Anomalous Dynamical Screening of Relativistic Plasma in a Magnetic Field

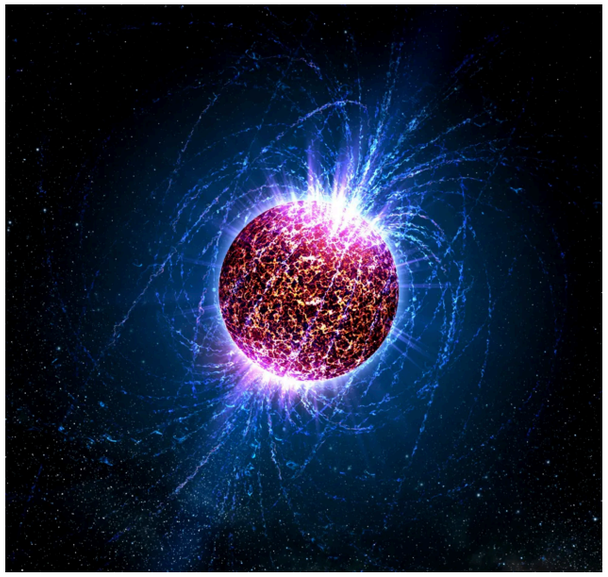
Sota Hanai (Academia Sinica)

Sota Hanai, PTEP 033B04 (2026) [arXiv:2509.16652]

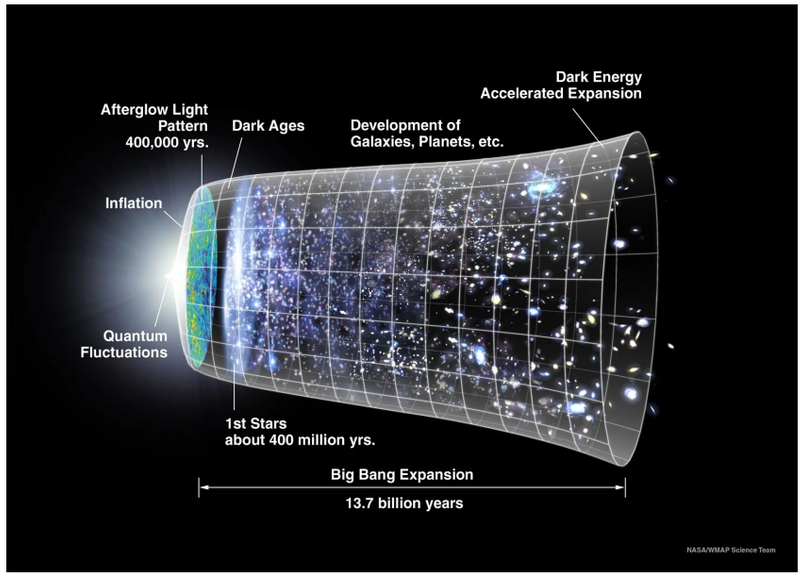
$$c = \hbar = k_{\mathrm{B}} = \varepsilon_0 = \mu_0 = 1$$

