Quantum simulations of lattice field theory with qubits and qumodes

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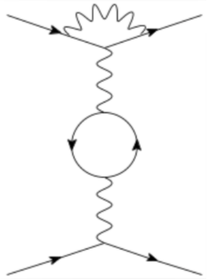
Quantum chromodynamics

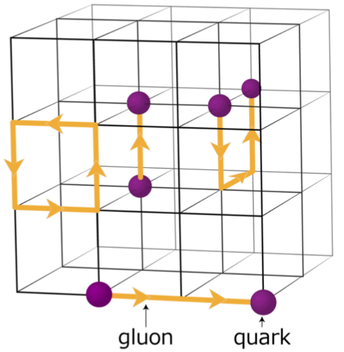
1. Perturbative QCD at high energies

$$\sigma = \sigma^{(0)} + \alpha_s \, \sigma^{(1)} + \alpha_s^2 \, \sigma^{(2)} + \ldots$$

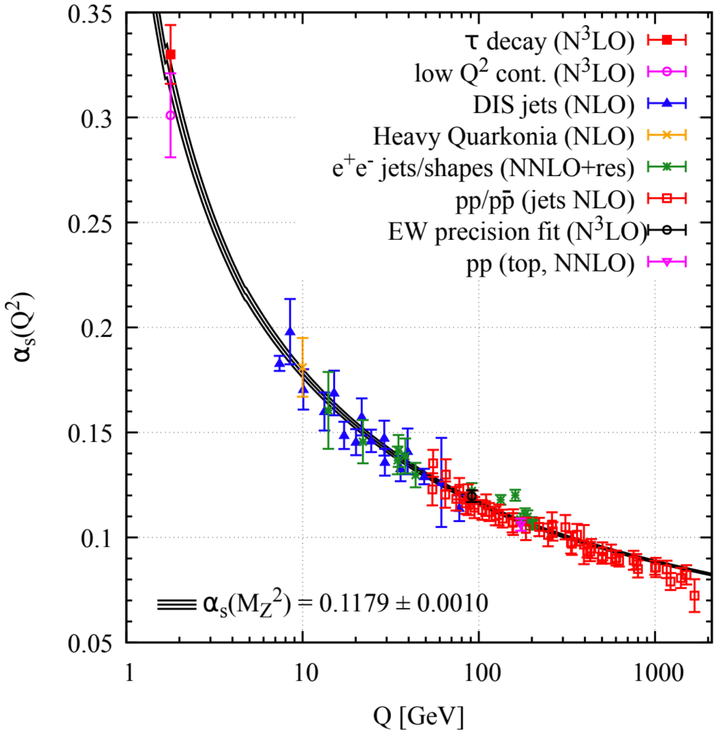


2. Nonperturbative at low energies

- Simulations using a lattice discretization
- Path integral and Hamiltonian formulation



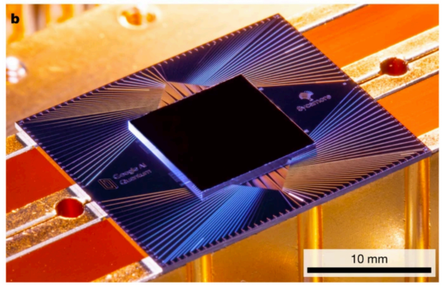
Wilson; Kogut, Susskind `70s



Huston, Rabbertz, Zanderighi (PDG)

Quantum computing for nuclear physics

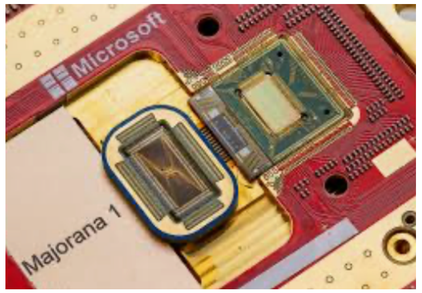
- Quantum advantage for random circuit sampling (2019)



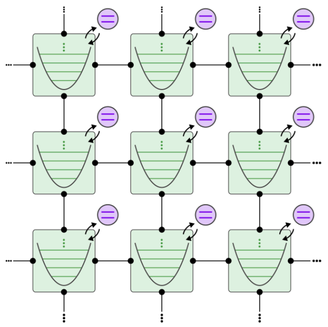
- Progress toward long-lived logical qubits using quantum error correction

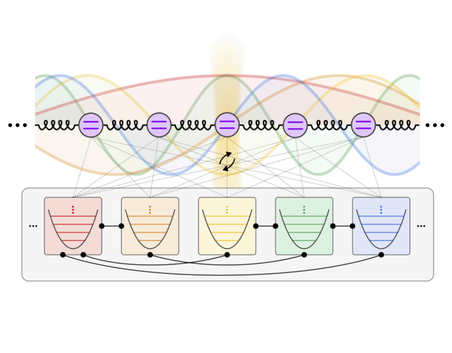


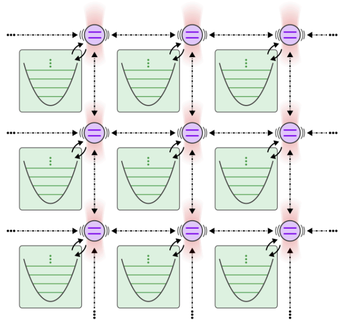


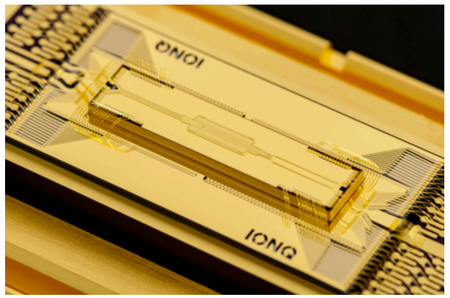


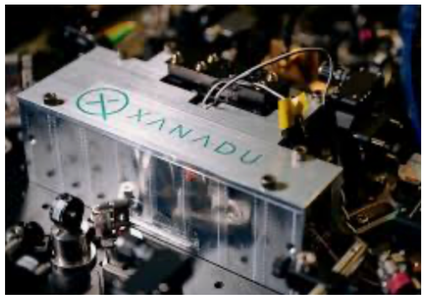
- Hybrid quantum hardware



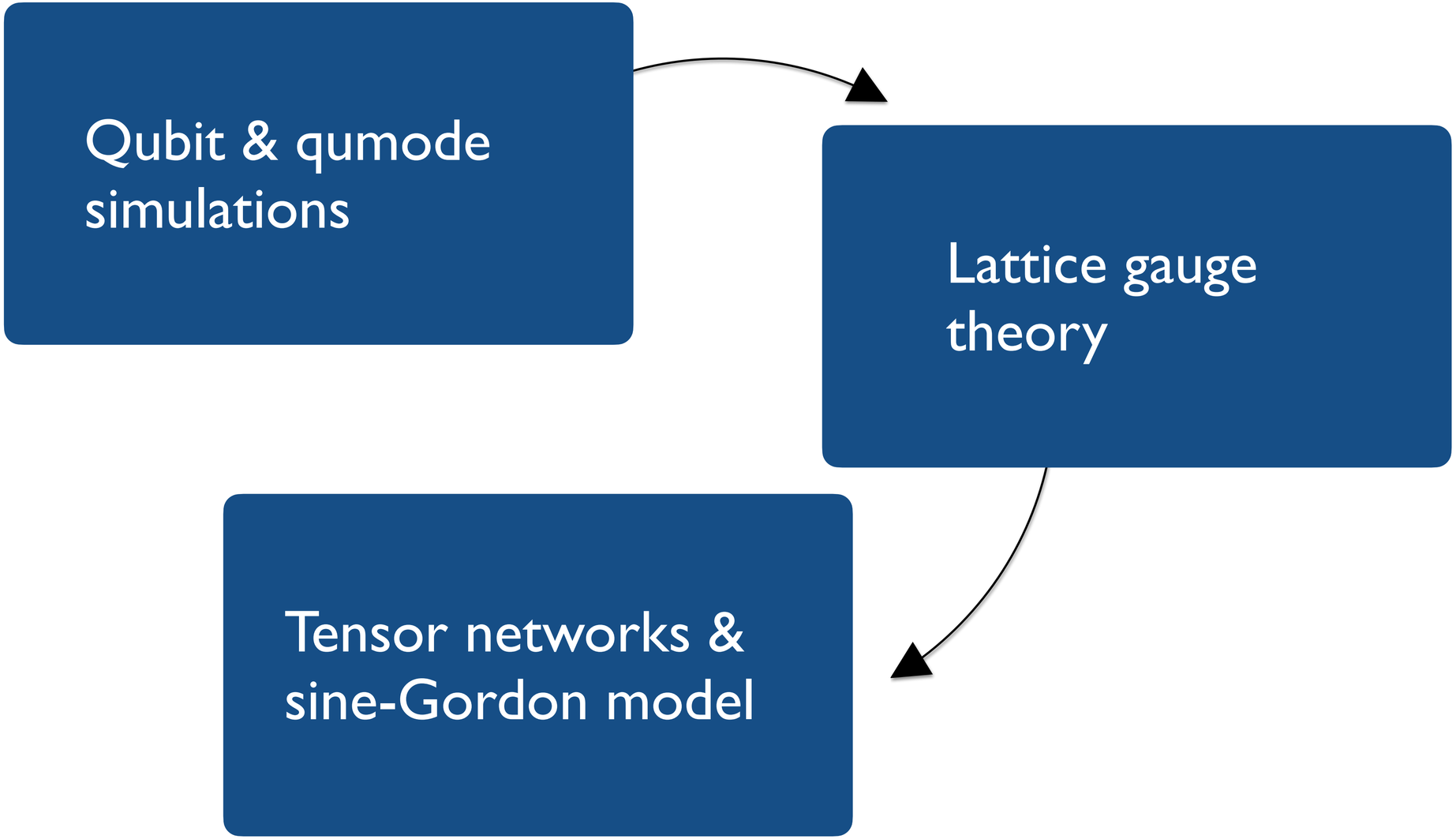








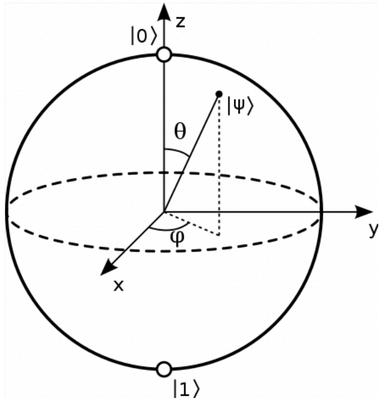
Outline

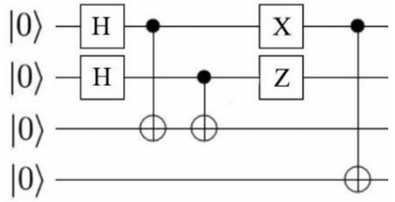

Qubit & qumode simulations
Lattice gauge theory
Tensor networks & sine-Gordon model

