

Quark orbital angular momentum from one-loop GTMD-GPD matching

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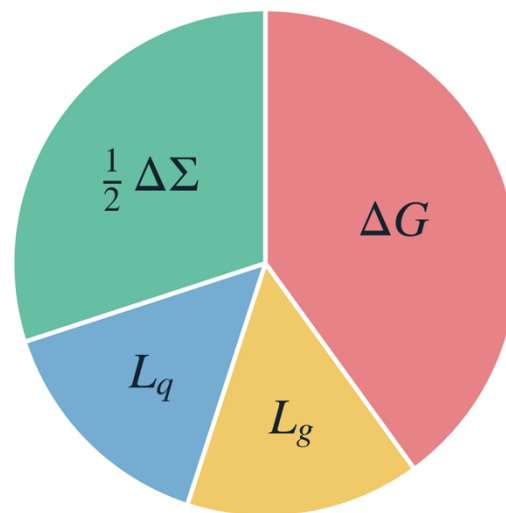
1. Introduction ---- Proton Spin Decomposition

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Spin decomposition

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

Jaffe-Manohar decomposition (canonical)



$$\Delta\Sigma = 0.32 \pm 0.02 \pm 0.04 \pm 0.05$$

$$(\mu^2 = 3 \text{ GeV}^2)$$

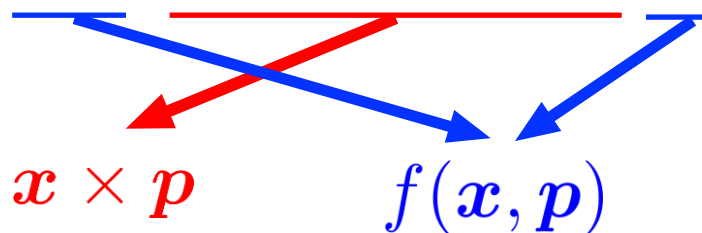
[COMPASS, (2017)]

$$\Delta G = 0.20^{+0.06}_{-0.07} \quad (\mu^2 = 10 \text{ GeV}^2)$$

[DSSV14, (2014)]

Precise determination of the fraction carried by OAM is a key goal of EIC.

$$L_q^{\lambda\mu\nu} = \bar{q}\gamma^\lambda(x^\mu i\partial^\nu - x^\nu i\partial^\mu)q,$$



Wigner, GTMDs

$$L_g^{\lambda\mu\nu} = -F_a^{\lambda\alpha}(x^\mu \partial^\nu - x^\nu \partial^\mu)A_\alpha^a.$$

→ OAM is related to GTMDs

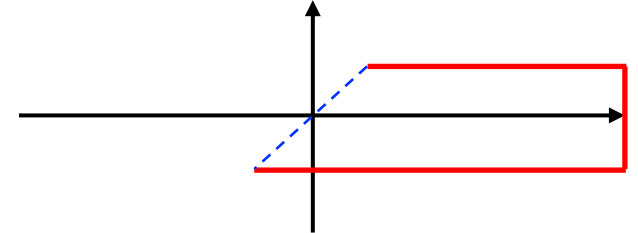
1. Introduction ---- OAM and GTMD

- Quark OAM is

$$L_q = \int dr^- d^2 r_T \bar{q}(r) \gamma^+ \left[\mathbf{r}_T \times \left(-\frac{i}{2} \overleftrightarrow{\nabla}_T \right) \right] q(r).$$

- GTMD correlator at zero skewness is

$$\begin{aligned} W_{++}^{[\gamma^+],q}(x, \mathbf{k}_T, \Delta_T) &= \frac{1}{2} \int \frac{dz^- d^2 z_T}{(2\pi)^3} e^{ixP^+ - i\mathbf{k}_T \cdot \mathbf{z}_T} \langle p', + | \bar{q}(-z/2) \gamma^+ q(z/2) | p, + \rangle \\ &= F_{11}^q + \frac{i(\mathbf{k}_T \times \Delta_T)_z}{M^2} F_{14}^q. \end{aligned}$$



