The IR Phase

New Frontiers in QCD
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Literature: [with degrees of separation]

0-th	0-th	1-st	1-st	2-nd
1906.08047	2305.09459	1405.2968	hep-lat/0607031	1807.03995
1502.07732	2310.03621	1412.1777	hep-lat/0610121	1809.07249
2103.05607	2404.12298		hep-lat/0703010	2110.11266
2110.04833			0803.2744	2205.11520
2509.03509				2207.13569
				2212.09806
basics	important newer evidence (thermal)	evidence (large N_f)	$< F^2 >$ & Dirac spectrum	math tools

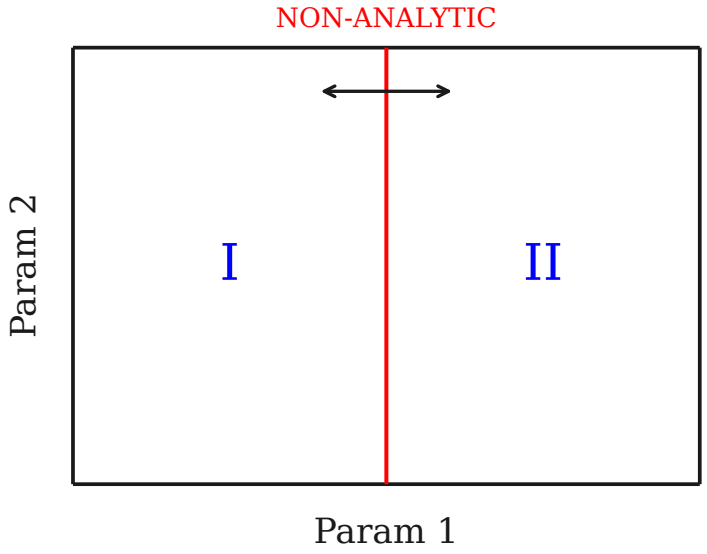
I. Context & Motivation

$$S = -\frac{1}{2g^2}\,\mathrm{tr}\,F_{\mu\nu}F_{\mu\nu} + \sum_{f=1}^{N_f}\bar{\psi}_f(D + m_f)\psi_f$$

- Euclidean formulation
- fundamental quarks
- SU(3)
- any T & μ_B=0

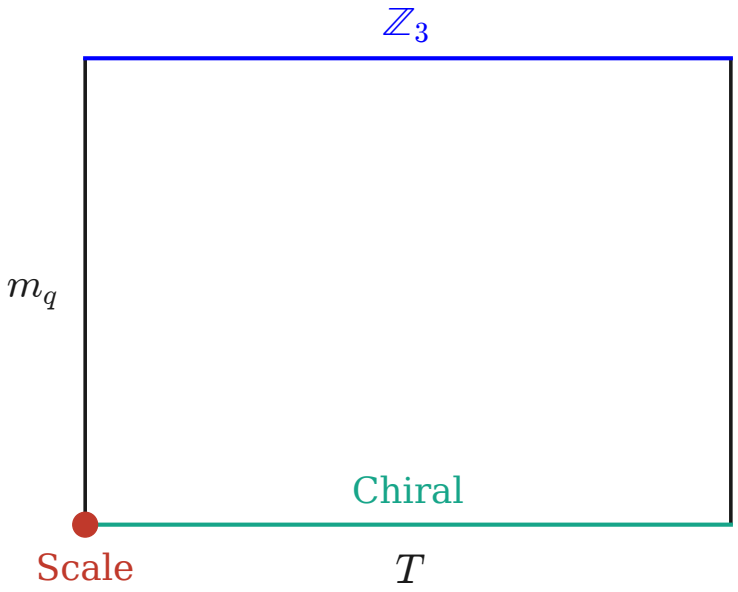
USEFUL CLASSIFICATION OF PHASES IN THE ENTIRE SET OF THESE THEORIES???

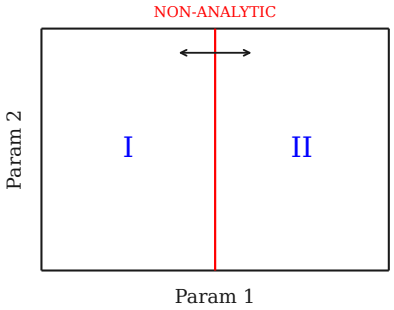
How do you mean?



- I ∪ II = entire set partition
- across boundary non-analytic

I. Context & Motivation…





Example:

- thermal N_f=2
- symmetries in the interior but not useful

STANDARD SYMMETRY APPROACH w ORDER PARAMETERS NOT LIKELY TO WORK!

⟶ If symmetry then in different guise and/or something different

II. Phases of SU(3) Gauge Theories w Fundamental Quarks

AA & IH 1906.08047

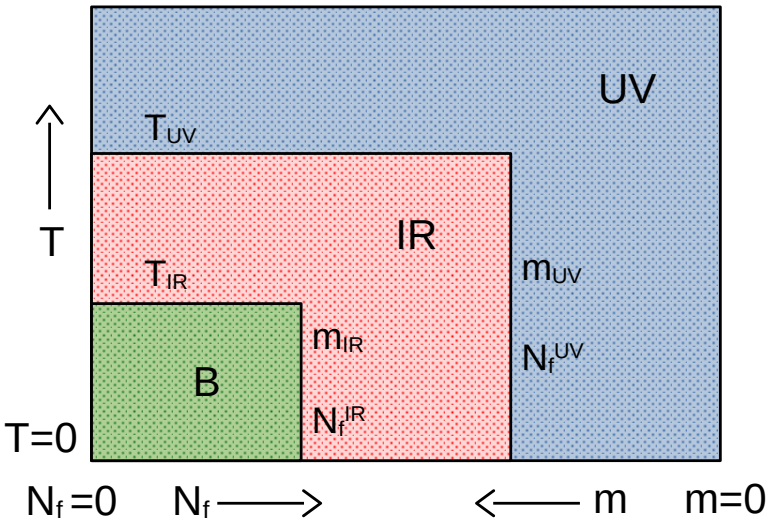
$$\text{phase} = \begin{cases} \text{B} & \text{if} \quad p = 0 \\ \text{IR} & \text{if} \quad p < 0 \\ \text{UV} & \text{if} \quad p > 0 \end{cases} \qquad \rho(\lambda) \propto \lambda^p \ , \ \lambda \to 0$$

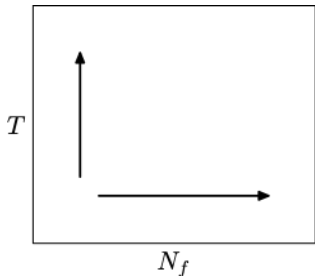
B = IR-broken

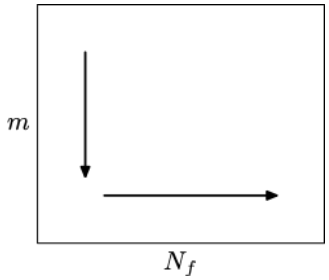
IR = IR-symmetric

UV = IR-trivial

SU(3) theories with fundamental quarks



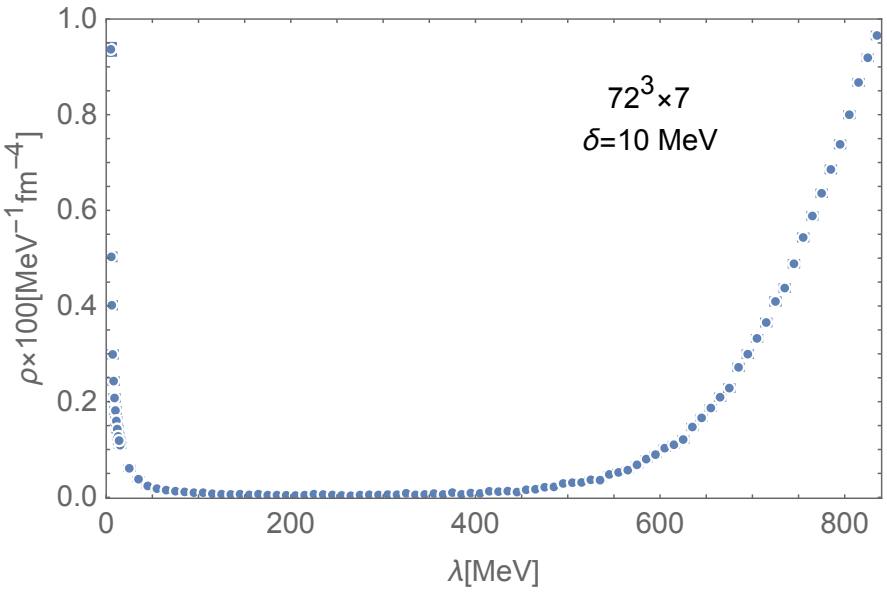




Changes in indicated directions can induce transitions B→IR or IR→UV. Also 1502.07732

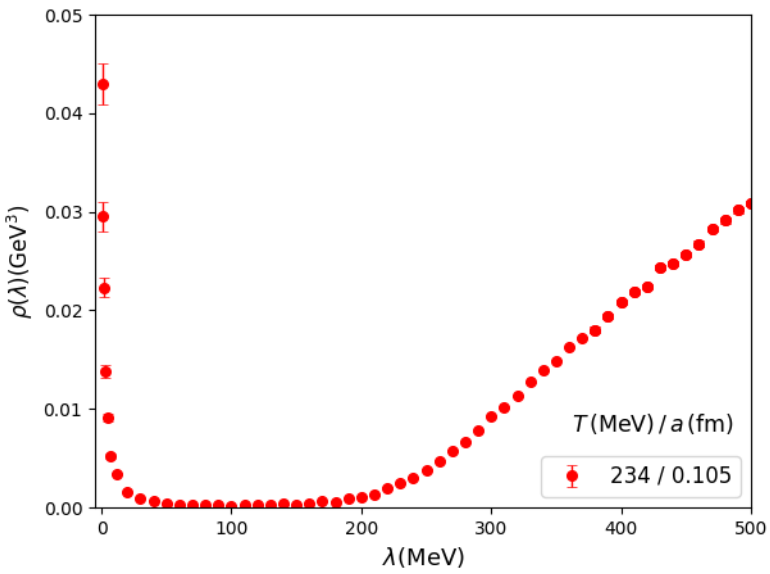
II. Phases… But is there such thing as p<0?