Qubits, qudits and qumodes

Elementary units for computing

- **Qubits**
  - Superconducting circuits, cold atoms, trapped ions, topological qubits
  - Digital gate-based computing
  - Superposition $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$
  - Entanglement





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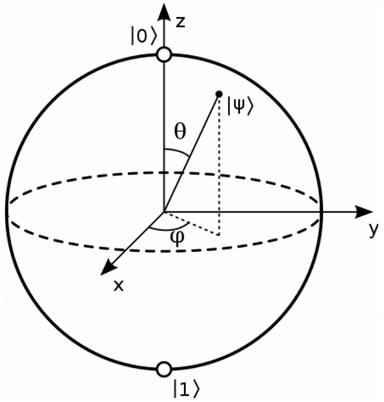
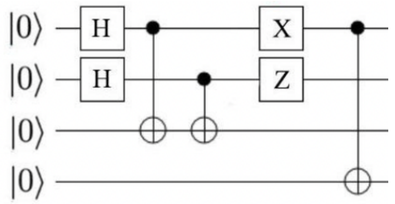
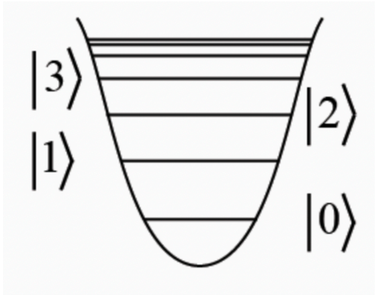




- **Qudits**
  - Multi-level $d > 2$ computational units
  - Gates e.g. $X_d,\ Z_d,\ C_2[R_d]$
  - Various hardware platforms



see e.g. Savage et al., Zoller et al.

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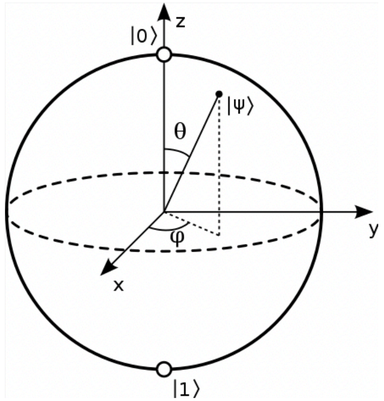
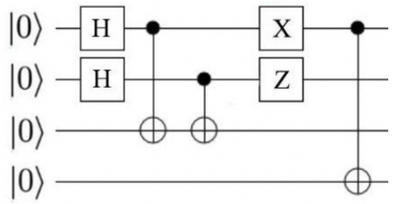
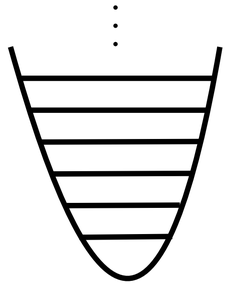
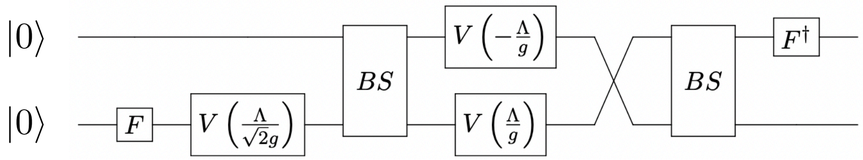




- **Qumodes**
  - Photonics, trapped ions, superconducting circuits
  - Infinite-dimensional Hilbert space
  - Gate based but with continuous variables





Quantum simulations with qumodes

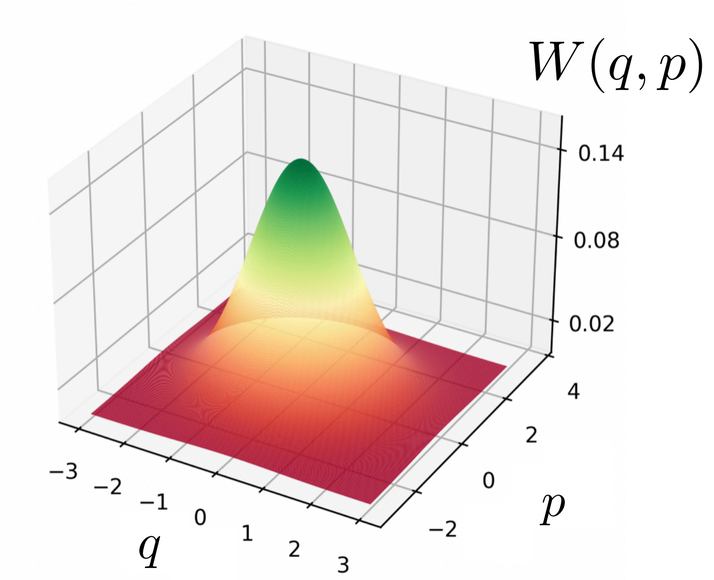
see Lloyd, Braunstein `99

- Bosonic raising/lowering operators $\hat{a}, \hat{a}^\dagger$ and Fock states $|n\rangle \sim \left(a^\dagger\right)^n |0\rangle$
- Quadratures $\hat{q}, \hat{p}$ with $[\hat{q}, \hat{p}] = i$ and state $|\psi\rangle = \int_{\mathbb{R}} \psi(q) \, |q\rangle \, \mathrm{d}q$
- Visualize using Wigner functions

$$W(q,p) \sim \text{ F.T. } \psi^*(q+q')\psi(q-q')$$

Ground state $|0\rangle$

Phase space

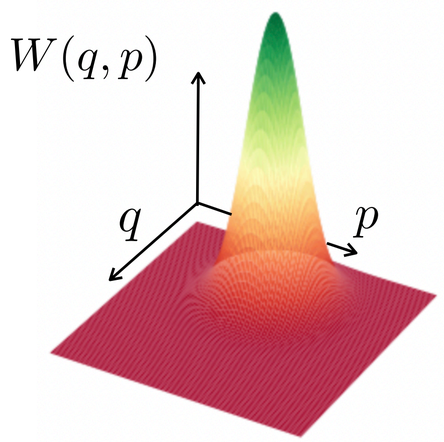


Quantum simulations with qumodes

see Lloyd, Braunstein `99

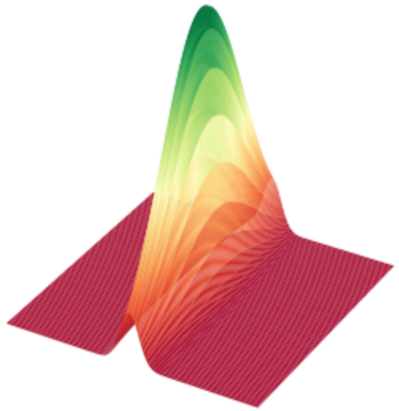
Universal gate set

- Displacement $D(z) = e^{z\hat{a}^\dagger - z^*\hat{a}}$



- Squeezing

$$S(z) = e^{\frac{1}{2}(z^*\hat{a}^2 - z\hat{a}^{\dagger 2})}$$

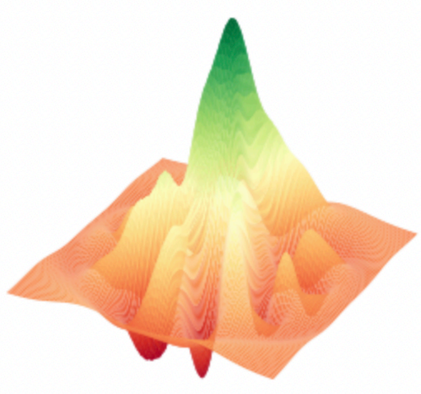


- Two-qumode beam splitter

$$U_{\text{bs}}(z) = e^{z\hat{a}\hat{b}^\dagger - z^*\hat{a}^\dagger\hat{b}} \qquad z = \theta e^{i\phi}$$

