

YITP long-term workshop

New Frontiers in QCD 2026

October 12 - November 13, 2026

Yukawa Institute for Theoretical Physics, Kyoto University

Three-Dimensional Structure of Nucleons: Overview of TMDs and GPDs

- ☐ New Frontiers in QCD
 - Emergence of Hadrons and Properties
- ☐ Hadron structure – see it by breaking it
 - TMDs: Correlation of parton motion and hadron properties
- ☐ Tomography – see it without breaking it
 - GPDs: Correlation of hadron property and parton spatial distribution
- ☐ Summary and Outlook

In collaboration with N. Sato, T. Uji, Z. Yu, Y. Zeng, ...

Jefferson Lab

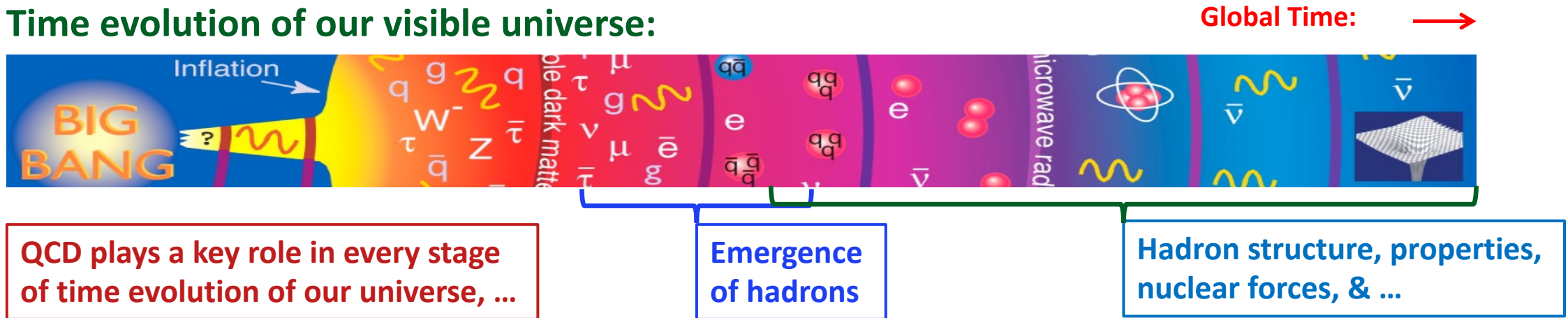


U.S. DEPARTMENT
of ENERGY

Jianwei Qiu
Theory Center

New Frontiers in QCD – Focusing on the Energy Frontier

□ Time evolution of our visible universe:



□ Emergence of hadrons from quarks and gluons - where we came from:

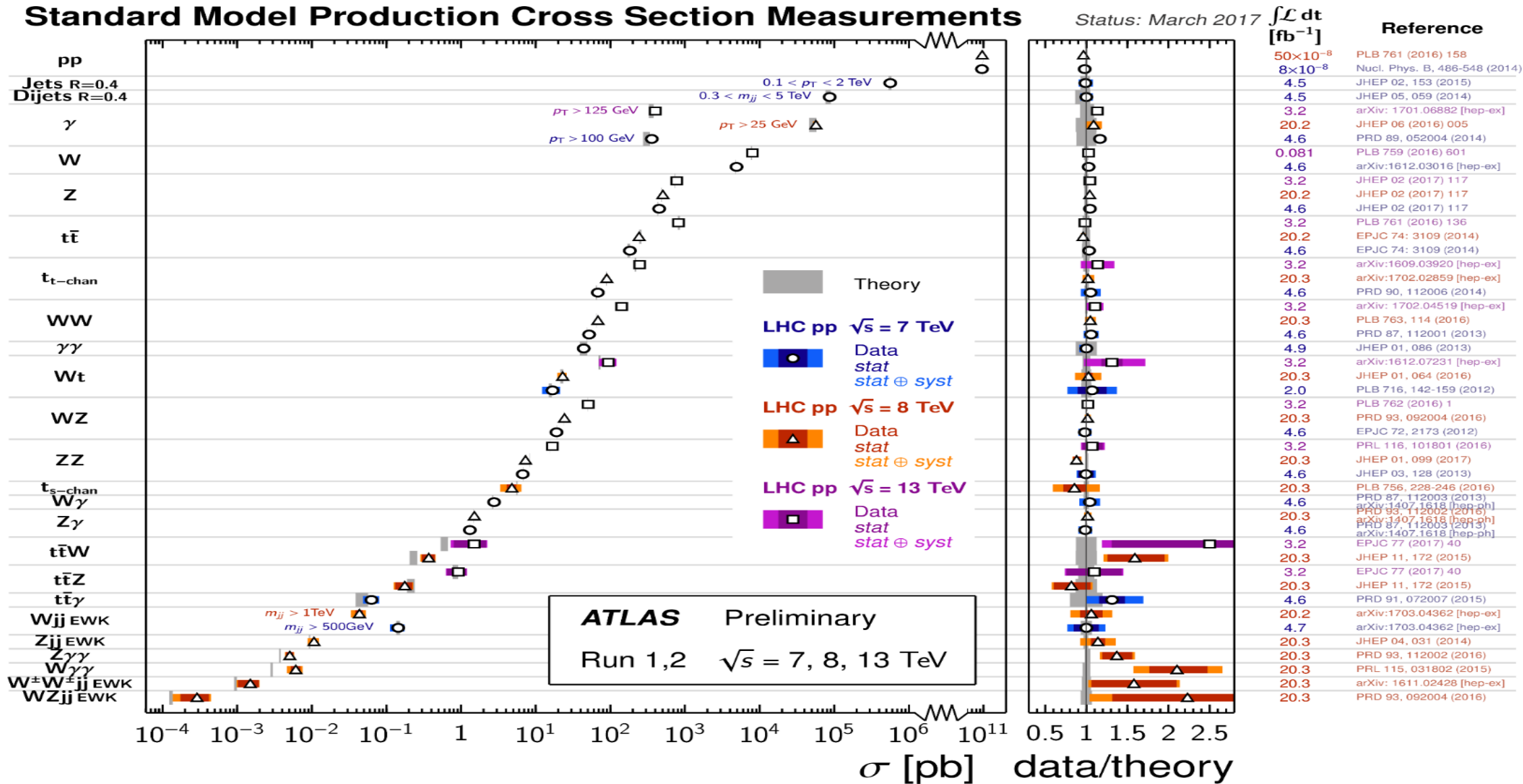
- High-energy scattering processes to produce quarks and gluons in the form of “jets”
- How produced quarks and gluons evolved into confined hadrons – real time evolution – quantum computers and quantum computations, ...

□ Hadron structure and emerging hadron properties – Who we are and where we are going:

- High-energy scattering processes to break the hadrons to “see” partons inside, PDFs, TMDs, QCFs, ...
- High-energy scattering without breaking the hadrons – 3D QCD tomography, GPDs, GTMDs, ...

Focus of this talk

Unprecedented Success of the Standard Model (including QCD)!

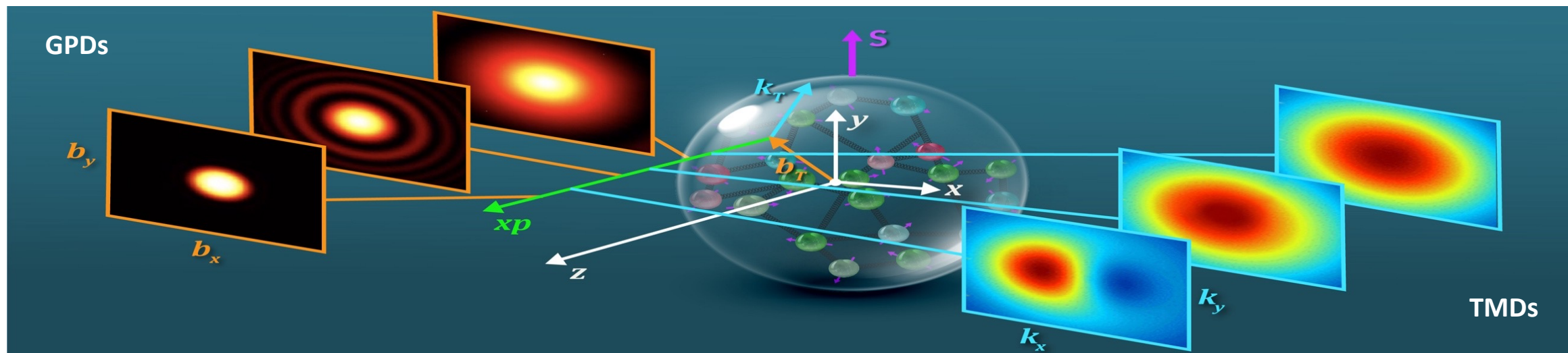


FACT: Electroweak processes + QCD perturbation theory works!

Exploring Hadron's 3D Partonic Structure at the Femtoscale

□ 3D hadron structure encoded in TMDs & GPDs:

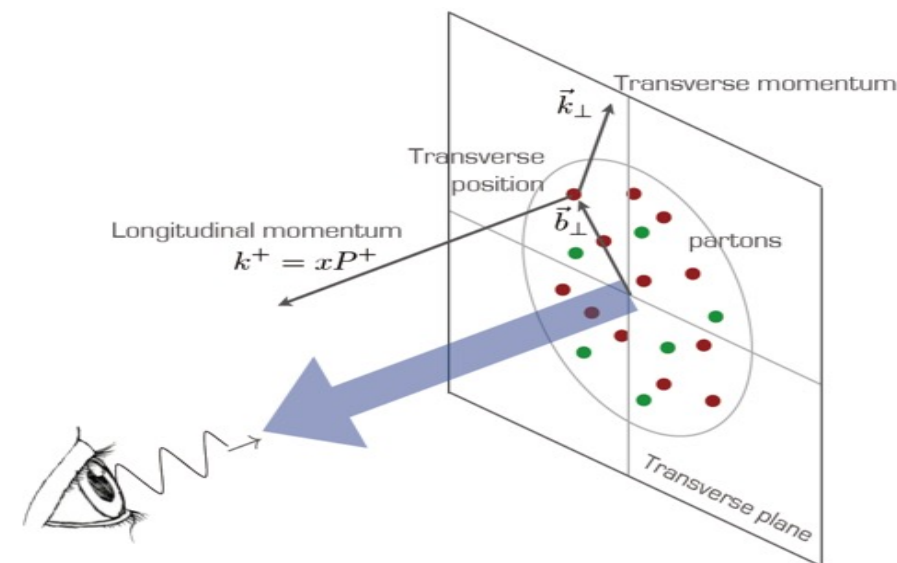
Single hard scattering is too localized in space!



□ Need new observables with two distinctive scales:

$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\text{QCD}}$$

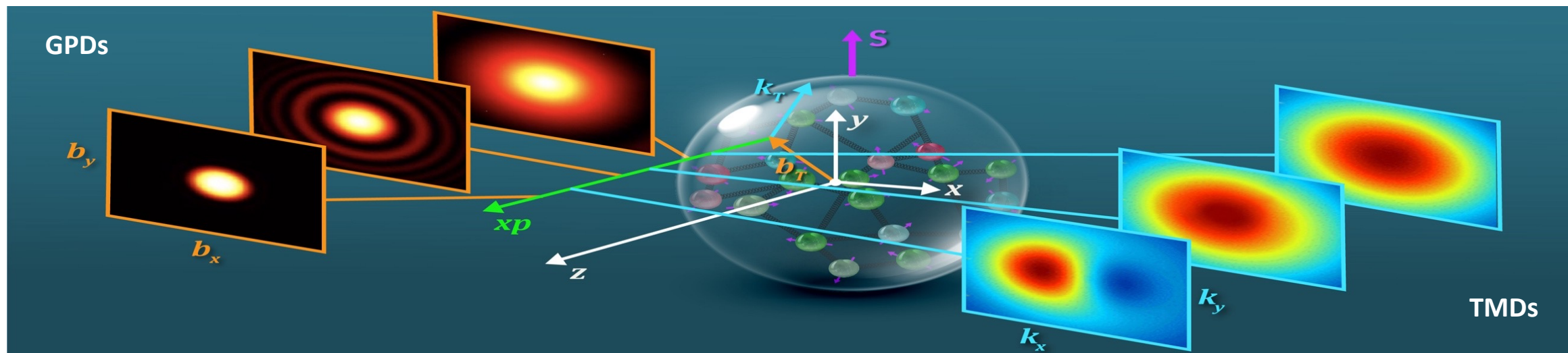
- **Hard scale:** Q_1 to localize the probe to see the particle nature of quarks/gluons
- **“Soft” scale:** Q_2 to be more sensitive to the emergent regime of hadron structure $\sim 1/\text{fm}$



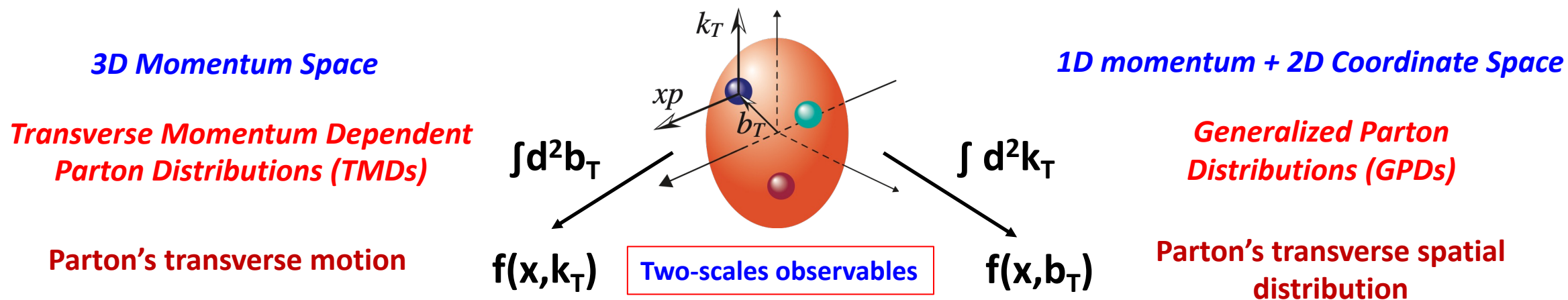
Exploring Hadron's 3D Partonic Structure at the Femtoscale

□ 3D hadron structure encoded in TMDs & GPDs:

Single hard scattering is too localized in space!



□ A unified description (Wigner distributions in 5D) – Formally:

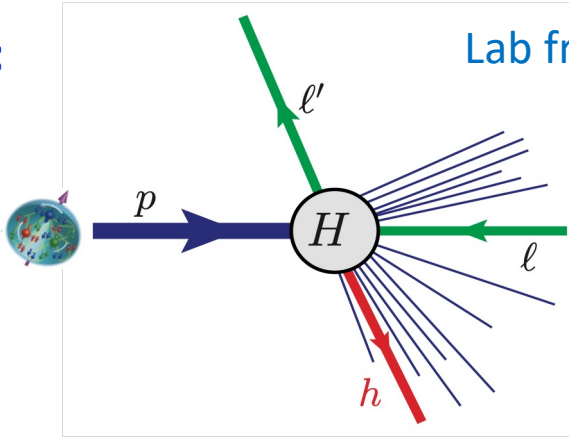


TMDs and Multidimensional Hadron Structure

□ Break the hadron to see partons inside:

■ Amplitude:

SIDIS



Lab frame: $|\vec{\ell}'_T| \sim |\vec{P}_{h_T}|$
 $\gg |\vec{\ell}'_T + \vec{P}_{h_T}|$

Breit frame:
 $Q^2 \gg P_{h_T}^2$

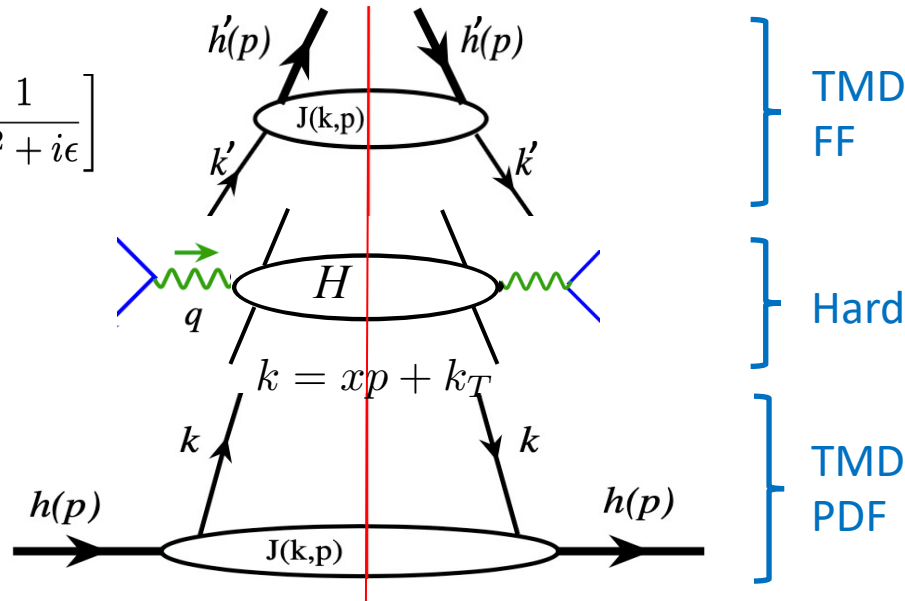
■ Cross Section:

$$\int dk^2 \left[J(k, p) \frac{1}{k^2 - i\epsilon} H(k, Q) \frac{1}{k^2 + i\epsilon} \right]$$

$\rightarrow \infty$ Perturbatively!

■ Pinch:

- Separation of scales
- Suppression quantum interference



TMD Handbook

A modern introduction to the physics of
Transverse Momentum Dependent distributions

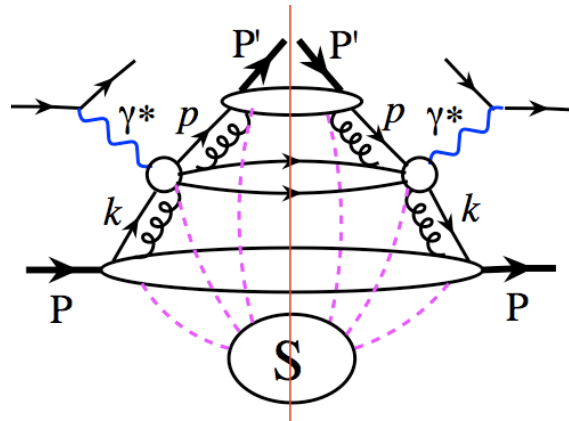


arXiv:2304.03302
to be published
by MIT Press

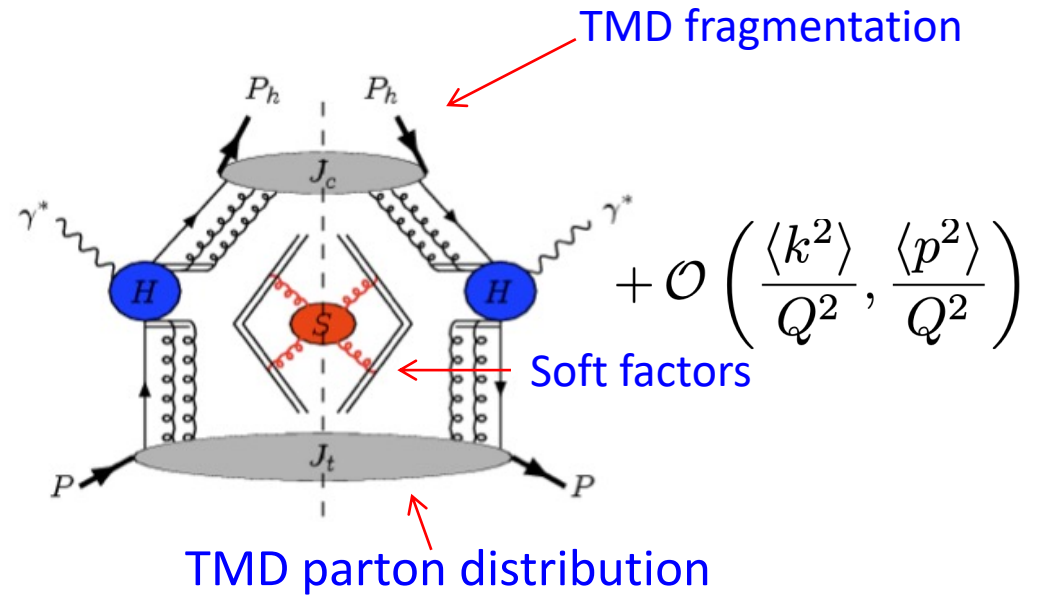
Renaud Boussarie
 Matthias Burkardt
 Martha Constantinou
 William Detmold
 Markus Ebert
 Michael Engelhardt
 Sean Fleming
 Leonard Gamberg
 Xiangdong Ji
 Zhong-Bo Kang
 Christopher Lee
 Keh-Fei Liu
 Simonetta Liuti
 Thomas Mehen *
 Andreas Metz
 John Negele
 Daniel Pitonyak
 Alexei Prokudin
 Jian-Wei Qiu
 Abha Rajan
 Marc Schlegel
 Phiala Shanahan
 Peter Schweitzer
 Iain W. Stewart *
 Andrey Tarasov
 Raju Venugopalan
 Ivan Vitev
 Feng Yuan
 Yong Zhao
 * - Editors

TMD Factorization for SIDIS

□ TMD factorization for SIDIS is solid ($P_{hT}^2 \ll Q^2$):



TMD Factorization



□ Factorization formalism in momentum space:

$$\sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q) \otimes \Phi_f(x, k_\perp) \otimes \mathcal{D}_{f \rightarrow h}(z, p_\perp) \otimes \mathcal{S}(k_{s\perp}) + \mathcal{O}\left[\frac{P_{h\perp}}{Q}\right]$$

Soft factor can be absorbed into TMDs

F.T. of TMDs:

$$\int d^2\mathbf{k}_{1T} d^2\mathbf{k}_{2T} f_{i/p}(x, \mathbf{k}_{1T}; \mu, Q^2) D_{h/j}(z, z\mathbf{k}_{2T}; \mu, Q^2) \delta^{(2)}(\mathbf{P}_{hT} + \mathbf{k}_{1T} - \mathbf{k}_{2T})$$

$$= \int \frac{d^2\mathbf{b}_T}{(2\pi)^2} e^{-i\mathbf{P}_{hT} \cdot \mathbf{b}_T} \tilde{f}_{i/p}(x, \mathbf{b}_T; \mu, Q^2) \tilde{D}_{h/j}(z, \mathbf{b}_T; \mu, Q^2)$$

Unpolarized:

$$\tilde{f}_{i/p}(x, \mathbf{b}_T; \mu, Q^2) = \int \frac{dy^-}{4\pi} e^{-ixP^+y^-} \langle h(P) | \bar{\psi}_i(0^+, y^-, \mathbf{b}_T) \gamma^+ \Phi(0^+y^-, \mathbf{b}_T, 0) \psi_i(0) | h(P) \rangle$$

Physics of TMDs

□ The quark TMD correlator:

$$\Phi_{ij}^{[U]}(x, k_T; P, S) = \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3} e^{ixP^+\xi^- - ik_T \cdot \xi_T} \langle P, S | \bar{\psi}_j(0) U_{[0,\xi]} \psi_i(\xi) | P, S \rangle \Big|_{\xi^+=0}$$

$U_{[0,\xi]}$ denotes a process-dependent gauge connection, color flow of the probe, ...

