Thermodynamics and Polyakov-Loop Potential in Accelerated QCD at Finite Temperature

Yusuke Shimada (YITP, Kyoto University)





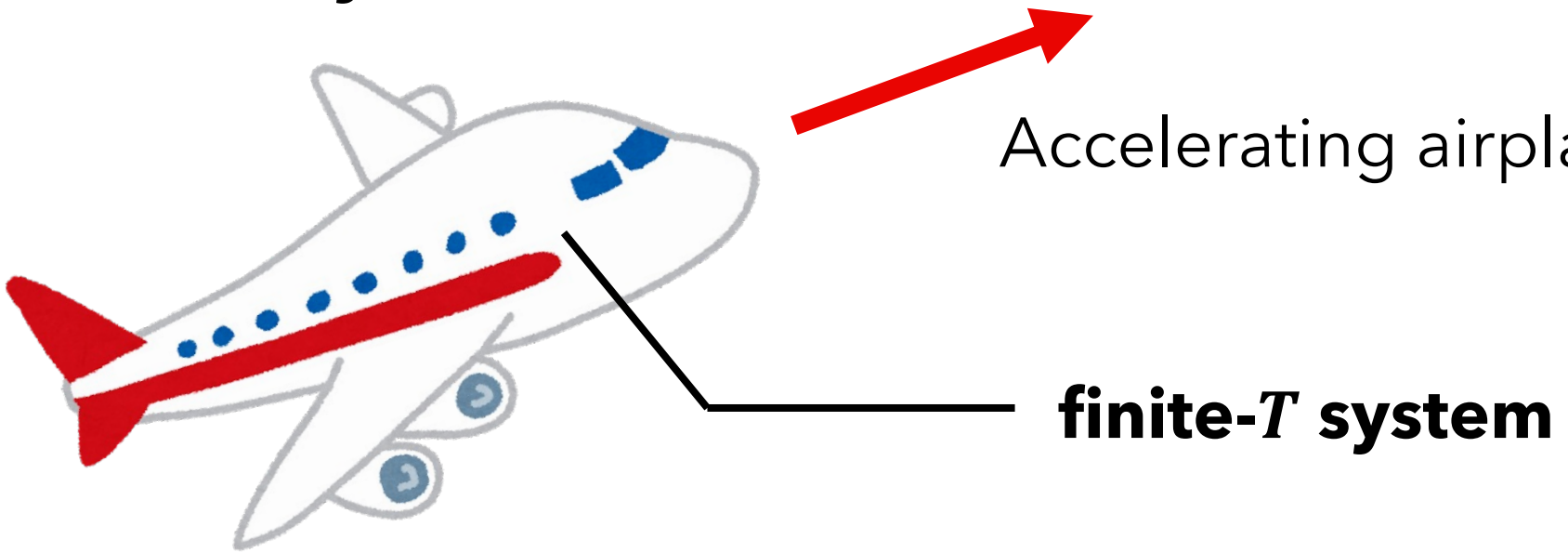
Based on Phys. Rev. D 114, 054012 (arXiv:2605.29503)
with Hao-Lei Chen, Kenji Fukushima, Yu-Han Gao,
Xu-Guang Huang, Zhi-Bin Zhu.





Motivation

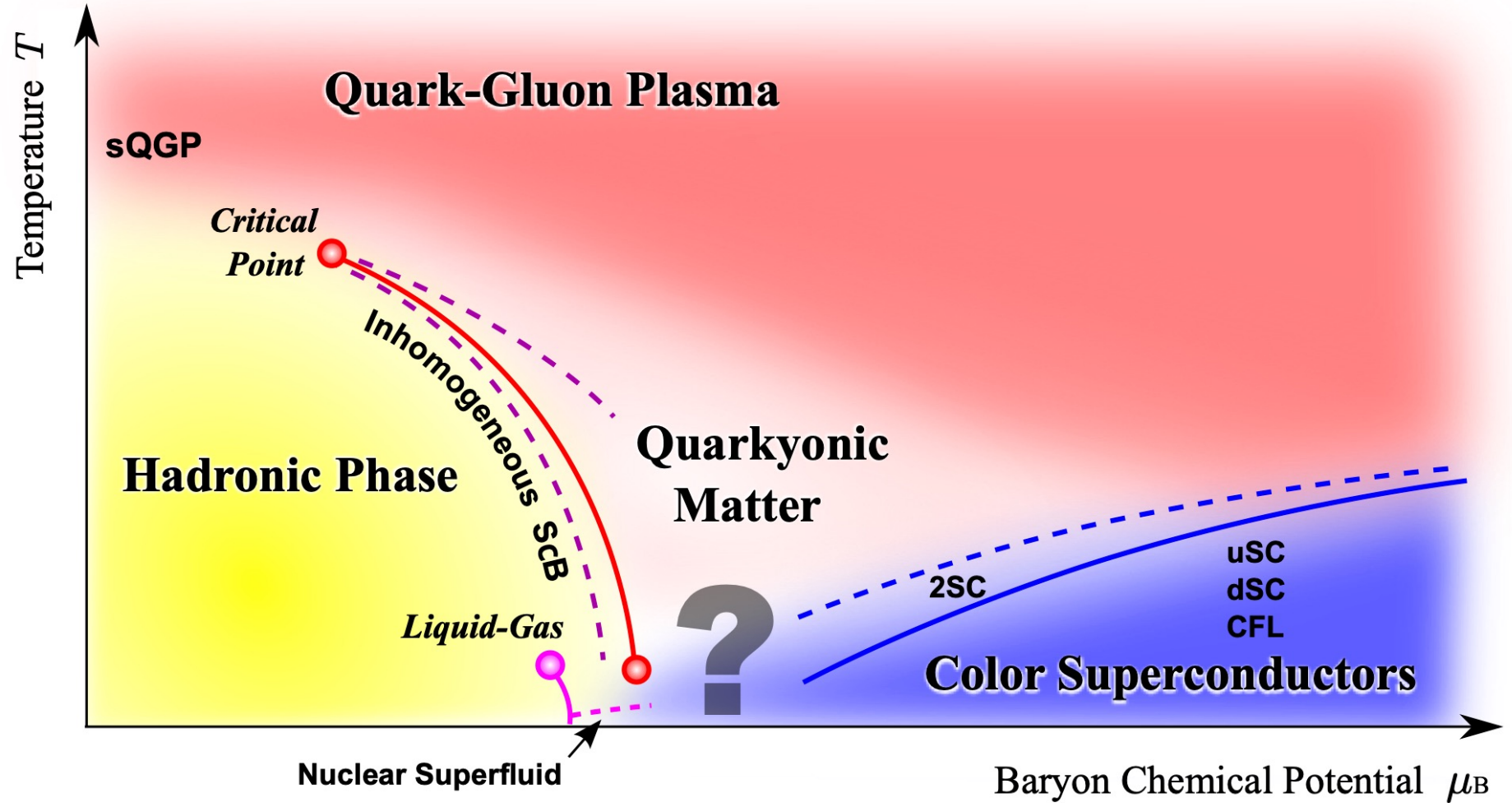
Accelerated System：



・What occurs in accelerated QCD systems?