This is *a posteriori* appetizer:



N_f=0
a=0.085 fm
T=1.12 T_c

AA & IH unpublished

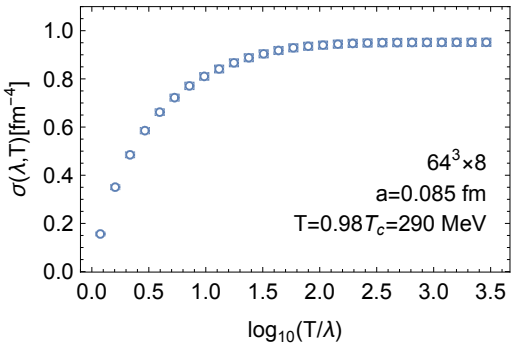


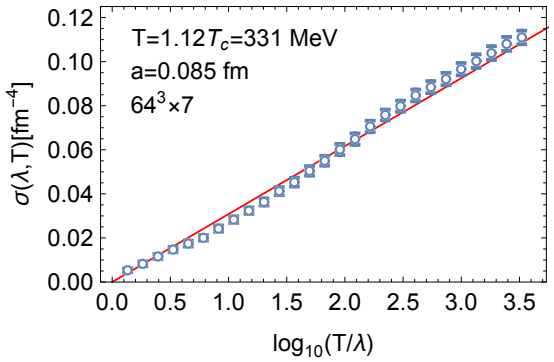
N_f=2+1 (real world)
L=5.0 fm
Clover dynamical q-s

2305.09459

II. Phases… But is there such thing as p<0 ?

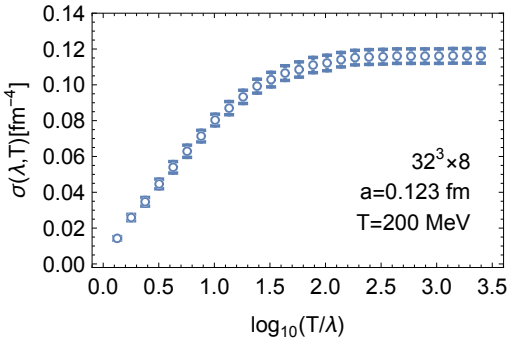
N_f=0

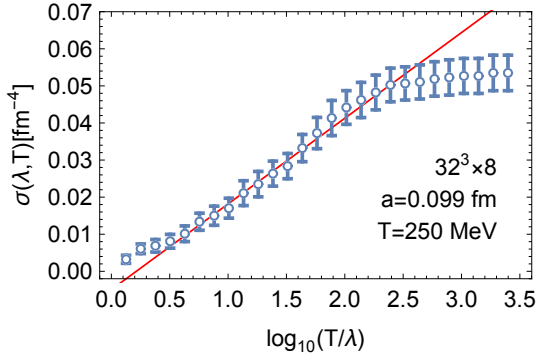




N_f=2+1

physical quark masses

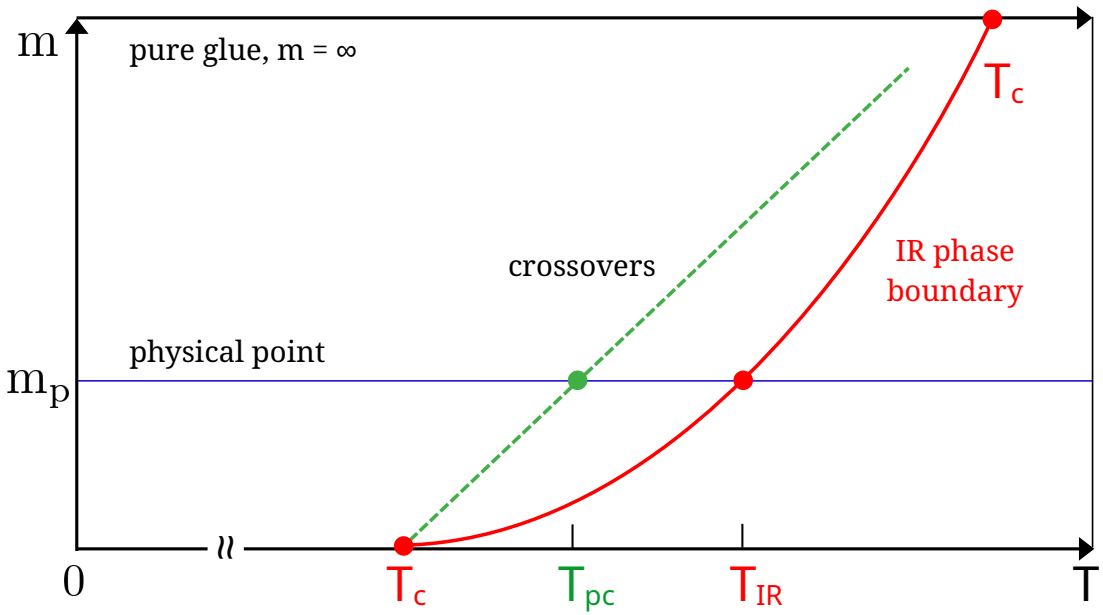




Proposition: REAL-WORLD QCD HAS IR PHASE WITH $p \approx -1$ AA & IH 1906.08047

current estimate: $T_{\mathrm{IR}} = 215(15)\,\mathrm{MeV}$ 2305.09459, 2404.12298

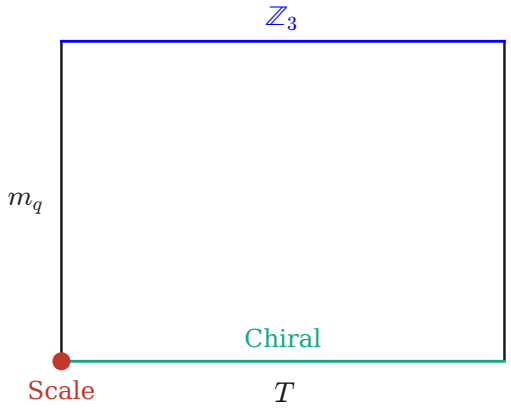
II. Phases… Thermal QCD



1906.08047
2509.03509

current estimate: $T_{\rm IR} = 215(15)\,{\rm MeV}$

1906.08047 2305.09459 2404.12298

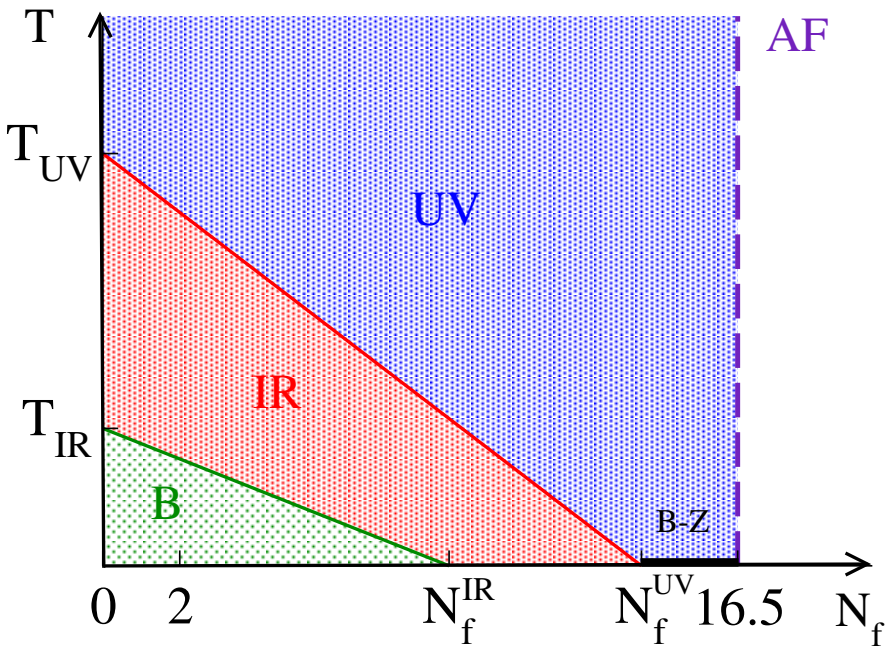


II. Phases… Many Flavors

Important B➞IR light-flavor case (T=0):