- Non-Gaussian operations

  e.g. Kerr gate



Qubit & qumode universality

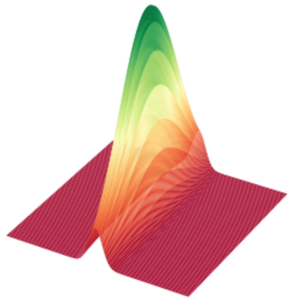
- Minimal gate sets

Qubits	Hybrid	Qumodes
$\text{Universal} \begin{cases} \text{Clifford} \begin{cases} S = \sigma_z^{1/2} \\ H \\ \text{CNOT} \end{cases} \\ \text{Magic} \;\; T = e^{-i\frac{\pi}{8}\sigma_z} \end{cases}$	$\text{Universal} \begin{cases} \text{CD}(\theta) = e^{\sigma_z(\theta\hat{a}^\dagger - \theta^*\hat{a})} \\ R_i(\theta) \\ \text{BS}(\theta,\phi) \end{cases}$	$\text{Universal} \begin{cases} \text{Gaussian} \begin{cases} \text{S}(\theta) \\ \text{R}(\theta) \\ \text{D}(\theta) \\ \text{BS}(\theta,\phi) \end{cases} \\ \text{Non-Gaussian} \;\; V(\theta) = e^{-i\theta\hat{q}^3} \end{cases}$
Pauli string σ_i	$\sigma_i \otimes P(\hat{q},\hat{p})$	Polynomial $P(\hat{q},\hat{p})$ e.g. $\sim \hat{q}_1^3\hat{p}_2^2$
Barenco et al., Gottesman, Knill	*Girvin, Wiebe et al. `24*	*Lloyd, Braunstein `97*

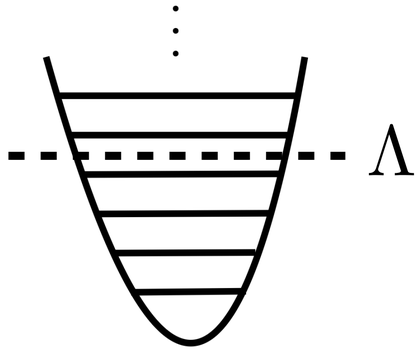
Qubit & qumode universality

- Minimal gate sets

- Physical qumodes
- Finite squeezing



- Fock cutoff



- For example $\Lambda = 100$

→ Qubit equivalent: 7 qubits

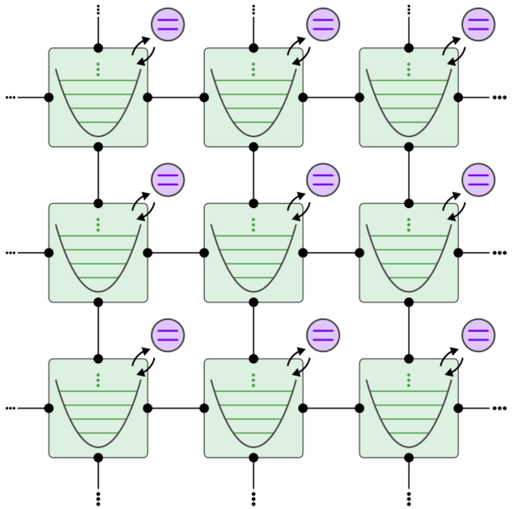
but a native qumode gate
can require O(100-10,000)
two-qubit gates

Barenco et al., Gottesman, Knill *Girvin, Wiebe et al. `24* *Lloyd, Braunstein `97*

Qubit & qumodes hardware platforms

Chuang, Girvin, Wiebe, et al. `24

Superconducting

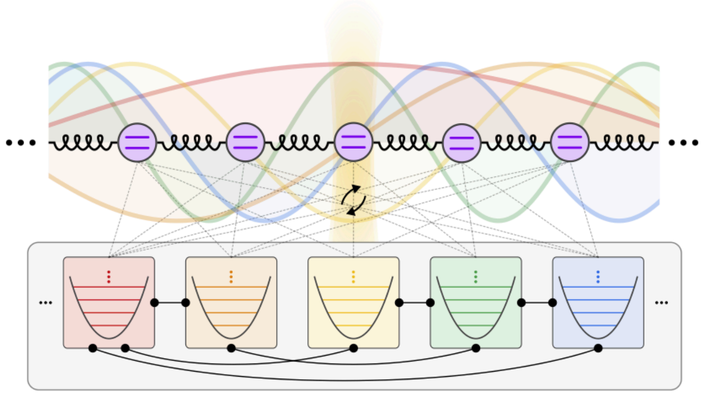


Microwave resonator

Superconducting qubit

Dispersive interaction

Trapped ion

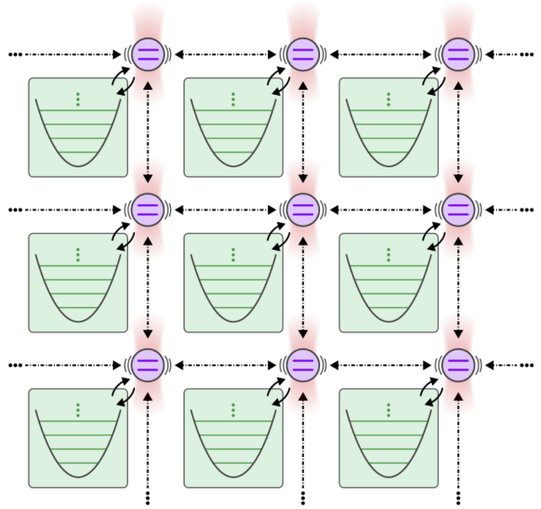


Collective motional modes

Ion qubit

Sideband interaction

Neutral atom



Atomic motional modes

Neutral atom qubit

Sideband interaction

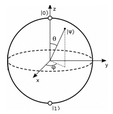
Platform dependence: • Native gate sets • Coherence times • Connectivity

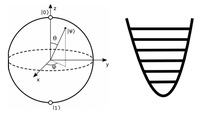
Quantum chromodynamics

- Simulate quarks and gluons with different quantum resources?

- QCD Lagrangian

$$\mathcal{L} = \bar{\psi}\left(i\partial^{\mu}\gamma_{\mu} - m\right)\psi + g_s\bar{\psi}\gamma^{\mu}T_a\psi A_{\mu}^{a} - \frac{1}{4}F_{a}^{\mu\nu}F_{\mu\nu}^{a}$$

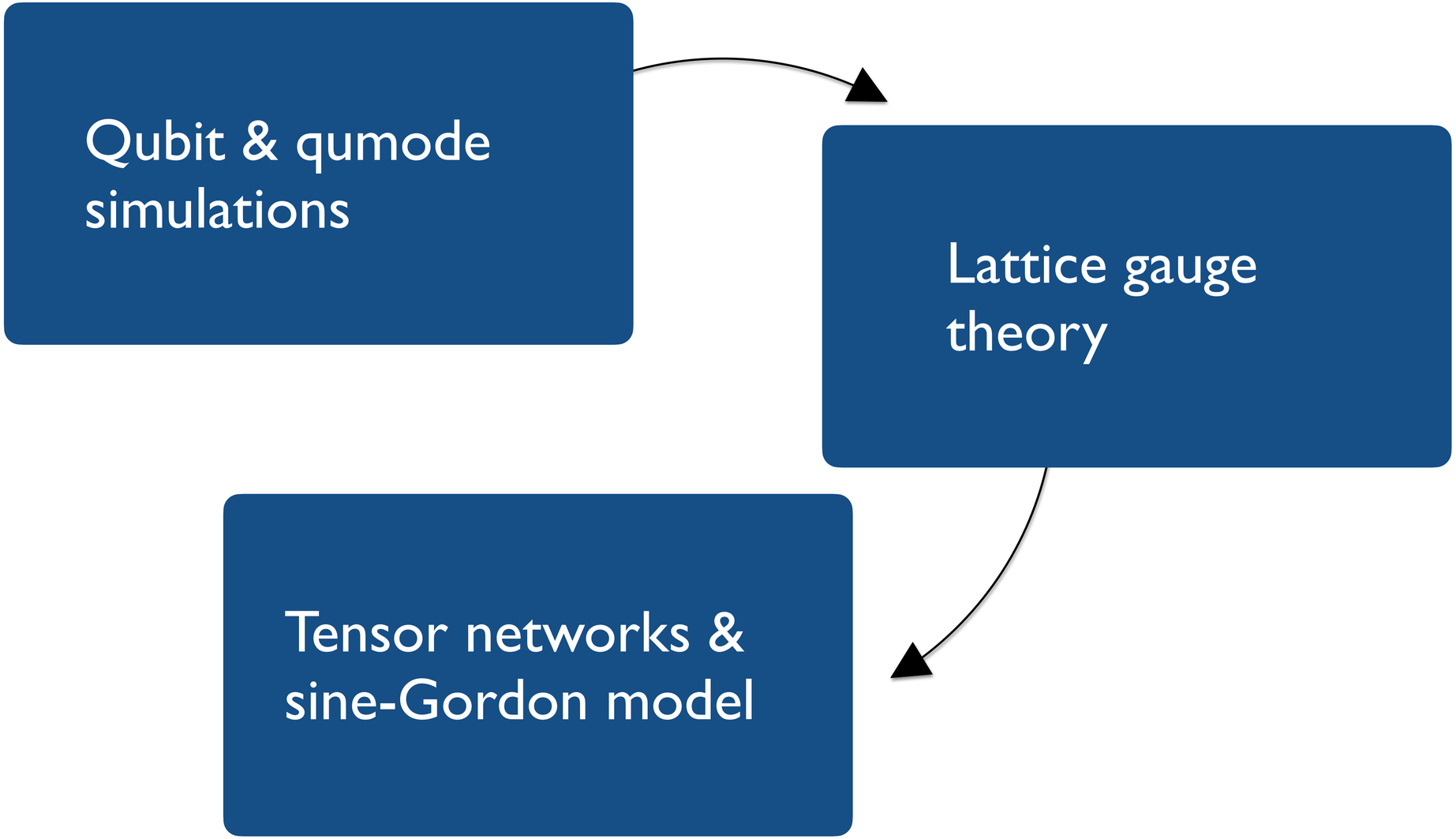
Qubits  Qumodes 

Qubit-qumode coupling 

→ **Develop a hybrid qubit/qumode approach**

$$\mathcal{H}_{\text{qumode}}^{m} \otimes \mathcal{H}_{\text{qubit}}^{n}$$

Outline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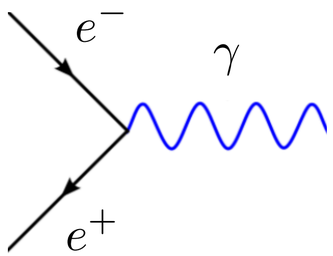
Quantum electrodynamics in 2+1d