- A properly defined TMD is a renormalized, gauge-invariant QCD correlation function – involving Wilson lines and a subtraction of overlapping soft radiation, before it can enter a factorization formalism.
- Its precise operator definition matters – determining the role of initial- and final-state color interactions, the dependence on rapidity scales, and the conditions under which the distribution can be used in different reactions

□ Physics of TMDs is much broader than improved descriptions of PDFs with a KT spectra:

- Polarized TMDs could reveal quantum correlations that disappear when transverse momentum is integrated out
- TMDs are probes of QCD dynamics that connect the active parton to the rest of the hadron – form of the gauge link, overlap of soft radiations, ...

□ Relation between TMDs and PDFs:

Commonly quoted: $\int d^2k_T f_1(x, k_T) = f_1(x)$

Better, more controlled relation – small b_T expansion

Naive integral of the renormalized TMD over all k_T equals a collinear PDF is NOT valid without additional ultraviolet treatment – formal statement of a fact:
“Single active parton links to cross section”

Power of the Spin – Many more TMDs:

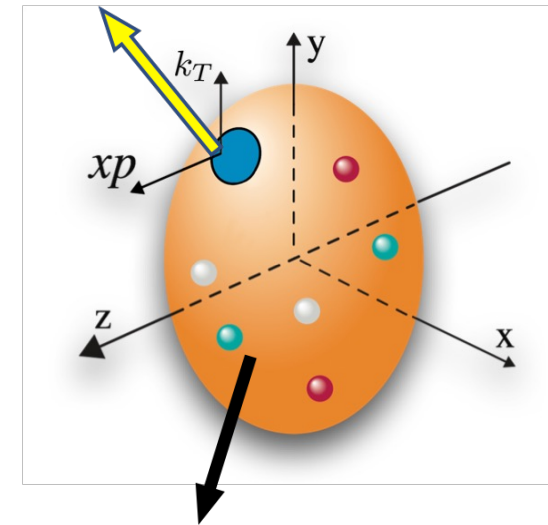
Correlations between hadron spin and parton spin:

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$ 		$h_1^\perp =$  —  Boer-Mulders
	L		$g_{1L} =$  —  Helicity	$h_{1L}^\perp =$  — 
	T	$f_{1T}^\perp =$  —  Sivers	$g_{1T}^\perp =$  — 	$h_1 =$  —  Transversity $h_{1T}^\perp =$  — 
		 → Nucleon Spin	 → Quark Spin	

Analogous tables for:

- Gluons
- Fragmentation functions
- Nuclear targets

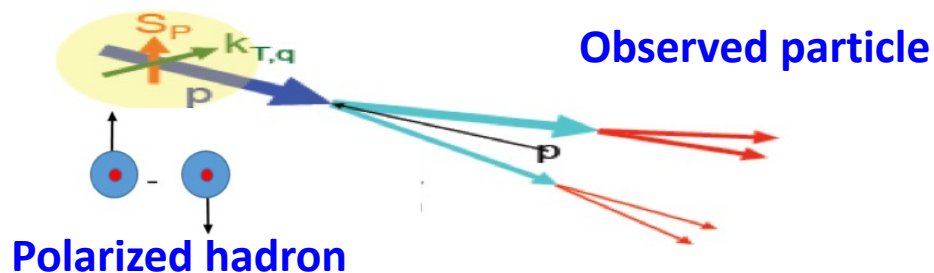
Quark Polarization



Nucleon Polarization

TMDs Introduce Sensitivity to Transverse Motion and Spin Correlations

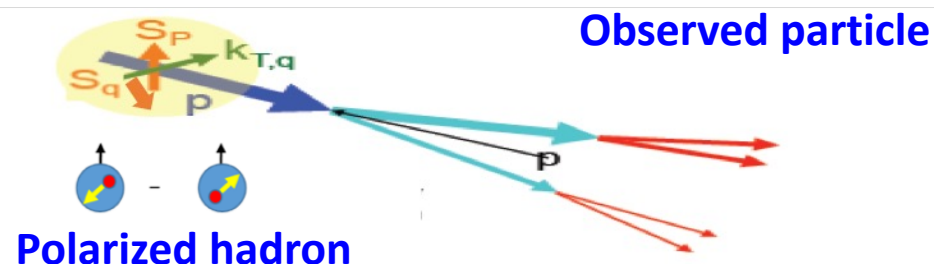
- Quantum correlation between hadron spin and parton motion:



Sivers effect – Sivers function

Hadron spin influences
parton's transverse motion

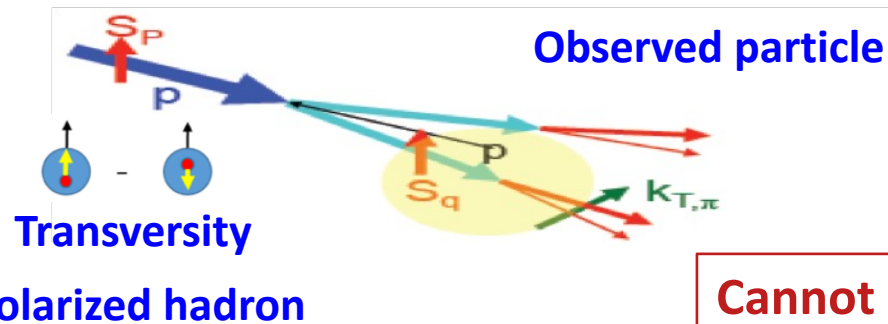
- Quantum correlation between hadron spin and parton spin:



Pretzelosity – model OAM

Hadron spin and parton spin
influence
parton's transverse motion

- Quantum correlation between parton's spin and its hadronization:



Collins effect – Collins function

Parton's transverse polarization
influences its hadronization

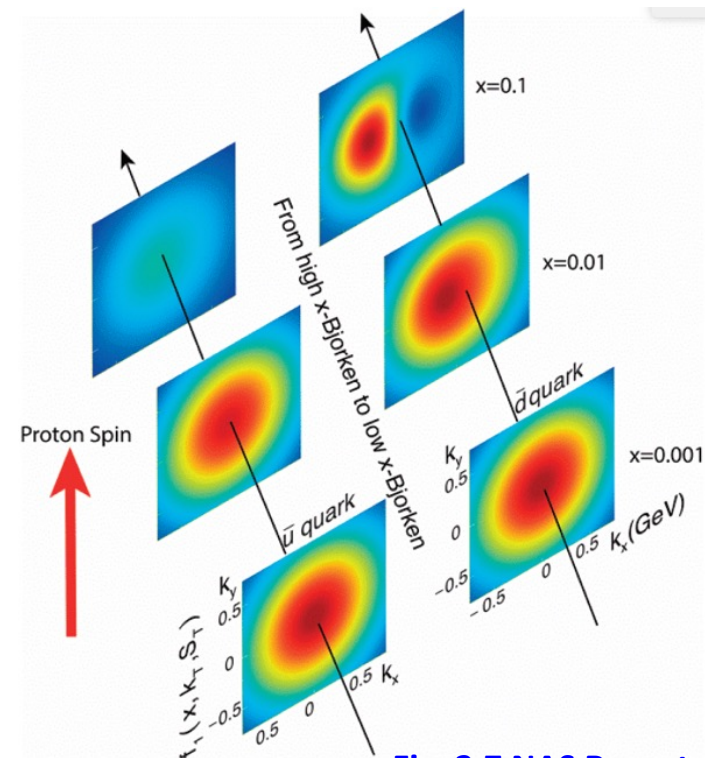


Fig. 2.7 NAS Report

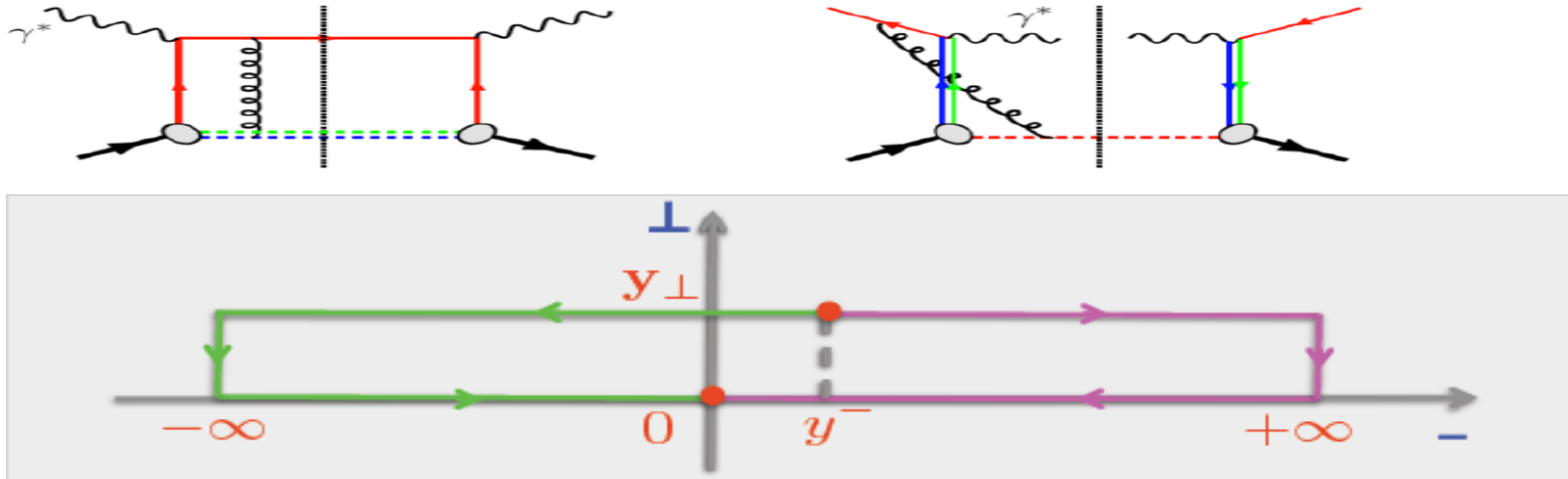
Boer–Mulders function $h_1^\perp$
can generate characteristic
azimuthal modulations in
unpolarized Drell–Yan, ...

TMDs Introduce Sensitivity of Color Flow of the Active Parton

□ Definition of quark TMD:

$$f_{q/h\uparrow}(x, \mathbf{k}_\perp, \vec{S}) = \int \frac{dy^- d^2 y_\perp}{(2\pi)^3} e^{ixp^+ y^- - i\mathbf{k}_\perp \cdot \mathbf{y}_\perp} \langle p, \vec{S} | \bar{\psi}(0^-, \mathbf{0}_\perp) \boxed{\text{Gauge link}} \frac{\gamma^+}{2} \psi(y^-, \mathbf{y}_\perp) | p, \vec{S} \rangle$$

□ Choice of the Gauge link – to be consistent with TMD factorization:



Immediate result of
PT symmetry of QCD

Collins, PLB 2002
Kang, Qiu, PRL 2009

TMD-factorization is
violated for hadronic
production of high
PT particles!

□ Modified universality of TMDs:

$$f_{q/h\uparrow}^{\text{SIDIS}}(x, \mathbf{k}_\perp, \vec{S}) \neq f_{q/h\uparrow}^{\text{DY}}(x, \mathbf{k}_\perp, \vec{S})$$

Collins Qiu, PRD 2007

Note: Collinear PDFs are process independent – key for the predictive power of QCD factorization

Modified Universality for TMDs

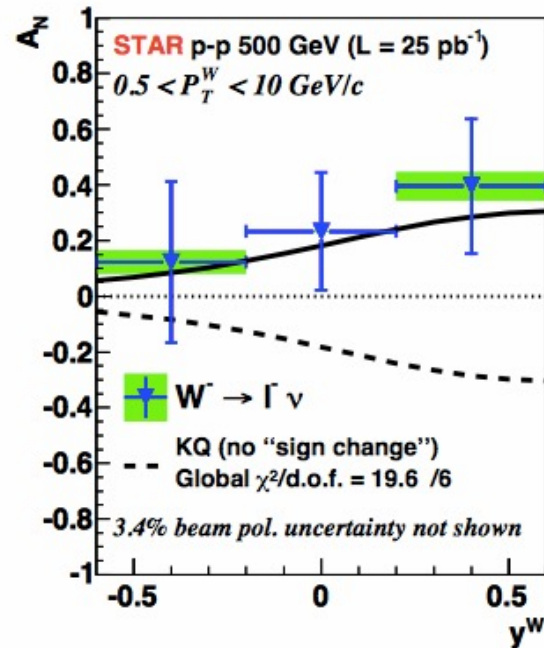
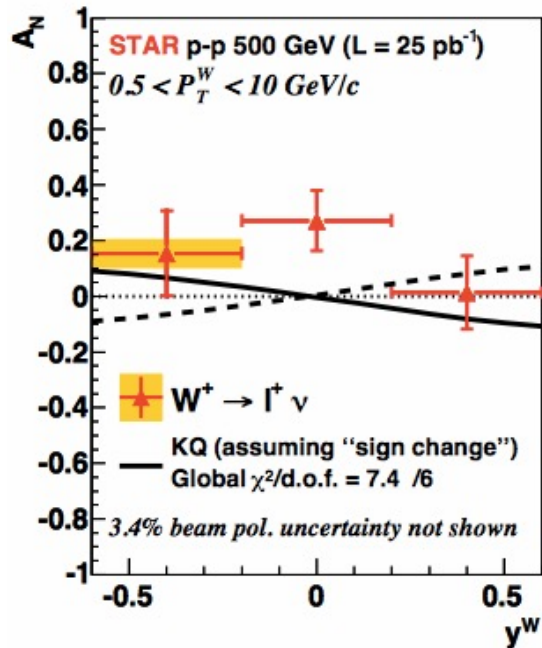
□ Parity – Time reversal invariance:

$$f_{q/h\uparrow}^{\text{SIDIS}}(x, \mathbf{k}_\perp, \vec{S}) = f_{q/h\uparrow}^{\text{DY}}(x, \mathbf{k}_\perp, -\vec{S})$$

□ Modified universality – Sivers function:

$$\Delta^N f_{q/h\uparrow}^{\text{SIDIS}}(x, k_\perp) = -\Delta^N f_{q/h\uparrow}^{\text{DY}}(x, k_\perp)$$

□ RHIC measurements:



Modified Universality for TMDs

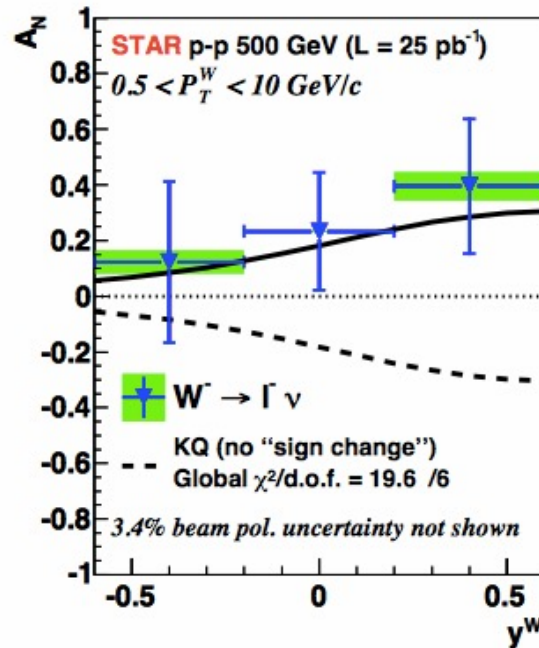
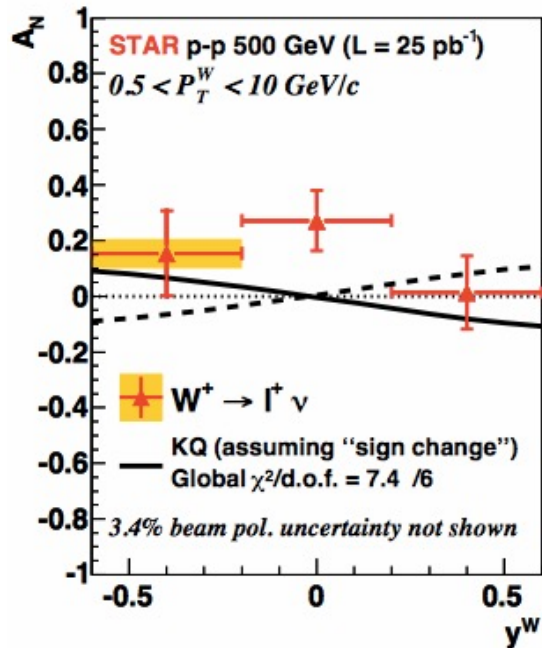
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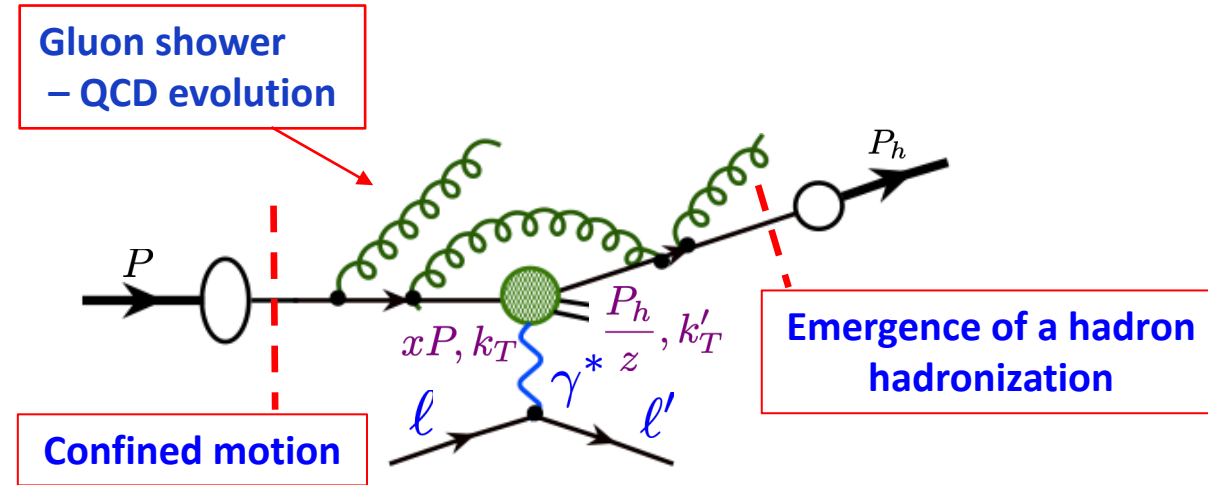
$$\Delta^N f_{q/h^\uparrow}^{\text{SIDIS}}(x, k_\perp) = -\Delta^N f_{q/h^\uparrow}^{\text{DY}}(x, k_\perp)$$

□ RHIC measurements:



STAR Collab. PRL116, 132301 (2016)

□ Experimental test is still inconclusive



Measured k_T of TMDs
 $\neq$ the *confined* motion inside the hadron!

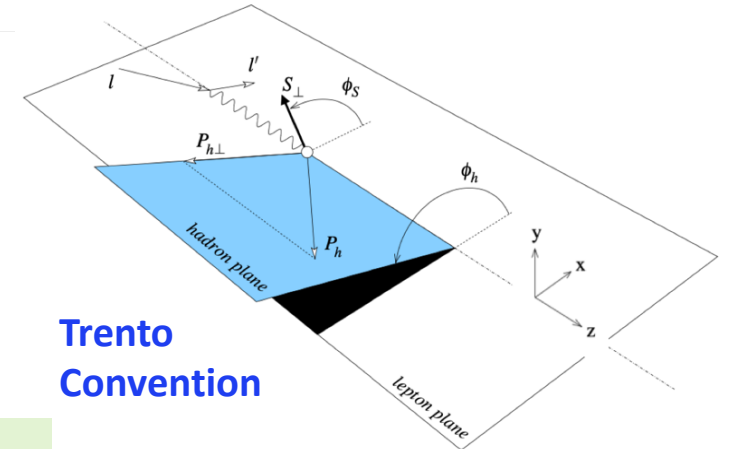
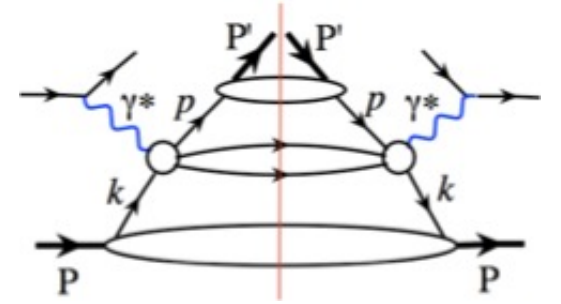
Collision induced shower dilutes the correlations between the hadron spin and parton motion

SIDIS Might be the Best for Probing TMDs

□ SIDIS with the “one-photon approximation”:

- 18 Structure functions
- A_N = at least one of 6 F_{UT} structure functions needs to be finite!

$$\begin{aligned} \frac{d\sigma}{dx dy d\psi dz d\phi_h dP_{h\perp}^2} = & \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x}\right) \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos\phi_h F_{UU}^{\cos\phi_h} \right. \\ & + \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} + \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \sin\phi_h F_{LU}^{\sin\phi_h} \\ & + S_{\parallel} \left[\sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_h F_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h) F_{UL}^{\sin 2\phi_h} \right] \\ & + S_{\parallel} \lambda_e \left[\sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_h F_{LL}^{\cos\phi_h} \right] \\ & + |S_{\perp}| \left[\sin(\phi_h - \phi_S) \left(F_{UT,T}^{\sin(\phi_h - \phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h - \phi_S)} \right) \right. \\ & \quad + \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} + \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} \\ & \quad + \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_S F_{UT}^{\sin\phi_S} + \sqrt{2\varepsilon(1+\varepsilon)} \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)} \left. \right] \\ & + |S_{\perp}| \lambda_e \left[\sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_S F_{LT}^{\cos\phi_S} \right. \\ & \quad \left. + \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)} \right] \left. \right\} \end{aligned}$$



Trento
Convention

$$\varepsilon = \frac{1 - y - \frac{1}{4}\gamma^2 y^2}{1 - y + \frac{1}{2}y^2 + \frac{1}{4}\gamma^2 y^2}$$

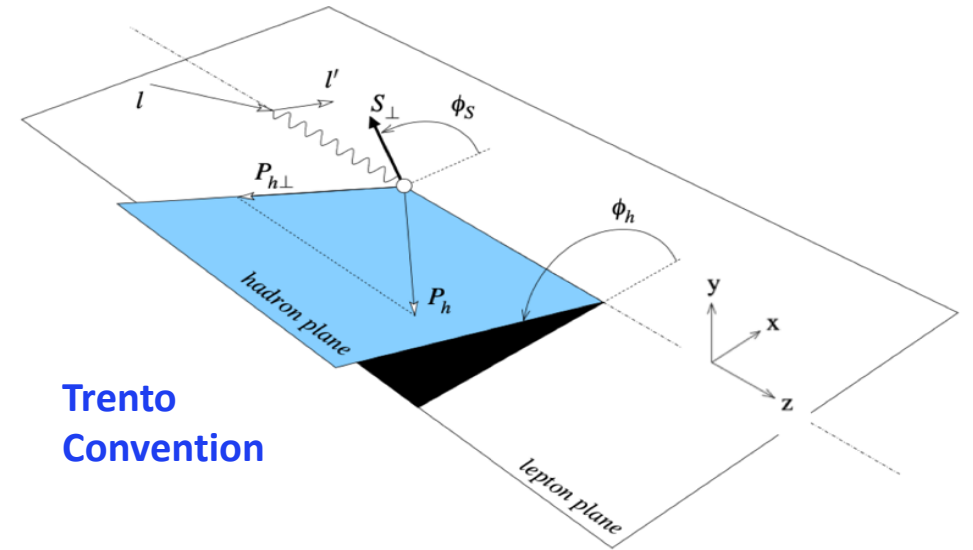
SIDIS Might be the Best for Probing TMDs

□ Single transverse-spin asymmetry:

$$\begin{aligned}
 A_{UT}(\phi_h^l, \phi_S^l) &= \frac{1}{P} \frac{N^\uparrow - N^\downarrow}{N^\uparrow + N^\downarrow} \\
 &= A_{UT}^{\text{Collins}} \sin(\phi_h + \phi_S) + A_{UT}^{\text{Sivers}} \sin(\phi_h - \phi_S) \\
 &\quad + A_{UT}^{\text{Pretzelosity}} \sin(3\phi_h - \phi_S)
 \end{aligned}$$

□ Angular modulations – Separation of TMDs:

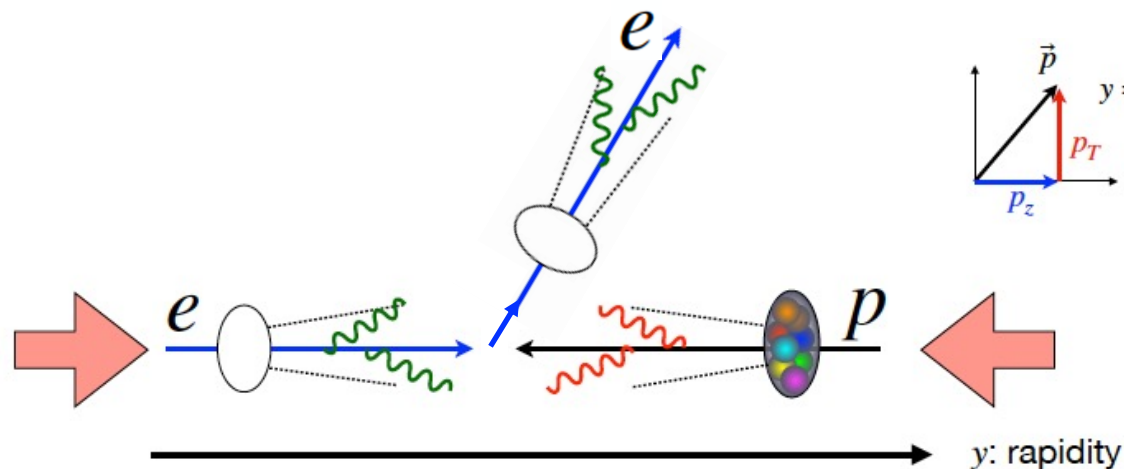
$$\begin{aligned}
 A_{UT}^{\text{Collins}} &\propto \langle \sin(\phi_h + \phi_S) \rangle_{UT} \propto h_1 \otimes H_1^\perp \\
 A_{UT}^{\text{Sivers}} &\propto \langle \sin(\phi_h - \phi_S) \rangle_{UT} \propto f_{1T}^\perp \otimes D_1 \\
 A_{UT}^{\text{Pretzelosity}} &\propto \langle \sin(3\phi_h - \phi_S) \rangle_{UT} \propto h_{1T}^\perp \otimes H_1^\perp
 \end{aligned}$$



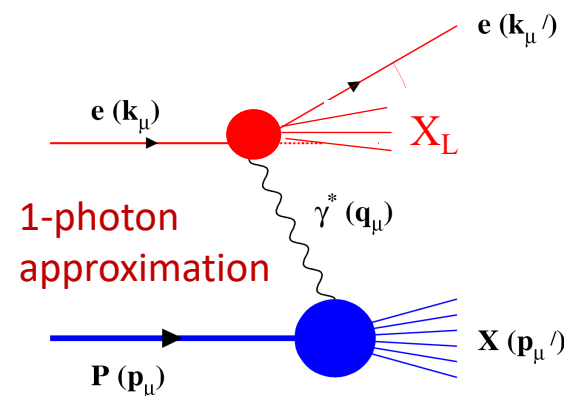
Collins fragmentation functions,
extracted from e^+e^- collisions

But, collision-induced QED radiation can change both the direction and magnitude of the exchanged virtual photon – the “probe”, and the angular modulation!

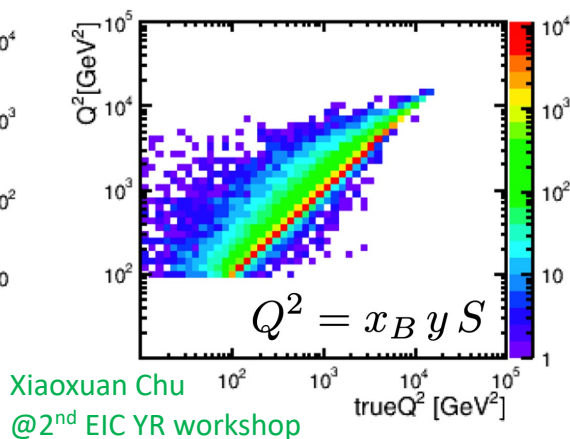
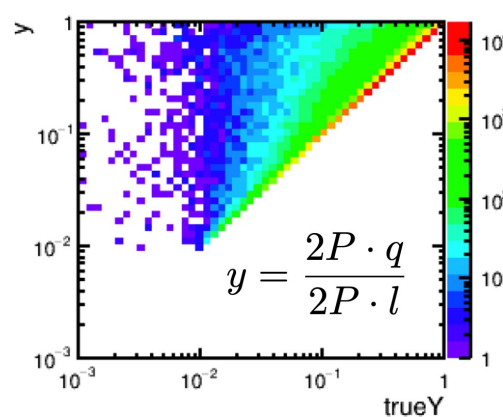
Hard Collision Induces both QCD and QED Radiations



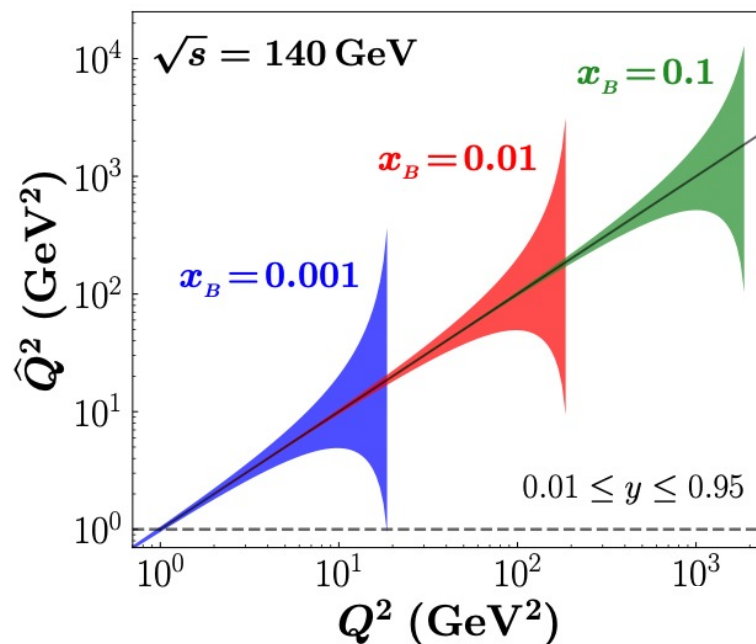
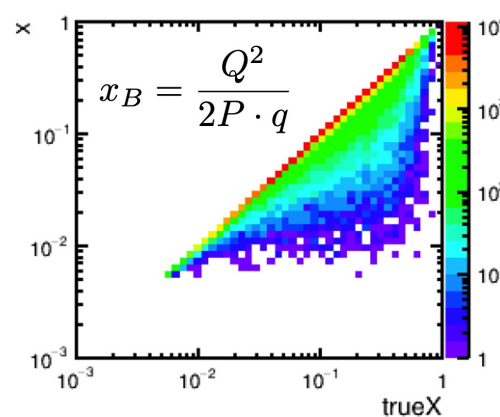
Historically, 1-photon approximation
+ radiative corrections (radiation from leptons)



“We know how to calculate QED radiation perturbatively!”



Xiaoxuan Chu
@2nd EIC YR workshop



$$x_B \rightarrow \hat{x}_B \in [x_B, 1]$$

$$\hat{Q}_{\min}^2 = Q^2 \frac{(1-y)}{(1-x_B y)}$$

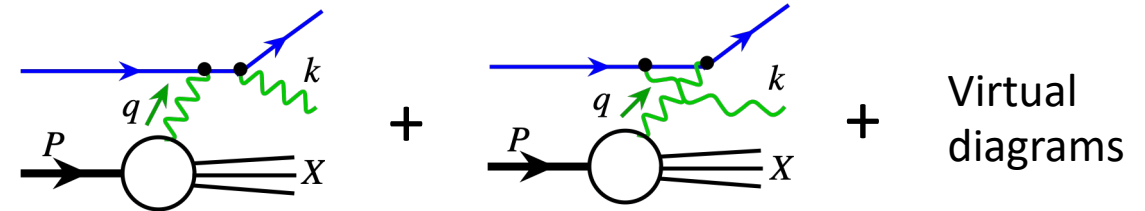
$$\hat{Q}_{\max}^2 = Q^2 \frac{1}{(1-y + x_B y)}$$

Must Introduce Unknown Parameter(s) in Traditional RC Approach

□ Radiative correction – Born kinematics:

$$\sigma_{\text{Measured}} \equiv \text{RC} \otimes \sigma_{\text{No QED Radiation}}$$

Necessary requirement: RC – Radiative correction factor
does not depend on the hadronic physics that we want to extract!

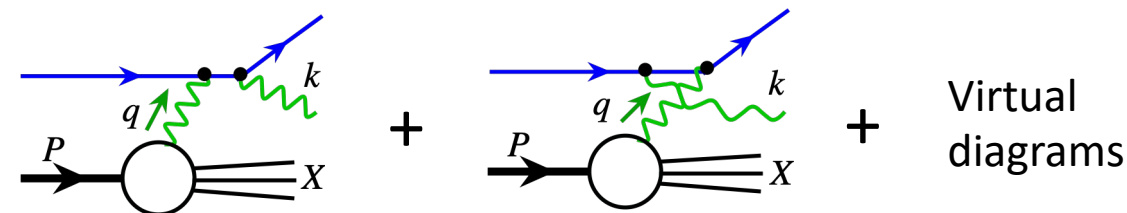


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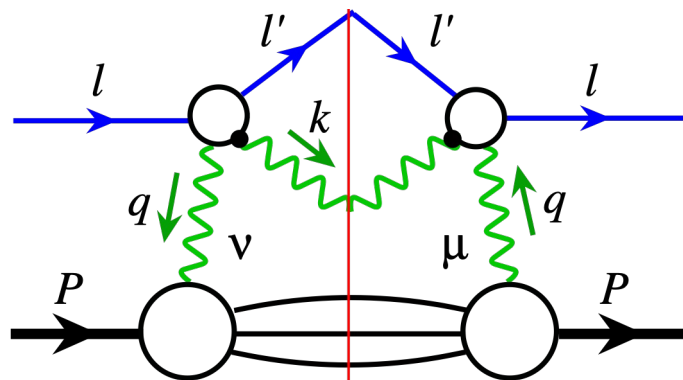
$$\sigma_{\text{Measured}} \equiv \text{RC} \otimes \sigma_{\text{No QED Radiation}}$$

Necessary requirement: RC – Radiative correction factor
does not depend on the hadronic physics that we want to extract!



❑ Leading Order Radiative Correction:

Contribution to the cross section – cut diagram notation:



The virtuality of the exchange photon q^2 depends on the phase-space of unobserved photon of momentum k

$$E' \frac{d\sigma_{eh \rightarrow eX}^{\text{RC}}}{d^3\ell'} \propto \int d^4q \left[W^{\mu\nu}(P, q) \frac{1}{q^2 + i\epsilon} L_{\mu\nu}^{(1)}(\ell, \ell', q) \frac{1}{q^2 - i\epsilon} \right] \rightarrow \infty$$

The q^2 is perturbatively pinched to $q^2 = 0$ if the phase space allows!

➡ **Impose “cutoff/limit” on the radiated and un-observed photon(s) to keep the q^2 “virtual”**

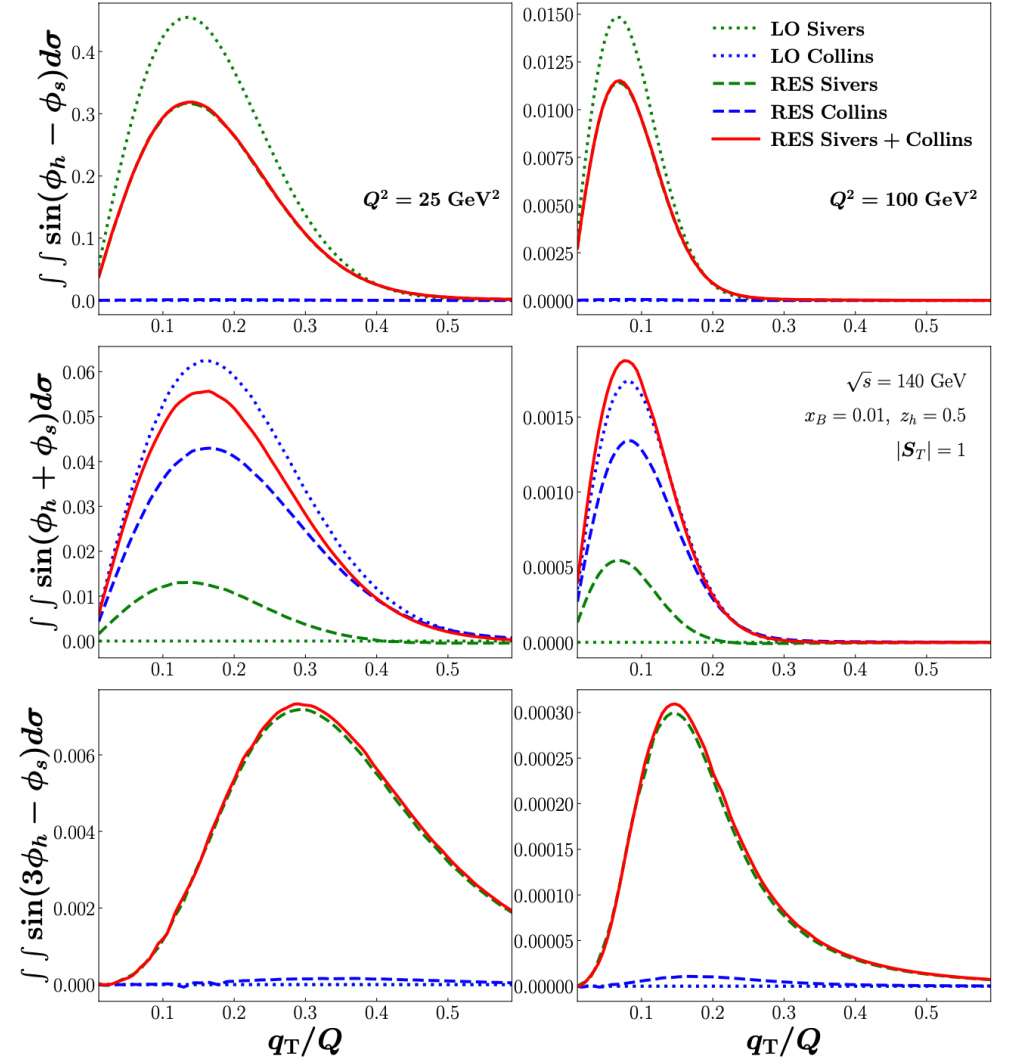
Need parameter(s) – tuning parameter(s) in MC for RC – Predictive power?