→ gluonic free energy: Polyakov-loop potential

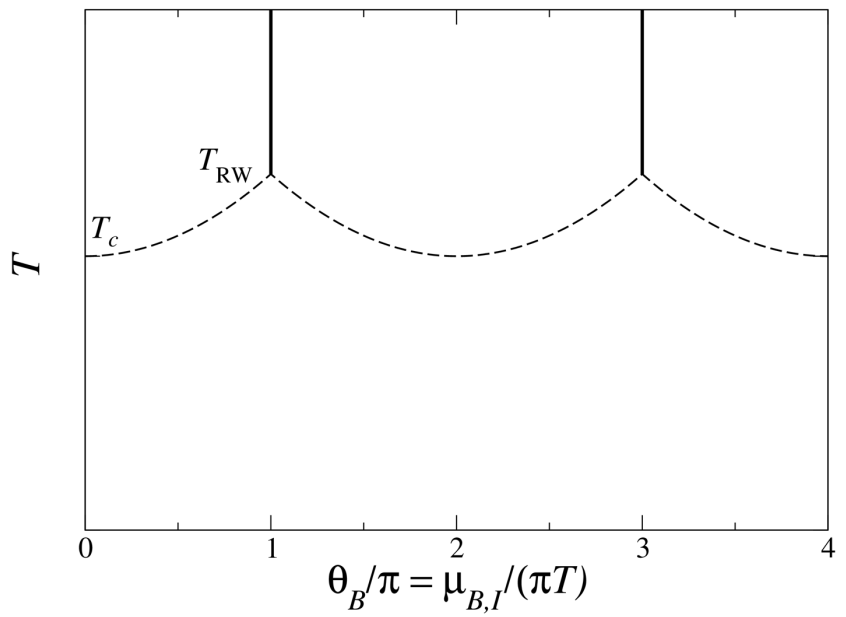
QCD Phase Diagram



Fukushima, Hatsuda (2011)

QCD Phase Diagram

imag. chem. potential



Bonati et al. (2016)

luon Plasma

Quarkyonic Matter

ous ScB

Liquid-Gas

?

2SC

uSC
dSC
CFL

Color Superconductors

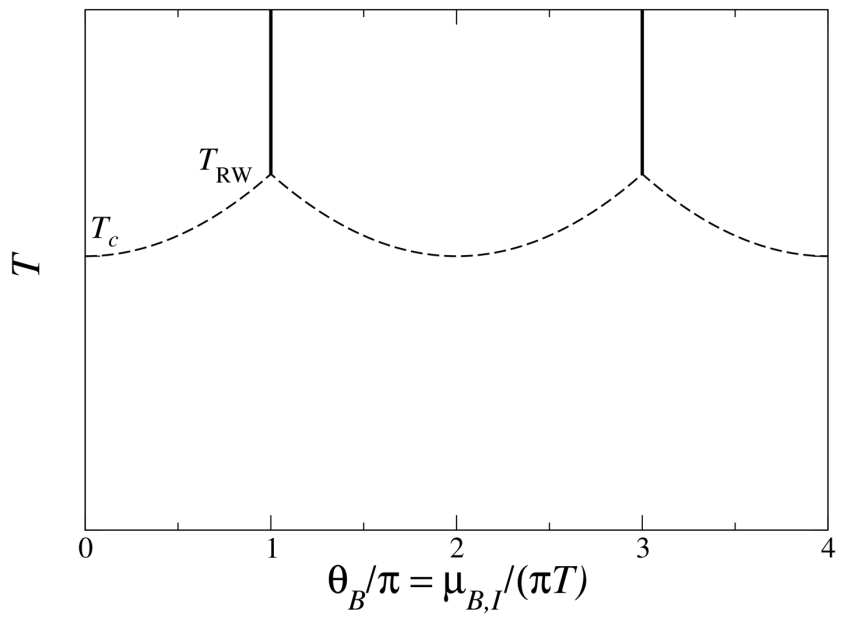
Nuclear Superfluid

Baryon Chemical Potential μ_B

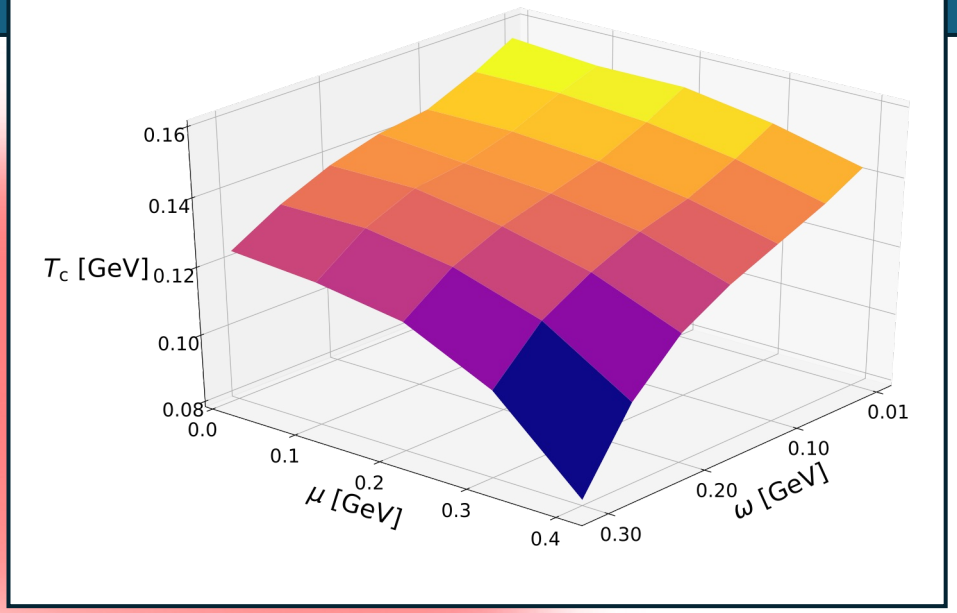
Fukushima, Hatsuda (2011)

