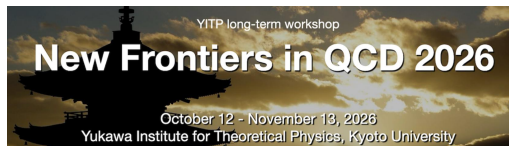


Analysis of Symmetry of 2-point Correlation Functions in Finite Temperature QCD

Using Möbius Domain-Wall Fermions



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for JLQCD Collaboration

Outline

- Research Background & Motivation
- Theoretical Background
- Lattice QCD Simulation Setup
- Chiral Condensate & Susceptibility Analysis
- Meson Screening Masses & Symmetry Restoration
- Summary and Outlook

Research Background

- $U(1)_A$ restoration at T_c affects the order of the chiral transition
[Pisarski & Wilczek (1984)]
- Staggered fermion behave incorrectly at finite lattice spacing a
 $\Rightarrow$ HISQ study showed $U(1)_A$ symmetry restored at temperature higher than pseudo critical temperature
- $N_f = 2$ MDWF (JLQCD): chiral flavor and $U(1)_A$ symmetry both restored near T_c
[Phys. Rev. D **111**, 114506 (2025)]

This Work

- $N_f = 2 + 1$ QCD with **Möbius domain-wall fermions**
- Physical point, $48^3 \times 16$, $T = 138\text{--}242$ MeV
 - Chiral condensate & susceptibility $\Rightarrow T_c$
 - Meson screening masses
- Fixed $\beta = 4.17$, $N_t = 12\text{--}18$
 - chiral limit

Symmetry of QCD

$$\mathcal{Z} = \int \mathcal{D}A \mathcal{D}\bar{q} \mathcal{D}q e^{-S}, \quad S = \int d^4x \left[-\frac{1}{2g^2} \text{Tr} F^{\mu\nu} F_{\mu\nu} + \bar{q}_a (i \not{D} - m_a) q_a \right]$$

For the limit of $m_{u,d} \rightarrow 0$:

$SU(2)_L \times SU(2)_R$ chiral symmetry:

$$q \rightarrow \exp(i\alpha_a T^a + i\beta_a T^a \gamma_5) q, \quad \bar{q} \rightarrow \bar{q} \exp(-i\alpha_a T^a + i\beta_a T^a \gamma_5)$$

$U(1)_A$ symmetry (anomalous):

$$q \rightarrow \exp(i\epsilon \gamma_5) q, \quad \bar{q} \rightarrow \bar{q} \exp(i\epsilon \gamma_5)$$

Meson Correlation Functions

Meson operators:

$$O_{\Gamma}(x) = \bar{q}(x)\Gamma F^a q(x),$$

$$\Gamma \in \{\mathbf{1}(S), \gamma_5(PS), \gamma_{\mu}(V), \gamma_{\mu}\gamma_5(A), \sigma_{\mu\nu}(T), \sigma_{\mu\nu}\gamma_5(X)\}$$

Spatial correlators define the screening mass m_{Γ} :

$$C_{\Gamma}(x) \simeq A \cosh\left(m_{\Gamma} \left(x - \frac{N_s}{2}\right)\right)$$

Symmetry of Correlation Functions

- Flavored chiral symmetry $SU(2)_L \times SU(2)_R$: $\delta_C O_\Gamma^a = \frac{i}{2} \beta_b \bar{q} \left(\{\Gamma, \gamma_5\} \frac{1}{2} \delta^{ab} \mathbf{1} + [\Gamma, \gamma_5] i \epsilon^{bac} \frac{\tau^c}{2} \right) q$
- $U(1)_A$ Symmetry : $\delta_A O_\Gamma^a = i \epsilon \bar{q} \{\Gamma, \gamma_5\} \frac{\tau^a}{2} q$

Γ	$[\Gamma, \gamma_5]$	$\{\Gamma, \gamma_5\}$
$\mathbf{1}(S)$		γ_5
$\gamma_5(P)$		$\mathbf{1}$
$\gamma_\mu(V)$	$\gamma_\mu \gamma_5$	
$\gamma_\mu \gamma_5(A)$	γ_μ	
$\sigma_{\mu\nu}(T)$		$\sigma_{\mu\nu} \gamma_5$
$\sigma_{\mu\nu} \gamma_5(X)$		$\sigma_{\mu\nu}$

