

New Frontiers in QCD 2026
Yukawa Institute for Theoretical Physics
QCD Matter under Extreme Conditions, Oct. 22 2026

The screening spectrum of QCD up to Electroweak-scale temperatures

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Outline

I Screening masses at high temperature

- Why screening masses? Why up to $T = 165$ GeV?

II Computational strategy and lattice setup

- Lines of constant physics and ensembles
- Effective mass analysis
- Combined continuum limits

III Discussion of results

- Temperature dependence and comparison with perturbation theory
- The hyperfine splitting
- On the restoration of chiral and $U(1)_A$ symmetry

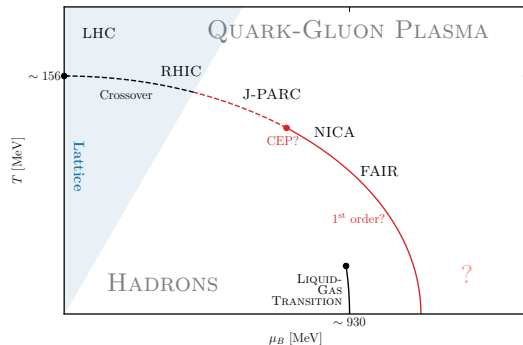
IV Closing

Thermal QCD

Understanding how QCD behaves at **non-zero temperature** is crucial for several reasons:

QCD phase diagram

- **Chiral crossover** at $T_c \simeq 156$ MeV
- **Heavy Ion collision** experiments probe **Quark-Gluon Plasma** (QGP), $T \lesssim 350$ MeV

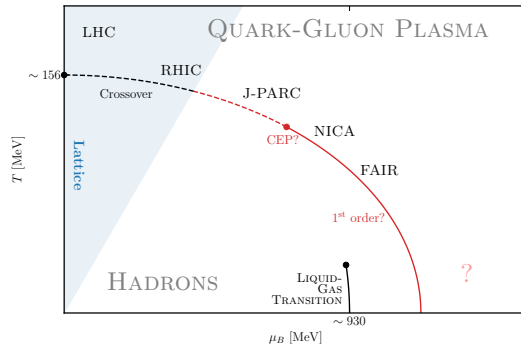
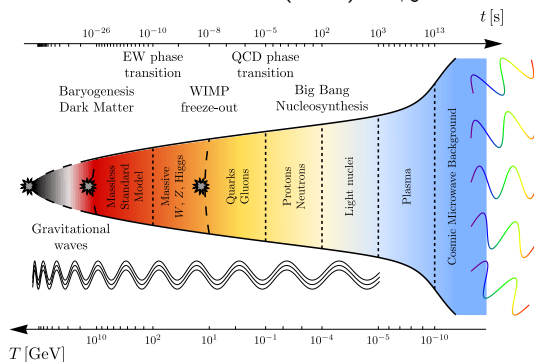


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Early Universe cosmology

- **Expansion** and evolution of the Universe after the Big Bang largely influenced by **QCD thermodynamics** (Equation of state, ...)
- Higher temperatures $\longleftrightarrow$ earlier times

Screening masses

Investigate the **spatial structure** of the QGP at equilibrium

- Over what **length scales** are interactions in the plasma **screened**?
- How does the medium respond to the insertion of a given particle?

Screening masses – inverse **correlation lengths** related to different channels (quantum numbers)

$$C_O^{(n)}(x_3) = \int_0^{L_0} dx_0 e^{i\omega_n x_0} \int dx_1 dx_2 \langle O(x) O^\dagger(0) \rangle \propto \exp\left\{-m_O^{(n)} x_3\right\} \left(1 + \mathcal{O}\left(\exp\left\{-\Delta_m^{(n)} x_3\right\}\right)\right)$$

$$m_O^{(n)} = \lim_{x_3 \rightarrow \infty} -\frac{d}{dx_3} \ln C_O^{(n)}(x_3)$$

Mesonic, $\omega_n = 0, 2\pi T, 4\pi T$ not discussed today

Baryonic, $\omega_n = \pi T$

$$O_\Gamma = \bar{\psi} T^a \Gamma \psi$$

$$N_\alpha^\pm = \epsilon^{abc} (u^{aT} C \gamma_5 d^b) d_\alpha^c P^\pm$$

Label	S^a	P^a	$V_\perp^a$	$A_\perp^a$	V_0^a	A_0^a
Γ	$\mathbb{1}$	γ_5	$\gamma_{1,2}$	$\gamma_5 \gamma_{1,2}$	γ_0	$\gamma_5 \gamma_0$

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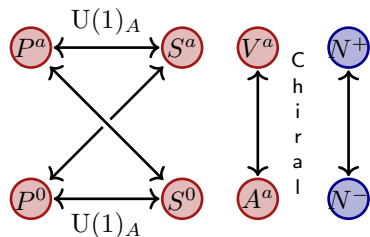
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Screening spectrum of QCD



Probe **symmetry restoration** at high T : [talk by Nie]

- Non-abelian **chiral symmetry**: degeneracy of V^a / A^a and N^+ / N^- channels
- $U(1)_A$ (topology suppression, “effective restoration”): degeneracy of P^a and S^a
- Larger **emergent** symmetries?

[Aoki et al. 2025; Rohrhofer et al. 2019]

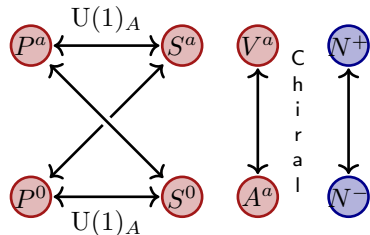
► At **very high T** can be computed in **perturbation theory**:

- Mesons: $m_{O_r}^{(n)} = \begin{cases} 2\pi T & n = 0 \\ 2n\pi T & n > 0 \end{cases} + c_{O_r}^{(n)} g^2 T + \mathcal{O}(g^3)$ [Laine and Vepsalainen 2004]
[Brandt et al 2012]
- Baryons: $m_{N^\pm} = 3\pi T (1 + 0.0046 \dots g^2 + \mathcal{O}(g^3))$ [Giusti, ..., PR 2024]
- **Spin-splitting**: $(m_{V_\perp}^{(0)} - m_P^{(0)}) / 2\pi T = 0.002376 \dots g^4 + \mathcal{O}(g^5)$ [Cé, ..., PR 2025]

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[Linde 1980] [Ginsparg 1980] [Appelquist and Pisarski 1981]

Non-perturbative study required
also at **very high temperatures**

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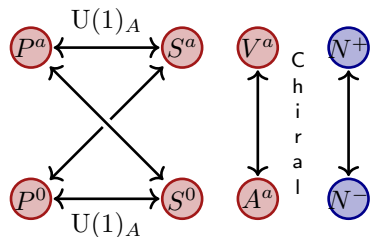
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Lattice QCD up to the Electroweak scale

Hadronic schemes face a **window problem** at high temperature:

$$L^{-1} \ll M_{\text{Had}} \ll T \ll a^{-1}, \quad \text{unfeasible for } T > 1 \text{ GeV}$$

Instead: fix the temperature imposing $\bar{g}^2(g_0, a\mu) = \bar{g}^2(\mu)$ with $a\mu \ll 1, \mu \sim T$

Finite volume scheme + step scaling $\Rightarrow$ Non-perturbative running up to EW-energies

[Lüscher et al. 1992, ALPHA 2016-2018, Bruno 2017]

$$\bar{g}^2(\mu) = \begin{cases} T \sim 3 - 165 \text{ GeV: Schrödinger functional, } \mu_{\text{SF}} = \frac{1}{L_0} = \sqrt{2}T \\ T \sim 1 - 3 \text{ GeV: Gradient flow, } \mu_{\text{GF}} = \frac{1}{2L_0} = \frac{T}{\sqrt{2}} \end{cases}$$



