

EFFECT OF TMD EVOLUTION ON SIVERS ASYMMETRY AT EIC

Asmita Mukherjee

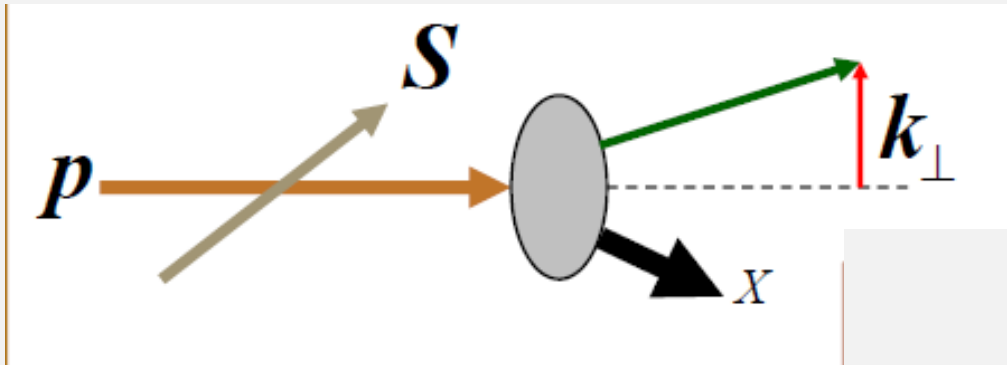
Indian Institute of Technology Bombay



In collaboration with Arghya Jana, Raj Kishore and Sangem Rajesh

NFQCD, YITP Kyoto, October 2026

Sivers Function TMD



$$f_{q/p,S}(x, k_{\perp}) = f_{q/p}(x, k_{\perp}) + \frac{1}{2} \Delta^N f_{q/p\uparrow}(x, k_{\perp}) S \cdot (\hat{p} \times \hat{k}_{\perp})$$

$$= f_{q/p}(x, k_{\perp}) - \frac{k_{\perp}}{M} f_{1T}^q(x, k_{\perp}) S \cdot (\hat{p} \times \hat{k}_{\perp})$$

Sivers function TMD (D. Sivers. (1990)) is related to the probability to find an unpolarized quark inside a transversely polarized nucleon

It includes the correlation between the quark intrinsic transverse momentum and the transverse spin of the proton

Gives the distribution of unpolarized quarks/ gluons in a transversely polarized nucleon, which is not left-right symmetric with respect to the plane formed by the transverse momentum and spin of the nucleon.

In some models it is related to the orbital angular momentum of the quarks

M. Burkardt, Nucl. Phys.A735, 185 (2004)

T-odd function ; combines with another T-odd effect to give an observable asymmetry

NON-ZERO SIVERS ASYMMETRY

The Sivers function introduces an asymmetry, for example, in the azimuthal angle of the observed final state hadron in semi-inclusive deep inelastic scattering (SIDIS) and in the azimuthal angle correlations of the lepton pair in Drell-Yan process or back-to-back jets in pp collision, called the Sivers asymmetry.

The first transverse moment of the Sivers function is related to the twist-three Qiu-Sterman function.

Qiu and Sterman, PRL (1991)

Experimental information on a nonzero Sivers function for quarks was first obtained from HERMES and COMPASS

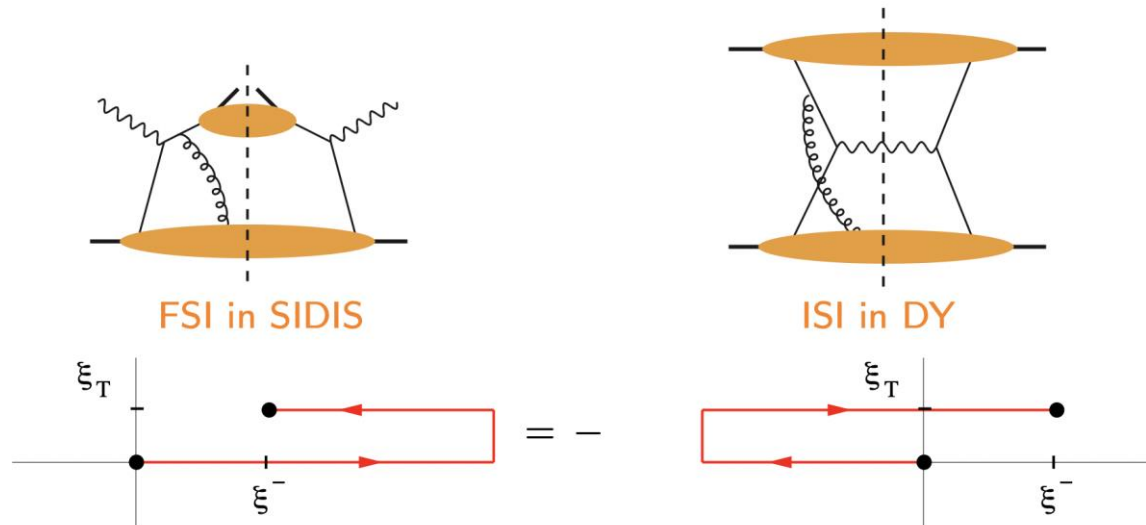
HERMES Collaboration, PRL94, 012002 (2005).

COMPASS Collaboration, PLBB 717, 383 (2012)

Parametrization of the Sivers function incorporating the TMD evolution has been obtained

M. G. Echevarria, A. Idilbi, Z.-B. Kang, and I. Vitev, PRD 89, 074013 (2014).

SIVERS FUNCTION: GAUGE LINK



$$f_{1T}^{\perp [DY]}(x, k_{\perp}^2) = -f_{1T}^{\perp [SIDIS]}(x, k_{\perp}^2)$$

However, more complex processes have complex gauge link structure, and factorization is not always guaranteed

Sivers function is a T-odd object and initial and final state interactions play an important role in Sivers asymmetry. They are resummed into the gauge link or Wilson line in the operator definition of the Sivers function that is needed for color gauge invariance.

Sivers function in Drell-Yan process is same in magnitude but opposite in sign compared to the Sivers function probed in semi-inclusive DIS

Collins, PLB (2002); Boer, Mulders, Pijlman, Nucl. Phys. B (2003)

GLUON SIVERS FUNCTION

GLUONS	<i>unpolarized</i>	<i>circular</i>	<i>linear</i>
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	$h_{1T}^g, h_{1T}^{\perp g}$

Very little is known about GSF apart from a positivity bound

P.J. Mulders and J. Rodrigues,
Phys. Rev. D 63, 094021 (2001).

Gluon TMDs need two gauge links for color gauge invariance unlike quark TMDs where there is one gauge link in the operator

Depending on the gauge link in the operator structure there can be two different gluon Sivers function, f-type and d-type

Bomhof and Mulders, JHEP 02, 029 (2007),
Buffing, AM, Mulders, PRD 88, 054027 (2013)

Burkardt's sum rule, which states that the total transverse momentum of all quarks and gluon in a transversely polarized proton is zero, still leaves some room for GSF (30 %), moreover d type GSF is not constrained by it.

M. Burkardt, Phys. Rev. D 69, 091501 (2004)

GLUON SIVERS FUNCTION IN J/ψ PRODUCTION PROCESSES

Semi-inclusive J/ψ production in eP collision is a good channel to probe gluon TMDs including gluon Sivers function, only f type gluon TMD contributes

For low transverse momentum region, TMD factorization is expected to hold and for large transverse momentum collinear factorization is applicable. In the intermediate region, results from these two formalisms should match

TMD factorized description of the process needs smearing effects to be taken into account in the form of TMD shape functions. The perturbative tail of the shape function can be obtained through a matching procedure.

M. G. Echevarria, JHEP (2019), Boer et al, JHEP (2023)

Also gluon TMDs can be probed in back-to-back production of J/ψ and photon/jet/pion, TMD factorization is expected to be valid. The small scale is provided by the transverse momentum of the pair. By varying the invariant mass of the pair scale evolution of the TMDs can be studied

Gluon Sivers function in pp collision at RHIC

U. D'Alesio, F. Murgia, and C. Pisano, Journal of High Energy Physics 2015, 1 (2015); U. D'Alesio, C. Flore, F. Murgia, C. Pisano, and P. Taels, Physical Review D 99, 036013 (2019)

Gluon Sivers function in di-jet production at EIC

Echevarria, Garcia, Scimemi; e-Print: [2603.00375](#) [hep-ph]

PRODUCTION OF J/ψ IN NRQCD

In NRQCD the heavy quark pair is produced in the hard process either in color octet or in color singlet configuration

Then they hadronize to form a color singlet quarkonium state of given quantum numbers through soft gluon emission

Hard process is calculated perturbatively and soft process is given in terms of long distance matrix elements (LDMEs) that are determined from data

The LDMEs are categorized by performing an expansion in terms of the relative velocity of the heavy quark v in the limit $v \ll 1$

The theoretical predictions are arranged as double expansions in terms of v as well as α_s .

C. E. Carlson and R. Suaya, Phys. Rev. D 14, 3115 (1976).
E. L. Berger and D. L. Jones, Phys. Rev. D 23, 1521 (1981).
R. Baier and R. Ruckl, Phys. Lett. B 102B, 364 (1981).
R. Baier and R. Ruckl, Nucl. Phys. B201, 1 (1982).
E. Braaten and S. Fleming, Phys. Rev. Lett. 74, 3327 (1995).
P. L. Cho and A. K. Leibovich, Phys. Rev. D 53, 150 (1996).

PRODUCTION OF J/ψ IN NRQCD

J/ψ is a bound state of charm quark and anti-quark ($Q\bar{Q}$)

Long distance matrix elements (LDMEs) : Describes hadronization of $Q\bar{Q}[n]$ states into final quarkonium state

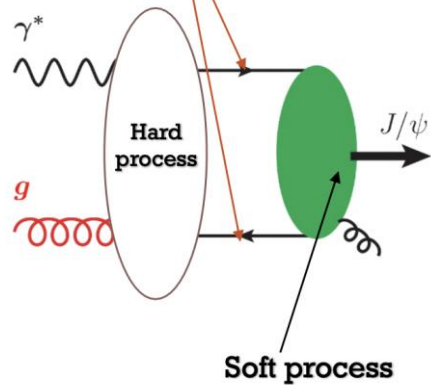
NRQCD factorization

$$d\sigma^{ab \rightarrow J/\psi} = \sum_n d\hat{\sigma}[ab \rightarrow c\bar{c}(n)] \langle 0 | \mathcal{O}_n^{J/\psi} | 0 \rangle$$

Perturbative short distance coefficient

Subprocess cross section for formation of heavy quark pair in particular color, angular momentum and spin state “n” : $^{2S+1}L_J$, calculated by perturbative QCD

$Q\bar{Q}$ pair with $[^{2S+1}L_J^{(1,8)}]$ quantum number



G. T. Bodwin et al, PRD51 (1995),
Lepage 95

Sivers Asymmetry in J/ψ Electroproduction

$$e(l) + p \uparrow(P) \rightarrow e(l') + J/\psi(P_h) + X,$$

Proton and virtual photon moving along $-z$ and $+z$ direction

$$P = n_- + \frac{M_p^2}{2} n_+ \approx n_- \quad \text{and} \quad q = -x_B n_- + \frac{Q^2}{2x_B} n_+ \\ \approx -x_B P + (P \cdot q) n_+,$$

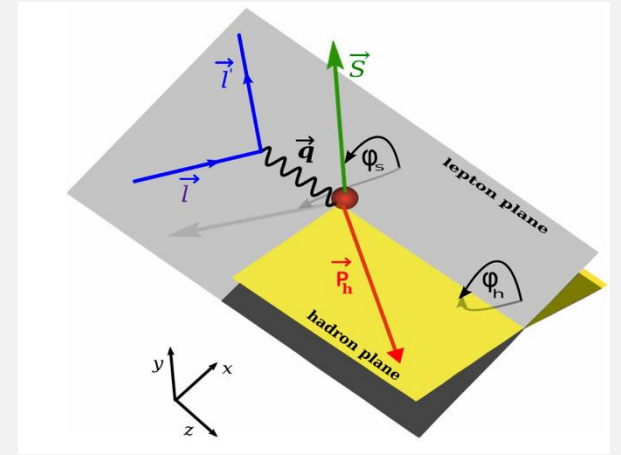
Kinematical variables are

$$y = \frac{P \cdot q}{P \cdot l}, \quad x_B = \frac{Q^2}{2P \cdot q}, \quad z = \frac{P \cdot P_h}{P \cdot q}$$

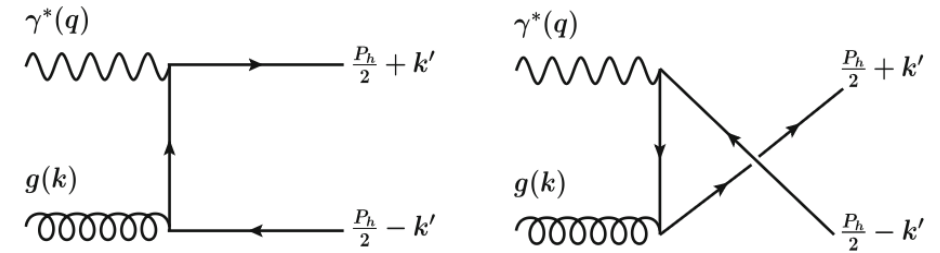
At LO, dominant subprocess is virtual photon-gluon fusion.
Assume TMD factorization

