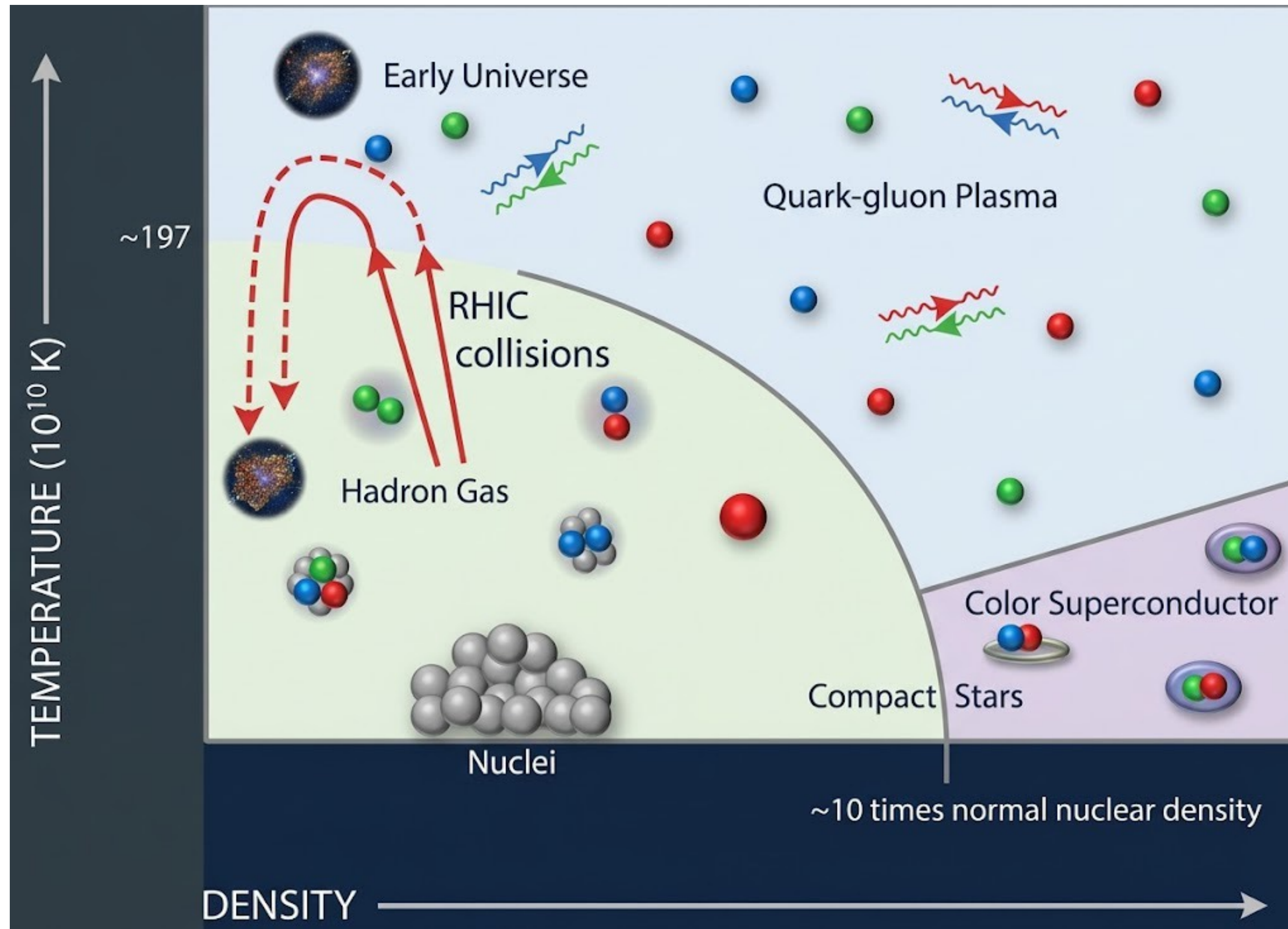


Bulk viscosity effects in compact star mergers: quark matter

Cristina Manuel



NFQCD, Kyoto University October 2026



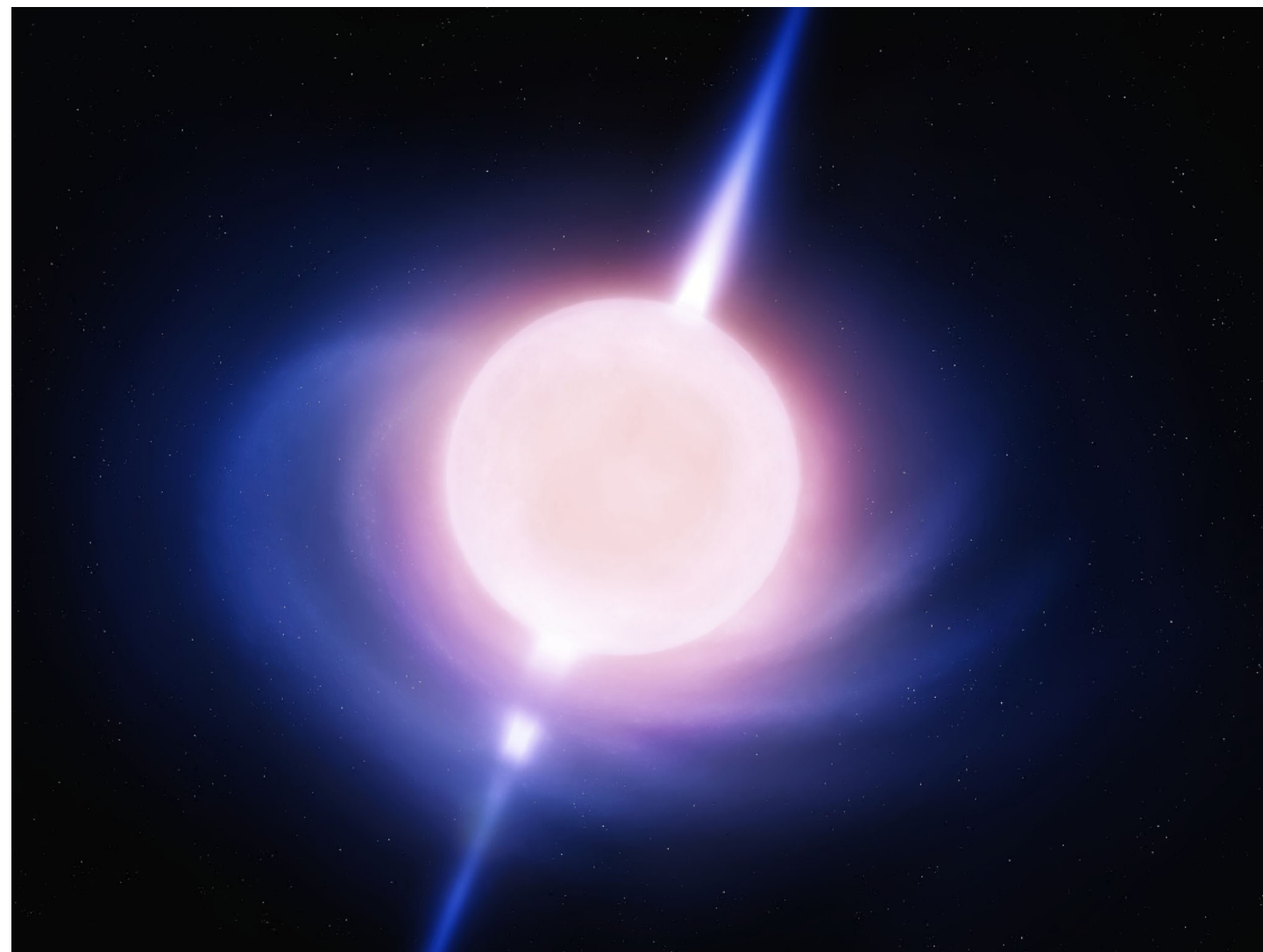
Can we detect deconfined phases at extreme conditions?

OUTLINE

- Introduction and motivation
- Bulk viscosity and quark matter
- Tidal heating in quark stars

based on work with **J.L. Hernandez**, L.Tolos and S. Säppi, S. Ghosh, B. Pradhan,
D. Chatterjee

Neutron stars: remnants of massive stars after supernova explosions



Source: Nazarii Neshcherenskyi / Getty Images

Mass:	$M \sim 1-2 M_{\odot},$
Radius:	$R \sim 10-12 \text{ km}$
Density:	$n \sim 10^{14}-10^{15} \text{ g/cm}^3$
Magnetic field:	$B \sim 10^8 \dots 10^{16} \text{ G}$
Temperature:	$T \sim 10^6 \dots 10^{11} \text{ K}$
Rotational periods:	$P \sim \text{ms} \dots \text{s}$