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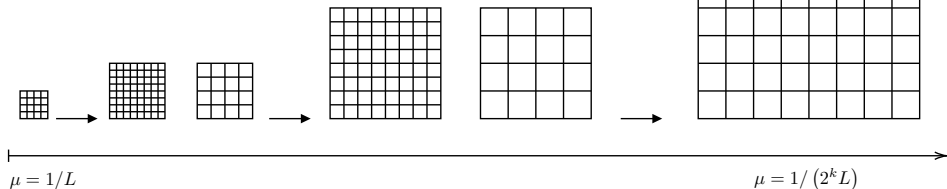
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Ensembles

T	$T[\text{GeV}]$	L_0/a	n_{mdu}
T_0	164.6(5.6)	4	300
		6	390
T_1	82.3(2.8)	4	300
		6	310
		8	500
		10	500
T_2	51.4(1.7)	4	300
		6	320
		8	490
		10	500
T_3	32.8(1.0)	4	300
		6	340
		8	490
		10	500
T_4	20.63(63)	4	440
		6	310
		8	490
		10	500
T_5	12.77(37)	4	310
		6	310
		8	500
		10	500

T	$T[\text{GeV}]$	L_0/a	n_{mdu}
T_6	8.03(22)	4	300
		6	320
		8	500
		10	500
T_7	4.91(13)	4	320
		6	310
		8	500
		10	500
T_8	3.040(78)	4	320
		6	300
		8	500
		10	500
T_9	2.833(68)	4	400
		6	390
		8	390
T_{10}	1.821(39)	4	410
		6	400
		8	400
		8	400
T_{11}	1.167(23)	4	400
		6	390
		8	400

► 12 temperatures between ~ 1 GeV and ~ 165 GeV

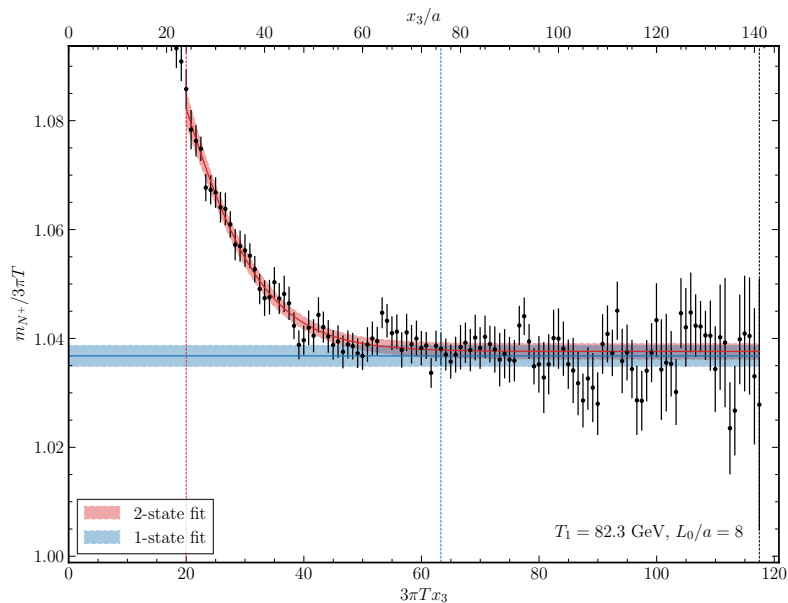
► $T_0 - T_8$: **Wilson** gauge action

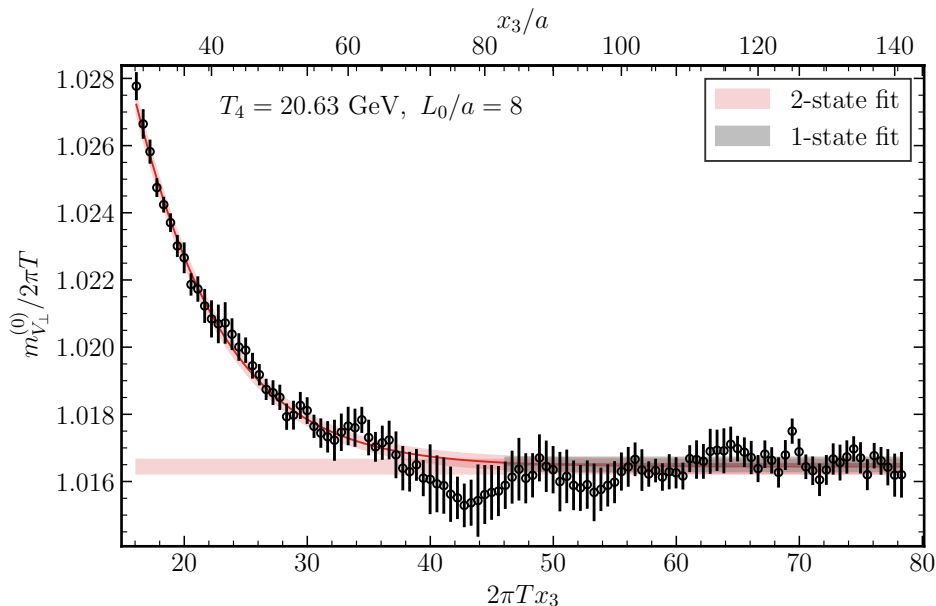
► $T_9 - T_{11}$: **Lüscher-Weisz** gauge action

► $N_f = 3$ $\mathcal{O}(a)$ -improved Wilson fermions in **chiral limit**

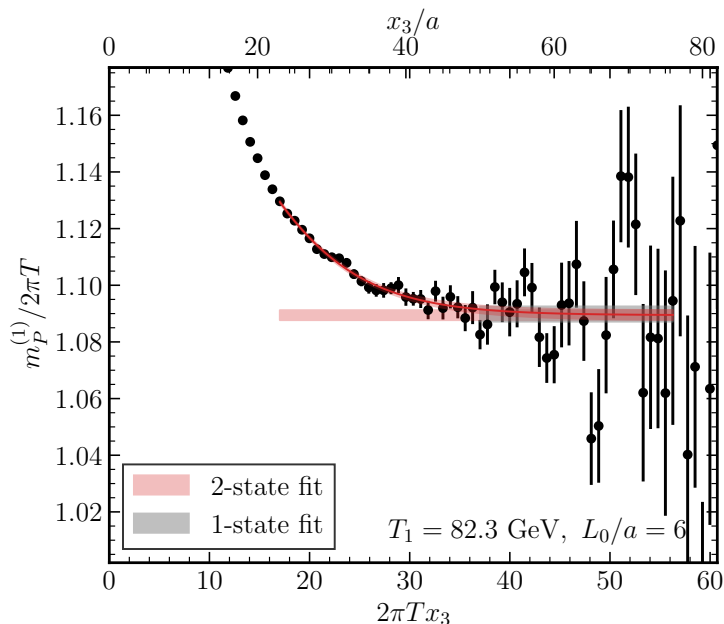
► $L_0 \times L^3$: $L/a = 288$,
 $L_0/a \in \{4, 6, 8, 10\}$ **for each**
 T : confident continuum limit

Effective masses – N^+

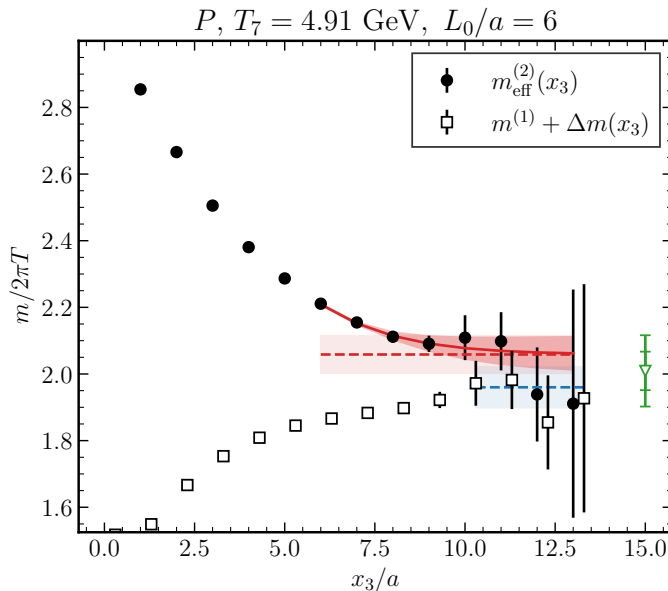


Effective masses – $V_\perp$, $n = 0$ 

Effective masses – P , $n = 1$



Effective masses – P , $n = 2$



Global continuum-temperature fit

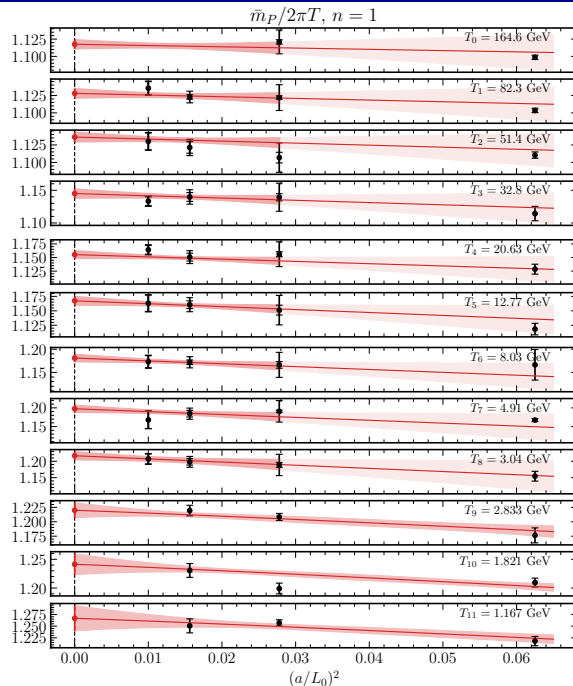
- Simultaneous **continuum extrapolation** and **interpolation of continuum values** in

$$\hat{g}(T) \equiv g_{\overline{\text{MS}}}^{(5\text{loop})}(2\pi T)$$

$$\begin{aligned} \frac{m}{2\pi T}(a/L_0, T) = & \left(1 + b_2 \hat{g}^2(T) + b_3 \hat{g}^3(T) + b_4 \hat{g}^4(T)\right) \\ & + \left(\frac{a}{L_0}\right)^2 (d_{2,2}^X \hat{g}^2(T) + d_{2,3}^X \hat{g}^3(T)) \\ & + d_{3,2}^X \left(\frac{a}{L_0}\right)^3 \hat{g}^2(T). \end{aligned}$$

- Different discretization effects $d_{i,j}^X$ for high / low temperatures

- **Final fit:** $L_0/a > 4$ for $T_0 - T_8$, no $(a/L_0)^3$, full form for $T_9 - T_{11}$

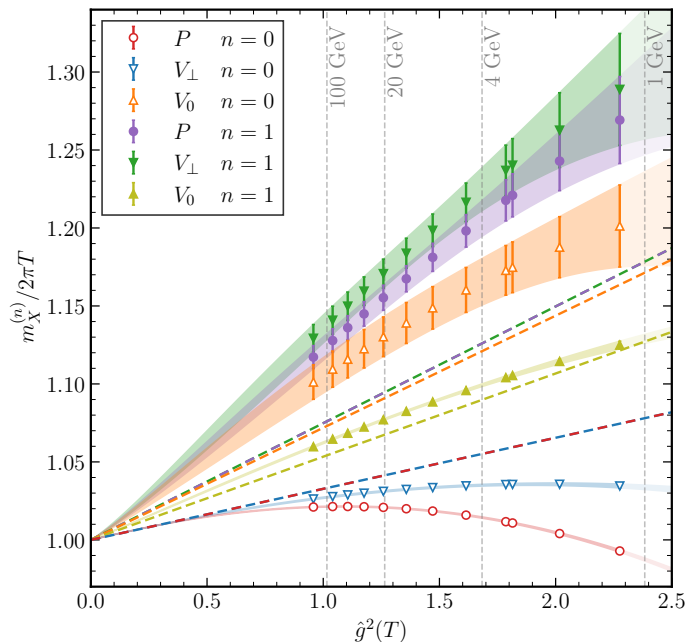


Temperature dependence

- Precise parameterization for all $T \geq 1 \text{ GeV}$

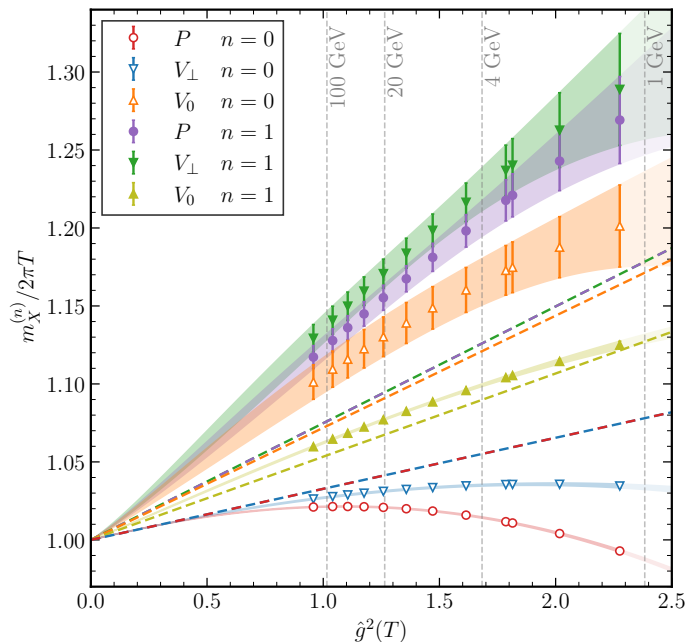
$\mathcal{O}(g^2)$ PT qualitatively reproduces ordering, but **sizeable** discrepancies persist over the whole temperature range

- Quantitative agreement only at **asymptotically high temperatures**, much larger than the Electroweak scale



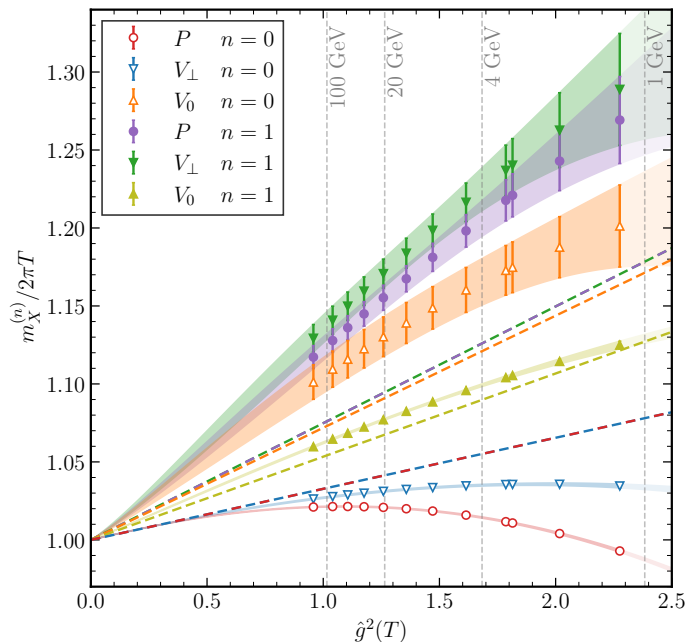
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Hyperfine splitting

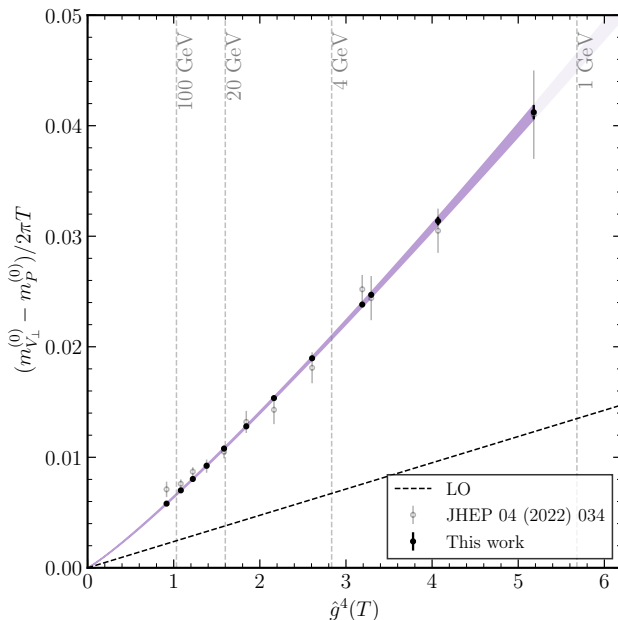
- $\mathcal{O}(g^4)$ PT compared with point-source data: effective slope is $\sim 3\times$ larger than PT

[Cè, Giusti, Laudicina, Pepe, **PR** 2025]

- New data, up to $10\times$ more precise, **can not be fit** with a single $\mathcal{O}(g^4)$ term: combined fit to

$$\frac{m_{V_\perp} - m_P}{2\pi T} = 0.002\,376\,\hat{g}^4 + s_5\hat{g}^5 + s_6\hat{g}^6$$

- Dominated up to $T \sim 165$ GeV by (effective) **higher-order terms** (**non-perturbative** string term in the interquark potential contributes at $\mathcal{O}(g^5)$)



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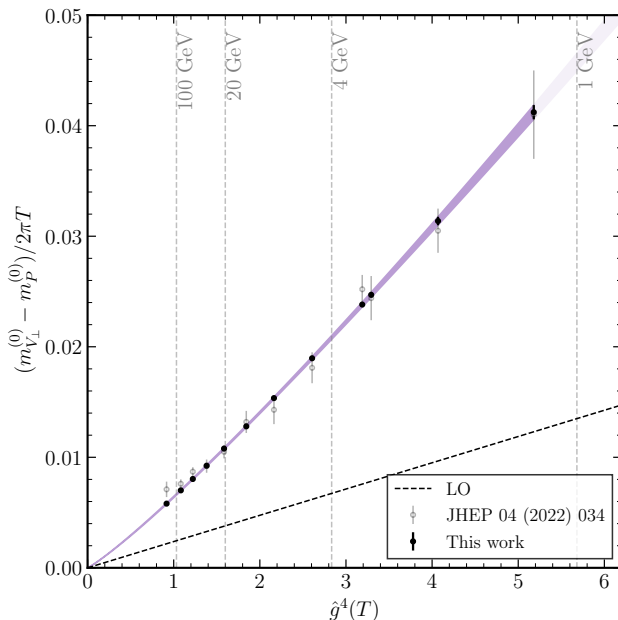
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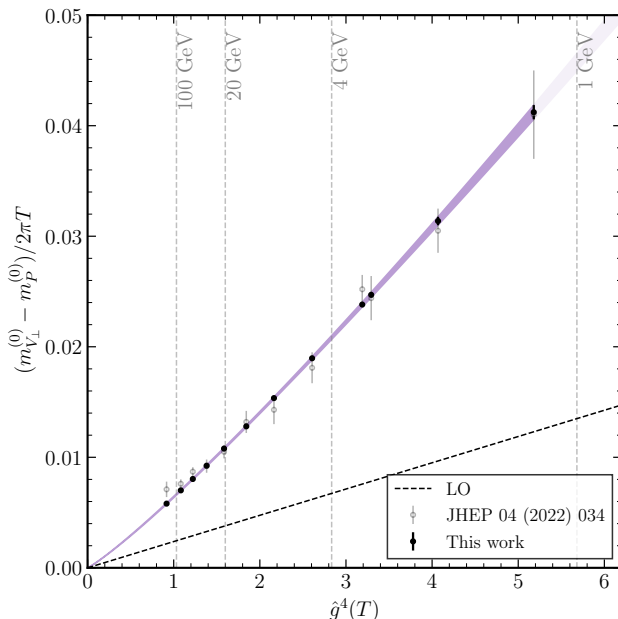
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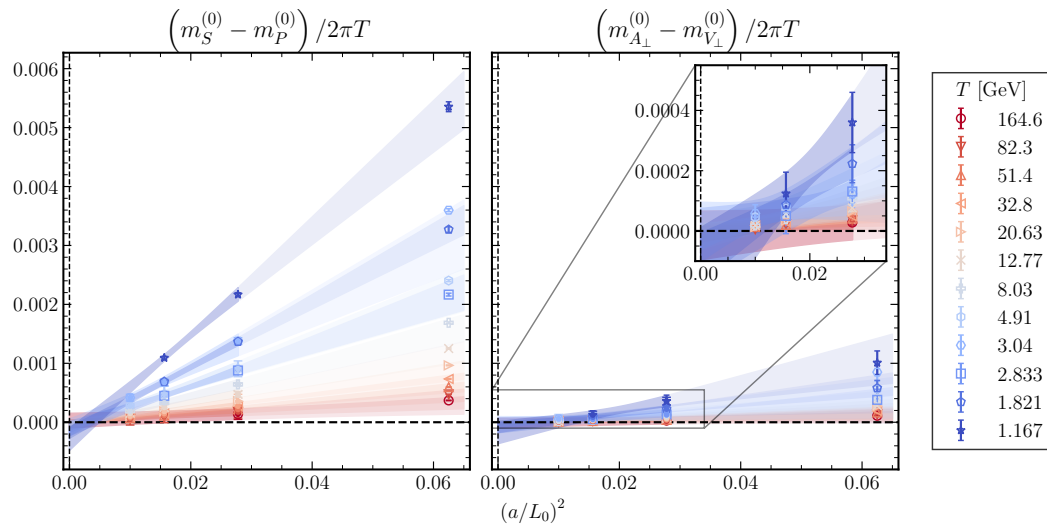
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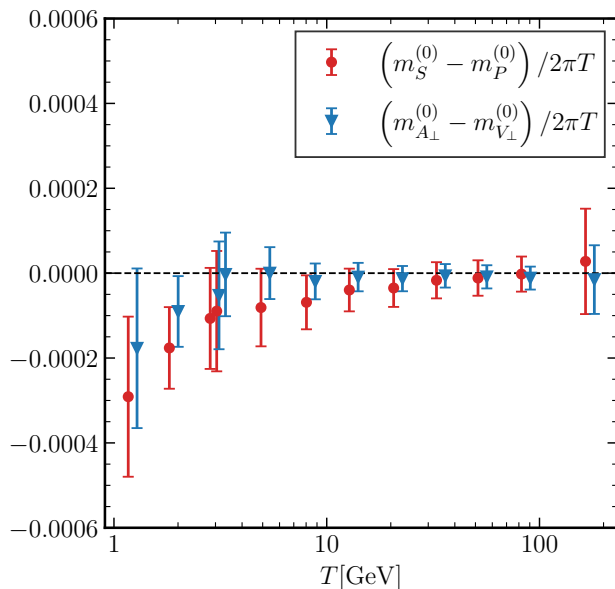
Chiral and $U(1)_A$ symmetry



$(n = 1, 2 : m_S - m_P \sim m_A - m_V \sim 0 \text{ at all } a \text{ and } T)$

Chiral and $U(1)_A$ symmetry

- **Chiral symmetry** restored over whole temperature range
- Simulations with **fixed** $Q = 0$: confirm the **expected** S/P degeneracy



Conclusions

- Carried an extensive study of the QCD **screening spectrum** at high temperatures in several **mesonic** channels and different **Matsubara sectors**
- **First results** up to the **Electroweak scale** for non-static mesonic channels
- Perturbation theory is **useful** to interpret and parameterize the temperature-dependence of **non-perturbative data**. However, it **does not offer a quantitative description** over the whole temperature range and in most channels. Similar behavior observed in the QCD Equation of State [Bresciani et al. 2025] [Talk by Pepe next week]
- **Consolidated strategy** to obtain non-perturbative thermal QCD predictions up to the Electroweak scale. Our **high-precision** results indicate:
 - ▶ **continuum limit** and **high precision** are crucial to draw conclusions about **structure** of the spectrum and **symmetries** of the QGP;
 - ▶ **perturbative expansion** of QCD_3 not satisfactory up to the Electroweak temperatures;
 - ▶ for the quark-gluon plasma, **weakly coupled** $\neq$ **perturbative**.



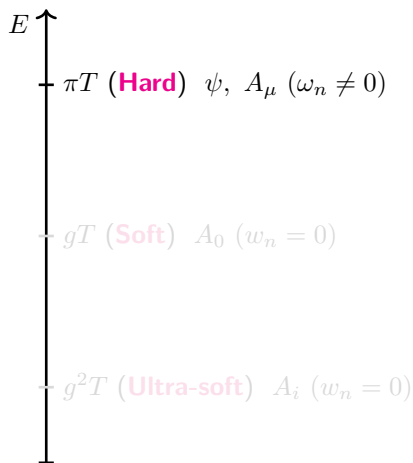
Thank you for your attention!



Backup

Relevant energy scales

Three relevant scales are developed at high T



Ultrasoft gluons cause **breakdown** of PT:

Observables computable only up to a **finte order** after which **infinite** # of diagrams are needed

E.g. QCD Equation of state computable **only up to $\mathcal{O}(g^6)$** [Kajante et al., 2003]

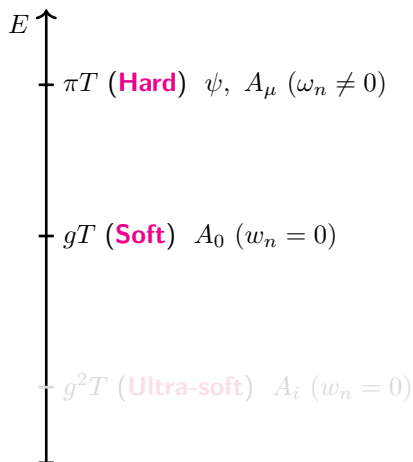
Infrared problem of non-Abelian Yang-Mills gas [Linde, 1980]



Non-perturbative treatment required also at high T

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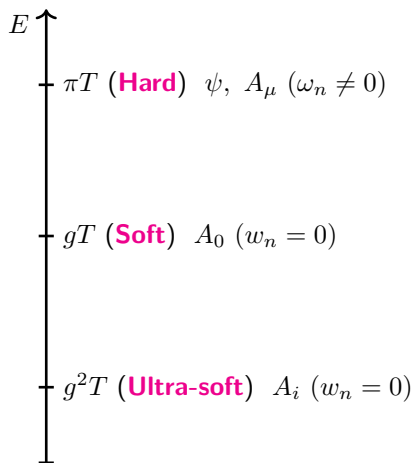
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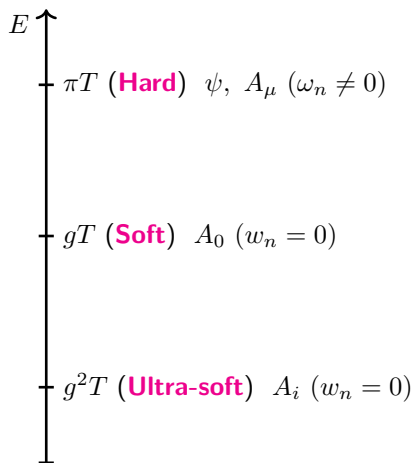
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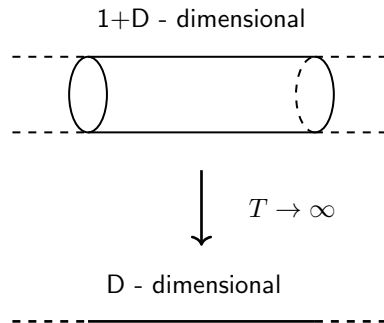
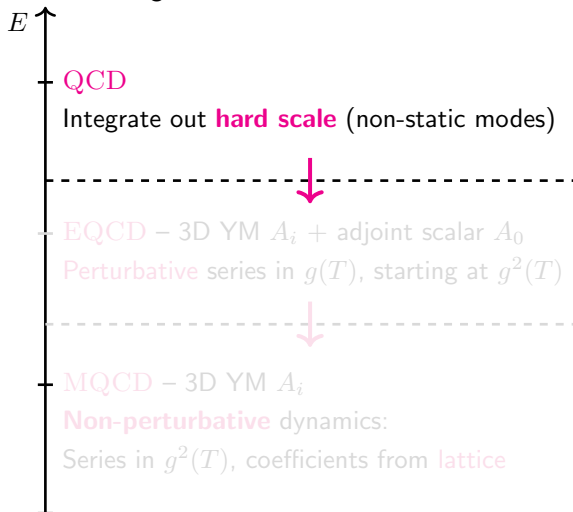


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3-Dimensional EFT [Braaten and Nieto, 1994]

At high temperature, **hierarchy** sets in: $\frac{g^2 T}{\pi} \ll gT \ll \pi T$

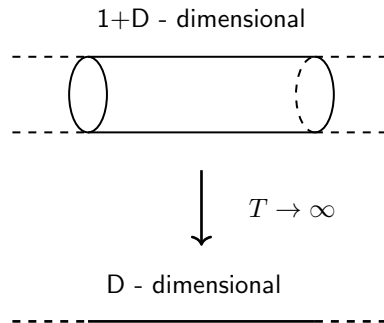
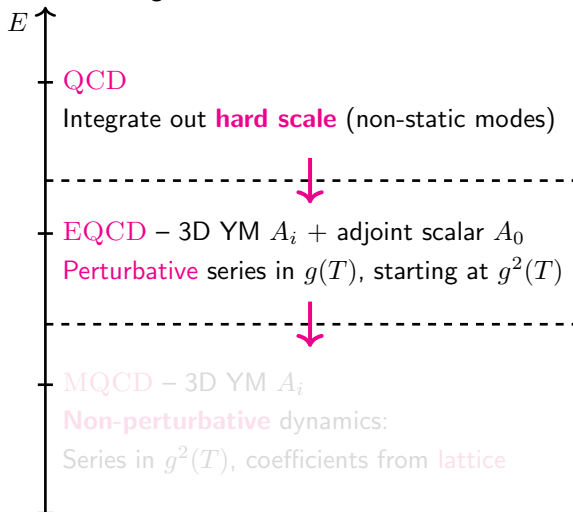
QCD undergoes **dimensional reduction** to an Effective Field Theory of the lightest modes



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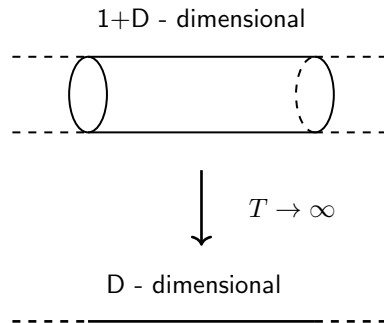
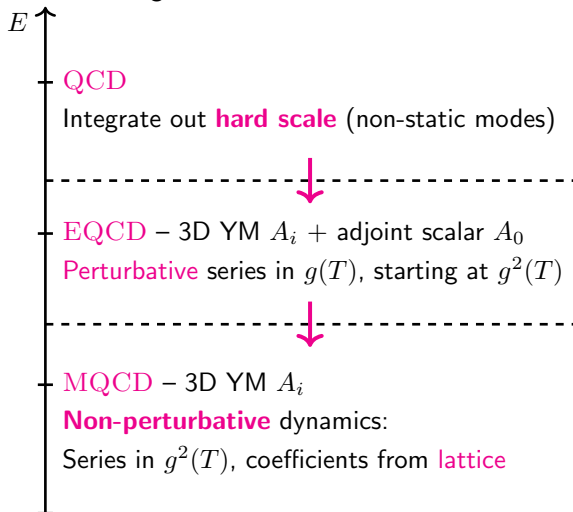
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3-Dimensional EFT [Braaten and Nieto, 1994]

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Valence heavy quarks for fermionic observables – **NRQCD**, perturbative [Brambilla et al. 2005]

- $$S_{\text{NRQCD}} = i \int d^3x \left\{ \bar{\chi} \left[M - g_E A_0 + D_3 - \frac{1}{2\pi T} \left(D_\perp^2 + \frac{g_E}{4i} [\sigma_j, \sigma_k] F_{jk} \right) \right] \chi \right. \\ \left. + \bar{\phi} \left[M - g_E A_0 - D_3 - \frac{1}{2\pi T} \left(D_\perp^2 + \frac{g_E}{4i} [\sigma_j, \sigma_k] F_{jk} \right) \right] \phi \right\} + \dots,$$

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Hyperfine splitting at LO

- **Hyperfine splitting** in static mesonic screening masses was observed to be $\mathcal{O}(g^4)$ over the whole temperature range [Giusti et al. 2022]
- **Increased statistical precision** calls for a **detailed investigation**: computed in perturbation theory to interpret the non-perturbative data [Cè, Giusti, Laudicina, Pepe, PR - JHEP 05 (2025) 044]

At $\mathcal{O}(g^2)$, $m_O = 2\pi T (1 + 0.032\,740 \dots g^2)$, $O = \{P, V_\perp\}$ [Laine and Vepsalainen, 2004]

First non-zero term: $\frac{\Delta m_{\text{VP}}^{\text{LO}}}{2\pi T} = 0.002\,376 \cdot g^4 + \mathcal{O}(g^5)$

► **Non-perturbative** string term in the interquark potential contributes at $\mathcal{O}(g^5)$! [Bala et al. 2025]

Perturbation theory *de facto* **limited at LO**

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Hyperfine splitting: computation

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Comes from **Schrödinger equation** with **spin-independent** potential

$$\left[-\frac{\nabla_{\mathbf{r}}^2}{\pi T} + 2M + U_0(\mathbf{r}) - E_0 \right] \psi_0(\mathbf{r}) = 0$$

$$U_0(\mathbf{r}) = g_{\text{E}}^2 \frac{C_F}{2\pi} \left[\ln\left(\frac{m_{\text{E}} r}{2}\right) + \gamma_{\text{E}} - K_0(m_{\text{E}} r) \right]$$

Leading Spin-dependent potential

The first **spin-dependent** potential has the form [Cè, Giusti, Laudicina, Pepe, PR - JHEP 05 (2025) 044]
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$$U_{\mp}(\mathbf{r}) = \mp g_E^2 \frac{C_F}{(2\pi T)^2} \delta(\mathbf{r}), \quad \mp \text{ for } P \text{ and } V_{\perp}$$

Temperature-suppressed: treat it as a **perturbation** to $U_0(\mathbf{r})$

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Solved numerically in **dimensionless units**: $\hat{r} = m_E r$, $\hat{\psi}_0(\hat{r}) = \psi_0(r) \sqrt{2\pi}/m_E$

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Wavefunction analysis

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- This induces a $\mathcal{O}(g)$ correction in $|\psi_0(\mathbf{0})|^2$ and thus an $\mathcal{O}(g^5)$ term in the hyperfine splitting

- **Non-perturbative effects** are needed to go beyond LO

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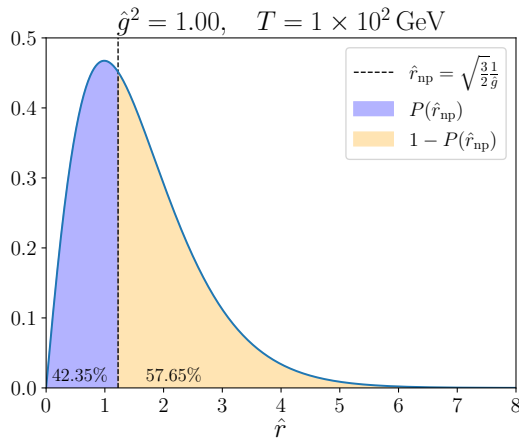
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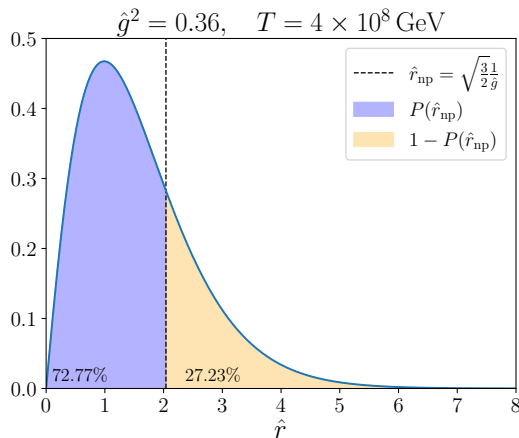
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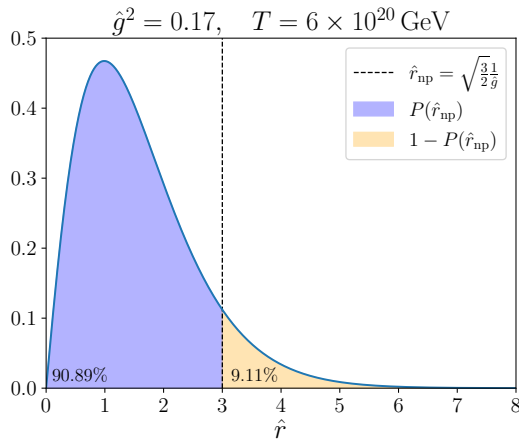
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Interpolating operator with **nucleon** quantum numbers N , opposite parity correlators

$$C_{N\pm}(x_3) = \sum_{x_0, x_1, x_2} e^{i\pi T x_0} \langle \text{Tr}[N(x)\bar{N}(0)P_{\pm}] \rangle$$

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Degenerate screening masses if chiral symmetry is **restored**

- Correlator Signal-to-Noise ratio decays **exponentially** with distance **at $T = 0$**
[Parisi 1984, Lepage 1989]

- In the **infinite temperature limit** there is **no exponential problem**:

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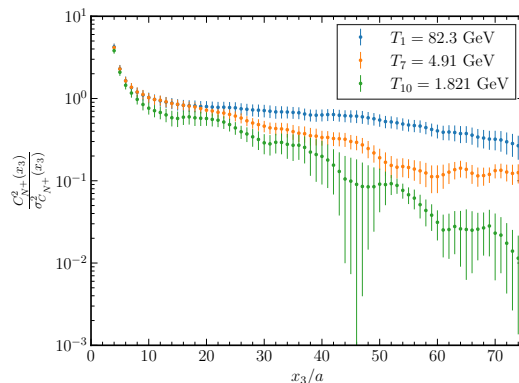
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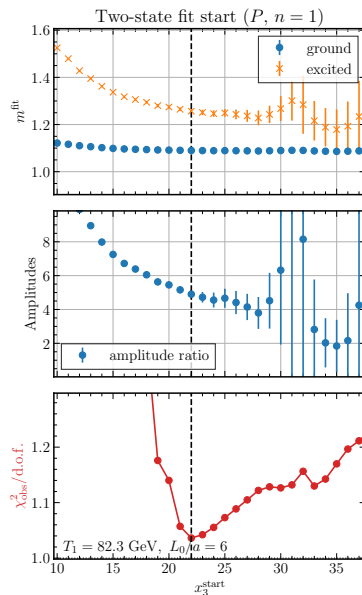
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- Scan start point, look for **stability** and good $\chi^2/\text{d.o.f.}$

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Compare excited state contribution relative to ground state (band) with relative error on correlator

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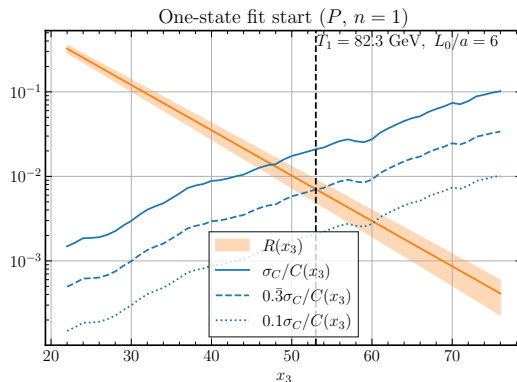
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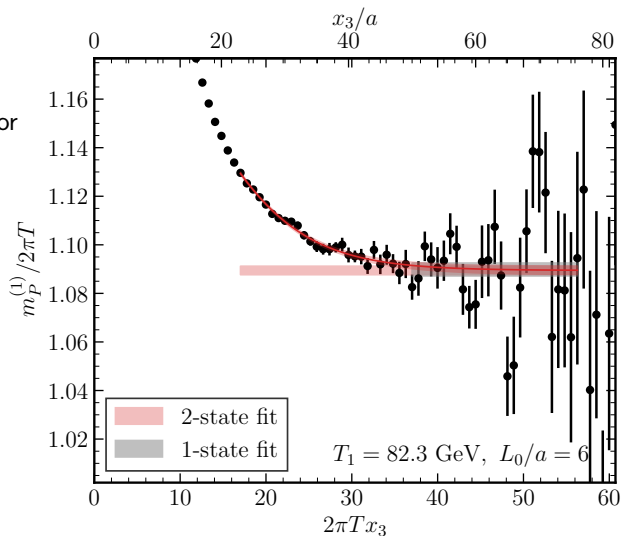
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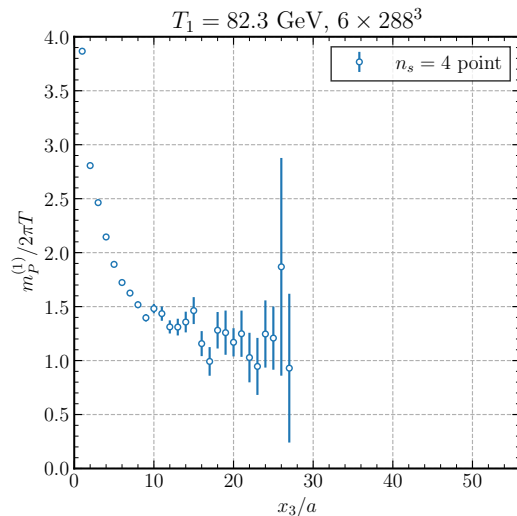
The exponential Signal-to-Noise problem

$$C_{O_\Gamma}^{(n)}(x_3) = \sum_{x_0, x_1, x_2} e^{i\omega_n x_0} \langle O_\Gamma(x) O_{O_\Gamma}^\dagger(0) \rangle$$

- $n \geq 1$ sectors suffer from **exponential** depletion of the **Signal-to-Noise** ratio

$$\frac{C_{O_\Gamma}^{(n)}}{\sigma_{C_{O_\Gamma}^{(n)}}}(x_3) \stackrel{x_3 \rightarrow \infty}{\sim} \exp\left\{-\left(m_{O_\Gamma}^{(n)} - m_P^{(0)}\right)x_3\right\}$$

- Projection of **point-source** static correlators on desired ω_n is **not a feasible way** to extract the screening masses



- Employ and **algorithmic improvement**: estimator with **same expectation value** but **reduced variance**

Random sources

Exploiting **translational invariance** and our large lattices $V = L_0 L^2/a^3 \sim 3 - 8 \times 10^5$:

$$\overline{C}(x_3 - y_3) \equiv \frac{a^3}{V} \sum_y C(x_3, y) = C(x_3, y) \Rightarrow \sigma_{\overline{C}} \propto \frac{1}{\sqrt{V}} \sigma_C$$

With **local point sources** requires V inversions, each inversion costing $\propto V$
 $\mathcal{O}(V^2)$ for $\mathcal{O}(V)$ gain in variance!

Random (wall) sources: $\eta_i^a(y) = \delta_{y_3 \bar{y}_3} \eta_i(\mathbf{y})$, $i = 1 \dots n_{\text{src}}$, $a = 0 \dots 11$

$$\langle \eta_i(\mathbf{x}) \eta_j^\dagger(\mathbf{y}) \rangle = \delta_{ij} \delta_{\mathbf{x}\mathbf{y}}$$

$$C(x_3, \bar{y}_3) \equiv \frac{a^3}{V} \frac{1}{n_{\text{src}}} \sum_{i=1}^{n_{\text{src}}} \sum_{\mathbf{x}, \mathbf{y}, z} e^{i\omega_n(x-y)} \eta_i^b(\mathbf{y})^* \Gamma^{ba} S^{ac}(x, y) \Gamma'^{cd} S^{db}(z, x) \eta_i^b(z) \Big|_{z_3=y_3=\bar{y}_3} = \overline{C}(x_3 - \bar{y}_3)$$

$$\sigma_{\overline{C}}^2 \propto \frac{a^3}{V} \left(\sigma_C^2 + \frac{1}{n_{\text{src}}} \sigma_{\text{noise}}^2 \right)$$

► **Efficient way** to implement volume average **if** one can **suppress** noise variance with moderate number of sources $n_{\text{src}} \ll V$

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$$\overline{C}(x_3 - y_3) \equiv \frac{a^3}{V} \sum_y C(x_3, y) = C(x_3, y) \Rightarrow \sigma_{\overline{C}} \propto \frac{1}{\sqrt{V}} \sigma_C$$

With **local point sources** requires V inversions, each inversion costing $\propto V$
 $\mathcal{O}(V^2)$ for $\mathcal{O}(V)$ gain in variance!

Random (wall) sources: $\eta_i^a(y) = \delta_{y_3 \bar{y}_3} \eta_i(\mathbf{y})$, $i = 1 \dots n_{\text{src}}$, $a = 0 \dots 11$

$$\langle \eta_i(\mathbf{x}) \eta_j^\dagger(\mathbf{y}) \rangle = \delta_{ij} \delta_{\mathbf{x}\mathbf{y}}$$

$$C(x_3, \bar{y}_3) \equiv \frac{a^3}{V} \frac{1}{n_{\text{src}}} \sum_{i=1}^{n_{\text{src}}} \sum_{\mathbf{x}, \mathbf{y}, \mathbf{z}} e^{i\omega_n(x-y)} \eta_i^b(y)^* \Gamma^{ba} S^{ac}(x, y) \Gamma'^{cd} S^{db}(z, x) \eta_i^b(z) \Big|_{z_3=y_3=\bar{y}_3} = \overline{C}(x_3 - \bar{y}_3)$$

$$\sigma_C^2 \propto \frac{a^3}{V} \left(\sigma_C^2 + \frac{1}{n_{\text{src}}} \sigma_{\text{noise}}^2 \right)$$

► **Efficient way** to implement volume average **if** one can **suppress** noise variance with moderate number of sources $n_{\text{src}} \ll V$

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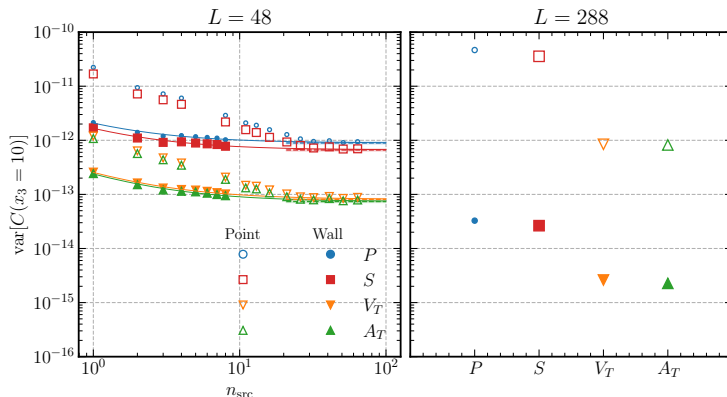
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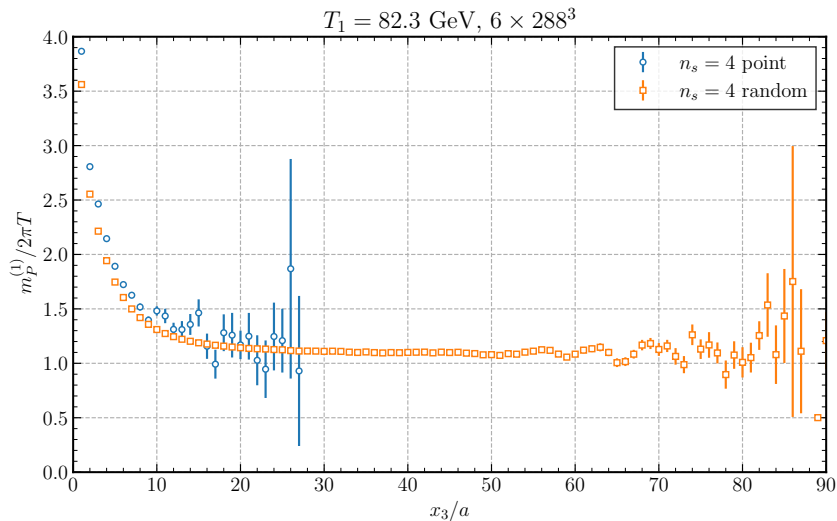
Implement $U(1)$ noise, study variance on a fixed slice for different n_{src}



► Expected behavior in n_{src} and V **correctly reproduced**, significant gain even with a single random source

► Variance reduction works also for **vector channels**, differently than what observed at $T = 0$ [Giusti et al., 2019]

Effective mass: $P, n = 1$



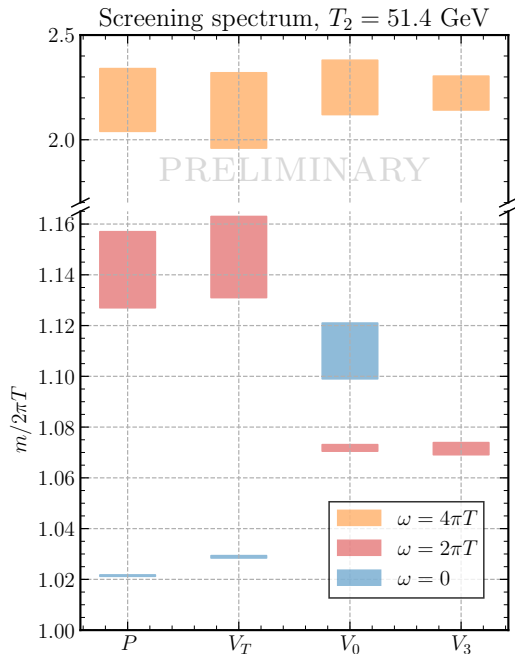
Structure of the spectrum

- Continuum precision varies significantly across channels:

- ▶ $\ll 0.1\%$ for P , $V_\perp$ at $\omega = 0$ (StN $\sim$ **constant**)
- ▶ $\lesssim 0.1\%$ for V_0 , V_3 at $\omega = 2\pi T$
- ▶ $\sim 0.5\%$ for P , $V_\perp$ at $\omega = 2\pi T$
- ▶ $\sim 1\%$ for V_0 at $\omega = 0$
- ▶ $\sim 5 - 10\%$ in the $\omega = 4\pi T$ sector

- Vector current conservation:** $m_{V_0}^{(n)} \approx m_{V_3}^{(n)}$ for $n \neq 0$. $m_{V_3}^{(0)}$ not defined

- Spin-splitting in the $\omega = 2\pi T, 4\pi T$ sectors **compatible with 0** within errors



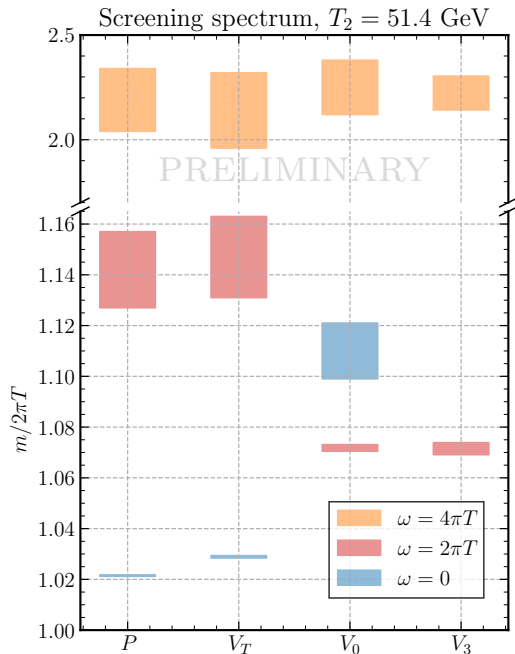
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