Cross section is written as

$$d\sigma = \frac{1}{2s} \frac{d^3 l'}{(2\pi)^3 2E_l'} \frac{d^3 P_h}{(2\pi)^3 2E_h} \int dx d^2 k_\perp (2\pi)^4 \delta^4(q + k - P_h) \\ \times \frac{1}{Q^4} L^{\nu\nu'}(l, q) \Phi_g^{\mu\mu'}(x, k_\perp) \mathcal{M}_{\mu\nu}^{\gamma^* g \rightarrow J/\psi} \mathcal{M}_{\mu'\nu'}^{* \gamma^* g \rightarrow J/\psi}.$$



$$n_- = P \quad \text{and} \quad n_+ = n = (q + x_B P) / P \cdot q$$



SIVERS ASYMMETRY IN J/ψ ELECTROPRODUCTION

$$\Phi_g^{\mu\mu'}(x, \mathbf{k}_\perp) = \frac{1}{2x} \left\{ -g_T^{\mu\mu'} f_1^g(x, \mathbf{k}_\perp^2) + \left(\frac{k_\perp^\mu k_\perp^{\mu'}}{M_p^2} + g_T^{\mu\mu'} \frac{\mathbf{k}_\perp^2}{2M_p^2} \right) h_1^{\perp g}(x, \mathbf{k}_\perp^2) \right\},$$

$$\Phi_g^{T\mu\mu'}(x, \mathbf{k}_\perp) = -\frac{1}{2x} g_T^{\mu\mu'} \frac{\epsilon_T^{\rho\sigma} k_{\perp\rho} S_{T\sigma}}{M_p} f_{1T}^{\perp g}(x, \mathbf{k}_\perp^2),$$

Unpolarized proton

Term containing Sivers function when proton is transversely polarized

Construct the weighted Sivers asymmetry, with the weight factor $\sin(\phi_h - \phi_S)$

Numerator dependent on the gluon Sivers function

Denominator contains unpolarized cross section

$$\begin{aligned} & \frac{d\sigma^\uparrow}{dy dx_B d^2\mathbf{P}_{hT}} - \frac{d\sigma^\downarrow}{dy dx_B d^2\mathbf{P}_{hT}} \\ &= \frac{\alpha}{8sx Q^4} [\mathcal{A}_0 + \mathcal{A}_1 \cos \phi_h] \Delta^N f_{g/p^\uparrow}(x, \mathbf{P}_{hT}). \end{aligned}$$

$$\begin{aligned} & \frac{d\sigma^\uparrow}{dy dx_B d^2\mathbf{P}_{hT}} + \frac{d\sigma^\downarrow}{dy dx_B d^2\mathbf{P}_{hT}} \\ &= \frac{2\alpha}{8sx Q^4} [\mathcal{A}_0 + \mathcal{A}_1 \cos \phi_h] f_{g/p}(x, \mathbf{P}_{hT}^2), \end{aligned}$$

Gluon Sivers function in Trento Convention

$$\Delta^N f_{g/p^\uparrow}(x, \mathbf{P}_{hT}, Q_f) = -2 f_{1T}^{\perp g}(x, \mathbf{P}_{hT}, Q_f) \frac{(\hat{\mathbf{P}} \times \mathbf{P}_{hT}) \cdot \mathbf{S}}{M_p}.$$

SIVERS ASYMMETRY IN J/ψ ELECTROPRODUCTION

J/ψ production is calculated in NRQCD based color octet framework. Color octet states $^1S_0$, $^3P_0$, $^3P_1$ and $^3P_2$ are taken into account

Sivers asymmetry depends on the LDME sets chosen, also on the parametrization of the Sivers function

We use a Gaussian parametrization of the gluon Sivers function

$$\Delta^N f_{g/p\uparrow}(x, k_\perp) = 2\mathcal{N}_g(x) f_{g/p}(x, \mu) h(k_\perp) \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle};$$

$$\mathcal{N}_g(x) = N_g x^\alpha (1-x)^\beta \frac{(\alpha + \beta)^{(\alpha + \beta)}}{\alpha^\alpha \beta^\beta}.$$

U. D'Alesio, F. Murgia, C. Pisano, JHEP **09**, 119 (2015).

M. Anselmino, M. Boglione, U. D'Alesio, F. Murgia, A. Prokudin, JHEP **04**, 046 (2017).

SIDIS1/SIDIS2

Best fit parameters are obtained from pion production data at RHIC

$$\begin{aligned} \mathcal{A}_0 = & \left[1 + (1-y)^2 \right] \frac{\mathcal{N} Q^2}{y^2 M} \left\{ \langle 0 | \mathcal{O}_8^{J/\psi} (^1S_0) | 0 \rangle \right. \\ & + \frac{4}{3M^2} \frac{(3M^2 + Q^2)^2}{(M^2 + Q^2)^2} \langle 0 | \mathcal{O}_8^{J/\psi} (^3P_0) | 0 \rangle \\ & + \frac{8Q^2}{3M^2(M^2 + Q^2)^2} \left(\frac{4M^2(1-y)}{1 + (1-y)^2} \right. \\ & \left. \left. + Q^2 \right) \langle 0 | \mathcal{O}_8^{J/\psi} (^3P_1) | 0 \rangle \right. \\ & \left. + \frac{8}{15M^2(M^2 + Q^2)^2} \left(6M^4 + Q^4 \right. \right. \\ & \left. \left. + 12M^2 Q^2 \frac{1-y}{1 + (1-y)^2} \right) \langle 0 | \mathcal{O}_8^{J/\psi} (^3P_2) | 0 \rangle \right\}, \\ \mathcal{A}_1 = & (2-y) \sqrt{1-y} \frac{4\mathcal{N} Q^3}{y^2 M} \left\{ - \langle 0 | \mathcal{O}_8^{J/\psi} (^1S_0) | 0 \rangle \right. \\ & - \frac{2}{3M^2 Q^2} \frac{(3M^2 + Q^2)^2}{M^2 + Q^2} \langle 0 | \mathcal{O}_8^{J/\psi} (^3P_0) | 0 \rangle \\ & - \frac{8Q^2}{3M^2(M^2 + Q^2)^2} \langle 0 | \mathcal{O}_8^{J/\psi} (^3P_1) | 0 \rangle \\ & \left. - \frac{4}{15M^2} \frac{7M^2 + Q^2}{M^2 + Q^2} \langle 0 | \mathcal{O}_8^{J/\psi} (^3P_2) | 0 \rangle \right\} \frac{k_\perp}{M^2 + Q^2} \end{aligned}$$

SIVERS ASYMMETRY IN J/ψ ELECTROPRODUCTION

- (a) $\mathcal{N}_g(x) = (N_u(x) + N_d(x))/2$,
 (b) $\mathcal{N}_g(x) = N_d(x)$.

BV-a

D. Boer, W. Vogelsang, Phys. Rev. D **69**, 094025 (2004).

BV-b

$$f_{g/p}(x, k_{\perp}^2) = f_{g/p}(x, \mu) \frac{1}{\pi \langle k_{\perp}^2 \rangle} e^{-k_{\perp}^2 / \langle k_{\perp}^2 \rangle}.$$

In order to estimate the asymmetry,
 the best fit parameters of the Sivers function
 corresponding to u and d quark was used

Best fit parameters

Evolution	a	N_a	α	β	ρ	$M_1^2 \text{ GeV}^2$	$\langle k_{\perp}^2 \rangle \text{ GeV}^2$	Notation
DGLAP	g [49]	0.65	2.8	2.8	0.687		0.25	SIDIS1
	g [49]	0.05	0.8	1.4	0.576		0.25	SIDIS2
	u [67]	0.18	1.0	6.6		0.8	0.57	BV-a
	d [67]	-0.52	1.9	10.0		0.8	0.57	BV-b
TMD	u [70]	0.106	1.051	4.857			0.38	TMD-a
	d [70]	-0.163	1.552	4.857			0.38	TMD-b

TMD EVOLUTION

TMDs depend on both a renormalization scale (μ) and a rapidity scale (ζ). These are introduced to regulate the ultraviolet and rapidity divergences respectively

Evolution of TMDs wrt these scales are called TMD evolution

TMD evolution is usually done in impact parameter (b_T) space which is obtained by Fourier transforming the TMDs

There are several approaches in the literature to perform TMD evolution, these differ in the way perturbative and non-perturbative Sudakov factors are calculated

The perturbative Sudakov factor takes care of the evolution in the small b_T region. Usually a prescription call b^* prescription is used to ensure that the perturbative evolution is restricted to the small b_T region

The non-perturbative evolution at large b_T are performed using a non-perturbative Sudakov kernel which is parametrized using data

J. Collins, Foundations of Perturbative QCD (Cambridge University Press, Cambridge, England, 2013); S. M. Aybat, J. C. Collins, J.-W. Qiu, and T. C. Rogers, [Phys. Rev. D 85, 034043 \(2012\)](#).

INCORPORATING TMD EVOLUTION

$$f(x, b_{\perp}, \mu) = \int d^2 \mathbf{k}_{\perp} e^{-i \mathbf{b}_{\perp} \cdot \mathbf{k}_{\perp}} f(x, k_{\perp}, \mu)$$

TMDs are evolved from the initial scale Q_i to the final scale Q_f

$$Q_i = c/b_*(b_{\perp})$$

$$c = 2e^{-\gamma_{\epsilon}}$$

$$Q_f = \sqrt{\zeta} = M$$

Use b^* prescription to avoid hitting Landau pole, or to ensure that evolution stays in the perturbative region

$$b_*(b_{\perp}) = \frac{b_{\perp}}{\sqrt{1 + (b_{\perp}/b_{\max})^2}} \quad \begin{array}{ll} b^*(b_{\perp}) \approx b_{\max} & \text{as } b_{\perp} \rightarrow \infty \\ b^*(b_{\perp}) \approx b_{\perp} & \text{as } b_{\perp} \rightarrow 0 \end{array}$$

$$f(x, b_{\perp}, Q_f, \zeta) = f(x, b_{\perp}, Q_i) R_{\text{pert}}(Q_f, Q_i, b_*) R_{NP}(Q_f, Q_i, b_{\perp}).$$

Collins-Soper-Sterman (CSS) TMD evolution formalism

Anomalous dimensions can be expanded

$$A_g(\alpha_s(\eta)) = \sum_{k=1}^{\infty} \left(\frac{\alpha_s(\eta)}{\pi} \right)^k A_g^{(k)}, \quad B_g(\alpha_s(\eta)) = \sum_{k=1}^{\infty} \left(\frac{\alpha_s(\eta)}{\pi} \right)^k B_g^{(k)}.$$

Perturbative evolution kernel is given by

$$R_{\text{pert}}(Q_f, b_*) = \exp \left\{ - \int_{c/b_*}^{Q_f} \frac{d\mu}{\mu} \left(A \log \left(\frac{Q_f^2}{\mu^2} \right) + B \right) \right\},$$

J. Collins, Foundations of Perturbative QCD (Cambridge University Press, Cambridge, England, 2013); S. M. Aybat, J. C. Collins, J.-W. Qiu, and T. C. Rogers, [Phys. Rev. D 85, 034043 \(2012\)](#).

INCORPORATING TMD EVOLUTION

Nonperturbative part of the evolution kernel is parametrized as

$$R_{NP} = \exp \left\{ - \left[g_1^{\text{TMD}} + \frac{g_2}{2} \log \frac{Q_f}{Q_0} \right] b_{\perp}^2 \right\}.$$

S. M. Aybat, J. C. Collins, J.-W. Qiu, and T. C. Rogers,
[Phys. Rev. D 85, 034043 \(2012\)](#).

While g_2 is universal, g_1 depends on the TMD; may be interpreted as the intrinsic transverse momentum width of the TMD at scale Q_0

Best fit parameters are taken from

M.G. Echevarria, A. Idilbi, Z.-B. Kang, I. Vitev, Phys. Rev. D
89, 074013 (2014).

$$g_1 = \frac{\langle k_{\perp}^2 \rangle^2}{4}, \quad g_2 = \frac{\langle k_{s\perp}^2 \rangle^2}{4}, \quad Q_0 = \sqrt{2.4} \text{ GeV}$$

Unpolarized gluon TMD and derivative of gluon Sivers TMD at initial scale can be expressed as

$$f_1^g(x, b_{\perp}, Q_i) = f_{g/p}(x, c/b_*) + \mathcal{O}(\alpha_s),$$

$$f_{1T}^{\prime\perp g}(x, b_{\perp}, Q_i) \simeq \frac{M_p b_{\perp}}{2} T_{g,F}(x, x, Q_i),$$

At LO

Qiu-Sterman function

INCORPORATING TMD EVOLUTION

Unpolarized gluon TMD

$$f_{g/p}(x, b_{\perp}, Q_f) = f_1^g(x, b_{\perp}, Q_i) \times \exp\left\{-\int_{c/b_*}^{Q_f} \frac{d\mu}{\mu} \left(A \log\left(\frac{Q_f^2}{\mu^2}\right) + B\right)\right\} \times \exp\left\{-\left[g_1^{\text{pdf}} + \frac{g_2}{2} \log \frac{Q_f}{Q_0}\right] b_{\perp}^2\right\}$$

Derivative of Sivers function

$$f_{1T}^{\perp g}(x, b_{\perp}, Q_f) = f_{1T}^{\perp g}(x, b_{\perp}, Q_i) \times \exp\left\{-\int_{c/b_*}^{Q_f} \frac{d\mu}{\mu} \left(A \log\left(\frac{Q_f^2}{\mu^2}\right) + B\right)\right\} \times \exp\left\{-\left[g_1^{\text{Sivers}} + \frac{g_2}{2} \log \frac{Q_f}{Q_0}\right] b_{\perp}^2\right\}.$$

Qiu-Sterman function is proportional to the collinear pdf

$$T_{g,F}(x, x, Q_i) = \mathcal{N}_g(x) f_{g/p}(x, Q_i),$$

C. Kouvaris, J.-W. Qiu, W. Vogelsang, F. Yuan, Phys. Rev. D **74**, 114013 (2006).

- (a) $\mathcal{N}_g(x) = (N_u(x) + N_d(x))/2$,
- (b) $\mathcal{N}_g(x) = N_d(x)$.