QCD Phase Diagram

imag. chem. potential



rotation



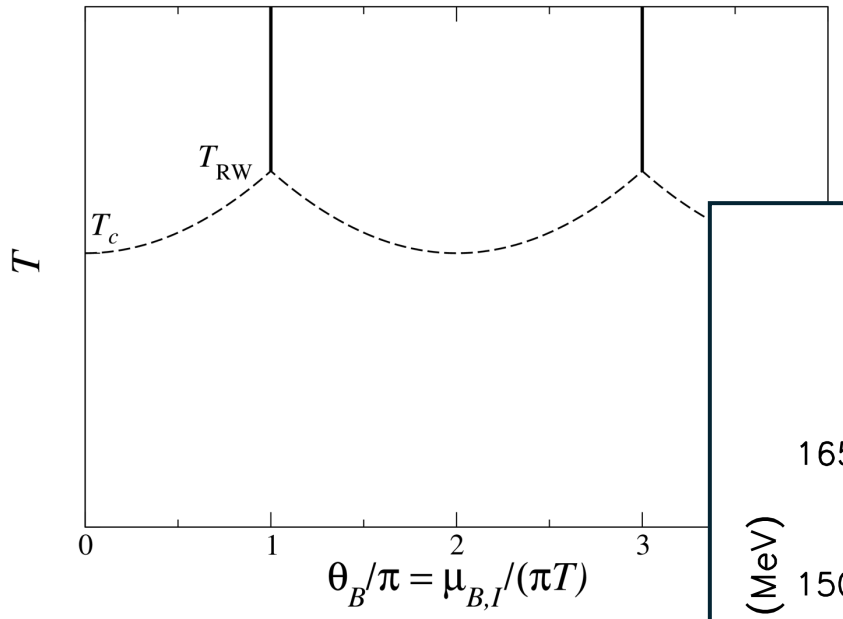
Bonati et al. (2016)

Fujimoto et al. (2021)

Fukushima, Hatsuda (2011)

QCD Phase Diagram

imag. chem. potential



Bonati et al. (2016)

Fujimoto et al. (2021)

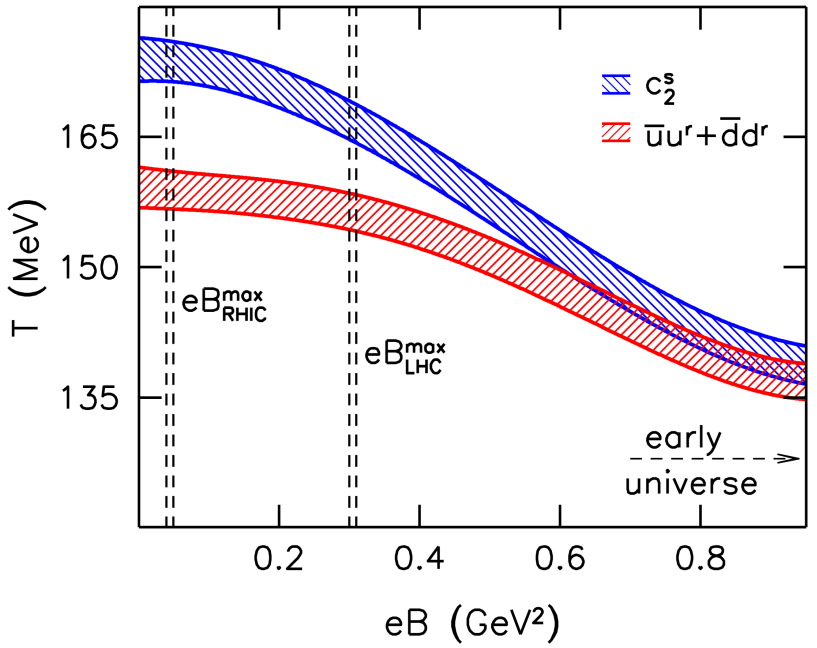
Bali et al. (2012)

Fukushima, Hatsud

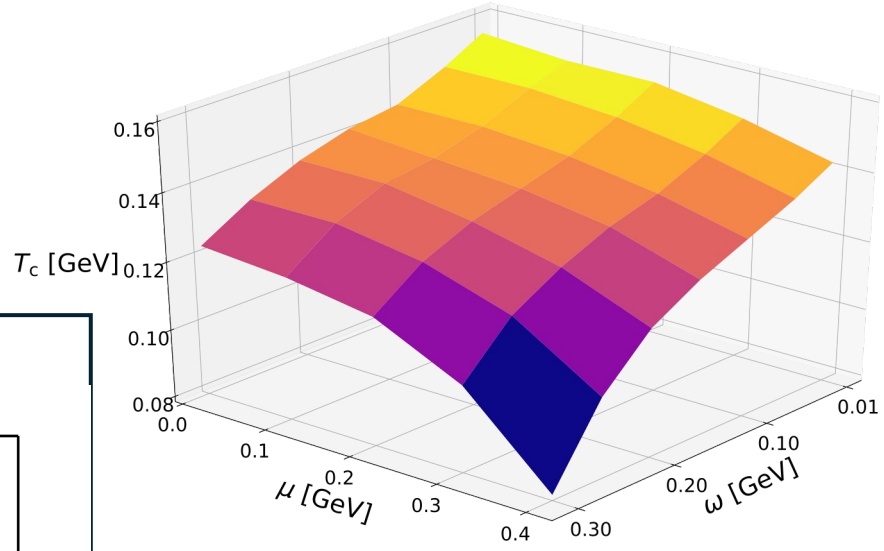
luon Plasma

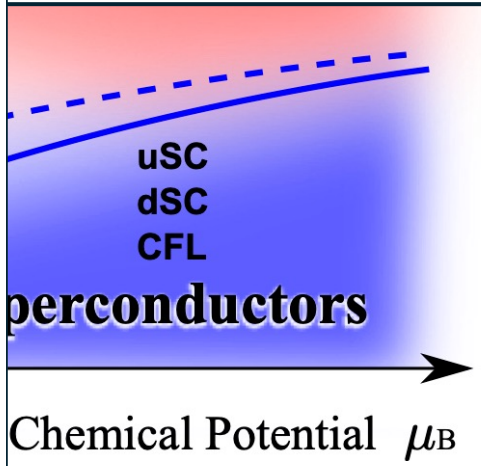
Nucle

magnetic field



rotation





Physics under Acceleration 1/2

New Axis for Phase Diagram: **Acceleration**

How does acceleration appear?