1405.2968, 1412.1777, 1906.08047

IR PHASE = STRONGLY COUPLED PART OF CONFORMAL WINDOW



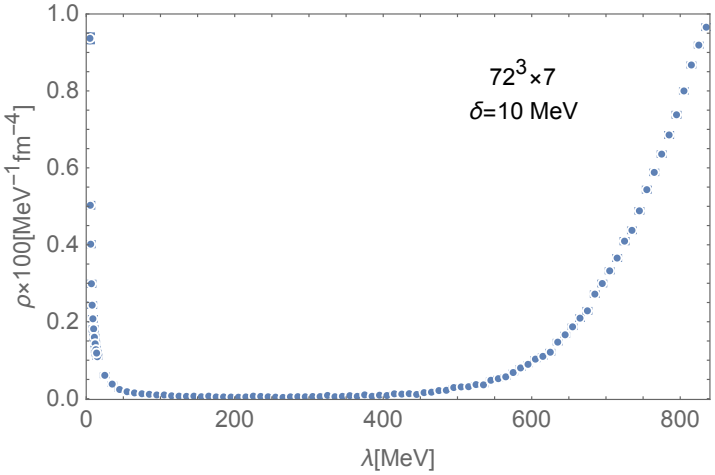
III. What Does the IR Phase Entail? Thermal…

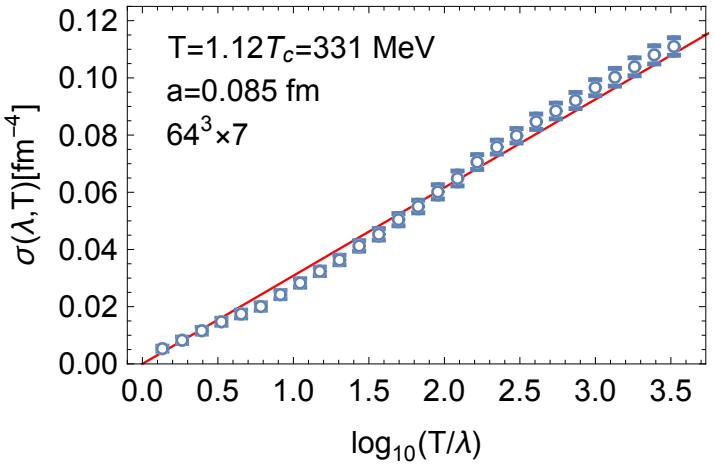
SCALE INVARIANCE + IR SEPARATION/DECOUPLING + RANDOMNESS

Scale invariance???

SCALE INV is possible due to DECOUPLING which is possible due to RANDOMNESS

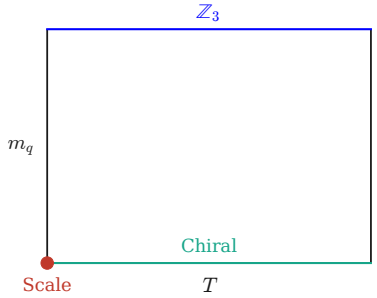
SCALE INV INHERITS ITS NATURE FROM BOTH





ρ(λ) is an almost pure power law but only in the decoupled IR component

Loophole to

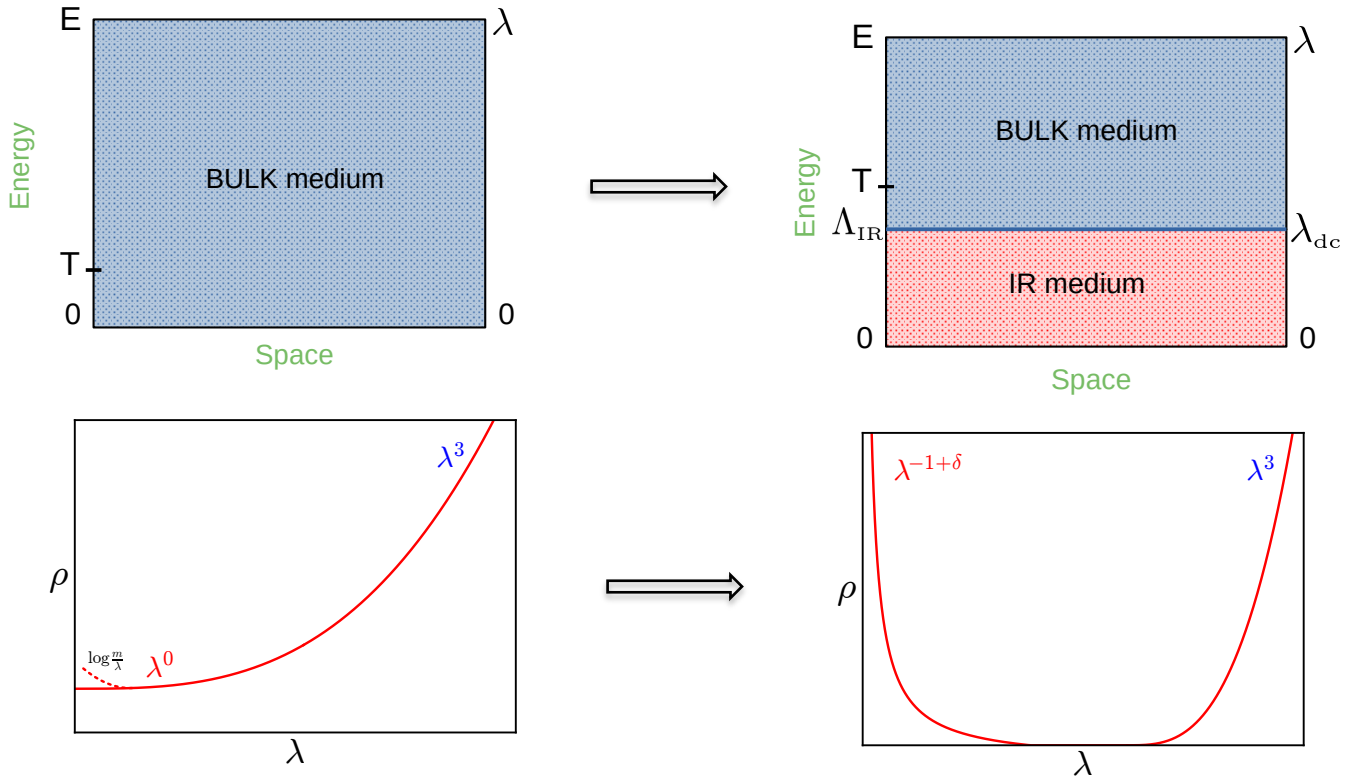


III. What Does the IR Phase Entail? Scale Inv + Decoupling

IR Transition at T_{IR}:

- single-component ⟶ multi-component
- scale invariance restored in IR component

1906.08047 1502.07732
2103.05607 2110.04833



III. What Does the IR Phase Entail? Scale Inv…

$$\rho(\lambda) \propto \lambda^{-1+\delta} \quad \Longrightarrow \quad \text{scale invariant glue field}$$

AA & IH 1906.08047

[Argument turned out to be perfectly valid – see on randomness.]

But how to make it more formal?

$$\langle F^2 \rangle_{a,L} = \frac{a^2}{c_{\rm S}\Delta} \int_0^{(\frac{2\Delta}{a})^-} d\sigma\, \sigma^2\, \rho_{\rm eff}(\sigma, a, L) + T\, \frac{2\Delta}{c_{\rm S}}\, \frac{\langle n_0 \rangle_{a,L}}{L^3}$$

IH 2509.03509

$$\rho_{\rm eff}(\sigma) \equiv \rho(\sigma) - \rho_0(\sigma) \qquad n_0 = \text{number of zeromodes}$$

overlap spectral density

→ If $\rho_{\rm eff}(\sigma)$ is a pure power, then the contribution of the interval $[0, \sigma_{\rm dc}]$ is a pure power in thermodynamic limit

→ Decoupled IR component doesn't contribute to scale anomaly

III. What Does the IR Phase Entail? Decoupling…

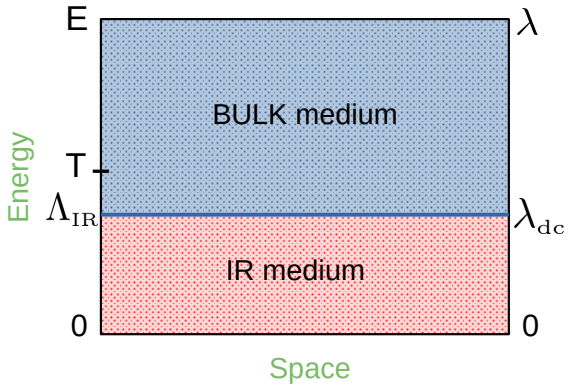
$$\langle F^2 \rangle(\sigma_1, \sigma_2) \propto \int_{\sigma_1}^{\sigma_2} d\sigma\, \sigma^2\, \rho_{\rm eff}(\sigma) \qquad \langle F^2 \rangle = \langle F^2 \rangle_{\rm IR} + \langle F^2 \rangle_{\rm B}$$

Decoupling:

There exists σ_{dc} such that for the associated scale parts of F^2

$$\left\langle F^2_{\rm IR} F^2_{\rm B} \right\rangle_c \longrightarrow 0 \quad \text{for } L \to \infty$$

THE STATEMENT OF DECOUPLING IS THE STATEMENT ON DECORRELATION OF IR & BULK



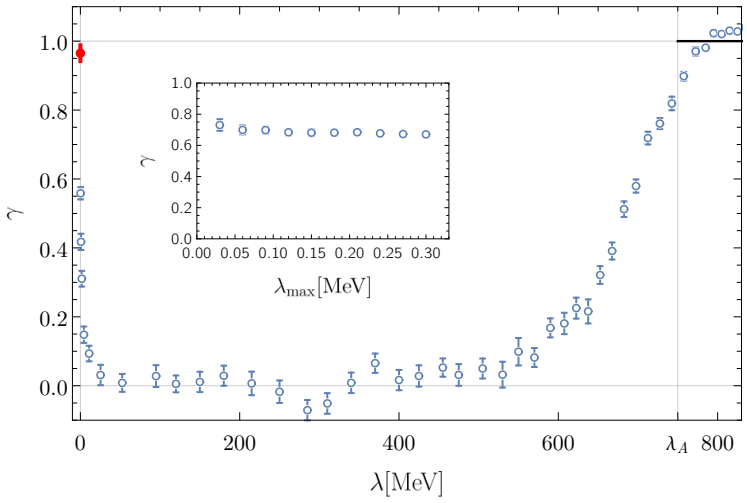
IR phase transition is also a transition
a from single-component system to
a multicomponent system.