- Then,

$$\begin{aligned} L_q(x) &= \int d^2 k_T i(\mathbf{k}_T \times \nabla_{\Delta_T})_z W_{++}^{[\gamma^+],q}(x, \mathbf{k}_T, \Delta_T) \Big|_{\Delta_T=0} \\ &= - \int d^2 k_T \frac{\mathbf{k}_T^2}{M^2} \underline{F_{14}^q(x, \mathbf{k}_T^2)} \\ &\quad \sim 1/k_T^4 \end{aligned}$$

[Lorcé, Pasquini, Xiong, Yuan, PRD (2012)]

[Kanazawa, Lorcé, Metz, Pasquini, Schlegel, PRD (2014)]

1. Introduction ---- OAM and GPD

OAM is also determined by GPDs

[Hatta, Yoshida, JHEP (2012)]

$$L_q(x) = x \int_x^1 \frac{dy}{y} [H^q(y) + E^q(y)] - x \int_x^1 \frac{dy}{y^2} \tilde{H}^q(y) \\ - x \int_x^1 dy_1 \int_{-1}^1 dy_2 \left[\mathcal{P} \frac{3y_1 - y_2}{y_1^2 (y_1 - y_2)^2} \Phi_F(y_1, y_2) + \mathcal{P} \frac{1}{y_1^2 (y_1 - y_2)} \tilde{\Phi}_F(y_1, y_2) \right].$$

- GPDs we need are $\underbrace{H^q(y), E^q(y), \tilde{H}^q(y)}_{\text{twist-2 GPDs (WW)}}$, $\underbrace{\Phi_F(y_1, y_2), \tilde{\Phi}_F(y_1, y_2)}_{\text{genuine twist-3 GPDs}}$

cf. genuine twist-3 GPDs (quark-gluon-quark)

$$\int \frac{d\lambda}{2\pi} \frac{d\tau}{2\pi} e^{iP^+[\lambda(x_1+x_2)/2+\tau(x_2-x_1)]} \left\langle \bar{q} \left(-\frac{\lambda n}{2} \right) \gamma^+ \left[-\frac{\lambda n}{2}, \tau n \right] gF_{i+}(\tau n) \left[\tau n, \frac{\lambda n}{2} \right] q \left(\frac{\lambda n}{2} \right) \right\rangle = \epsilon_T^{ij} \Delta^j \Phi_F(x_1, x_2).$$

- OAM is given by convolution of GPDs and some coefficients functions

$$-L_q(x) = C_H \otimes H^q + C_E \otimes E^q + C_{\tilde{H}} \otimes \tilde{H}^q + C_{\Phi} \otimes \Phi_F + C_{\tilde{\Phi}} \otimes \tilde{\Phi}_F.$$

1. Introduction ---- GTMD-GPD matching

We have two expressions for OAM

$$\left\{ \begin{array}{l} -L_q(x) = \int d^2 k_T \frac{k_T^2}{M^2} F_{14}^q(x, \mathbf{k}_T^2) = \mathcal{F}_{14}^q(x, \mathbf{b} = 0). \\ -L_q(x) = C_H \otimes H^q + C_E \otimes E^q + C_{\tilde{H}} \otimes \tilde{H}^q + C_{\Phi} \otimes \Phi_F + C_{\tilde{\Phi}} \otimes \tilde{\Phi}_F. \end{array} \right.$$

$$\rightarrow \mathcal{F}_{14}(x, \mathbf{b} = 0) \stackrel{?}{=} C_H \otimes H^q + C_E \otimes E^q + C_{\tilde{H}} \otimes \tilde{H}^q + C_{\Phi} \otimes \Phi_F + C_{\tilde{\Phi}} \otimes \tilde{\Phi}_F.$$

- Naïve $\mathbf{b} \rightarrow 0$ limit has divergence. We need small-b OPE to match GTMD and GPD.
- Where is the rapidity renormalization scale dependence of the right-hand side?
- Our goal is to determine C_X at NLO: $C_X = C_X^{[0]} + \alpha_s C_X^{[1]} + O(\alpha_s^2)$.

