_c^2 - 1)}{8\pi^2 N_c} \right) \omega^\mu$$



■ For massive fermions :

S. Fang, S. Pu, DY, PRD 112 (2025) 1, 014038

$$J_5^\mu = \frac{N_c N_f T^2 \omega^\mu}{\pi^2} \left[I_0(\beta m) + \frac{g^2(N_c^2 - 1)}{8N_c} \left(1 + \frac{\mu^2}{\pi^2 T^2} \right) I_1(\beta m) \right]$$

See also M. Buzzegoli, D. E. Kharzeev, PRD 103, 116005 (2021)
for different approximations.

■ Massless limit :

$$J_{5,m=0}^\mu = \frac{N_c N_f T^2 \omega^\mu}{6} \left[1 + \frac{3(N_c^2 - 1)g^2}{16N_c\pi^2} + \frac{(N_c^2 - 1)g^2}{8N_c\pi^2} \left(1 + \frac{\mu^2}{\pi^2 T^2} \right) \int_{\Lambda_{\text{IR}}}^{+\infty} \frac{d|\mathbf{p}|}{|\mathbf{p}|} f_V^{(0)} \right]$$

HTL approx.

Spin correlations from color fields

- Spin density matrix can be directly related to Wigner functions of the coalesced quark and antiquark through the quark-meson interaction.

❖ Kinetic theory of vector mesons :

$$q \cdot \partial f_{\lambda}^{\phi} = \epsilon_{\mu}^{*}(\lambda, \mathbf{q}) \epsilon_{\nu}(\lambda, \mathbf{q}) [\mathcal{C}_{\text{coal}}^{\mu\nu}(q, x)(1 + f_{\lambda}^{\phi}) - \mathcal{C}_{\text{diss}}^{\mu\nu}(q, x)f_{\lambda}^{\phi}] \approx \boxed{\epsilon_{\mu}^{*}(\lambda, \mathbf{q}) \epsilon_{\nu}(\lambda, \mathbf{q}) \mathcal{C}_{\text{coal}}^{\mu\nu}(q, x)}$$

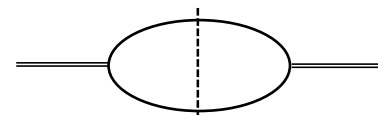
➡

$$\rho_{00}(q) = \frac{\int d\Sigma_X \cdot q f_0^{\phi}(q, X)}{\int d\Sigma_X \cdot q (f_0^{\phi}(q, X) + f_{+1}^{\phi}(q, X) + f_{-1}^{\phi}(q, X))}$$

$$= \frac{1 - \text{Tr}_c \langle \hat{\mathcal{P}}_q^y(\mathbf{q}/2) \hat{\mathcal{P}}_{\bar{q}}^y(\mathbf{q}/2) \rangle_{q=0}}{3 - \sum_{i=x,y,z} \text{Tr}_c \langle \hat{\mathcal{P}}_q^i(\mathbf{q}/2) \hat{\mathcal{P}}_{\bar{q}}^i(\mathbf{q}/2) \rangle_{q=0}}$$

quark-meson int. :

$$\mathcal{L}_{\text{int}} = g_{\phi} \bar{\psi} \gamma^{\mu} V_{\mu} \psi$$

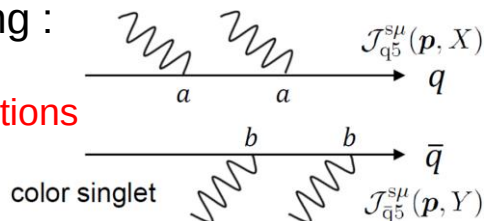


❖ Spin correlations in the non-relativistic limit :