Observed as: magnetars, radio pulsars, x-ray pulsars, magnetar+pulsar,

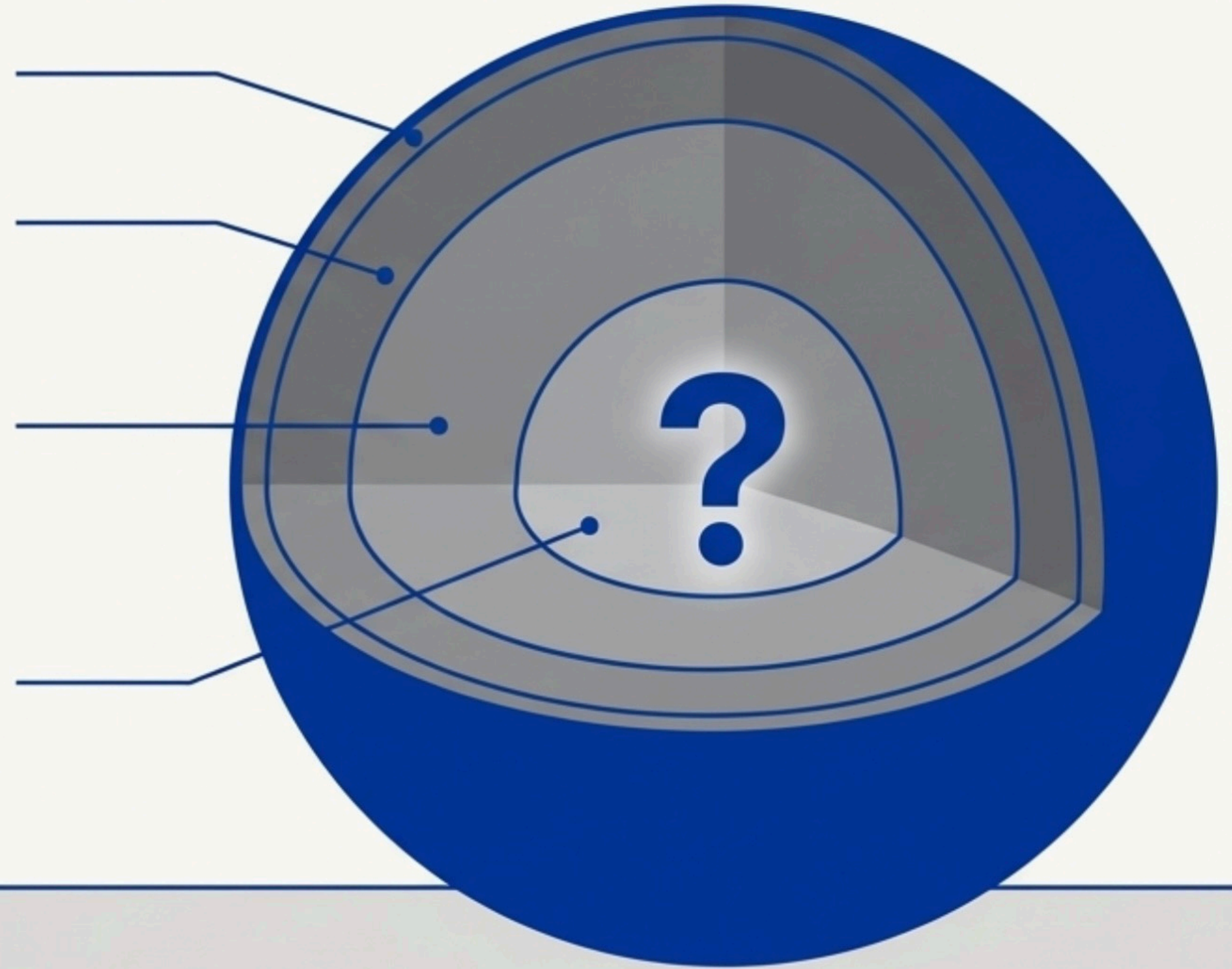
Structure of a Neutron Star

Outer Crust: A solid lattice of atomic nuclei and a sea of free electrons.

Inner Crust: Heavier nuclei, free electrons, and a superfluid of “dripped” neutrons. Exotic “pasta” phases may exist here.

Outer Core: A quantum liquid of superfluid neutrons, superconducting protons, and electrons. Densities are several times that of an atomic nucleus ($\sim 0.5 - 2 \rho_0$).

Inner Core: The ultimate unknown. Densities may reach $10-15 \rho_0$. Is it hyperons? A quark-gluon plasma? A color superconductor?



Central Challenge

- The main puzzle: strong nuclear interactions (QCD) have never been tested at so high densities.