TMD-a

TMD-b

MSTW2008 collinear pdf

Gluon Sivers function is given by

$$f_{1T}^{\perp g}(x, \mathbf{P}_{hT}, Q_f) = -\frac{1}{2\pi P_{hT}} \int_0^{\infty} db_{\perp} b_{\perp} J_1(P_{hT} b_{\perp}) f_{1T}^{\perp g}(x, b_{\perp}, Q_f),$$

SIVERS ASYMMETRY AT EIC

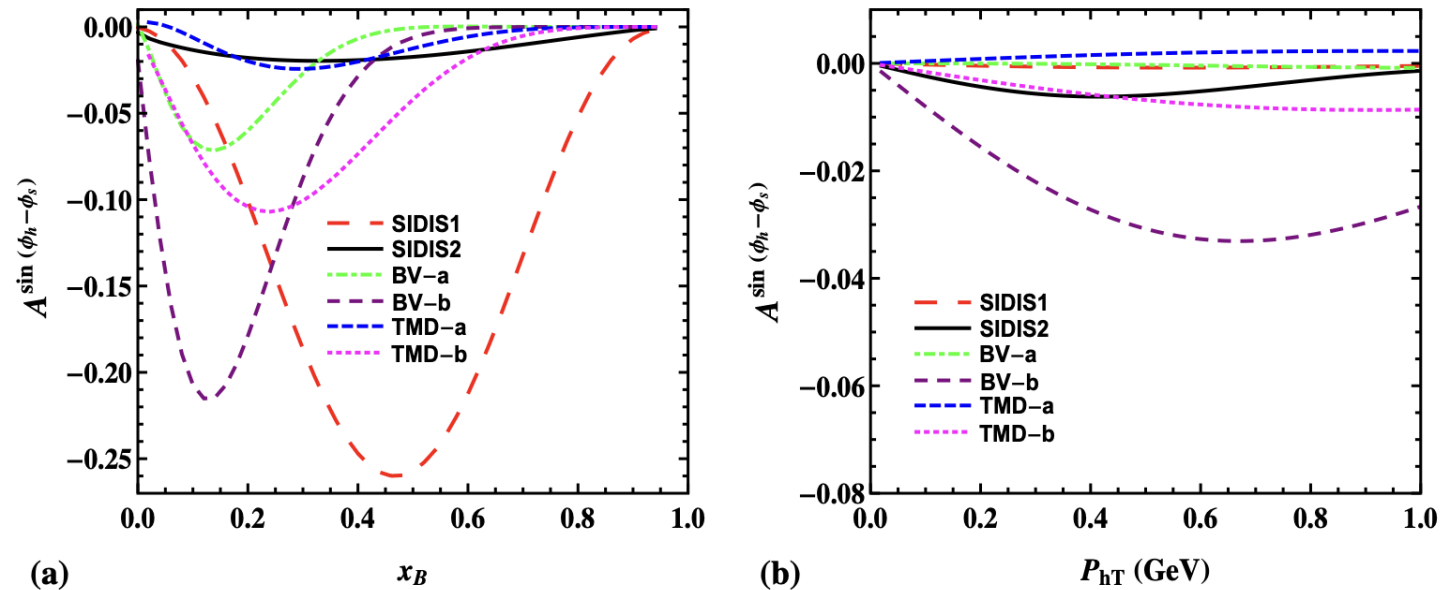


Fig. 2 Single spin asymmetry in $e + p^{\uparrow} \rightarrow e + J/\psi + X$ process as function of **a** x_B (left panel) and **b** P_{hT} (right panel) at $\sqrt{s} = 45.0$ GeV (EIC) using DGLAP (SIDIS1, SIDIS2, BV-a and BV-b) and TMD (TMD-a and TMD-b) evolution approaches. The integration ranges are $0 < P_{hT} < 1.0$ GeV, $0.1 < y < 0.9$ and $0.0001 < x_B < 0.9$

Asymmetry in the kinematics of EIC ; compared with and without TMD evolution and different TMD parametrization

J/ψ production is calculated using NRQCD: color octet states $^1S_0$, $^3P_0$, $^3P_1$ and $^3P_2$ are included

Due to final state interaction Sivers asymmetry in J/ψ production is zero when the quarkonium is produced in color singlet state

SIVERS ASYMMETRY

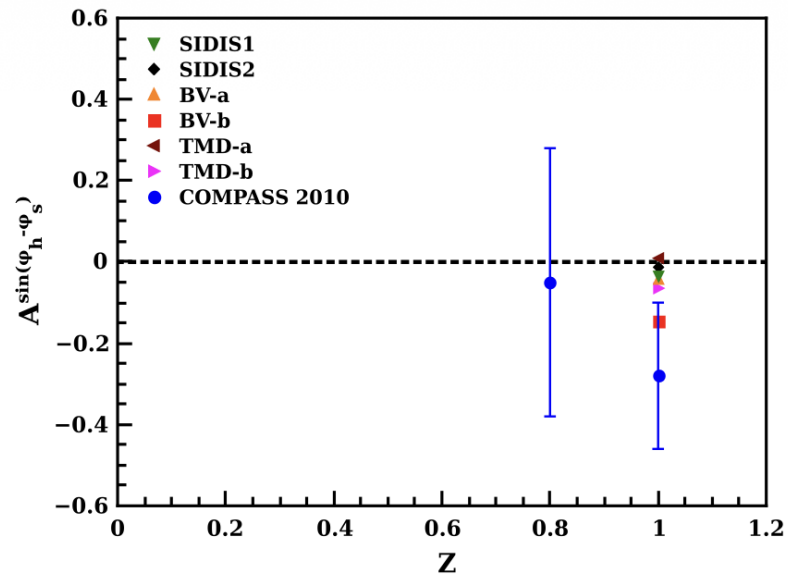


Fig. 6 Single spin asymmetry in $e + p^\uparrow \rightarrow e + J/\psi + X$ process at $z = 1$ with $\sqrt{s} = 17.2$ GeV (COMPASS) using DGLAP (SIDIS1, SIDIS2, BV-a and BV-b) and TMD (TMD-a and TMD-b) evolution approaches. The integration ranges are $0 < P_{hT} < 1.0$ GeV, $0.1 < y < 0.9$ and $0.0001 < x_B < 0.65$. Data from [77]

Sivers asymmetry in single inclusive J/ψ electroproduction compared with COMPASS (2010) data

$$z = \frac{P \cdot P_h}{P \cdot q}$$

At leading order $z=1$, so our theoretical predictions are at $z=1$

Directly dependent on gluon Sivers function. Effect of TMD evolution decreases the asymmetry

Used LDMEs from

AM, S. Rajesh, EPJC (2017) 77

P. Ko, J. Lee, H.S. Song, Phys. Rev. D **54**, 4312 (1996) [Erratum: Phys. Rev. D **60**, 119902 (1999)]

SIVERS ASYMMETRY IN INCLUSIVE PHOTOPRODUCTION OF J/ψ

$$e(l) + p^\uparrow(P) \rightarrow J/\psi(P_h) + X.$$

Consider single spin asymmetry

Consider forward limit when the virtuality of the photon is almost zero

$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow},$$

Dominant subprocess for heavy quarkonium production is photon-gluon fusion

$$\gamma(q) + g(k) \rightarrow J/\psi(P_h) + g(p_g).$$

This takes place through the Weizsacker-Williams photon distributions in the electron

$$f_{\gamma/e}(x_\gamma) = \frac{\alpha}{2\pi} \left[2m_e^2 x_\gamma \left(\frac{1}{Q_{\min}^2} - \frac{1}{Q_{\max}^2} \right) + \frac{1 + (1 - x_\gamma)^2}{x_\gamma} \ln \frac{Q_{\max}^2}{Q_{\min}^2} \right]$$

Leading order photon-gluon fusion process gives elastic J/ψ production at $z=1$, and for low z there are resolved photon contribution

Consider $0.3 < z < 0.9$ for inelastic photoproduction

SIVERS ASYMMETRY IN INCLUSIVE PHOTOPRODUCTION OF J/ψ

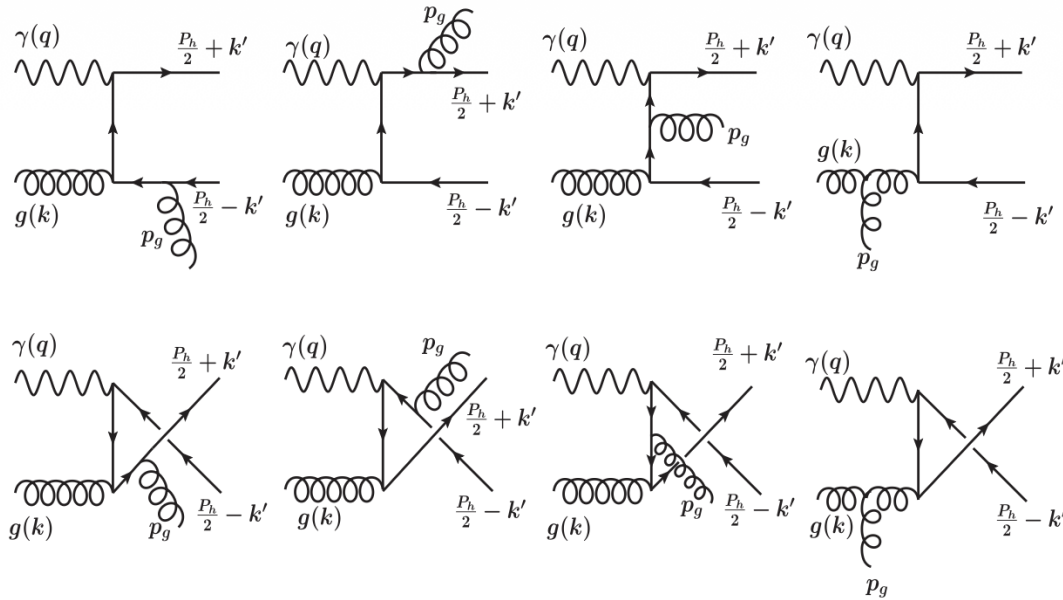


FIG. 2. Feynman diagrams for $\gamma + g \rightarrow J/\psi + g$ process.

J/ψ production is calculated in NRQCD

Used Gaussian parametrization for gluon TMDs

Used different LDME sets

$$E_h \frac{d\sigma}{d^3\mathbf{P}_h} = \frac{1}{2(2\pi)^2} \int dx_\gamma dx_g d^2\mathbf{k}_{\perp g} f_{\gamma/e}(x_\gamma) f_{g/p}(x_g, \mathbf{k}_{\perp g}) \times \delta(\hat{s} + \hat{t} + \hat{u} - M^2) \frac{1}{2\hat{s}} |\mathcal{M}_{\gamma+g \rightarrow J/\psi+g}|^2.$$

Approach of Boer and Pisano, PRD (2012)

UNPOLARIZED CROSS SECTION

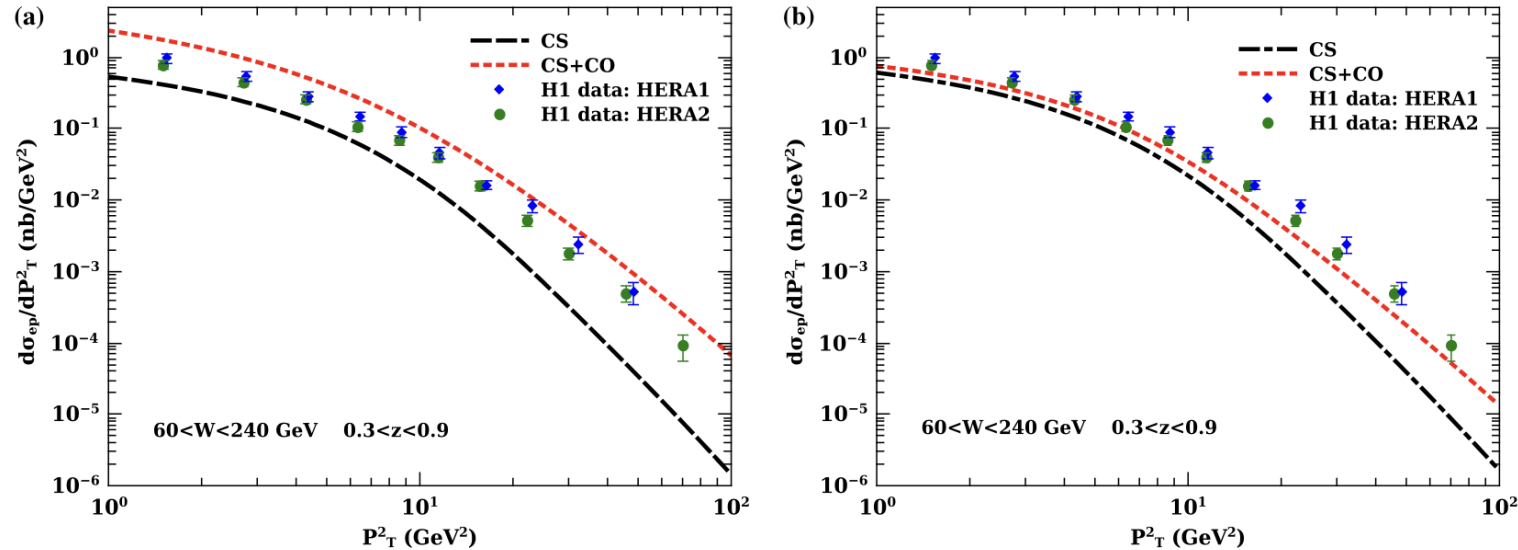


FIG. 8. Unpolarized differential cross section in $e + p \rightarrow J/\psi + X$ process as function of P_T at HERA ($\sqrt{s} = 318$ GeV) using the LDMEs from (a) Ref. [45] (left panel) and (b) Ref. [46] (right panel) with $\langle k_{\perp g}^2 \rangle = 1$ GeV². The H1 data from [39,40]. The integration ranges are $1 < P_T \leq 10$ GeV, $60 < W < 240$ GeV and $0.3 < z < 0.9$. The convention of lines is same as Fig. 7.