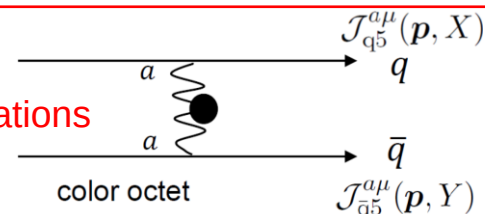
$$\text{Tr}_c \langle \hat{\mathcal{P}}_q^i(\mathbf{p}) \hat{\mathcal{P}}_{\bar{q}}^i(\mathbf{p}) \rangle \approx \frac{4 \int d\Sigma_X \cdot p (\langle \mathcal{A}_q^{\text{si}}(\mathbf{p}, X) \mathcal{A}_{\bar{q}}^{\text{si}}(\mathbf{p}, X) \rangle + \langle \mathcal{A}_q^{\text{ao}}(\mathbf{p}, X) \mathcal{A}_{\bar{q}}^{\text{ao}}(\mathbf{p}, X) \rangle) / (2N_c)}{\int d\Sigma_X \cdot p f_{Vq}^{\text{si}}(\mathbf{p}, X) f_{V\bar{q}}^{\text{si}}(\mathbf{p}, X)}$$

➤ Weak coupling :

4-field correlations
 $\propto F^4$

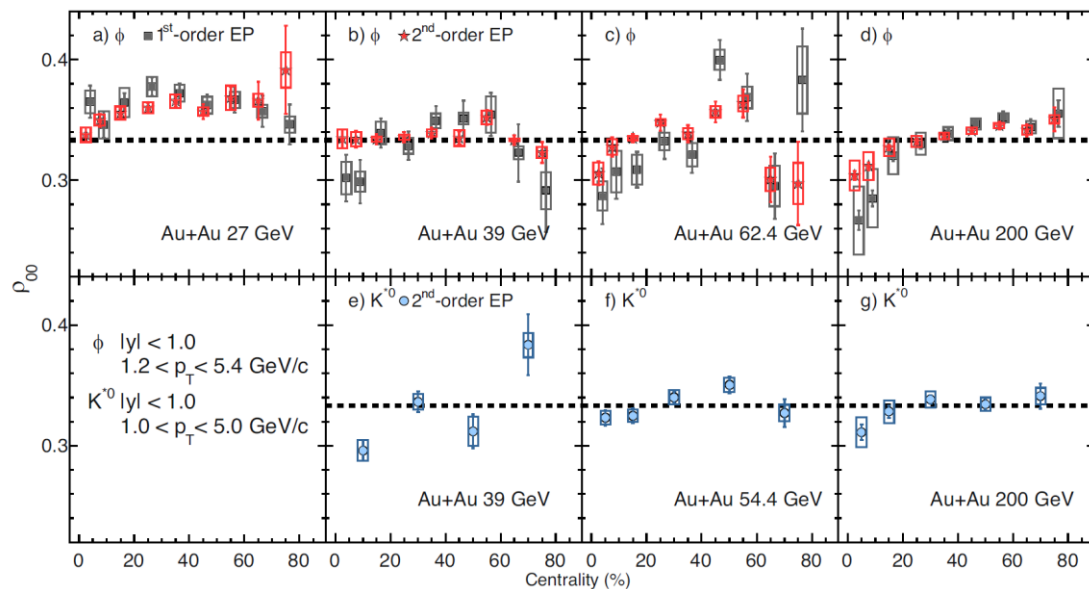
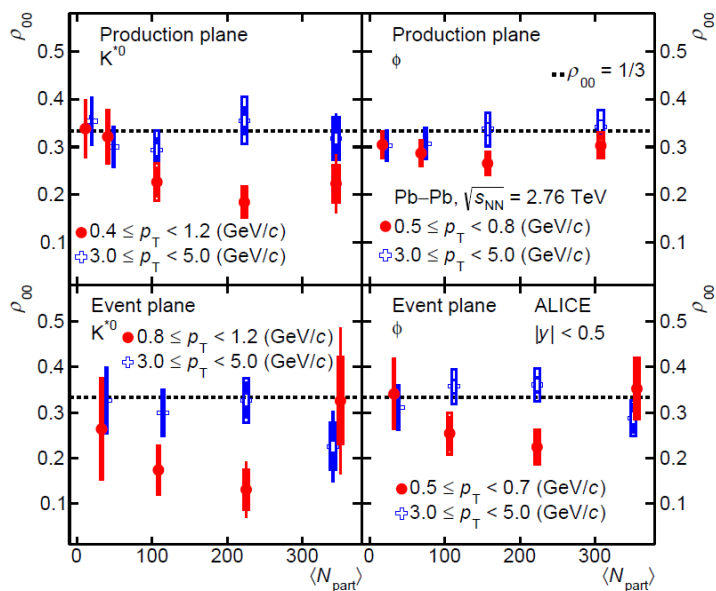