2026 Nov. 2-6 New Frontier in QCD 4th week

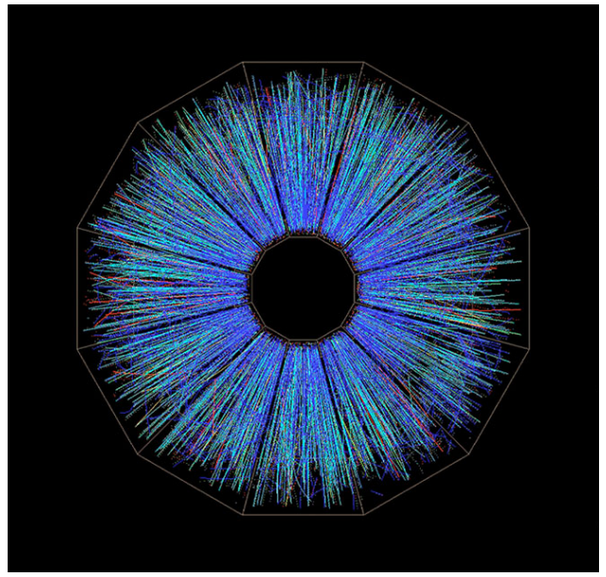
Relativistic plasma



Neutron star



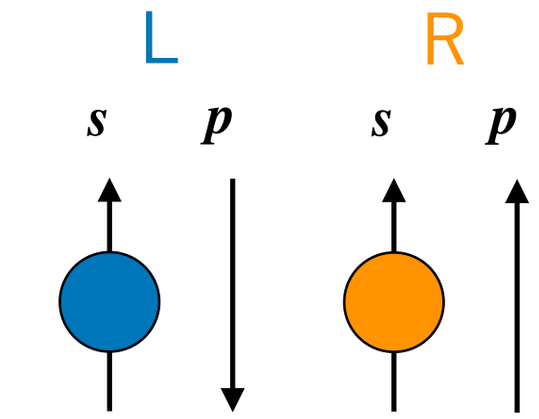
Early Universe



Heavy-ion collision

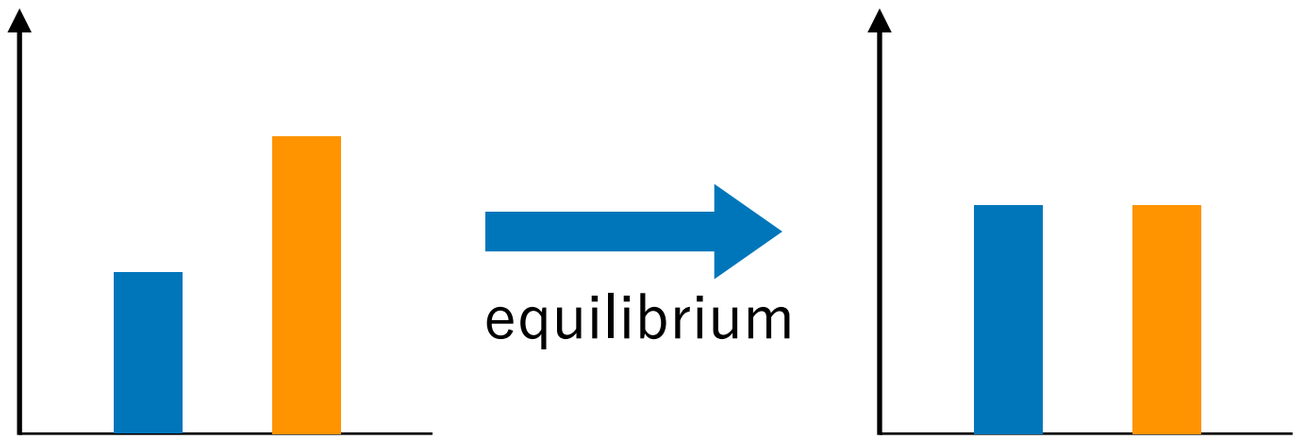
- Magnetic fields can also exist in these systems
- **Chirality** plays a crucial role in relativistic plasmas

Chirality



chiral charge density

$n_5 \equiv n_{\mathrm{R}} - n_{\mathrm{L}}$



$n_5 \neq 0$

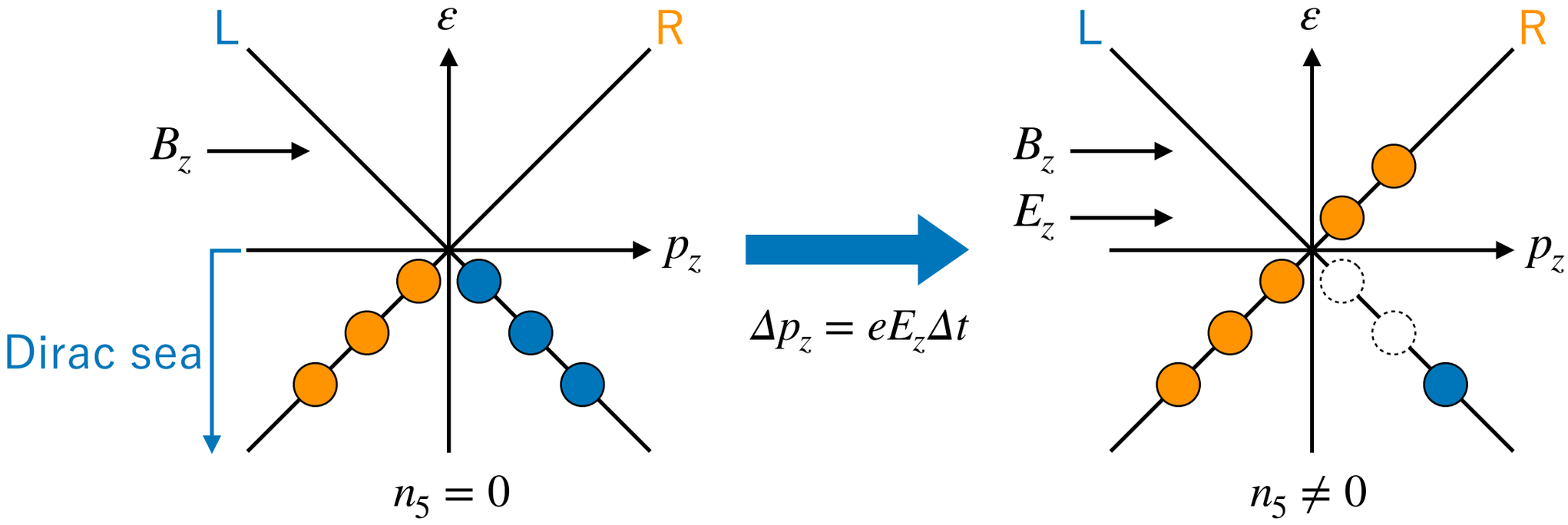
$\bar{n}_5 = 0$

- Difficult to capture net chiral charge density
- Fluctuation of chiral charge density δn_5 affects on responses
  → observations

The chiral charge density fluctuation gives rise to
a novel electromagnetic response via the chiral anomaly

Chiral anomaly

- Massless fermions under magnetic field in vacuum
- Focus on the lowest Landau level (LLL) Nielsen, Ninomiya (1983)