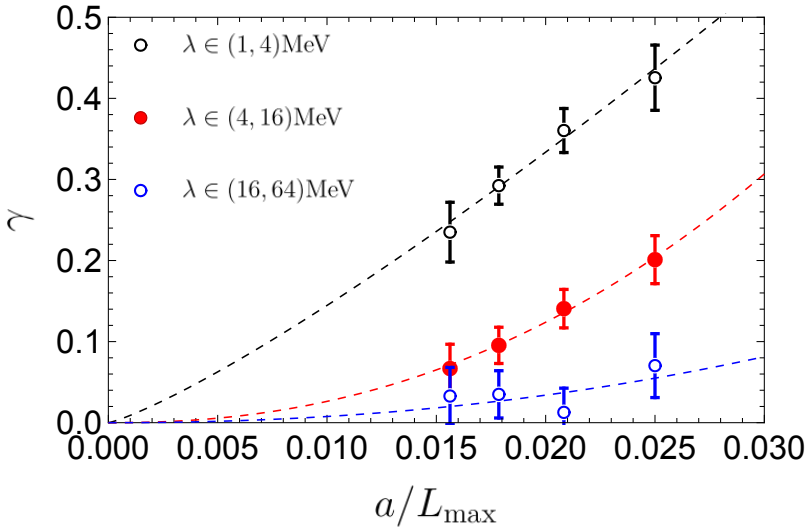
No symmetry needed a priori although scale invariance of a component involved.

III. What Does the IR Phase Entail? Randomness

2110.04833

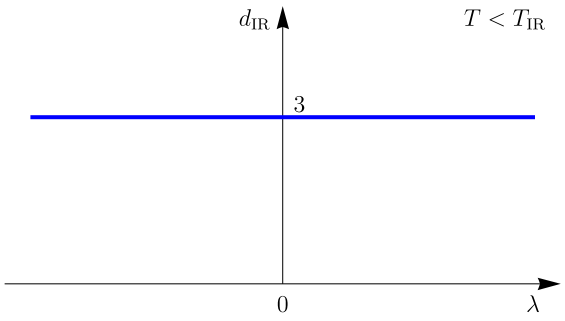
2103.05607 2110.04833 2310.03621

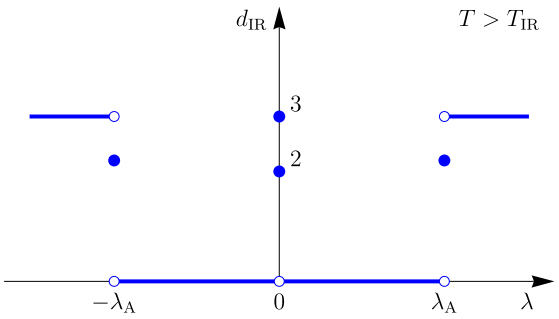




$$\left\langle\, \ell[\psi] \,\right\rangle_{a,L,\lambda} \propto L^{\gamma(a,\lambda)} \quad \text{for} \quad L \to \infty$$

IR: $\quad \left\langle\, \mathcal{N}_\star \,\right\rangle_{a,L} \propto L^{d_{\rm IR}(a)} \;, \quad L \to \infty$

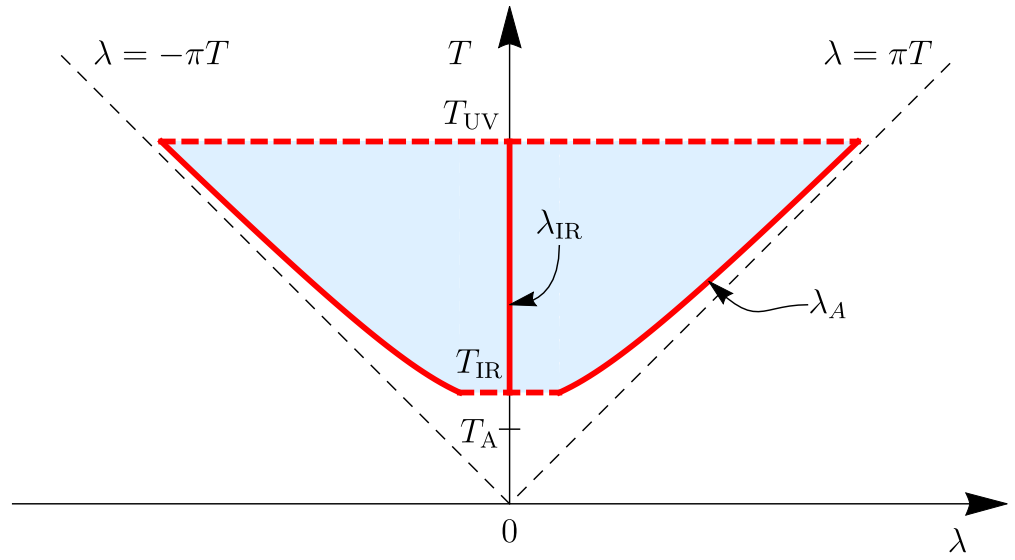


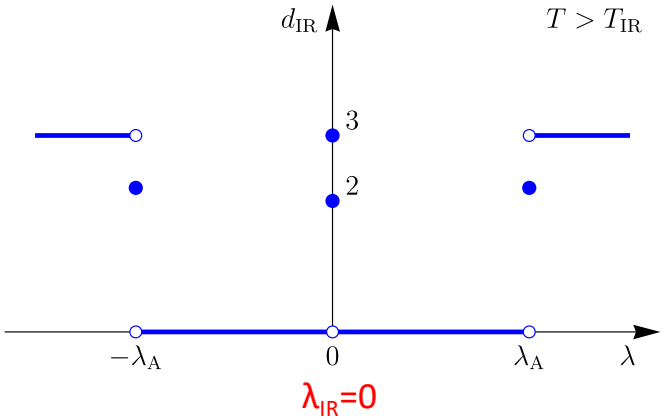


Collapse of size and dimensional collapse of volume: Very ``Anderson-like''

III. What Does the IR Phase Entail? Randomness…

Metal-to-Critical picture: AA&IH 2110.04833





λ_{IR}=0

λ_{IR}=0 AA & IH 2103.05607 2110.04833, 2310.03621
singular mobility edge
responsible for long-range nature

λ_A>0 Garcia-Garcia, Osborn 2007
Kovacs, Pitler 2010
Giordano, Kovacs, Pitler 2014
regular mobility edge

III. What Does the IR Phase Entail? Randomness…Take this to the extreme

One particular type of Anderson mobility edge:

Evangelou, Katsanos cond-mat/0206089

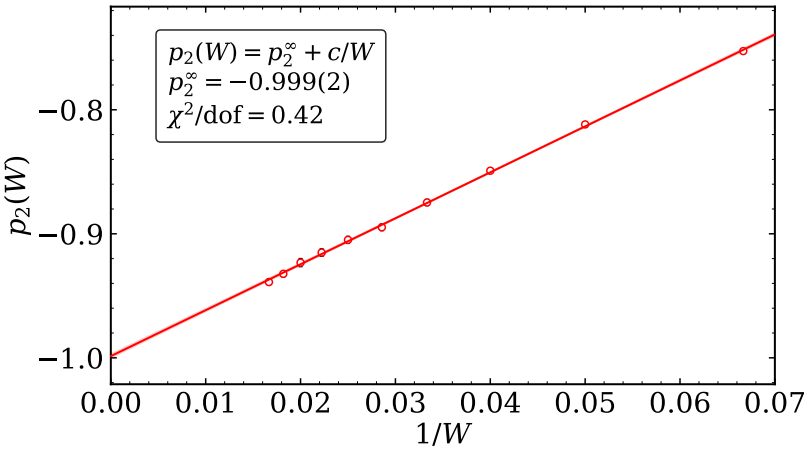
Chiral orthogonal bipartite model with off-diagonal logarithmic disorder in (d=3)

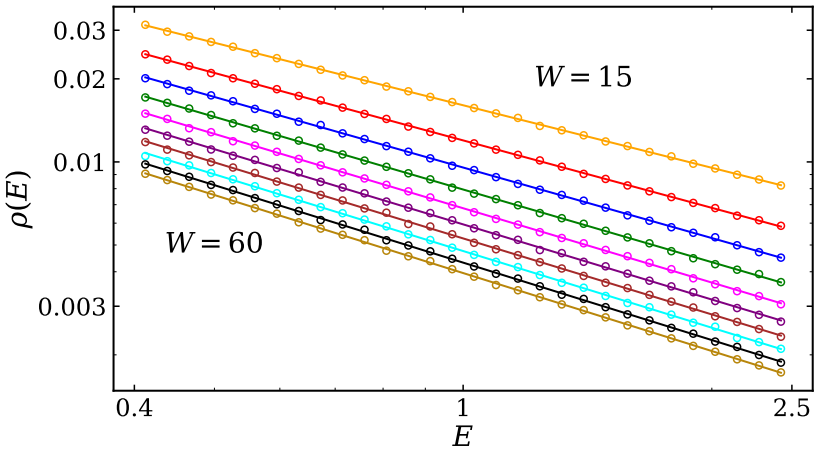
$$H = \sum_{r\in\mathcal{L},i} t_{r,r+i}\left(c_r^\dagger c_{r+i} + c_{r+i}^\dagger c_r\right)$$