Γ	Reference Name	Abbr.	Symmetry Correspondences
$\mathbb{I}$	Scalar	S	$\left. \begin{array}{l} U(1)_A \end{array} \right\}$
γ_5	Pseudo Scalar	PS	
γ_1, γ_2	Vector	V	$\left. \begin{array}{l} SU(2)_L \times SU(2)_R \end{array} \right\}$
$\gamma_1 \gamma_5, \gamma_2 \gamma_5$	Axial Vector	A	
$\gamma_4 \gamma_3$	Tensor	T_t	$\left. \begin{array}{l} U(1)_A \end{array} \right\}$
$\gamma_4 \gamma_3 \gamma_5$	Axial Tensor	X_t	
			$SU(2)_{CS}$

Expectations at High Temperature

- NRQCD₃ study: spin-independent screening mass [Laine & Vepsäläinen (2004)]

$$m_{\text{scr}} \approx 2\pi T + 0.1872 g^2 T$$

- $U(1)_A$: expected to be restored as topological fluctuations are suppressed.
- Chiral-spin symmetry $SU(2)_{CS}$: Possible larger emergent symmetry

$$q \rightarrow \exp(i\Sigma^i \epsilon_i) q, \quad \Sigma^i = (\gamma_k, -i\gamma_5 \gamma_k, \gamma_5), \quad k = 1, 2$$

Domain-Wall Fermions (DWF)

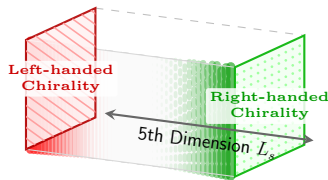
- effective 4D operator satisfied GW relationship:

$$D_{\text{DW}}^{4\text{D}}(m) = \frac{1+m}{2} + \frac{1-m}{2} \gamma_5 \tanh\left(L_s \tanh^{-1}(H_M)\right)$$

- Finite $L_s \Rightarrow$ small residual chiral breaking m_{res} :

$$\bar{\psi} \Delta_L \psi \simeq m_{\text{res}} \bar{\psi} \psi + \mathcal{O}(a^2)$$

- We use $L_s = 12$, $b_5 = 1.5$, $c_5 = 0.5$.



$$D_{s,s'}^{5\text{D}}(m) = D_+^{(s)} \delta_{s,s'} + D_-^{(s)} (P_+ \delta_{s-1,s'} + P_- \delta_{s+1,s'}) \\ - m (D_-^{(1)} P_+ \delta_{s,1} \delta_{s',L_s} + D_-^{(L_s)} P_- \delta_{s,L_s} \delta_{s',1})$$

$$\text{where } D_+^{(s)} = b_5 D_W + 1, \quad D_-^{(s)} = c_5 D_W - 1,$$

$$P_{\pm} = \frac{1 \pm \gamma_5}{2} \quad (b_5 = 1.5, c_5 = 0.5).$$

Residual Mass & Renormalization

Residual mass from the midpoint density J_{5q} :

$$am_{\text{res}} = \frac{\sum_{\vec{x}} \langle J_{5q}(\vec{x}, t) P(\vec{0}, 0) \rangle}{\sum_{\vec{x}} \langle P(\vec{x}, t) P(\vec{0}, 0) \rangle}$$

β	4.13	4.15	4.17	4.18	4.20	4.23	4.30	4.405
T (MeV)	138.23	145.75	153.31	157.03	164.55	176.43	202.52	241.60
am_{res}	7.31e-4	4.98e-4	3.40e-4	2.80e-4	1.91e-4	1.08e-4	2.81e-5	3.75e-6
m_{res}/m_l	0.9087	0.5360	0.3394	0.2744	0.1836	0.1041	0.0299	0.0049
m_{res} (MeV)	1.61	1.16	0.832	0.703	0.503	0.304	0.0908	0.0145
$Z_m(\beta)$	0.9377	0.9524	0.9662	0.9729	0.9857	1.0038	1.0422	1.0927