2-field correlations
 $\propto F^2$

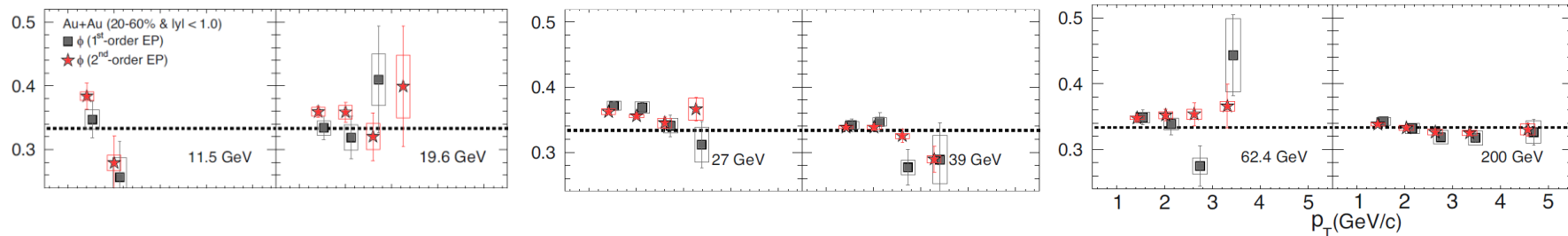


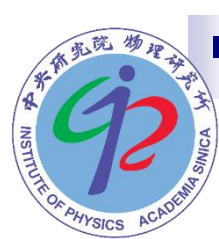
More experimental data

❖ Centrality dependence :



❖ p_T dependence in RHIC :





Axial kinetic theory with color fields

- Incorporation of background color fields into Wigner functions and kinetic equations.

- Color decomposition : $O = O^s I + O^a t^a$

U. W. Heinz, Phys. Rev. Lett. 51, 351 (1983)

H. T. Elze, M. Gyulassy, D. Vasak, Nucl. Phys. B276, 706 (1986).

e.g., $\mathcal{A}^\mu(p, x) = \mathcal{A}^{s\mu}(p, x)I + \mathcal{A}^{a\mu}(p, x)t^a$, $f_V(p, x) = f_V^s(p, x)I + f_V^a(p, x)t^a$,
 $\tilde{a}^\mu(p, x) = \tilde{a}^{s\mu}(p, x)I + \tilde{a}^{a\mu}(p, x)t^a$.

- Kinetic equations : DY, JHEP 06, 140 (2022)

SKEs : $p^\rho \left(\partial_\rho f_V^s + \frac{g}{2N_c} F_{\nu\rho}^a \partial_p^\nu f_V^a \right) = \mathcal{C}_s$, $p^\rho \left(\partial_\rho f_V^a + g F_{\nu\rho}^a \partial_p^\nu f_V^s + \frac{d^{bca}}{2} g F_{\nu\rho}^b \partial_p^\nu f_V^c \right) = \mathcal{C}_o^a$,

diffusion

dynamical spin polarization

AKEs : $p^\rho \partial_\rho \tilde{a}^{s\mu} + \frac{g}{2N_c} \left(p^\rho F_{\nu\rho}^a \partial_p^\nu \tilde{a}^{a\mu} + F^{a\nu\mu} \tilde{a}_\nu^a \right) - \frac{\hbar}{4N_c} \epsilon^{\mu\nu\rho\sigma} p_\rho (\partial_\sigma g F_{\beta\nu}^a) \partial_p^\beta f_V^a = \mathcal{C}_s^\mu$,

$p^\rho \partial_\rho \tilde{a}^{a\mu} + g \left(p^\rho F_{\nu\rho}^a \partial_p^\nu \tilde{a}^{s\mu} + F^{a\nu\mu} \tilde{a}_\nu^s \right) + \frac{d^{bca}}{2} g \left(p^\rho F_{\nu\rho}^b \partial_p^\nu \tilde{a}^{c\mu} + F^{b\nu\mu} \tilde{a}_\nu^c \right) - \frac{\hbar}{2} \epsilon^{\mu\nu\rho\sigma} p_\rho (\partial_\sigma g F_{\beta\nu}^a) \partial_p^\beta f_V^s = \mathcal{C}_o^{a\mu}$.