$$\mathcal{F}_{14}(x, \mathbf{b}; \mu, \zeta) = \sum_X C_X(x, \mathbf{b}; \mu, \zeta) \otimes X(\mu) + O(\mathbf{b}^2 \Lambda_{\text{QCD}}^2), \quad X = \{H, E, \tilde{H}, \Phi_F, \tilde{\Phi}_F\}$$

1. Introduction ---- Related works

Sivers-ETQS relations for SSA [Ji, Qiu, Vogelsang, Yuan, PRL (2006)]

$$T_F^q(x, x) = - \int d^2 k_T \frac{k_T^2}{M} f_{1T}^{\perp q}(x, \mathbf{k}_T^2), \quad \text{cf. } L_q(x) = - \int d^2 k_T \frac{k_T^2}{M^2} F_{14}^q(x, \mathbf{k}_T^2).$$

- Twist-2 TMD and PDF matching at NLO for quarks is done except pretzelosity

[Bacchetta, Prokudin, Nucl.Phys.B (2013)]

[Rein, Rodini, Schäfer, Vladimirov, JHEP (2023)]

[Scimemi, Tarasov, Vladimirov, JHEP (2019)]

- For gluons, twist-2 TMD and twist-2 PDF are matched, but genuine twist 3 is not...

[Echevarría, Kasemets, Mulders, Pisano, JHEP (2015)]

[Alvaro, Kato, Pasquini, Pisano, Rodini, PRD (2026)]

- For quark and gluon GTMDs, twist-2 GTMD and twist-2 GPD are matched

[Bertone, Eur.Phys.J (2022)]

[Bertone, Echevarría, del Rio, Rodini, JHEP (2025)]

→ Our matching contains the genuine twist-3 GPD for the first time

2. Method ---- Background-field method

- To perform small- b OPE, we decompose fields into hard and soft (background field)

$$q \rightarrow \underbrace{q}_{\text{soft}} + \underbrace{\psi}_{\text{hard}}, \quad A^\mu \rightarrow \underbrace{A^\mu}_{\text{soft}} + \underbrace{B^\mu}_{\text{hard}}. \quad A^\mu(zn + b) = A^\mu(zn) + b^i \partial_i A^\mu(zn) + \dots$$

[Scimemi, Tarasov, Vladimirov, JHEP (2019)]

- Operators are calculated by integrating out fast modes:

$$\tilde{\mathcal{O}}(z, \mathbf{b})[q, A] \propto \int D\psi D\bar{\psi} DB \mathcal{O}(z, \mathbf{b}; q + \psi, A + B) \exp \{iS[q + \psi, A + B] - iS[q, A]\}.$$

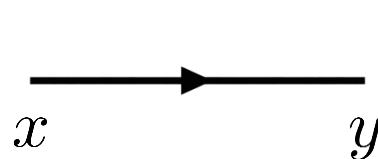
- We fix the gauge:

$$\underbrace{\partial_\mu B^\mu - ig[A_\mu, B^\mu] = 0}_{\text{background Feynman gauge } (\alpha = 1)}, \quad \underbrace{A^+ = 0}_{\text{light-cone gauge}}$$

2. Method --- Background-field method

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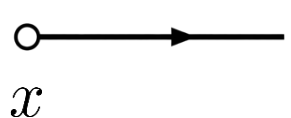
Feynman rules we need



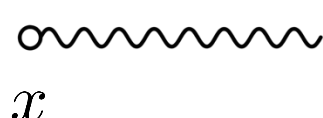
$$\frac{\Gamma(2 - \epsilon)}{(2\pi)^{d/2}} \frac{i(\not{x} - \not{y})}{[-(x - y)^2 + i\epsilon]^{2-\epsilon}}$$



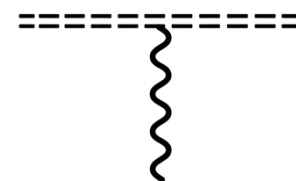
$$-g_{\mu\nu} \frac{\Gamma(1 - \epsilon)}{4\pi^{d/2}} \frac{1}{[-(x - y)^2 + i\epsilon]^{1-\epsilon}}$$