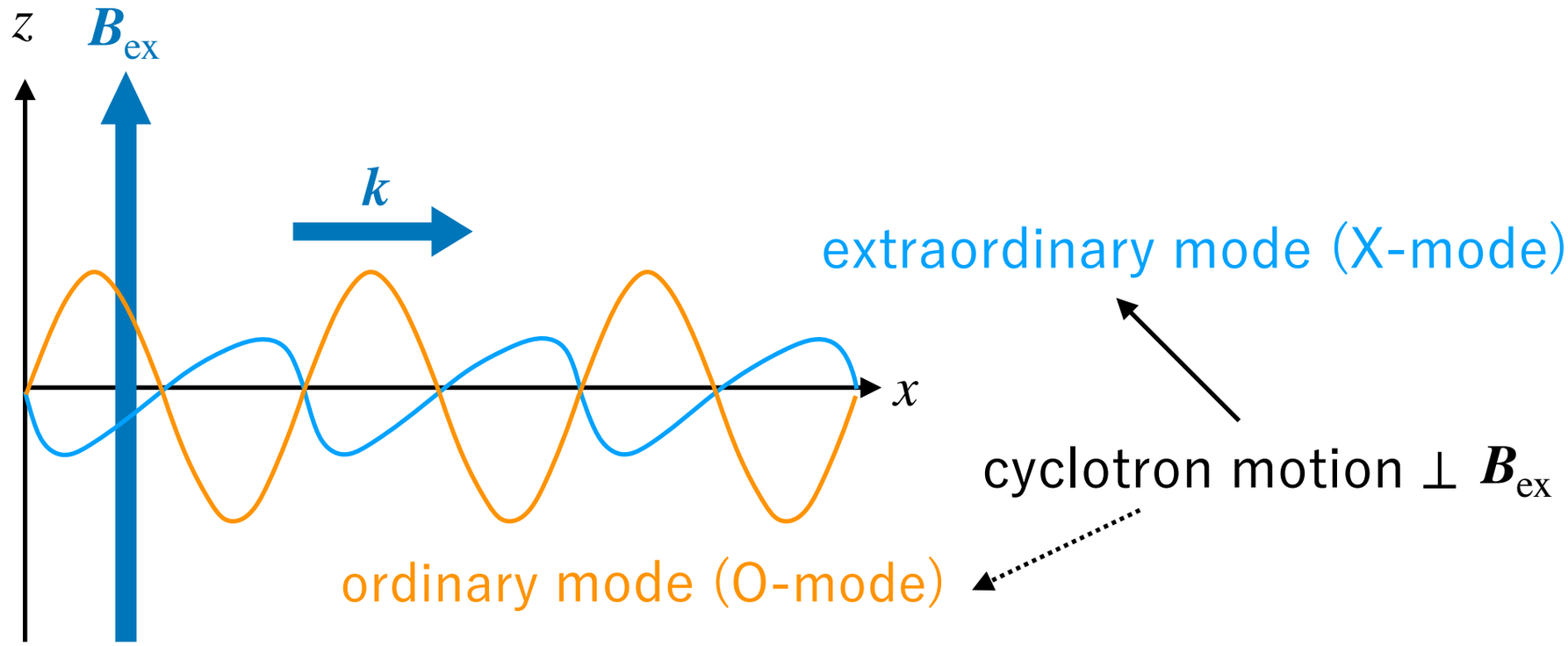


- Chiral charge is generated due to the quantum effect

Adler (1969); Bell, Jackiw (1969)

$$\partial_\mu j_5^\mu = \frac{e^2}{2\pi^2}\boldsymbol{E}\cdot\boldsymbol{B}\,, \qquad j_5^\mu \equiv j_{\mathrm{R}}^\mu - j_{\mathrm{L}}^\mu$$

Why chiral anomaly can affect electromagnetic response



- Chiral anomaly is induced by $\boldsymbol{E} \cdot \boldsymbol{B}$

$$\delta n_5 \propto B_{\mathrm{ex}} \delta E_z$$

O-mode receives the back reaction induced by δn_5

Classical electromagnetism in media

- Dispersion relation is obtained from Maxwell equations

$$\boldsymbol{k}\times\delta\boldsymbol{E}=\omega\delta\boldsymbol{B}\,,\qquad \boldsymbol{k}\times\delta\boldsymbol{B}=-\omega\delta\boldsymbol{E}+\underline{e\delta\boldsymbol{j}}$$

→ medium effect

- Transport coefficient → microscopic framework is needed

→ kinetic theory

- Collisionless Boltzmann equation

$$\frac{\partial f}{\partial t}+\dot{\boldsymbol{x}}\cdot\frac{\partial f}{\partial \boldsymbol{x}}+\dot{\boldsymbol{p}}\cdot\frac{\partial f}{\partial \boldsymbol{p}}=0\,,\qquad \dot{\boldsymbol{p}}=e\boldsymbol{E}+e\dot{\boldsymbol{x}}\times\boldsymbol{B}$$