Axial Wigner functions :

$\mathcal{A}^{s\mu}(p, x) = \frac{1}{2\epsilon_p} \left[\tilde{a}^{s\mu} - \frac{\hbar}{4N_c} \tilde{F}^{a\mu\nu} \left(\partial_{p\nu} f_V^a - \frac{\epsilon_p}{2} \partial_{p\perp\nu} (f_V^a / \epsilon_p) \right) \right]_{p_0=\epsilon_p}$,

$\mathcal{A}^{a\mu}(p, x) = \frac{1}{2\epsilon_p} \left[\tilde{a}^{a\mu} - \frac{\hbar}{2} \tilde{F}^{a\mu\nu} \left(\partial_{p\nu} f_V^s - \frac{\epsilon_p}{2} \partial_{p\perp\nu} (f_V^s / \epsilon_p) \right) \right]_{p_0=\epsilon_p}$.

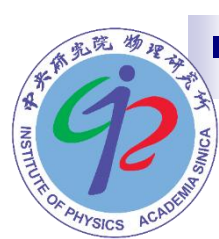
dynamical (w/ memory effect)

non-dynamical (w/o memory effect)

Spin

polarization:

$\mathcal{P}^\mu(p) = \frac{\int d\Sigma \cdot p \text{Tr}_c \mathcal{A}^\mu(p, x)}{2m \int d\Sigma \cdot p (2\epsilon_p)^{-1} f_V^s(p, x)} = \frac{\int d\Sigma \cdot p \mathcal{A}^{s\mu}(p, x)}{2m \int d\Sigma \cdot p (2\epsilon_p)^{-1} f_V^s(p, x)}$.



Spin “polarization” from glasma fields

- Generalized AKT with color fields : $\tilde{a}^\mu(p, x) = \tilde{a}^{s\mu}(p, x)I + \boxed{\tilde{a}^{a\mu}(p, x)} t^a$.
 DY, JHEP 06, 140 (2022)
 B. Müller, DY, PRD 105, L011901 (2022) (more dominant in the perturbative approach)

- Assuming the (s) quark is produced from the glasma : given $f_V^s \Rightarrow$ induced $\tilde{a}_\mu^a$

- Glasma phase ($t_i \leq t \leq t_{th}$) :

- Color-octet AKE with the **weak-field** approx. :

$$p^\rho \partial_\rho \tilde{a}^{a\mu} - \frac{\hbar}{2} \epsilon^{\mu\nu\rho\sigma} p_\rho (\partial_\sigma g F_{\beta\nu}^a) \partial_p^\beta f_V^s = 0$$

$$\xrightarrow{\quad} \tilde{a}^{ai}(p, x)|_{t=t_{th}} \approx -\frac{\hbar g}{2} \left[\left(B^{ai}(t_i) - \frac{\epsilon^{ijk} p_j E_k^a(t_i)}{p_0} \right) \partial_{p_0} f_V^s(p_0, t_i) \right. \\ \left. - \left(B^{ai}(t_{th}) - \frac{\epsilon^{ijk} p_j E_k^a(t_{th})}{p_0} \right) \partial_{p_0} f_V^s(p_0, t_{th}) \right]$$

$f_V^s(p, x) = f_V^s(p_0, t)$

suppressed

- QGP phase ($t_{th} \leq t$) :

- Relaxation-time approx. : $p^\rho \partial_\rho \tilde{a}^{a\mu} = -\frac{p_0 \tilde{a}^{a\mu}}{\tau_R^0} \xrightarrow{\quad} \tilde{a}^{a\mu}(p, x) = \tilde{a}^{a\mu}(t_{th}) e^{-\frac{(t-t_{th})}{\tau_R^0}}$