$t_{r,r+l}$ analogous to $U_{x,\mu}$

$$P(t) = \frac{1}{W}\frac{1}{t} \qquad t \in \left[e^{-W/2}, e^{W/2}\right]$$

regularized scale-invariant randomness





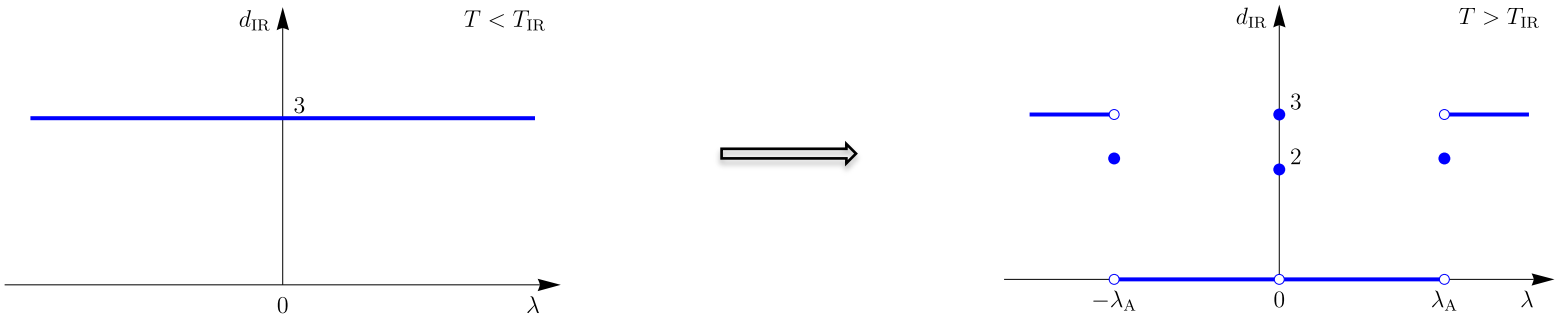
$$\lim_{W\to\infty} p(W) = -1$$

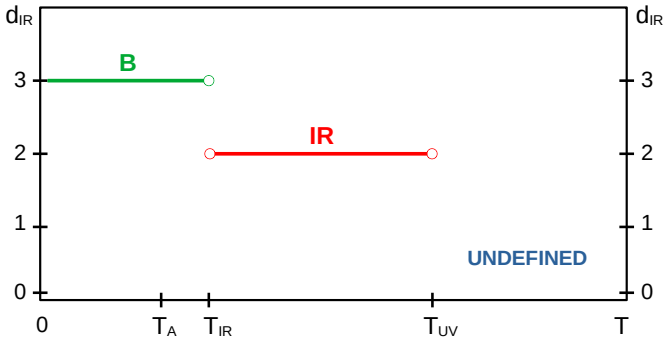
Upon removing the regularization, quantum mechanics transforms self-similar noise into scale-invariant spectrum

Recall our interpretation of scale-invariant density wrt glue fields…

III. What Does the IR Phase Entail? Non-Analyticity

Descends from non-analyticity of Anderson-like structures…

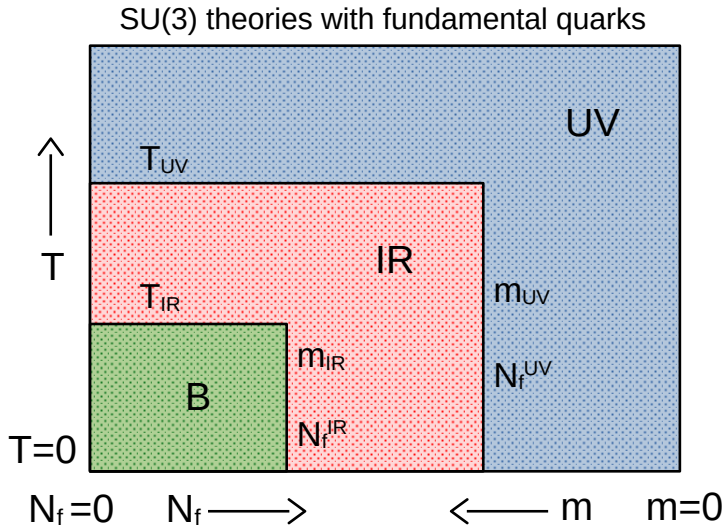




- d_{IR} is the infrared spatial dimension
- source of non-analyticity in T is the dimensional collapse

IV. Take-Homes

$$\text{phase} = \begin{cases} \text{B} & \text{if} \quad p = 0 \\ \text{IR} & \text{if} \quad p < 0 \\ \text{UV} & \text{if} \quad p > 0 \end{cases} \qquad \rho(\lambda) \propto \lambda^p \ , \ \lambda \to 0$$



This satisfies what I set to define as a phase structure…

(i) describes a multicomponent system with scale-invariant IR component

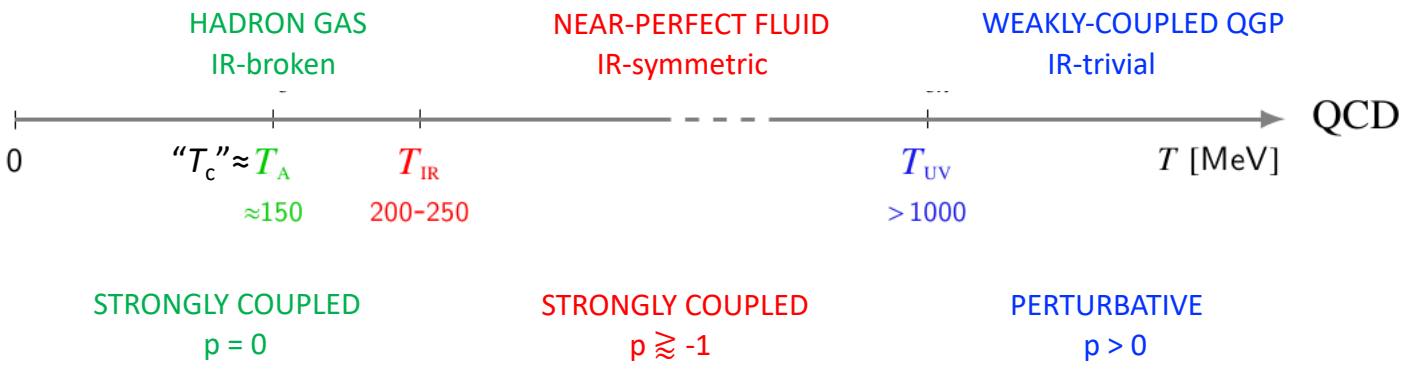
(ii) IR component is long-range: long-range of noisy type (really different)

(iii) transitions non-analytic

[Would not be true only if iAnderson-like connection totally flawed – unlikely.]

Take-Homes… Thermal

AA & IH 1906.08047



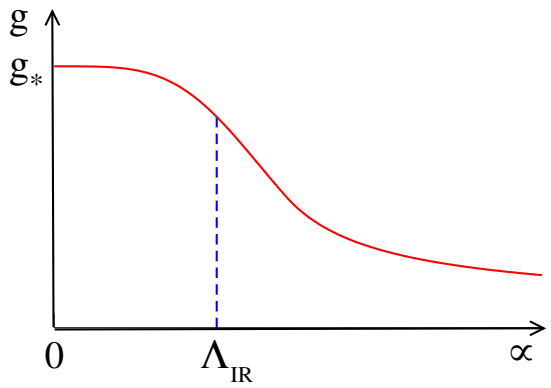
$$T_c < T_{IR} < T < T_{UV}$$

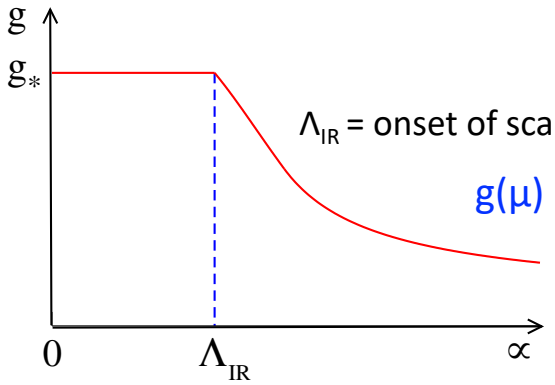
≈155 MeV 200-230 MeV perturb

BACKUPS

How did the connection to Anderson-like localization arise? 1906.08047, 2103.05607, 2110.04833

Came from early considerations concerning IR scale invariance: asymptotic or exact?





Λ_{IR} = onset of scale invariance

g(μ) non-analytic at Λ_{IR}

Q: If exact, how does the non-analyticity arise?