- gravitational system near a heavy and large object
  uniform gravity ~ acceleration

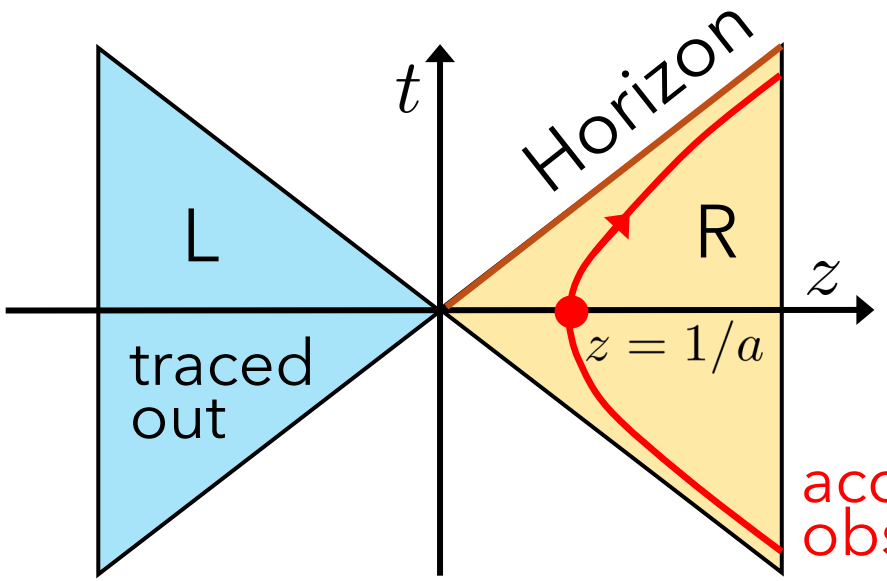
- Strong acceleration (deceleration) in heavy-ion collisions
  thermal effects at the QCD scale can appear

What occurs in high-T accelerated (pure YM) systems?

Physics under Acceleration 2/2

New Axis for Phase Diagram: **Acceleration**

Unruh Effect:



$$\rho_{\mathrm{R}} = \mathrm{Tr}_{\mathrm{L}} \left(|\Omega_{\mathrm{M}}\rangle \langle \Omega_{\mathrm{M}}| \right) = \frac{1}{Z} e^{-(2\pi/a) H_{\mathrm{R}}}$$

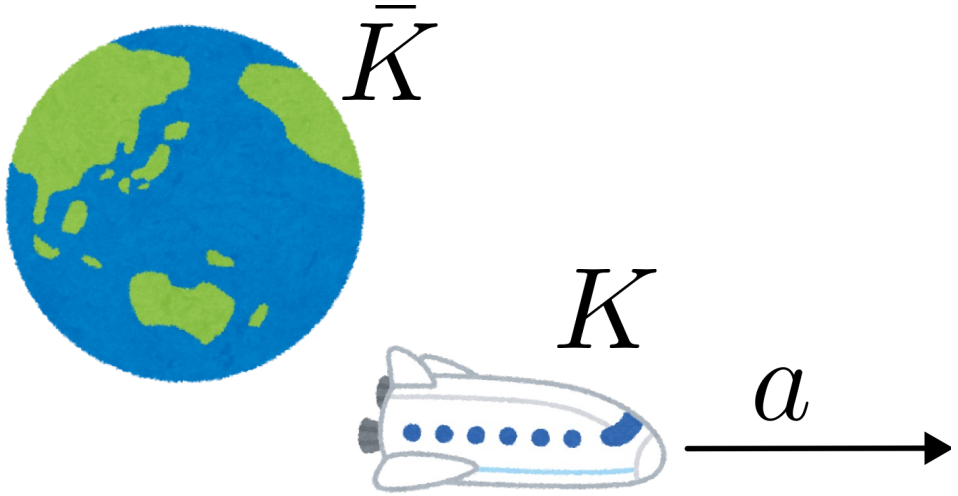
$$= \frac{1}{Z} e^{-H_{\mathrm{R}}/T_{\mathrm{U}}}$$

$$T_{\mathrm{U}} = a/2\pi$$

From an accelerated observer, the Minkowski vacuum is seen as a finite temperature system.

Co-Accelerated System

- Consider the system accelerated by a ,
  with an observer co-accelerated with the system.



$$a = \left[1 - \bar{v}^2\right]^{-3/2} \frac{d\bar{v}}{d\bar{t}} \, .$$

$$\bar{t} = \left(z + \tfrac{1}{a}\right) \sinh(at) \, , \quad \bar{z} + \tfrac{1}{a} = \left(z + \tfrac{1}{a}\right) \cosh(at) \, .$$

Rindler Metric 1/2

Trsf. between the inertial and accelerated systems provides

$$ds^2 = d\bar{t}^2 - d\bar{x}^2 - d\bar{y}^2 - d\bar{z}^2$$
$$= a^2\left(z + \tfrac{1}{a}\right)^2 dt^2 - dx^2 - dy^2 - dz^2 \,.$$

Or, under $z + 1/a \to z$,

$$ds^2 = a^2 z^2 dt^2 - dx^2 - dy^2 - dz^2 \,.$$

(Rindler Metric)

$z = 1/a$ feels acceleration a, and generally $a(z) = 1/z$.

Rindler Metric 2/2

For finite temperature cases,

$$ds^2 = z^2 d(a\tau)^2 + dx^2 + dy^2 + dz^2 \,.$$

(τ, z) corresponds to a “circle”. ref. $ds^2 = r^2 d\theta^2 + dr^2$.

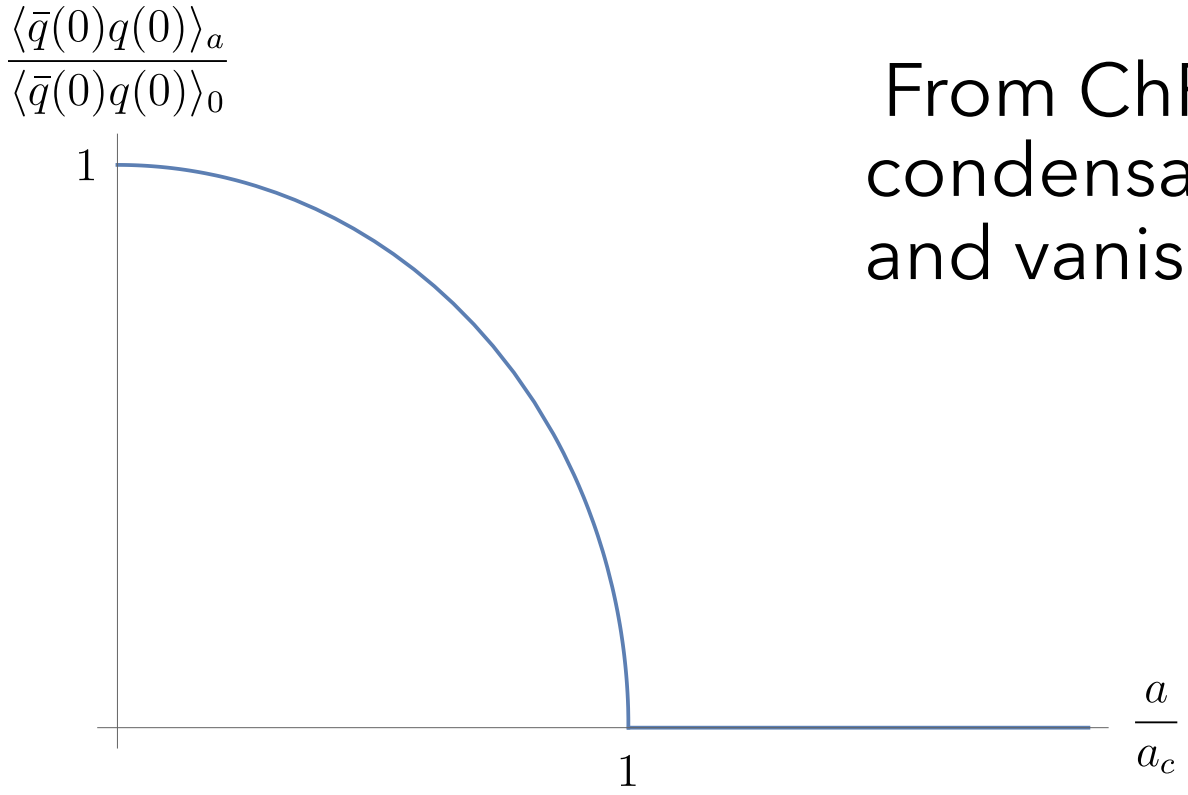
τ direction is periodic at $\tau \sim \tau + 2\pi/a$.