- ❖ Axial WF : $\mathcal{A}^{ai}(p, x) \approx -\frac{\hbar g}{4\epsilon_p} e^{\frac{-(t-t_{th})}{\tau_R^0}} \left[B^{ai}(t_i) - \frac{\epsilon^{ijk} p_j E_k^a(t_i)}{\epsilon_p} \right] \partial_{\epsilon_p} f_V^s(\epsilon_p, t_i)$
 (no contribution to Λ pol.)
blue relaxation spin polarization

Color fields from the glasma

- Solving linearized Yang-Mills eqs. : $[D_\mu, F^{\mu\nu}] = J^\nu$

P. Guerrero-Rodriguez, T. Lappi, PRD 104, 014011 (2021)

- Color-field correlators in the glasma :

evolution in time

$$\text{e.g. } \langle E_T^{ai}(X') E_T^{aj}(X'') \rangle = -\bar{N}_c \epsilon^{in} \epsilon^{jm} \int_{\perp; q, u}^{X'} \int_{\perp; l, v}^{X''} \Omega_-(u_\perp, v_\perp) \frac{q^n l^m}{ql} \times \boxed{J_1(qX'_0) J_1(lX''_0)},$$

$$\langle B_T^{ai}(X') B_T^{aj}(X'') \rangle = -\bar{N}_c \epsilon^{in} \epsilon^{jm} \int_{\perp; q, u}^{X'} \int_{\perp; l, v}^{X''} \Omega_+(u_\perp, v_\perp) \frac{q^n l^m}{ql} \times J_1(qX'_0) J_1(lX''_0),$$

$$\bar{N}_c \equiv \frac{1}{2} g^2 N_c (N_c^2 - 1),$$

$$\Omega_\mp(u_\perp, v_\perp) = [G_1(u_\perp, v_\perp) \boxed{G_2(u_\perp, v_\perp)} \mp h_1(u_\perp, v_\perp) \boxed{h_2(u_\perp, v_\perp)}],$$

$$\int_{\perp; q, u}^{X'} \equiv \int \frac{d^2 q_\perp}{(2\pi)^2} \int d^2 u_\perp e^{iq_\perp (X' - u)_\perp}.$$

unpolarized & linearly polarized
Gluon distribution functions

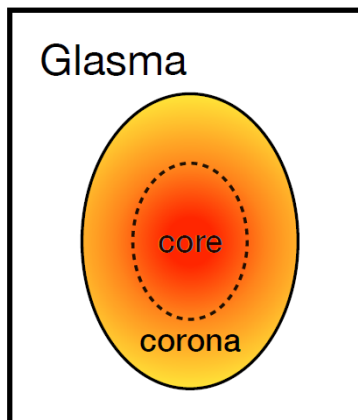
- Golec-Biernat Wusthoff (GBW) distribution : K. J. Golec-Biernat and M. Wusthoff, PRD 59, 014017 (1998)

$$\Omega_\pm(u_\perp, v_\perp) = \Omega(u_\perp, v_\perp) = \frac{Q_s^4}{g^4 N_c^2} \left(\frac{1 - e^{-Q_s^2 |u_\perp - v_\perp|^2 / 4}}{Q_s^2 |u_\perp - v_\perp|^2 / 4} \right)^2$$