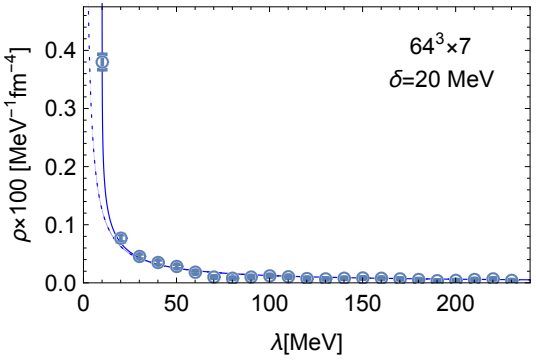
A: IR-Bulk decoupling ---> IR glue fluctuates independently of Bulk glue
---> mismatch/non-analyticity natural

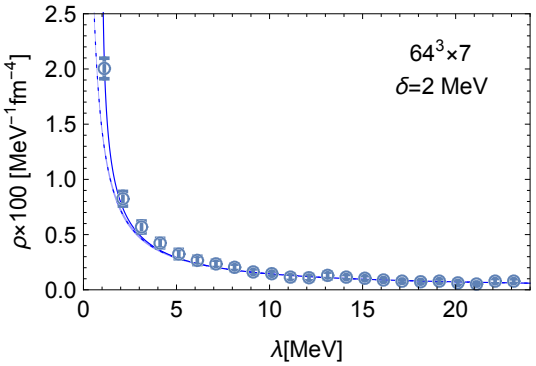
Q: Is there a known non-analytic structure in the Dirac spectrum that could do that?

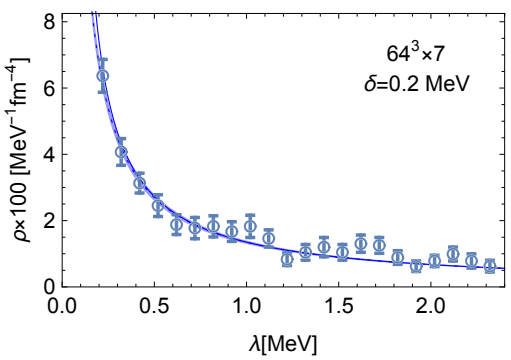
A: Anderson-like mobility edge $\lambda_A > 0$ proposed/studied in

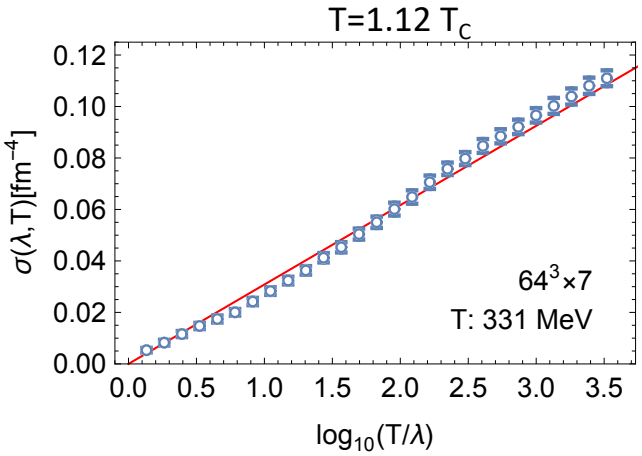
Garcia-Garcia & Osborn hep-lat/0611019 Kovacs & Pitler 1006.1205 Giordano, Kovacs & Pitler 1312.1179

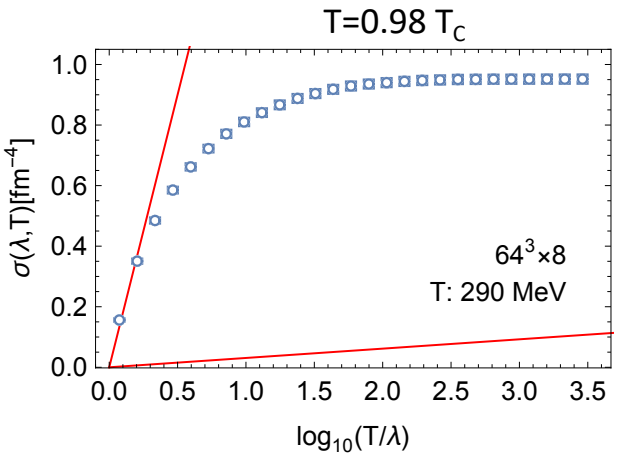
Fits to $\rho(\lambda) \propto 1/\lambda$ [N_f=0, T=1.12 T_c] AA & IH 1906.08047











Data: (1) IR SCALE-INVARIANT DENSITY ($\lambda < T$) OVER 3 ORDERS OF MAGNITUDE IN SCALE

(2) NEGATIVE POWER-LAW ACCUMULATION OF DIRAC MODES IN IR: $\rho(\lambda) \propto \lambda^p$ $p \gtrsim -1$

Proposal: THIS REFLECTS IR SCALE-INVARIANT GLUE: IR PHASE [p<0] 1906.08047

T=0 classically scale invariant theory

quantum fluctuations

scale anomaly

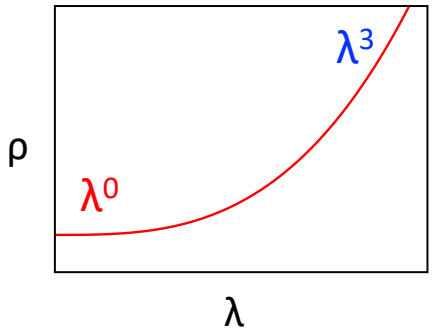
scales generated world of hadrons

scale-broken T=0 theory

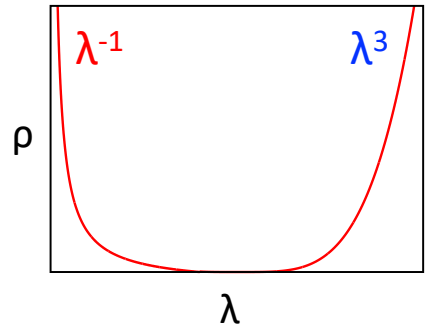
thermal fluctuations

increasing T

scale-invariant but only for $\Lambda < \Lambda_{IR} < T$



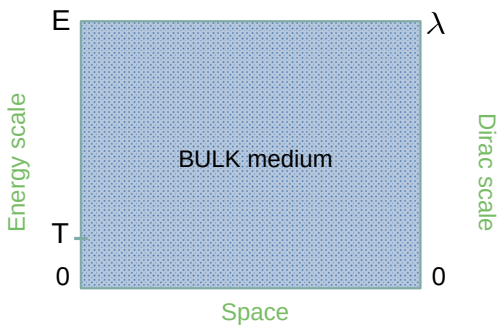
thermal agitation



$\lambda^{-1} \rightarrow \lambda^{-1+\delta}$

$\delta=\delta(a) \rightarrow 0$?

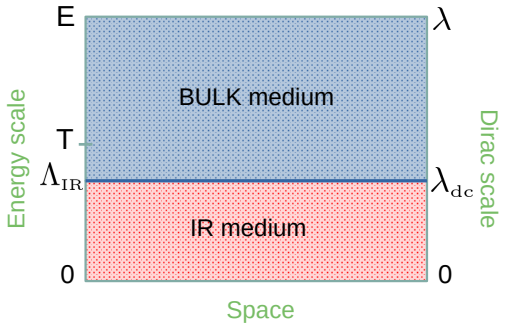
$a \rightarrow 0$



thermal agitation

IR-BULK SEPARATION

AA & IH 1906.08047



NON-INVARIANT

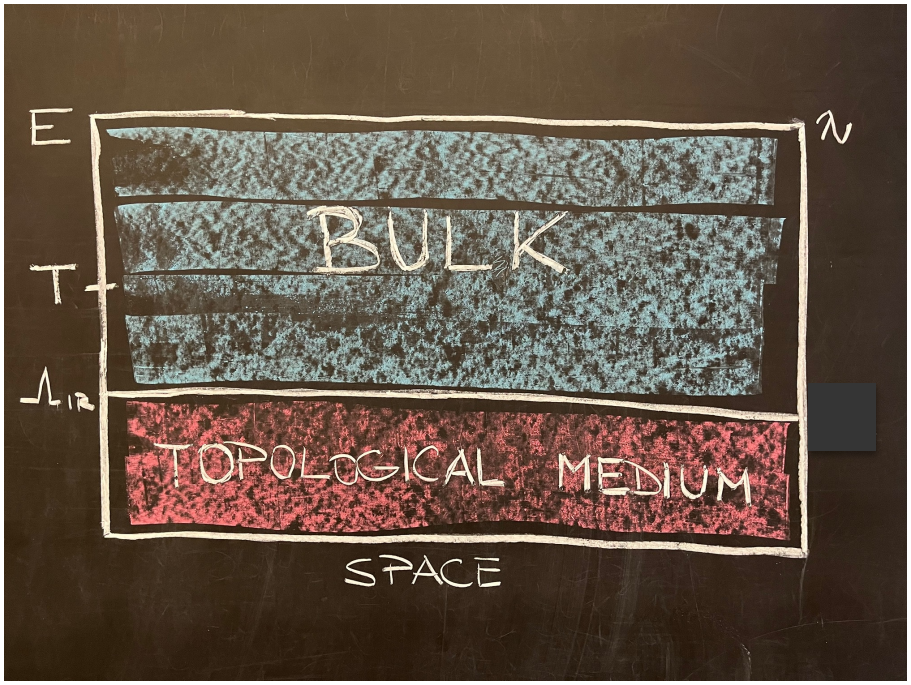
INVARIANT

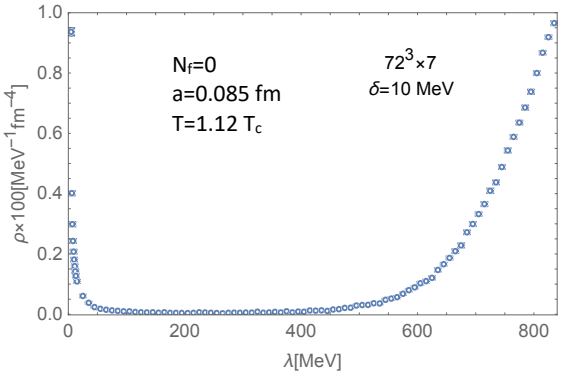
AT $T=T_{IR}$ THERMAL QCD BECOMES 2-COMPONENT SYSTEM: IR MEDIUM AN AUTONOMOUS SUBSYSTEM

Hypothesis: IR phase describes the near-perfect fluid [RHIC, ALICE] state of matter

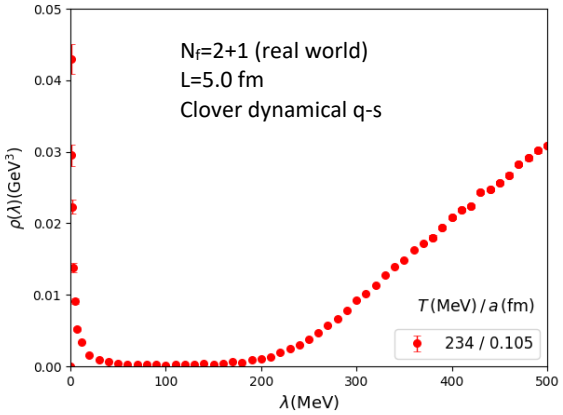
Experimental signatures on ALICE3?