The Rindler coordinate system contains temperature

$T_U = a/2\pi$ (Unruh Temperature)

Symmetry Restoration under Acceleration

Unruh temperature leads to the restoration of chiral symmetry.



From ChPT calculations, the quark condensate decreases with acceleration and vanishes at $a = a_{\mathrm{c}}$.

Here,

$$T_{\mathrm{U}}(a_{\mathrm{c}}) = T_{\mathrm{c}}(a=0)\,,$$
$$a_{\mathrm{c}} = 2\pi T_{\mathrm{c}}(a=0)\,.$$

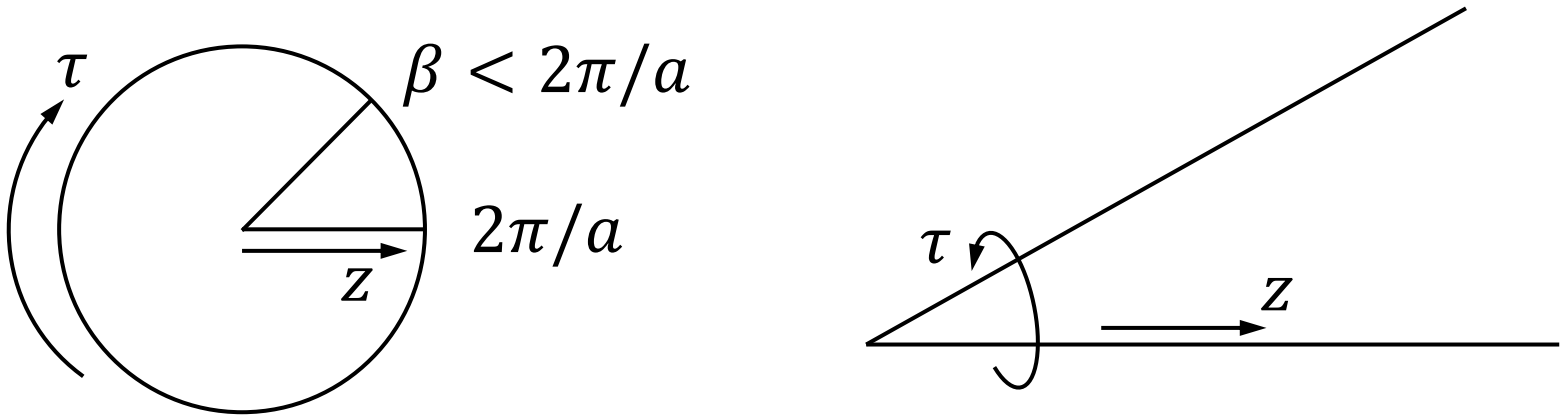
Casado-Turrión and Dobado (2019)

Finite Temperature Accelerated System

How about finite temperature $T \neq a/2\pi$?

$$ds^2 = z^2 d(a\tau)^2 + dx^2 + dy^2 + dz^2 \,.$$

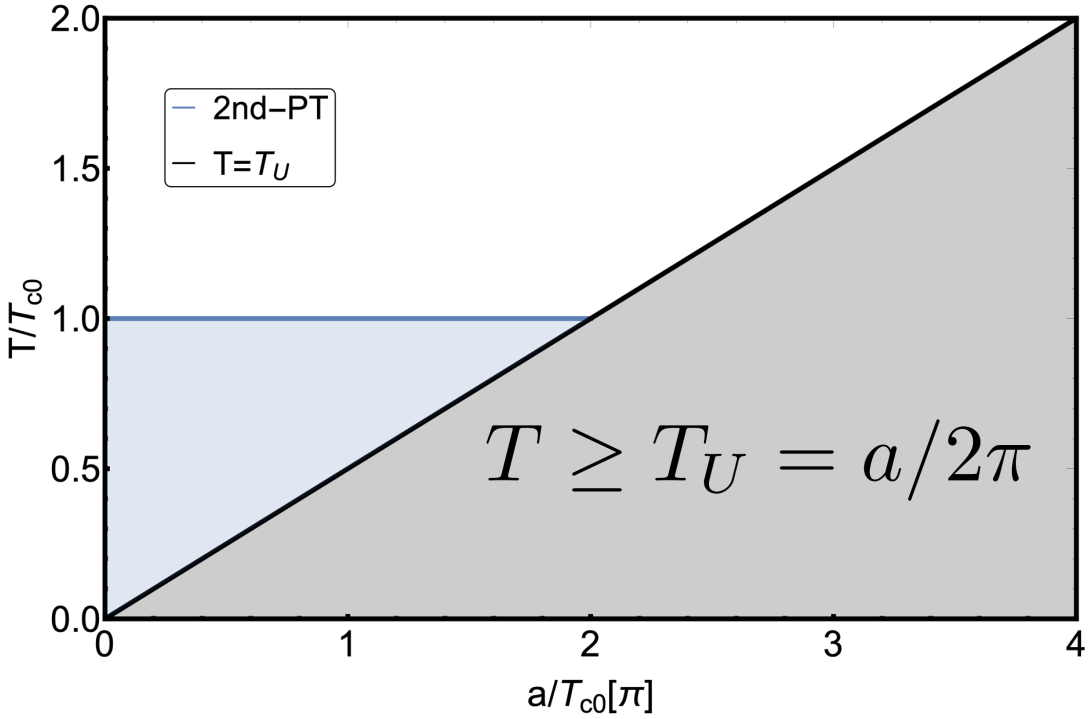
Impose the Euclidean-time period $\beta = 1/T \leq 2\pi/a$.



Circle with smaller angle → **cone** cf. cosmic string

Phase Transition in Acc. Systems 1/2

Critical temperature (of chiral transition) in finite-temperature accelerated system from NJL model.



At acceleration a,

$$T_c = az\, T_c(a=0)$$

and the critical temperature is always the same at $z = 1/a$.

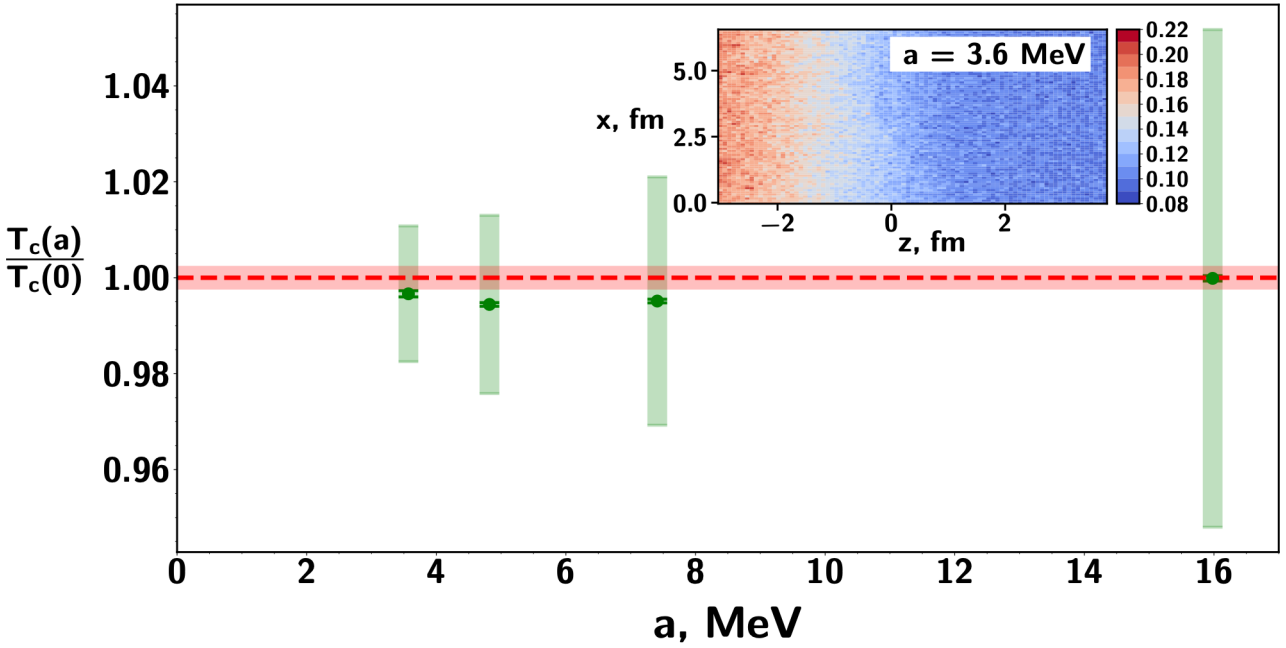
Zhu, Chen, and Huang (2026)

Phase Transition in Acc. Systems 2/2

Lattice calculation: from the character of an accelerated system

$$T(z) = T(z = 1/a)/az \ ,$$

we can consider accelerated systems by taking $T = T(z)$.



Acceleration dependence of $T_{c(\mathrm{pc})}$ is very weak.

The green bands show the width of the crossover.

Chernodub, Goy, et al. (2025)

Point until/from Now

Accelerated vacuum feels “temperature”.

→ Even SSB restoration occurs (Unruh Effect).

At finite temperature, the critical temperature does not seem to change under acceleration.

Next question：

How does acceleration change thermodynamics?

We consider a high-temperature pure gluonic system, and see deconfinement behavior under acceleration.

High-Temp. pure Yang–Mills 1/3

Expand the $SU(2)$ Yang–Mills gauge field as follows:

$$A_\mu = A_{\mathrm{B}\mu} + \mathcal{A}_\mu\,, \qquad A_{\mathrm{B}\tau} = \frac{1}{2g\beta}\phi\cdot\sigma_3\,.$$

The background part determines the Polyakov loop at LO,

$$\langle L\rangle = \frac{1}{2}\,\mathrm{tr}\,\mathcal{P}\,\exp\left(-ig\int_0^\beta \mathrm{d}\tau\, A_{B\tau}\right) = \cos\left(\frac{\phi}{2}\right).$$

Free energy becomes a function of ϕ , and then is a function of the Polyakov loop (Polyakov-loop potential).

High-Temp. pure Yang–Mills 2/3

(Local) Free energy is given by the partition function as

$$\int \mathrm{d}^3x \sqrt{g}\, V(x;\phi) = \Omega = -T \ln Z \,.$$

The partition function at one-loop order

$\sim$ The determinant of the coefficient of dynamical gauge field in the action with the Rindler metric $g_{\mu\nu}$.

e.g.) with $D_{\mathrm{B}\mu} A^\mu = 0$, we have

$$\frac{1}{2} \operatorname{tr} F^{\mu\nu} F_{\mu\nu} = \mathcal{A}_\mu (-g^{\nu\rho} D_{\mathrm{B}\nu} D_{\mathrm{B}\rho}) \mathcal{A}^\mu + \cdots \,.$$

High-Temp. pure Yang–Mills 3/3

However, there is a subtle point in using the Path Integral.

Fursaev (1998), Diakonov (2024)

When a Killing horizon exists, PI calculations of the partition function **do not** coincide with $Z = \mathrm{tr}\exp\left(-\beta H\right)$

Earlier studies for Rindler systems (1994-1998) calculated the *energy-momentum tensor* or considered *Weyl trsf. to optical spacetime*, which is free from this problem.

We also consider these different methods to obtain Z .

PLP in Accelerated Systems (EMT) 1/2

Calculate EMT in the Euclidean Rindler system.

The EMT corresponds to $T^{\mu}_{\ \nu} = \mathrm{diag}(\rho, -P_x, -P_y, -P_z)$.

$$Z = \mathrm{tr}\exp\left(-\beta \int \mathrm{d}^3x\,\sqrt{g}\,T^{\tau}_{\tau}\right), \quad -\frac{\partial}{\partial\beta}\ln Z = \int \mathrm{d}^3x\,\sqrt{g}\ \langle T^{\tau}_{\tau}\rangle\ .$$

The expectation value of the EMT is obtained from the action.

$$T^{\mu\nu}(x) = \frac{2}{\sqrt{g}}\frac{\delta S}{\delta g_{\mu\nu}(x)}, \quad \langle T^{\mu\nu}(x)\rangle = -\frac{2}{\sqrt{g}}\frac{\delta \ln Z}{\delta g_{\mu\nu}(x)}\ .$$

PLP in Accelerated Systems (EMT) 2/2

Here, we can not obtain the free energy directly from P_z , even though $P_x = P_y = P_z$.

Ambrus, Gecić (2025)

The EMT pressure and thermodynamic pressure are different.

$$P_z = -\langle T^z_z \rangle = \langle T^\tau_\tau \rangle /3 = \rho/3\,, \quad \partial_\beta(\beta V) = \rho\,.$$

$P_{\rm th} = -V$ coincides with P_z if and only if $P \propto T^4$.

Our potential is V, obtained from the EMT using thermodynamic relations.

The path-integral formalism provides $-P_z$ as free energy.

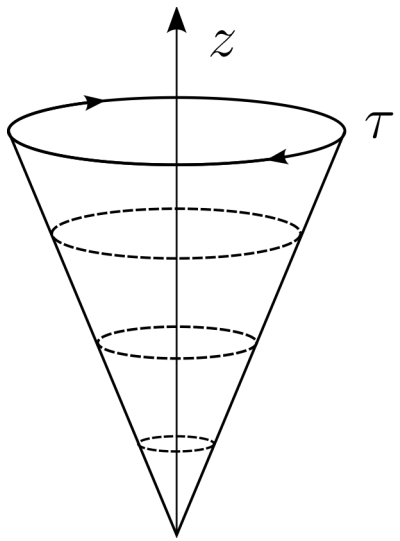
PLP in Accelerated Systems (Weyl) 1/2

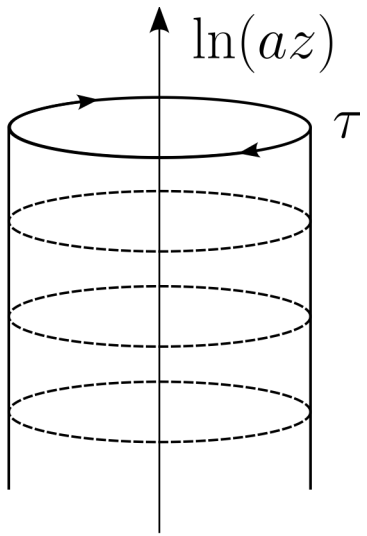
Weyl Trsf. $g_{\mu\nu} \to \Omega^2 g_{\mu\nu} =: g'_{\mu\nu}$: free from defect and singularity

$$ds^2 = a^2 z^2 d\tau^2 + dx^2 + dy^2 + dz^2$$

$$\to ds^2 = d\tau^2 + \frac{1}{a^2 z^2}\left(dx^2 + dy^2 + dz^2\right)$$

(Optical Metric)





The problem in the Rindler metric is a conical singularity at $z \to 0$.

This problem disappears after the Weyl transformation. (optical metric)

PLP in Accelerated Systems (Weyl) 2/2

Optical Metric : $ds^2 = d\tau^2 + \frac{1}{a^2 z^2}\left(dx^2 + dy^2 + dz^2\right)$

Spacetime is $S^1 \times H^3_{1/a}$ (Hyperbolic spacetime w/ radius $1/a$).

Since the time direction is flat, the free energy is easily given by the heat kernel method.

The Weyl anomaly exists but it changes the free energy only by a constant.

PLP in Accelerated Systems

As a result of our calculations, with $(\tilde{a} = a/T)$,

$$V_{\text{gluon}}(x;\phi) = \frac{T^4}{a^4 z^4} \sum_{\pm} \left[-\frac{\pi^2}{45} - \frac{\tilde{a}^2}{6} + \frac{\tilde{a}^2}{2\pi}|\phi| + \left(\frac{1}{6} - \frac{\tilde{a}^2}{4\pi^2}\right)\phi^2 - \frac{|\phi|^3}{6\pi} + \frac{\phi^4}{24\pi^2} - \frac{11\tilde{a}^4}{240\pi^2} \right],$$

$$V_{\text{fermion}}(x;\phi) = \frac{T^4}{a^4 z^4} \sum_{\pm} \left[-\frac{7\pi^2}{180} - \frac{\tilde{a}^2}{24} + \frac{\phi^2}{24} + \frac{\tilde{a}^2\phi^2}{32\pi^2} - \frac{\phi^4}{192\pi^2} - \frac{17\tilde{a}^4}{5 \cdot 192\pi^2} \right].$$

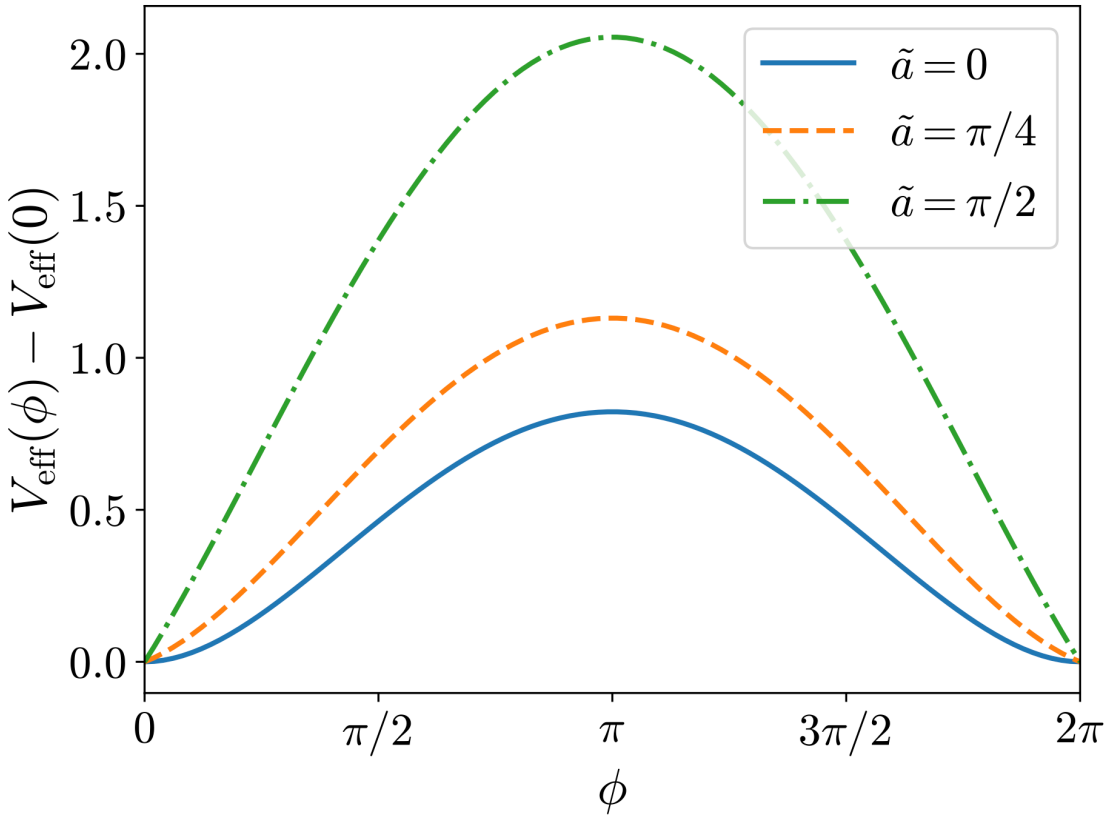
This reproduces the well-known free energy at $z = 1/a$ as $a \to 0$.