W is invariant mass of photon-proton system

Unpolarized cross section compared with H1 data

LDME sets from

K.-T. Chao, Y.-Q. Ma, H.-S. Shao, K. Wang, and Y.-J. Zhang, Phys. Rev. Lett. 108, 242004 (2012) (left)

M. Butenschoen and B. A. Kniehl, Phys. Rev. D 84, 051501 (2011) (right)

C. Adloff et al. (H1 Collaboration), Eur. Phys. J. C 25, 25 (2002).

F. D. Aaron et al. (H1 Collaboration), Eur. Phys. J. C 68, 401 (2010).

SINGLE SPIN ASYMMETRY

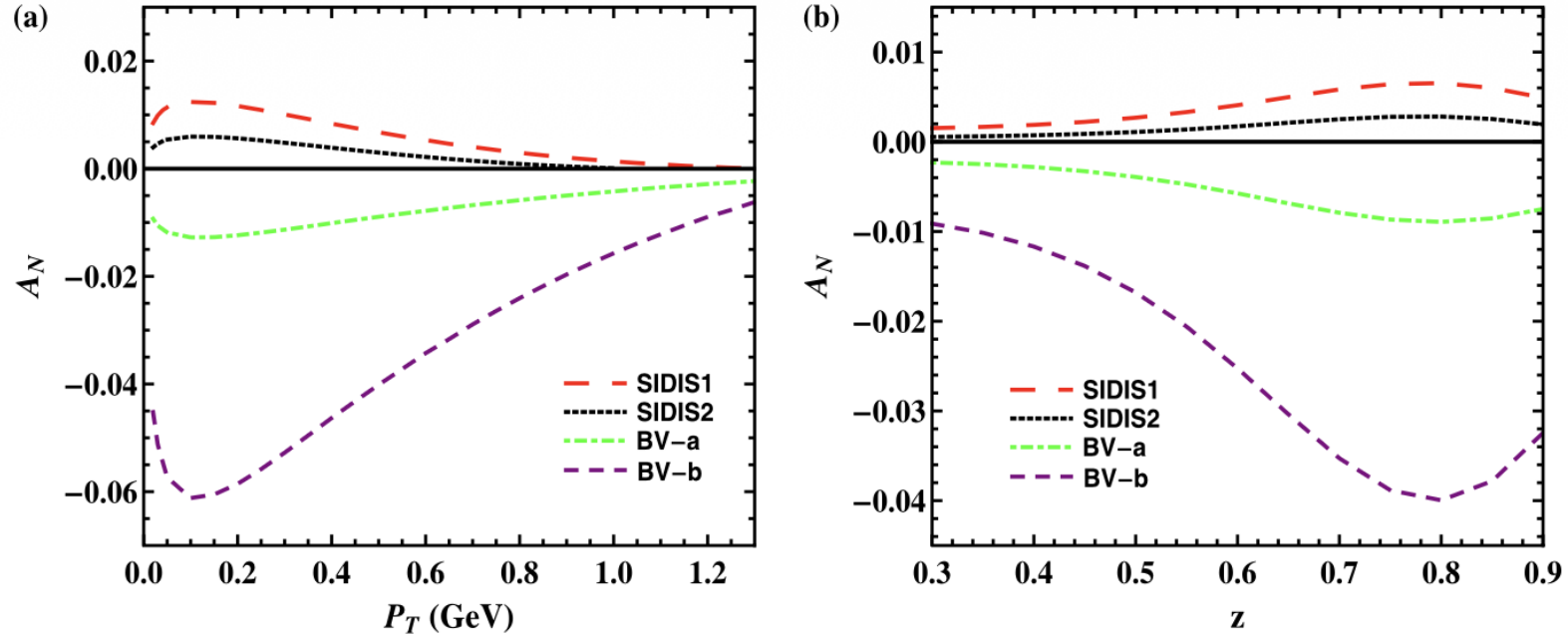


FIG. 4. Single spin asymmetry in $e + p^\uparrow \rightarrow J/\psi + X$ process as function of (a) P_T (left panel) and (b) z (right panel) at $\sqrt{s} = 45$ GeV (EIC). The integration ranges are $0 < P_T \leq 1$ GeV and $0.3 < z < 0.9$. For convention of lines see the text.

SIVERS ASYMMETRY IN ALMOST BACK-TO-BACK PHOTOPRODUCTION OF J/ψ AND JET

$$e(l) + p^\uparrow(P) \rightarrow J/\psi(P_\psi) + \text{jet}(P_j) + X,$$

$$q^2 \approx -2E_e E'_e (1 - \cos \theta)$$

Consider the limit when 4 momentum of virtual photon $q^2 = -Q^2 \rightarrow 0$

$$\begin{aligned} E_\psi E_j \frac{d\sigma}{d^3\mathbf{P}_\psi d^3\mathbf{P}_j} &= \frac{d\sigma}{d^2\mathbf{P}_{\psi\perp} dz d^2\mathbf{P}_{j\perp} dz_1} \\ &= \frac{1}{4(2\pi)^2} \frac{1}{z z_1} \sum_a \int dx_\gamma dx_a d^2\mathbf{p}_{a\perp} f_{\gamma/e}(x_\gamma) \\ &\quad \times f_{a/p}(x_a, p_{a\perp}) \\ &\quad \times \delta^4(q + p - P_\psi - P_j) \frac{1}{2\hat{s}} |\mathcal{M}_{\gamma a \rightarrow J/\psi a}|^2, \end{aligned}$$

When the exchanged photon is quasi-real, the interaction takes place through the Weizsäcker-Williams distribution function of the electron, this describes the density of photons inside the electron and is given by

$$\begin{aligned} f_{\gamma/e}(x_\gamma) &= \frac{\alpha}{2\pi} \left[2m_e^2 x_\gamma \left(\frac{1}{Q_{\min}^2} - \frac{1}{Q_{\max}^2} \right) \right. \\ &\quad \left. + \frac{1 + (1 - x_\gamma)^2}{x_\gamma} \ln \frac{Q_{\max}^2}{Q_{\min}^2} \right], \end{aligned}$$

$$Q_{\min}^2 = m_e^2 \frac{x_\gamma^2}{1 - x_\gamma},$$

$$z = \frac{P \cdot P_h}{P \cdot q} \text{ and } z_1 = \frac{P \cdot P_j}{P \cdot q}$$

Both CS and CO contributions are
Considered

$$^3S_1^{(1,8)}, \quad ^1\tilde{S}_0^{(8)} \quad \text{and} \quad ^3\hat{P}_J^{(8)}$$

Dominant subprocess

$$\gamma(q) + g(p) \rightarrow J/\psi(P_\psi) + g(P_j)$$

SIVERS ASYMMETRY IN BACK-TO-BACK PHOTOPRODUCTION OF J/ψ AND JET

Outgoing J/ψ and jet are almost back

$$\mathbf{q}_\perp = \mathbf{P}_{\Psi\perp} + \mathbf{P}_{j\perp}$$

$$\mathbf{K}_\perp = (\mathbf{P}_{\Psi\perp} - \mathbf{P}_{j\perp})/2.$$

$$|\mathbf{q}_\perp| \ll |\mathbf{K}_\perp|$$

For transversely polarized proton

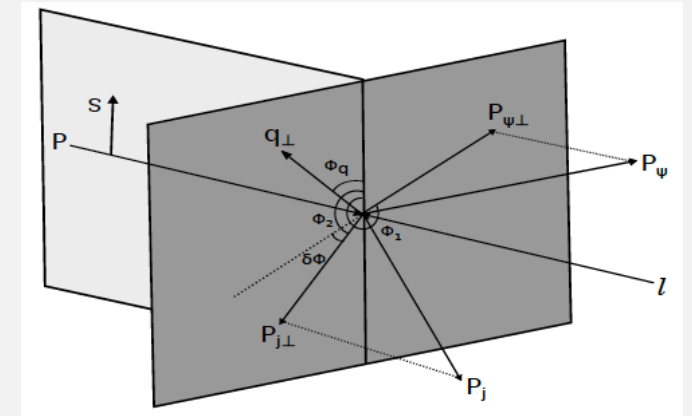
$$\frac{d\sigma^{\uparrow(\downarrow)}}{d^2\mathbf{q}_\perp dz d^2\mathbf{K}_\perp} = \frac{1}{2(2\pi)^2} \frac{1}{z(1-z)s} \sum_a \int \frac{dx_\gamma}{x_\gamma} f_{\gamma/e}(x_\gamma) f_{a/p^{\uparrow(\downarrow)}}(x_a, \mathbf{q}_\perp) \frac{1}{2\hat{s}} |\mathcal{M}_{\gamma a \rightarrow J/\psi a}|^2.$$

Weighted Sivers asymmetry

$$A_N^{W(\phi_q)} \equiv \frac{\int d\phi_q W(\phi_q) (d\sigma^\uparrow - d\sigma^\downarrow)}{\int d\phi_q (d\sigma^\uparrow + d\sigma^\downarrow)} \equiv \frac{\int d\phi_q W(\phi_q) d\Delta\sigma(\phi_q)}{\int d\phi_q 2d\sigma},$$

Weight factor for Sivers asymmetry

$$-\sin(\phi_q) = \frac{(\mathbf{S} \times \hat{\mathbf{P}}) \cdot \mathbf{q}_\perp}{|\mathbf{S} \times \hat{\mathbf{P}}| |\hat{\mathbf{P}} \times \mathbf{q}_\perp|}.$$



SIVERS ASYMMETRY

Cross section for transversely polarized proton

$$\begin{aligned} d\Delta\sigma &\equiv \frac{d\sigma^\uparrow}{d^2\mathbf{q}_\perp dz d^2\mathbf{K}_\perp} - \frac{d\sigma^\downarrow}{d^2\mathbf{q}_\perp dz d^2\mathbf{K}_\perp} \\ &= \frac{1}{2(2\pi)^2} \frac{1}{z(1-z)s} \sum_a \int \frac{dx_\gamma}{x_\gamma} f_{\gamma/e}(x_\gamma) \Delta\hat{f}_{a/p}(x_a, \mathbf{q}_\perp) \\ &\quad \times \frac{1}{2\hat{s}} |\mathcal{M}_{\gamma a \rightarrow J/\psi a}|^2, \end{aligned}$$

$$\gamma + g \rightarrow J/\psi + g$$

$$\begin{aligned} |\mathcal{M}[^1S_0^{(8)}]|^2 &= \frac{3\pi^3 e_c^2 \alpha_s^2 \alpha}{4M} \langle 0 | \mathcal{O}_8^{J/\psi}(^1S_0) | 0 \rangle \frac{128}{s_1^2 t_1^2 u_1^2 (M^2 + u_1)^2} \{ 8M^{14} + 4M^{12} \\ &\quad \times (4(s_1 + t_1) + 7u_1) + 2M^{10}(8s_1^2 + 17u_1(s_1 + t_1) + 12s_1t_1 + 8t_1^2 + 19u_1^2) \\ &\quad + 2M^8(7s_1^3 + 4u_1(s_1^2 + 5s_1t_1 + t_1^2) + 5s_1^2t_1 + 5s_1t_1^2 + 6u_1^2(s_1 + t_1) + 7t_1^3 \\ &\quad + 13u_1^3) + 2M^6(2s_1^4 + 4u_1(s_1^3 + t_1^3) + s_1^3t_1 + u_1^2(-17s_1^2 + 7s_1t_1 - 17t_1^2) \\ &\quad + 2s_1^2t_1^2 + s_1t_1^3 - 8u_1^3(s_1 + t_1) + 2t_1^4 + 5u_1^4) + 2M^4(6u_1^3(-3s_1^2 + s_1t_1 - 3t_1^2) \\ &\quad - 6u_1^2(s_1 + t_1)(s_1^2 + t_1^2) + u_1(s_1 - t_1)^2(s_1^2 + 4s_1t_1 + t_1^2) - 6u_1^4(s_1 + t_1) \\ &\quad - 3s_1t_1(s_1 - t_1)^2(s_1 + t_1) + u_1^5) + M^2(-2u_1^4(5s_1^2 - 11s_1t_1 + 5t_1^2) + u_1^2 \\ &\quad \times (-2s_1^4 + s_1^3t_1 - 5s_1^2t_1^2 + s_1t_1^3 - 2t_1^4) - 2u_1^5(s_1 + t_1) - 6u_1^3(s_1 - t_1)^2(s_1 + t_1) \\ &\quad + s_1t_1u_1(s_1 - t_1)^2(s_1 + t_1) - s_1t_1(s_1 - t_1)^2(2s_1 + t_1)(s_1 + 2t_1)) \\ &\quad + s_1t_1u_1(3u_1^2(s_1^2 + s_1t_1 + t_1^2) + (s_1 - t_1)^2(s_1^2 + s_1t_1 + t_1^2) + 8u_1^3(s_1 + t_1) + 8u_1^4) \} \end{aligned}$$

Some of the subprocess contributions are given below :

$$\gamma + q(\bar{q}) \rightarrow J/\psi + q(\bar{q})$$

$$|\mathcal{M}(^3S_1^{(1)})|^2 = 0,$$

$$|\mathcal{M}(^3S_1^{(8)})|^2 = \frac{-2(4\pi)^3 e_c^2 \alpha_s^2 \alpha}{9M^3 \hat{s} \hat{t}} \langle 0 | \mathcal{O}_8^{J/\psi}(^3S_1) | 0 \rangle [\hat{s}^2 + \hat{t}^2 + 2\hat{u}M^2],$$

$$|\mathcal{M}(^1S_0^{(8)})|^2 = \frac{-4(4\pi)^3 e_c^2 \alpha_s^2 \alpha}{3M} \langle 0 | \mathcal{O}_8^{J/\psi}(^1S_0) | 0 \rangle \frac{\hat{s}^2 + \hat{t}^2}{\hat{u}(\hat{s} + \hat{t})^2},$$

$$|\mathcal{M}(^3P_0^{(8)})|^2 = \frac{-16(4\pi)^3 e_c^2 \alpha_s^2 \alpha}{9M^3} \langle 0 | \mathcal{O}_8^{J/\psi}(^3P_0) | 0 \rangle \frac{(\hat{s}^2 + \hat{t}^2)(\hat{u} - 3M^2)^2}{\hat{u}(\hat{s} + \hat{t})^4},$$

$$|\mathcal{M}(^3P_1^{(8)})|^2 = \frac{-32(4\pi)^3 e_c^2 \alpha_s^2 \alpha}{9M^3} \langle 0 | \mathcal{O}_8^{J/\psi}(^3P_1) | 0 \rangle \frac{(\hat{s}^2 + \hat{t}^2)\hat{u} + 4M^2\hat{s}\hat{t}}{(\hat{s} + \hat{t})^4},$$

Rajesh, Kishore, AM, PRD 98, 014007(2018);
Kishore, AM, Rajesh, PRD 101, 054003 (2020)