Lattice Setup & Ensembles

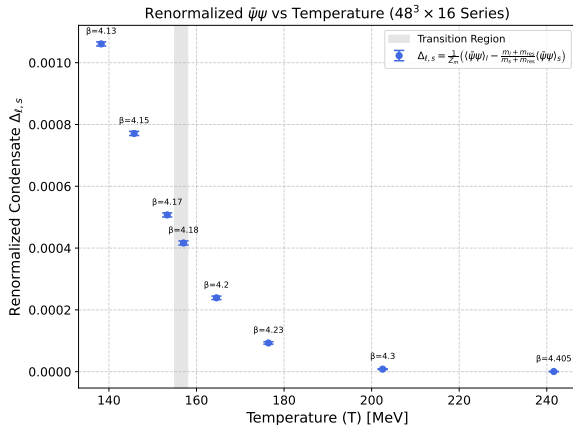
Physical Point LCP Setup ($48^3 \times 16$, $N_f = 2 + 1$)

β	T (MeV)	am_s	am_l	$N_{\text{cfg}}^{\text{cond}}$	$N_{\text{cfg}}^{\text{meson}}$ (16-src)
4.13	138.23	0.043547	0.000805	2000	395 (6320)
4.15	145.75	0.040843	0.000930	2024	398 (6368)
4.17	153.31	0.038400	0.001001	2008	396 (6336)
4.18	157.03	0.037265	0.001022	2029	402 (6432)
4.20	164.55	0.035150	0.001041	2072	407 (6512)
4.23	176.43	0.032315	0.001033	2012	395 (6320)
4.30	202.52	0.026930	0.000939	2052	404 (6464)
4.405	241.60	0.021032	0.000760	4855	—

Fixed $\beta = 4.17$ Setup ($a^{-1} = 2453$ MeV)

Lattice Size	am_s	am_l	T (MeV)
$32^3 \times 12$	0.040	0.0020, 0.0035, 0.0070, 0.0120	204.00
$32^3 \times 14$	0.040	0.0020, 0.0035, 0.0070, 0.0120	174.86
$40^3 \times 16$	0.040	0.0020, 0.0035, 0.0070, 0.0120	153.00
$32^3 \times 16$	0.040	0.0020, 0.0035, 0.0070, 0.0120	153.00
$48^3 \times 18$	0.040	0.0020, 0.0035, 0.0070, 0.0120	136.00
$36^3 \times 18$	0.040	0.0020, 0.0035, 0.0070, 0.0120	136.00

Chiral Condensate (Lattice Units)

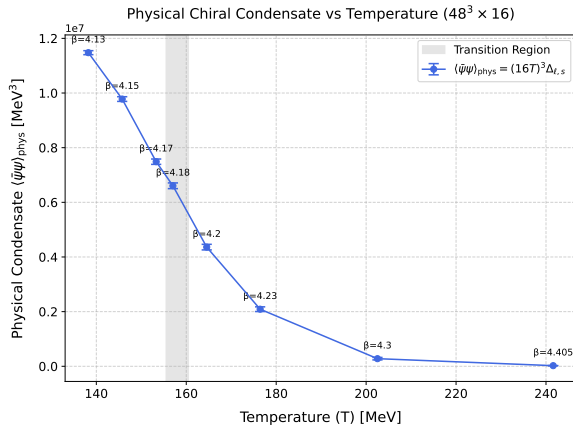


Dimensionless subtracted condensate:

$$\Delta_{\ell,s}^R = \frac{1}{Z_m(\beta)} \left[\langle \bar{\psi}\psi \rangle_\ell - \frac{m_\ell + m_{\text{res}}}{m_s + m_{\text{res}}} \langle \bar{\psi}\psi \rangle_s \right]$$

- Evaluated directly on lattice in dimensionless units.
- Strange-quark subtraction cancels additive UV divergence.
- Normalized by scalar renormalization factor $Z_m(\beta)$.