Our results also coincide with previous calculations for massless Dirac fermions (with the correspondence $\phi \sim i\mu_I$).

ref. Ambrus, Gecić (2025)

Deconfinement of finite T accel. systems

Minima of the free energy → Polyakov loop $\langle L \rangle = \cos(\phi/2)$



$\langle L \rangle = 1$ deconfined

$\langle L \rangle = 0$ confined

$\langle L \rangle = -1$ deconfined

Minima always at $\langle L \rangle = \pm 1$.
→ always deconfined

Larger acceleration makes valleys deeper.
→ more deconfined

Summary

Q: What occurs for high-temperature accelerated systems?

Our answer is:

- At finite temperature, acceleration enhances deconfinement.
- Acceleration adds a term to the potential,

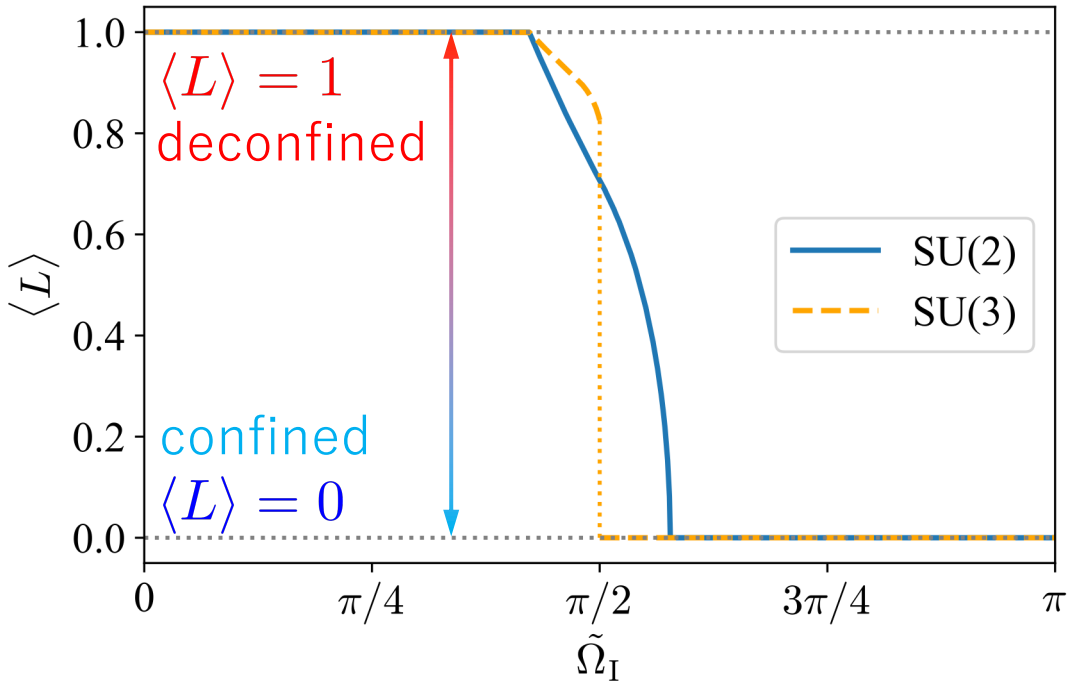
  leading to different definitions of pressure,
  and singular behavior in the Polyakov-loop potential.

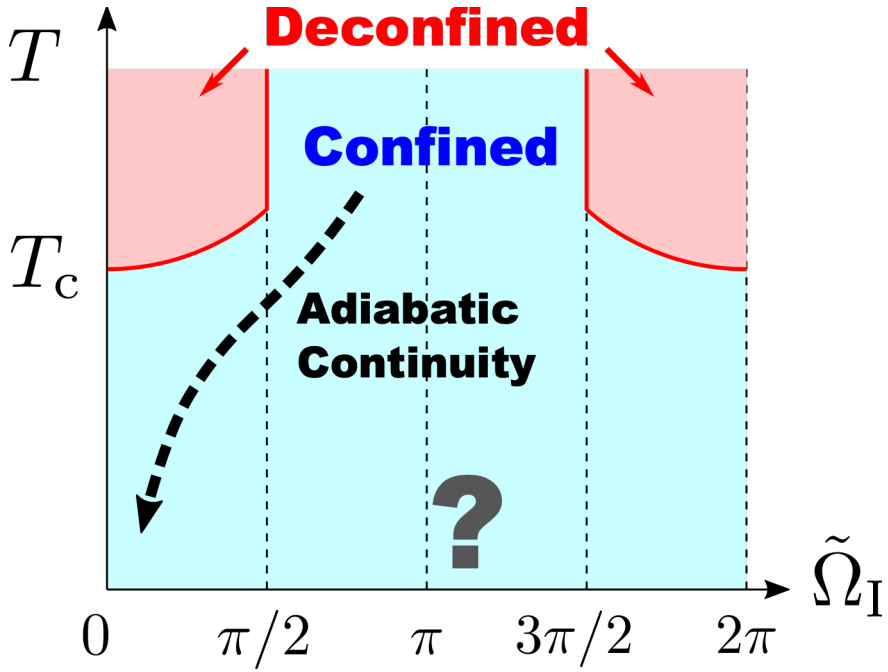
Next Q :

- What happens in full QCD?
- Why does only PI give P_z ?
- Why do PI and other methods provide different results?

Appendix 1/2

In a rotating system, *imaginary angular velocity*, a way to avoid the sign problem, leads to interesting physics.





A high-temperature but confined phase appears.

Appendix 2/2

We also consider systems with *imaginary acceleration*, which also exhibit a high-temperature confined phase.