SIVERS FUNCTION PARAMETRIZATION

Gaussian parametrization

$$\Delta^N f_{a/p^\uparrow}(x_a, q_\perp) = 2 \frac{\sqrt{2e}}{\pi} \mathcal{N}_a(x_a) f_{a/p}(x_a) \sqrt{\frac{1-\rho}{\rho}} q_\perp \frac{e^{-q_\perp^2/\rho \langle q_\perp^2 \rangle}}{\langle q_\perp^2 \rangle^{3/2}}.$$

$$f_{a/p}(x_a, q_\perp) = \frac{1}{\pi \langle q_\perp^2 \rangle} f(x_a) e^{-q_\perp^2/\langle q_\perp^2 \rangle}.$$

U. D'Alesio, F. Murgia, and C. Pisano, *J. High Energy Phys.* (2015) 119 ; U. D'Alesio, C. Flore, F. Murgia, C. Pisano, and P. Tael, *Phys. Rev. D* 99, 036013 (2019).

$$\mathcal{N}_a(x_a) = N_a x_a^\alpha (1-x_a)^\beta \frac{(\alpha+\beta)^{(\alpha+\beta)}}{\alpha^\alpha \beta^\beta},$$

Satisfies positivity bound

Best fit parameters for Sivers function are obtained from SIDIS and RHIC data

Evolution	a	N_a	α	β	ρ	$\langle q_\perp^2 \rangle$ GeV ²	Notation
DGLAP	g [7]	0.65	2.8	2.8	0.687	0.25	SIDIS1
	g [7]	0.05	0.8	1.4	0.576	0.25	SIDIS2
	g [8]	0.25	0.6	0.6	0.1	1.0	SIDIS3
TMD	u [13]	0.106	1.051	4.857		0.38	TMD-a
	d [13]	-0.163	1.552	4.857		0.38	TMD-b

INCORPORATING TMD EVOLUTION

Renormalization group and Collins-Soper equations are obtained by taking scale evolution with respect to μ and ζ

light-cone (rapidity) divergences in TMD factorization is regulated by the the rapidity scale ζ

$$f(x_a, b_\perp, \mu) = \int d^2 \mathbf{p}_{a\perp} e^{-i \mathbf{b}_\perp \cdot \mathbf{p}_{a\perp}} f(x_a, p_{a\perp}, \mu),$$

J. Collins, Foundations of Perturbative QCD (Cambridge University Press, Cambridge, England, 2013); S. M. Aybat, J. C. Collins, J.-W. Qiu, and T. C. Rogers, [Phys. Rev. D 85, 034043 \(2012\)](#).

$$f(x_a, b_\perp, Q_f, \zeta) = f(x_a, b_\perp, Q_i) R_{\text{pert}}(Q_f, Q_i, b_*) R_{NP}(Q_f, b_\perp)$$

— A

Perturbative and non-perturbative parts are given by

$$R_{\text{pert}}(Q_f, b_*) = \exp \left\{ - \int_{c/b_*}^{Q_f} \frac{d\mu}{\mu} \left(A \log \left(\frac{Q_f^2}{\mu^2} \right) + B \right) \right\},$$

$$R_{NP}(Q_f, b_\perp) = \exp \left\{ - \left[g_1^{\text{TMD}} + \frac{g_2}{2} \log \frac{Q_f}{Q_0} \right] b_\perp^2 \right\}$$

Anomalous dimensions are obtained order by order in perturbative series

$$A_1 = C_A, \quad A_2 = \frac{1}{2} C_F \left(C_A \left(\frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{5}{9} C_A N_f \right)$$

$$B_1 = -\frac{1}{2} \left(\frac{11}{3} C_A - \frac{2}{3} N_f \right),$$

Gluons

$$A_1 = C_F, \quad A_2 = \frac{1}{2} C_F \left(C_A \left(\frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{5}{9} N_f \right) \quad B_1 = -\frac{3}{2} C_F$$

Quarks

TMD EVOLUTION

Initial scale of TMDs $Q_i = c/b_*(b_\perp)$ $c = 2e^{-\gamma_\epsilon}$ Large $b_\perp$ region perturbation theory breaks down

b_* prescription allows us to use perturbation theory in the region where it is valid, while including nonperturbative effects separately.

$$b_*(b_\perp) = \frac{b_\perp}{\sqrt{1 + (\frac{b_\perp}{b_{\max}})^2}} \quad b^*(b_\perp) \approx b_{\max} \quad \text{as } b_\perp \rightarrow \infty$$

$$b^*(b_\perp) \approx b_\perp \quad \text{as } b_\perp \rightarrow 0$$

Unpolarized TMD

$$f(x_a, b_\perp, Q_i) = \sum_{i=g,q} \int_x^1 \frac{d\hat{x}}{\hat{x}} C_{i/a}(x_a/\hat{x}, b_\perp, \alpha_s, Q_i) f_{i/p}(\hat{x}, c/b_*) + \mathcal{O}(b_\perp \Lambda_{\text{QCD}}),$$



perturbatively calculated process independent coefficient function, different for each type of TMD.

Derivative of Siverson function related to Qiu-Sterman function

$$f'_{1T}(x_a, b_\perp, Q_i) \simeq \frac{M_p b_\perp}{2} T_{a,F}(x_a, x_a, Q_i),$$

Assumed to be given in terms of collinear pdfs

$$T_{a,F}(x_a, x_a, Q_i) = \mathcal{N}_a(x_a) f_{a/p}(x_a, Q_i).$$

Sivers function

$$f'_{1T}(x_a, p_{a\perp}, Q_f) = -\frac{1}{2\pi p_{a\perp}} \int_0^\infty db_\perp b_\perp J_1(p_{a\perp} b_\perp) f'_{1T}(x_a, b_\perp, Q_f),$$

Derivative of Siverson function follows the same evolution equation as (A)

SIVERS FUNCTION

$$(a) \mathcal{N}_g(x_g) = (\mathcal{N}_u(x_g) + \mathcal{N}_d(x_g))/2,$$

$$(b) \mathcal{N}_g(x_g) = \mathcal{N}_d(x_g).$$

Boer and Vogelsang, [Phys. Rev. D 69, 094025 \(2004\)](#).

$$d\Delta\sigma = -\frac{1}{\pi M_p} \frac{1}{2(2\pi)^2} \frac{1}{z(1-z)s} \sum_a \int \frac{dx_\gamma}{x_\gamma} db_\perp b_\perp J_1(q_\perp b_\perp) f'_{1T}(x_a, b_\perp, Q_f) f_{\gamma/e}(x_\gamma) \frac{1}{2\hat{s}} |\mathcal{M}_{\gamma a \rightarrow J/\psi a}|^2 \sin(\phi_q),$$

$$2d\sigma = \frac{1}{2(2\pi)^2 \pi} \frac{1}{z(1-z)s} \sum_a \int \frac{dx_\gamma}{x_\gamma} db_\perp b_\perp J_0(q_\perp b_\perp) f_{a/p}(x_a, b_\perp, Q_f) f_{\gamma/e}(x_\gamma) \frac{1}{2\hat{s}} |\mathcal{M}_{\gamma a \rightarrow J/\psi a}|^2.$$

Assuming contributions only from u and d quarks and gluons, we have checked that TMD-(a) parametrization satisfies the Burkardt sum rule, and violation is about 1%; TMD-(b) violates the sum rule by about 19 %

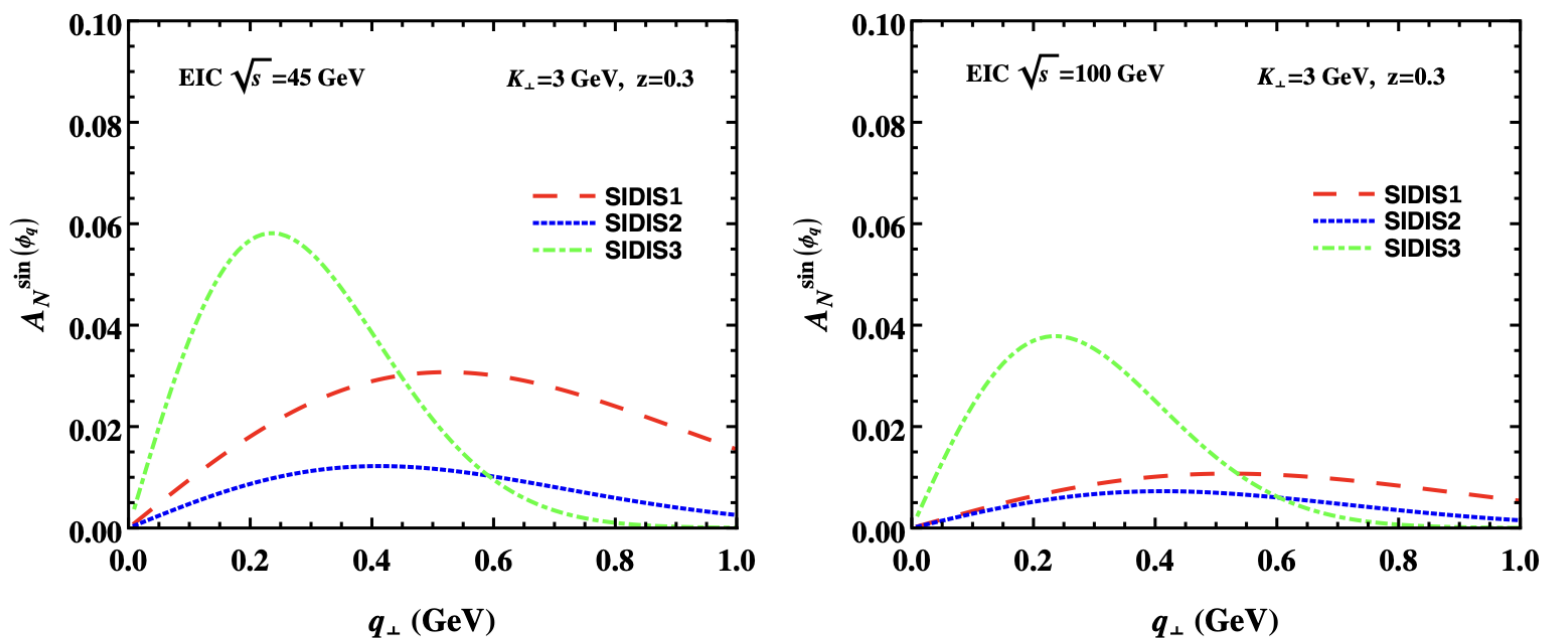
Upper bound of Sivers asymmetry is obtained by saturating the positivity constraint 

$$\begin{aligned} \Delta \hat{f}_{a/p^\uparrow}(x_a, \mathbf{q}_\perp) &\equiv \hat{f}_{a/p^\uparrow}(x_a, \mathbf{q}_\perp) - \hat{f}_{a/p^\downarrow}(x_a, \mathbf{q}_\perp) \\ &= \Delta^N f_{a/p^\uparrow}(x_a, \mathbf{q}_\perp) \hat{\mathbf{S}} \cdot (\hat{\mathbf{P}} \times \hat{\mathbf{q}}_\perp) \\ &= -\Delta^N f_{a/p^\uparrow}(x_a, \mathbf{q}_\perp) \sin(\phi_q) \\ &= -\frac{2}{M_p} f_{1T}^\perp(x_a, \mathbf{q}_\perp) \hat{\mathbf{S}} \cdot (\hat{\mathbf{P}} \times \mathbf{q}_\perp). \end{aligned}$$

 Sivers function in Trento convention

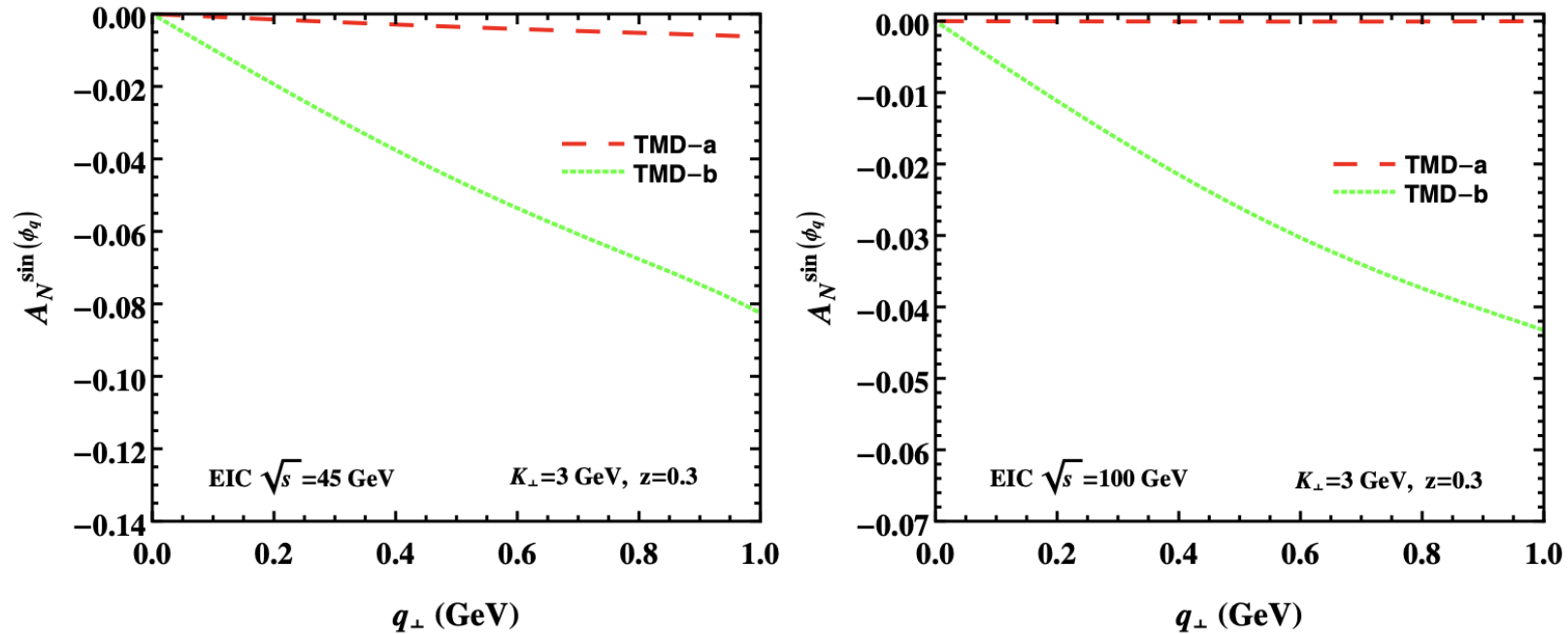
$$\begin{aligned} |\Delta^N f_{a/p^\uparrow}(x_a, \mathbf{q}_\perp)| &\leq 2f_{a/p}(x_a, \mathbf{q}_\perp), \\ \frac{q_\perp}{M_p} |f_{1T}^\perp(x_a, \mathbf{q}_\perp)| &\leq f_{a/p}(x_a, \mathbf{q}_\perp). \end{aligned}$$

SIVERS ASYMMETRY IN BACK-TO-BACK PHOTOPRODUCTION OF J/ψ AND JET



The weighted Sivers asymmetry in the $e + p \rightarrow J/\psi + \text{jet} + X$ process as a function of $q_\perp$ at EIC (a) $\sqrt{s} = 45$ GeV and (b) $\sqrt{s} = 100$ GeV using the DGLAP evolution approach for SIDIS1, SIDIS2, and SIDIS3 GSF parametrization sets which are given in Table I.