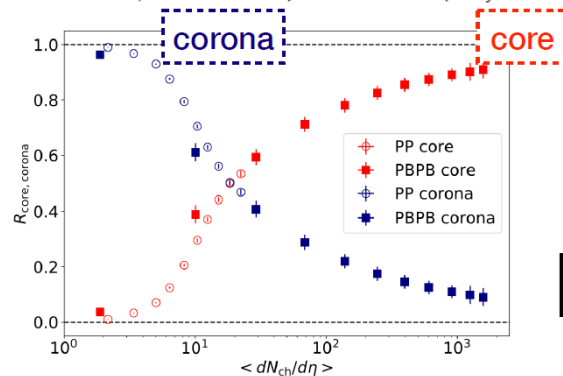
Core-corona scanario

Corona effect

Haesom Sung, ISMD 2026



Y. Kanakubo, Y. Tachibana, and T. Hirano, Phys. Rev. C 105, 024905 (2022), arXiv:2108.07943

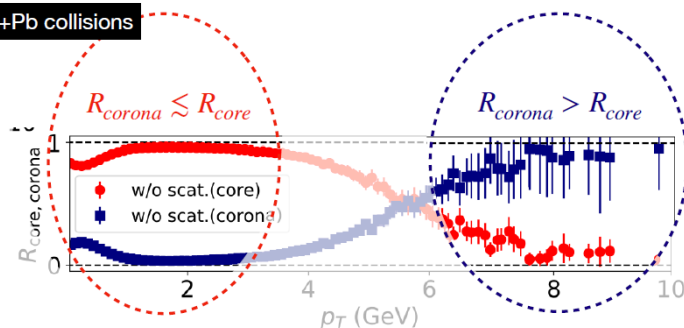


p+p collisions at $\sqrt{s} = 7 \text{ TeV}$

Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76 \text{ TeV}$

$$R_{corona} \sim 0.5 \text{ at } \langle dN_{ch}/d\eta \rangle \sim 11$$

Pb+Pb collisions



In high p_T : $R_{corona} > R_{core}$

In small p_T : $R_{corona} < R_{core}$

Motivation

Introduction

Calculation

Result

Haesom Sung

Longitudinal Local Polarization

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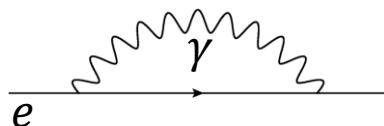


Self-energy corrections

■ We previously consider $\Sigma_{\mu}^{\leq}$ related to $\text{Im}(\Sigma_{\mu}^{r/a})$ for scattering.

■ How about $\bar{\Sigma}_{\mu} = \text{Re}(\Sigma_{\mu}^{r/a})$?

➤ changing onshell condition : $q^2 = 0 \rightarrow q^2 = 2q \cdot \bar{\Sigma} = m_{\text{th}}^2$



➤ KB equation : $(\not{D} - m)S^< + \gamma^{\mu} \frac{i\hbar}{2} \nabla_{\mu} S^< + \bar{\Sigma} \star S^< = \frac{i\hbar}{2} (\Sigma^< \star S^> - \Sigma^> \star S^<)$

$\propto e^2$ $\propto e^4$

■ QKT with the self-energy corrections :

N. Yamamoto, DY, PRD 109, 056010 (2024)

S. Fang, S. Pu, DY, PRD 109, 034034 (2024)

❖ For $\bar{\Sigma}_V^{\mu} \neq 0 \Rightarrow F_{\mu\nu}^{\text{eff}} : \bar{F}_{\mu\nu} = \partial_{[\mu} \bar{\Sigma}_{V\nu]}$ **effective background EM fields**

WFs $\left\{ \begin{array}{l} \mathcal{V}_{\mu}^<(p, x) = 2\pi\epsilon(p_0)\delta(p^2 - m^2 - 2p \cdot \bar{\Sigma}_V)(p_{\mu} - \bar{\Sigma}_{V\mu})f_V(p, x), \\ \mathcal{A}_{\mu}^<(p, x) = 2\pi\epsilon(p_0)[\delta(p^2 - m^2 - 2p \cdot \bar{\Sigma}_V)\tilde{a}_{\mu}^< + \hbar\delta'(p^2 - m^2)f_V^<\tilde{\tilde{F}}_{\mu\nu}p^{\nu}] \end{array} \right.$

non-dynamical pol.

KEs $\left\{ \begin{array}{l} p^{\rho}(\partial_{\rho} - \bar{\Sigma}_{V\rho})f_V^< - p^{\rho}[\bar{\Sigma}_{V\rho}, f_V^<]_{\text{P.B.}} = \mathcal{C}[f_V], \\ p^{\rho}(\partial_{\rho} - \bar{\Sigma}_{V\rho})\tilde{a}_{\mu}^< - p^{\rho}[\bar{\Sigma}_{V\rho}, \tilde{a}_{\mu}^<]_{\text{P.B.}} - \bar{F}_{\mu\nu}\tilde{a}^{<\nu} + \hbar\frac{p^{\beta}}{2}[\tilde{\tilde{F}}_{\mu\beta}, f_V^<]_{\text{P.B.}} = \mathcal{C}_{\mathcal{A}}^{\mu}. \end{array} \right.$