Impact on SIDIS – TMD extraction

□ Case study – single transverse spin asymmetry:

$$\begin{aligned}
 \frac{d\sigma}{dx dy d\psi dz d\phi_h dP_{h\perp}^2} = & \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x}\right) \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos\phi_h F_{UU}^{\cos\phi_h} \right. \\
 & + \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} + \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \sin\phi_h F_{LU}^{\sin\phi_h} \\
 & + S_{\parallel} \left[\sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_h F_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h) F_{UL}^{\sin 2\phi_h} \right] \\
 & + S_{\parallel} \lambda_e \left[\sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_h F_{LL}^{\cos\phi_h} \right] \\
 & + |S_{\perp}| \left[\sin(\phi_h - \phi_S) \left(F_{UT,T}^{\sin(\phi_h - \phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h - \phi_S)} \right) \right. \\
 & + \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} + \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} \\
 & + \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_S F_{UT}^{\sin\phi_S} + \sqrt{2\varepsilon(1+\varepsilon)} \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)} \left. \right] \\
 & + |S_{\perp}| \lambda_e \left[\sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_S F_{LT}^{\cos\phi_S} \right. \\
 & \left. + \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)} \right] \left. \right\}
 \end{aligned}$$

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371



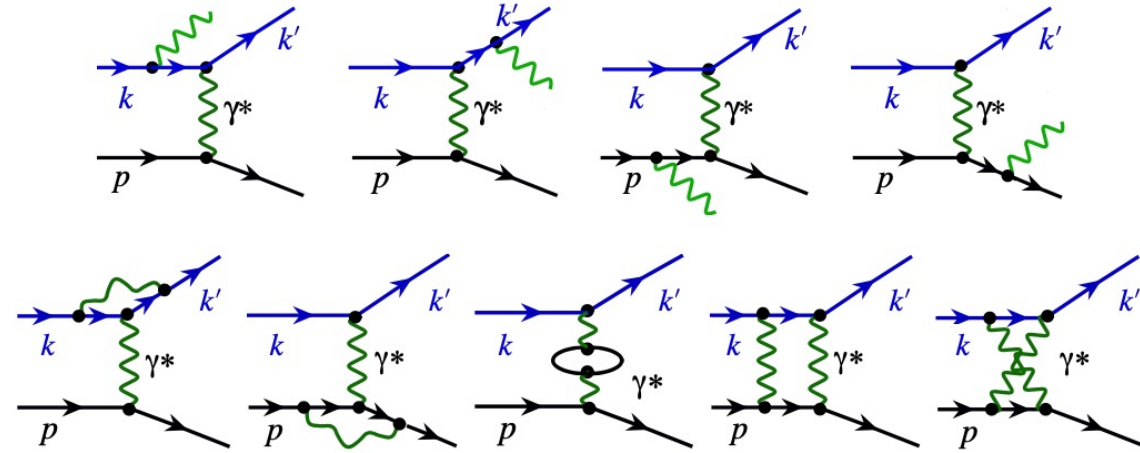
Joint QCD and QED Factorization for Lepton-Hadron DIS

Cammarota, Qiu, Watanabe, Zhang
PRD 112 (2025) 056007

□ **NLO QED contribution to the leading subprocess:** $e(k) + q(p) \rightarrow e(k') + X$

$$\begin{aligned} \hat{H}_{eq \rightarrow eq}^{(3,0)}(\hat{s}, \hat{t}, \hat{u}) = & e_q^2 (4\alpha_{em}^2) \frac{\alpha_{em}}{\pi} \frac{1}{(1-\hat{v})^2 \hat{s}} \\ & \times \left\{ e_l^2 \frac{2(1+\hat{v}^2)}{9\hat{v}} \left(3 \ln \left[\frac{(1-\hat{v})\hat{s}}{\mu^2} \right] - 5 \right) \delta(1-\hat{w}) \right. \\ & + e_q \left(a_1 \delta(1-\hat{w}) + a_2 \frac{1}{[1-\hat{w}]_+} + a_4 \right) \\ & \left. + e_q^2 \left(b_1 \delta(1-\hat{w}) + b_2 \frac{1}{[1-\hat{w}]_+} + b_3 \left[\frac{\ln(1-\hat{w})}{1-\hat{w}} \right]_+ \right. \right. \\ & \left. \left. + b_4 \right) \right\} \end{aligned} \quad (53)$$

$$\left\{ + c_1 \delta(1-\hat{w}) + c_2 \frac{1}{[1-\hat{w}]_+} + c_3 \left[\frac{\ln(1-\hat{w})}{1-\hat{w}} \right]_+ + c_4 \right\}$$



a_1, a_2, a_3, a_4
 b_1, b_2, b_3, b_4
 c_1, c_2, c_3, c_4

are analytic functions
of $\hat{v}$ and $\hat{w}$.

$$\hat{v} = 1 - \frac{x_B}{x} \frac{y}{\zeta}$$

$$\hat{w} = \frac{1-y}{\xi (\zeta - (x_B/x) y)}$$

IR-safe
No free parameter!

e_q - Dependence helps to isolate contributions from different diagrams and cancelation of perturbative divergences

e_q^4 - Same as NLO QCD

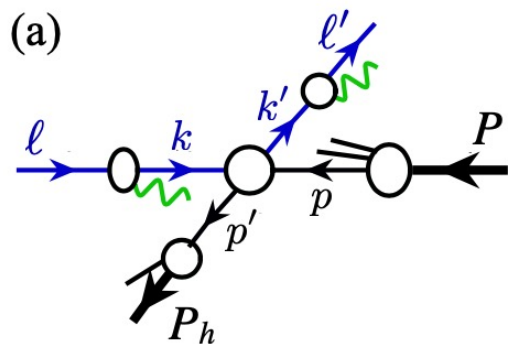
e_q^3 - Two-photon exchange

e_q^2 - RC from electron

Hybrid Factorization for Lepton-Hadron SIDIS

□ Inclusive production of a lepton and a hadron:

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371



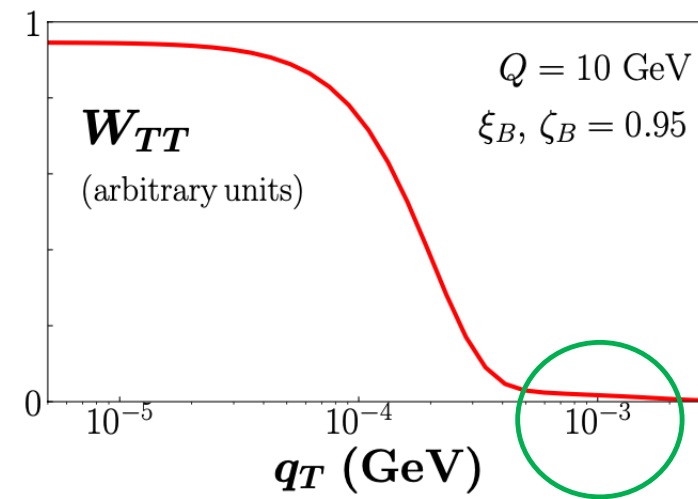
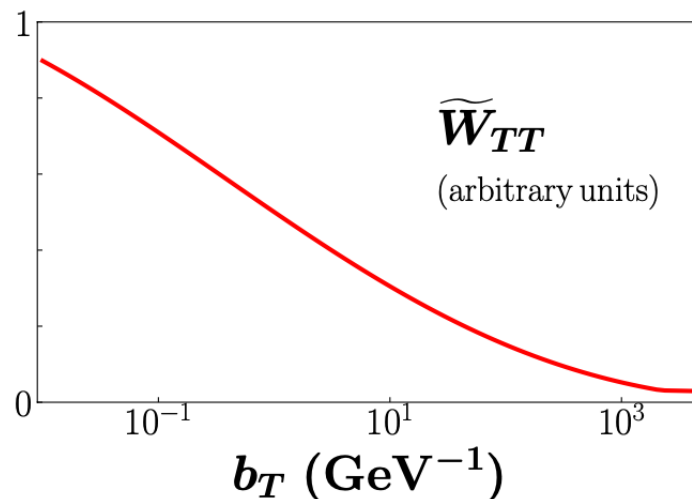
$$e(\ell) + N(P) \rightarrow e(\ell') + h(P_h) + X$$

Momentum imbalance between the lepton and the hadron could be sensitive to both parton TMDs and lepton TMDs

Typical parton transverse momentum: $k_T^2 \sim \Lambda_{\text{QCD}}^2 + \langle k_T^2 \rangle_{\text{generated by QCD shower}}$

□ Estimate of lepton transverse momentum generated by QED shower:

Resummation
to lepton TMD



QED broadening for lepton is so much smaller than typical parton k_T !

Collinear factorization for high order QED contributions

Hybrid Factorization for Lepton-Hadron SIDIS

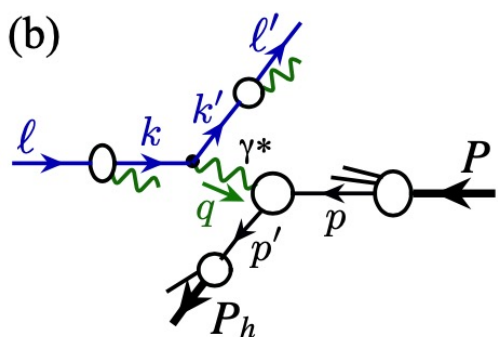
QED factorization of collision induced radiation – collinear:

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

$$E_{\ell'} E_{P_h} \frac{d^6 \sigma_{\ell(\lambda_\ell) P(S) \rightarrow \ell' P_h X}}{d^3 \ell' d^3 P_h} \approx \sum_{ij\lambda_k} \int_{\zeta_{\min}}^1 \frac{d\zeta}{\zeta^2} D_{e/j}(\zeta) \int_{\xi_{\min}}^1 d\xi f_{i(\lambda_k)/e(\lambda_\ell)}(\xi) \left[E_{k'} E_{P_h} \frac{d^6 \hat{\sigma}_{k(\lambda_k) P(S) \rightarrow k' P_h X}}{d^3 k' d^3 P_h} \right]_{k=\xi \ell, k'=\ell'/\zeta} + \mathcal{O}\left(\frac{m_e^n}{Q^n}\right)$$

- Leading power IR sensitive contribution is universal, as $m_e/Q \rightarrow 0$, factorized into LDFs and LFFs
- IR safe contributions are calculated order-by-order in powers of α
- Neglect m_e/Q power suppressed contributions
- Collinear QED factorization for both inclusive DIS and SIDIS, or e^+e^- , ... [global fits of LDFs, LFFs]

“One photon”-approximation:



$$\frac{d^6 \sigma_{\ell(\lambda_\ell) P(S) \rightarrow \ell' P_h X}}{dx_B dy d\psi dz_h d\phi_h dP_{hT}^2} = \sum_{ij\lambda_k} \int_{\zeta_{\min}}^1 \frac{d\zeta}{\zeta^2} \int_{\xi_{\min}}^1 \frac{d\xi}{\xi} f_{i(\lambda_k)/e(\lambda_\ell)}(\xi) D_{e/j}(\zeta) \times \frac{\hat{x}_B}{x_B \xi \zeta} \left[\frac{\alpha^2}{\hat{x}_B \hat{y} \hat{Q}^2} \frac{\hat{y}^2}{2(1-\hat{\epsilon})} \left(1 + \frac{\hat{\gamma}^2}{2\hat{x}_B} \right) \sum_n \hat{w}_n F_n^h(\hat{x}_B, \hat{Q}^2, \hat{z}_h, \hat{P}_{hT}^2) \right]$$

Apply a (ξ, ζ) -dependent Lorentz transformation:

Evaluated in a “virtual photon-hadron” frame

$$\{\hat{q}, P, \hat{P}_h\} \xrightarrow{(\xi, \zeta)} \{q, P, P_h\}$$

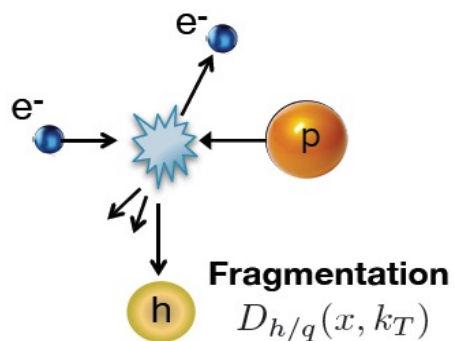
In a frame to compare with exp. measurements

Extract TMDs from Experimental Data (an inverse problem)

❑ Need multiple and complementary observables depending on the same TMDs:

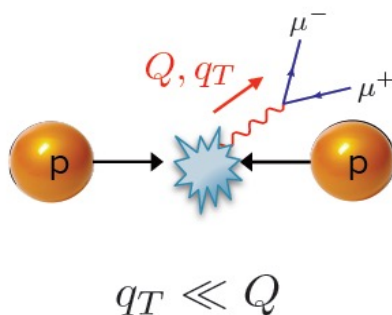
Semi-Inclusive DIS

$$\sigma \sim f_{q/P}(x, k_T) D_{h/q}(x, k_T)$$



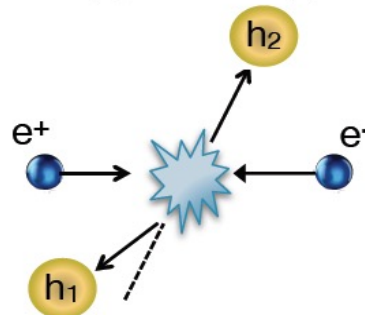
Drell-Yan

$$\sigma \sim f_{q/P}(x, k_T) f_{q/P}(x, k_T)$$



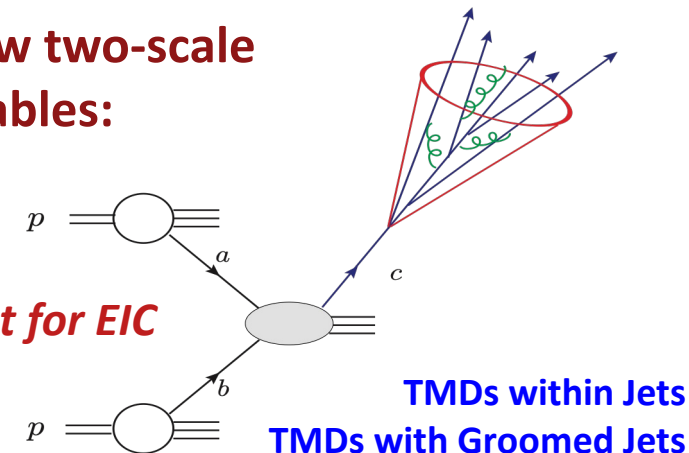
Dihadron in e^+e^-

$$\sigma \sim D_{h_1/q}(x, k_T) D_{h_2/q}(x, k_T)$$



Plus new two-scale observables:

Relevant for EIC



See Nobuo's talk

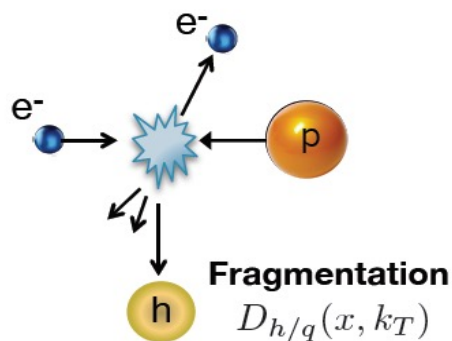
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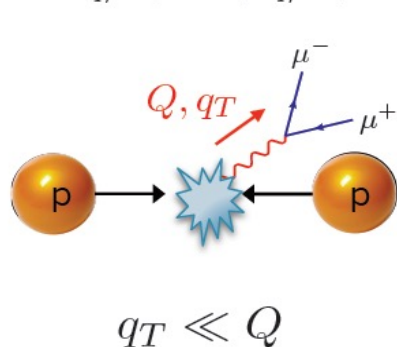
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$$\sigma \sim f_{q/P}(x, k_T) D_{h/q}(x, k_T)$$



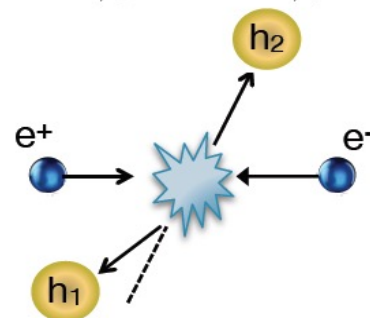
Drell-Yan

$$\sigma \sim f_{q/P}(x, k_T) f_{q/P}(x, k_T)$$



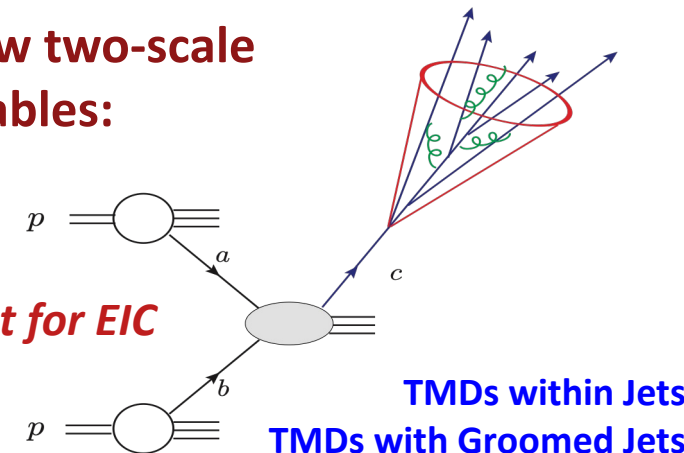
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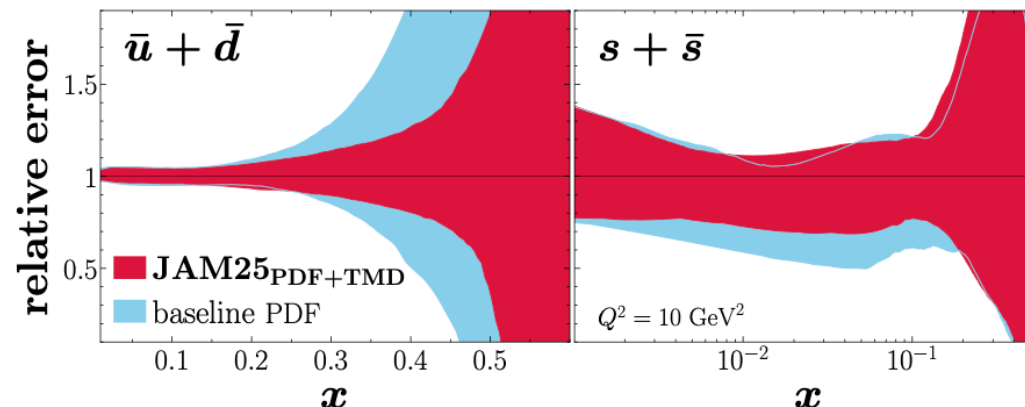
❑ First Simultaneous Analysis of TMDs and Collinear PDFs:

$$\tilde{f}_{i/H}(x, b_T; \mu, \zeta) = \sum_j \int_x^1 \frac{d\xi}{\xi} C_{i \leftarrow j} \left(\frac{x}{\xi}, b_T; \mu, \zeta \right) f_{j/H}(\xi, \mu) + \mathcal{O}(b_T^2 \Lambda_{\text{QCD}}^2), \text{ -- OPE relation}$$

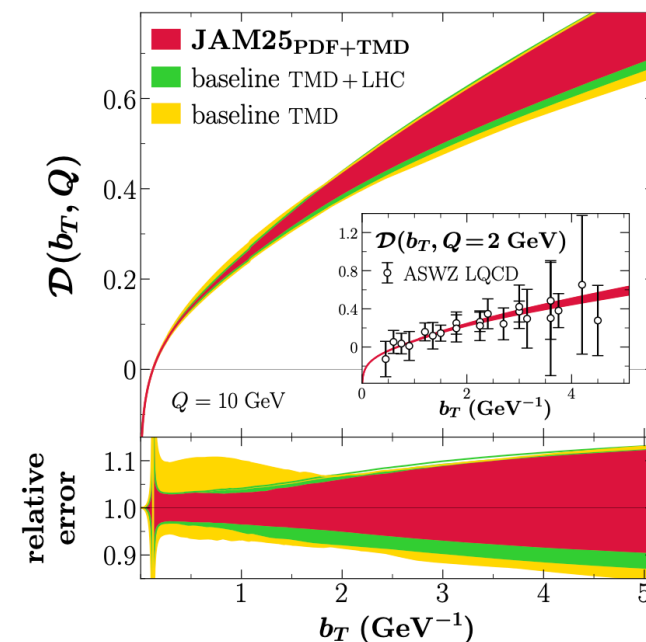
Making PDFs essential inputs
to many TMD analyses, ...

Improves knowledge of both
collinear and TMD & PDFs

for the poorly constrained
sea-quark distributions



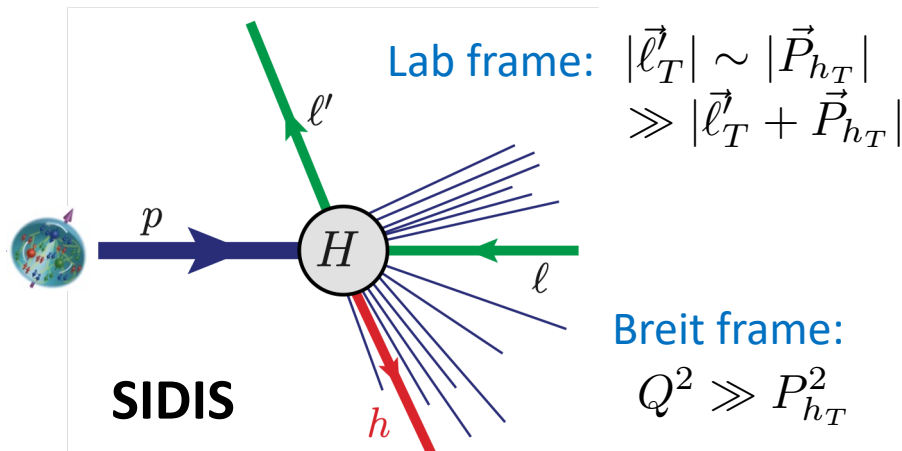
JAM, PRL 2026



Explore Multidimensional Structure without Breaking the Hadron

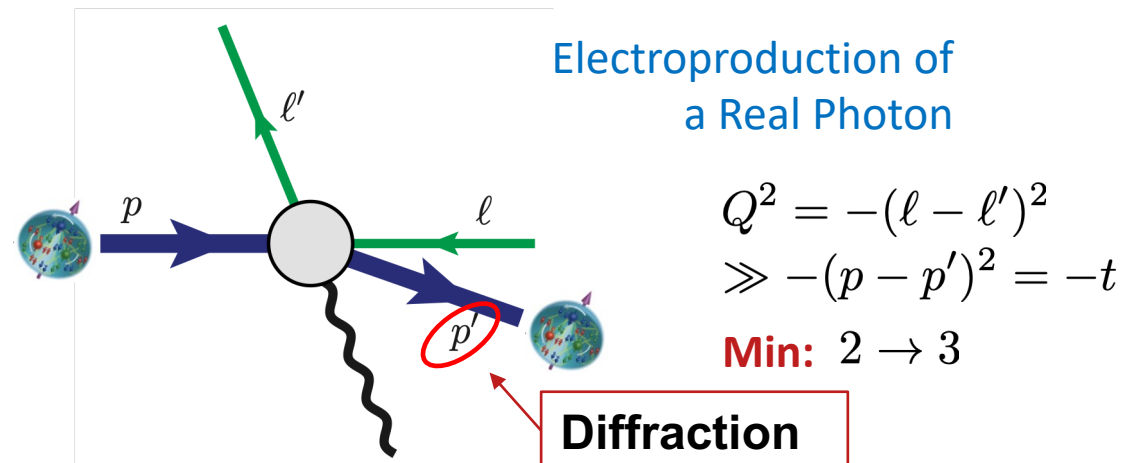
Inclusive scattering

Scattering
Amplitude



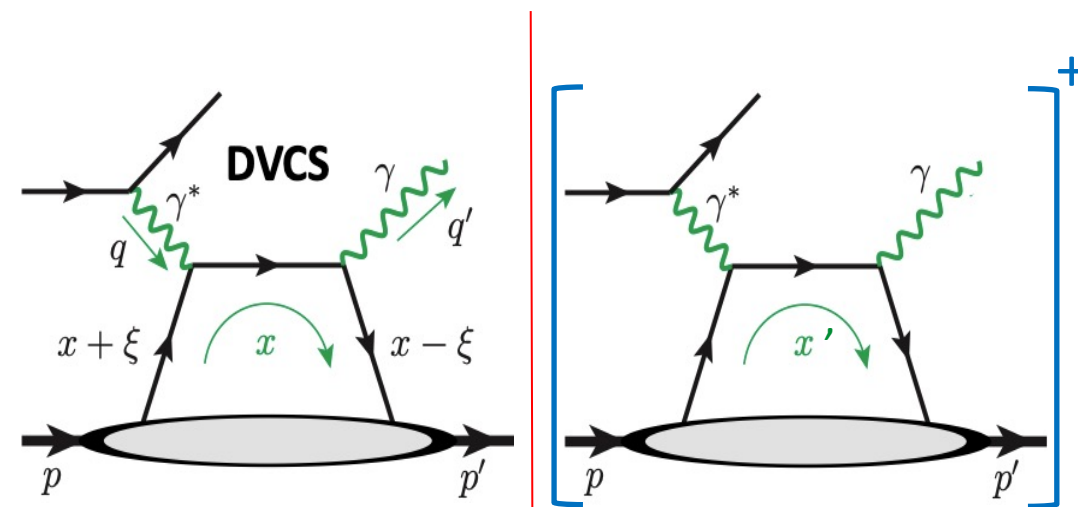
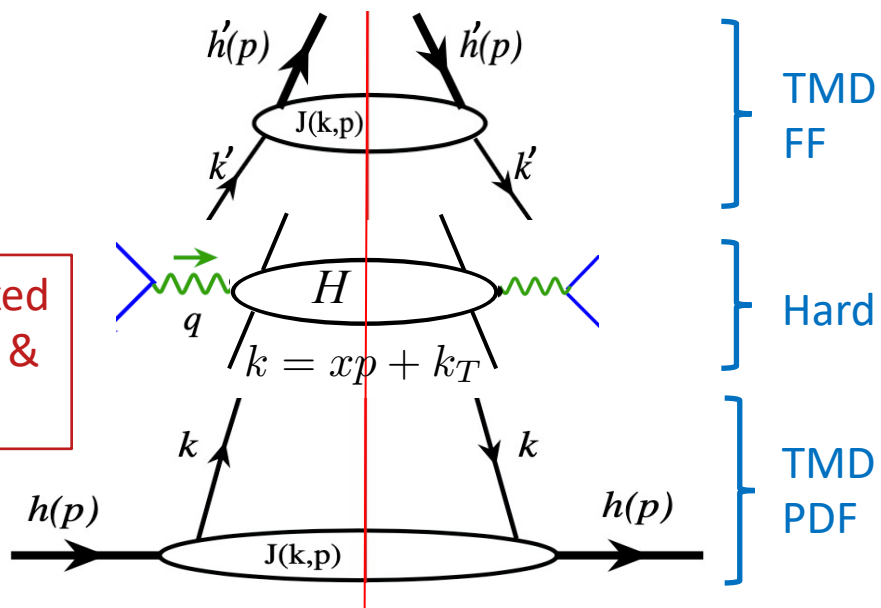
Exclusive diffraction

Electroproduction of
a Real Photon



Cross
Section

Range of x fixed
by hard scale &
kinematics!



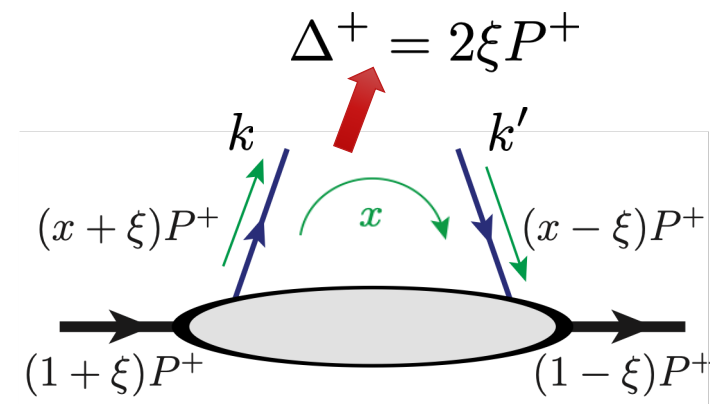
Range of x is always the same!

Generalized Parton Distributions (GPDs)

□ Definition:

$$\begin{aligned}
 F^q(x, \xi, t) &= \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p' | \bar{q}(z^-/2) \gamma^+ q(-z^-/2) | p \rangle \\
 &= \frac{1}{2P^+} \left[\tilde{H}^q(x, \xi, t) \bar{u}(p') \gamma^+ u(p) - \tilde{E}^q(x, \xi, t) \bar{u}(p') \frac{i\sigma^{+\alpha} \Delta_\alpha}{2m} u(p) \right], \\
 \tilde{F}^q(x, \xi, t) &= \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p' | \bar{q}(z^-/2) \gamma^+ \gamma_5 q(-z^-/2) | p \rangle \\
 &= \frac{1}{2P^+} \left[\tilde{H}^q(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) - \tilde{E}^q(x, \xi, t) \bar{u}(p') \frac{\gamma_5 \Delta^+}{2m} u(p) \right].
 \end{aligned}$$

D. Müller, D. Robaschik, B. Geyer, F.-M. Dittes, J. Hořejši,
Fortsch. Phys. 42 (1994) 101



$$\begin{aligned}
 P^+ &= \frac{p^+ + p'^+}{2} \\
 \Delta &= p - p' \quad t = \Delta^2
 \end{aligned}$$

Similar definition
 for gluon GPDs

Generalized Parton Distributions (GPDs)

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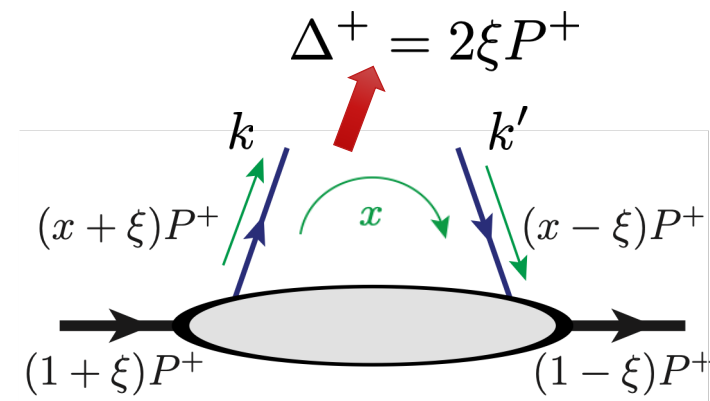
D. Müller, D. Robaschik, B. Geyer, F.-M. Dittes, J. Hořejši,
Fortsch. Phys. 42 (1994) 101

$$F^q(x, \xi, t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p' | \bar{q}(z^-/2) \gamma^+ q(-z^-/2) | p \rangle$$

$$= \frac{1}{2P^+} \left[H^q(x, \xi, t) \bar{u}(p') \gamma^+ u(p) - E^q(x, \xi, t) \bar{u}(p') \frac{i\sigma^{+\alpha} \Delta_\alpha}{2m} u(p) \right],$$

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$$= \frac{1}{2P^+} \left[\tilde{H}^q(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) - \tilde{E}^q(x, \xi, t) \bar{u}(p') \frac{\gamma_5 \Delta^+}{2m} u(p) \right].$$

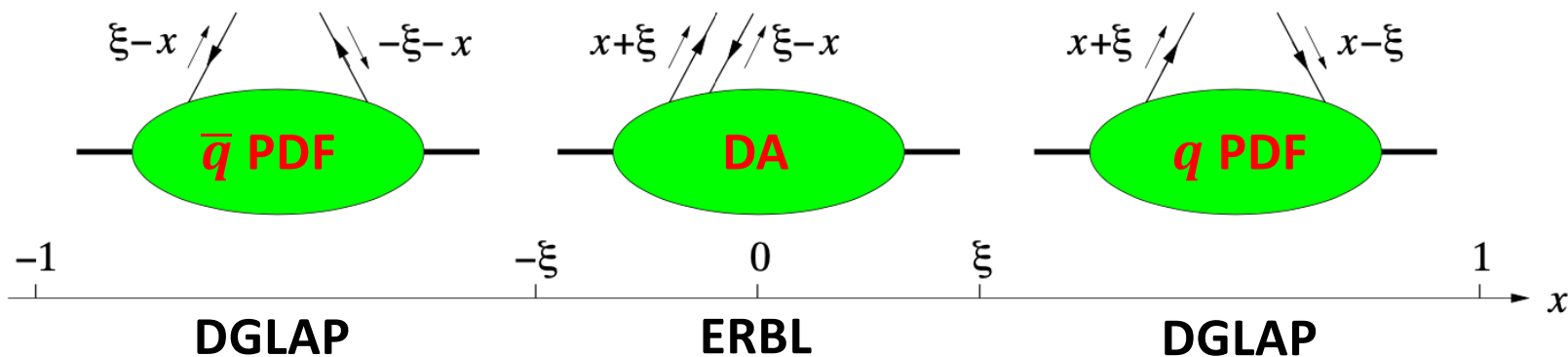


Combine PDFs ($p'=p$) and Form Factors ($z=0$):

Forward limit $\xi = t = 0$: $H^q(x, 0, 0) = q(x)$, $\tilde{H}^q(x, 0, 0) = \Delta q(x)$

$$P^+ = \frac{p^+ + p'^+}{2}$$

$$\Delta = p - p' \quad t = \Delta^2$$



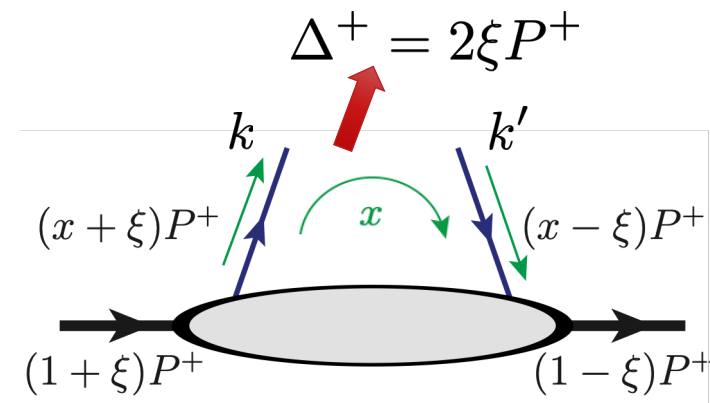
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D. Müller, D. Robaschik, B. Geyer, F.-M. Dittes, J. Hořejši,
Fortsch. Phys. 42 (1994) 101

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 \end{aligned}$$



□ GPDs depend on the Choice of “+” for given p and p’ :

$$\begin{aligned}
 P^+ &= \frac{p^+ + p'^+}{2} \\
 \Delta &= p - p' \quad t = \Delta^2
 \end{aligned}$$

- Need to choose a light-cone vector: $n^\mu = (n^+, n^-, \mathbf{n}^\perp) = (0, 1, \mathbf{0}^\perp)$
with $n^2 = 0$, $n \cdot v = v^+$ for any 4-vector $v^\mu = (v^+, v^-, \mathbf{v}^\perp)$
- For a single vector, p , there is a unique light-cone vector “ n ” to pick up the largest component of p , as p^+ .
- When $p \neq p'$, we have infinite choices for the light-cone vector “ n ”!

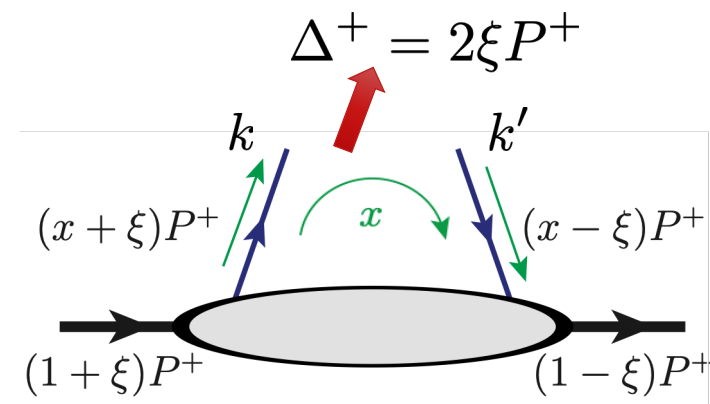
if $\sqrt{|\Delta^2|} = \sqrt{|(p - p')^2|} \ll p^0 \sim p'^0$, GPDs with different choice of “ n ” differ by powers of $|\Delta^2|/\hat{s}$
with the hard scale $\hat{s}$, e.g., $\hat{s} \sim Q^2$ for DVCS.

Generalized Parton Distributions (GPDs)

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 \end{aligned}$$

D. Müller, D. Robaschik, B. Geyer, F.-M. Dittes, J. Hořejši,
Fortsch. Phys. 42 (1994) 101



□ Variables of GPDs – x, ξ, t :

- $t = \Delta^2 = (p - p')^2$ is uniquely fixed by the momentum of scattered hadron p' .
- $\xi = (p - p') \cdot n / (p + p') \cdot n$ is uniquely fixed once we make a choice of the light-cone vector “ n ”
- $x = (k + k') \cdot n / (p + p') \cdot n$ carries partonic information and need to be probed by the hard probe!

$$\begin{aligned}
 P^+ &= \frac{p^+ + p'^+}{2} \\
 \Delta &= p - p' \quad t = \Delta^2
 \end{aligned}$$

Questions:

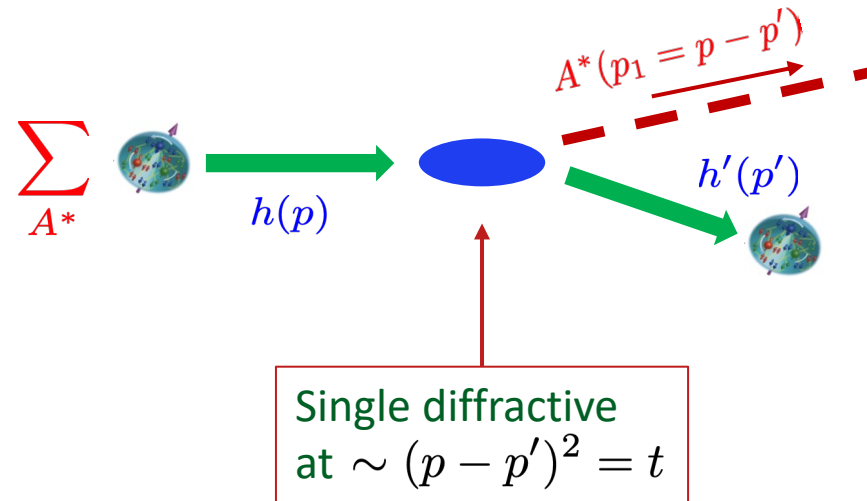
- For extracting universal GPDs via QCD global analysis, is there an “ideal” choice of the light-cone vector “ n ” ?
- What kind of hard probes have better access to the x -dependence of GPDs ?

How to Find Physical Processes to be Sensitive to GPDs?

❑ Need an exclusive process – Not to break the hadron:

Single diffractive: $h(p) \rightarrow h'(p') + A^*(p_1 = p - p')$ ← Momentum, spin, composition of particles (or partons)

A “soft” scale: $t = (p - p')^2 \equiv \Delta^2$



How to Find Physical Processes to be Sensitive to GPDs?

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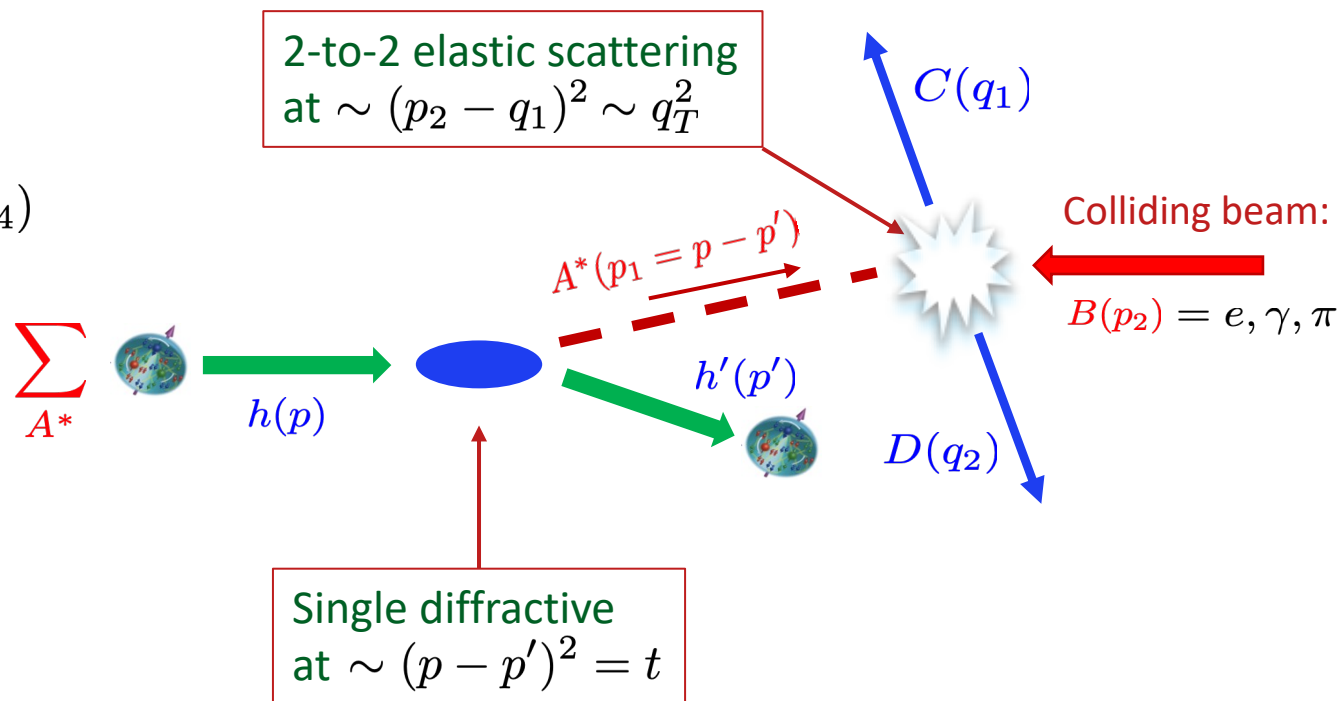
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A “soft” scale: $t = (p - p')^2 \equiv \Delta^2$

❑ Need a hard collision – to see the parton(s):

Hard exclusive: $A^*(p_1) + B(p_2) \rightarrow C(p_3) + D(p_4)$
($2 \rightarrow 2$ minimum)

A “hard” scale: $\hat{s} = (p_1 + p_2)^2 \gg \Lambda_{\text{QCD}}^2$
 $\hat{t} = (p_1 - q_1)^2 \sim (p_1 - q_2)^2 = \hat{u}$



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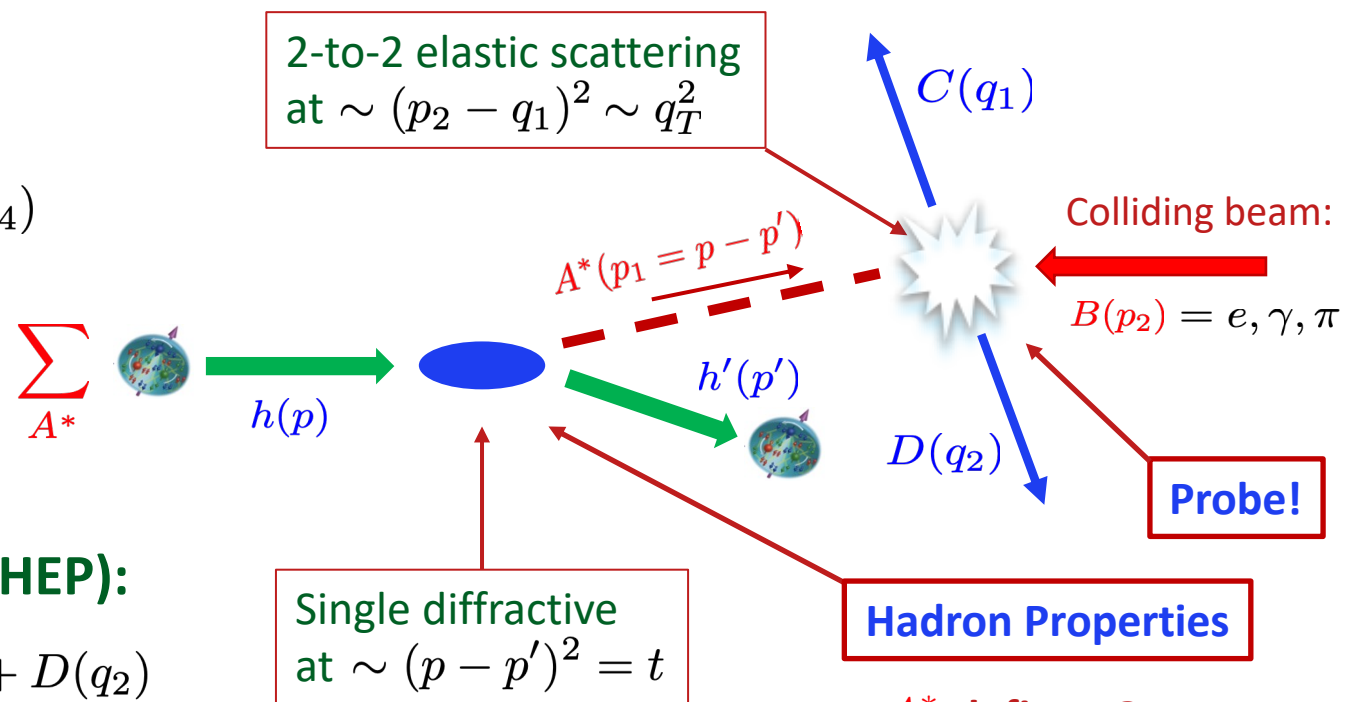
A “hard” scale: $\hat{s} = (p_1 + p_2)^2 \gg \Lambda_{\text{QCD}}^2$
 $\hat{t} = (p_1 - q_1)^2 \sim (p_1 - q_2)^2 = \hat{u}$

❑ Single diffractive hard exclusive process (SDHEP):

SDHEP Process: $h(p) + B(p_2) \rightarrow h'(p') + C(q_1) + D(q_2)$
(Need a minimum $2 \rightarrow 3$ SDHEPs)

Factorization: $Q^2 \sim \hat{s} \sim -\hat{t} \sim -\hat{u} \sim q_T^2 \gg -t$

A necessary condition!



How to Choose the Light-cone Vector “n” to Define GPDs?

□ Choose the z-axis of the hard collisions – a choice of frame:

Recall – Inclusive DIS: $h(p) + l(\ell) \rightarrow l'(\ell') + X$

Let p and ℓ define the **z**-axis of the collision & p along the **+z** direction and ℓ long the **-z** direction,

$$p^\mu = (E, \mathbf{0}_\perp, p_z) \rightarrow (p^+, p^\perp, p^-) \text{ with}$$

$$p^+ = (E + p_z)/\sqrt{2} \text{ and } p^- = (E - p_z)/\sqrt{2} = m^2/2p^+$$

➡ Light-cone vector: $n^\mu = (n^+, \mathbf{n}^\perp, n^-) = (0, \mathbf{0}^\perp, 1)$
along the direction of $l(\ell)$

How to Choose the Light-cone Vector “n” to Define GPDs?

□ Choose the z-axis of the hard collisions – a choice of frame:

Recall – Inclusive DIS: $h(p) + l(\ell) \rightarrow l'(\ell') + X$

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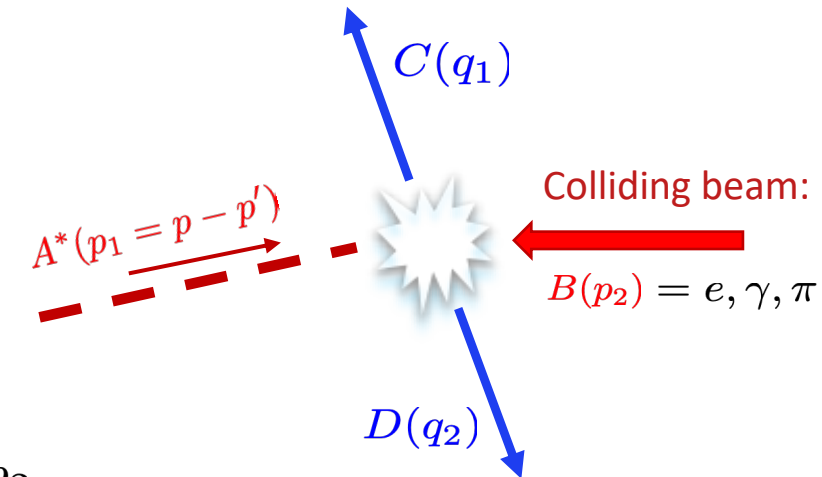
$$p^+ = (E + p_z)/\sqrt{2} \text{ and } p^- = (E - p_z)/\sqrt{2} = m^2/2p^+$$

➡ Light-cone vector: $n^\mu = (n^+, \mathbf{n}^\perp, n^-) = (0, \mathbf{0}^\perp, 1)$
along the direction of $l(\ell)$ to pick up large component of p

□ Choose the z-axis of the hard exclusive collision – SDHEP frame:

Let momentum of A^* , $p_1 = p - p'$ and the momentum of colliding beam, p_2 define the z-axis of the hard exclusive collision with p_1 along with +z direction

➡ Light-cone vector: $n^\mu = (n^+, \mathbf{n}^\perp, n^-) = (0, \mathbf{0}^\perp, 1)$
along the direction of $B(p_2)$ to pick up large component of $p_1 = p - p'$
 $p_1^+ = (p - p')^+ = 2\xi P^+ = \xi(p + p')^+ \text{ and } p_1^- = t/2p_1^+$

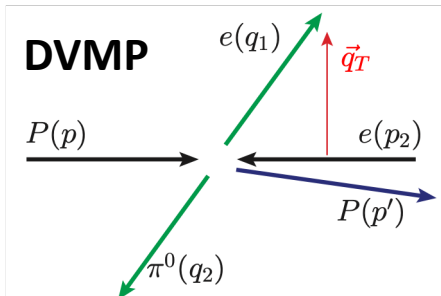
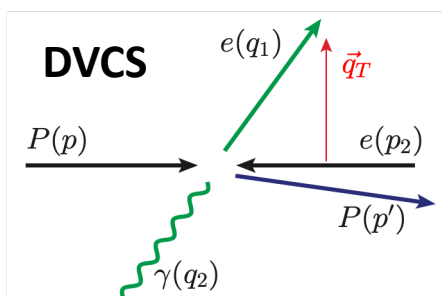


A^* defines GPDs
& what we are
probing!

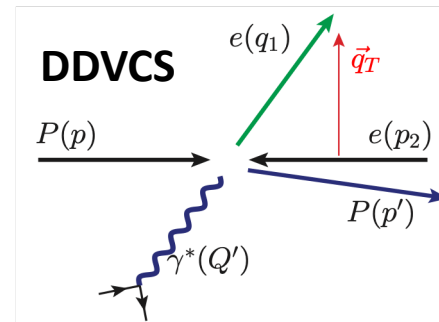
All Known Processes for Extracting GPDs – Fit the Kinematics of SDHEPs

□ Electro-production (JLab, EIC, ...)

Qiu & Yu, JHEP 08 (2022) 103
PRD 107 (2023) 014007
PRL 131 (2023) 161902

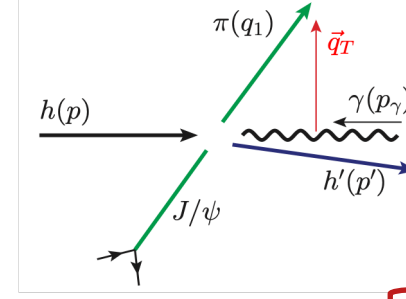
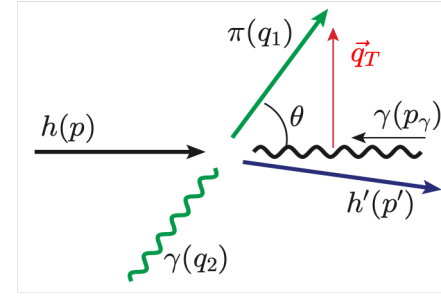
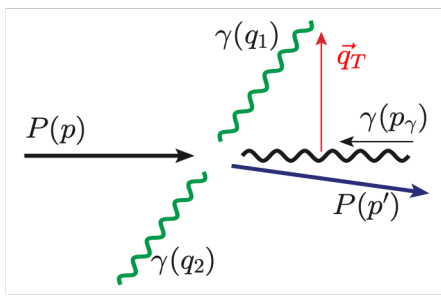
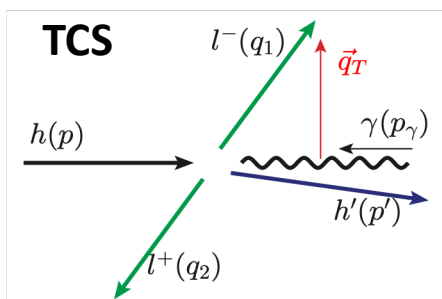


add
virtuality



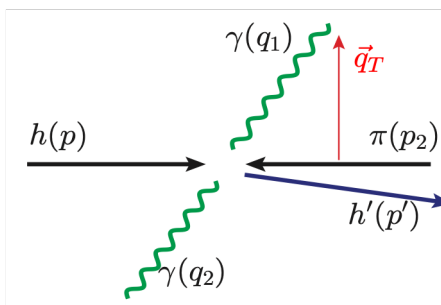
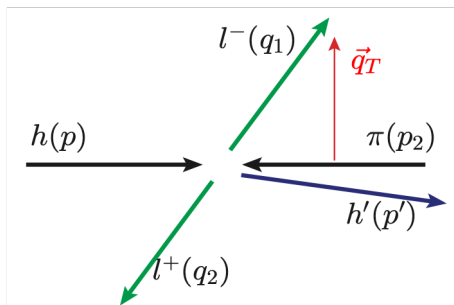
DHQP...

□ Photo-production (JLab, EIC, ...)



All of them are
2 → 3 exclusive
Processes !

□ Meso-production (AMBER, J-PARC, ...)



...



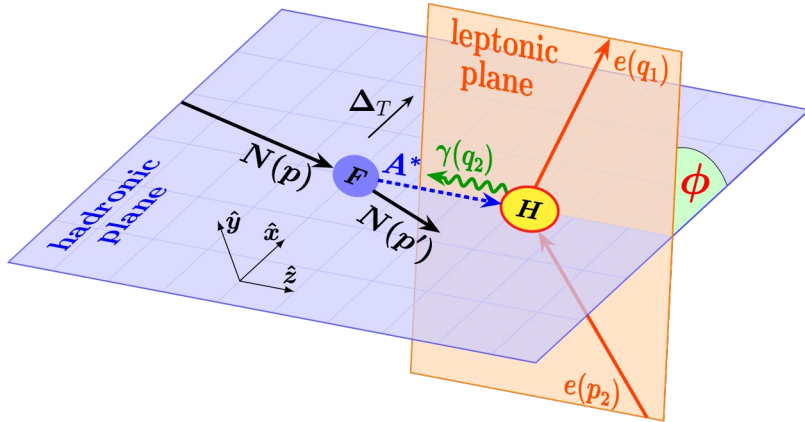
Unified frame for all
these observables

Same "n" along p_2

- Unified definition for all GPDs
- Ready for Global analysis of GPDs
- Easy to implement experimentally

SDHEP Frame vs. Breit Frame – Use DVCS as an Example

□ **Breit Frame** = γ^* - **Beam Frame**:



$$h(p) + e(p_2) \rightarrow h(p') + e(q_1) + \gamma(q_2)$$

$$e(p_2) \rightarrow e(q_1) + \gamma^*(q)$$

DVCS

$$h(p) + \gamma^*(q) \rightarrow h(p') + \gamma(q_2)$$

$$h(p) \rightarrow h(p') + \gamma^*(\Delta)$$

BH

$$e(p_2) + \gamma^*(\Delta) \rightarrow e(q_1) + \gamma(q_2)$$

$$q = p_2 - q_1$$

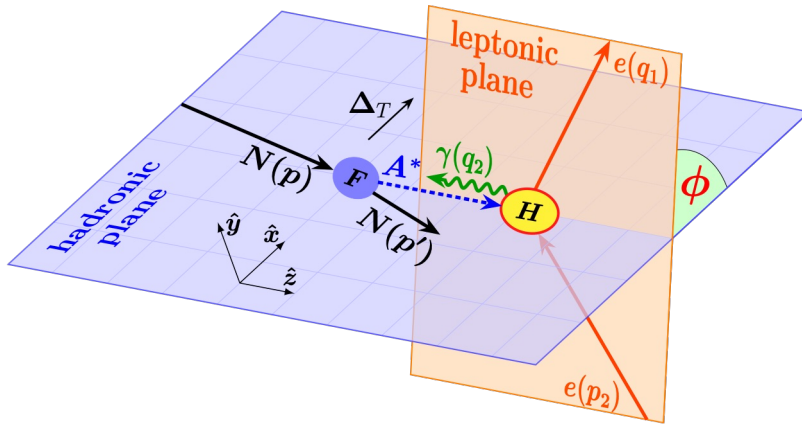
$$Q^2 \equiv -q^2 = -(p_2 - q_1)^2$$

Physics at **both** Q^2
and $t = (p - p')^2$

$$\Delta = p - p'$$

SDHEP Frame vs. Breit Frame – Use DVCS as an Example

□ **Breit Frame** = γ^* - Beam Frame:



$$h(p) + e(p_2) \rightarrow h(p') + e(q_1) + \gamma(q_2)$$

$$e(p_2) \rightarrow e(q_1) + \gamma^*(q)$$

DVCS

$$h(p) + \gamma^*(q) \rightarrow h(p') + \gamma(q_2)$$

$$h(p) \rightarrow h(p') + \gamma^*(\Delta)$$

BH

$$e(p_2) + \gamma^*(\Delta) \rightarrow e(q_1) + \gamma(q_2)$$

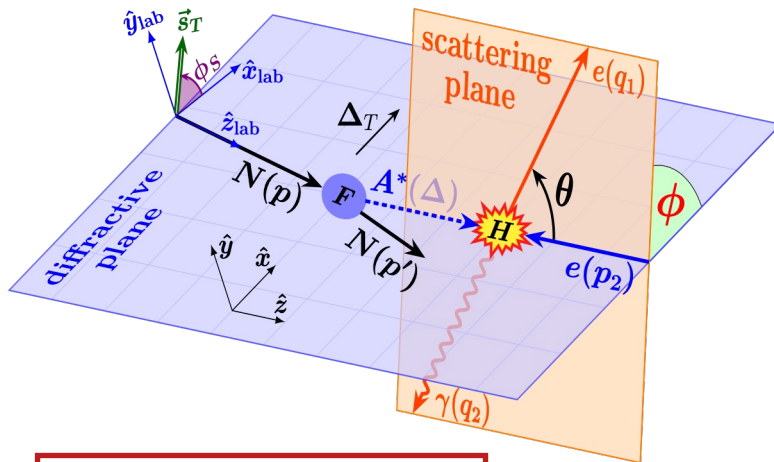
$$q = p_2 - q_1$$

$$Q^2 \equiv -q^2 = -(p_2 - q_1)^2$$

Physics at **both** Q^2
and $t = (p - p')^2$

$$\Delta = p - p'$$

□ **SDHEP Frame** = A^* - Beam Frame:



$$h(p) \rightarrow h(p') + A^*(\Delta)$$

SDHEP

Include both DVCS & BH subprocesses

$$e(p_2) + A^*(\Delta) \rightarrow e(q_1) + \gamma(q_2)$$

Physics at t

Physics at Q^2

Hadron plays
the role of
the lepton in
Breit frame

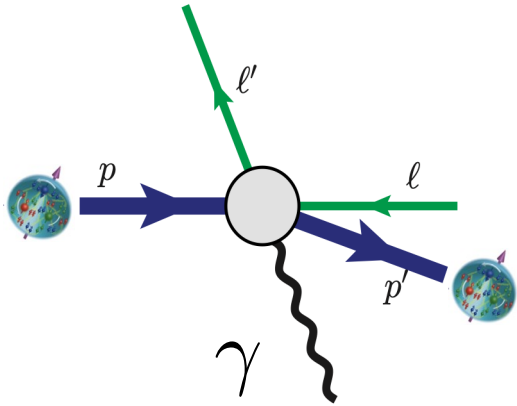
- Easy to implement
- A^* defines the GPDs
- Angular modulation to separate GPDs
- Same for all GPD finding processes!

ϕ -dependence
 $\Leftrightarrow$ quantum # of A^*

$$\gamma^*(q) \rightarrow A^*(\Delta)$$

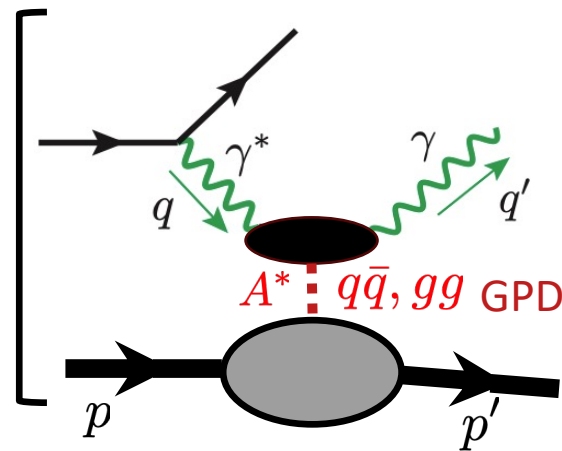
SDHEP Frame vs. Breit Frame – Use DVCS as an Example

□ Breit Frame = γ^* - Beam Frame:

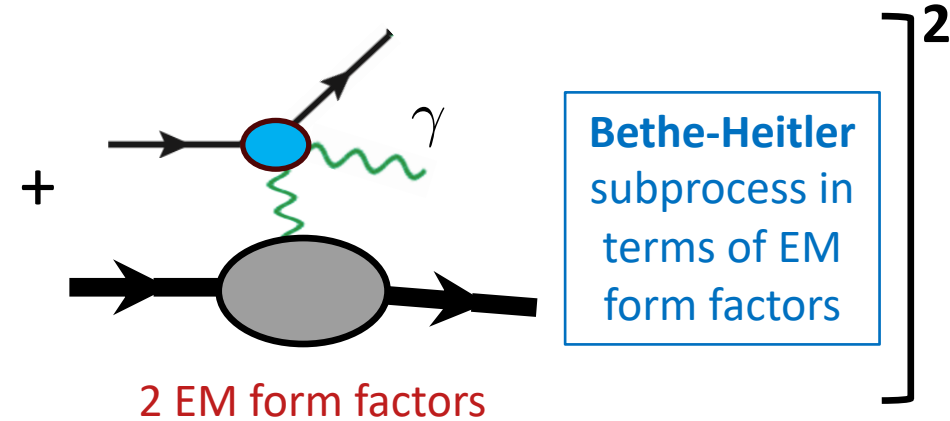


Breit frame is a natural extension from the treatment of DIS, SIDIS

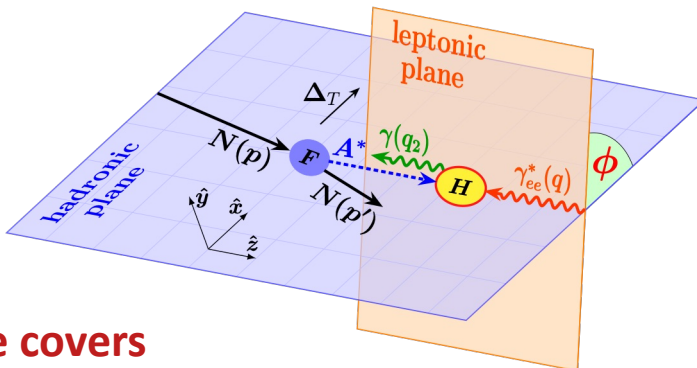
Angular distribution between lepton plane ($\ell \rightarrow \ell'$) and hadron plane ($p \rightarrow p'$) DOES not have a simple relation to GPDs to help their extraction, due to mixing of BH!



DVCS: in terms of factorized GPDs

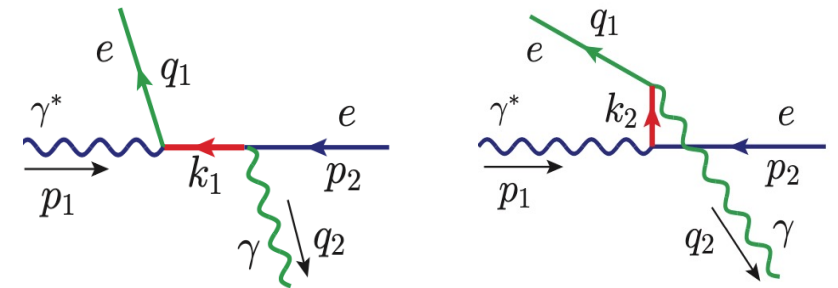


Bethe-Heitler subprocess in terms of EM form factors



Hadronic plane covers Both hard and soft physics

BH is not a "t"-channel process:



Propagators of k_1 & k_2 have ϕ -dependence!

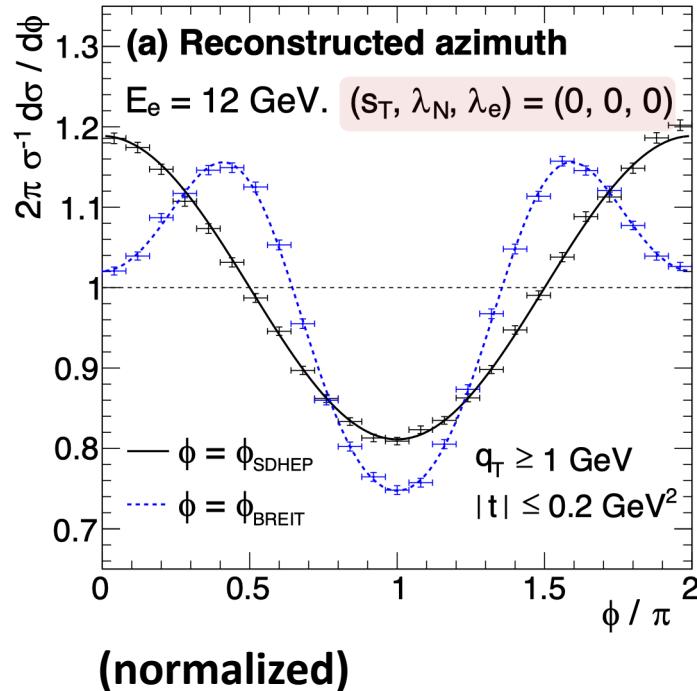
Simple Numerical Examples for Angular Distribution/Modulation

$$\frac{d\sigma}{dt d\xi d\phi_S d\cos\theta d\phi} = \frac{1}{(2\pi)^2} \frac{d\sigma^{\text{unpol}}}{dt d\xi d\cos\theta} \cdot \left[1 + \lambda_e \lambda_N A_{LL}^{\text{LP}} + \lambda_e s_T A_{TL}^{\text{LP}} \cos\phi_S \right.$$

Generate 10^6 events and reconstruct

$$\begin{aligned} &+ (A_{UU}^{\text{NLP}} + \lambda_e \lambda_N A_{LL}^{\text{NLP}}) \cos\phi + (\lambda_e A_{UL}^{\text{NLP}} + \lambda_N A_{LU}^{\text{NLP}}) \sin\phi \\ &+ s_T (A_{TU,1}^{\text{NLP}} \cos\phi_S \sin\phi + A_{TU,2}^{\text{NLP}} \sin\phi_S \cos\phi) \\ &+ \lambda_e s_T (A_{TL,1}^{\text{NLP}} \cos\phi_S \cos\phi + A_{TL,2}^{\text{NLP}} \sin\phi_S \sin\phi) \end{aligned} \Bigg]$$

unpolarized



SDHEP fit to $1.00 + 0.190 \cos\phi \Rightarrow \langle A_{UU}^{\text{NLP}} \rangle = 0.190$

Breit fit to $1.00 + 0.15 \cos\phi - 0.12 \cos 2\phi - 0.01 \cos 3\phi + 0.01 \cos 4\phi$

- No straightforward interpretation of the coefficients.
- Need to introduce more gears in GPD extraction.

[A.V. Belitsky et al., 2002]

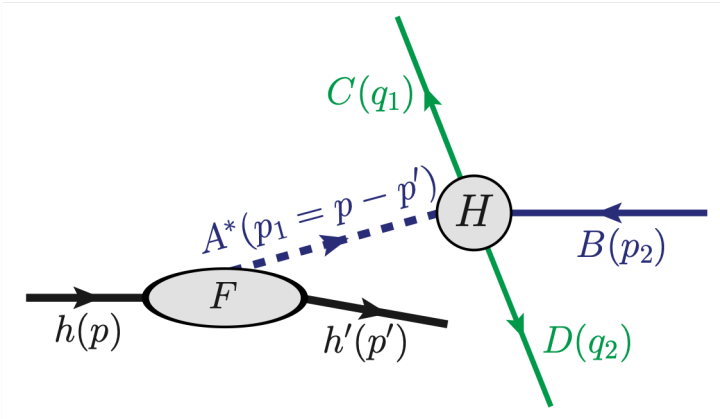
[B. Kriesten et al., 2020, 2022]

[Y. Guo et al., 2021, 2022]

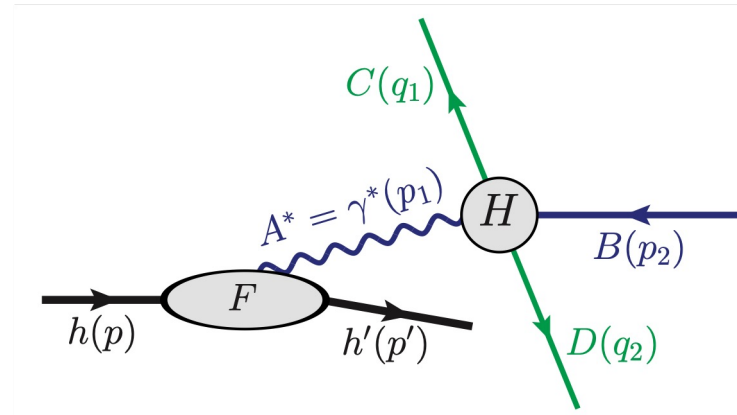
...

Nature of the Exchange State $A^* = \text{An Expansion in Powers of } \sqrt{-t}/q_T$

$\sum_{A^*}$

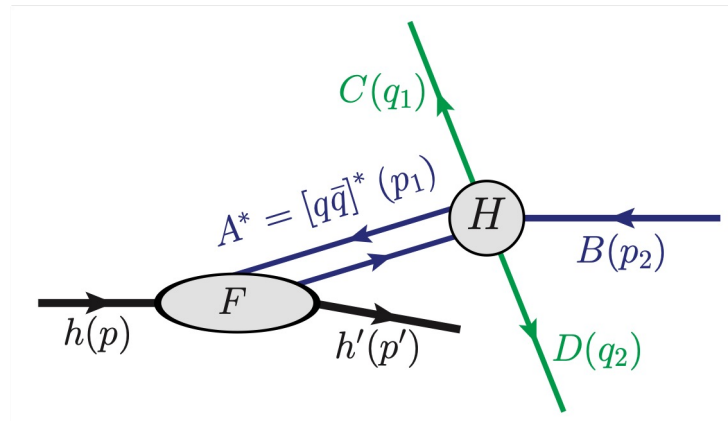


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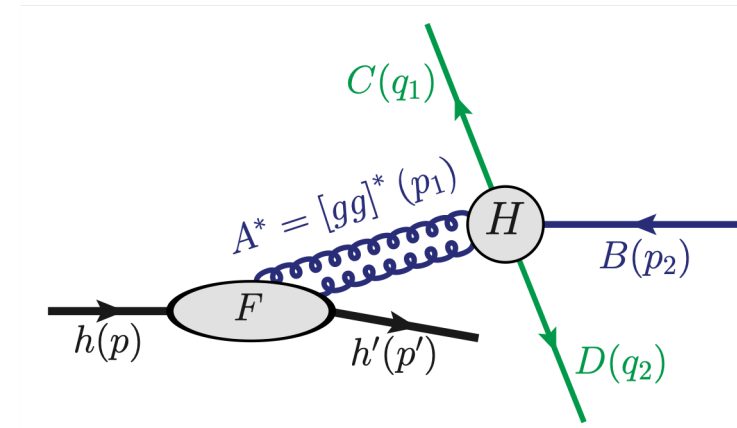


Leading Power in $\frac{\sqrt{-t}}{Q}$

+



+



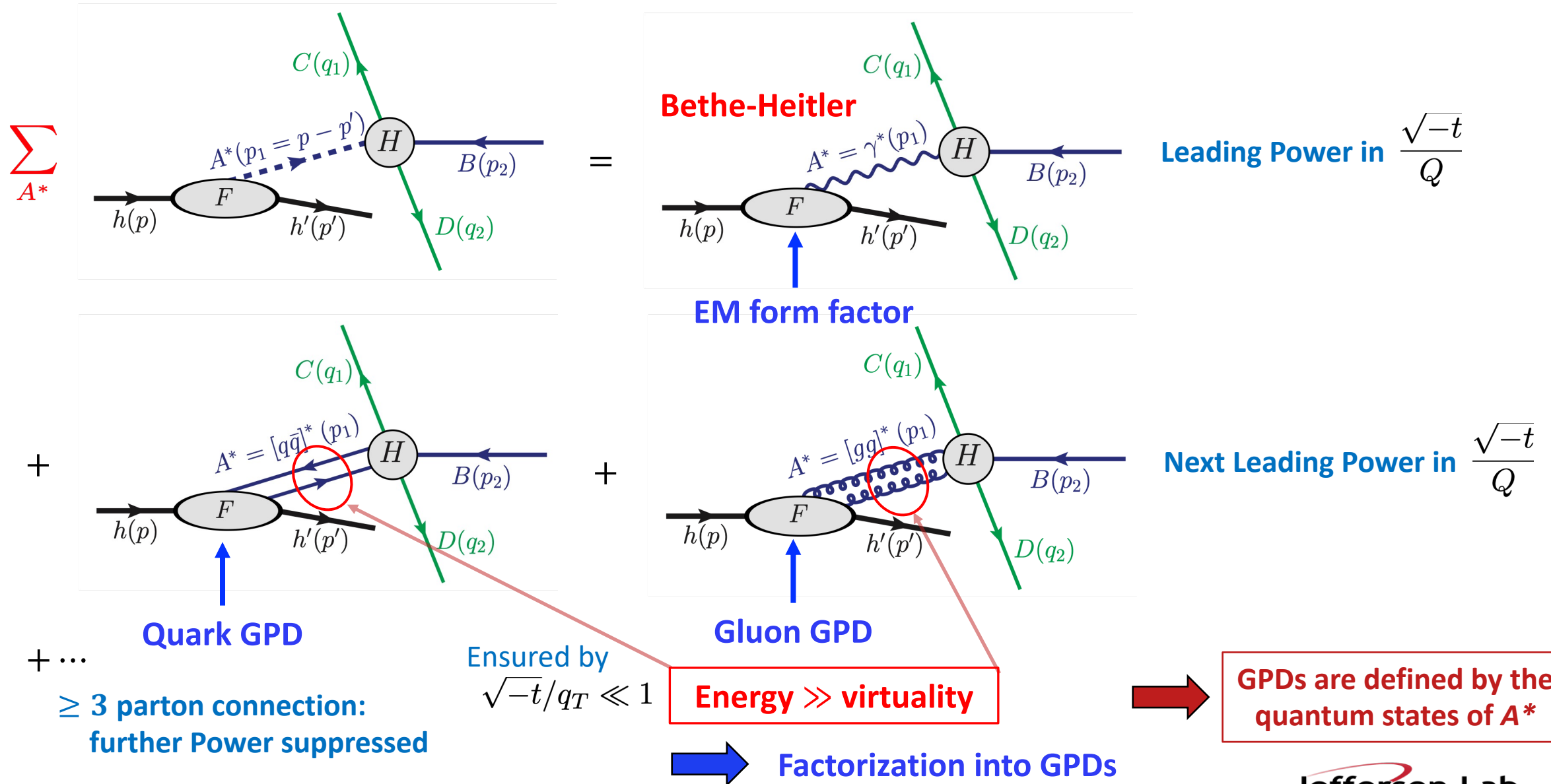
Next Leading Power in $\frac{\sqrt{-t}}{Q}$

+ ...

≥ 3 parton connection:
further Power suppressed

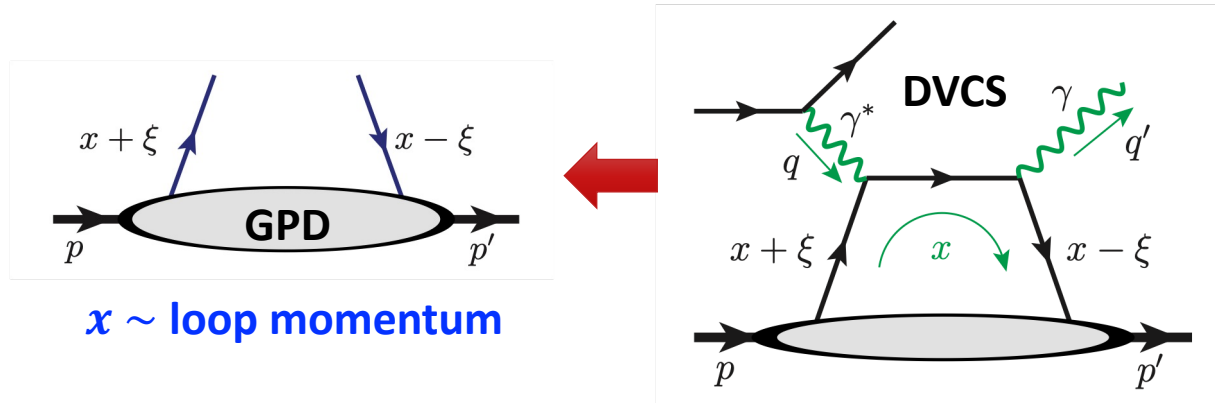
If A^* cannot be factorized in terms of partons, one might interpretate them as quasi-states, like pomeron, odderon, ..., from the diffractive scattering

Nature of the Exchange State A^* - An Expansion in Powers of $\sqrt{-t}/q_T$



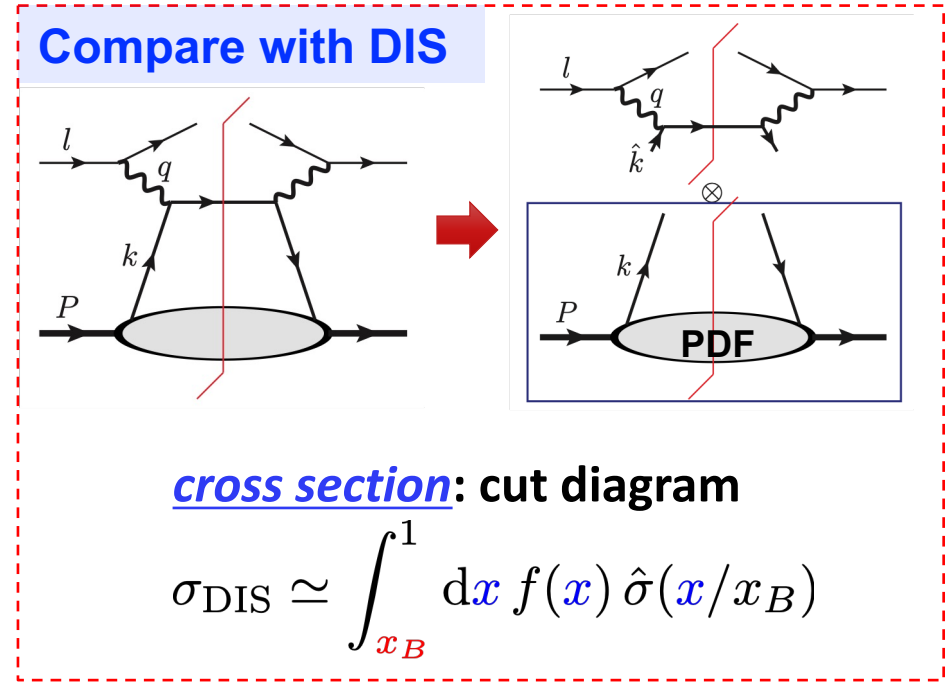
Why the GPD's x -Dependence is so *Difficult* to Measure?

□ **Amplitude** nature: exclusive processes



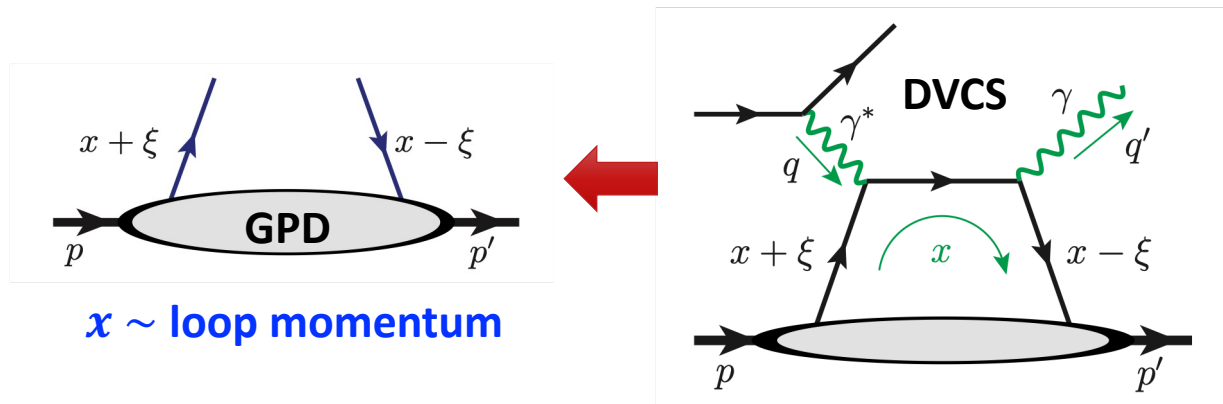
$$i\mathcal{M} \sim \int_{-1}^1 d\mathbf{x} F(\mathbf{x}, \xi, t) \cdot C(\mathbf{x}, \xi; Q/\mu)$$

Full range of x , including $x = 0$; $x = \pm\xi$



Why the GPD's x -Dependence is so *Difficult* to Measure?

□ Amplitude nature: exclusive processes



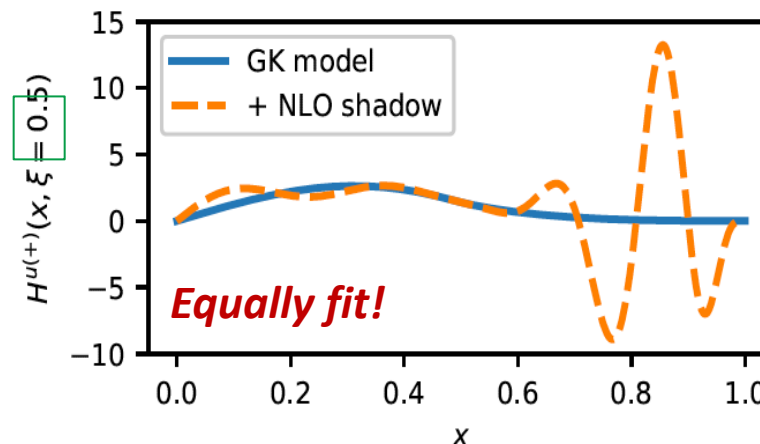
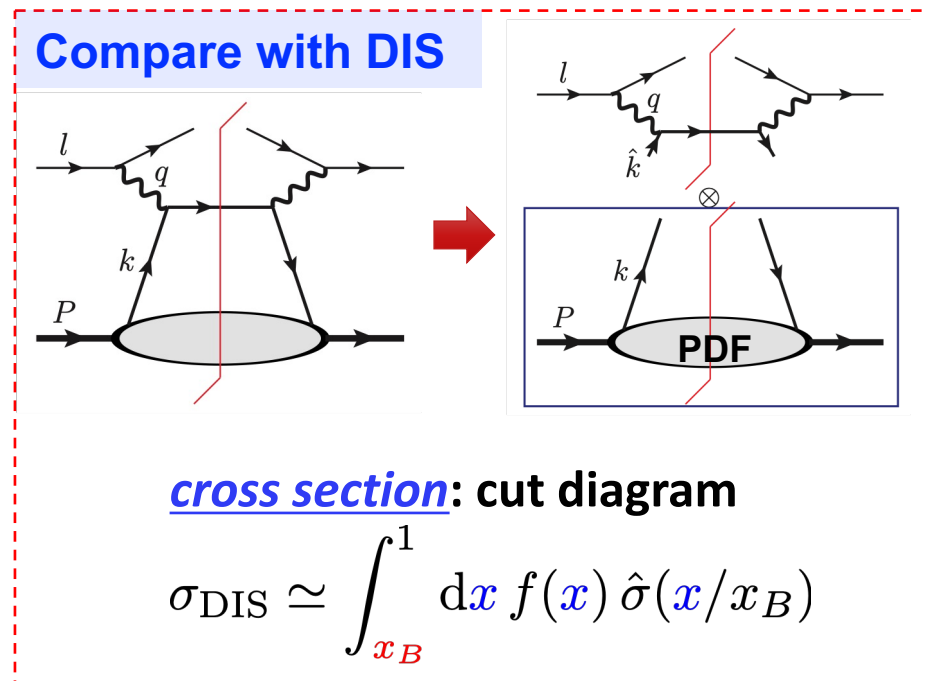
$$i\mathcal{M} \sim \int_{-1}^1 d\mathbf{x} F(\mathbf{x}, \xi, t) \cdot C(\mathbf{x}, \xi; Q/\mu)$$

Full range of x , including $x = 0$; $x = \pm\xi$

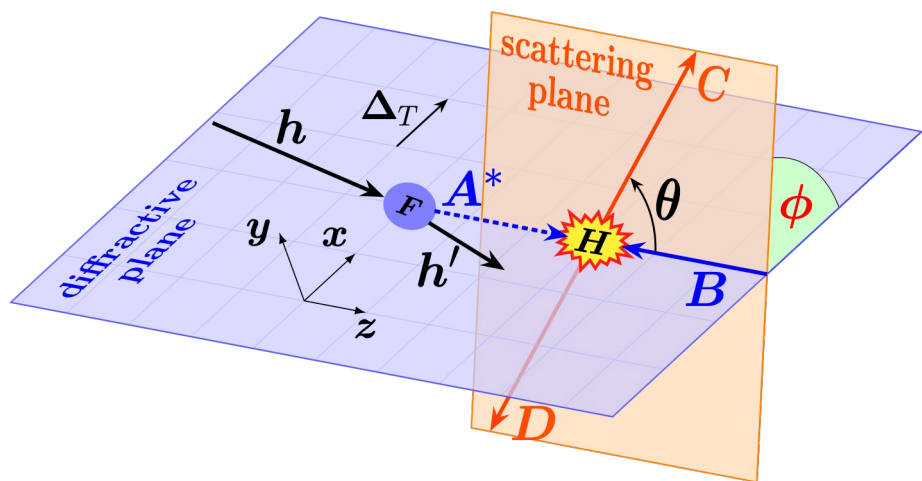
□ Sensitivity to x : comes from $C(x, \xi; Q/\mu)$

$$C(x, \xi; Q/\mu) = T(Q/\mu) \cdot G(x, \xi) \propto \frac{1}{x - \xi + i\epsilon} \dots$$

$$\Rightarrow i\mathcal{M} \propto \int_{-1}^1 d\mathbf{x} \frac{F(\mathbf{x}, \xi, t)}{x - \xi + i\epsilon} \equiv "F_0(\xi, t)" \quad "x\text{-moment}"$$



Where Does the x -sensitivity Come From?



□ x -sensitivity $\Leftrightarrow 2 \rightarrow 2$ hard scattering:

Kinematics:

1. $\hat{s} = 2 \xi s / (1 + \xi)$ $\leftarrow \xi$
2. θ or $q_T = (\sqrt{\hat{s}}/2) \sin \theta$ $\longleftrightarrow x$
3. ϕ $\leftarrow (A^* B)$ spin states

$$\mathcal{M}(Q, \phi) \simeq \sum_A e^{i(\lambda_A - \lambda_B)\phi} \cdot \int_{-1}^1 dx F_A(x) C_A(x; Q) \quad (Q = \theta \text{ or } q_T)$$

[suppressing t and ξ dependence]

■ **Moment-type sensitivity:** $C(x; Q) = G(x) \cdot T(Q)$ $\longrightarrow F_G = \int_{-1}^1 dx G(x) F(x, \xi, t)$ **Independent of Q**
Scaling for F_G

$\longrightarrow$ **Inversion problem:** shadow GPD $S_G = \int_{-1}^1 dx G(x) S(x, \xi) = 0$ [Bertone et al. PRD '21]

■ **Enhanced sensitivity:** $C(x; Q) \neq G(x) \cdot T(Q)$ $\longrightarrow d\sigma/dQ \sim |C(x; Q) \otimes_x F(x, \xi, t)|^2$

What Kind of Process Could be Sensitive to the x -Dependence?

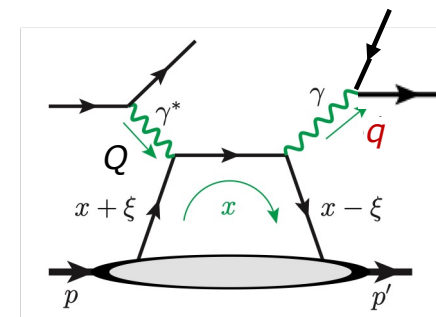
- ❑ Create an entanglement between the internal x and an externally measured variable?

$$i\mathcal{M} \propto \int_{-1}^1 d\mathbf{x} \frac{F(\mathbf{x}, \xi, t)}{x - \mathbf{x}_p(\xi, \mathbf{q}) + i\varepsilon}$$

Change external $\mathbf{q}$ to sample different part of $\mathbf{x}$.

- Double DVCS (two scales):

$$\mathbf{x}_p(\xi, \mathbf{q}) = \xi \left(\frac{1 - \mathbf{q}^2/Q^2}{1 + \mathbf{q}^2/Q^2} \right) \rightarrow \xi \text{ same as DVCS if } \mathbf{q} \rightarrow 0$$

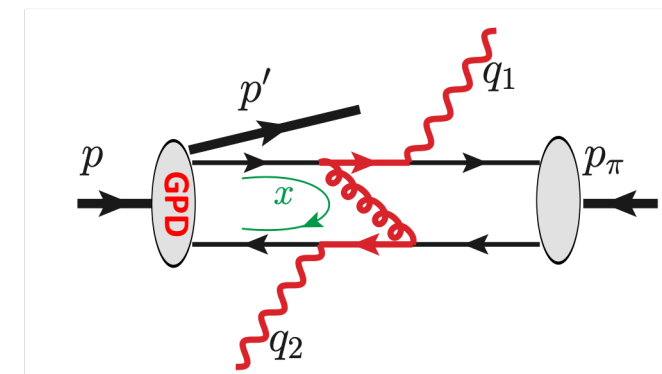


- Production of two back-to-back high p_T particles (say, two photons):

$$\pi^-(p_\pi) + P(p) \rightarrow \gamma(q_1) + \gamma(q_2) + N(p')$$

Hard scale: $q_T \gg \Lambda_{\text{QCD}}$ Soft scale: $t \sim \Lambda_{\text{QCD}}^2$

Qiu & Yu
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- Factorization:

$$\mathcal{M}(t, \xi, q_T) = \int_{-1}^1 d\mathbf{x} F(\mathbf{x}, \xi, t; \mu) \cdot C(\mathbf{x}, \xi; q_T/\mu) + \mathcal{O}(\Lambda_{\text{QCD}}/q_T) \quad [\text{suppressing pion DA factor}]$$



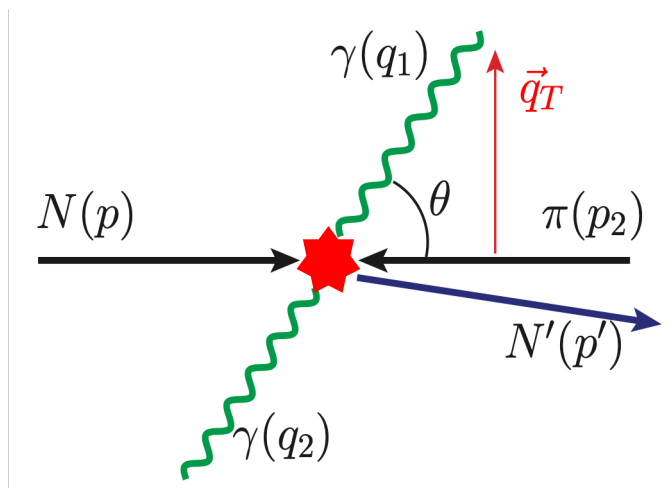
$$\frac{d\sigma}{dt d\xi dq_T} \sim |\mathcal{M}(t, \xi, q_T)|^2$$

q_T distribution is “conjugate” to x distribution

$$x \leftrightarrow q_T$$

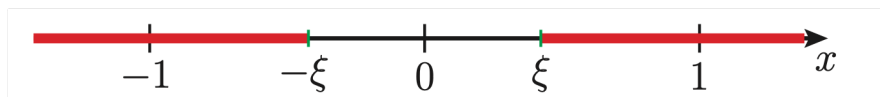
New Physical Processes to have Enhanced x -Sensitivity

□ **Diphoton process** $N\pi \rightarrow N'\gamma\gamma$: (1) $p\pi^- \rightarrow n\gamma\gamma$; (2) $n\pi^+ \rightarrow p\gamma\gamma$



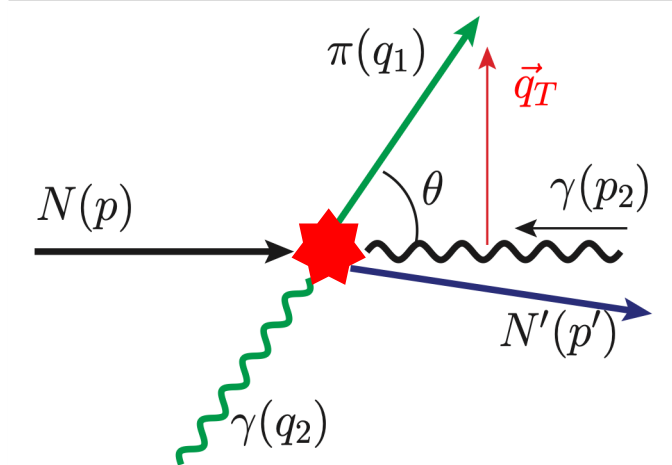
$i\mathcal{M}$ also contains $I(t, \xi; z, \theta) = \int_{-1}^1 \frac{dx F(x, \xi, t)}{x - \rho(z; \theta) + i\epsilon \operatorname{sgn} [\cos^2(\theta/2) - z]}$

$$\rho(z; \theta) = \xi \cdot \left[\frac{1 - z + \tan^2(\theta/2) z}{1 - z - \tan^2(\theta/2) z} \right] \in (-\infty, -\xi] \cup [\xi, \infty)$$



Covers the DGLAP region

□ **γ - π Pair Photoproduction:**



$i\mathcal{M}$ also contains: $I'(t, \xi; z, \theta) = \int_{-1}^1 \frac{dx F(x, \xi, t)}{x - \rho'(z; \theta) + i\epsilon}$

$$\rho'(z; \theta) = \xi \cdot \left[\frac{\cos^2(\theta/2) (1 - z) - z}{\cos^2(\theta/2) (1 - z) + z} \right] \in [-\xi, \xi]$$



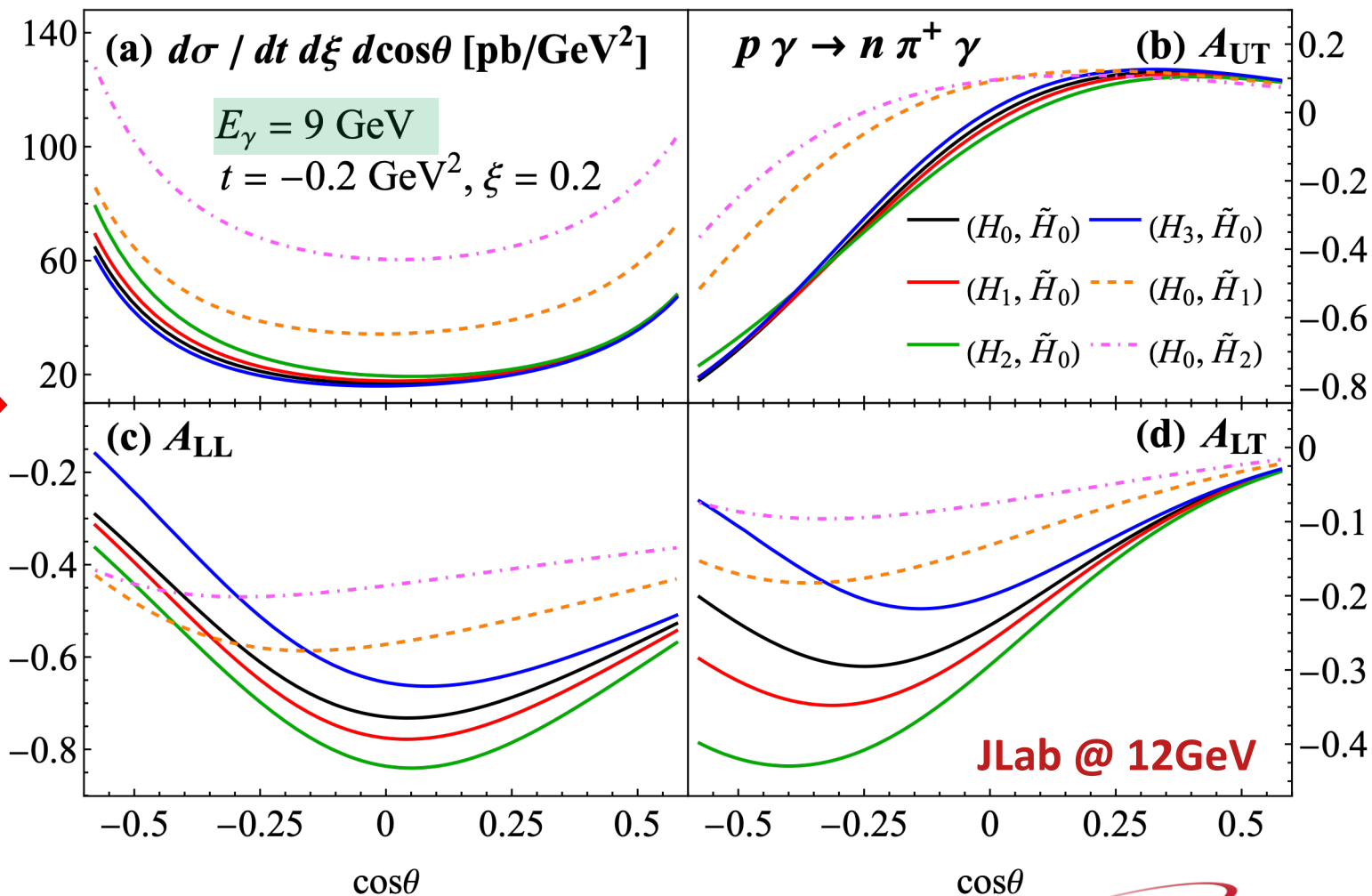
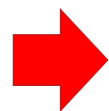
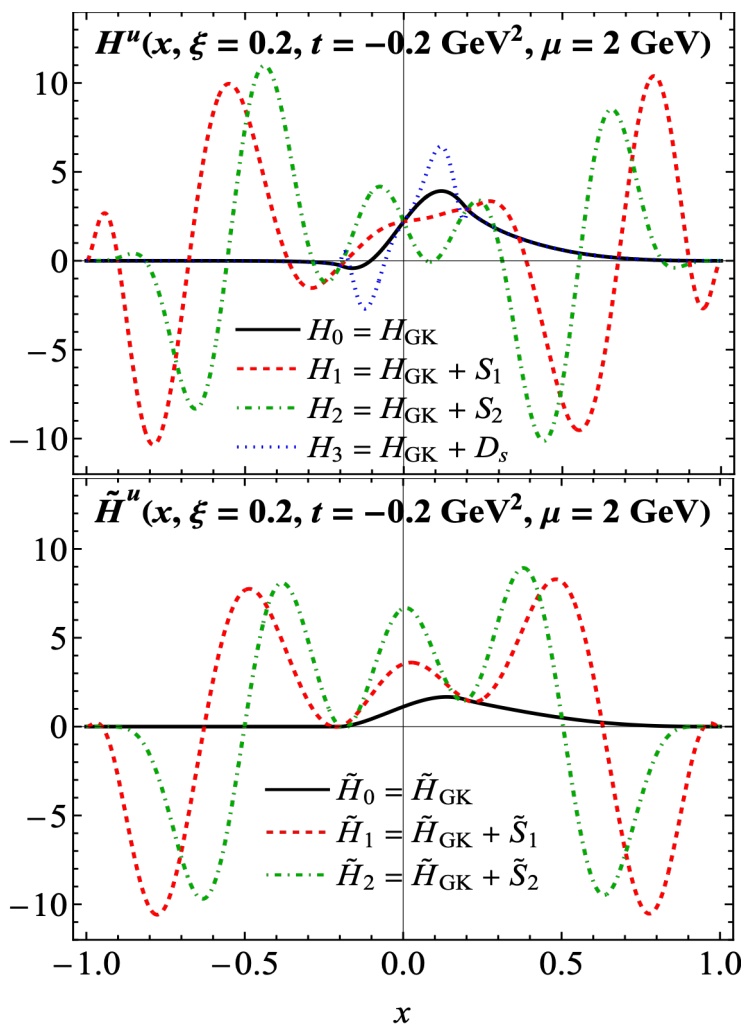
Covers the ERBL region

Test the Sensitivity on the GPDs' x -Dependence

GPD models = GK model + shadow GPDs

$$\int_{-1}^1 \frac{dx S(x, \xi)}{x - \xi \pm i\epsilon} = 0$$

Goloskokov, Kroll, '05, '07, '09
Bertone et al. '21
Moffat et al. '23

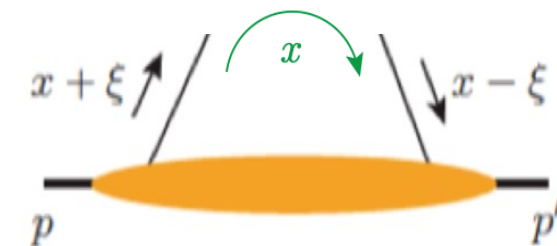


What Do We Learn from GPDs – Partonic ?

□ Impact parameter dependent parton density distribution:

$$q(x, b_{\perp}, Q) = \int d^2 \Delta_{\perp} e^{-i \Delta_{\perp} \cdot b_{\perp}} H_q(x, \xi = 0, t = -\Delta_{\perp}^2, Q)$$

➡ Quark density in $dx d^2 b_T$

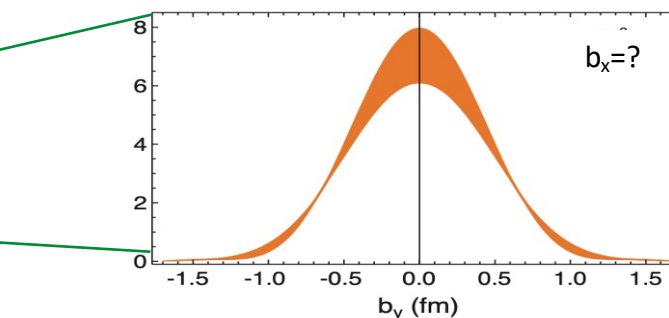
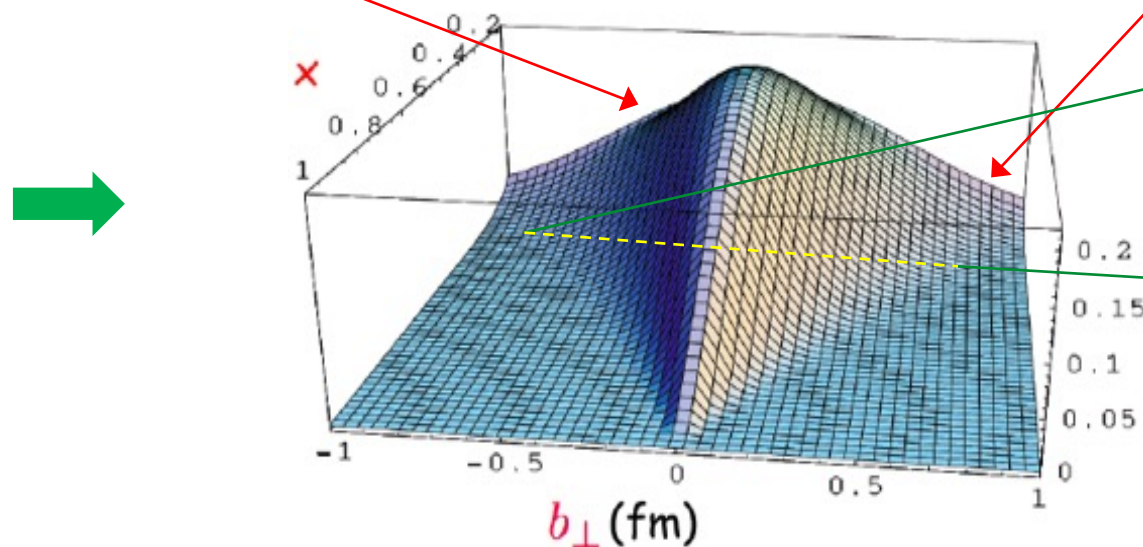


Measurement of p' fixes (t, ξ)
 x = momentum flow between the pair

How fast does glue density fall?

Tomographic image of hadron in slice of x

How far does glue density spread?



Modeled by
 M. Burkardt,
 PRD 2000

Slice in (x, Q)

$$\langle q_{\perp}^N \rangle \equiv \int db_{\perp} b_{\perp}^N q(x, b_{\perp}, Q)$$

➡ Proton radii, $r_q(x)$ & $r_g(x)$, from q & g spatial density

Need to know the x -dependence of GPDs to construct the proper radii !

- Should $r_q(x) > r_g(x)$, or vice versa?
- How do they compare with known radius (EM charge radius, mass radius, ...), & why?

GPDs – Partonic Origin of Hadronic Properties

QCD energy-momentum tensor:

$$T^{\mu\nu} = \sum_f \bar{\psi}_f \gamma^{(\mu} i D^{\nu)} \psi_f - F^{\mu\rho} F^{\nu}_{\rho} + \frac{1}{4} g^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta}$$

“Gravitational” form factors (GFFs) of the nucleon:

$$\langle p' | T^{\mu\nu} | p \rangle = \bar{u}(p') \left[\gamma^{(\mu} P^{\nu)} A(t) + \frac{i P^{(\mu} \sigma^{\nu)\rho} \Delta_{\rho}}{2M} B(t) + \frac{D(t)}{4M} (\Delta^{\mu} \Delta^{\nu} - g^{\mu\nu} \Delta^2) \right] u(p).$$

$T^{00}(x)$	$T^{01}(x)$	$T^{02}(x)$	$T^{03}(x)$
$T^{10}(x)$	$T^{11}(x)$	$T^{12}(x)$	$T^{13}(x)$
$T^{20}(x)$	$T^{21}(x)$	$T^{22}(x)$	$T^{23}(x)$
$T^{30}(x)$	$T^{31}(x)$	$T^{32}(x)$	$T^{33}(x)$

Pressures

Stress tensor

Connection to GPD moments:

$$\int_{-1}^1 dx x F_i(x, \xi, t) \propto \langle p' | T_i^{++} | p \rangle \propto \bar{u}(p') \left[\underbrace{(A_i + \xi^2 D_i)}_{\int_{-1}^1 dx x H_i(x, \xi, t)} \gamma^+ + \underbrace{(B_i - \xi^2 D_i)}_{\int_{-1}^1 dx x E_i(x, \xi, t)} \frac{i \sigma^{+\Delta}}{2m} \right] u(p)$$

Angular momentum sum rule:

$$J_i = \lim_{t \rightarrow 0} \int_{-1}^1 dx x [H_i(x, \xi, t) + E_i(x, \xi, t)]$$

$i = q, g$

3D tomography
Relation to GFFs
Angular Momentum

x-moments
of GPDs!
Input for
GFFs!

Related to pressure
& stress force inside h

Polyakov, schweitzer,
Inntt. J. Mod. Phys.
A33, 1830025 (2018)
Burkert, Elouadrhiri, Girod
Nature 557, 396 (2018)

Need to extract GPD and its x-dependence from DATA to construct the proper moments!

Summary and Outlook

- ❑ The emergence of hadrons and their properties
 - Is deeply rooted in color confinement in QCD
 - Understanding it fully is a challenge and a frontier in QCD
- ❑ TMDs are much broader than improved descriptions of PDFs:
 - Reveal quantum correlations that disappear when k_T is integrated out
 - Connect the active parton and its color & spin to the rest of the hadron
- ❑ GPDs connect parton tomography to hadron properties:
 - Reveal proton radius in many distributions, charge, quark/gluon density, mass, ...
 - Connect hadron properties to its partonic origins, ...
- ❑ GTMDs provide a framework for phase-space-like parton correlations
 - By retaining both transverse partonic momentum and hadron transverse momentum transfer – leading to a plan to search for answers to
Who we are, where we came from and where are we going?

A long but challenging & exciting way to go!

Thanks!