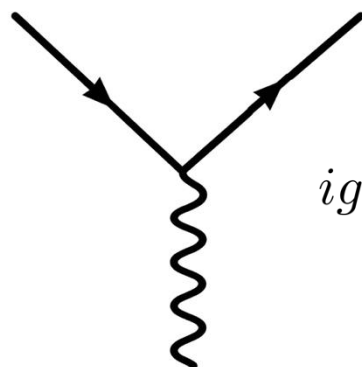
$$q(x)$$



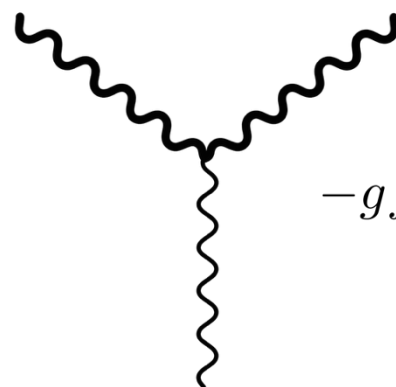
$$A^\mu(x)$$



$$igt^a n^\mu$$



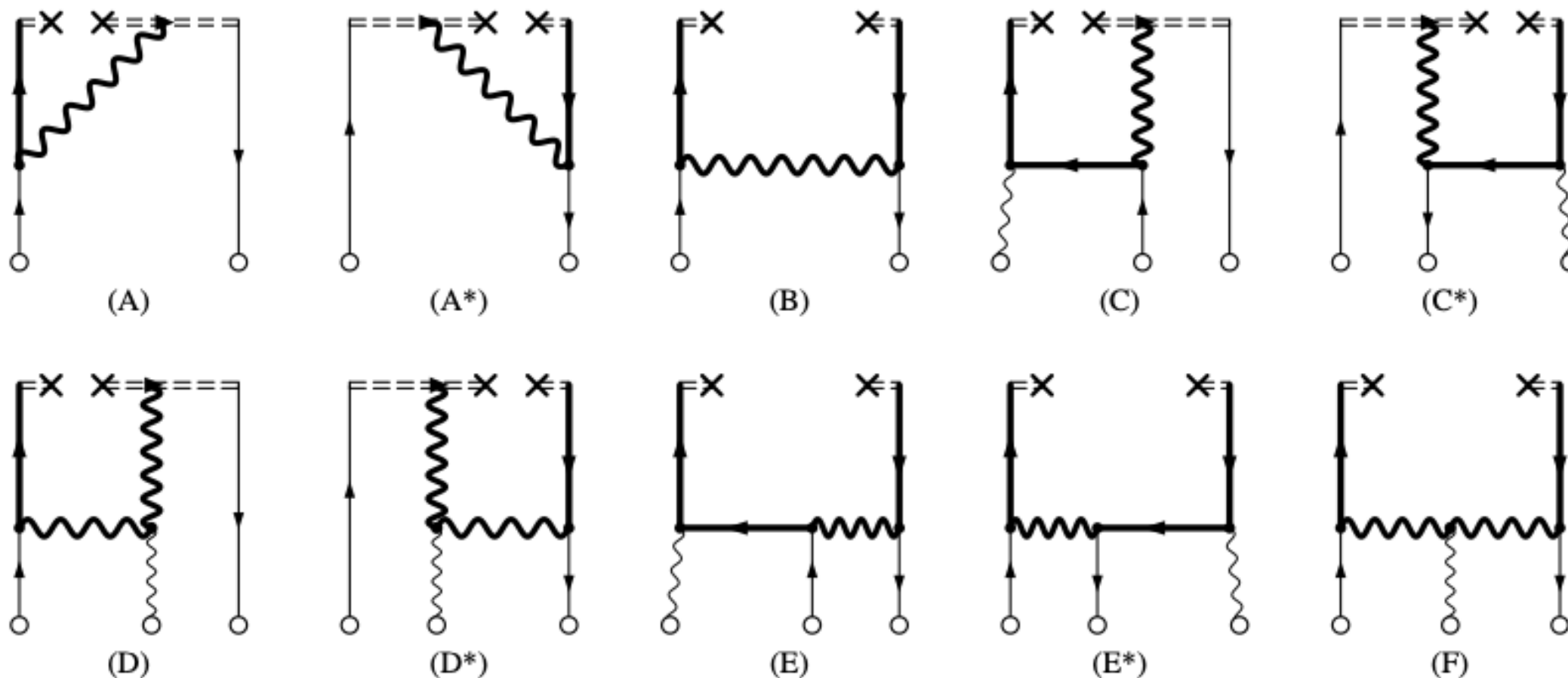
$$ig\gamma^\mu t^a$$



$$-gf^{abc} A_\mu^a(y) (\partial_\alpha B_\beta^b(y)) B_\gamma^c(y) [2g^{\mu\beta} g^{\alpha\gamma} - g^{\mu\alpha} g^{\beta\gamma} - g^{\mu\gamma} g^{\alpha\beta}]$$

3. Calculation ---- One-loop diagrams

[Scimemi, Tarasov, Vladimirov, JHEP (2019)]



These diagrams contain the rapidity, UV, and collinear singularities

3. Calculation ---- Rapidity divergence

Diagram calculation gives coefficient functions

$$\mathcal{F}_{14}(x, b) = \sum_X C_X(x, b) \otimes X + O(b^2 \Lambda_{\text{QCD}}^2), \quad X = \{H, E, \tilde{H}, \Phi_F, \tilde{\Phi}_F\}$$

- Rapidity-divergent part is

$$C_X^{[1], \text{rap. div.}} = -4C_F \Gamma(-\epsilon) e^{\gamma_E \epsilon} \left(\frac{\mu^2 b^2}{4} \right)^\epsilon \ln \frac{\delta}{P^+ x} C_X^{[0]}.$$

↖ δ -regulator

- Soft factor cancels this divergence