Chiral Condensate (Physical Units)

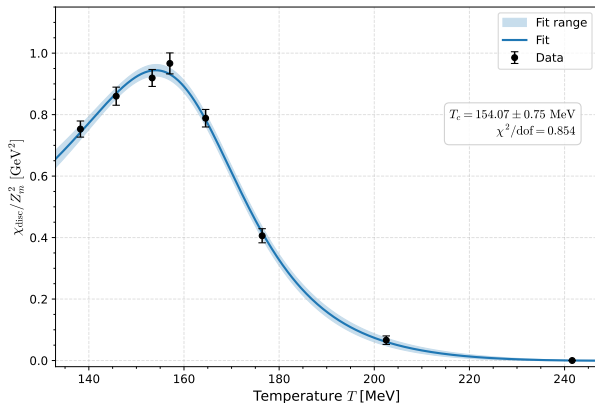


Conversion to physical units:

$$\langle \bar{\psi}\psi \rangle_{\text{phys}} = (N_t T)^3 \Delta_{\ell,s}^R = \frac{1}{a^3} \Delta_{\ell,s}^R$$

- Converted using lattice scale setting $a(\beta)$ via $T = 1/(N_t a)$.
- Approaches zero at high temperature ($T \gtrsim 200$ MeV).

Chiral Susceptibility & T_{pc}



Disconnected susceptibility ($48^3 \times 16$):

$$\chi_{\text{disc}} = \frac{T}{V} (\langle (\text{Tr } D_\ell^{-1})^2 \rangle - \langle \text{Tr } D_\ell^{-1} \rangle^2)$$

- Clear peak near $T \approx 155$ MeV
- Fit to data:

$$T_{pc} = 154.1(8) \text{ MeV}$$

- Consistent with HotQCD
156.5(1.5) MeV

Extracting Screening Masses

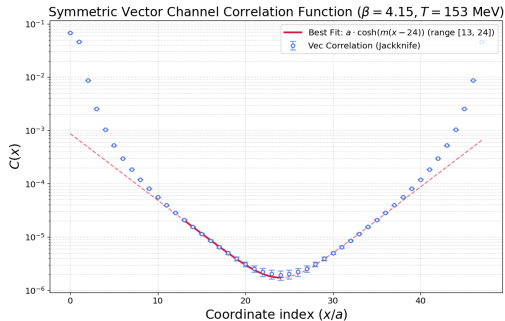
- Average over 16 sources and spatial directions (x, y, z)
- Effective mass to choose the plateau region:

$$\frac{C(x)}{C(x+1)} = \frac{\cosh(m_{\text{eff}}(x - N_s/2))}{\cosh(m_{\text{eff}}(x+1 - N_s/2))}$$

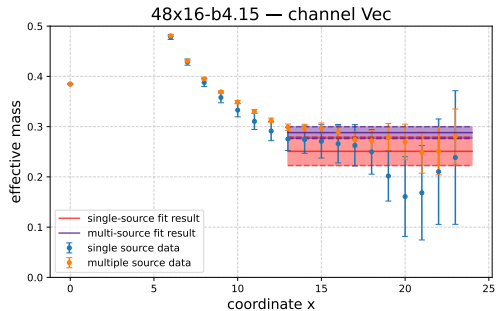
- cosh fit in the plateau, errors from jackknife:

$$C(x) = A \cosh \left(m_{\text{scr}} \left(x - \frac{N_s}{2} \right) \right)$$

Fit Example ($\beta = 4.15, 48^3 \times 16$)