- Distribution function leads to current

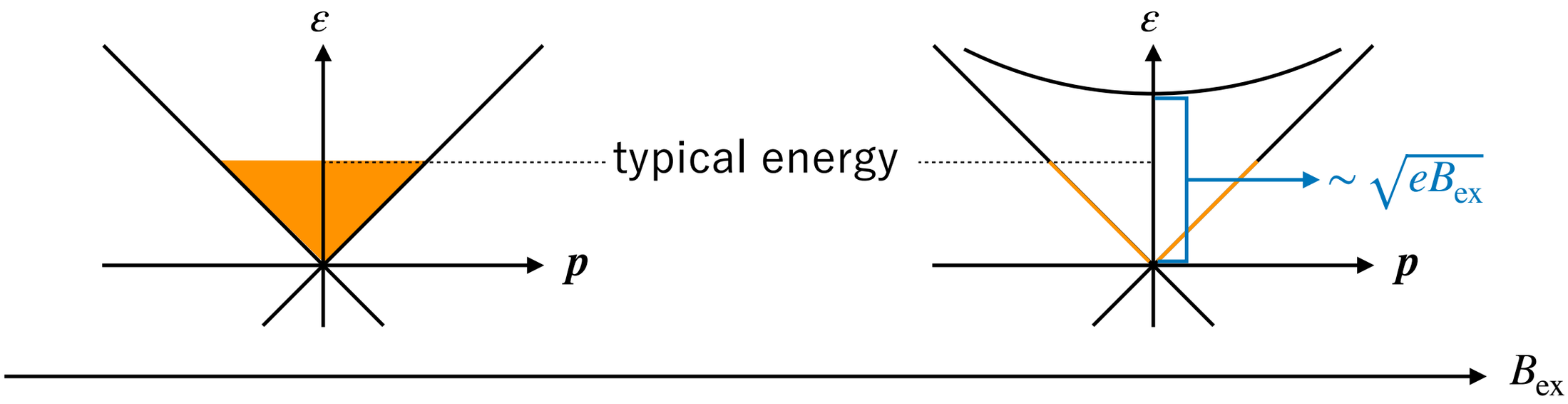
$$e\delta\boldsymbol{j}=\int\frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\dot{\boldsymbol{x}}\delta f(t,\boldsymbol{x},\boldsymbol{p})\quad\Rightarrow\quad \omega^2=|\boldsymbol{k}|^2+\underline{\Pi}$$

Debye screening,
Landau damping

Two limiting cases

- Need to take quantum effect into account

- Kinetic theory in two limiting cases



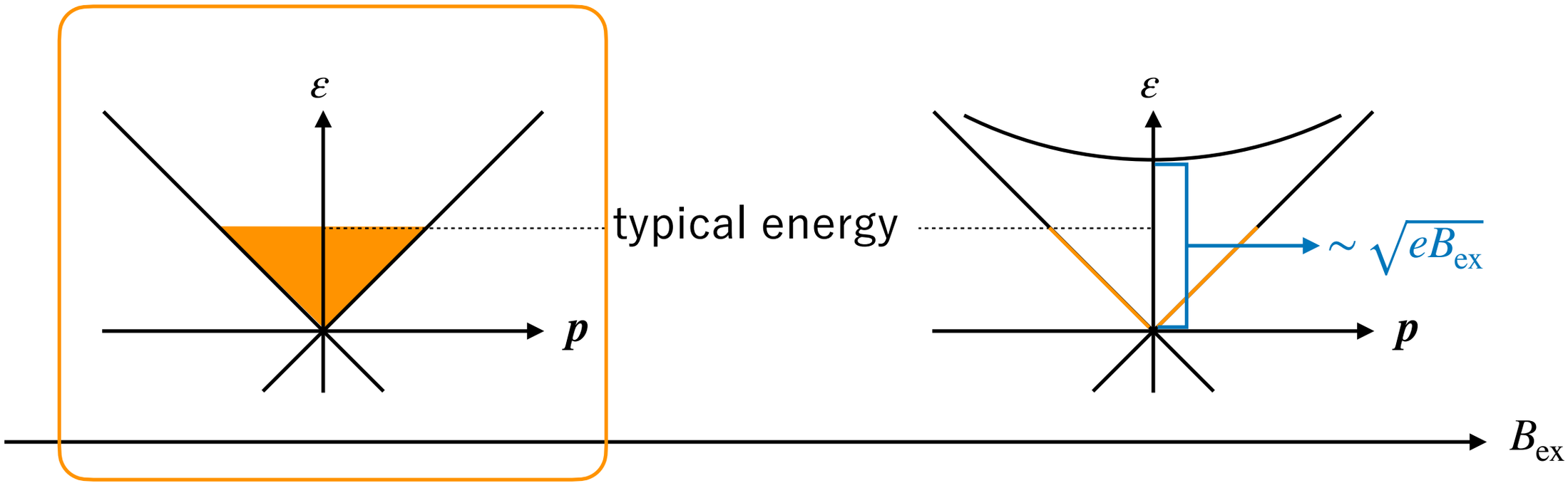
chiral kinetic theory
(weak magnetic field limit)

LLL approximation
(strong magnetic field limit)

Two limiting cases

- Need to take quantum effect into account

- Kinetic theory in two limiting cases



chiral kinetic theory
(weak magnetic field limit)

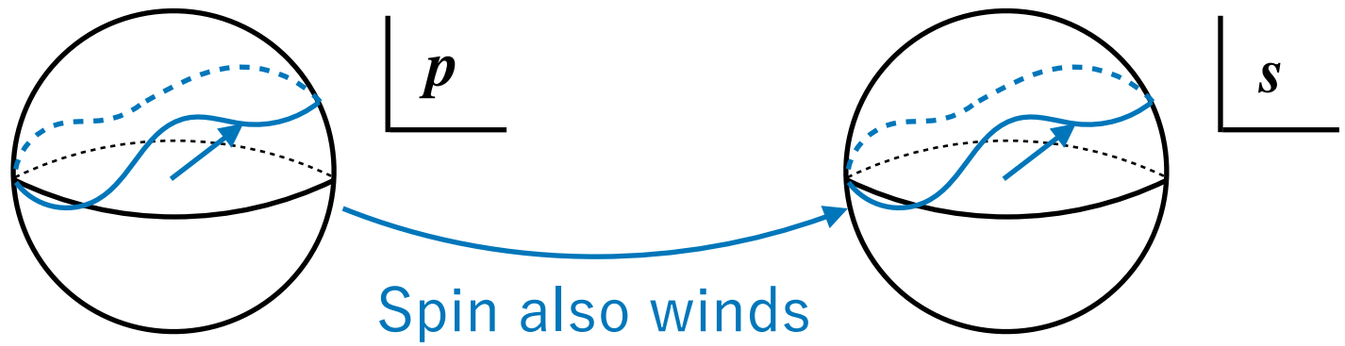
LLL approximation
(strong magnetic field limit)

Chiral kinetic theory

Son, Yamamoto (2012); Stephanov, Yin (2012)

- Topology of chiral fermions
  → spin locked to momentum

$$H = \boldsymbol{\sigma} \cdot \boldsymbol{p}$$



- Berry curvature： $\boldsymbol{\Omega} = \dfrac{\boldsymbol{p}}{2|\boldsymbol{p}|^3}$ (“magnetic monopole” in momentum space)

- Modified equations of motion

$$\dot{\boldsymbol{x}} = \boldsymbol{v} + \dot{\boldsymbol{p}} \times \boldsymbol{\Omega}$$

$$\dot{\boldsymbol{p}} = e\boldsymbol{E} + e\dot{\boldsymbol{x}} \times \boldsymbol{B}$$

$$\Rightarrow \quad \frac{\partial f_\lambda}{\partial t} + \dot{\boldsymbol{x}} \cdot \frac{\partial f_\lambda}{\partial \boldsymbol{x}} + \dot{\boldsymbol{p}} \cdot \frac{\partial f_\lambda}{\partial \boldsymbol{p}} = 0$$

($\lambda = \pm$ ：chirality)

Dispersion relation

Hanai (2026)

- Obtained current

$$e\delta j_z(\omega,k_x)=\mathrm{i}\frac{m_\mathrm{D}^2}{\omega}F(\xi)\delta E_z+\underbrace{\frac{e^2B_\mathrm{ex}}{2\pi^2\chi}G(\xi)\delta n_5}_{\text{comes from chirality}}+\mathrm{i}\frac{e^4B_\mathrm{ex}^2}{\pi^2\omega|k_x|^2}I(\xi)\delta E_z$$

m_D : Debye screening mass

F,G,I : dimensionless functions of $\xi\equiv\omega/|k_x|$

δn_5 is eliminated by combining the chiral anomaly

$$\omega\delta n_5=\mathrm{i}\frac{e^2B_\mathrm{ex}}{2\pi^2}\delta E_z$$

$$(k_x^2-\omega^2)\delta E_z=\mathrm{i}\omega e\delta j_z$$

$$\Rightarrow\quad \omega^2=|k_x|^2+\Pi_\mathrm{O}$$

$$\Pi_\mathrm{O}(\xi)=m_\mathrm{D}^2F(\xi)+\frac{e^4B_\mathrm{ex}^2}{\pi^2|k_x|^2}I(\xi)+\underbrace{m_\mathrm{a}^2G(\xi)}_{\text{quantum correction}}\qquad\left(m_\mathrm{a}^2\equiv\frac{e^4B_\mathrm{ex}^2}{4\pi^4\chi}\right)$$

Dispersion relation

Hanai (2026)

- Obtained current

$$e\delta j_z(\omega,k_x)=\mathrm{i}\frac{m_\mathrm{D}^2}{\omega}F(\xi)\delta E_z+\frac{e^2B_\mathrm{ex}}{2\pi^2\chi}G(\xi)\delta n_5+\mathrm{i}\frac{e^4B_\mathrm{ex}^2}{\pi^2\omega|k_x|^2}I(\xi)\delta E_z$$

comes from chirality

m_D : Debye screening mass

F,G,I : dimensionless functions of $\xi\equiv\omega/|k_x|$

δn_5 is eliminated by combining the chiral anomaly

$$\omega\delta n_5=\mathrm{i}\frac{e^2B_\mathrm{ex}}{2\pi^2}\delta E_z$$

$$(k_x^2-\omega^2)\delta E_z=\mathrm{i}\omega e\delta j_z$$

$$\omega^2=|k_x|^2+\Pi_\mathrm{O}$$

$$\Pi_\mathrm{O}(\xi)=m_\mathrm{D}^2F(\xi)+\frac{e^4B_\mathrm{ex}^2}{\pi^2|k_x|^2}I(\xi)+m_\mathrm{a}^2G(\xi)\qquad\left(m_\mathrm{a}^2\equiv\frac{e^4B_\mathrm{ex}^2}{4\pi^4\chi}\right)$$

quantum correction

Limiting expressions

Hanai (2026)

- Long wavelength limit ($\xi \gg 1$)

$$\omega^2 = \omega_{\mathrm{p}}^2 \left(1 + \frac{m_{\mathrm{a}}^2}{m_{\mathrm{D}}^2} \right) + \frac{6}{5} |k_x|^2 \left(1 - \frac{4m_{\mathrm{a}}^2}{3m_{\mathrm{D}}^2} \right)$$

correction to plasma frequency

- Quasi-static limit ($\xi \ll 1$)

$$\Pi_{\mathrm{O}} \simeq -\mathrm{i}\frac{\pi m_{\mathrm{D}}^2}{4}\xi - \mathrm{i}\frac{e^4 B_{\mathrm{ex}}^2}{16\pi |k_x|^2}\xi + m_{\mathrm{a}}^2 + \mathrm{i}\frac{\pi m_{\mathrm{a}}^2}{2}\xi$$

Landau damping

anomalous dynamical screening

O-mode, which is massless in classical theory, acquire an effective gap dynamically

Relaxation time

- Relaxation time of dense matter

$$\frac{1}{\tau} \sim \alpha^2 \frac{T^2}{k_{\mathrm{IR}}}$$ Pauli blocking

- Electron-electron scattering

$$\mathscr{M}_{\mathrm{O}} \propto \frac{1}{\omega^2 - |\boldsymbol{k}|^2 + \Pi_{\mathrm{O}}} \quad \Rightarrow \quad k_{\mathrm{IR}}^2 \sim \Pi_{\mathrm{O}}$$

- Temperature dependence changes

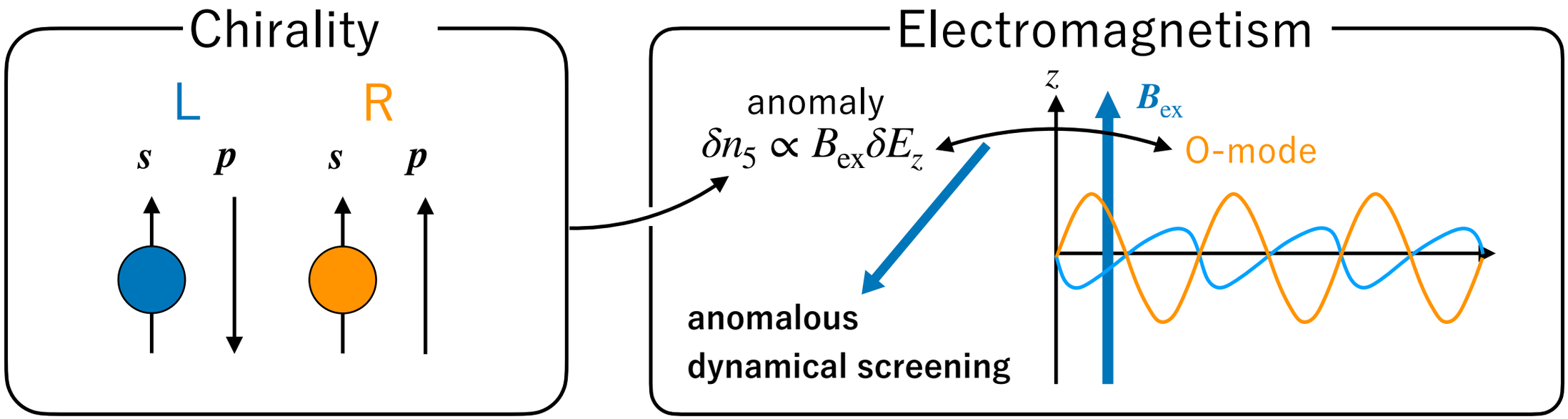
$$\frac{1}{\tau_{\mathrm{w}}} \sim \alpha^2 \frac{T^{5/3}}{m_{\mathrm{D}}^{2/3}} + \cdots \qquad \frac{1}{\tau_{\mathrm{s}}} \sim \alpha^2 \frac{T^2}{m_{\mathrm{a}}^2}$$

B_{ex}

$\sim 10^{18}$ G

Transport coefficients would have the magnetic field dependence

Conclusion and outlook



- Implications for quark matter such as quark-gluon plasma and color super conductor
- Understanding from the aspect of quantum field theory
- Behavior in the intermediate magnetic field region

Appendix

Electromagnetism in magnetic fields

See, e.g., Landau, Lifshitz vol.8

- Maxwell equations in media

$$\boldsymbol{k}\times\delta\boldsymbol{E}=\omega\delta\boldsymbol{B}\,,\qquad \boldsymbol{k}\times\delta\boldsymbol{B}=-\omega\delta\boldsymbol{E}-\mathrm{i}e\delta\boldsymbol{j}$$

- Linear response

$$e\delta\boldsymbol{j}=-\mathrm{i}\omega\delta\boldsymbol{P}=-\mathrm{i}\omega(\varepsilon-1)\delta\boldsymbol{E}$$

microscopic details

- Permittivity tensor in an external magnetic field

$$\varepsilon=\begin{pmatrix}\varepsilon_{xx} & \varepsilon_{xy} & 0\\ -\varepsilon_{xy} & \varepsilon_{xx} & 0\\ 0 & 0 & \varepsilon_{zz}\end{pmatrix}$$

Onsager's theorem: $\varepsilon^{ij}(B_{\mathrm{ex}})=\varepsilon^{ji}(-B_{\mathrm{ex}})$

・Diagonal：even in B_{ex}

・Off-diagonal：odd in B_{ex}

$$\boldsymbol{r}^2\delta\boldsymbol{E}-(\boldsymbol{r}\cdot\delta\boldsymbol{E})\boldsymbol{r}-\varepsilon\delta\boldsymbol{E}=0\qquad\left(\boldsymbol{r}\equiv\frac{\boldsymbol{k}}{\omega}\right)$$

$$\Rightarrow\quad r_x^2=\varepsilon_{zz}$$

O-mode

Counting in weak field

- Counting

$$A^{\mu}=\mathcal{O}(\epsilon)\,,\ \partial_{\mu}=\mathcal{O}(\delta)\,,\ e=\mathcal{O}(\delta)\,,\ \mu_5=\mathcal{O}(\delta^3\epsilon^2)$$

- Solve Boltzmann equation by expansion

$$f_{\lambda}(t,\boldsymbol{x},\boldsymbol{p})=f_{\lambda}^{(0)}+f_{\lambda}^{(\delta\epsilon)}+f_{\lambda}^{(\delta^2\epsilon)}+f_{\lambda}^{(\delta^2\epsilon^2)}+f_{\lambda}^{(\delta^3\epsilon^3)}$$

- Currents at each order

$$j_{\lambda z}^{(\delta\epsilon)}(\omega,\boldsymbol{k})=\mathrm{i}\frac{m_{\mathrm{D}\lambda}^2}{e\omega}F(\xi)E_z\,,$$

$$j_{\lambda z}^{(\delta^2\epsilon)}(\omega,\boldsymbol{k})=\lambda\frac{e\mu_{\lambda}}{4\pi^2}G(\xi)B_z\,,$$

$$j_{\lambda z}^{(\delta^3\epsilon^3)}(\omega,\boldsymbol{k})=\mathrm{i}\frac{e^3B_{\mathrm{ex}}^2}{2\pi^2\omega\left|k_x\right|^2}I(\xi)E_z$$

Counting in strong field

- Counting

$$B_{\mathrm{ex}} = \mathcal{O}(1)\,,\ \delta A^{\mu} = \mathcal{O}(\epsilon)\,,\ \partial_{\mu} = \mathcal{O}(\delta)\,,\ e = \mathcal{O}(\delta^{2/3})\,,\ \mu_5 = \mathcal{O}(\delta^{4/3}\epsilon)$$

- Solve Boltzmann equation by expansion

$$f_{\lambda}(t, z, p_z) = f_{\lambda}^{(0)} + f_{\lambda}^{(\delta\epsilon)}$$

- Currents at each order

$$j_{\lambda z}^{(\delta)}(\omega, k_z) = \lambda \frac{\mu_{\lambda}}{4\pi^2} |\, eB_{\mathrm{ex}} \,|$$

$$j_{\lambda z}^{(\delta^2\epsilon)}(\omega, k_z) = \mathrm{i} \frac{|\, eB_{\mathrm{ex}} \,|}{2\pi^2} \frac{k_z}{\omega^2 - k_z^2} eE_z$$

Dimensionless functions

- Definitions

$$L(\xi) \equiv \frac{\xi}{2} \ln \left| \frac{1+\xi}{1-\xi} \right| - \mathrm{i}\frac{\pi\xi}{2}\theta(1-\xi)\,,$$

$$F(\xi) \equiv \frac{1}{2}[\xi^2 + (1-\xi^2)L(\xi)]\,,$$

$$G(\xi) \equiv (1-\xi^2)[1-L(\xi)]\,,$$

$$I(\xi) \equiv \xi \int \frac{\mathrm{d}^2\boldsymbol{v}}{4\pi} \left[\frac{v_x v_z^2}{(\xi - v_x + \mathrm{i}\eta)^4} - 3\frac{v_y^2 v_z^2}{(\xi - v_x + \mathrm{i}\eta)^5} \right]$$

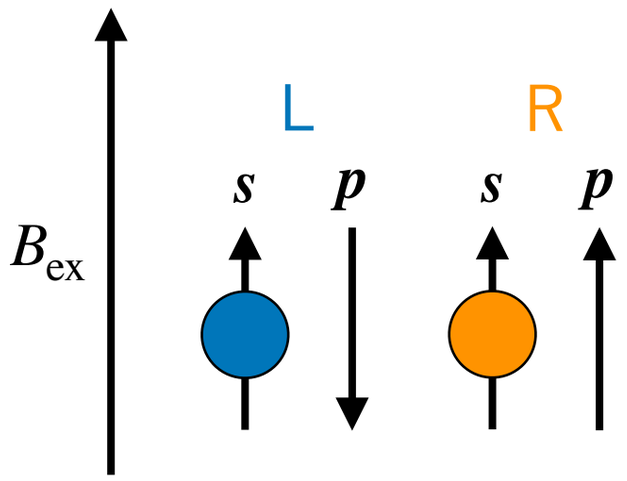
- The limiting expressions of

$$I(\xi) \simeq \begin{cases} -\mathrm{i}\frac{\pi}{16}\xi + \mathcal{O}(\xi^2) & (\xi \ll 1) \\ \mathcal{O}(\xi^{-3}) & (\xi \gg 1) \end{cases}$$

Strong magnetic field limit

- LLL approximation
  → effective (1+1)-dimensions

$$\dot{z} = \lambda\,, \qquad \dot{p}_z = eE_z$$



- Only anomalous term remains

- Obtained dispersion relation

$$\omega^2 = |k_x|^2 + \Pi_{\mathrm{O}}\,, \qquad \Pi_{\mathrm{O}} = m_{\mathrm{a}}^2$$

anomalous screening

In both cases, O-mode is screened by anomalous effect

Hanai (2025)

Implications for neutron star phenomenology

Hanai (2026)

- Consider dense electron matter in neutron star core

- Implication 1：relaxation time

$$\mathscr{M}_{\mathrm{O}} \propto \frac{1}{\omega^2 - |\boldsymbol{k}|^2 + \Pi_{\mathrm{O}}}$$

IR cutoff

$$\frac{1}{\tau_{\mathrm{w}}} \sim \alpha^2 \frac{T^{5/3}}{m_{\mathrm{D}}^{2/3}} + \cdots \qquad \frac{1}{\tau_{\mathrm{s}}} \sim \alpha^2 \frac{T^2}{m_{\mathrm{a}}^2}$$

B_{ex}

$\sim 10^{18}\ \mathrm{G}$

→ Temperature dependence of shear viscosity changes as B_{ex} increases

- Implication 2：screening length

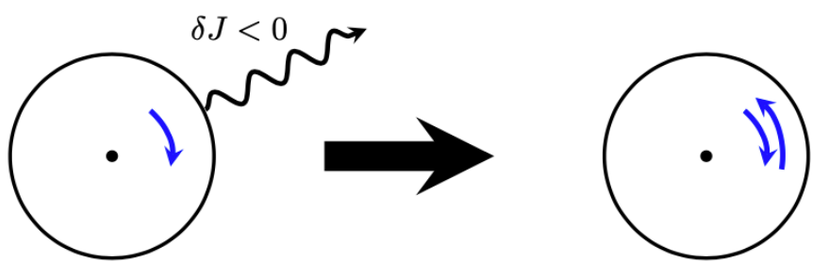
$$l_{\mathrm{a}} \equiv \frac{1}{m_{\mathrm{a}}} \sim 10^{-9}\ \mathrm{m} \left(\frac{B_{\mathrm{ex}}}{10^{14}\ \mathrm{G}} \right)^{-1} \left(\frac{\mu_{\mathrm{e}}}{10^{2}\ \mathrm{MeV}} \right)$$

→ Dynamical magnetic fields are partially screened at large scales

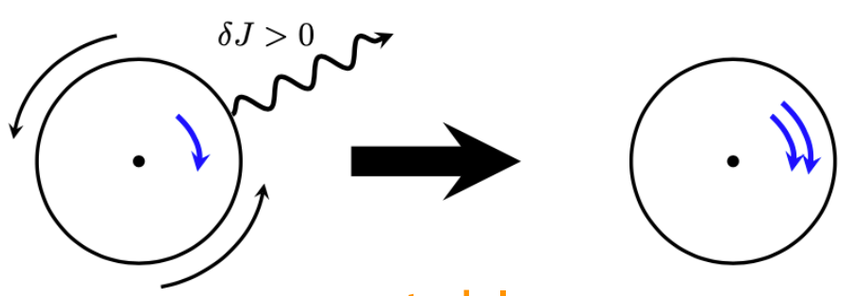
r-mode instability

- Chandrasekhar-Friedman-Schutz (CFS) instability

Chandrasekhar (1970); Friedman, Schutz (1978)



stable



unstable

- Coriolis force induces the r-mode

- r-mode is unstable due to CFS instability (r-mode instability)

Andersson (1998)

Limiting expressions

Hanai (2026)

- Quasi-static limit ($\xi \equiv \omega/|\boldsymbol{k}| \ll 1$)

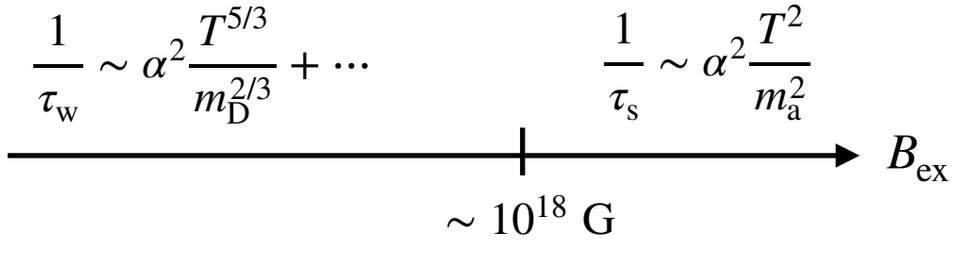
$$\Pi_{\mathrm{O}} \simeq \underline{-\mathrm{i}\frac{\pi m_{\mathrm{D}}^2}{4}\xi} - \mathrm{i}\frac{e^4 B_{\mathrm{ex}}^2}{16\pi|k_x|^2}\xi + \underline{m_{\mathrm{a}}^2} + \mathrm{i}\frac{\pi m_{\mathrm{a}}^2}{2}\xi \qquad \left(m_{\mathrm{a}}^2 \equiv \frac{e^4 B_{\mathrm{ex}}^2}{4\pi^4\chi}\right)$$

Landau damping

anomalous dynamical screening

O-mode, which is massless in classical theory, acquire an effective gap dynamically

- Relaxation time of dense matter : $\frac{1}{\tau} \sim \alpha^2 \frac{T^2}{\Pi_{\mathrm{O}}^{1/2}}$ Pauli blocking



Transport coefficients would have the magnetic field dependence