$$\mathcal{F}_{14}^{q, \text{soft. sub.}} = R_q \mathcal{F}_{14}^{q, \text{bare}}, \quad C_X^{\text{soft. sub.}} = R_q C_X^{\text{bare}}, \quad R_q = S_q^{-1/2}.$$

ex. $C_{H,E}^{[1], \text{soft-sub.}} = 2C_F \Gamma(-\epsilon) e^{\gamma_E \epsilon} \left(\frac{\mu^2 b^2}{4} \right)^\epsilon \left[2 \ln \left(1 - \frac{x}{y} \right) + \ln \left(\frac{b^2 \zeta}{4} \right) - \psi(-\epsilon) + \gamma_E \right] C_{H,E}^{[0]}$

↖ rapidity scale ζ

$$+ \underbrace{2C_F \Gamma(-\epsilon) e^{\gamma_E \epsilon} \left(\frac{\mu^2 b^2}{4} \right)^\epsilon (1 - \epsilon) \left(\ln \frac{y}{x} + \frac{x}{y} - 1 \right)}_{\text{UV divergence}} C_{H,E}^{[0]},$$

3. Calculation ---- UV divergence of GTMD

- UV renormalization of GTMD [Echevarria, Idilbi, Kanazawa, Lorcé, Metz, Pasquini, Schlegel, PLB (2016)]

$$\mathcal{F}_{14}^{\text{ren}} = Z_2^{-1} Z_q^{\text{TMD}} \mathcal{F}_{14}^{\text{soft.sub.}}, \quad C_X^{\text{GTMD-ren.}} = Z_2^{-1} Z_q^{\text{TMD}} C_X^{\text{soft.sub.}},$$

$$Z_2^{-1} Z_q^{\text{TMD}} = 1 - a_s C_F \left[\frac{2}{\epsilon^2} + \frac{3 + 2 \ln(\mu^2/\zeta)}{\epsilon} \right] + O(a_s^2).$$

- We get
$$C_{H,E}^{[1],\text{GTMD-ren.}} = 2C_F \left\{ -\frac{1}{\epsilon} \left[2 \ln \left(1 - \frac{x}{y} \right) + \frac{3}{2} \right] - \frac{\pi^2}{12} \right. \\ \left. - \ln \left(\frac{\mu^2 \mathbf{b}^2}{4e^{-2\gamma_E}} \right) \left[2 \ln \left(1 - \frac{x}{y} \right) + \frac{1}{2} \ln \left(\frac{\mu^2 \mathbf{b}^2}{4e^{-2\gamma_E}} \right) - \ln \left(\frac{\mu^2}{\zeta} \right) \right] + O(\epsilon) \right\} C_{H,E}^{[0]} \\ + 2C_F \left[-\frac{1}{\epsilon} + 1 - \ln \left(\frac{\mu^2 \mathbf{b}^2}{4e^{-2\gamma_E}} \right) + O(\epsilon) \right] \left(\ln \frac{y}{x} + \frac{x}{y} - 1 \right) C_{H,E}^{[0]}$$

- Still divergent because

$$\mathcal{F}_{14}^{\text{ren.}}(x, \mathbf{b}; \zeta, \mu) = C_H^{\text{GTMD-ren.}}(x, \mathbf{b}; \zeta, \mu) \otimes H^{\text{bare}} + \dots$$

3. Calculation ---- GPD renormalization

- Renormalization of GPDs

$$f_2^{\text{bare}} = \left(1 + \frac{a_s}{\epsilon} P_{qq}\right) \otimes f_2, \quad f_2 \in \{H^q, E^q, \tilde{H}^q\}.$$

P_{qq} :DGLAP kernel

$$f_3^{\text{bare}} = \left(1 - \frac{a_s}{2\epsilon} \mathbb{K}\right) \otimes f_3, \quad f_3 \in \{\Phi_F, \tilde{\Phi}_F\}.$$

$\mathbb{K}$:BFLK kernel

[Bukhvostov, Frolov, Lipatov, Kuraev, NPB (1985)]

- All poles are cancelled!!

$$C_{H,E}^{[1]} = 2C_F \left\{ -\frac{\pi^2}{12} - \ln\left(\frac{\mu^2 \mathbf{b}^2}{4e^{-2\gamma_E}}\right) \left[2 \ln\left(1 - \frac{x}{y}\right) + \frac{1}{2} \ln\left(\frac{\mu^2 \mathbf{b}^2}{4e^{-2\gamma_E}}\right) - \ln\left(\frac{\mu^2}{\zeta}\right) \right] \right\} C_{H,E}^{[0]}$$

- Evolution equations for coefficients

$$\frac{\partial}{\partial \ln \zeta} C_X^{[1]} = -2C_F \ln\left(\frac{\mu^2 \mathbf{b}^2}{4e^{-2\gamma_E}}\right) C_X^{[0]},$$

$$\frac{\partial}{\partial \ln \mu^2} C_2^{[1]} = C_F \left[3 + 2 \ln\left(\frac{\mu^2}{\zeta}\right) \right] C_2^{[0]} - C_2^{[0]} \otimes P_{qq}$$

$$\frac{\partial}{\partial \ln \mu^2} C_3^{[1]} = C_F \left[3 + 2 \ln\left(\frac{\mu^2}{\zeta}\right) \right] C_3^{[0]} - C_3^{[0]} \otimes \mathbb{K}$$

Key results

- We improve on the naïve kT moment relation, $L_q(x) = - \int d^2 k_T \frac{k_T^2}{M^2} F_{14}^q(x, \mathbf{k}_T^2).$

using more controlled way, small-b operator product expansion at NLO

$$\mathcal{F}_{14}(x, \mathbf{b}; \mu, \zeta) = \sum_X C_X(x, \mathbf{b}; \mu, \zeta) \otimes X(\mu) + O(\mathbf{b}^2 \Lambda_{\text{QCD}}^2)$$

Outlook

- For the flavor singlet sector, we need the gluon counterpart
 - We need to finish TMD-PDF matching relation for gluon Sivers function
- I already completed the axial counterpart, spin-orbit correlations and G11 moment