SIVERS ASYMMETRY IN BACK-TO-BACK PHOTOPRODUCTION OF J/ψ AND JET



The weighted Sivers asymmetry in the $e + p^\uparrow \rightarrow J/\psi + \text{jet} + X$ process as a function of $q_\perp$ at EIC (a) $\sqrt{s} = 45$ GeV and (b) $\sqrt{s} = 100$ GeV using the TMD evolution approach for TMD-a and TMD-b GSF parametrization sets which are given in Table I.

SIVERS ASYMMETRY IN J/ψ AND PHOTON ELECTRO PRODUCTION

We consider the following process where the proton can be unpolarized or transversely polarized

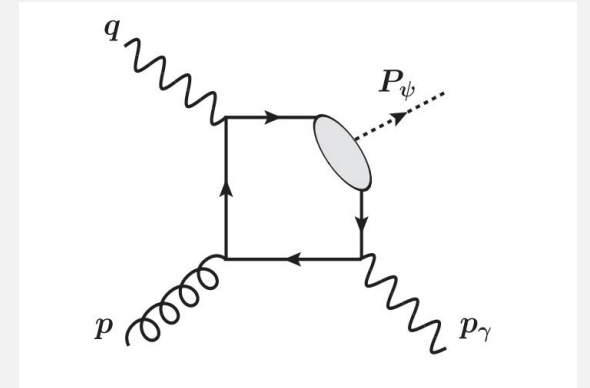
$$e(l) + p^\uparrow(P) \rightarrow e(l') + J/\psi(P_\psi) + \gamma(p_\gamma) + X,$$

$$P^\mu = n_-^\mu + \frac{M_p^2}{2} n_+^\mu \approx n_-^\mu,$$

$$q^\mu = -x_B n_-^\mu + \frac{Q^2}{2x_B} n_+^\mu \approx -x_B P^\mu + (P \cdot q) n_+^\mu,$$

$$l^\mu = \frac{1-y}{y} x_B n_-^\mu + \frac{1}{y} \frac{Q^2}{2x_B} n_+^\mu + \frac{\sqrt{1-y}}{y} Q \hat{l}_\perp^\mu,$$

$$Q^2 = x_b y S \quad S = (P + l)^2$$



$$\begin{aligned} d\sigma = & \frac{1}{2S} \frac{d^3 l'}{(2\pi)^3 2E_{l'}} \frac{d^3 P_\psi}{(2\pi)^3 2E_\psi} \frac{d^3 p_\gamma}{(2\pi)^3 2E_\gamma} \\ & \times \int dx d^2 \mathbf{p}_T (2\pi)^4 \delta^4(q + p - P_\psi - p_\gamma) \\ & \times \frac{1}{Q^4} L^{\mu\nu}(l, q) \Phi_g^{\rho\sigma}(x, \mathbf{p}_T) H_{\mu\rho} H_{\nu\sigma}^*. \end{aligned}$$

$$y = \frac{P \cdot q}{P \cdot l} \quad x_B = \frac{Q^2}{2P \cdot q}$$

Partonic subprocess : virtual photon-gluon fusion

J/ψ production in color singlet channel in virtual photon-photon fusion is suppressed due to a larger gluon density

Use TMD factorization

D. Chakrabarti, R. Kishore, AM, S. Rajesh,
Phys.Rev.D 107 (2023) 1, 014008

TMD EVOLUTION

TMDs are evolved from initial scale

$$\mu_b = \frac{b_0}{b_T},$$

$$b_0 = 2e^{-\gamma_E},$$

To the final scale

$$\mu = \sqrt{M_\psi^2 + Q^2},$$

Perturbative expression for the unpolarized gluon TMD valid in the region $|\mathbf{b}_T| \ll 1/\Lambda_{\text{QCD}},$

$$\tilde{f}_1^g(x, \mathbf{b}_T; \mu_b, \mu) = \sum_{a=q, \bar{q}, g} (C_{g/a} \otimes f_1^a)(x, \mu_b) \times \exp(-S_{\text{pert}}(\mu_b, \mu)),$$

Echevarria, Idilbi, Kang, and Vitev, Phys. Rev. D 89, 074013 (2014)

Pert coefficient function

Collinear pdf

Pert Sudakov factor

$$C_{g/a}(x, \mu_b) = \delta_{ga} \delta(1-x) + \sum_{k=1}^{\infty} C_{g/a}^{(k)}(x) \left(\frac{\alpha_s(\mu_b)}{\pi} \right)^k$$

Boer, D'Alesio, Murgia, Pisano, and Taels, JHEP 09, 040 (2020),

NLL :

$$\begin{aligned} \sum_{a=q, \bar{q}, g} (C_{g/a} \otimes f_1^a)(x, \mu_b) &= \sum_{a=q, \bar{q}, g} \int_x^1 \frac{dx'}{x'} C_{g/a}(x', \mu_b) f_1^a\left(\frac{x}{x'}, \mu_b\right) \\ &= \sum_{a=q, \bar{q}, g} \int_x^1 \frac{dx'}{x'} \delta_{ga} \delta(1-x') f_1^a\left(\frac{x}{x'}, \mu_b\right) \\ &= f_1^g(x, \mu_b). \end{aligned}$$

Unpolarized gluon TMD

$$\tilde{f}_1^g(x, \mathbf{b}_T; \mu_b, \mu) = f_1^g(x, \mu_b) \exp[-S_{\text{pert}}(\mu_b, \mu)].$$

Perturbative Sudakov factor :

$$S_{\text{pert}}(\mu_b, \mu) = \int_{\mu_b^2}^{\mu^2} \frac{d\eta^2}{\eta^2} \left[A_g(\alpha_s(\eta)) \log\left(\frac{\mu^2}{\eta^2}\right) + B_g(\alpha_s(\eta)) \right],$$

TMD EVOLUTION

$$A_g(\alpha_s(\eta)) = \sum_{k=1}^{\infty} \left(\frac{\alpha_s(\eta)}{\pi} \right)^k A_g^{(k)}, \quad B_g(\alpha_s(\eta)) = \sum_{k=1}^{\infty} \left(\frac{\alpha_s(\eta)}{\pi} \right)^k B_g^{(k)}.$$

At one loop perturbative Sudakov kernel

$$S_{\text{pert}}^{1\text{-loop}}(\mu^2, \mu_b^2) = -\frac{4A_g^{(1)}}{\beta_0} \left[\ln \left(\frac{\mu^2}{\mu_b^2} \right) - \left(\frac{B_g^{(1)}}{A_g^{(1)}} + L_\mu \right) \ln \left(\frac{L_\mu}{L_{\mu_b}} \right) \right].$$

$$\begin{aligned} A_g^{(1)} &= \frac{C_A}{2}, \\ A_g^{(2)} &= \frac{C_A}{4} \left[C_A \left(\frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{5}{9} n_f \right], \\ B_g^{(1)} &= -\frac{C_A}{2} \frac{\beta_0}{6}, \quad \beta_0 = 11 - \frac{2}{3} n_f. \end{aligned}$$

Maxia, Boer, and Bor, Phys. Rev. D 112, 076034 (2025),

And at NLL it can be written as

$$S_{\text{pert}}^{\text{NLL}}(\mu^2, \mu_b^2) = S_{\text{pert}}^{1\text{-loop}} - \frac{16A_g^{(2)}}{\beta_0^2} \left[\ln \left(\frac{L_\mu}{L_{\mu_b}} \right) - \frac{1}{L_{\mu_b}} \ln \left(\frac{\mu^2}{\mu_b^2} \right) \right], \quad L_\eta = \ln(\eta^2/\Lambda_{\text{QCD}}^2).$$

Perturbative
expansion valid for
 $|\mathbf{b}_T| \ll 1/\Lambda_{\text{QCD}}.$

Fourier integral extends from small to large values of $\mathbf{b}_T$. In the large- $\mathbf{b}_T$ region, perturbation theory breaks down due to the presence of large logarithmic contributions from higher-order terms : nonperturbative large- $\mathbf{b}_T$ region cannot be described using perturbation theory alone.

$\mathbf{b}_*$ prescription allows us to use perturbation theory in the region where it is valid, while including nonperturbative effects separately.

TMD EVOLUTION

Introduce modified impact parameter

$$b_*(b_T) = \frac{b_T}{\sqrt{1 + \left(\frac{b_T}{b_{\max}}\right)^2}},$$

$$b_{\max} = 1.5 \text{ GeV}^{-1}.$$

$b_T \ll b_{\max}, b_* \approx b_T$ Remains perturbative

$b_T \gg b_{\max}, b^* \rightarrow b_{\max}$ Prevents scale from entering non-perturbative region

In small b_T region scale may be larger than hard scale, to prevent this introduce a lower cutoff

$$b_{\min} = \frac{b_0}{\mu}.$$

Also use a regulated scale

$$\mu'_{b_*} = \frac{b_0}{\sqrt{b_*^2 + b_{\min}^2}},$$

which ensures the scale associated with b_T always remains a physically meaningful perturbative range.

Maxia, Boer, and Bor, Phys. Rev. D 112, 076034 (2025),

$$\begin{aligned} \tilde{f}_1^g(x, \mathbf{b}_T; \mu'_{b_*}, \mu) &= \tilde{f}_1^g(x, \mathbf{b}_*; \mu'_{b_*}, \mu) e^{-S_{\text{NP}}} \\ &= f_1^g(x, \mu'_{b_*}) \exp[-S_{\text{pert}}(\mu'_{b_*}, \mu)] e^{-S_{\text{NP}}} \end{aligned}$$

Non-perturbative effects are incorporated

$$\tilde{f}_{1T}^{\prime \perp g}(x, \mathbf{b}_T; \mu'_{b_*}, \mu) = \frac{b_T M_p}{2} T_{g,F}(x, x, \mu'_{b_*}) \exp[-S_{\text{pert}}(\mu'_{b_*}, \mu)],$$

Use same prescription as before for derivative of Sivers function

$$\begin{aligned} S_{\text{NP}}^{\text{pdf}}(b_T, \mu) &= b_T^2 \left(g_1^{\text{pdf}} + \frac{g_2}{2} \ln \frac{\mu}{Q_0} \right), \\ S_{\text{NP}}^{\text{Sivers}}(b_T, \mu) &= b_T^2 \left(g_1^{\text{Sivers}} + \frac{g_2}{2} \ln \frac{\mu}{Q_0} \right), \end{aligned}$$

Best fit parameters in non-pert part from

SIVERS ASYMMETRY IN BACK-TO-BACK J/ψ AND JET ELECTRO PRODUCTION

$$e^-(l) + p(P) \rightarrow e^-(l') + J/\psi(P_\psi) + \text{jet}(P_j) + X,$$

$$Q^2 = -q^2, \quad s = (P + l)^2, \quad W^2 = (P + q)^2,$$

$$x_B = \frac{Q^2}{2P \cdot q}, \quad y = \frac{P \cdot q}{P \cdot l}, \quad z = \frac{P \cdot P_\psi}{P \cdot q}.$$

Use TMD factorization in the kinematics where the outgoing J/ψ and (gluon) jet are almost back-to back

Use NRQCD to calculate the J/ψ production

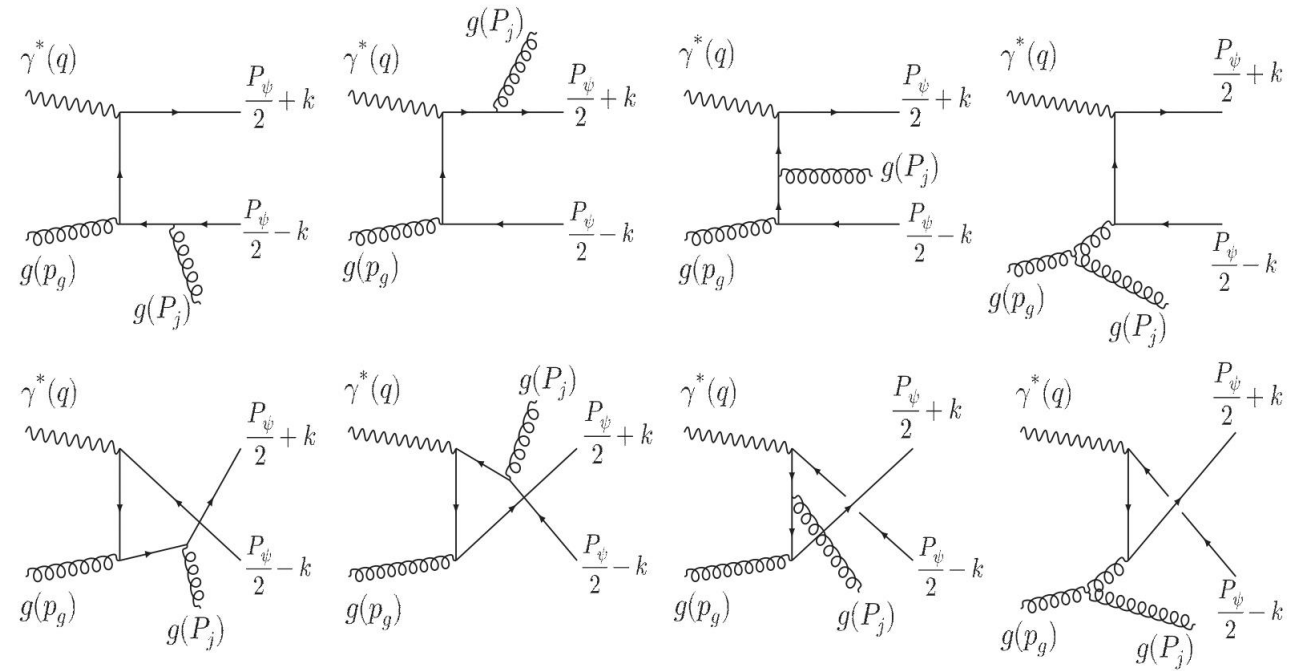


FIG. 1. Feynman diagrams for the partonic process $\gamma^*(q) + g(p_g) \rightarrow J/\psi(P_\psi) + g(P_j)$.

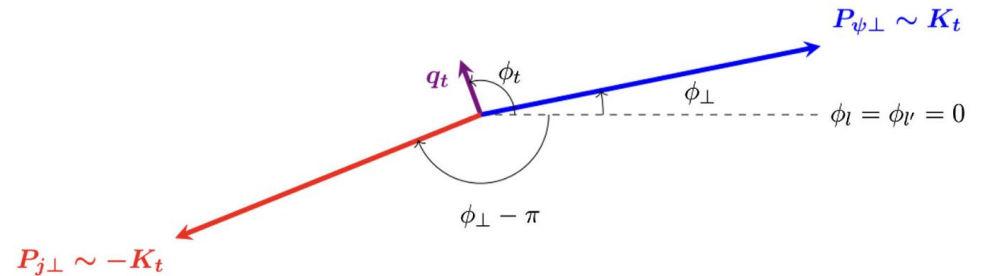
BACK-TO-BACK PRODUCTION OF J/ψ AND JET

$$d\sigma = \frac{1}{2s} \frac{d^3 l'}{(2\pi)^3 2E_{l'}} \frac{d^3 \mathbf{P}_\psi}{2E_\psi (2\pi)^3} \frac{d^3 \mathbf{P}_j}{2E_j (2\pi)^3} \\ \times \int dx d^2 \mathbf{p}_T (2\pi)^4 \delta^4(q + p_g - \mathbf{P}_j - \mathbf{P}_\psi) \\ \times \frac{1}{Q^4} L^{\mu\mu'}(l, q) \Phi_g^{\nu\nu'}(x, \mathbf{p}_T^2) \mathcal{M}_{\mu\nu}^{g\gamma^* \rightarrow J/\psi g} \mathcal{M}_{\mu'\nu'}^{*g\gamma^* \rightarrow J/\psi g}.$$

$$\mathcal{M}(\gamma^* g \rightarrow Q\bar{Q} [^{2S+1}L_J^{(1,8)}] g) \\ = \sum_{L_z S_z} \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \Psi_{LL_z}(\mathbf{k}) \langle LL_z; SS_z | JJ_z \rangle \\ \times \text{Tr}[O(q, p_g, \mathbf{P}_\psi, k) \mathcal{P}_{SS_z}(\mathbf{P}_\psi, k)],$$

Contribution comes from the color singlet state $(^3S_1^{(1)})$ And color octet states $(^3S_1^{(8)}, ^1S_0^{(8)}, ^3P_{J(0,1,2)}^{(8)})$

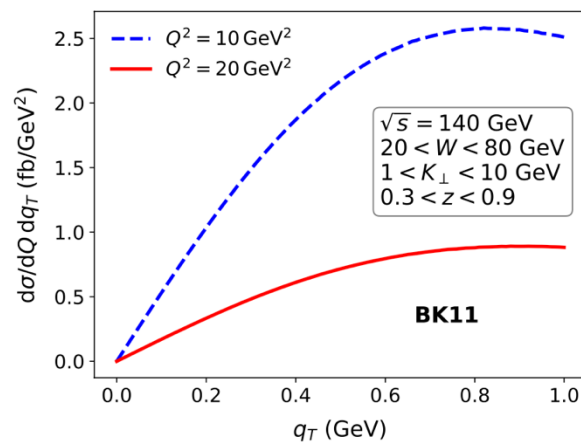
In NRQCD, k , the relative momentum of the charm quark is small. Taylor expanded the amplitude about $k=0$. The first term gives the S wave contribution and second term the p wave contribution. Results depend on LDME set chosen



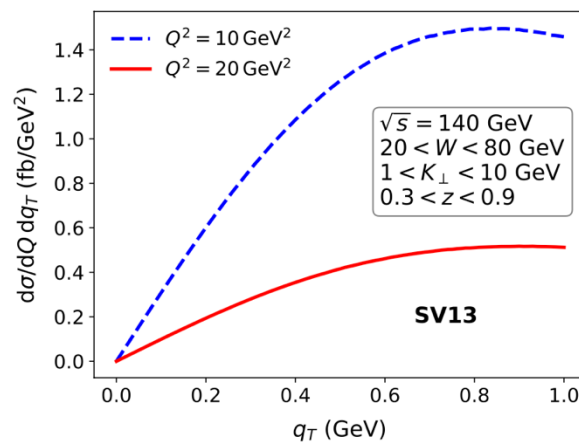
U. D'Alesio, F. Murgia, C. Pisano, and P. Tael *Phys.Rev.D* 100 (2019) 9, 094016

Raj Kishore, AM, Amol Pawar, M. Siddiqah *Phys.Rev.D* 106 (2022) 3, 034009

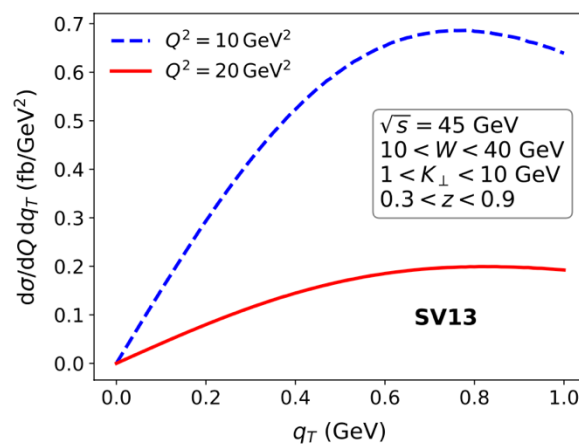
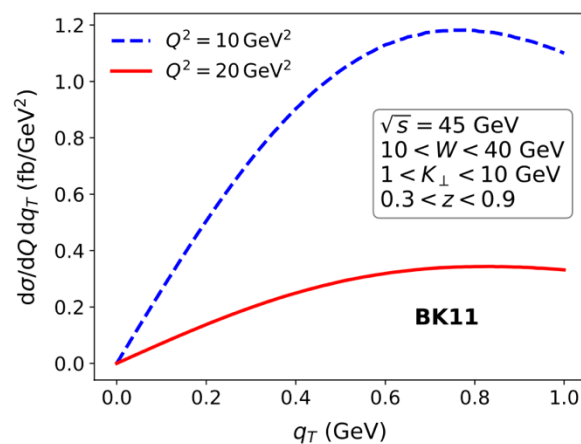
UNPOLARIZED CROSS SECTION IN BACK-TO-BACK ELECTROPRODUCTION OF J/ψ AND PHOTON



(a)



(b)



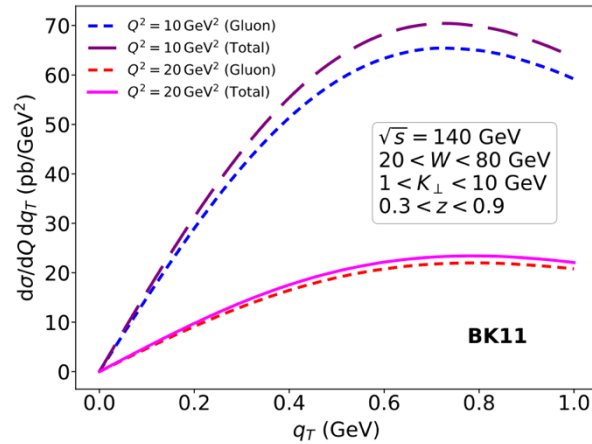
M. Butenschoen and B. A. Kniehl, Physical Review D—Particles, Fields, Gravitation, and Cosmology 84, 051501 (2011)

R. Sharma and I. Vitev, Physical Review C 87, 044905 (2013)

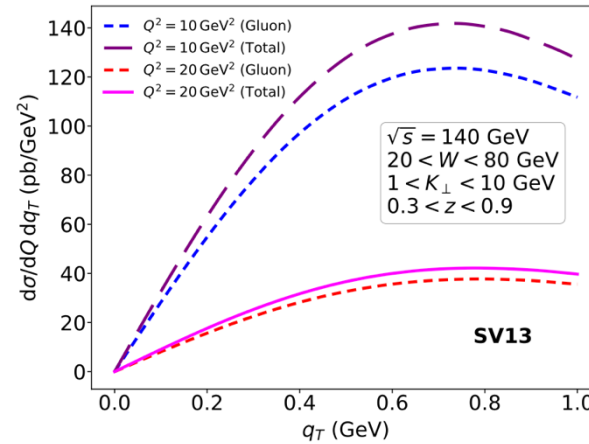
Only the CO^3S_1 state contributes. LDME value of this state in the BK11 set is 1.73 times larger than that of SV13.

Arghya Jana, AM, Sangem Rajesh, e-Print: 2606.31124 [hep-ph]

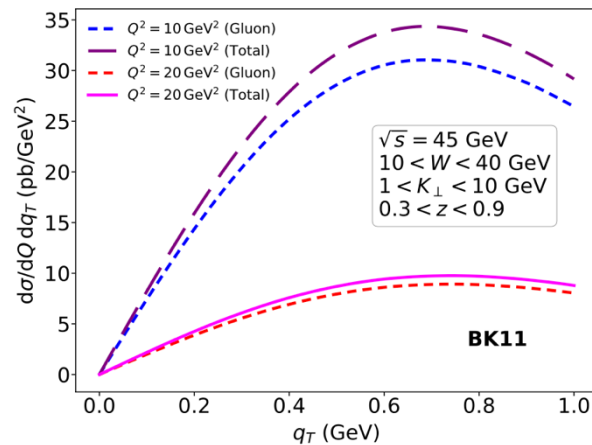
UNPOLARIZED CROSS SECTION IN BACK-TO-BACK ELECTROPRODUCTION OF J/ψ AND JET



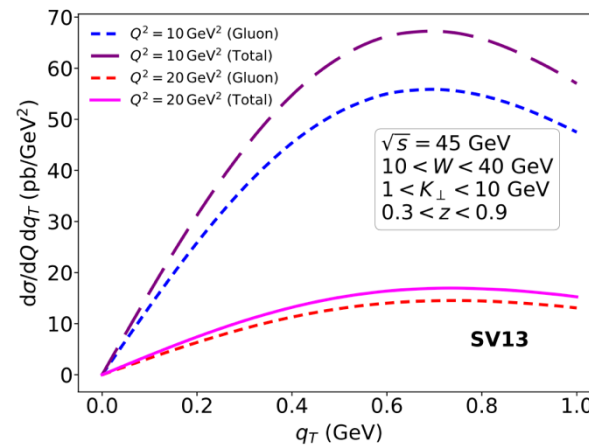
(a)



(b)



(c)



(d)

M. Butenschoen and B. A. Kniehl, Physical Review D—Particles, Fields, Gravitation, and Cosmology 84, 051501 (2011)

R. Sharma and I. Vitev, Physical Review C 87, 044905 (2013)

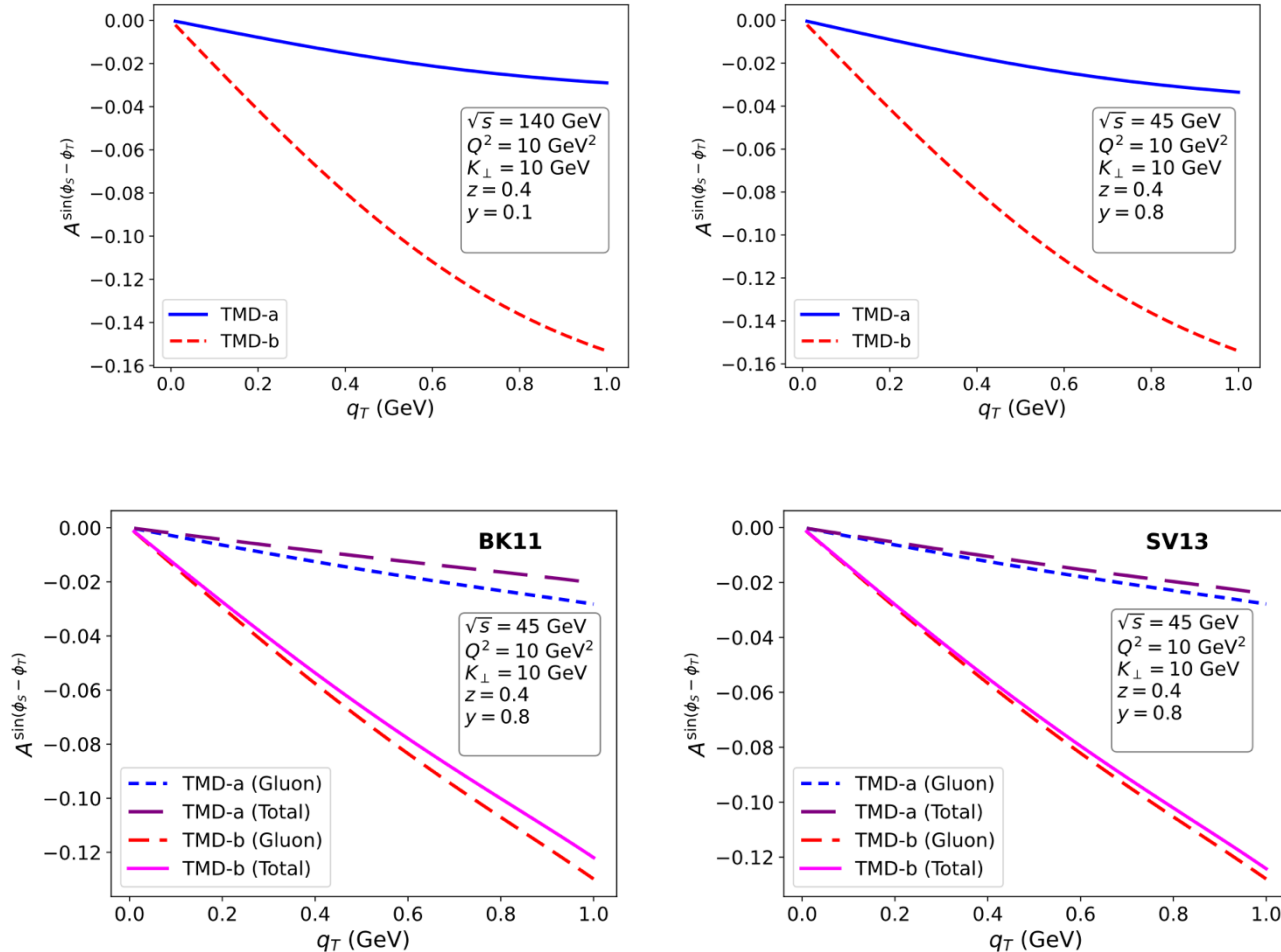
Cross section larger

CS and CO contributions are included, also both quark and gluon channels

Gluon channel contribution dominates over quark channel : ideal to probe gluon TMD