AA & IH 1906.08047





AA & IH
unpublished

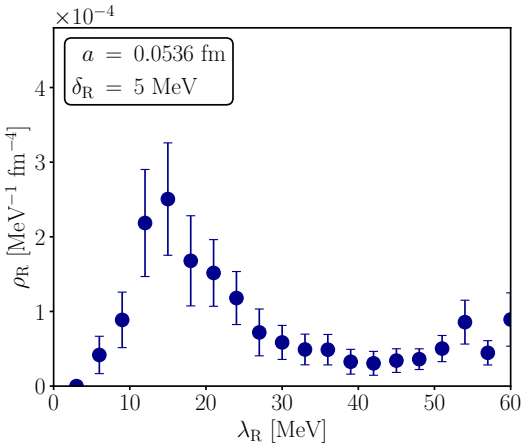


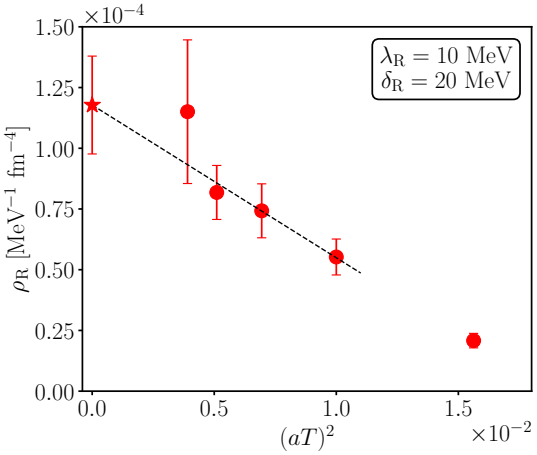
X. Meng et al
2305.09459

- System in IR Phase a 2-component medium: invariant IR and non-invariant bulk
- Tisza/Landau 2-component theory of liquid helium? Maybe in some dual form.

A. IR Phase: Real-World QCD…

AA, Bonanno, D`Elia, IH 2404.12298





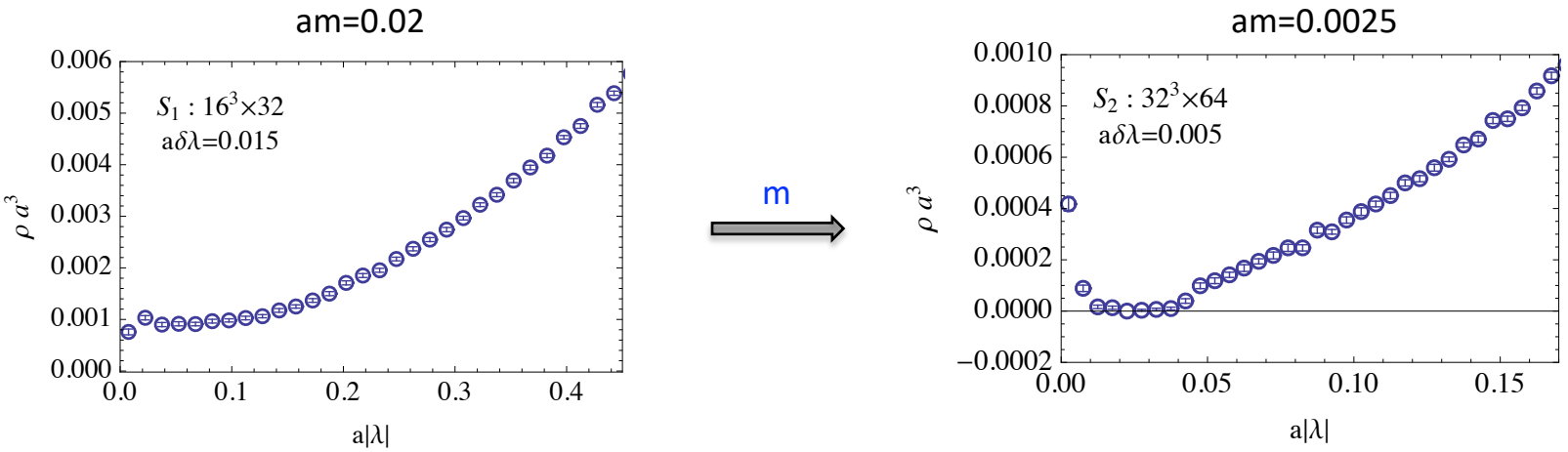
Real-world QCD is N_f=2+1 at physical quark masses of stouted staggered quarks here.

Lattice Dirac operator = stouted staggered [not overlap]

- IR structure exists in Dirac operator describing dynamical quarks
- not a lattice artifact
- IR medium is a quark-glue medium
- green light to study IR phase using overlap: correct & efficient

A. IR Phase: Theories with Many Flavors?

N_f=12, T=0 Configss: A. Hasenfratz et al, 1207.7162 staggered with nHYP AA & IH 1405.2968 1411.1777

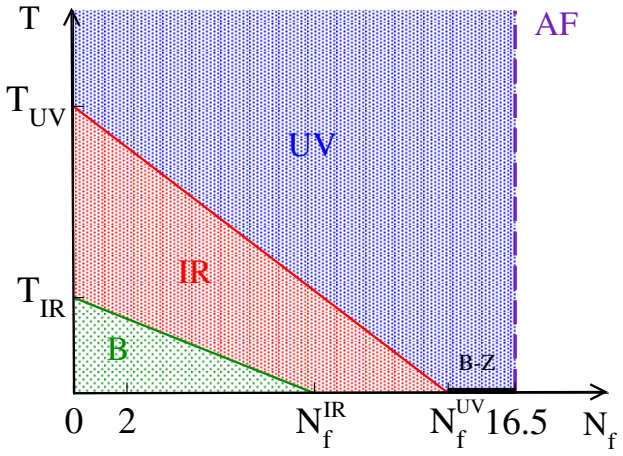


Lowering quark mass at sufficient number of flavors can generate IR phase

Conjecture: AA & IH 1906.08047

Conformal window has a strongly coupled part with p < 0.

$$N_f^c \equiv N_f^{\rm IR} < N_f < N_f^{\rm UV} \leq 16.5$$



WHAT WE HAVE HERE IS THE LACK OF COMMUNICATION

N_F=0 easier to communicate the point [same in N_f=2+1 just more awkward.]