Ale, Rainaldi, Rico, FR, Siopsis `25

- QED Lagrangian

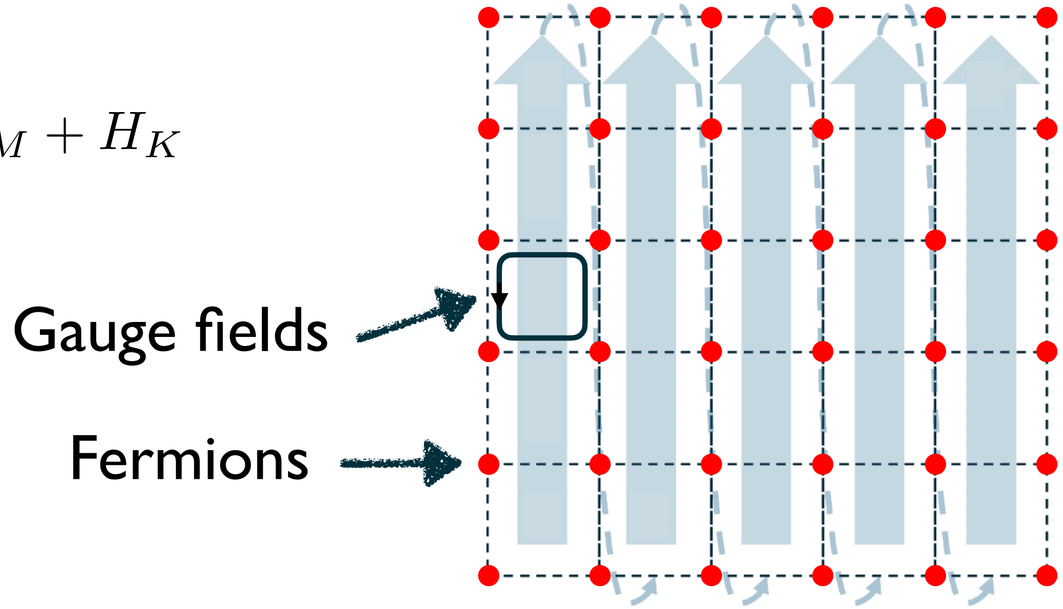
$$\mathcal{L} = \bar{\psi}\left(i\partial^\mu\gamma_\mu - m\right)\psi + e\bar{\psi}\gamma^\mu\psi A_\mu - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$



- Discretize on a spatial lattice $H = H_E + H_B + H_M + H_K$

$$H_E = \frac{g^2}{2}\sum_{\boldsymbol{n},i} E^2_{\boldsymbol{n},i}, \quad H_B = -\frac{1}{2g^2a^2}\sum_{\boldsymbol{n}}\left(P_{\boldsymbol{n}} + P^\dagger_{\boldsymbol{n}} - 2\right)$$

with $P_{\boldsymbol{n}} \equiv U_{\boldsymbol{n},x}U_{\boldsymbol{n}+\boldsymbol{e}_x,y}U^\dagger_{\boldsymbol{n}+\boldsymbol{e}_y,x}U^\dagger_{\boldsymbol{n},y}$



See also e.g. Girvin, Wiebe et al., Zoller et al., Davoudi et al.

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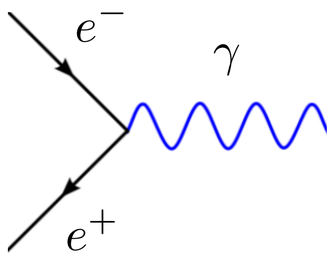

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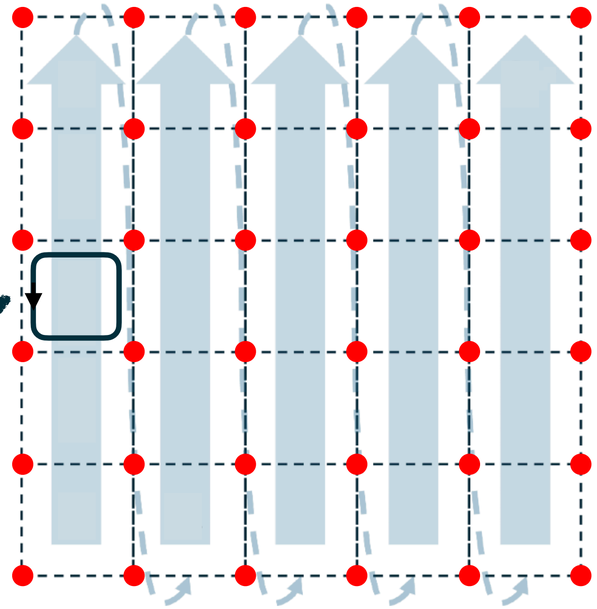
with $$P_{\boldsymbol{n}} \equiv U_{\boldsymbol{n},x}U_{\boldsymbol{n}+\boldsymbol{e}_x,y}U^\dagger_{\boldsymbol{n}+\boldsymbol{e}_y,x}U^\dagger_{\boldsymbol{n},y}$$

$$H_M = m_0\sum_{\boldsymbol{n}}(-)^{n_x+n_y}\Psi^\dagger_{\boldsymbol{n}}\Psi_{\boldsymbol{n}}, \quad H_K = \frac{1}{2}\sum_{\boldsymbol{n}}\sum_{i\in\{x,y\}}\left[\Psi^\dagger_{\boldsymbol{n}}U^\dagger_{\boldsymbol{n},\boldsymbol{e}_i}\Psi_{\boldsymbol{n}+\boldsymbol{e}_i} + \text{ h.c. }\right]$$

Gauge fields

Fermions



See also e.g. Girvin, Wiebe et al., Zoller et al., Davoudi et al.

Quantum electrodynamics in 2+1d

- Qumodes are non-compact $q \in (-\infty, \infty)$

  Ale, Bauer, Jha, FR, Siopsis `24
  Ale, Rainaldi, Rico, FR, Siopsis `25

  - Scalar field theory *Marshall, Pooser, Siopsis, Weedbrock `15*
  - sine-Gordon *Rainaldi, Ale, Grau, Kharzeev, Rico, FR, Shome, Siopsis `25*

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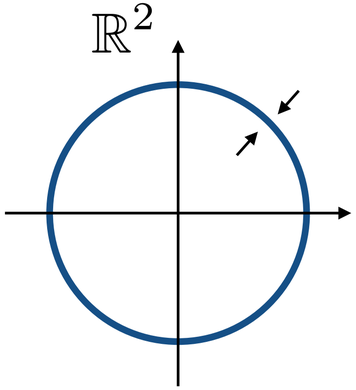
- Gauge theories typically involve compact variables
  - U(1) link variable $U_{\boldsymbol{n}} = e^{i\theta_{\boldsymbol{n}}}, \ \theta_{\boldsymbol{n}} \in [0, 2\pi)$
  - With qubits typically use either finite representation or finite subgroup
  - With qubits and 2 qumodes

$$q_0^2 + q_1^2 = 1 \longrightarrow$$ continuous and compact variable



Quantum electrodynamics in 2+1d

- More efficient formulation involving trigonometric gates
- Electric & magnetic part

$$Q_i = \Psi_i^\dagger \Psi_i - \frac{1-(-1)^i}{2}\mathbb{I}$$

$$\hat{H}_E = \frac{g^2}{2}\sum_{ij}\left(\hat{p}_i \mathcal{H}^{(2)}_{ij}\hat{p}_j + \hat{p}_i \mathcal{H}^{(1)}_{ij} Q_j + Q_i \mathcal{H}^{(0)}_{ij} Q_j\right)$$

$$\hat{H}_B = -\frac{1}{g^2}\sum_j \cos\left(\hat{q}_j\right)$$

Qubit/qumode universality *Girvin, Wiebe et al. `24*

- Non-compact $(\hat{q}_i, \hat{p}_i)$
- Non-polynomial trigonometric gate

Rainaldi, Ale, Grau, Kharzeev, Rico, FR, Shome, Siopsis `25, Kassel, Tan et al. `15
Ale, Rainaldi, Rico, FR, Siopsis `26