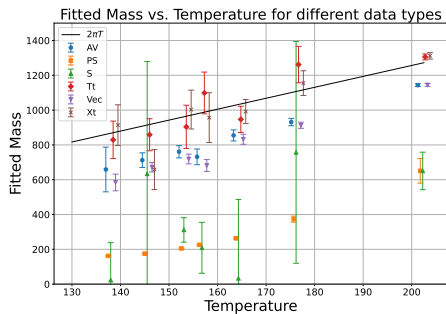
Correlator



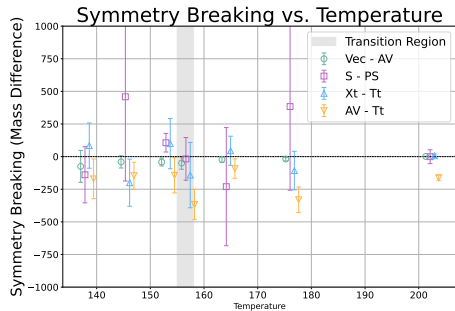
Effective mass: 16 sources vs 1 source

Numerical Results

Multi-source analysis results (grey band denotes $T_c = 156.5 \pm 1.5$ MeV crossover region):



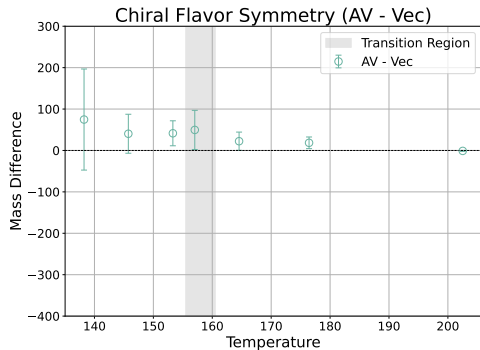
Simulated effective masses of different channels



Symmetry breaking of different channel pairs

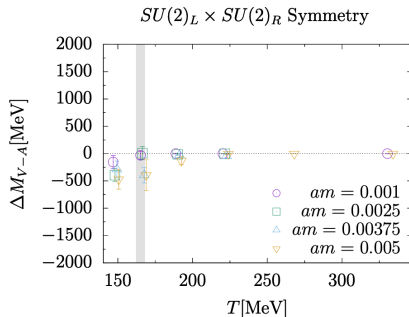
$SU(2)_L \times SU(2)_R$ Chiral Symmetry

Our Result ($N_f = 2 + 1$ MDWF)



Restoration behavior of chiral symmetry

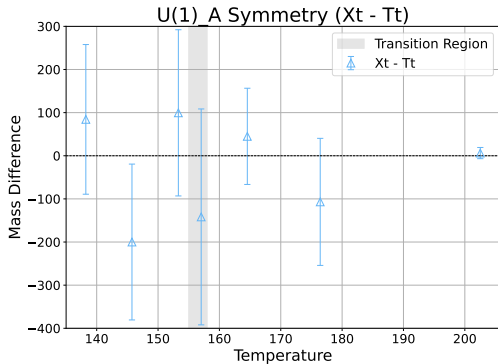
Previous Result ($N_f = 2$ MDWF)



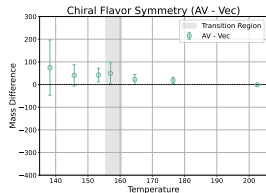
For comparison [PRD **111**, 114506 (2025)]

$U(1)_A$ Symmetry ($X_t - T_t$)

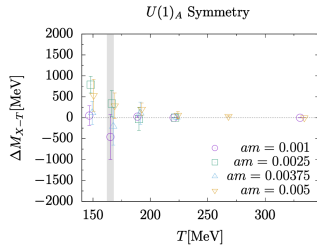
Our $U(1)_A$ ($N_f = 2 + 1$ MDWF)



Our Chiral ($N_f = 2 + 1$ MDWF)

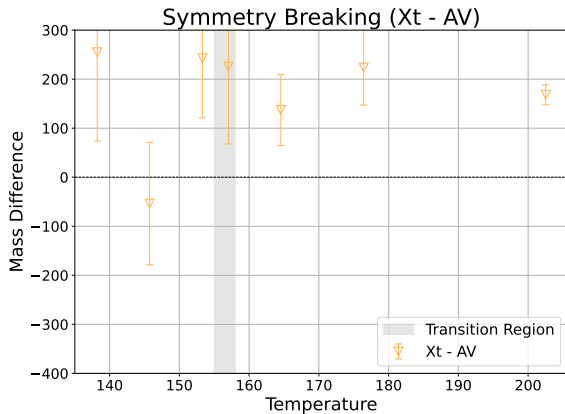


Previous $U(1)_A$ ($N_f = 2$ MDWF)



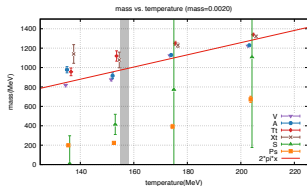
$SU(2)_{CS}$ Symmetry

- Tested via mass difference between X_t and AV channels.
- Broken up to $T = 202.5$ MeV ($\Delta M \approx 164(19)$ MeV).
- Chiral-spin symmetry is not observed in the analyzed temperature range.

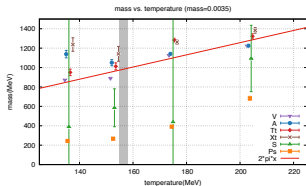


Screening Masses at Fixed $\beta = 4.17$

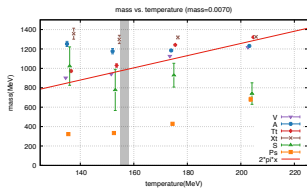
Temperature dependence of screening masses for different light quark masses ($a^{-1} = 2453$ MeV):



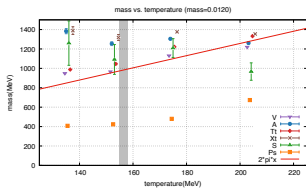
$$am_l = 0.0020$$



$$am_l = 0.0035$$



$$am_l = 0.0070$$

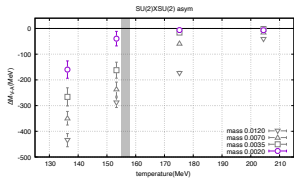


$$am_l = 0.0120$$

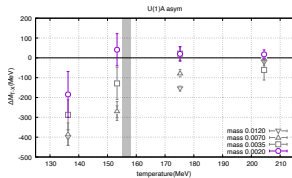
- As temperature increases, partner channels in PS-S and V-AV sectors become degenerate.
- This degeneration is observed for all simulated light quark masses.

Symmetry Restoration at Fixed $\beta = 4.17$

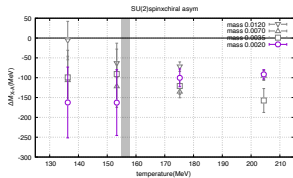
Asymmetry observables as a function of temperature for various light quark masses:



$SU(2)_L \times SU(2)_R$ (V-AV)



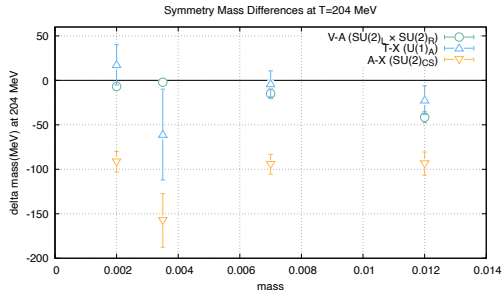
$U(1)_A$ (T-X)



$SU(2)_{CS}$ (X-AV)

- All symmetries show a clear trend towards restoration above T_C .
- $U(1)_A$ and chiral $SU(2)_L \times SU(2)_R$ symmetries show an almost simultaneous restoration as we approach the chiral limit.
- Chiral-spin symmetry $SU(2)_{CS}$ remains broken in the analyzed temperature range.

Chiral Limit Extrapolation ($T \approx 204$ MeV)



Mass differences between mesonic pairs as a function of am_l at $T \approx 204$ MeV

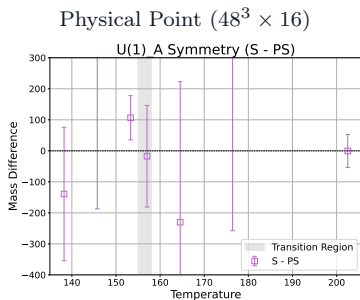
- As $am_l \rightarrow 0$, the mass differences between chiral pairs ($V - AV$) and axial pairs ($T - X$) extrapolate toward zero.
- Supports simultaneous restoration of $SU(2)_L \times SU(2)_R$ and $U(1)_A$ in the chiral limit.
- In contrast, $SU(2)_{CS}$ breaking ($X - AV$) remains non-zero even in the chiral limit.

Summary and Outlook

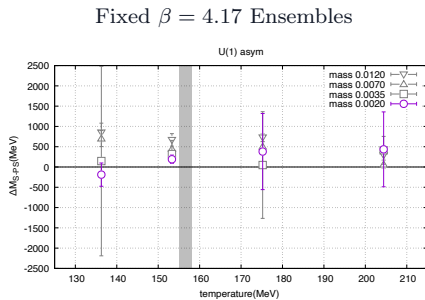
- **Physical Point:** Crossover transition with $T_{pc} = 154.1(8)$ MeV determined by chiral condensation and disconnected susceptibility. Above T_c , chiral flavor and $U(1)_A$ symmetry breakings are consistent with zero within errors. But the scalar channel is noisier than $N_f = 2$ case.
- **Chiral Limit Study:** Extrapolations to $am_l \rightarrow 0$ both of the breaking of chiral flavor and $U(1)_A$ symmetries consistent with zero within large noisy errors.
- **Chiral-Spin Symmetry:** Remains broken in the analyzed temperature range (up to 204 MeV).
- **Outlook:** try to implement more sophisticated statistical methods to improve data quality; measure higher-order fluctuations, etc.

Backup Slides

Backup: Scalar (S) Channel and $S - PS$ Splitting



$U(1)_A$ breaking in $S - PS$ channel



ΔM_{S-PS} across different quark masses

- The scalar (S) channel suffers from severe statistical fluctuations and large noise compared to other channels.
- Large error bars and oscillations in $S - PS$ splitting motivate using $X_t - T_t$ as the primary probe for $U(1)_A$ restoration.

Backup: Overlap Dirac Operator

The massless operator $D_{\text{ov}}(0) = \frac{1}{2}(1 + \gamma_5 \text{sgn}(H_W))$ with Hermitian H_W satisfies the Ginsparg-Wilson relation:

$$\gamma_5 D_{\text{ov}} + D_{\text{ov}} \gamma_5 = 2 D_{\text{ov}} \gamma_5 D_{\text{ov}}$$

The massive overlap operator is defined as:

$$D_{\text{ov}}(m) = D_{\text{ov}}(0) + m(1 - D_{\text{ov}}(0)) = \frac{1+m}{2} + \frac{1-m}{2} \gamma_5 \text{sgn}(H_W)$$

$$\hat{\gamma}_5 = \gamma_5 (1 - 2D_{\text{ov}}(0)) = -\gamma_5 \text{sgn}(H_W) \quad (1)$$

which is hermitian and satisfies $\hat{\gamma}_5^2 = 1$ by virtue of the Ginsparg-Wilson relation. The lattice chiral transformation is defined as:

$$\delta\psi = i\alpha \hat{\gamma}_5 \psi, \quad \delta\bar{\psi} = i\alpha \bar{\psi} \gamma_5 \quad (2)$$

Backup: Order of Chiral Phase Transition & Scenarios

- Perturbative $4 - \epsilon$ expansion RG analysis of effective Φ^4 linear sigma model shows no stable IR fixed point for $N_f > \sqrt{3}$ flavor symmetry [Pisarski-Wilczek (1984)].
- For $N_f \geq 3$, the anomaly determinant term doesn't change the order of the transition; for $N_f = 2$, it becomes a quadratic mass-like term which can smooth out the singularity and make the transition 2nd-order.
- **Previous Scenario:** 1st-order if $U(1)_A$ is restored at T_c ; 2nd-order $O(4)$ if $U(1)_A$ remains broken.
- **Recent Higher-Loop / FRG / Conformal Bootstrap Scenarios:** $O(4)$ if $U(1)_A$ remains broken; weak 1st-order or 2nd-order ($O(4) \times \mathbb{Z}_2$, $O(4) \times \mathbb{Z}_4$, $O(4) \times O(2)$) if $U(1)_A$ is restored [Pelissetto-Vicari (2013), Nakayama-Ohtsuki (2016), Fejos (2022)].

Backup: Condensate & Susceptibility Data Tables

Standard $48^3 \times 16$ LCP Scan

β	T [MeV]	$\Delta_{\ell,s}^R$	$\langle \bar{\psi}\psi \rangle_{\text{sub}}^R$ [GeV ³]	χ_{disc} [GeV ²]
4.13	138.23	$1.0608(67) \times 10^{-3}$	$1.148(7) \times 10^{-2}$	0.662(23)
4.15	145.75	$7.7109(68) \times 10^{-4}$	$9.779(86) \times 10^{-3}$	0.780(27)
4.17	153.31	$5.0728(66) \times 10^{-4}$	$7.487(98) \times 10^{-3}$	0.858(26)
4.18	157.03	$4.1649(66) \times 10^{-4}$	$6.606(105) \times 10^{-3}$	0.915(32)
4.20	164.55	$2.3896(56) \times 10^{-4}$	$4.361(102) \times 10^{-3}$	0.766(28)
4.23	176.43	$9.2886(381) \times 10^{-5}$	$2.089(86) \times 10^{-3}$	0.409(23)
4.30	202.52	$8.204(1330) \times 10^{-6}$	$2.791(451) \times 10^{-4}$	0.072(15)
4.405	241.60	$4.030(207) \times 10^{-7}$	$2.328(120) \times 10^{-5}$	$8.6(1.7) \times 10^{-5}$

Fixed $\beta = 4.17$ Scaling ($am_l = 0.0020$)

Size	T [MeV]	$\Delta_{\ell,s}^R$	χ_{disc} [GeV ²]
$48^3 \times 18$	136.28	$1.055(14) \times 10^{-3}$	0.317(37)
$40^3 \times 16$	153.31	$6.491(38) \times 10^{-4}$	0.486(80)
$32^3 \times 14$	175.21	$2.039(24) \times 10^{-4}$	0.426(61)
$32^3 \times 12$	204.41	$-2.10(242) \times 10^{-5}$	0.136(52)

Backup: Phenomenology of Symmetry Restoration

- **Chiral Symmetry ($SU(2)_L \times SU(2)_R$) & Condensate:**

- GMOR relation connects Goldstone pion mass and chiral condensate:

$$m_\pi^2 f_\pi^2 = -m_q \langle 0 | \bar{q}q | 0 \rangle$$

- Chiral susceptibility $\chi_{\text{disc}} = \frac{\partial \langle \bar{q}q \rangle}{\partial m_l} = \frac{1}{4}(\chi_\pi - \chi_\delta)$ peaks at T_{pc} , where $\langle \bar{q}q \rangle \rightarrow 0$.
- Condensate melting removes spontaneous chiral breaking $\Rightarrow$ parity pairs degenerate: $V \leftrightarrow A$, $S \leftrightarrow PS$.

- **Axial $U(1)_A$ Anomaly & Topological Susceptibility:**

- Witten-Veneziano relation links η' mass to topological susceptibility χ_t :

$$F_\pi^2 m_{\eta'}^2 = 2N_f \chi_t, \quad \chi_t = \int d^4x \langle q(x)q(0) \rangle \propto n_I(T)$$

- Debye screening suppresses instantons at high T : $n(\rho, T) \propto \exp(-\frac{1}{3}(2N_c + N_f)(\pi\rho T)^2)$.
- As $T > T_c$, $n_I(T) \rightarrow 0 \Rightarrow \chi_t \rightarrow 0$, restoring $U(1)_A$ symmetry ($S \leftrightarrow PS$, $T_t \leftrightarrow X_t$).

- **High- T Spatial Screening Mass (3D-NRQCD):**

- Lowest Matsubara mode $p_0 = \pi T$ gives spin/parity-independent screening mass:
 $m_{\text{scr}} \approx 2\pi T + 0.1872 g^2 T$.

Backup: Effective Mass Plateaus ($48^3 \times 18, \beta = 4.17$)

Ensemble Overview: 48×18 ($N_s = 48, N_t = 18, T = 136.00$ MeV) Across All 4 Quark Masses