T=0 classically scale invariant theory

quantum fluctuations

scale anomaly

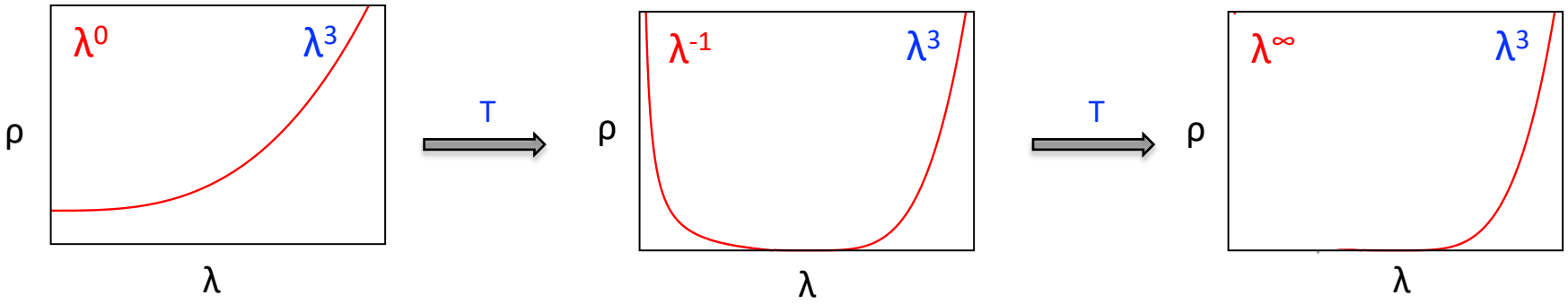
scales generated
world of hadrons etc

scale-broken
T=0 theory

thermal fluctuations

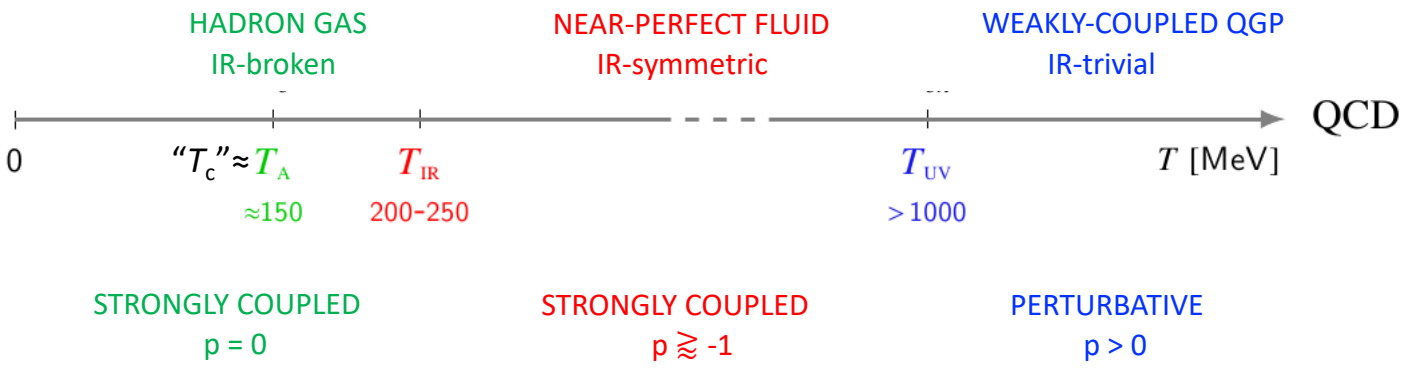
increasing T

scale-invariant
but only for $\Lambda \leq \Lambda_{\rm IR} < T$



WHAT WE HAVE HERE IS THE LACK OF COMMUNICATION…

PHASE STRUCTURE OF THERMAL QCD IN TERMS OF GLUE IR SCALE INVARIANCE [AA & IH 1906.08047]

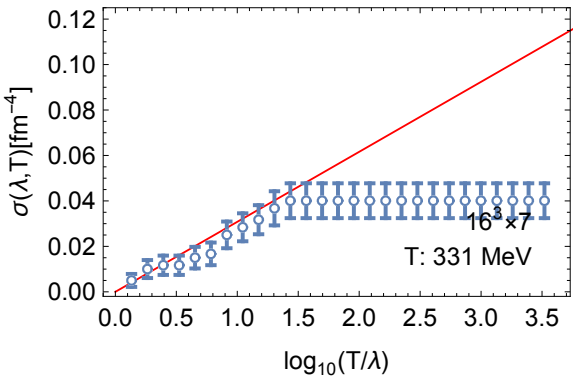


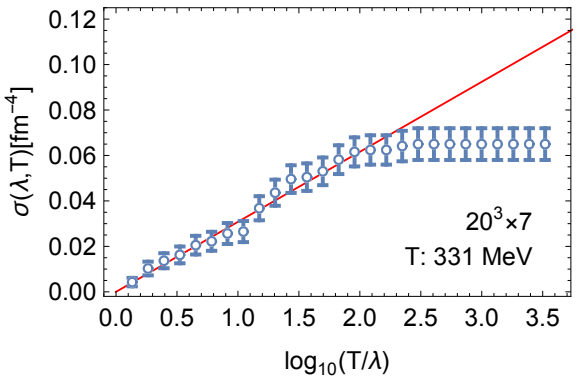
$$\text{phase} = \begin{cases} \text{B} & \text{if} \quad p = 0 \\ \text{IR} & \text{if} \quad p < 0 \\ \text{UV} & \text{if} \quad p > 0 \end{cases} \qquad \text{with} \qquad \rho(\lambda) \propto \lambda^p \quad \text{for} \quad \lambda \to 0$$

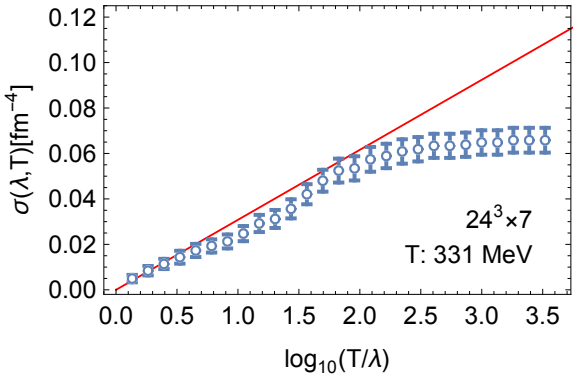
Original talk: https://indico.cern.ch/event/764552/contributions/3420459/attachments/1865996/3068382/WuHan_jun_2019_infra.pdf

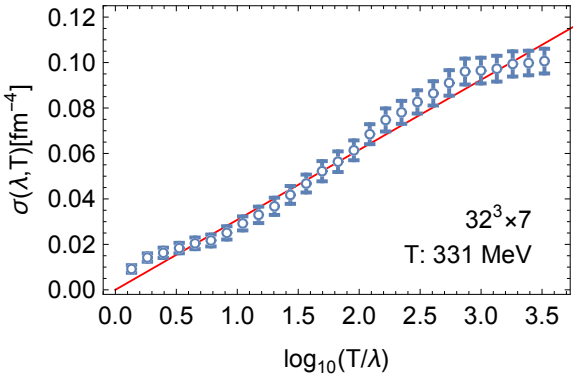
Useful talk: https://drive.google.com/file/d/1vZ0AY0WsZAfF9iV7-Br-E_2NiwaZzRGp/view

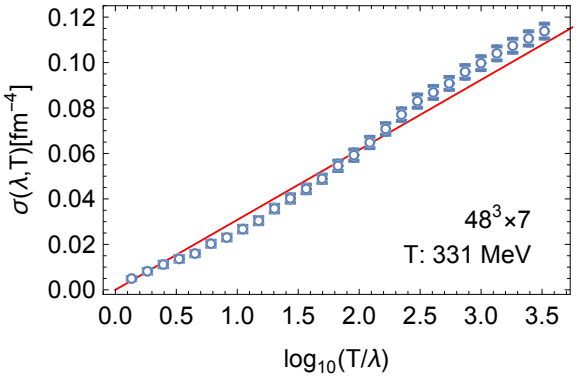
Check this further & better… $\sigma(\lambda_1, \lambda_2) \equiv \int_{\lambda_1}^{\lambda_2} d\lambda\, \rho(\lambda)$ $\rho(\lambda) = c/\lambda \longrightarrow \sigma(\lambda, T) = c \ln(T/\lambda)$

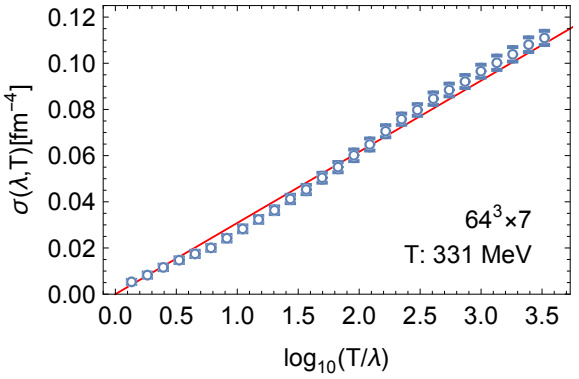






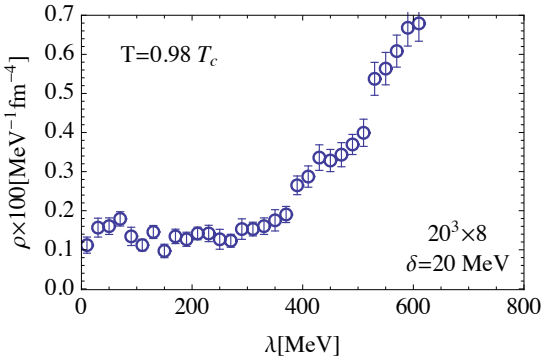


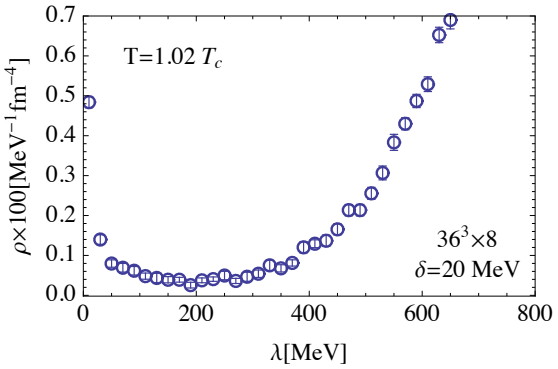


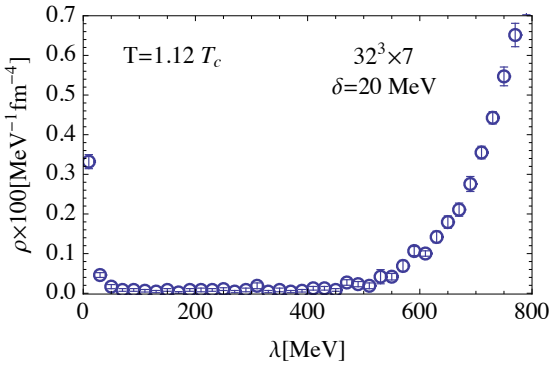


- Peak in IR overlap spectrum upon crossing T_c (pure glue) [Edwards, Heller, Narayanan, Kiskis, 1999]

- Our version of it [AA & IH, 1502.07732]







- knee-jerk reaction was: quenched chiral condensate may diverge in high-temperature pure glue
- knee-jerk reaction should be: what on earth is glue doing to produce this? [1502.07732]
- didn't know but went on with it, e.g., around chiral crossover we got this:

