Neutron Star Structure & Cooling: Thoughts on some open questions

Sanjay Reddy, Institute for Nuclear Theory, University of Washington, Seattle.

- EOS and Neutron Stars:
  - Implications of a robust bound on the sound speed.
  - Uncertainties at moderate density.
- Neutron star cooling:
  - Bridging 1- and 2-nucleon neutrino processes in degenerate matter.
- Conclusions

INSTITUTE for NUCLEAR THEORY
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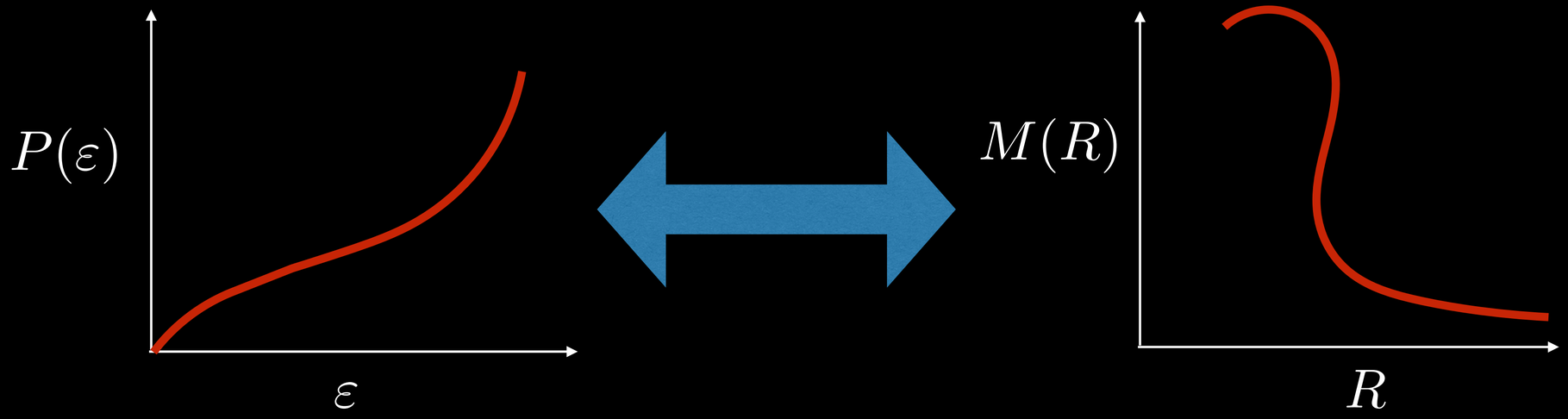
N3AS Network for Neutrinos, Nuclear Astrophysics, and Symmetries PHYSICS FRONTIER CENTER
NSF

Equation of State and Neutron Star Structure



$$P(\varepsilon) + \text{Gen.Rel.} = M(R)$$

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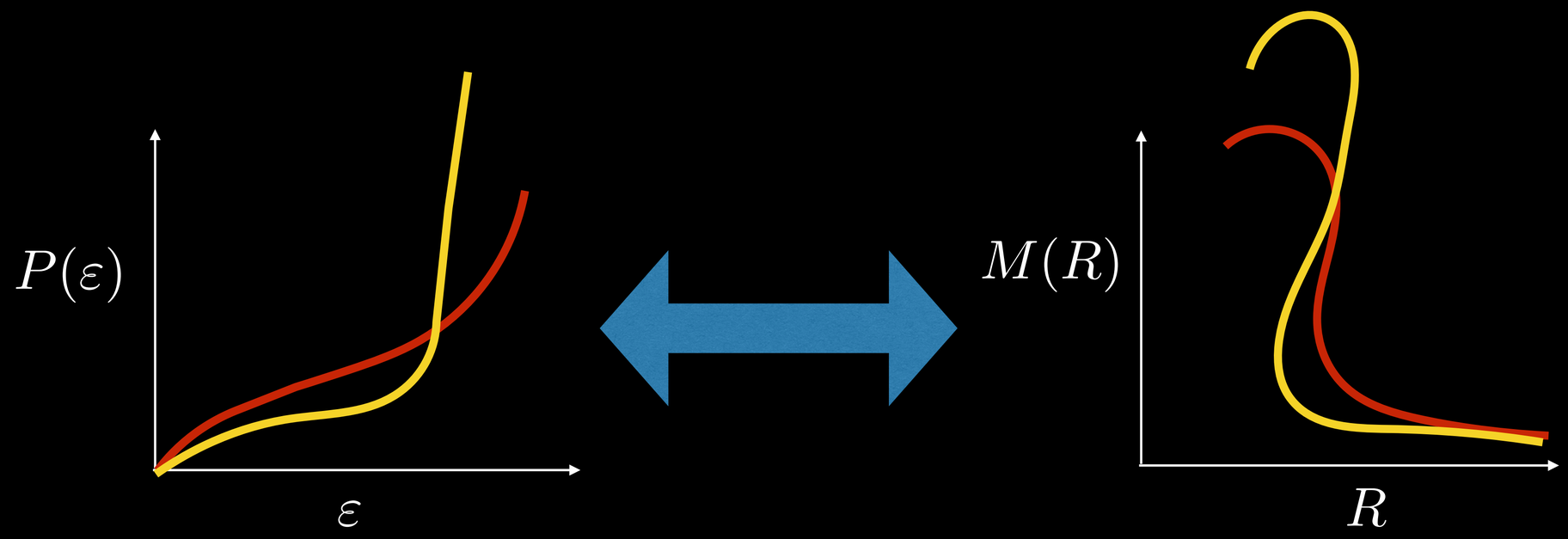

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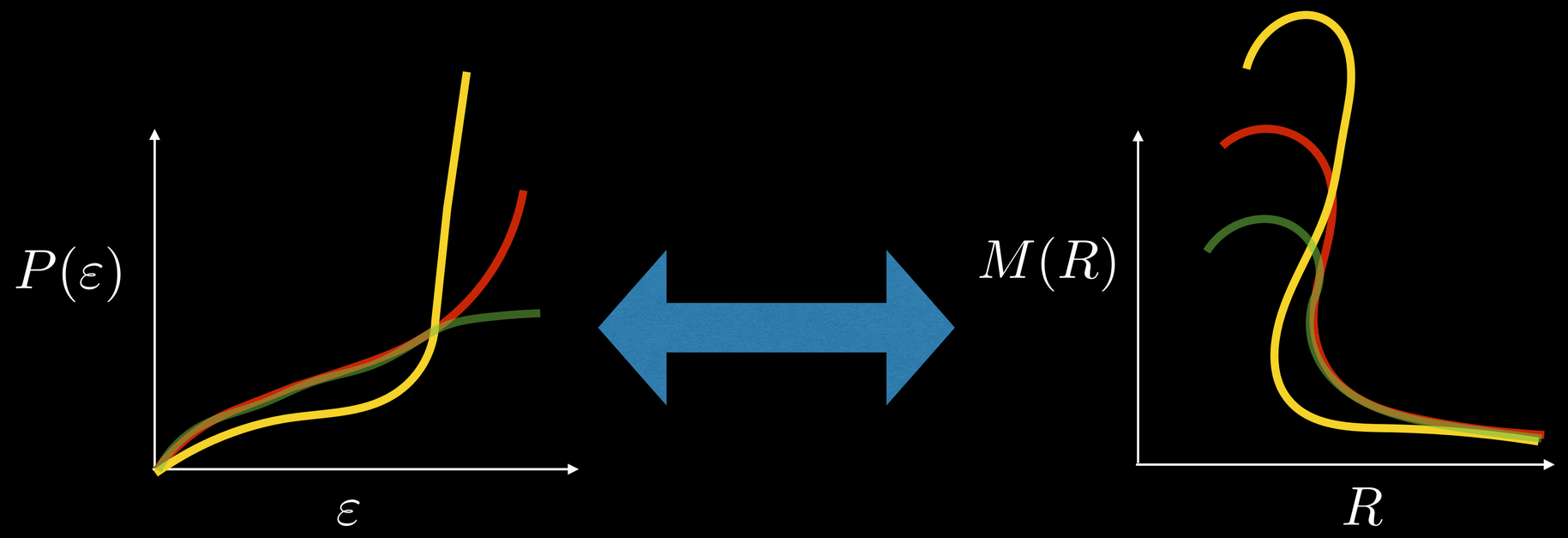

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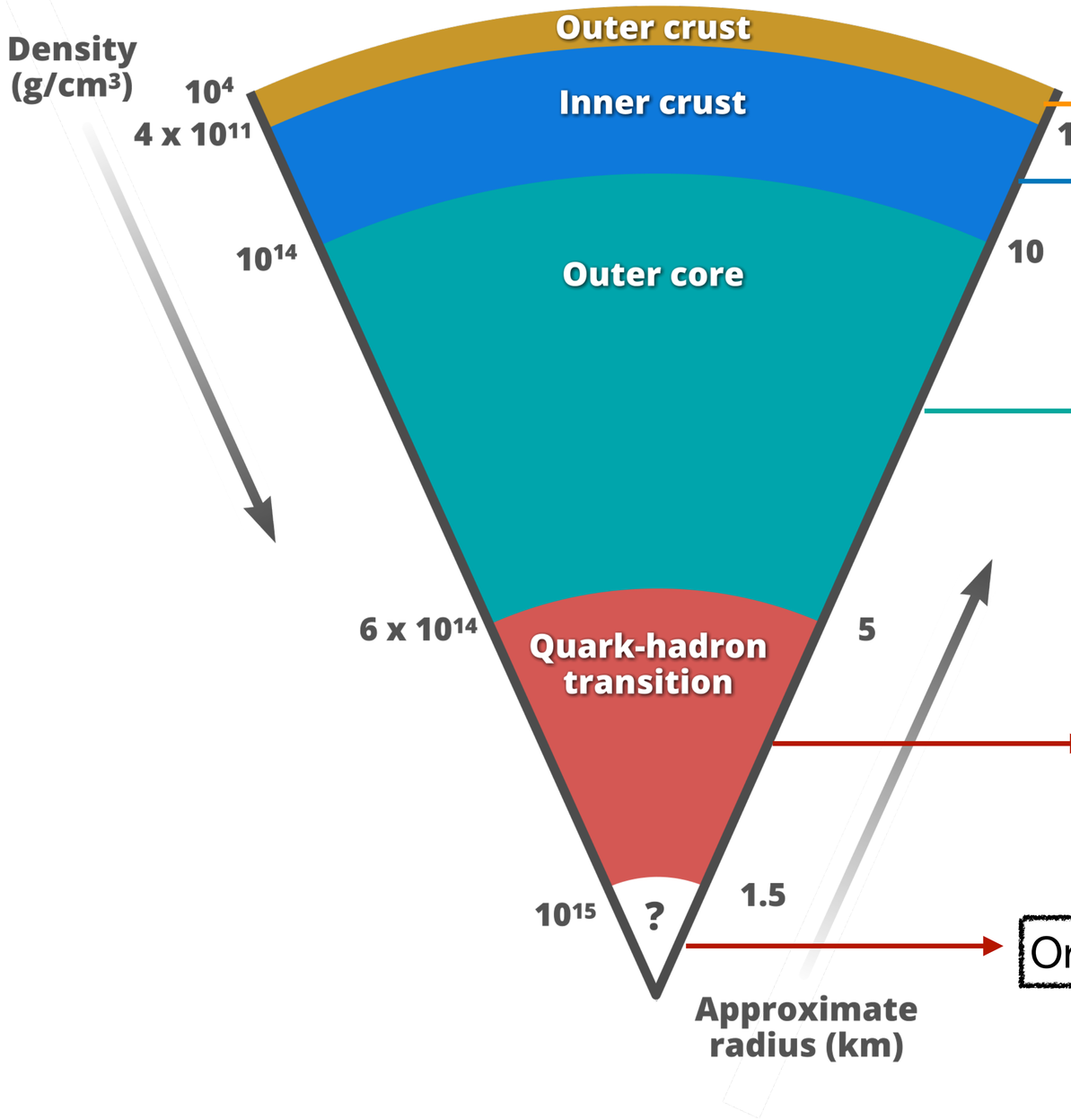
Inside Neutron Stars



Nuclei and relativistic electrons.

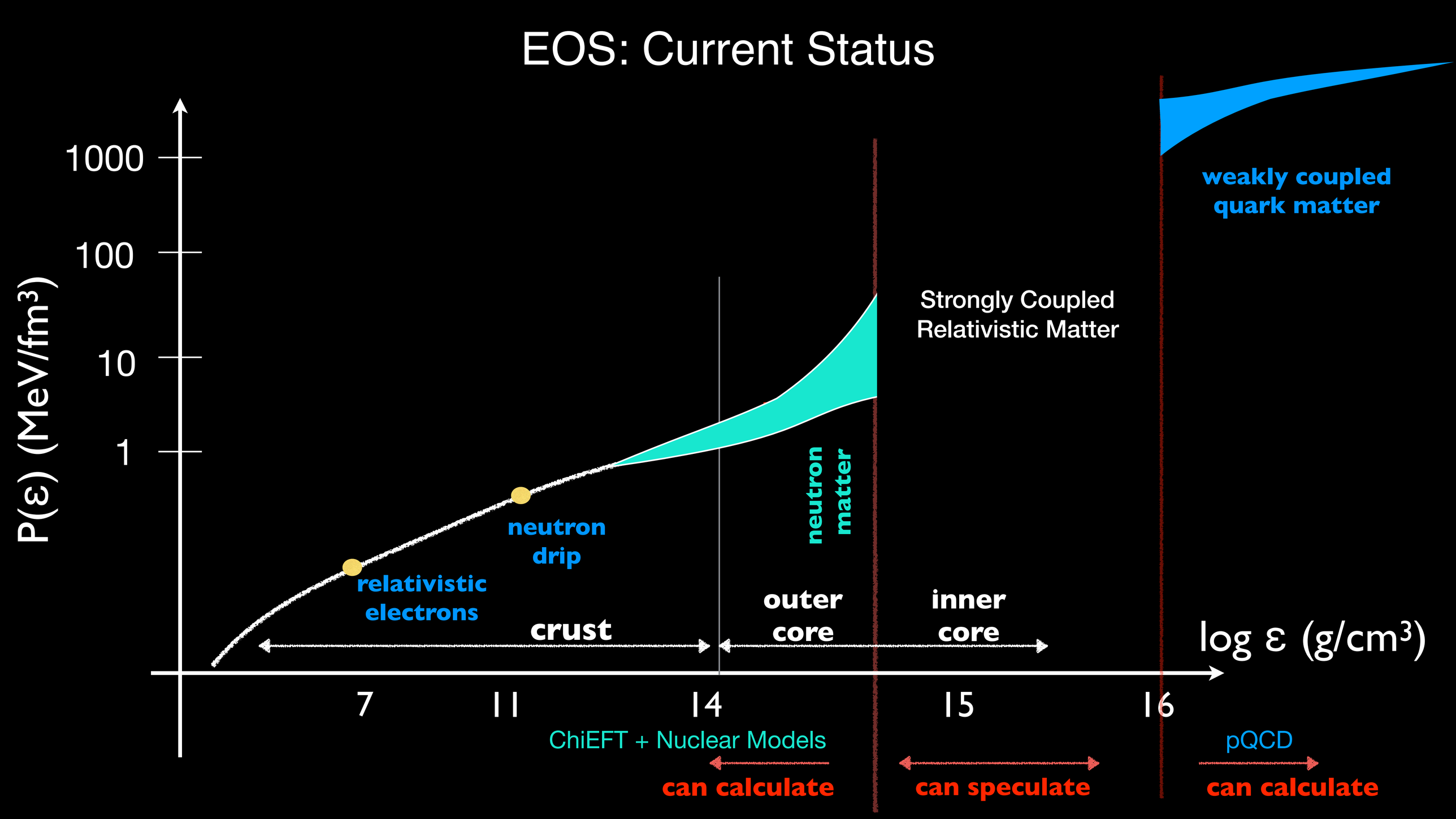
Neutron-rich nuclei, relativistic electrons, superfluid neutrons

Neutrons (~ 90%), protons, relativistic electrons, muons. Description in terms of baryons remains useful. Superfluid neutrons & superconducting protons.

Complex strongly interacting relativistic matter. Description not simple in terms of either baryons or quarks. Quarkyonic Matter?

Ordered quark matter? color superconductor?

EOS: Current Status



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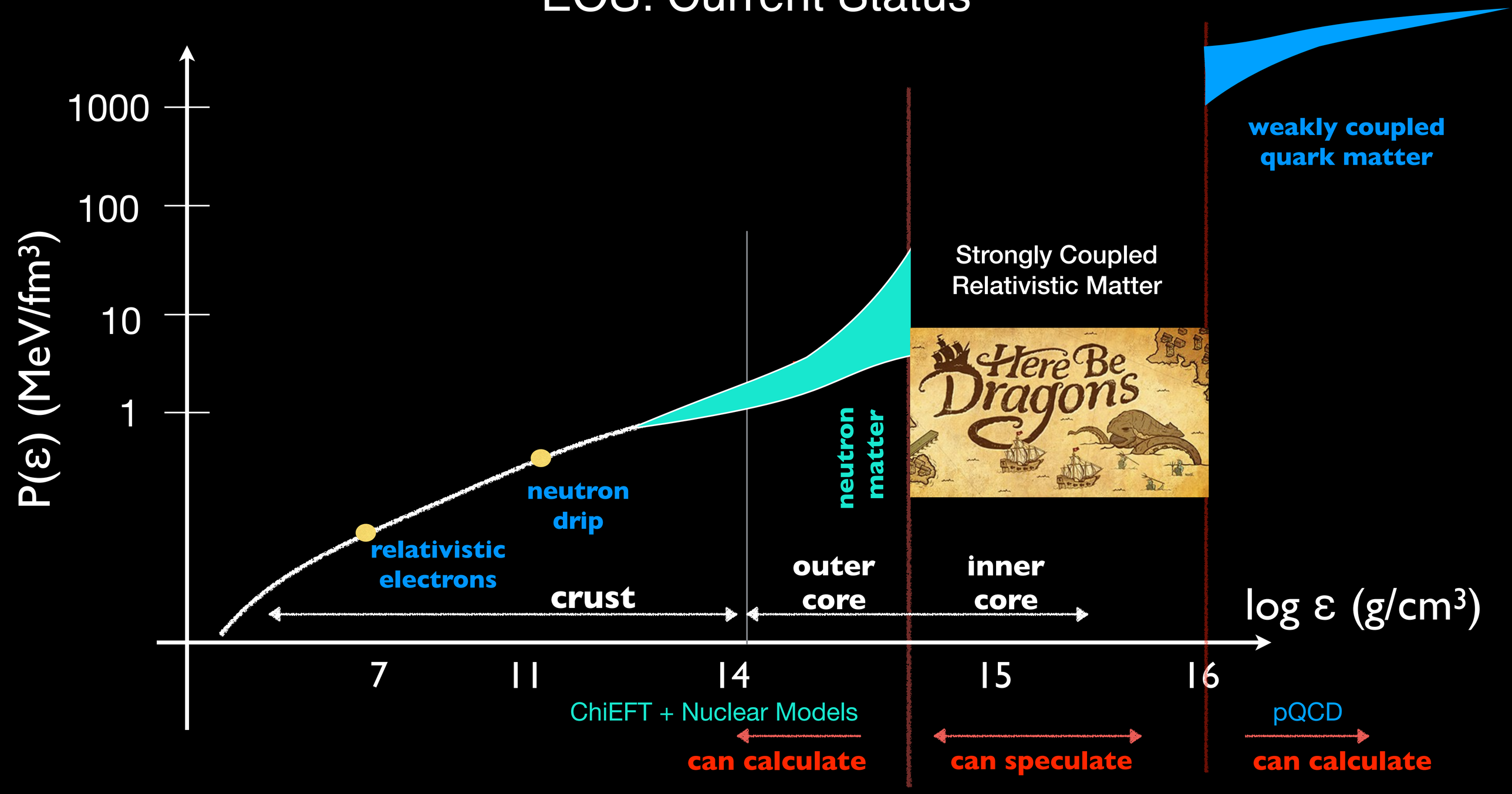

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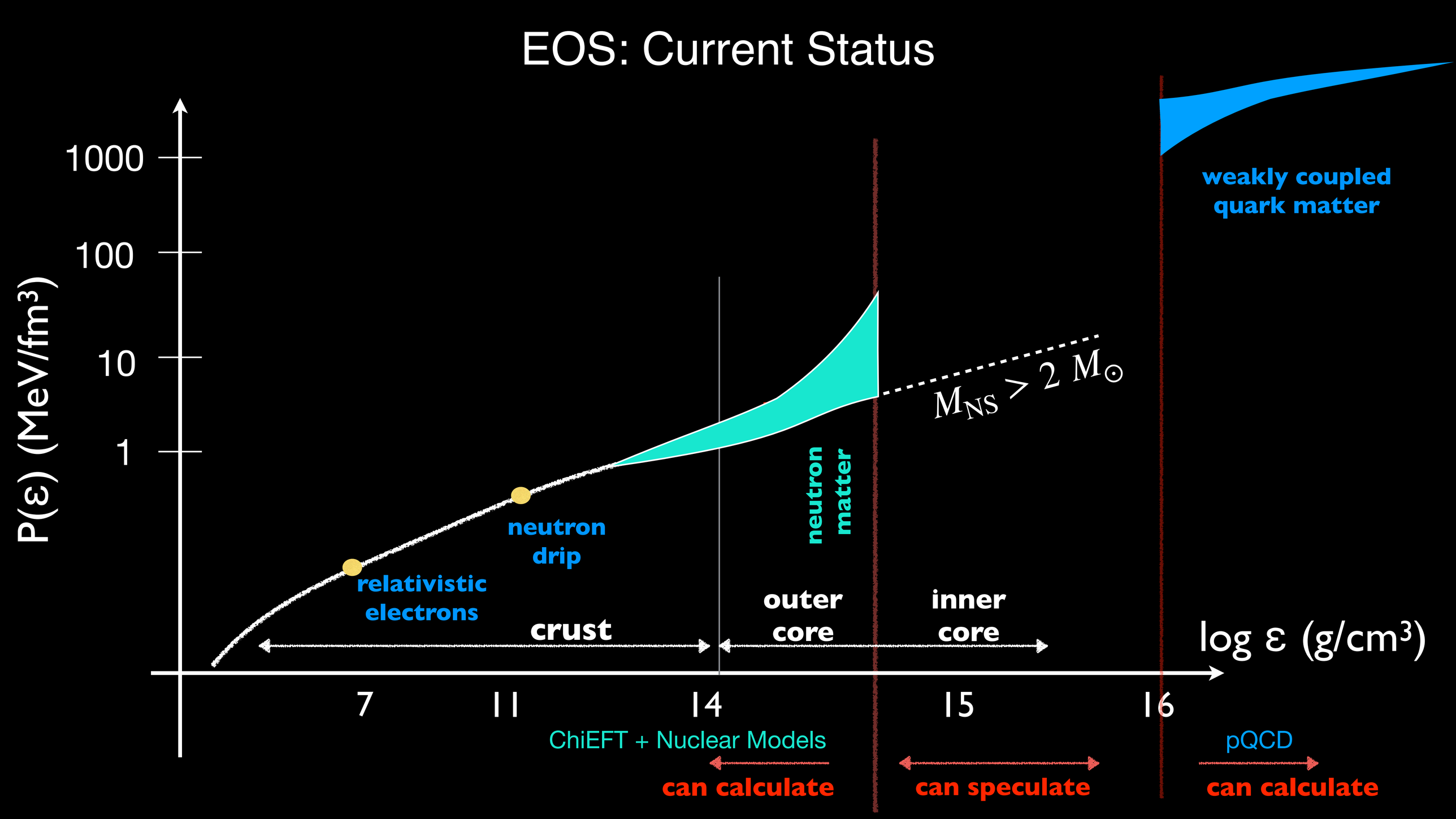

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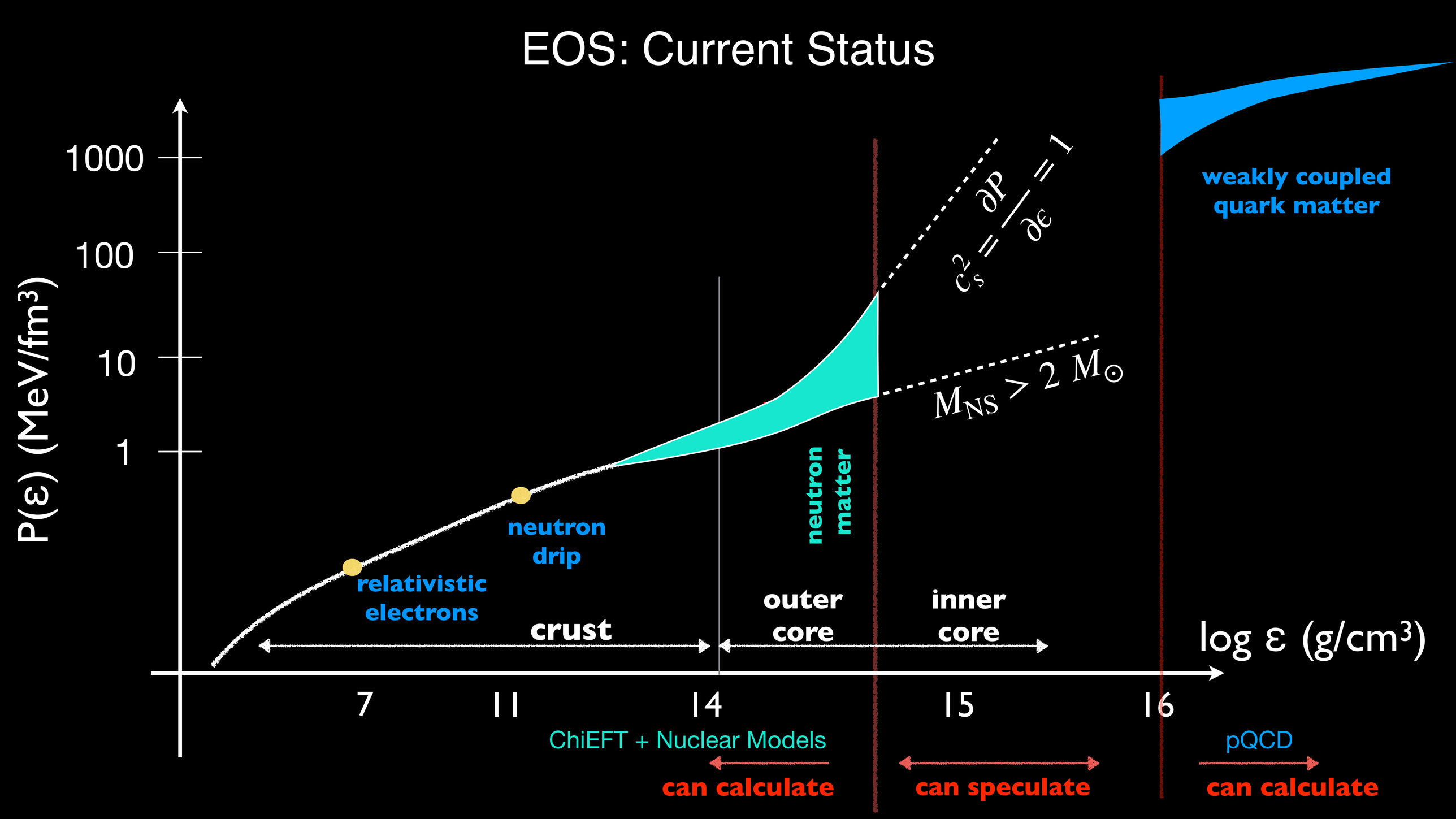
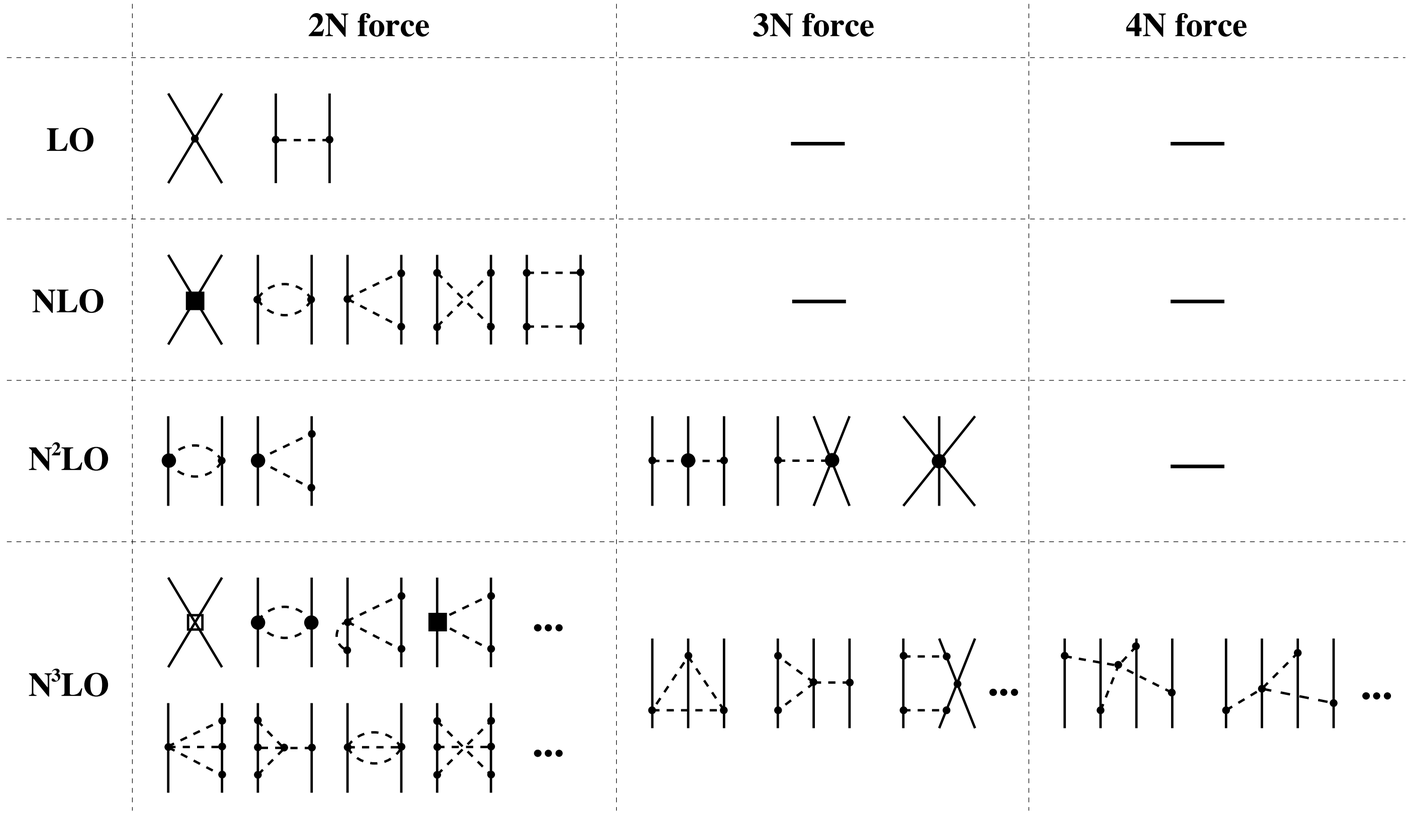
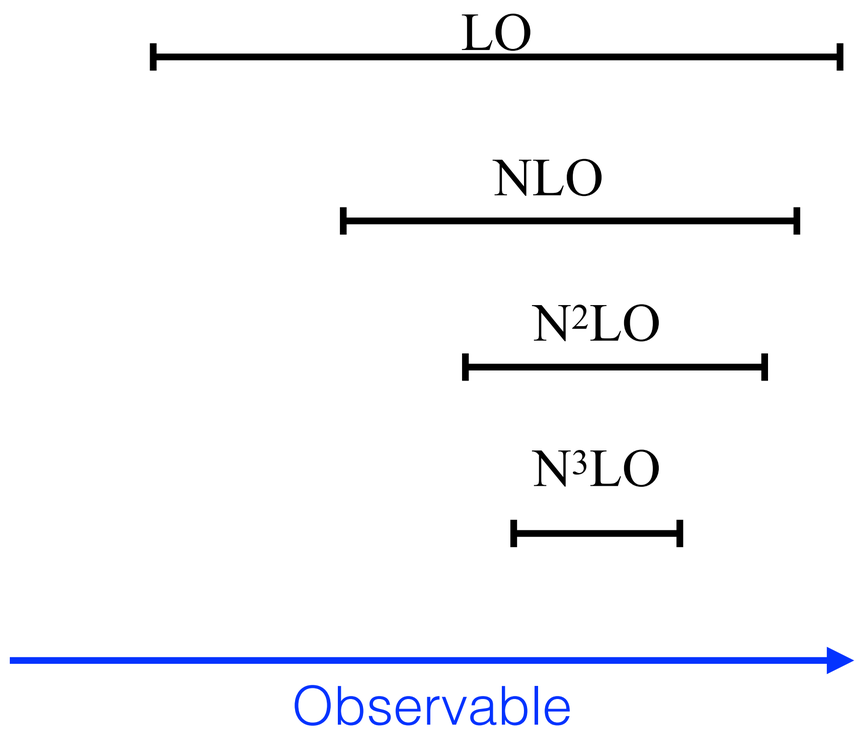


Nuclear Forces from Effective Field Theory (EFT)

EFT Hamiltonians organizes operators in powers of the momentum: $\frac{Q}{\Lambda_B}$





Allows for error estimation*. Provides guidance for the structure of three and many-body forces.

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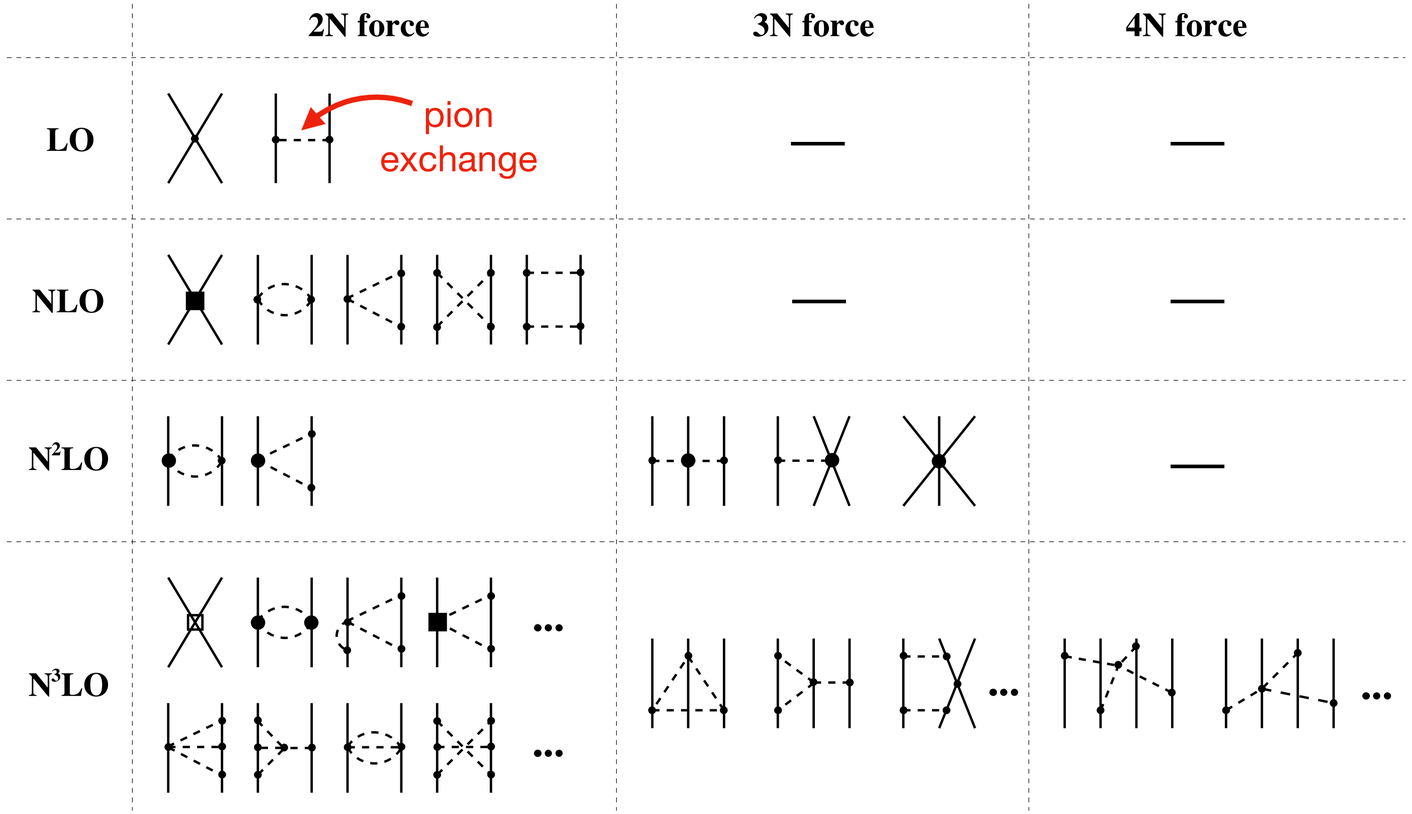

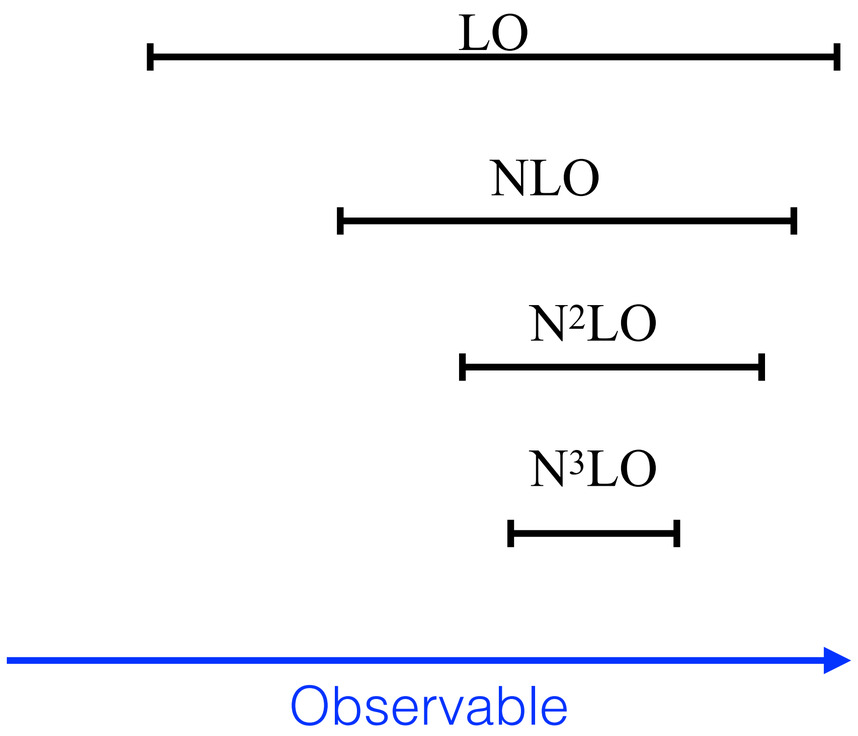

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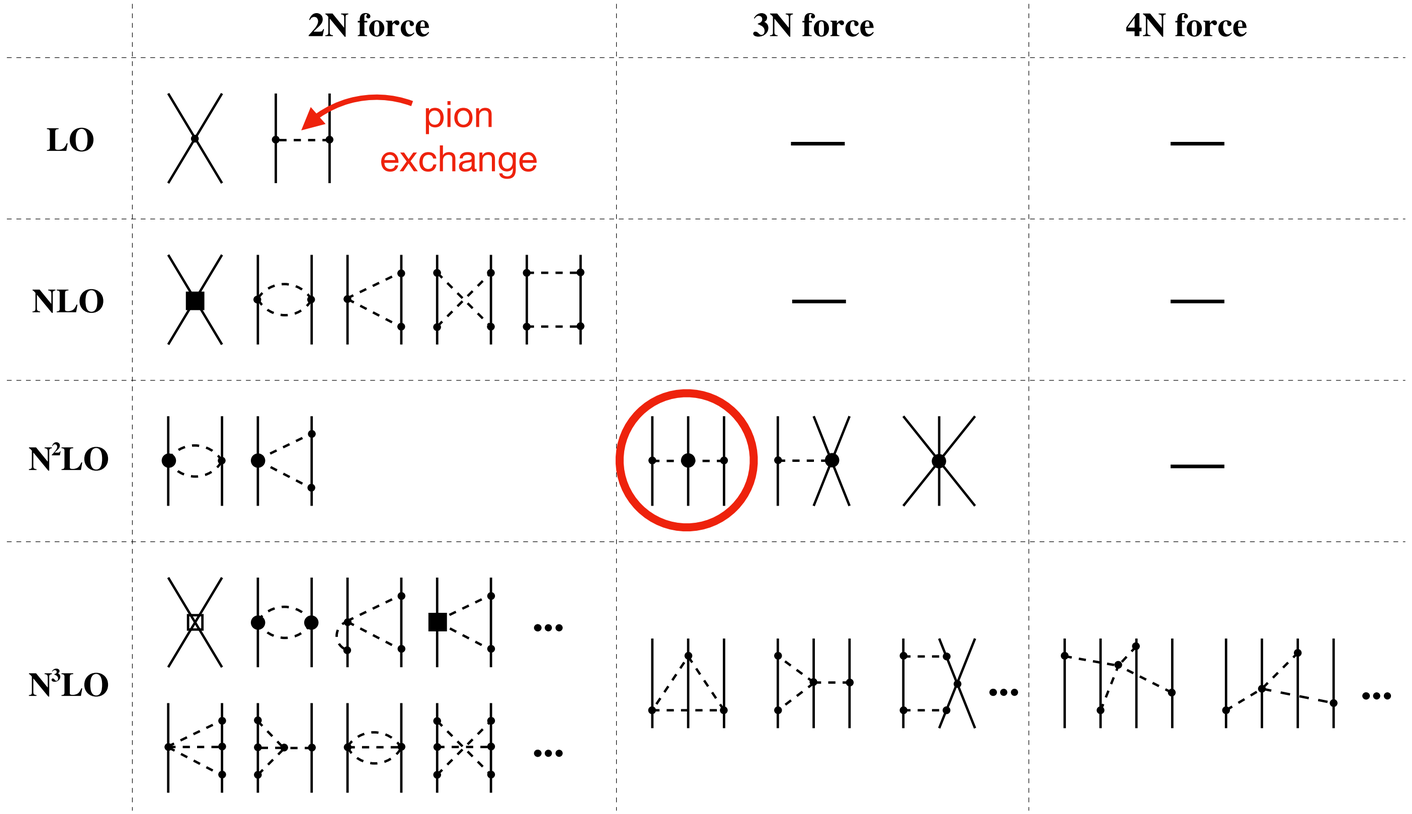

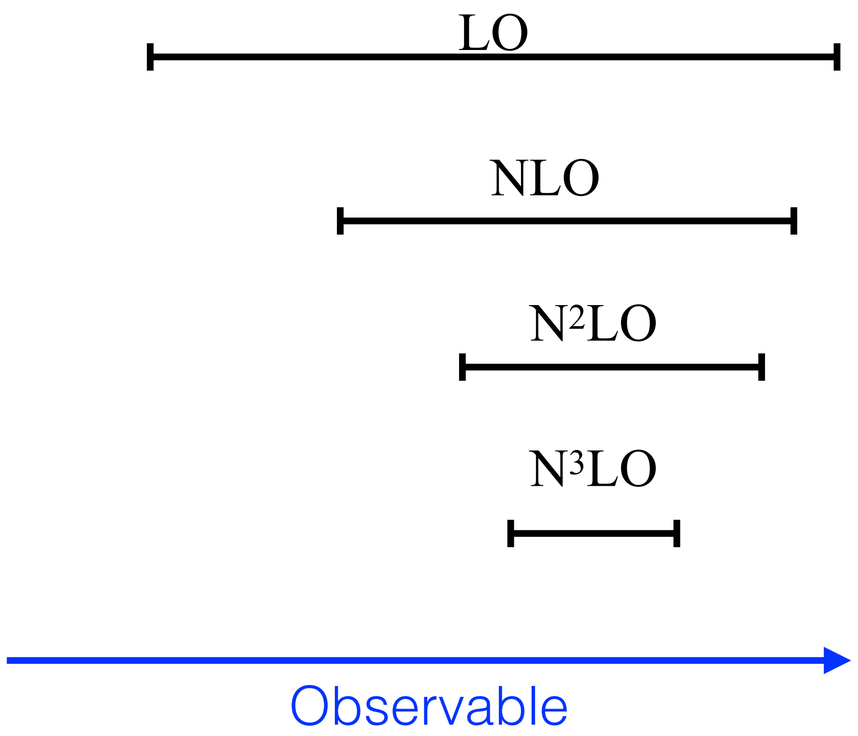

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Equation of State of Dense Nuclear Matter

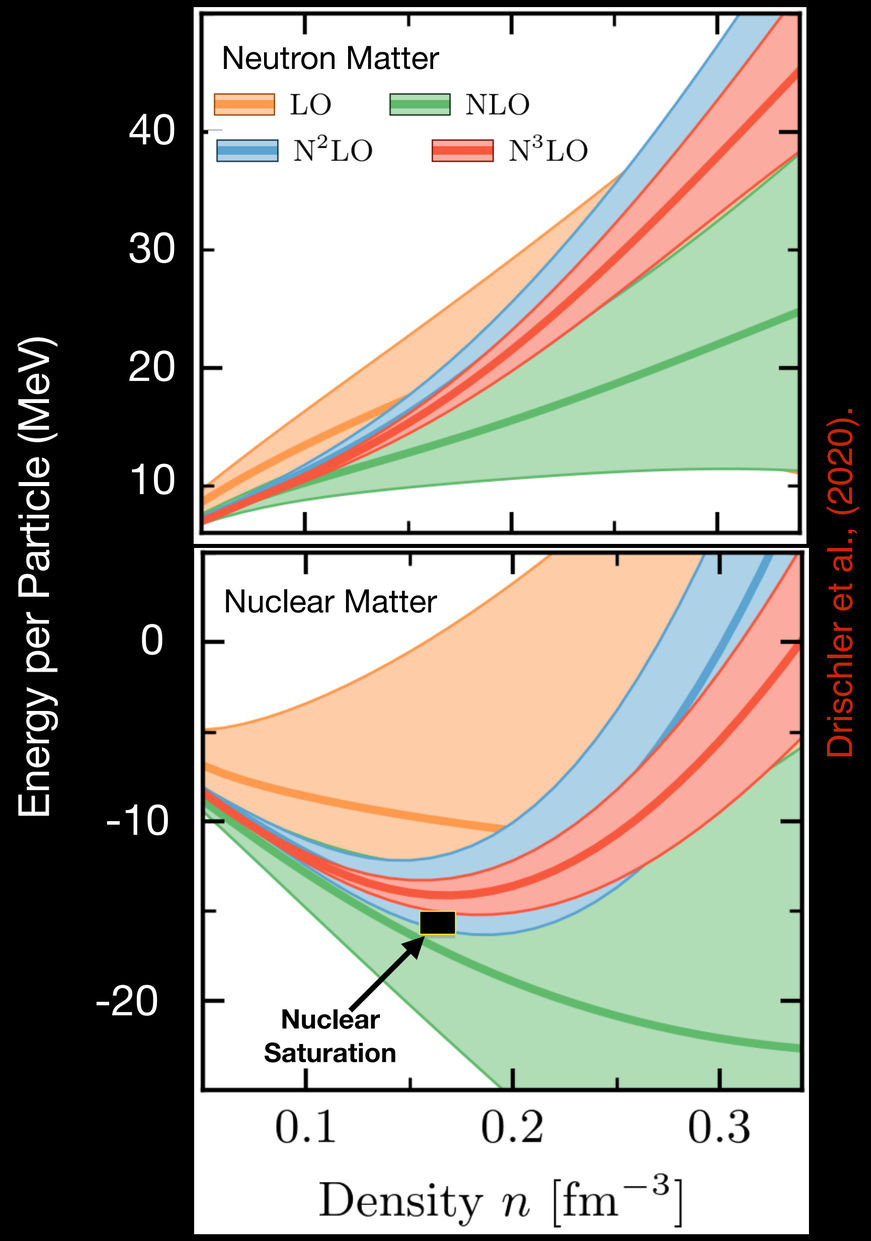
Quantum many-body calculations of neutron matter and nuclear matter using EFT potentials show convergence up to about twice nuclear saturation density.

Hebeler and Schwenk (2009), Gandolfi, Carlson, Reddy (2010), Gezerlis et al. (2013), Tews, Kruger, Hebeler, Schwenk (2013), Holt Kaiser, Weise (2013), Hagen et al. (2013), Roggero, Mukherjee, Pederiva (2014), Wlazlowski, Holt, Moroz, Bulgac, Roche (2014), Tews et al. (2018), Drischler et al., (2020).

Three-nucleon forces at N^{2}LO play a key role. They provide the repulsion needed to hold up neutron stars.

Drischler et al. used Bayesian methods to systematically estimate the EFT truncation errors in neutron and nuclear matter.

Drischler, Furnstahl, Melendez, Phillips, (2020).



Drischler et al., (2020).

Equation of State of Neutron Star Matter

In neutron stars, matter is in equilibrium with respect to weak interactions and contains a small fraction (about 5-10%) of protons, electrons and muons:

Many-body perturbation theory and Bayesian estimates of the EFT truncation errors predict:

$$P_{\rm NSM}(n_B = 0.16\ \text{fm}^{-3}) = 3.0 \pm 0.2\ \text{MeV/fm}^3$$

$$P_{\rm NSM}(n_B = 0.34\ \text{fm}^{-3}) = 20.0 \pm 5\ \text{MeV/fm}^3$$



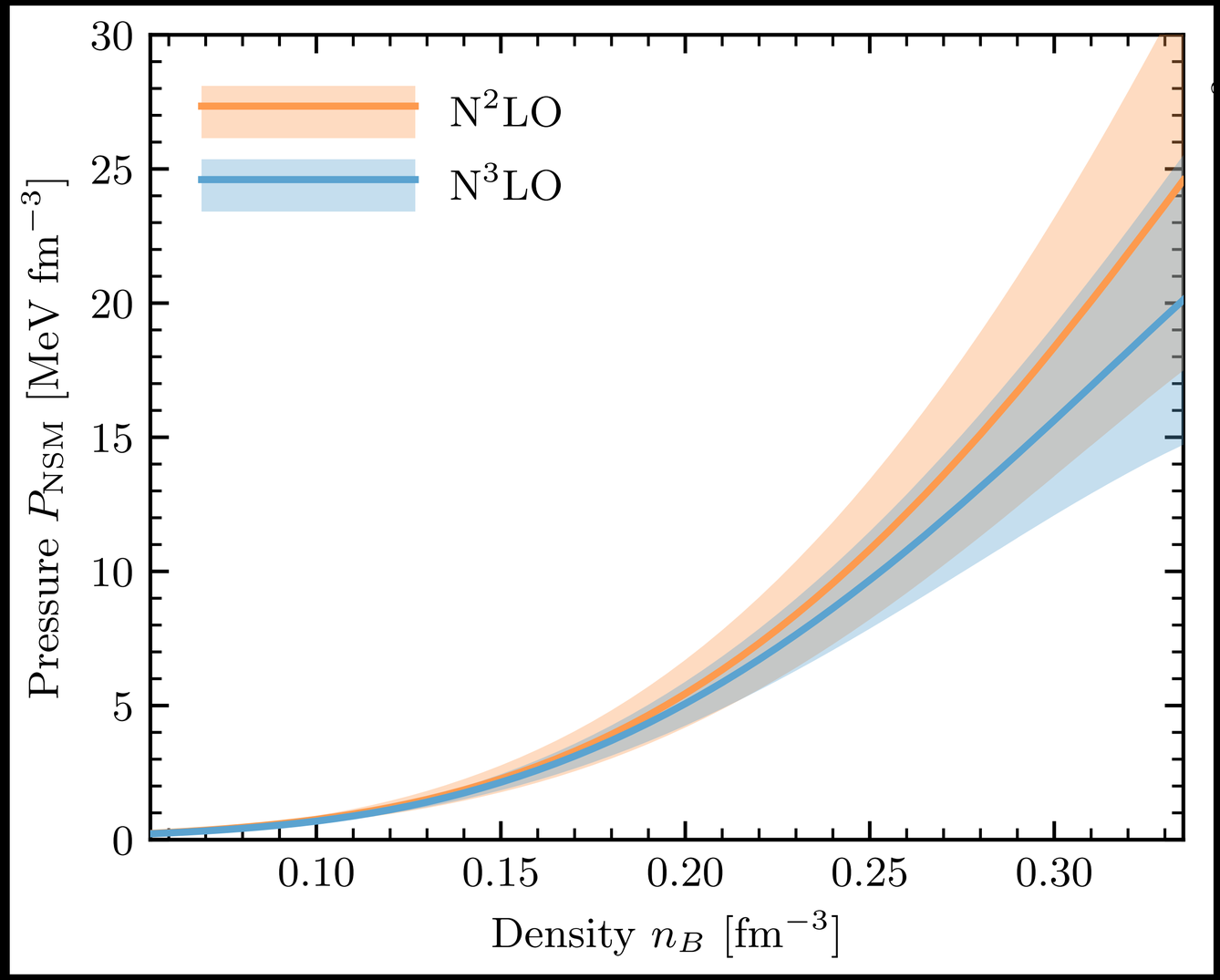
Christian Drischler



Sophia Han



Tianqi Zhao



Drischler, Han, Lattimer, Prakash, Reddy, Zhao (2020)

Bounds on Neutron Star Radii

EFT predictions for the EOS can be combined with an extreme high-density EOS (with $c_s^2 = 1$) to derive robust bounds on the radius of a NS of any mass.

The lower limit on the NS maximum mass obtained from observations strengthens these bounds:

- $M_{\rm max} > 2.0\ M_\odot$, 9.2 km < $R_{1.4}$ < 13.2 km
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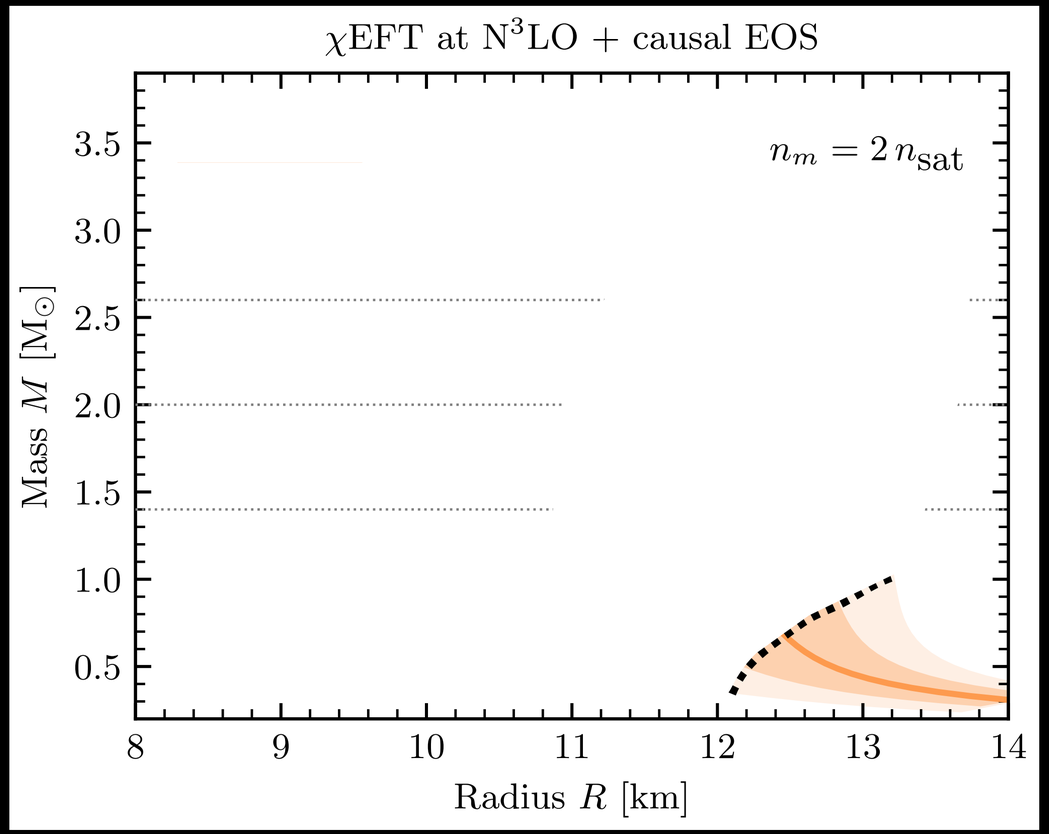

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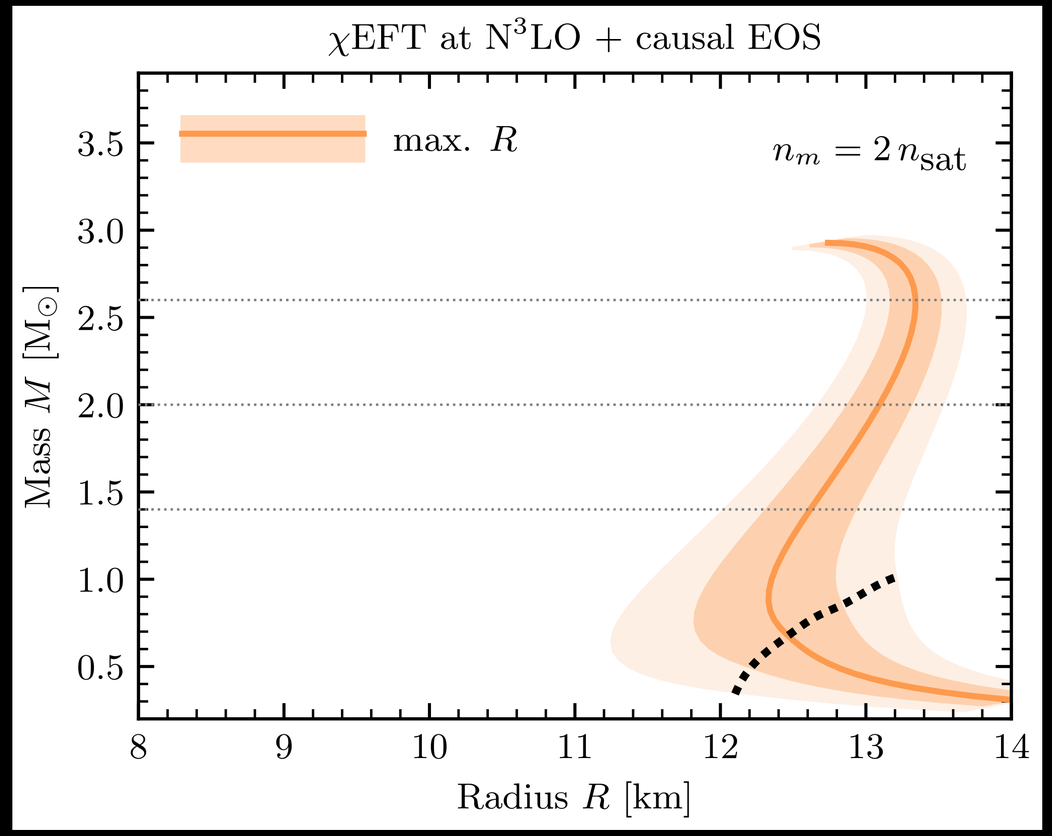

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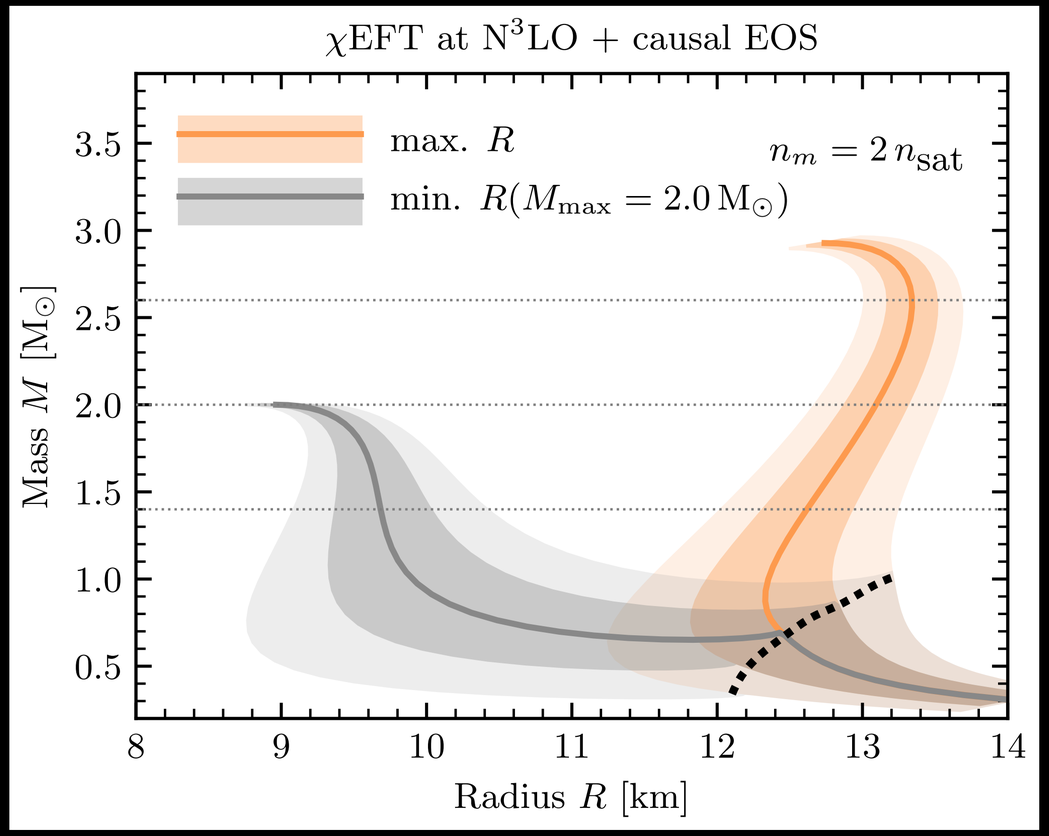

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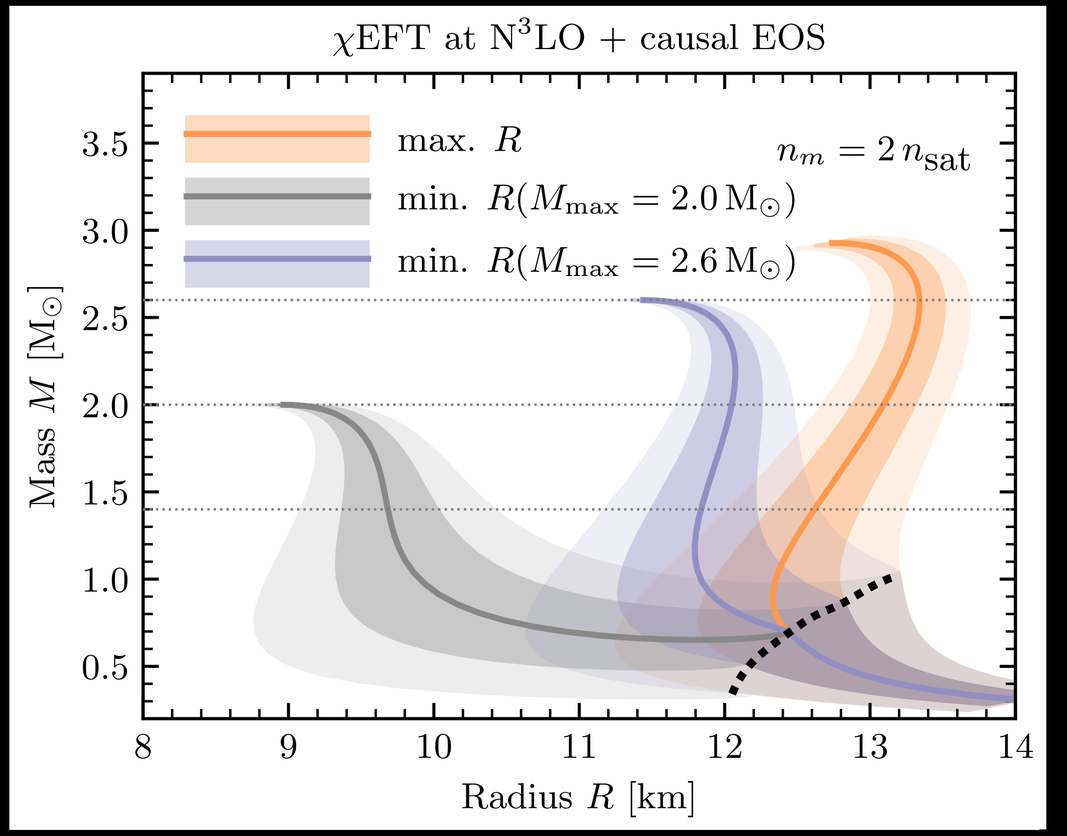

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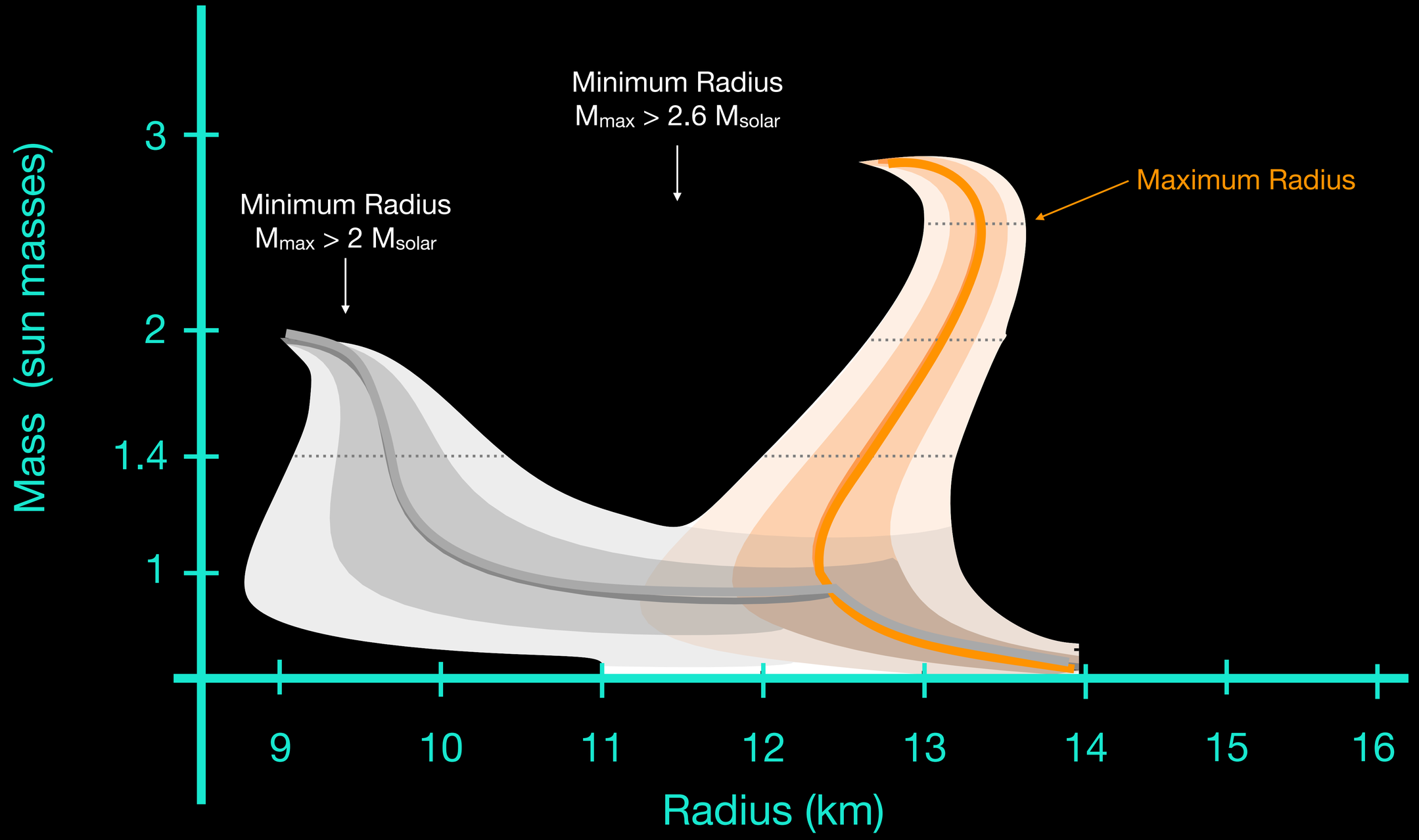

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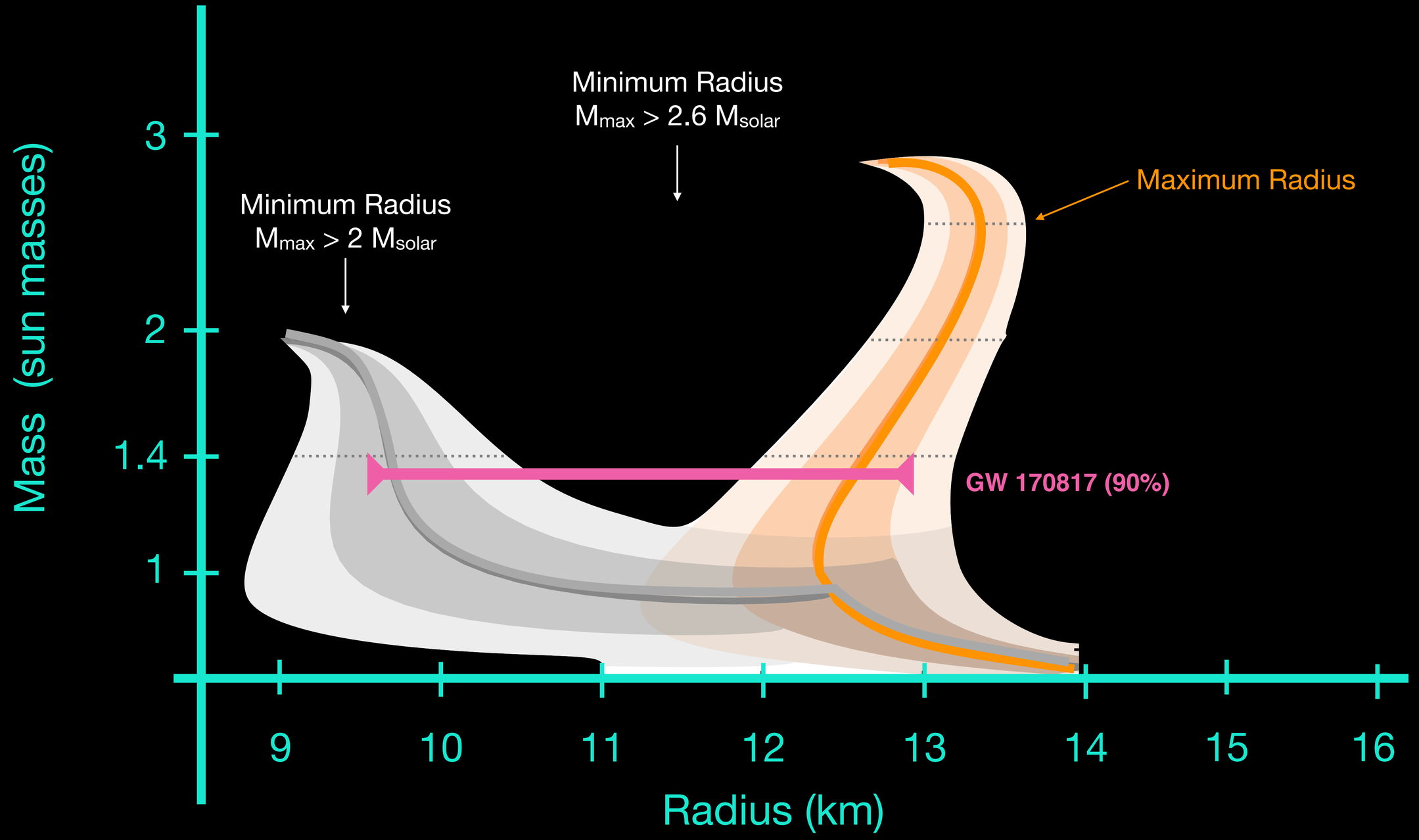

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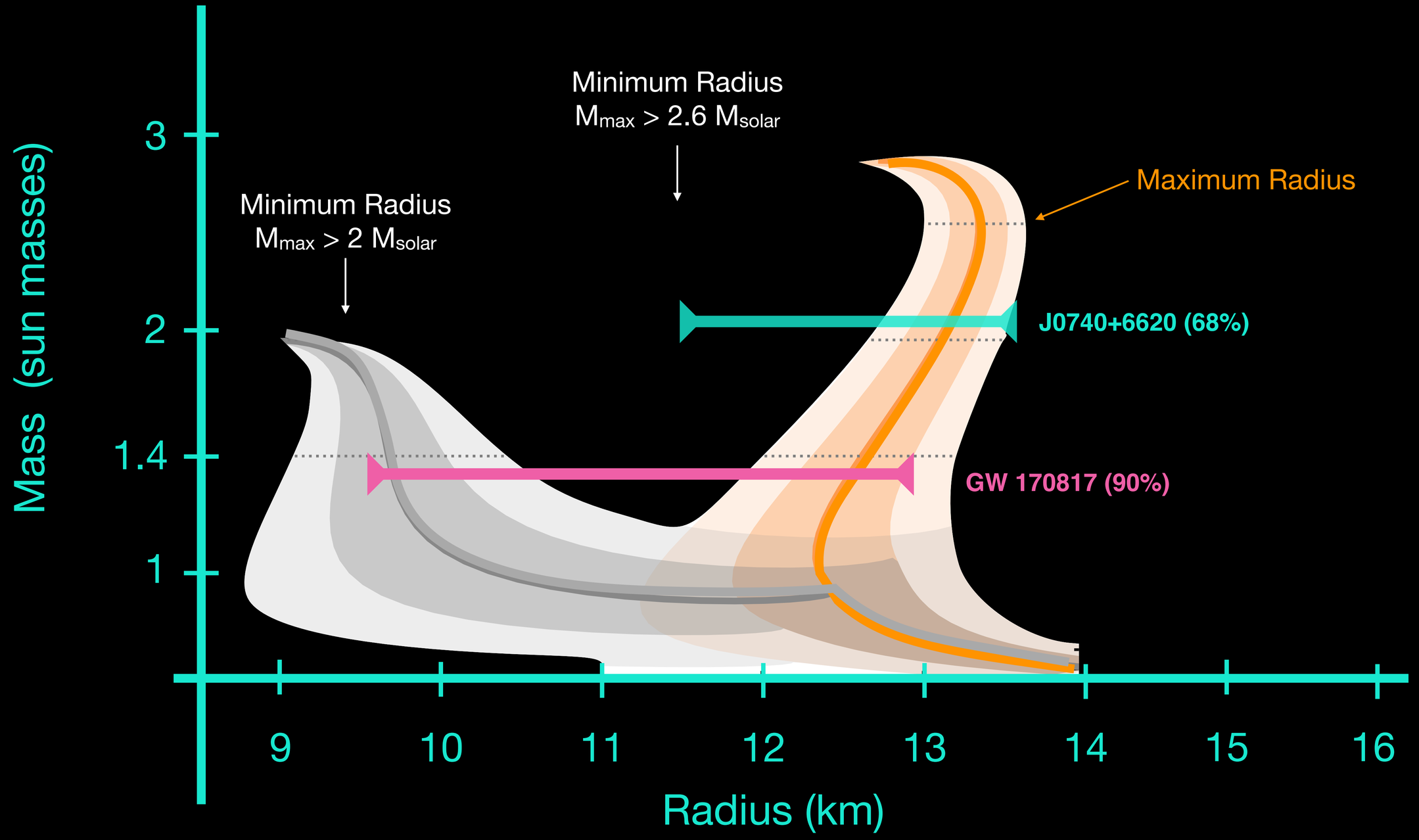

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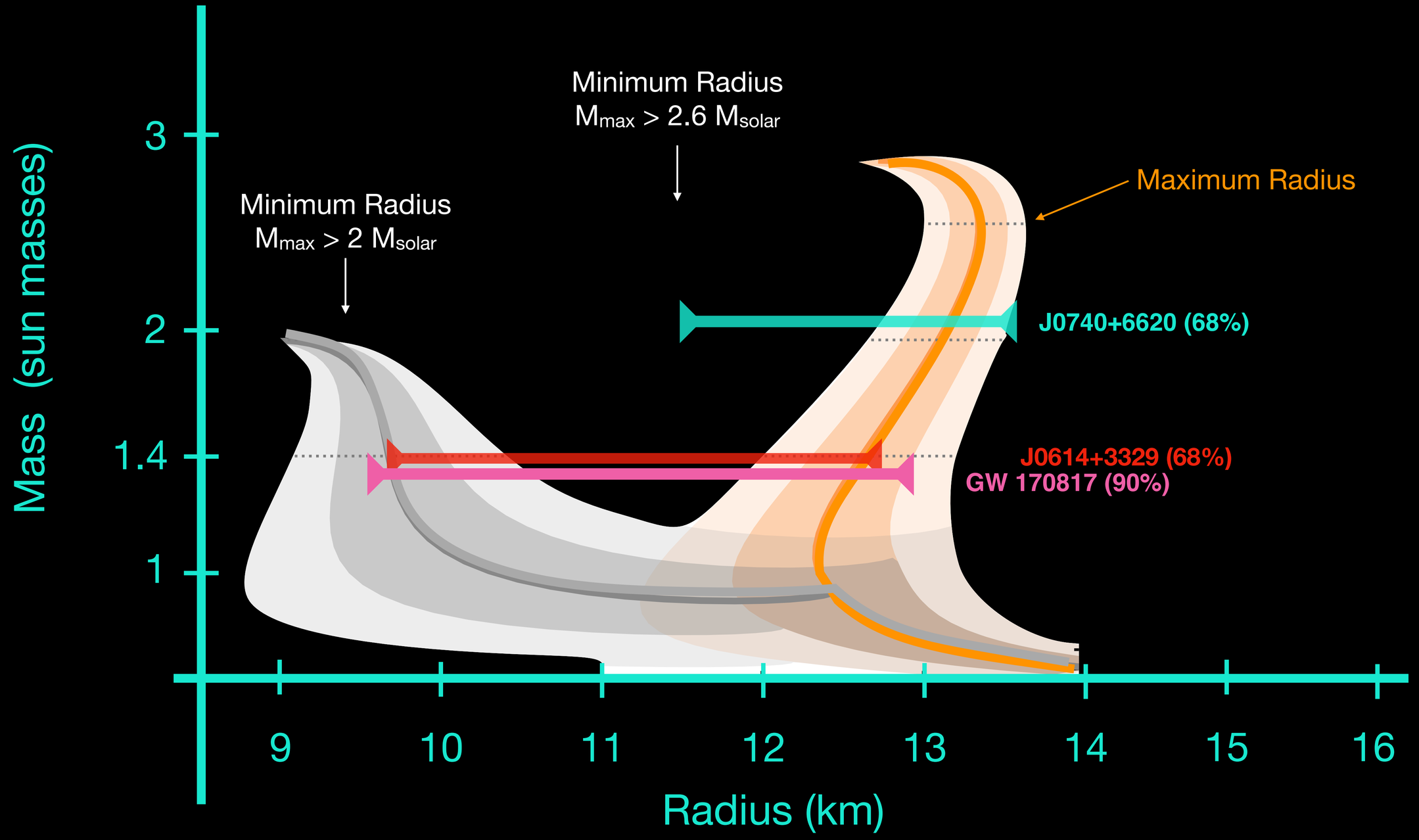

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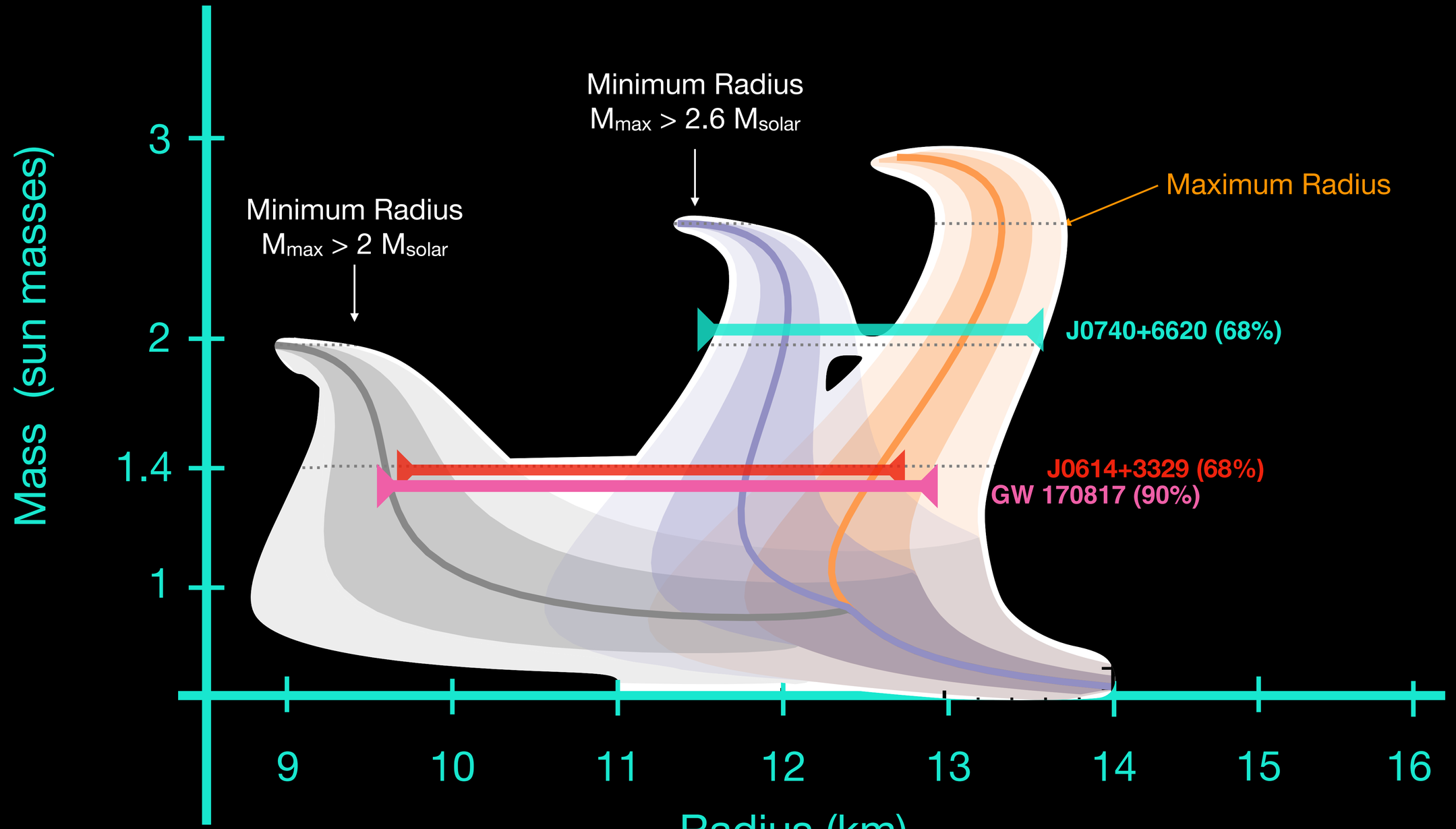
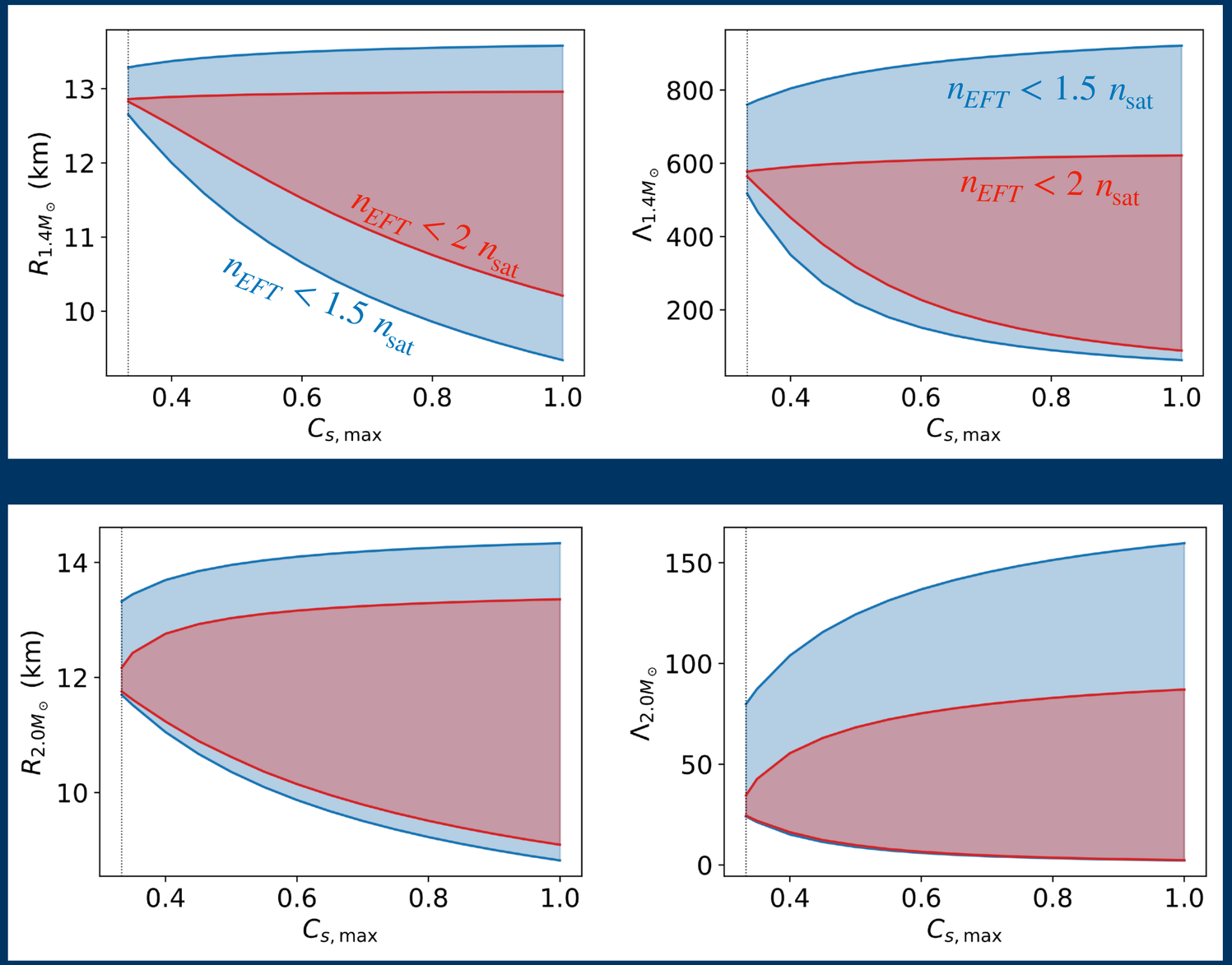


Impact of a Robust Upper Bound on the Sound Speed

EFT for $n_B < \alpha\ n_{\rm sat}$ where $\alpha : 1.5 - 2$ when combined with an assumption about the maximum sound speed in QCD yields useful bounds.

It is intriguing that if $C_{\rm s,max} = c_s^2 \simeq 1/3$, the radii and tidal deformability of neutron stars are tightly constrained.

Bedaque & Steiner (2015) Tews et al. (2018)

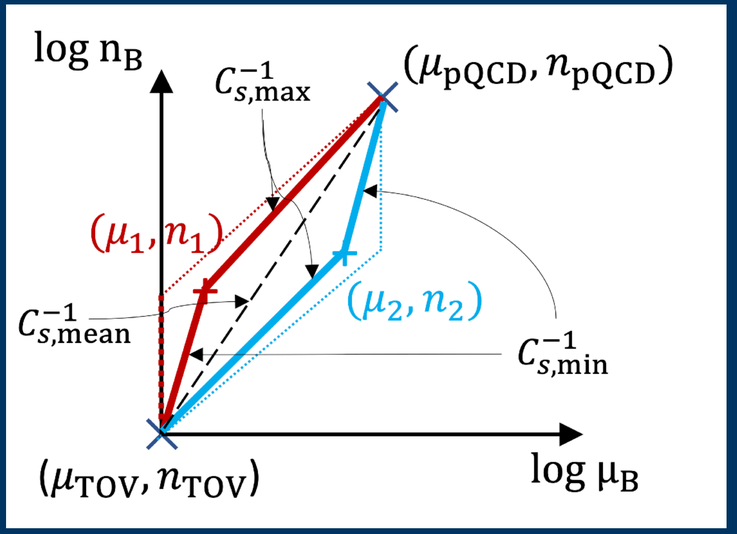


Figures from Dake Zhou (2026)

Inferring Properties Beyond Neutron Star Densities

Aleksi Kurkela and collaborators have emphasized constraints obtained by matching to pQCD.

Kurkela, Romatschke, Vuorinen (2010–2014); Komoltsev & Kurkela (2022); Somasundaram et al. (2024).



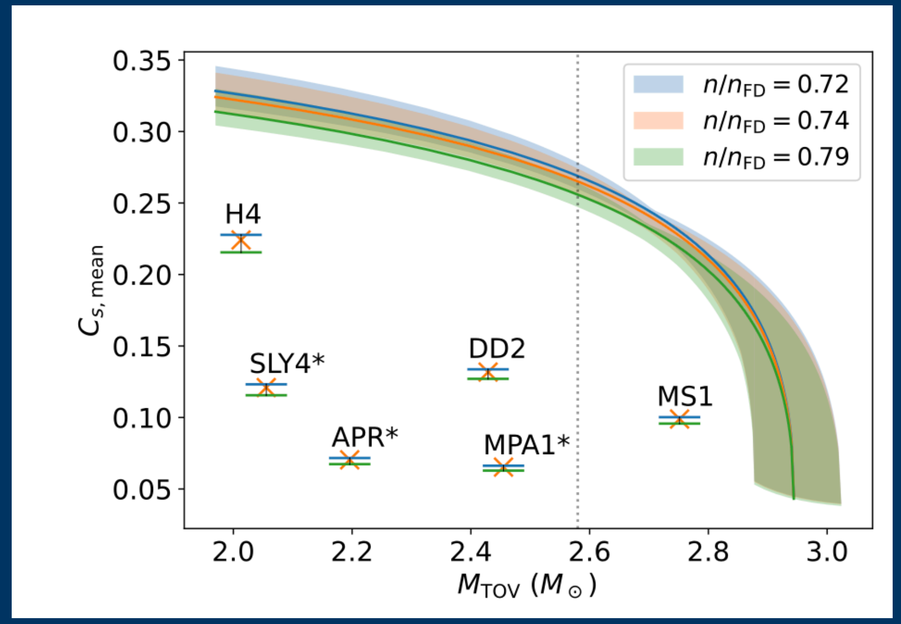
Dake Zhao has shown that a useful upper bound on the minimum sound speed at densities beyond those realized in neutron stars can also be obtained using pQCD and neutron stars. Dake Zhou (2025), arXiv:2408.16738.

$$C_s^{-1} = \frac{1}{c_s^2} = \frac{d\log n_B}{d\log \mu_B} \quad \Rightarrow \quad C_{s,\rm mean} = \frac{\log(\mu_{pQCD}/\mu_{\rm TOV})}{\log(n_{pQCD}/n_{\rm TOV})}$$

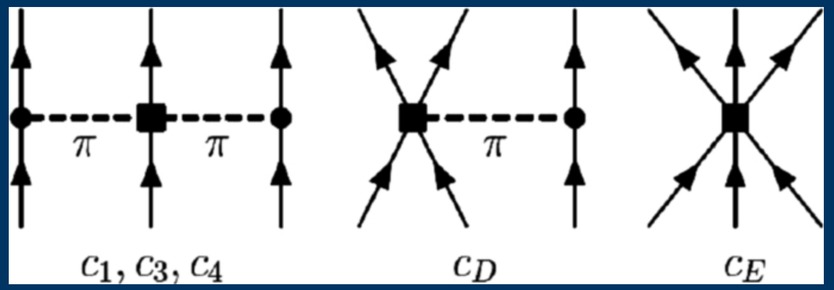
Mean value theorem guarantees that

$$C_{s,\min} < C_{s,\rm mean} < C_{s,\max}$$

The discovery of massive neutron stars, especially those that are also compact, would require softening at higher density.



Shorter Range Three-Nucleon Forces in Neutron Matter



- At N2LO, only the long-range part of the 3N force contributes. These LECs are determined fairly well.
- C_D contribution vanishes because the pion couples to spin.
- C_E contribution vanishes due to the Pauli principle.

  Hebeler and Schwenk (2009)

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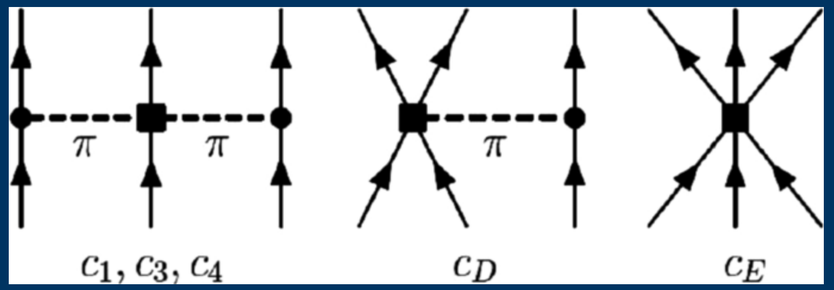

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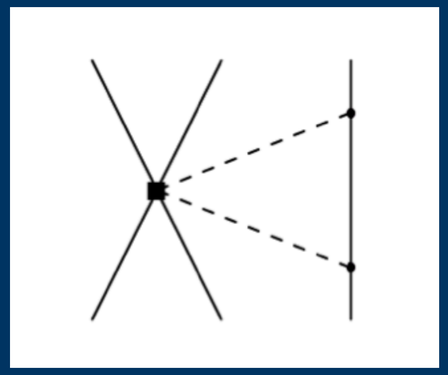
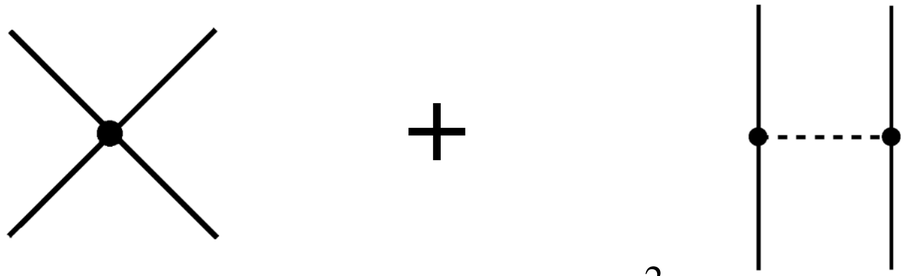


Shorter-range 3N forces from the coupling to two-nucleons do not vanish in neutron matter. But they occur at higher order (N^5L0) in the Chiral EFT expansion and are expected to be small in Weinberg counting.

But the anomalous RG scaling of NN interactions near unitarity challenges Weinberg's naturalness assumptions. This motivates a reassessment of the power counting and an exploration of operators that may be enhanced.

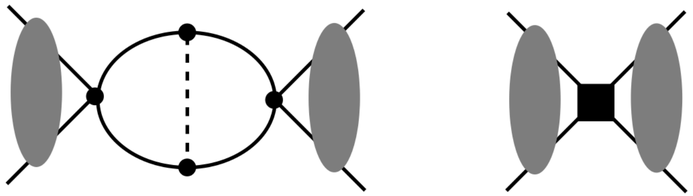
Pion mass-dependence of NN interaction in EFT



$$V_{\rm LO}(q) = \quad C_0 + D_2\, m_\pi^2 \quad + \quad \frac{g_A^2}{4f_\pi^2}\,\frac{\sigma_1\cdot\mathbf{q}\;\sigma_2\cdot\mathbf{q}}{\mathbf{q}^2+m_\pi^2}\,\tau_1\cdot\tau_2$$

Renormalization requires D_2 :

To obtain a scattering amplitude that is independent of regularization or cut-off Λ requires:



$$\Lambda\frac{d}{d\Lambda}\left(\frac{D_2}{C_0^2}\right)_{\rm KSW} = \frac{g_A^2 m_N^2}{64\pi^2 f_\pi^2} \quad \Rightarrow \quad \frac{|D_2|}{C_0^2} \approx \frac{g_A^2 m_N^2}{64\pi^2 f_\pi^2} \simeq \frac{1}{4} \quad \Rightarrow \quad |D_2| \simeq \frac{1}{4}C_0^2$$

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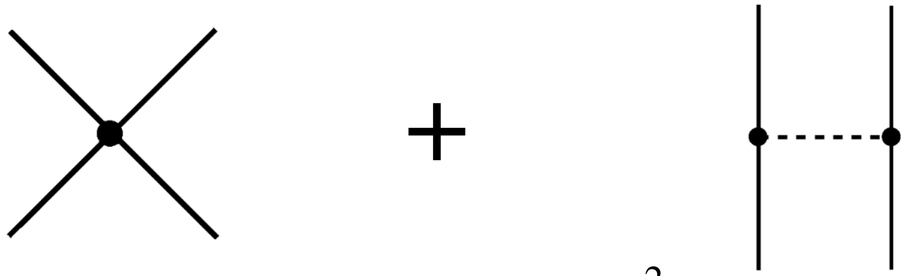

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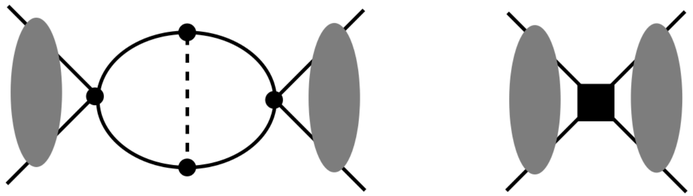

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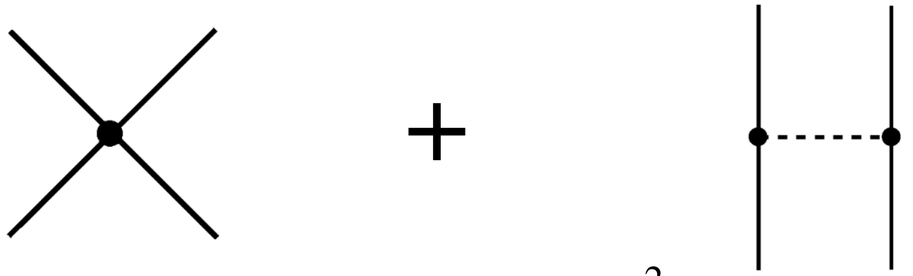

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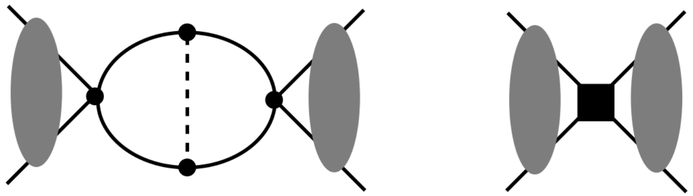

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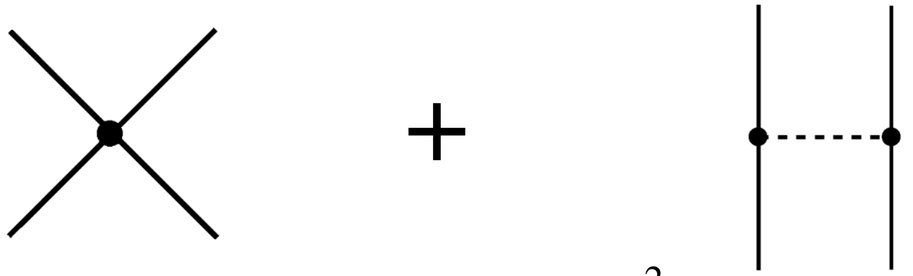
Analysis of 2-nucleon scattering in Lattice QCD as a function of m_π could determine D_2 .

Pion mass-dependence of NN interaction in EFT

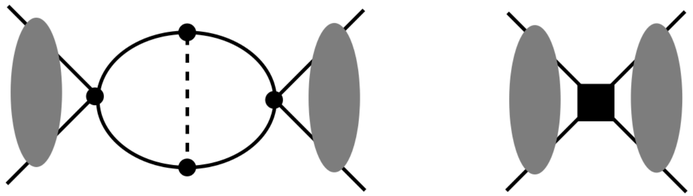


$$V_{\rm LO}(q) = \qquad C_0 + D_2\, m_\pi^2 \qquad + \qquad \frac{g_A^2}{4f_\pi^2}\,\frac{\sigma_1\cdot\mathbf{q}\;\sigma_2\cdot\mathbf{q}}{\mathbf{q}^2+m_\pi^2}\,\tau_1\cdot\tau_2$$

Renormalization requires D_2 :

Kaplan, Savage, Wise (1998)

To obtain a scattering amplitude that is independent of regularization or cut-off Λ requires:



$$\Lambda\frac{d}{d\Lambda}\left(\frac{D_2}{C_0^2}\right)_{\rm KSW} = \frac{g_A^2 m_N^2}{64\pi^2 f_\pi^2} \quad\Rightarrow\quad \frac{|D_2|}{C_0^2} \approx \frac{g_A^2 m_N^2}{64\pi^2 f_\pi^2} \simeq \frac{1}{4} \quad\Rightarrow\quad |D_2| \simeq \frac{1}{4}C_0^2$$

Analysis of 2-nucleon scattering in Lattice QCD as a function of m_π could determine D_2 .

Beane, Bedaque, Detmold, Savage (NPLQCD), Walker-Loud (Cal-Lat), Aoki, Hatsuda, Ishii (HAL QCD Collaboration), ….

D_2 and Coupling to Pions

Chiral symmetry requires that pion mass terms only appear in a specified form:

$$m_\pi^2 \quad \Rightarrow \quad m_\pi^2 \left(1 + \frac{\pi_a \pi_b}{2 f_\pi^2} \, \delta_{ab} + \cdots \right)$$

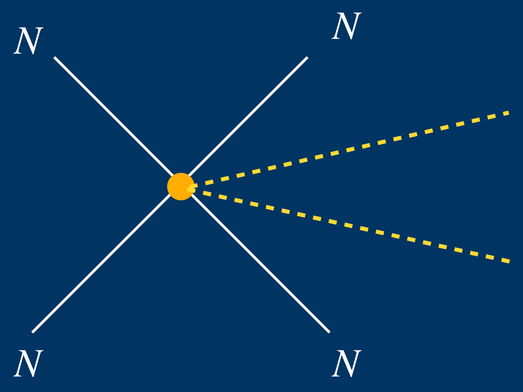
This induces a coupling of pions to two-nucleons:



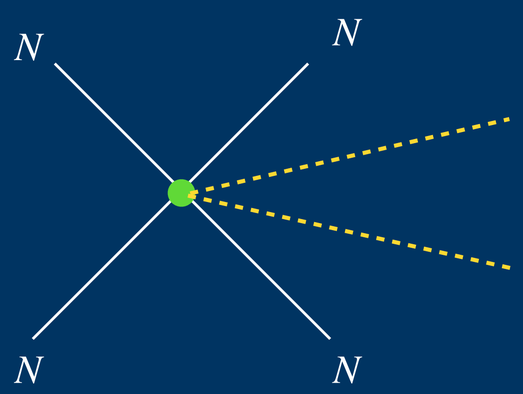
$D_2 \, m_\pi^2$ + $D_2 \, \dfrac{m_\pi^2}{2 f_\pi^2}$

Two more two-pion-two-nucleon couplings: E_2 and F_2

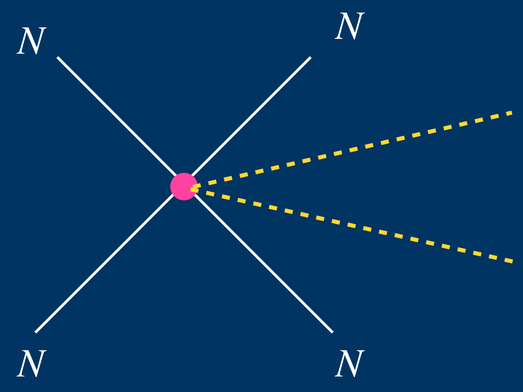
B. Borasoy and H. W. Griesshammer (2001), (2003)



$D_2\ m_\pi^2$



$E_2\ \omega^2$



$F_2\ q^2$

$D_2, E_2, \&\ F_2$ are enhanced for the same reason and are a priori expected to be of similar size.

The typical size of these LECs depends on the scaling of C_0 :

In the KSW-inspired scheme ($C_0 \sim 1/f_\pi^2$): $\quad D_2 \approx E_2 \approx F_2 \approx \dfrac{1}{5 f_\pi^4}$

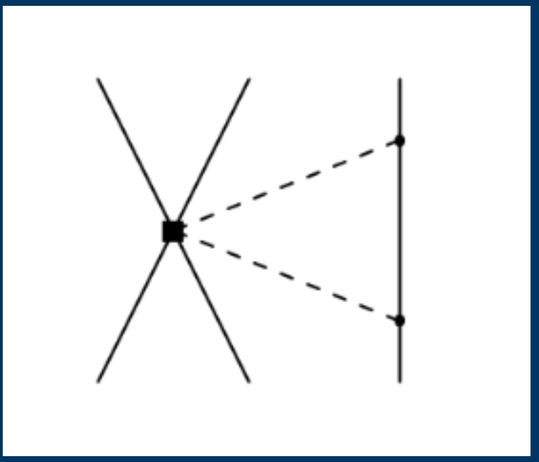
In the Weinberg scheme (Naive Dimensional Analysis $C_0 \sim 1/\Lambda f_\pi$): $\quad D_2 \approx E_2 \approx F_2 \approx \dfrac{1}{5 f_\pi^2 \Lambda^2}$

A New Class of Three Nucleon Forces

V. Cirigliano, M. Dawid, W. Dekens, S. Reddy (2025)

3NF due to pion coupling to two nucleons are large because:

- Enhanced D_2 & F_2 in RG-invariant regularization schemes
- Heavy nucleon intermediate states enhance the loop contribution



$$V_{ijk}^{i'j'k'}(\vec{q}_1,\vec{q}_2,\vec{q}_3) = -\frac{9g_A^2 D_2 m_\pi^3}{128\pi f_\pi^4}\kappa_{ij}^{i'j'}\delta_{kk'}\,\mathcal{I}\left(\frac{\vec{q}_3^{\,2}}{4m_\pi^2}\right) \quad \text{where} \quad \mathcal{I}(b) = \frac{2}{3}\left(1+\left(\frac{1}{2\sqrt{b}}+\sqrt{b}\right)\cot^{-1}(1/\sqrt{b})\right)$$

$$V_{ijk}^{i'j'k'}(\vec{q}_1,\vec{q}_2,\vec{q}_3) = -\frac{15F_2 g_A^2 m_\pi^3}{16\pi f_\pi^4}\delta_{kk'}\left(\bar{f}_2^S\delta_{ii'}\delta_{jj'}+\bar{f}_2^T\vec{\sigma}_{i'i}\cdot\vec{\sigma}_{j'j}\right)\mathcal{J}\left(\frac{\vec{q}_3^{\,2}}{4m_\pi^2}\right) \quad \text{where} \quad \mathcal{J}(b) = \frac{3}{5}\left((1+2b)\mathcal{I}(b)+\frac{2}{3}\right)$$



Maria Dawid



Wouter Dekens

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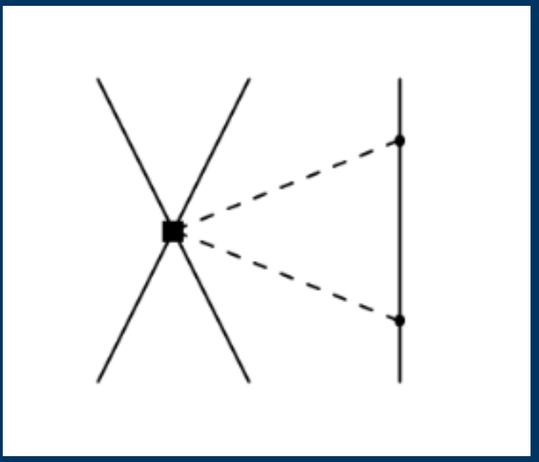

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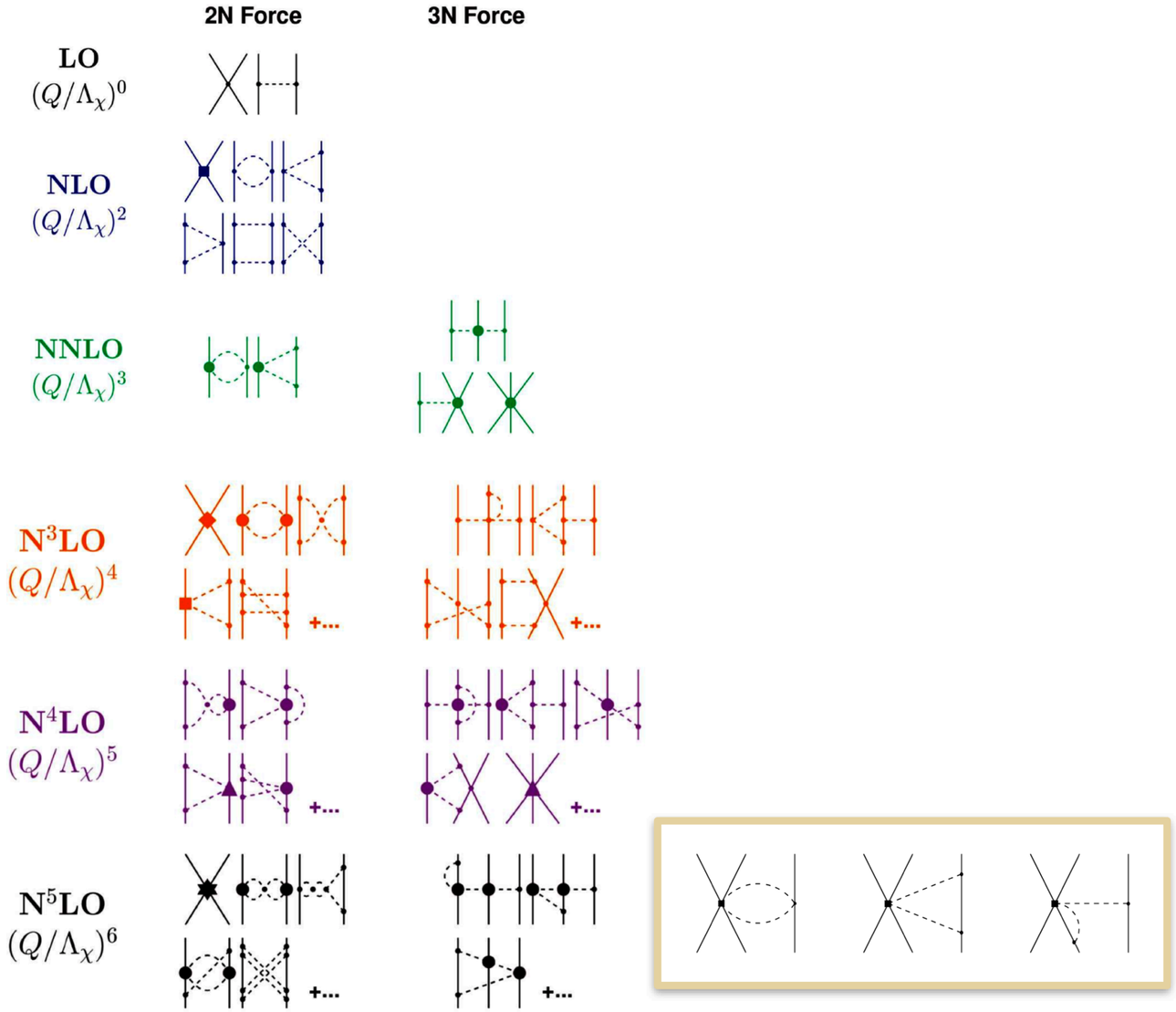
The New Three-Nucleon Force Can be Enhanced



- NDA or Weinberg power counting would imply D_2 & F_2 only appears at N^5LO
- If $C_0 \sim 1/f_\pi^2$ due to large scattering length, they are promoted to appear at N^3LO
- A loop enhancement amplifies the contribution by a further factor of π.

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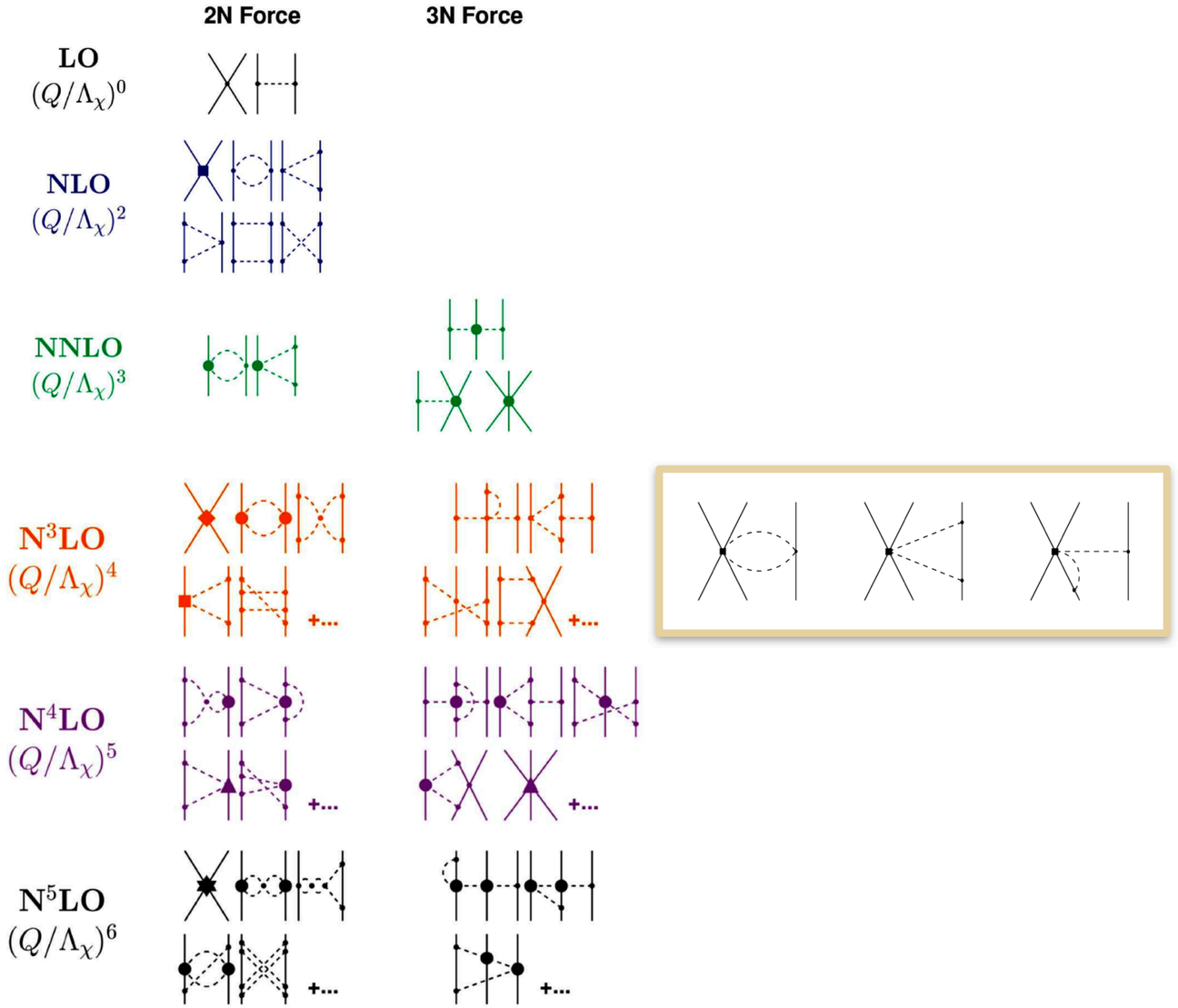

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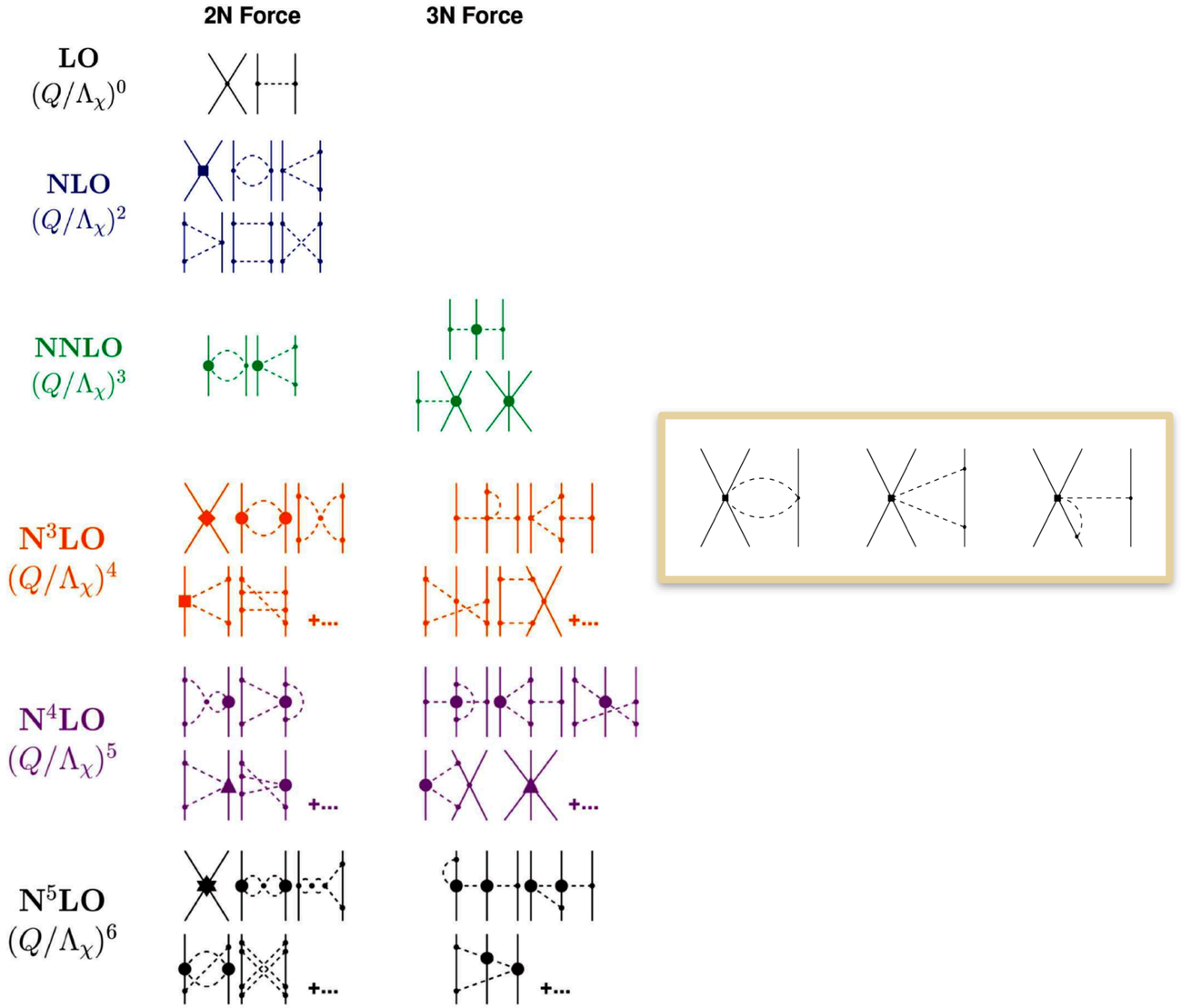

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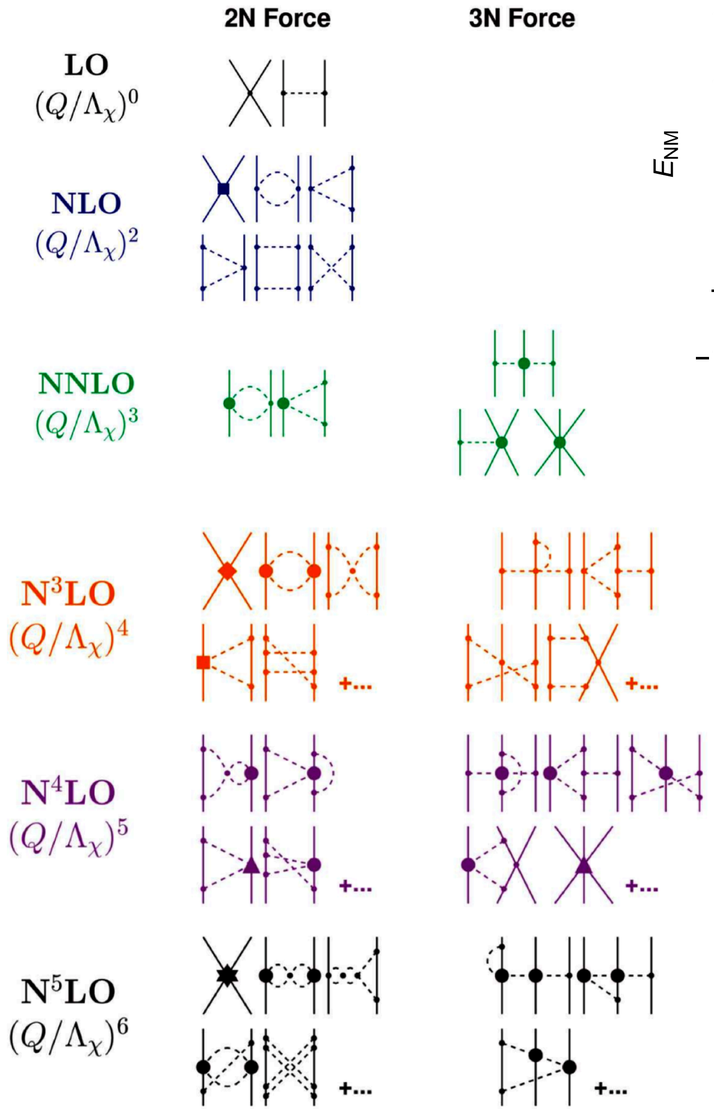

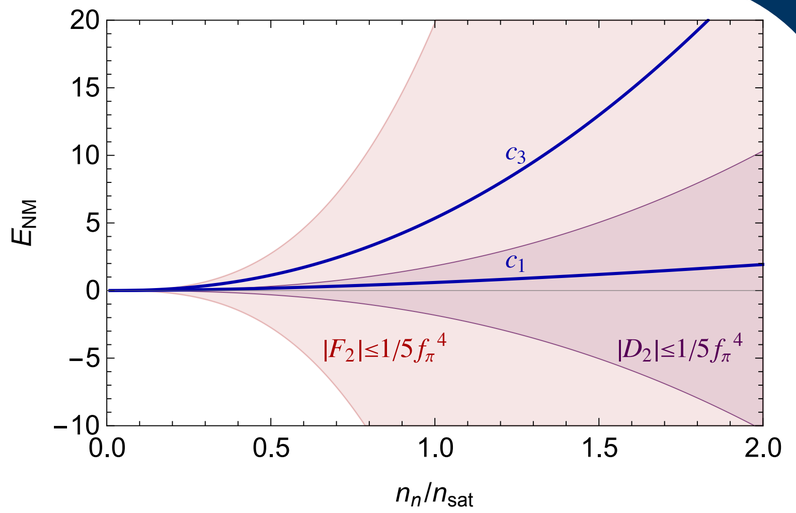

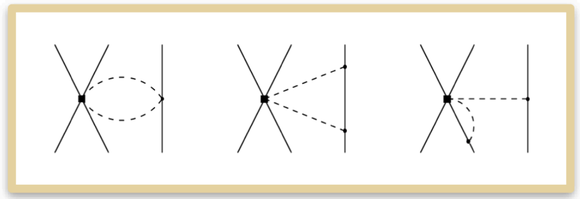

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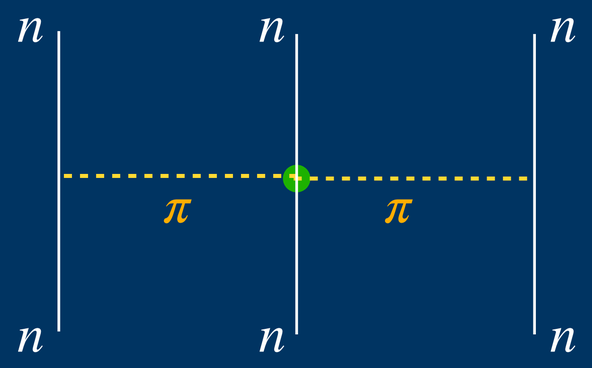
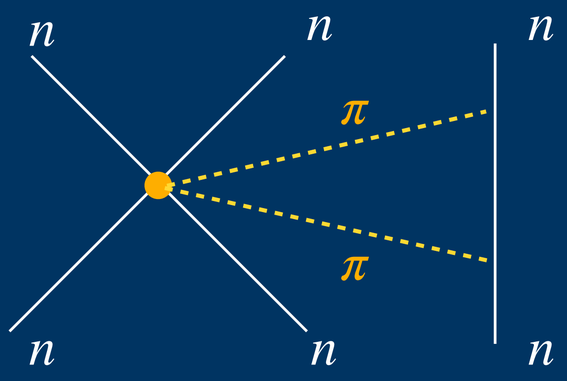
Neutron Matter: Underestimating Errors?



c_1 & c_3



D_2 & F_2

Current Paradigm:

Leading 3NF is determined by pion-nucleon scattering data. Independent of multi-nucleon information. Errors are small because there are no 3NF short-distance contributions.

Our calculation:

Pion coupling to two-nucleons can play a role. Information about two-nucleon dynamics influences 3NF to ensure proper renormalization. Error estimates will likely need revision.

Chiral EFT at N²L0 predicts

$$P(n_{\rm sat}) = 3.1 \pm 0.5 \ \ {\rm MeV/fm^3}$$

C. Drischler, R. J. Furnstahl, J. A. Melendez, D. R. Phillips (2021)

$$P(n_{\rm sat}) = 2.2 \pm 0.4 \ \ ({\rm MeV/fm^3})$$

I. Tews,R. Somasundaram,D. Lonardoni,H. Göttling, R. Seutin, J. Carlson S. Gandolfi,K. Hebeler, A. Schwenk (2024)

We estimate the contribution to the pressure from our new 3NFs to be:

$$\delta P_{\rm 3NF} = \left[0.7 \left(\frac{D_2}{D_2^{\rm ref}} \right) + 8.8 \left(\frac{F_2}{F_2^{\rm ref}} \right) \right] \ \frac{\rm MeV}{\rm fm^3}$$

where $\quad |D_2^{\rm ref}| = |F_2^{\rm ref}| = \dfrac{1}{5 f_\pi^4}$

V. Cirigliano, M. Dawid, W. Dekens, S. Reddy (2025)

Epelbaum et al. find our 3NFs are suppressed.

Scrutiny of the new class of three-nucleon forces E. Epelbaum,[1] A. M. Gasparyan,[1] J. Gegelia,[1,2] D. Hog,[1] and H. Krebs[1]

In the Weinberg scheme, the nuclear potential is regulated with a momentum cutoff and short-distance contributions from pion loops are absorbed into contact interactions.

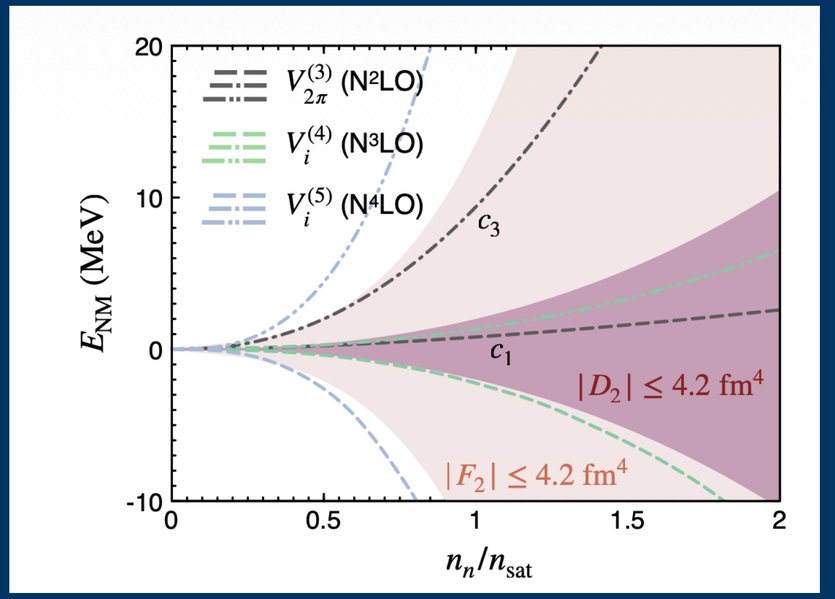
Compared with dimensional regularization, this subtraction prescription significantly reduces the pion-loop contribution to the potential.

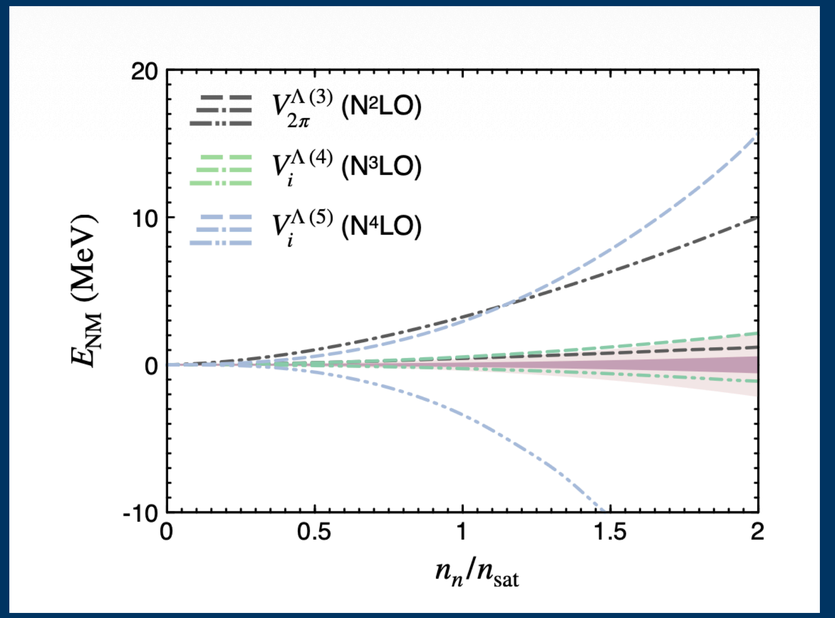
For commonly employed cutoff values, the fitted leading-order contact coupling is also smaller than the estimates adopted in our paper.

They find other 3NFs at N⁴L0 to be more important.

A complete accounting of all 3NFs at N⁴L0 would be valuable to assess EOS errors at $n_B \simeq 2\ n_{\rm sat}$.

$D_2 : 4.2\ \mathrm{fm}^4 \rightarrow 1\ \mathrm{fm}^4$





Cutoff regularization of the corresponding spectral integrals.

Part I: Take Aways

- The EOS up to $n_B \lesssim 2\ n_{\rm sat}$ can be calculated using Chiral EFT. The error estimates are critical and warrant further study.
- RG and scaling of operators are key to obtaining improved estimates.
- In the Weinberg scheme, other N^{4}L0 terms appear to be more relevant.
- When combined with bounds on c_s^2 from QCD, one can derive bounds on neutron star structure.
- If the upper bound on $c_s^2 \simeq 1/3$ in QCD, neutron star radii and tidal deformabilities are strongly constrained.

Part II:
Neutrino Interactions in Degenerate Nuclear Matter

Neutrino Processes in Degenerate Nuclear Matter

Neutral Current:

$$\nu + N \to \nu + N$$

$$\nu + n + X \to \nu + n + X$$

$$n + X \to n + X + \nu + \bar{\nu}$$

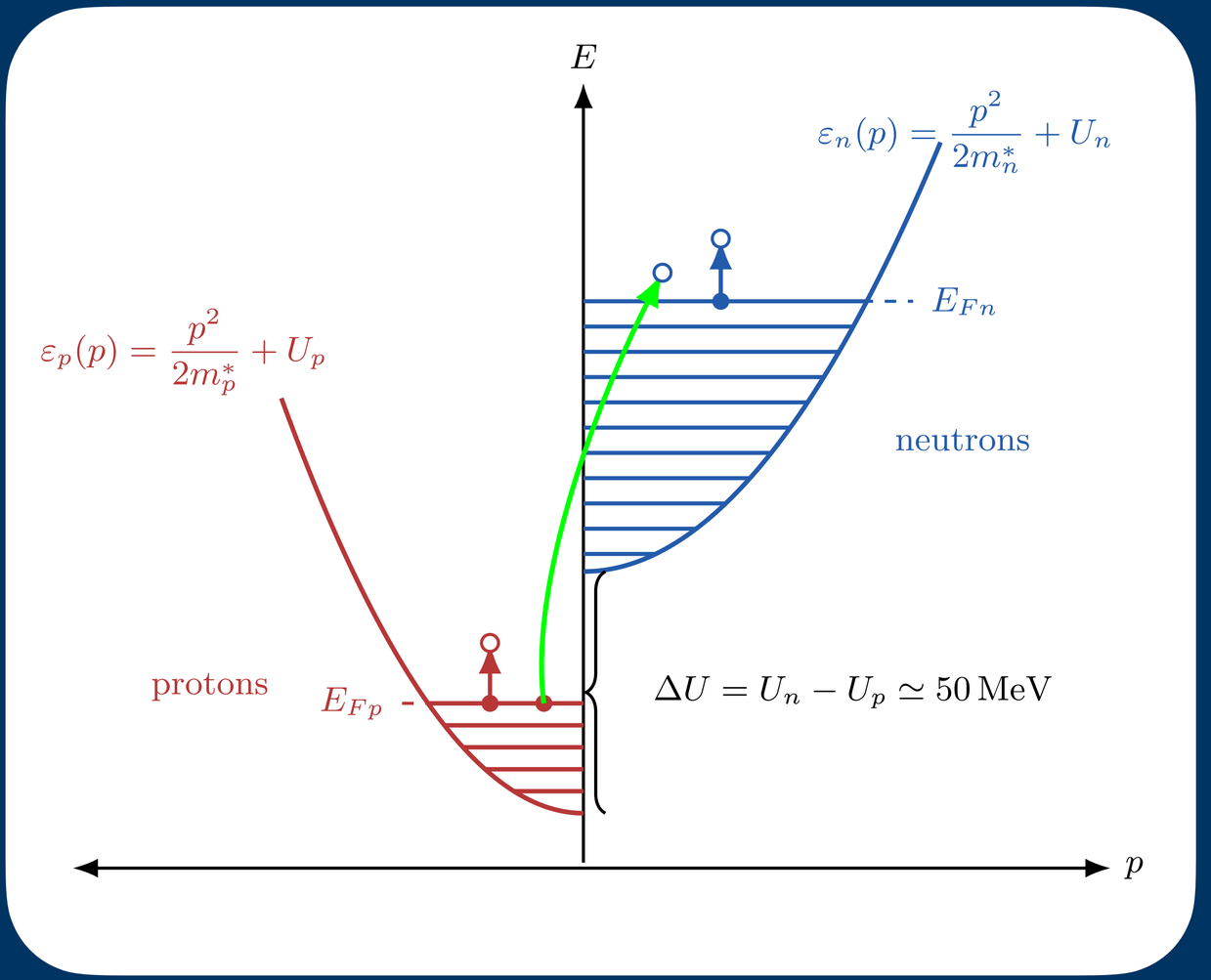
Charged Current:

$$e^- + p \to \nu_e + n$$

$$n \to e^- + p + \bar{\nu}_e$$

$$e^- + p + X \to \nu_e + n + X$$

$$n + X \to e^- + p + \bar{\nu}_e + X$$



Linear Response for Neutrino Interactions in Dense Matter

Low energy Lagrangian: $\mathcal{L} = \frac{G_F}{\sqrt{2}}\, l_\mu\, j^\mu \qquad l_1 + N_2 \to l_3 + N_4$

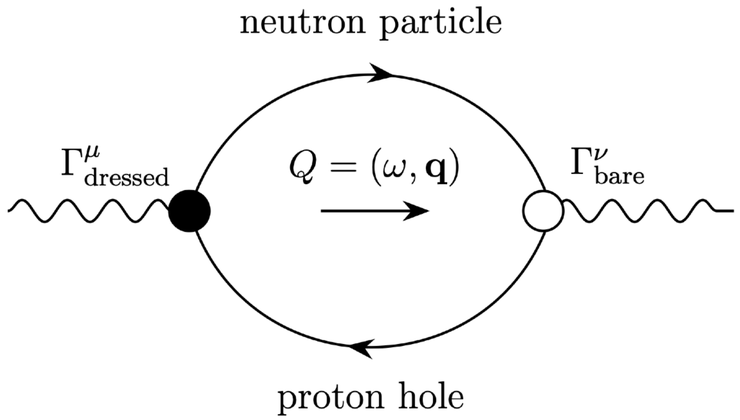
Neutrino scattering and absorption rates: $\frac{d\Gamma(E_1)}{dE_3 d\mu_{13}} = \frac{G_F^2}{32\pi^2}\frac{p_3}{E_1}(1 - f_3(E_3))L_{\mu\nu}\, \mathcal{S}^{\mu\nu}(q_0, q)$

Dynamic structure function: $\mathcal{S}^{\mu\nu}(q_0, q) = \frac{-2\ \mathrm{Im}\ \mathbf{\Pi}^{\mu\nu}(q_0, q)}{1 - \exp\left(-(q_0 + \Delta\mu)/T\right)}$

Current-current correlations functions: $\mathbf{\Pi}^{\mu\nu}(q_0, q) = -i \int dt\ d^3x\ \theta(t)\ e^{i(q_0 t - \vec{q}\cdot\vec{x})} \langle\ |[j_\mu\ (\vec{x}, t), j_\nu(\vec{0}, 0)]|\ \rangle$

difficult to calculate in general due to the non-perturbative nature of strong interactions.
Usually calculated in mean-field theory: Hartree + RPA.

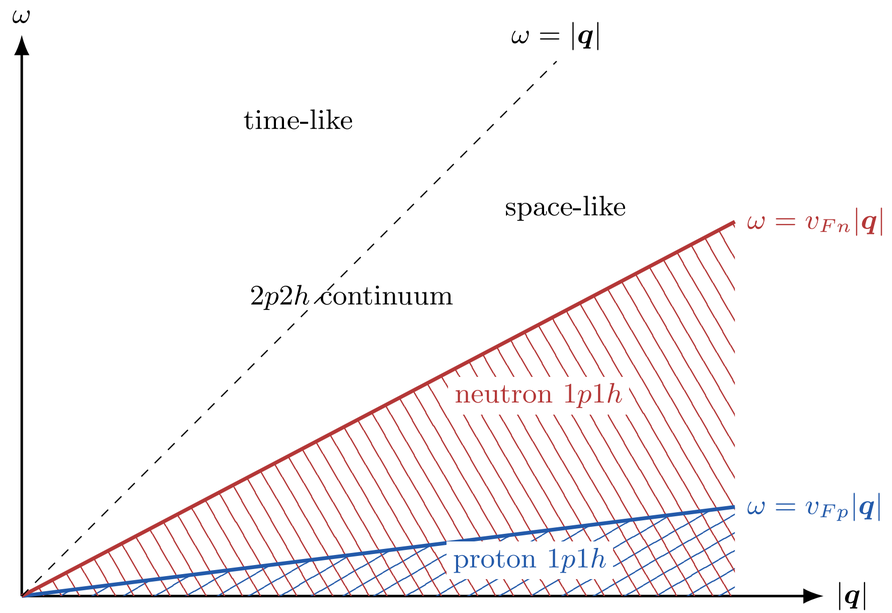
$$\Pi^{\mu\nu}_{pn}(Q) \sim \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}[G_p(p)\Gamma^\mu_{\mathrm{dressed}} G_n(p+Q)\Gamma^\nu_{\mathrm{bare}}]$$



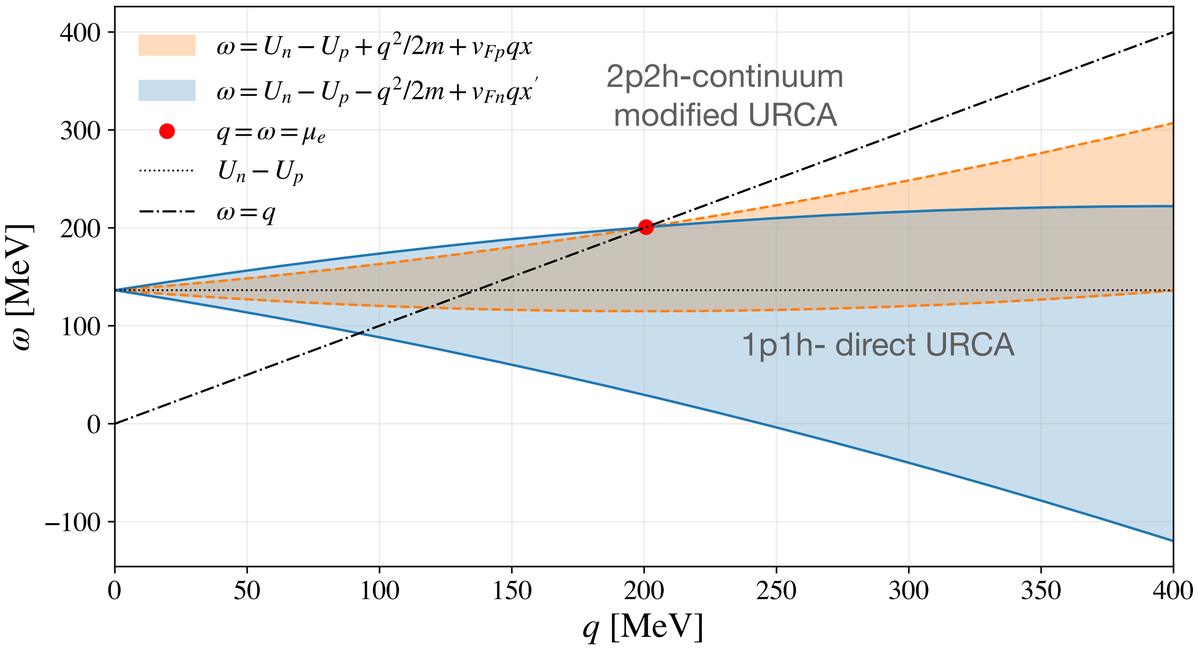
Iwamoto & Pethick (1980s), Horowitz & Wherberger, Raffelt et al., Reddy et al. (1990s), Carlson, Gandolfi, Horowitz, Pethick, Reddy, Roberts, Schwenk, and others (2000s)

2p-2h Excitations Alleviate Kinematic Constraints

Neutral Current:



Charged Current:



Allows greater energy transfer during neutral-current scattering.

Overcomes the momentum constraints on the direct URCA reaction using the spectator momentum (modified URCA).

Consistent Inclusion of 2p2h Response

Earlier work has included finite quasiparticle lifetimes, particularly for the spin response. These studies established the need for finite widths beyond a collisionless 1p1h picture.

Raffelt & Seckel (1991); Sedrakian & Dieperink (2000); Hanhart et al. (2001); Bacca et al. (2009)

Direct calculation of the 2p2h modified-URCA rate was redone correctly but without an introduction of a consistent width, found enhancement near the direct-URCA threshold due to the small energy denominator.

Shternin, Baldo & Haensel (2018)

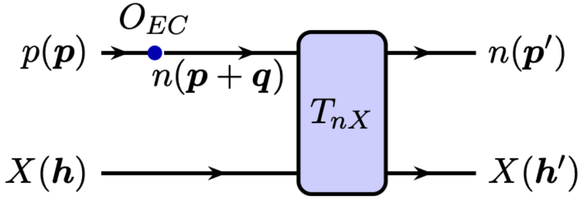
Alford, Haber & Zhang insert a nucleon width in the 1p1h response. This yields a smooth direct- to modified-Urca interpolation for total emissivity

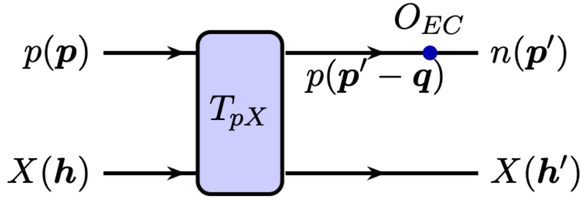
Alford, Haber & Zhang (2024)

A direct calculation of the 2p2h response and correct treatment of the resonant regions yeilkds a consitent treatement of both 1p1h and 2p2h.

Kumamoto, Reddy & Zhao (2026 .. in prep.)

2p2h Response from Fermi Golden Rule





$$\mathcal{H}_i^\mu = T_{nX}(\boldsymbol{p}', \boldsymbol{h}'; \boldsymbol{p}+\boldsymbol{q}, \boldsymbol{h}) \frac{O_{\mathrm{EC}}^\mu}{D_i}$$

$$\mathcal{H}_o^\mu = \frac{O_{\mathrm{EC}}^\mu}{D_o} T_{pX}(\boldsymbol{p}'-\boldsymbol{q}, \boldsymbol{h}'; \boldsymbol{p}, \boldsymbol{h})$$

The response contains two amplitudes. The interference between them is critical for current conservation.

The nuclear response function is calculated in the quasi-particle approximation

$$R_{pn}^{\mu\nu}(Q) = \int \frac{\mathrm{d}^3p\,\mathrm{d}^3p'\,\mathrm{d}^3h}{(2\pi)^9} f_p(\boldsymbol{p}) f_X(\boldsymbol{h}) \left[1 - f_n(\boldsymbol{p}')\right] \left[1 - f_X(\boldsymbol{p}-\boldsymbol{p}'+\boldsymbol{h})\right]$$
$$\times\, 2\pi\delta\left[\epsilon_p(\boldsymbol{p}) + \epsilon_X(\boldsymbol{h}) + \omega - \epsilon_n(\boldsymbol{p}') - \epsilon_X(\boldsymbol{p}-\boldsymbol{p}'+\boldsymbol{h})\right] \left\langle \mathcal{H}^\mu \mathcal{H}^{\nu *} \right\rangle,$$

The energy denominators include the quasi-particle widths Γ_n and Γ_p

$$D_i = \epsilon_p(\boldsymbol{p}) + \omega - \epsilon_n(\boldsymbol{p}+\boldsymbol{q}) + i\frac{\Gamma_n(\boldsymbol{p}+\boldsymbol{q})}{2},$$
$$D_o = \epsilon_n(\boldsymbol{p}') - \omega - \epsilon_p(\boldsymbol{p}'-\boldsymbol{q}) + i\frac{\Gamma_p(\boldsymbol{p}'-\boldsymbol{q})}{2}.$$

Electron Capture 2p2h Excitation

A direct calculation yields

$$\frac{\mathrm{d}\Gamma_{\mathrm{EC}}}{V} = \frac{G_F^2 |V_{ud}|^2}{2} \frac{\mathrm{d}^3 k_e}{(2\pi)^3\, 2E_e} \frac{\mathrm{d}^3 k_\nu}{(2\pi)^3\, 2E_\nu} f_e(E_e)[1 - f_{\nu_e}(E_\nu)] \; \left[g_V^2 \ell_F(\theta) S_{pn}^F(Q) + g_A^2 \ell_{GT}(\theta) S_{pn}^{GT}(Q) \right]$$

where the response functions are

$$\begin{aligned} S_{pn}^{\alpha}(Q) = \int & \frac{\mathrm{d}^3 p\, \mathrm{d}^3 p'\, \mathrm{d}^3 h}{(2\pi)^9} f_p(\boldsymbol{p})\, f_X(\boldsymbol{h})\, [1 - f_n(\boldsymbol{p}')]\, [1 - f_X(\boldsymbol{h} + \boldsymbol{p} + \boldsymbol{q} - \boldsymbol{p}')] \\ & \times 2\pi\delta[\epsilon_p(\boldsymbol{p}) + \epsilon_X(\boldsymbol{h}) + \omega - \epsilon_n(\boldsymbol{p}') - \epsilon_X(\boldsymbol{h} + \boldsymbol{p} + \boldsymbol{q} - \boldsymbol{p}')] \\ & \times \left[\frac{\mathcal{W}_{ii}^{\alpha}}{|D_i(\boldsymbol{p}; Q)|^2} + \frac{\mathcal{W}_{oo}^{\alpha}}{|D_o(\boldsymbol{p}'; Q)|^2} + 2\mathrm{Re}\left(\frac{\mathcal{W}_{io}^{\alpha}}{D_i(\boldsymbol{p}; Q) D_o^*(\boldsymbol{p}'; Q)} \right) \right], \end{aligned}$$

and the nucleon-nucleon scattering elements are

$$\mathcal{W}_{ii}^F = \overline{T_n T_n^\dagger}, \qquad \mathcal{W}_{oo}^F = \overline{T_p T_p^\dagger}, \qquad \mathcal{W}_{io}^F = \overline{T_n T_p^\dagger},$$

$$\mathcal{W}_{ii}^{GT} = \overline{T_n T_n^\dagger}, \qquad \mathcal{W}_{oo}^{GT} = \overline{T_p T_p^\dagger}, \qquad \mathcal{W}_{io}^{GT} = \frac{1}{3} \sum_a \overline{T_n \sigma^a T_p^\dagger \sigma^a}.$$

Resonant 2p2h is exactly the 1p1h response

Using consistent quasi-particle widths

$$[1 - f_n(\boldsymbol{p}+\boldsymbol{q})]\,\Gamma_n^{\rm qp}(\boldsymbol{p}+\boldsymbol{q}) = \int \frac{{\rm d}^3p'\,{\rm d}^3h}{(2\pi)^6} f_X(\boldsymbol{h})\,[1 - f_n(\boldsymbol{p}')]\,[1 - f_X(\boldsymbol{h}+\boldsymbol{p}+\boldsymbol{q}-\boldsymbol{p}')] \times 2\pi\delta[\epsilon_n(\boldsymbol{p}+\boldsymbol{q}) + \epsilon_X(\boldsymbol{h}) - \epsilon_n(\boldsymbol{p}') - \epsilon_X(\boldsymbol{h}+\boldsymbol{p}+\boldsymbol{q}-\boldsymbol{p}')]\;\mathcal{W}_{ii}^{\alpha}$$

The response in the vicinity of the two resonant $\mathrm{Re}[D_i] = 0$ and $\mathrm{Re}[D_o] = 0$ regions are given by

$$S^{\alpha}_{ii,\rm pole}(Q) = \int \frac{{\rm d}^3p}{(2\pi)^3} f_p(\boldsymbol{p})\,[1 - f_n(\boldsymbol{p}+\boldsymbol{q})]\left[\frac{\Gamma_n^{\rm qp}(\boldsymbol{p}+\boldsymbol{q})}{d_i^2 + \Gamma_n^{\rm qp}(\boldsymbol{p}+\boldsymbol{q})^2}\right] \qquad S^{\alpha}_{oo,\rm pole}(Q) = \int \frac{{\rm d}^3l}{(2\pi)^3} f_p(\boldsymbol{l})\,[1 - f_n(\boldsymbol{l}+\boldsymbol{q})]\left[\frac{\Gamma_p^{\rm qp}(\boldsymbol{l})}{d_o^2 + \Gamma_p^{\rm qp}(\boldsymbol{l})^2}\right]$$

In the narrow width limit, this is precisely the 1p1h contribution to the response

$$\frac{\Gamma_n^{\rm qp}}{[d_i]^2 + [\Gamma_n^{\rm qp}/2]^2} \longrightarrow 2\pi\;\delta(d_i), \qquad \frac{\Gamma_p^{\rm qp}}{[d_o]^2 + [\Gamma_p^{\rm qp}/2]^2} \longrightarrow 2\pi\;\delta(d_0)$$

Part II: Take away

- The 2p2h response can be calculated in a simple formalism that naturally accounts for the 1p1h response and respects conservation laws.
- Formulated using the Fermi golden rule, it allows for the incorporation of detailed nuclear matrix elements and in-medium effects.
- Numerical studies using an EFT description of the nucleon-nucleon interaction is underway and will be reported shortly.

Conclusions

- A bound on the speed of sound in QCD imposes useful constraints on the structure of neutron stars.
- If $c_{s,\max}^2 \simeq 1/3$ and EFT is reliable up to $n_B \simeq 2n_{\rm sat}$, the constraints on neutron star radii and tidal deformabilities are very stringent.
- Current EFT error estimates and their dependence on the regularization scheme warrant further study. The contributions, especially of higher-order 3NFs, can be large.
- Neutrino processes involving one and two nucleons in the initial and final states can be treated consistently within a simple formalism.