Quantum electrodynamics in 2+1d

- More efficient formulation involving trigonometric gates
- Electric & magnetic part

$$\hat{H}_E = \frac{g^2}{2} \sum_{ij} \left(\hat{p}_i \mathcal{H}^{(2)}_{ij} \hat{p}_j + \hat{p}_i \mathcal{H}^{(1)}_{ij} Q_j + Q_i \mathcal{H}^{(0)}_{ij} Q_j \right)$$

$$\hat{H}_B = -\frac{1}{g^2} \sum_j \cos\left(\hat{q}_j\right)$$

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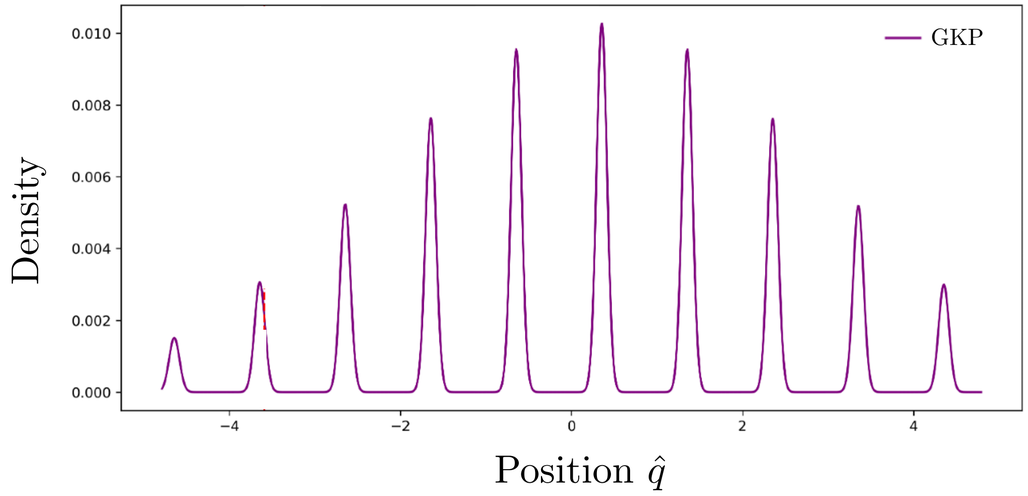
- Non-polynomial trigonometric gate



$$\psi_\beta(\chi) = \mathcal{N} \sum_{n \in \mathbb{Z}} \exp\left[-\frac{\beta}{2} (\chi + 2\pi n)^2 \right], \quad \chi \in [-\pi, \pi)$$

$$\rightarrow \quad L^2(\mathbb{R}) \to L^2(S^1), \; p \in \mathbb{Z}$$

Rainaldi, Ale, Grau, Kharzeev, Rico, FR, Shome, Siopsis `25, Kassel, Tan et al. `15
Ale, Rainaldi, Rico, FR, Siopsis `26

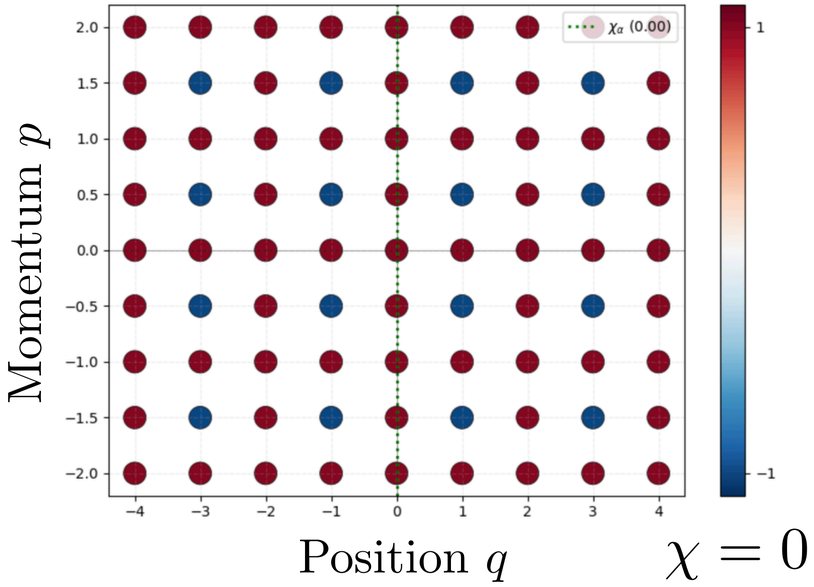
Quantum electrodynamics in 2+1d

- Electric & magnetic part

$$\hat{H}_E = \frac{g^2}{2} \sum_{ij} \left(\hat{p}_i \mathcal{H}^{(2)}_{ij} \hat{p}_j + \hat{p}_i \mathcal{H}^{(1)}_{ij} Q_j + Q_i \mathcal{H}^{(0)}_{ij} Q_j \right)$$

$$\hat{H}_B = -\frac{1}{g^2} \sum_j \cos\left(\hat{q}_j\right)$$

$$\hat{H}_J = -J \sum_j \cos\left(2\pi \hat{p}_j\right)$$



GKP stabilizers converting non-compact to compact variables + quantum error correction

GKP/qunaught Wigner function

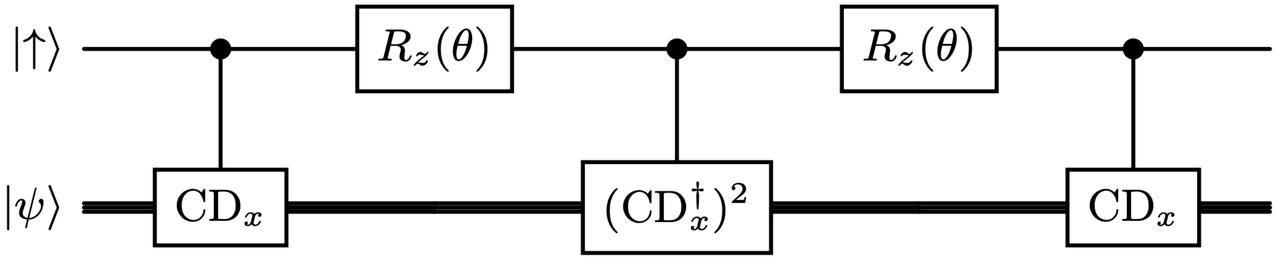
$$W(q,p) = \frac{1}{\pi} \int \mathrm{d}q' \, \psi^*(q+q')\psi(q-q')e^{2ipy}$$

Rainaldi, Ale, Grau, Kharzeev, Rico, FR, Shome, Siopsis `25, Kassel, Tan et al. `15
Ale, Rainaldi, Rico, FR, Siopsis `26

Trigonometric gates

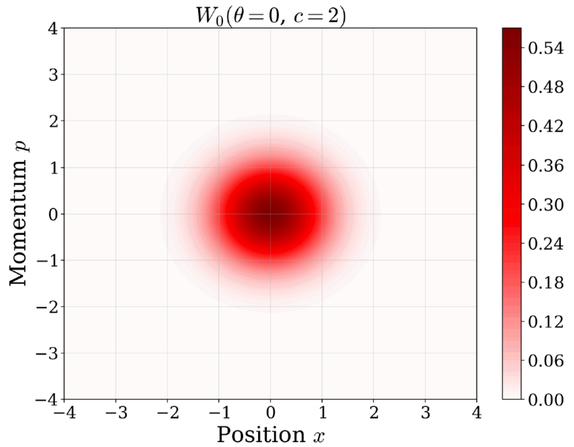
Kassal, Tan et al. `25
Rainaldi, Ale, Grau, Kharzeev, Rico, FR, Shome, Siopsis `25

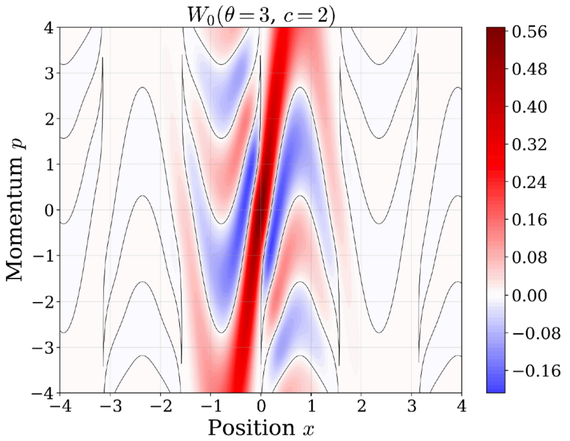
- Qubit/qumode circuit for $e^{-i\theta\cos(c\hat{q})}$