Arghya Jana, AM, Sangem Rajesh, e-Print: 2606.31124 [hep-ph]

SIVERS ASYMMETRY



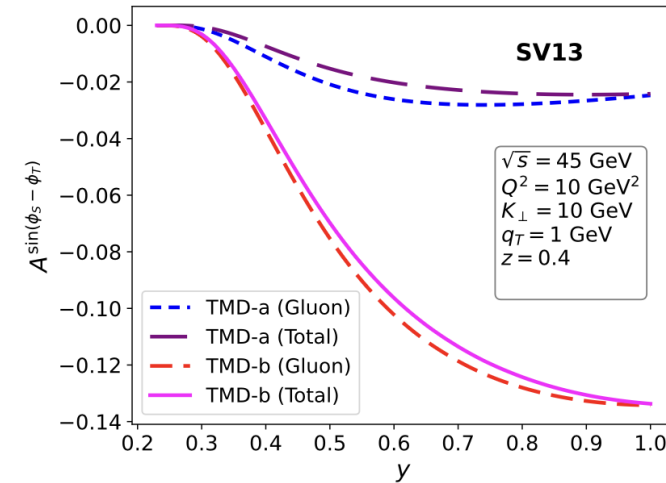
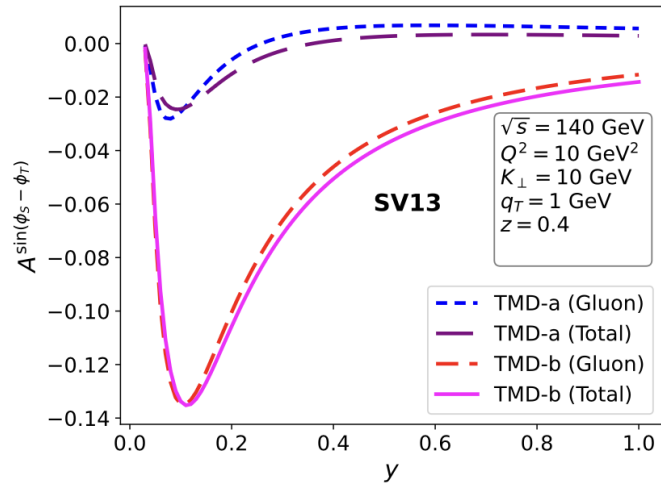
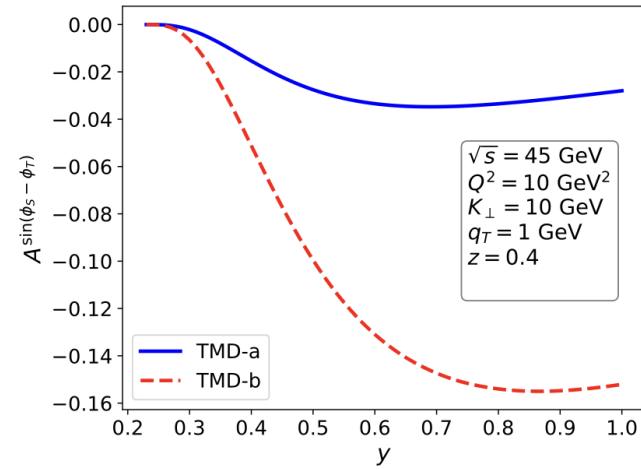
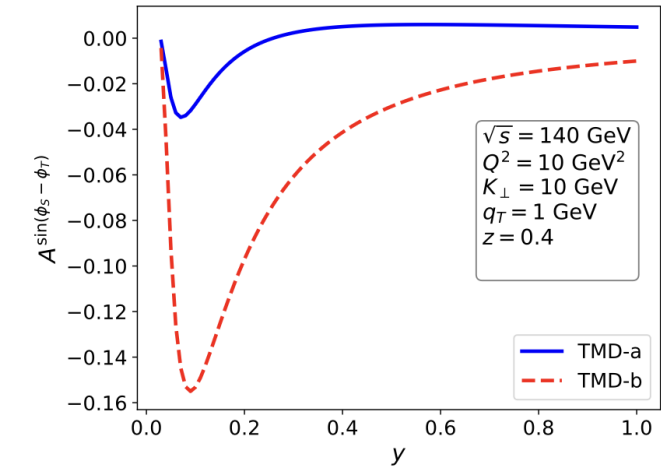
J/ψ and photon

TMD-a : $N_g(x)$ is defined by averaging the $N_u(x)$ and $N_d(x)$ of up-quark and down-quark Sivers functions. Best fit value $N_d(x)$ for the down-quark Sivers function is negative; $N_u(x)$ is positive for the up-quark : smaller value of $N_g(x)$ for GSF in TMD-a parametrization. As a result, the Sivers asymmetry is maximum for the TMD-b parametrization compared to the TMD-a parametrization in all the figures.

J/ψ and jet

Arghya Jana, AM, Sangem Rajesh, e-Print: 2606.31124 [hep-ph]

SIVERS ASYMMETRY



J/ψ and photon

Asymmetry rises at intermediate y , reaches a peak, and then decreases. The cm energy s strongly affects the position of this peak. For larger s peak appears at smaller y and is clearly visible. For smaller s , the peak may not be visible within the kinematic range, resulting in an asymmetry that appears to increase.

J/ψ and jet

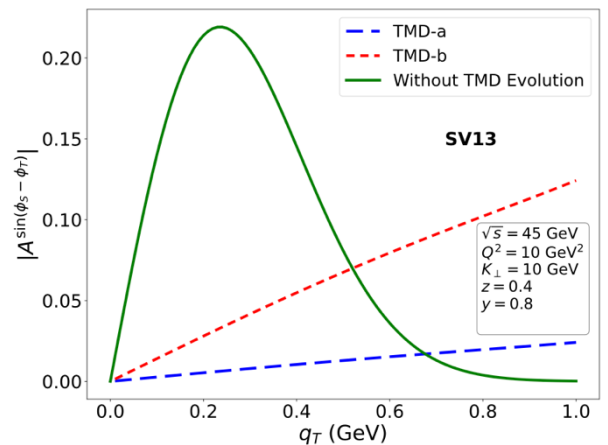
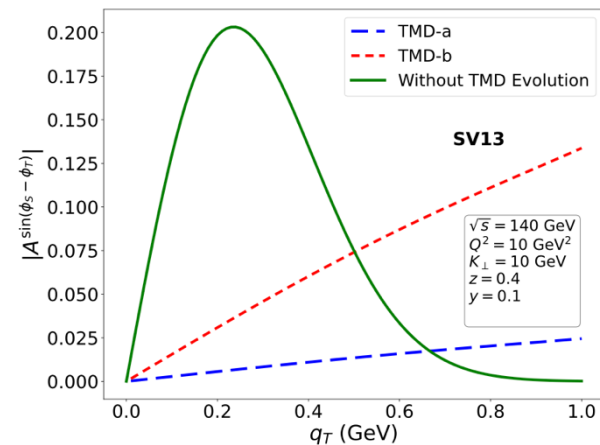
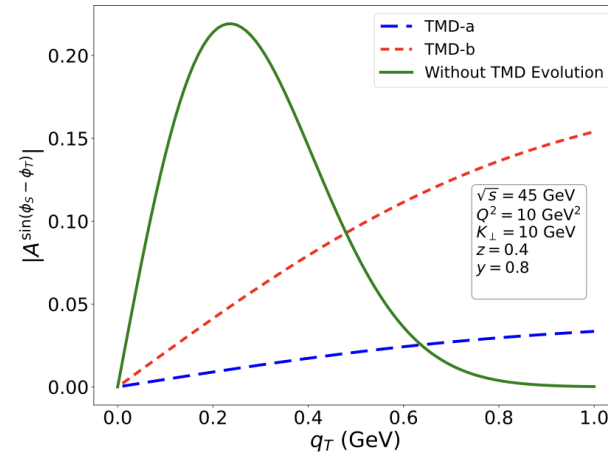
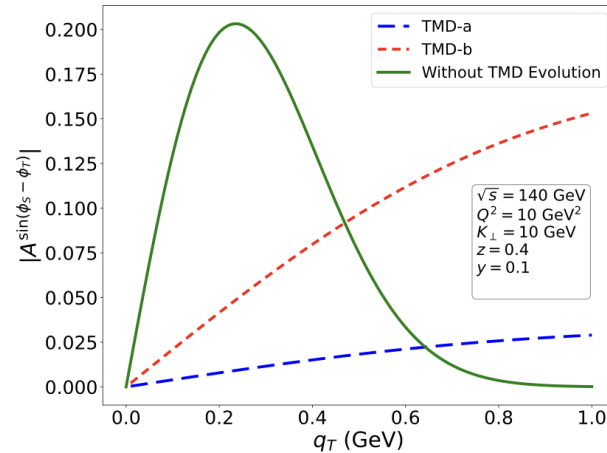
Arghya Jana, AM, Sangem Rajesh, e-Print: 2606.31124 [hep-ph]

SIVERS ASYMMETRY WITH AND WITHOUT TMD EVOLUTION

J/ψ and photon

J/ψ and jet

Arghya Jana, AM, Sangem Rajesh, e-
Print: 2606.31124 [hep-ph]



SUMMARY AND CONCLUSION

J/ψ production processes at the upcoming EIC are useful tools to probe gluon TMDs including gluon Sivers function.

We presented a study of the unpolarized cross section and Sivers asymmetry in $J\psi$ production processes, in particular in the kinematics of the upcoming EIC. We showed the results with and without incorporating TMD evolution. J/ψ production was calculated in NRQCD

We presented results using a Gaussian parametrization of the TMDs, and also using TMD evolution

Sign of the Sivers function depends on the parametrization, in general the results also depend on the choice of LDME set

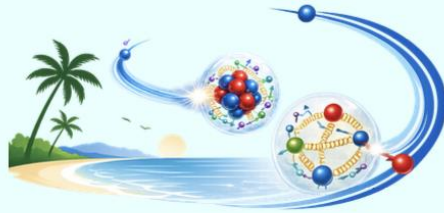
For J/ψ –photon production, cross section depends only on one LDME; asymmetry is independent of LDME and a robust probe of gluon TMD

An almost back-to-back production of J/ψ -photon(jet) are good channels to probe the gluon Sivers function, as in this kinematics TMD factorization is expected to hold. Systematic inclusion of TMD shape function is needed

Physics Opportunities

at an Electron-Ion Collider 2027

15-19 February, Bambolim Beach Resort, Goa, India



POETIC 2027

POETIC XII, the twelfth international conference on “Physics Opportunities at an Electron-Ion Collider”, will be held in Goa, India, continuing the established POETIC conference series. The meeting takes place at an important stage in the development of the future EIC program, marked by significant advances in detector technologies, software and computing, machine learning applications, and growing international participation.

Key topics include the structure of nucleons and nuclei in Quantum Chromodynamics, spin physics, jet physics, small-x dynamics, detector technologies, and AI-driven data analysis.

Local Organizing Committee:

Vinod Chandra (IIT Gandhinagar)
Santosh K. Das (IIT Goa), Convenor
Sabyasachi Ghosh (IIT Bhilai)
Venkatesha R. Hathwar (Goa University)
Satyabrata Mahapatra (IIT Goa)
Prabhakar Palni (IIT Mandi)
Nihar Ranjan Sahoo (IISER Tirupati)

International Advisory Committee

Nestor Armesto (Spain)
Elke Aschenauer (USA)
Alessandro Bacchetta (Italy)
Daniel Boer (Netherlands)
Wim Cosyn (USA)
Silvia Dalla Torre (Italy)
Abhay Deshpande (USA)
Bruno El-Bennich (Brasil)
Rolf Ent (USA)
Yuji Koike (Japan)
Bedangadas Mohanty (India)
Asmita Mukherjee (India)
Kenta Shigaki (Japan)
Marco Stratmann (Germany)
Tony Thomas (Australia)
Thomas Ullrich (USA)
Raju Venugopalan (USA)
Qinghua Xu (China)
Feng Yuan (USA)

National Advisory Committee:

Raktim Abir (AMU)
Dipankar Chakrabarti (IIT Kanpur)
Sadhana Dash (IIT Bombay)
Chitrasen Jena (IISER Tirupati)
Lokesh Kumar (Panjab University)
Nilmani Mathur (TIFR)
Md Nasim (IISER Berhampur)
Prabhat Pujahari (IIT Madras)
V Ravindran (IMSc)
Tobias Toll (IIT Delhi)

Webpage link: <https://indico.cern.ch/event/1685317>

Email ID: poetic2027@iitgoa.ac.in



POETIC 2027 in Goa, India

Registration closes on December 15 th 2026

Student support deadline October 15 th 2026