← Qubit decouples

- Using unitary & hermitian Pauli operator
- First order Trotter approximation

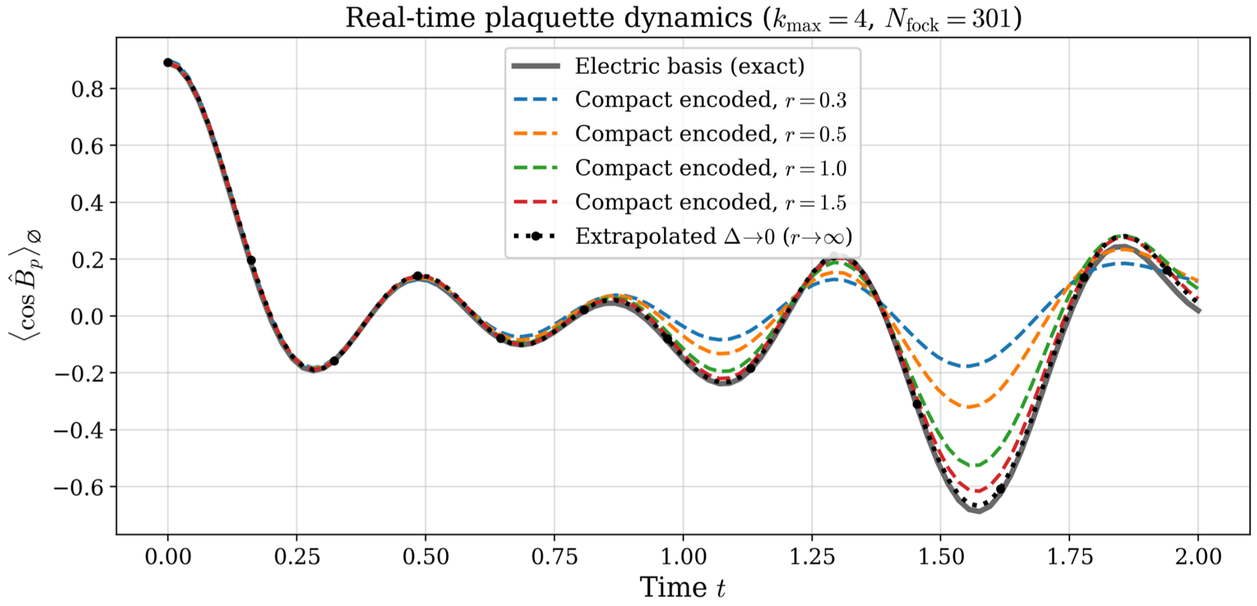


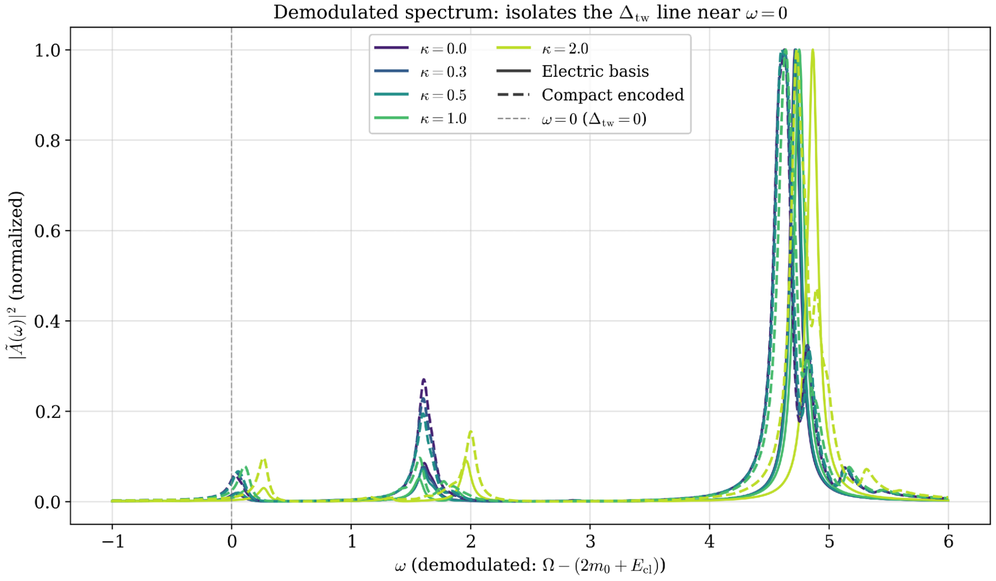


Quantum electrodynamics in 2+1d

Ale, Rainaldi, Rico, FR, Siopsis `26

- Real-time evolution e.g.
- Single plaquette, one-point function
- Topological twist energy using real-time spectroscopy

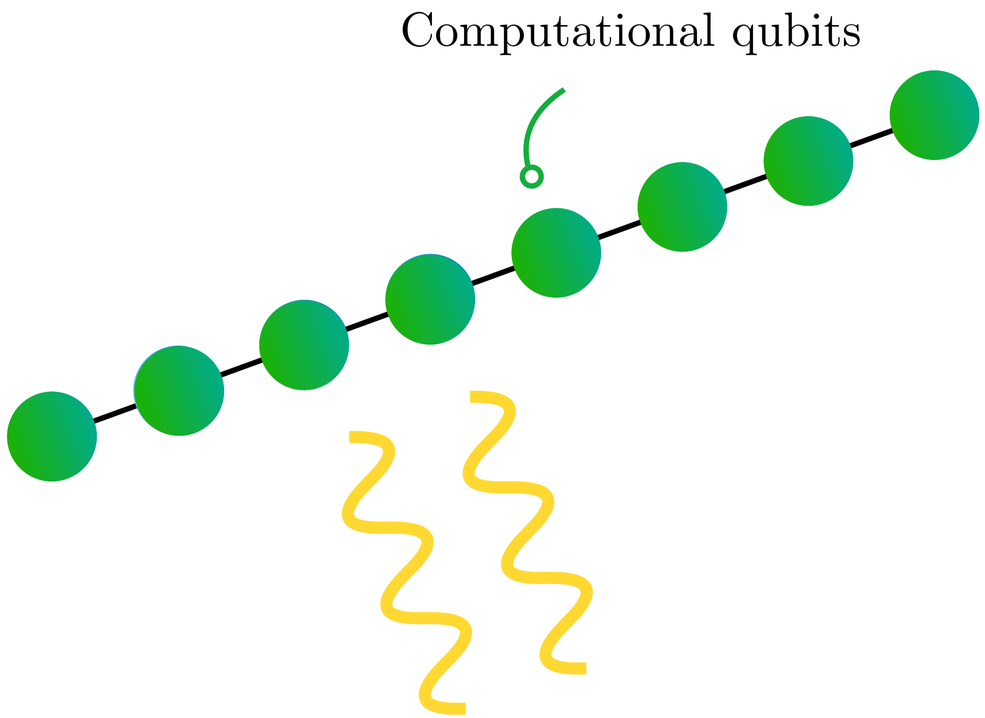




Qubit & qumodes with trapped ions

Araz, Grau, Montgomery, FR `24

- **Qubits:** electronic states of the trapped ion



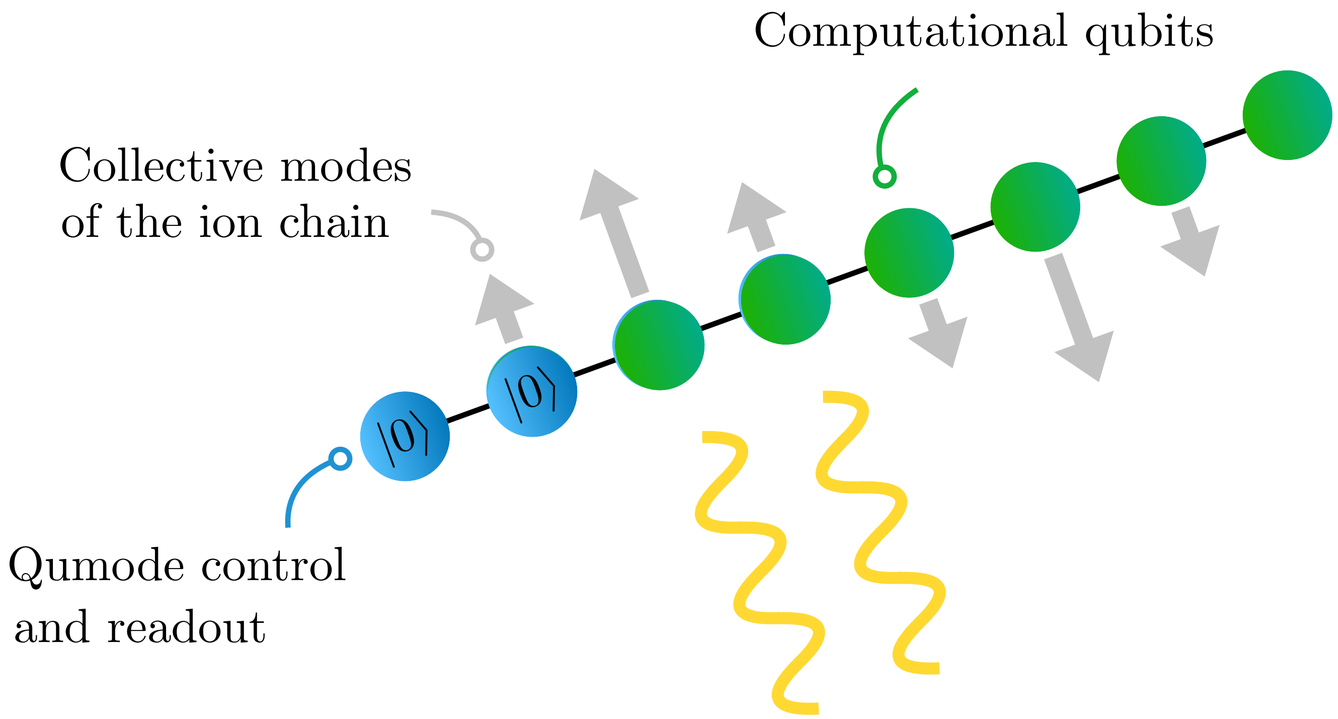
see also Monroe et al., Kim et al., Brown et al., Girvin et al.

Qubit & qumodes with trapped ions

Araz, Grau, Montgomery, FR `24

- **Qubits:** electronic states of the trapped ion

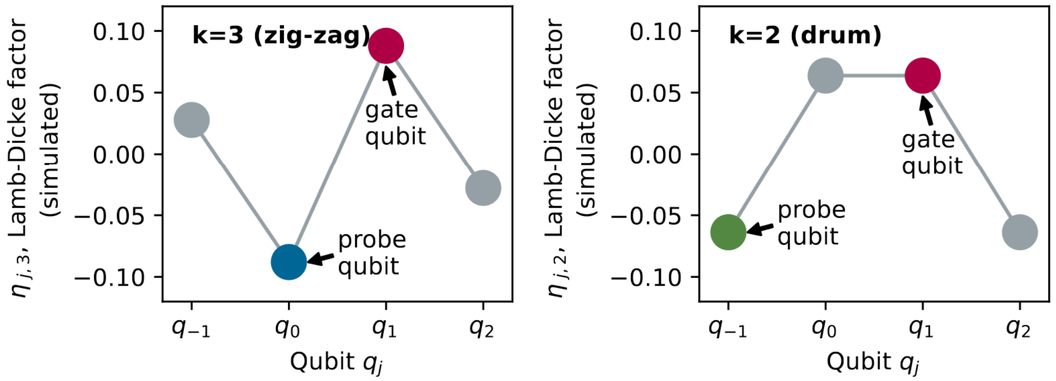
- **Qumodes:** collective vibrational modes of the ion chain (phonons)

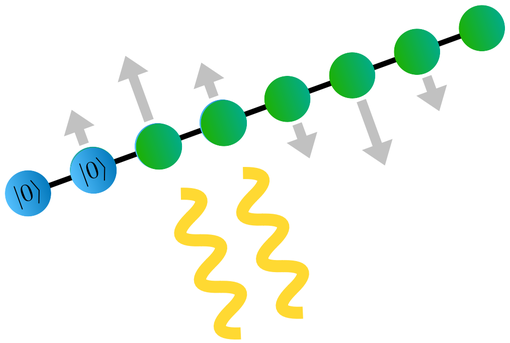


see also Monroe et al., Kim et al., Brown et al., Girvin et al.

Qubit & qumodes with trapped ions

- Real-time evolution $e^{-i\theta\cos(c\hat{q})}$
- Four-ion chain





- Account for device initialization, readout, and dephasing

e.g. $$\frac{\mathrm{d}\hat{\rho}}{\mathrm{d}t} = -i[\hat{H},\hat{\rho}] + \sum_i \left(\hat{C}_i\hat{\rho}\hat{C}_i^\dagger - \tfrac{1}{2}\{\hat{C}_i^\dagger\hat{C}_i,\hat{\rho}\}\right)$$

with $\hat{C} = \sqrt{\gamma}\hat{a}^\dagger\hat{a}$

QSCOUT

Sandia National Laboratories

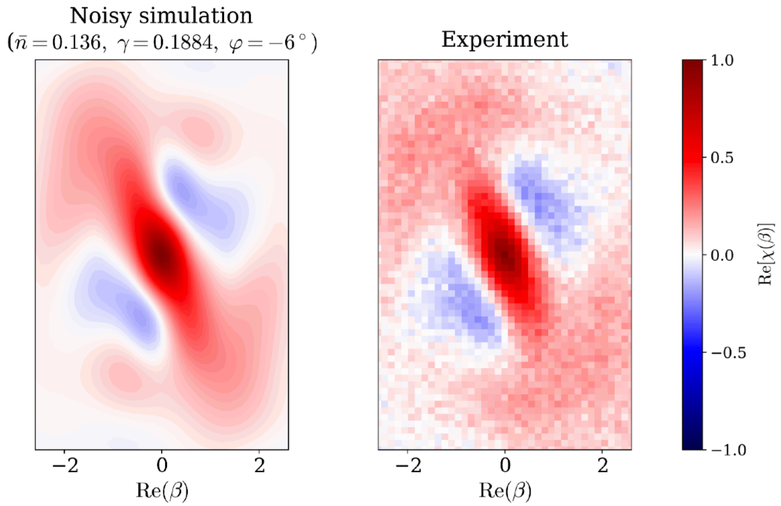
Rainaldi, Montgomery, Yale, Revelle, Lobser, Clarke, Grau, Siopsis, FR, et al. `26

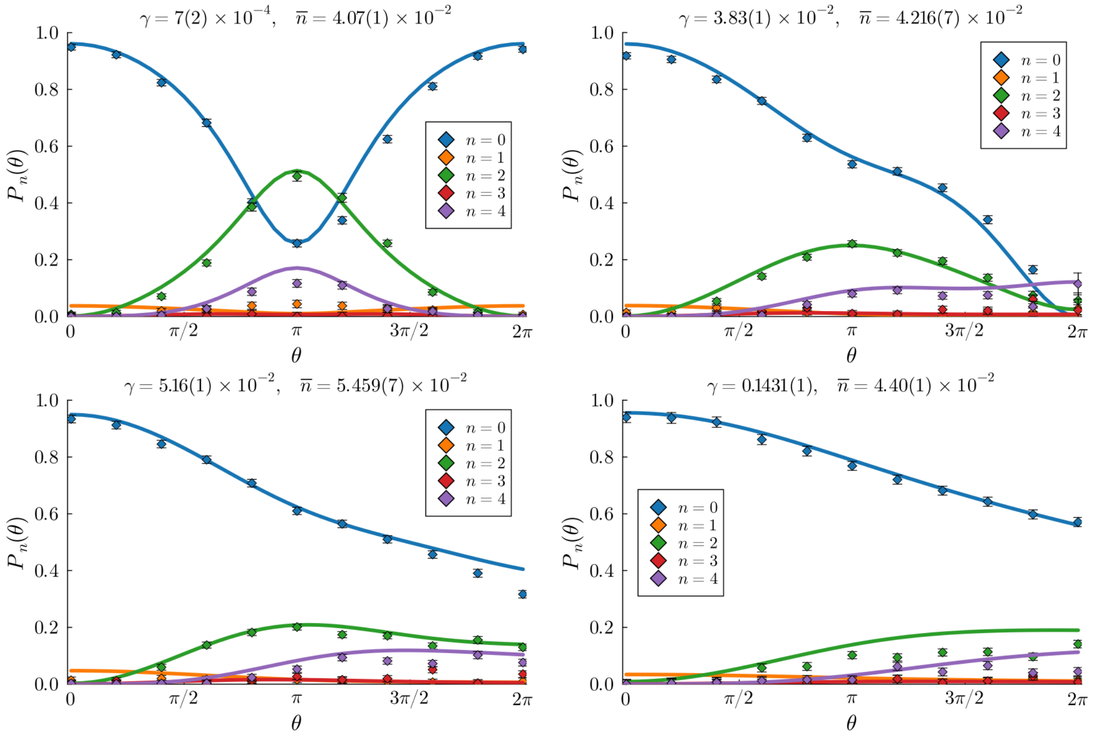
Trigonometric gates with trapped ions

- Transition probabilities

$$P_n(\theta) = |\langle n|e^{-i\theta\cos(c\hat{q})}|0\rangle|^2$$

- Wigner/characteristic function



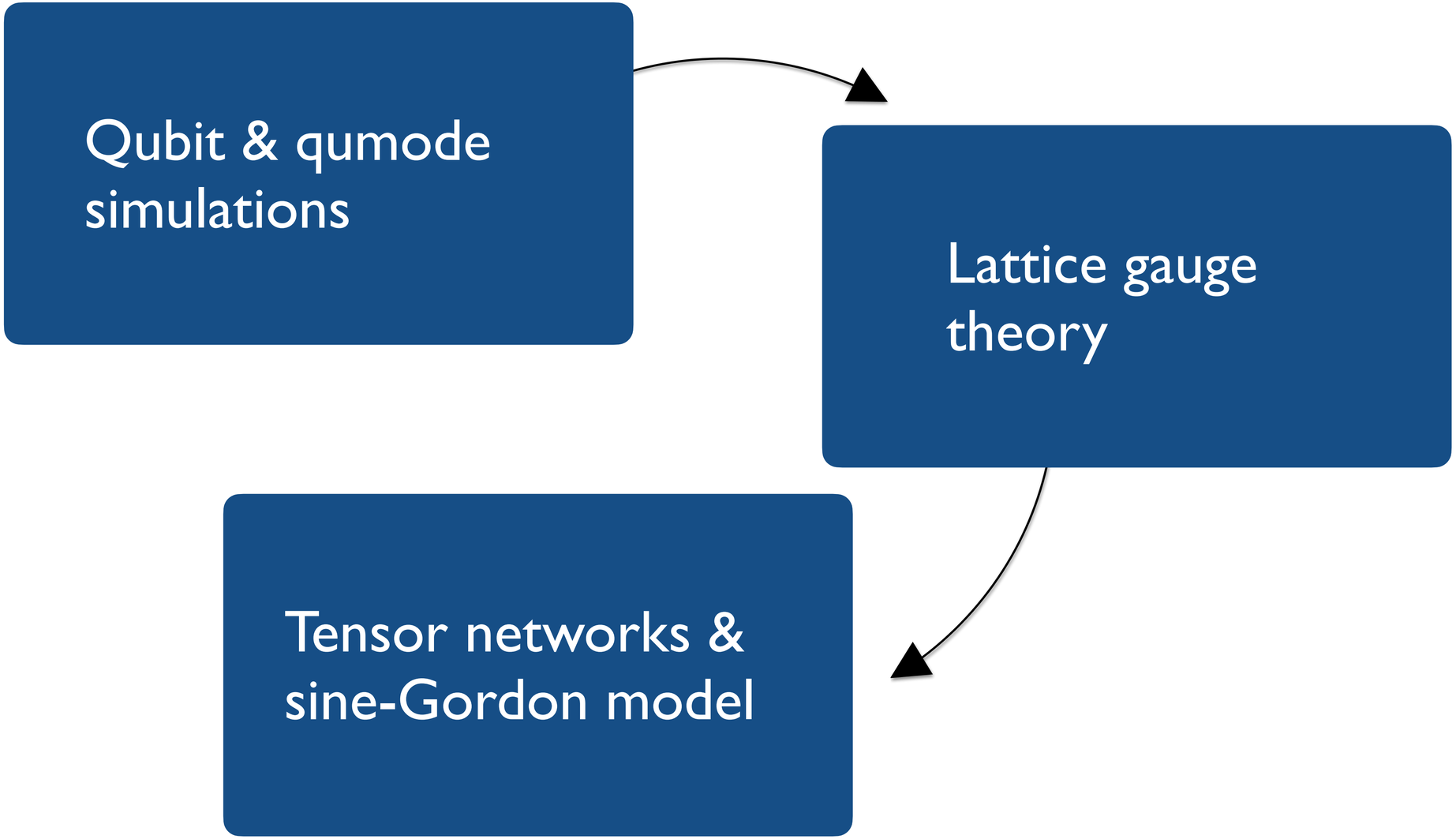


QSCOUT

Sandia National Laboratories

Rainaldi, Montgomery, Yale, Revelle, Lobser, Clarke, Grau, Siopsis, FR, et al. `26

Outline


Qubit & qumode simulations
Lattice gauge theory
Tensor networks & sine-Gordon model

sine-Gordon & bosonic tensor networks

Hechenberger, FR, Rainaldi `26

- Lagrangian $\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{m^2}{\beta^2}\left(\cos(\beta\phi) - 1\right)$

Real scalar field

- Lattice Hamiltonian $H = \sum_{j=1}^{N} \left[\frac{1}{2a}\Pi_j^2 + \frac{1}{a}\phi_j^2 - \frac{1}{a}\phi_j\phi_{j+1} + \frac{a m_{\text{bare}}^2}{\beta^2}\left(1 - \cos(\beta\phi_j)\right) \right]$

One qumode per site

sine-Gordon & bosonic tensor networks

Hechenberger, FR, Rainaldi `26

- Lagrangian $\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{m^2}{\beta^2}\left(\cos(\beta\phi) - 1\right)$

Real scalar field

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One qumode per site

- Spectrum, scattering phase shifts, etc. are known analytically

Zamolodchikov`79, `95

- E.g. soliton mass

$$M_{\text{sol}} = \frac{2}{\sqrt{\pi}} \frac{\Gamma\left(\xi/2\right)}{\Gamma\left((1+\xi)/2\right)} \left(\frac{\pi m_{\text{ren}}^2}{2\beta^2} \frac{\Gamma\left(1 - \beta^2/(8\pi)\right)}{\Gamma\left(\beta^2/(8\pi)\right)} \right)^{\frac{1}{2-\beta^2/(4\pi)}}$$

with $\xi = \frac{\beta^2}{8\pi - \beta^2}$

sine-Gordon & bosonic tensor networks

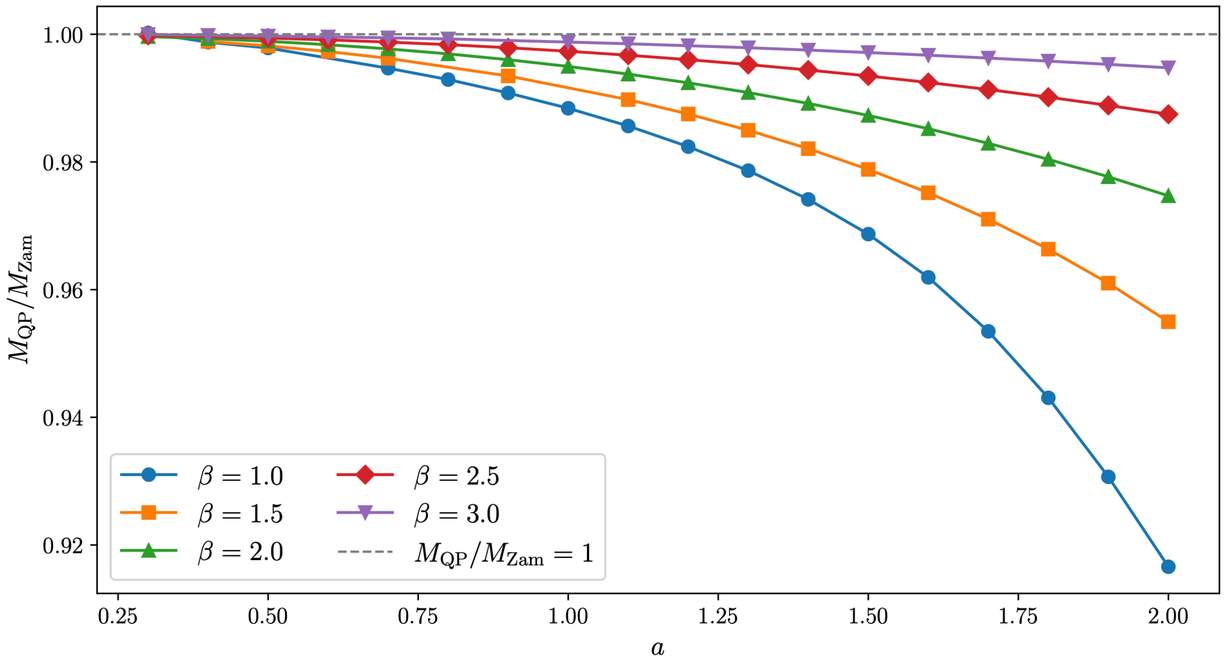
Hechenberger, FR, Rainaldi `26

- Lattice Hamiltonian $H = \sum_{j=1}^{N} \left[\frac{1}{2a}\Pi_j^2 + \frac{1}{a}\phi_j^2 - \frac{1}{a}\phi_j\phi_{j+1} + \frac{am_{\text{bare}}^2}{\beta^2}\left(1-\cos(\beta\phi_j)\right) \right]$

- One-to-one comparison requires renormalization using Zamolodchikov's scheme

  Zamolodchikov`79, `95

- Continuum extrapolation of uniform bosonic tensor network results

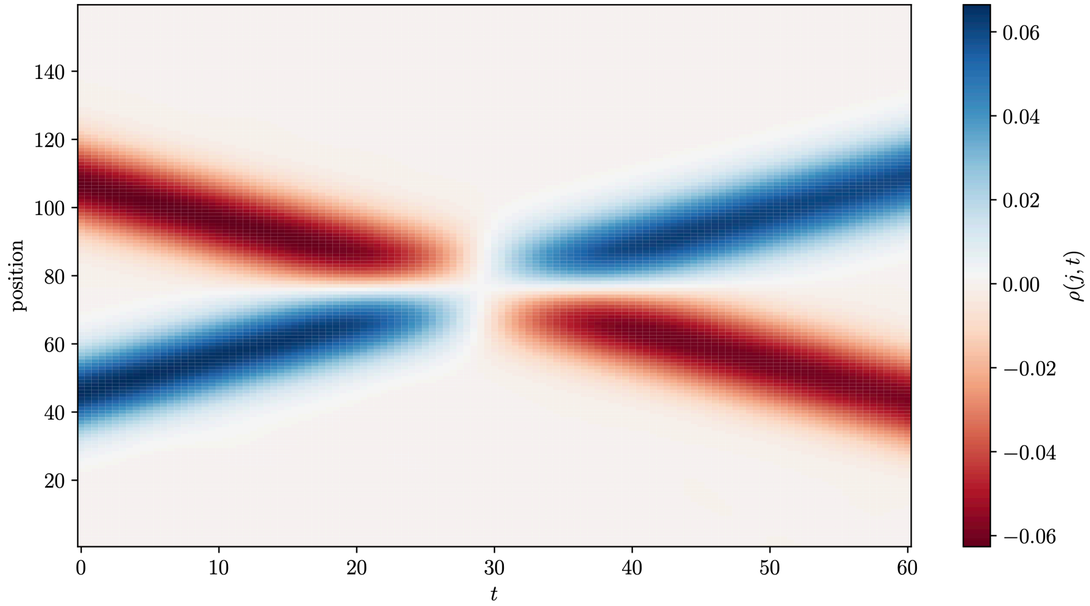


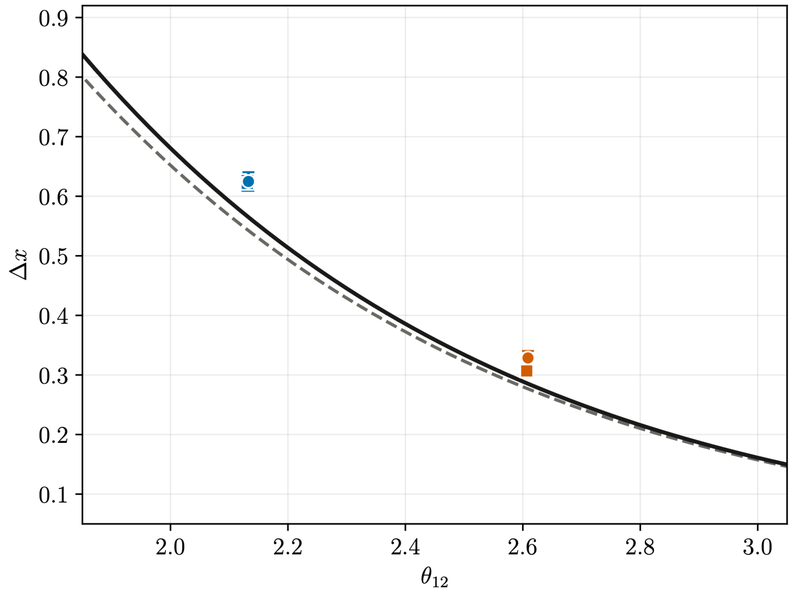
sine-Gordon & bosonic tensor networks

- Soliton Anti-Soliton wave packet scattering
- Track via topological charge
- Compare displacement due to elastic scattering to continuum result

Zamolodchikov`79, `95

Hechenberger, FR, Rainaldi `26

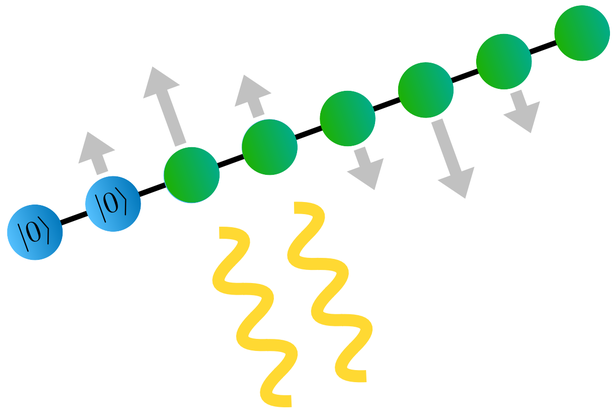




$$\Delta x_{S/\bar{S}} = -\frac{\partial \arg S_T(\theta_{12})}{\partial p_{S/\bar{S}}}$$

Conclusions

- Quantum computing is well-suited to address challenges in fundamental physics
- Exploration of qubits, qudits, qumodes



- Study e.g. scattering and lightcone correlation functions
- Build up toward quantum utility in fundamental physics