- Most of our predictions are based on models.

Einstein equations + Relativistic Hydrodynamics

M/R in the range 0.1 - 0.3

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu - g^{\mu\nu}$$

} EOS allows to fix M/R (TOV equations)

$$\delta(G_{\mu\nu} - 8\pi T_{\mu\nu}) = 0$$

$$\delta[u^\mu \nabla_\alpha T_\mu^\alpha] = 0$$

$$\delta[P_\rho^\mu \nabla_\alpha T_\mu^\alpha] = 0$$

$$\delta(\nabla_\mu(nu^\mu)) = 0$$

} Oscillation hydrodynamical modes

(p, f, g, r modes)

} Frequency fixed by EOS, lifetime by
transport coefficients

GRAVITATIONAL WAVE ASTRONOMY

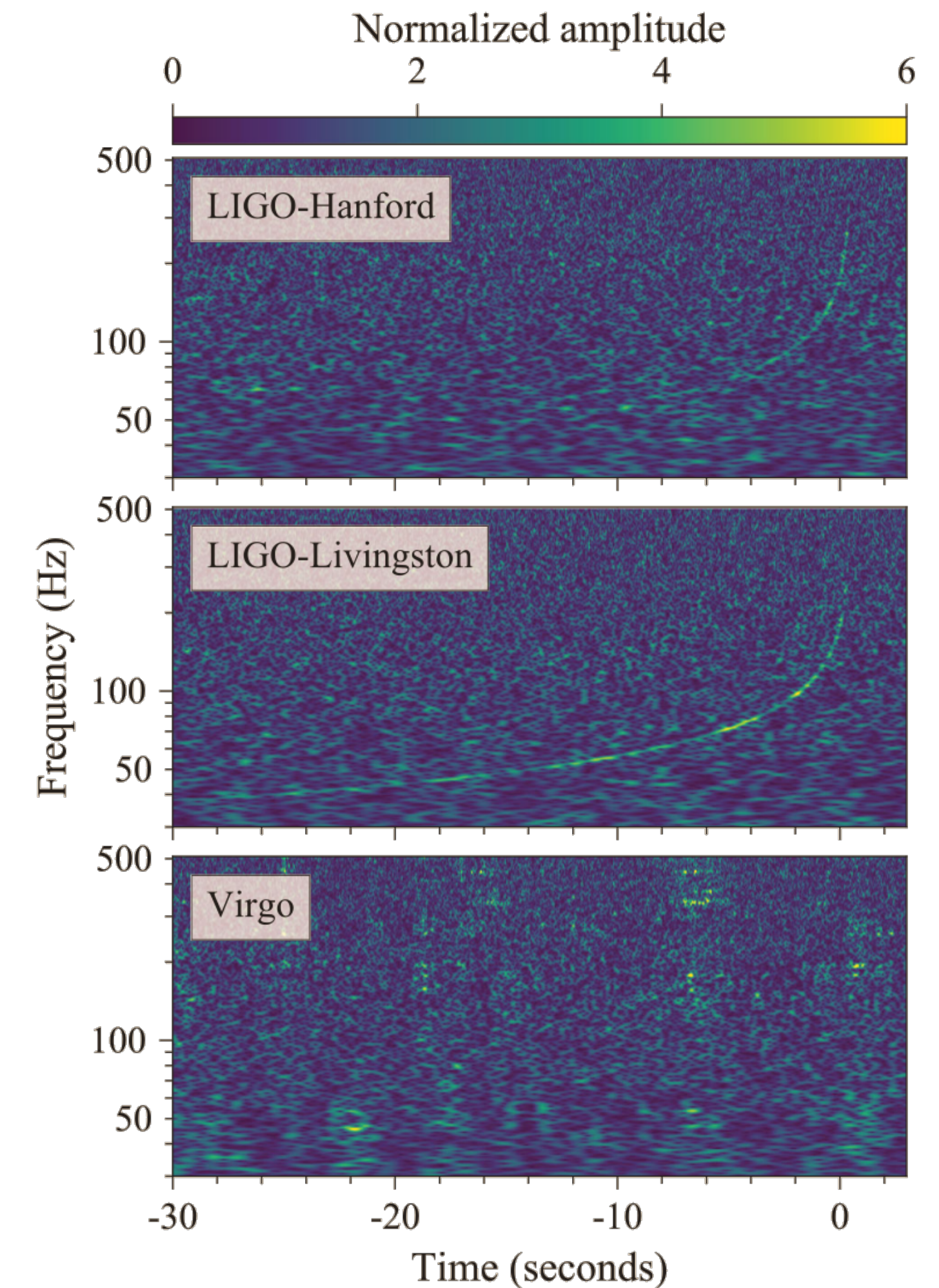
It can provide information on the M/R ratio (and thus EOS) and tidal deformability



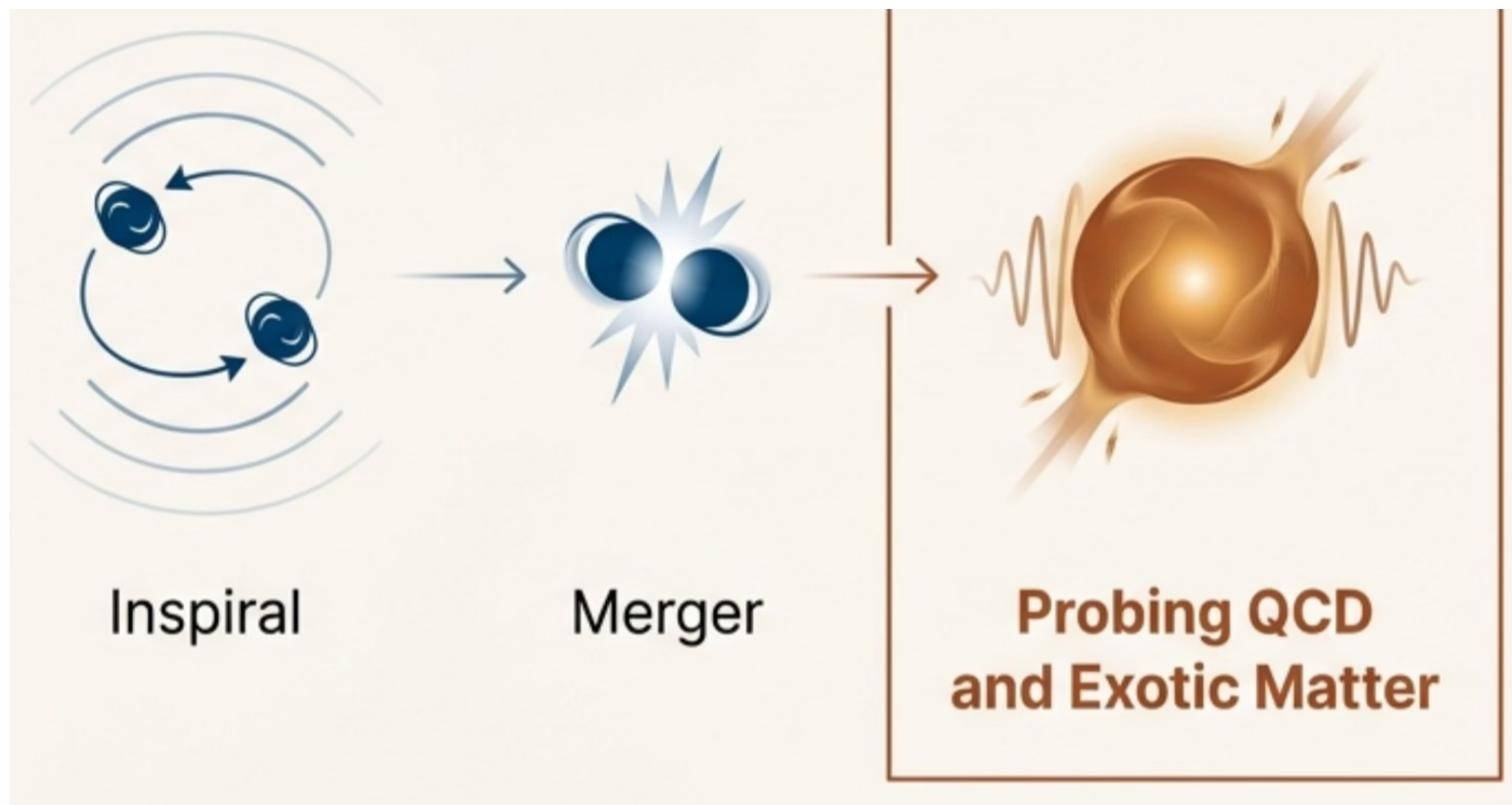
LIGO Scientific Collaboration and Virgo Collaboration

GW170817 first detection of a neutron
star merger

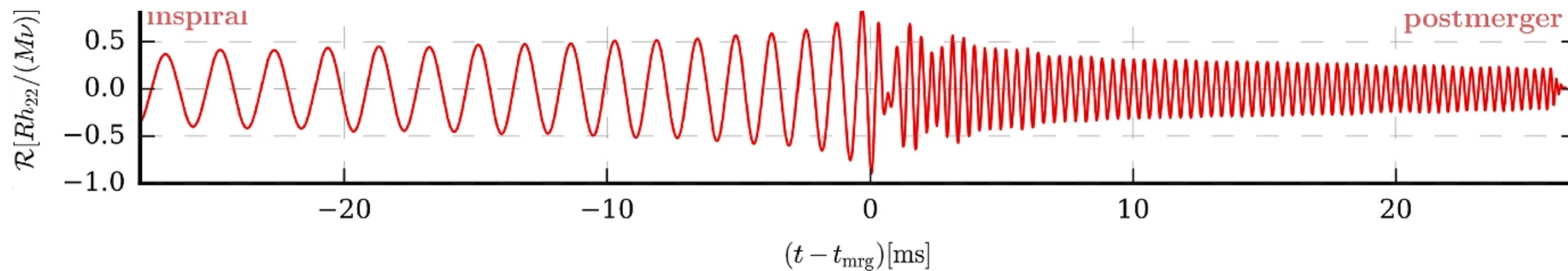
GW190425



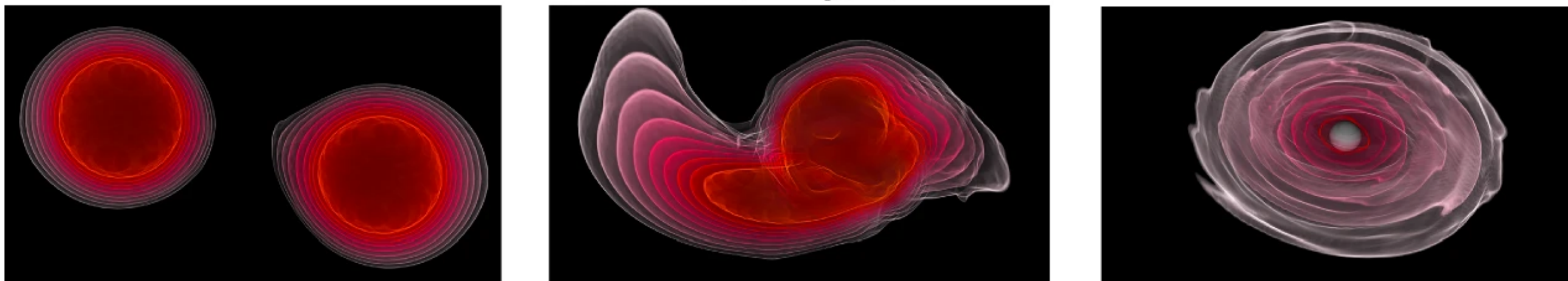
Can it provide information on transport properties of NS?



Numerical simulations emitted GW



Relevant times scales



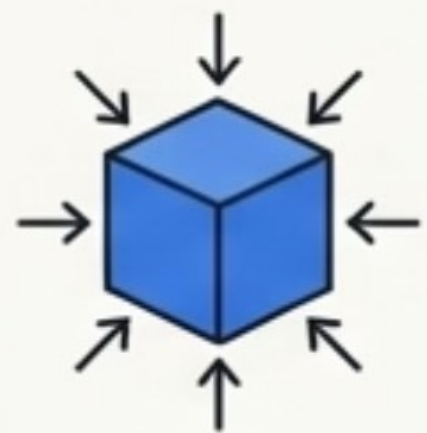
Estimates associated to shear viscosity, thermal conductivity and bulk viscosity, for nucleonic matter, and assuming neutrinos are not trapped for nuclear matter

Alford et al, Phys. Rev. Lett. **120**, 041101

$$\tau_{\eta}^e \approx 1.6 \cdot 10^8 \text{s} \left(\frac{z_{\text{typ}}}{1 \text{ km}} \right)^2 \left(\frac{T}{1 \text{ MeV}} \right)^{5/3} \left(\frac{0.1}{x_p} \right)^{14/9} \left(\frac{n_0}{n_B} \right)^{5/9}$$

$$\tau_{\kappa}^e \approx 5 \cdot 10^8 \text{s} \left(\frac{0.1}{x_p} \right)^{2/3} \left(\frac{m_n^*}{m_n} \right) \left(\frac{n_0}{n_B} \right)^{1/3} \left(\frac{z_{\text{typ}}}{1 \text{ km}} \right) \left(\frac{T}{1 \text{ MeV}} \right)$$

$$\tau_{\zeta}^m \approx 3 \text{ ms} \left(\frac{\tau_{\text{dens}}}{1 \text{ ms}} \right) \left(\frac{K}{250 \text{ MeV}} \right) \left(\frac{0.25 \text{ MeV}}{Y_{\zeta}} \right)$$



Bulk Viscosity (ζ): The fluid's resistance to compression or expansion. It's crucial during the violent oscillations of a merger.

RELATIVISTIC VISCOUS HYDRODYNAMICS

$$\Pi = P - P_0$$

Bulk viscous pressure

Second order transport hydrodynamics

$$\tau D_u \Pi + \Pi = -\zeta_0 \vartheta$$

$$D_u \equiv u^\mu \nabla_\mu$$

$$\vartheta \equiv \nabla_\mu u^\mu$$

Israel-Stewart (IS) equation

For harmonic oscillations, it is equivalent to consider a frequency-dependent bulk viscosity

$$\Pi \propto e^{-i\omega t}$$

$$\zeta(\omega) = -\frac{\text{Re}\Pi}{v\theta}$$

Effective frequency-dependent viscosity

Resonant behaviour

$$\zeta(\omega) = \frac{\zeta_0}{1 + \tau^2 \omega^2}$$


$$\omega\tau \ll 1$$

Navier-Stokes regime recovered

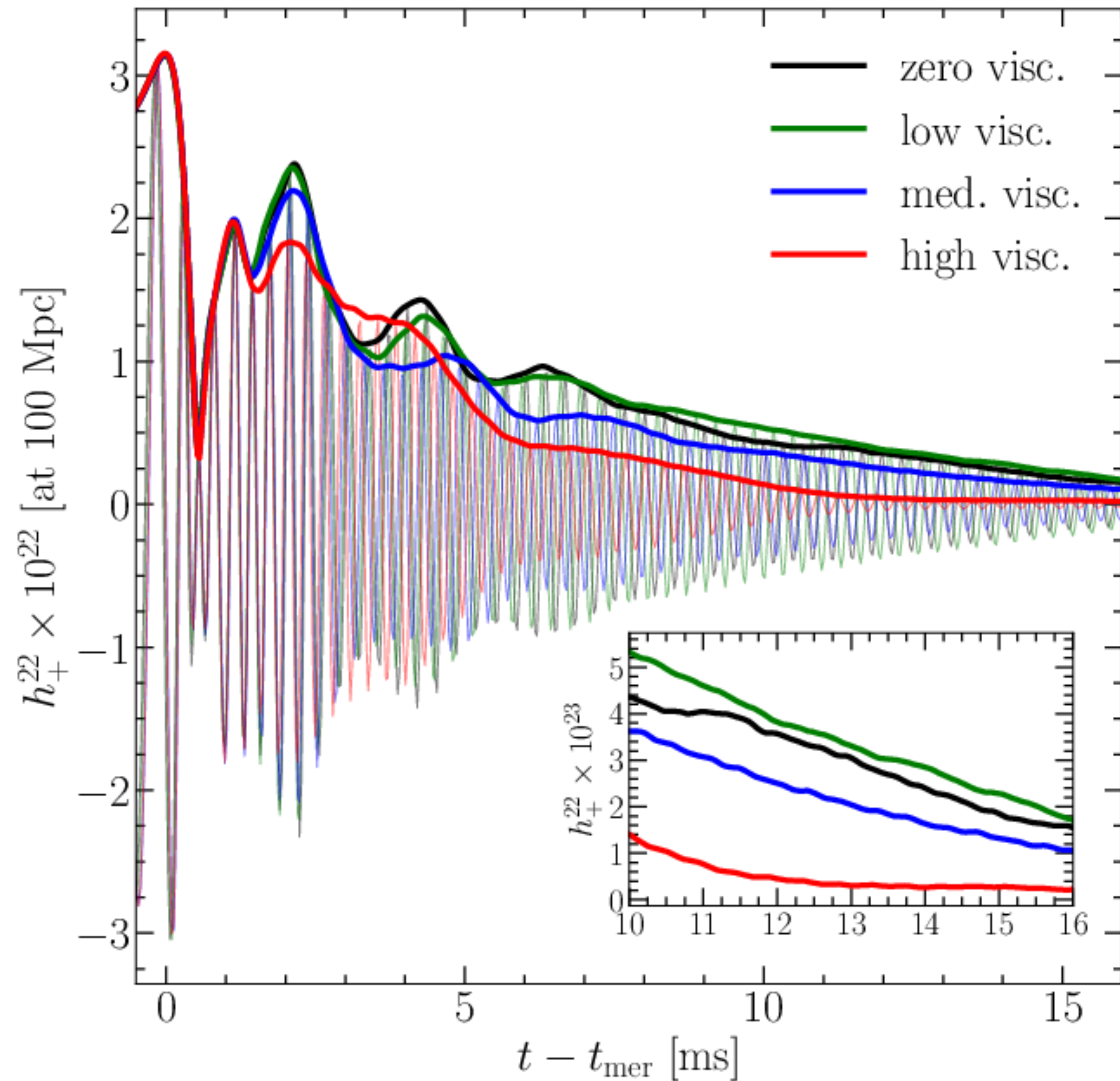
$$\omega\tau \sim 1$$

Maximum dissipation

$$\omega\tau \gg 1$$

Frozen: no dissipation

First attempts to include the effects of the bulk viscosity

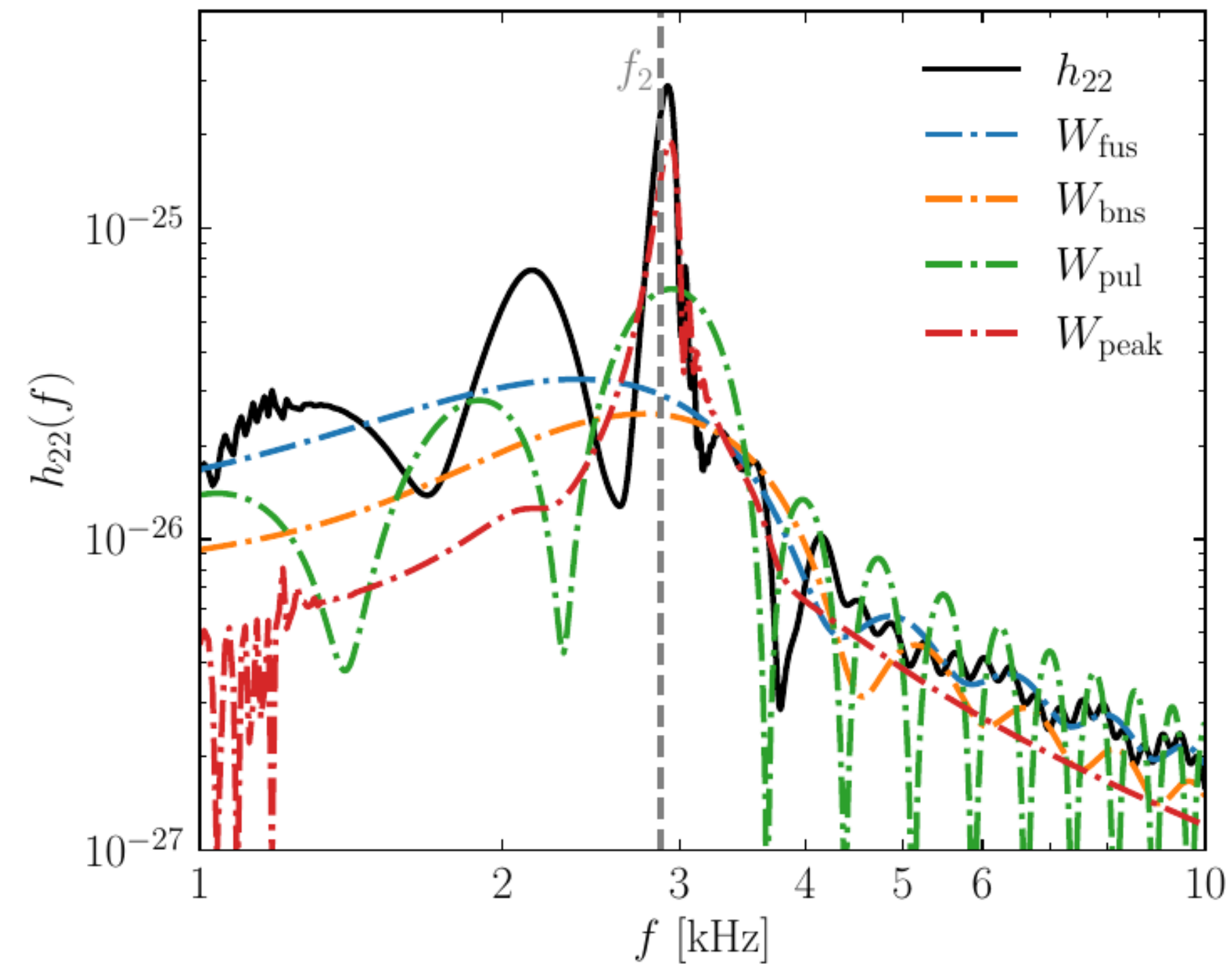


$$\zeta \in \zeta_0[10^{-4}, 0.4, 0.1, 2.4]$$

$$\zeta_0 = 10^{30} \text{ g cm}^{-1} \text{ s}^{-1}$$

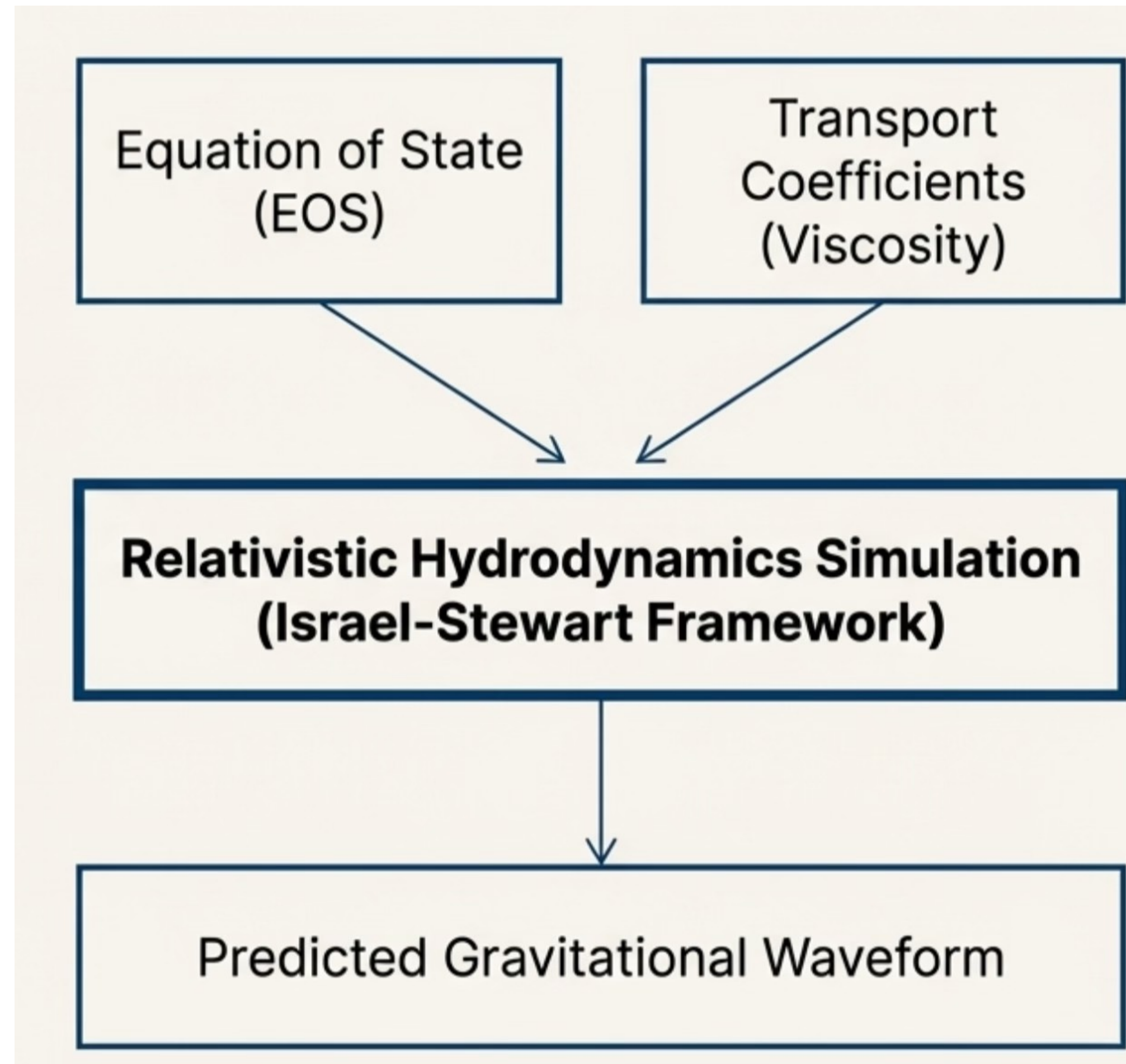
Viscosity would provide a quickly damped signal

Post-merger GW, neutron matter



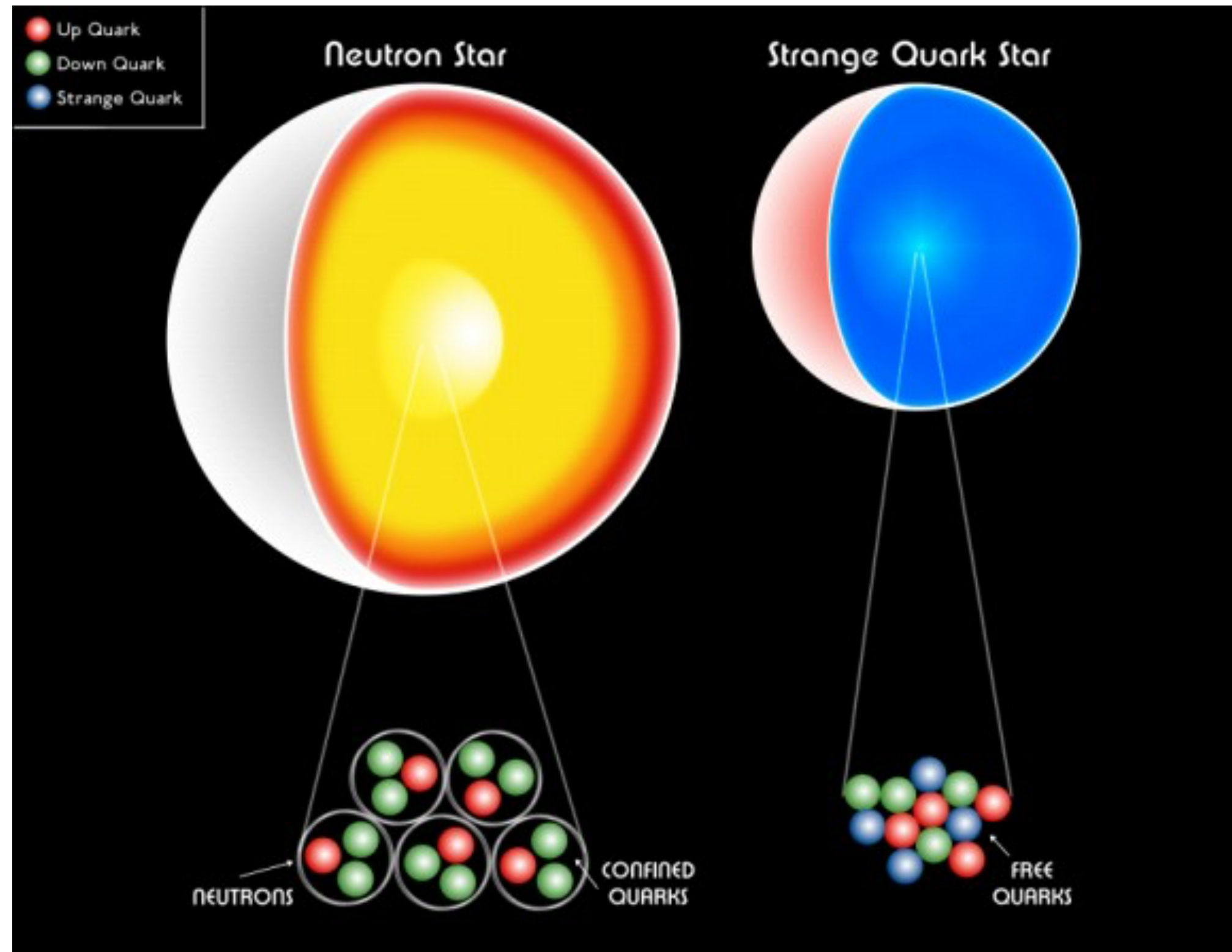
BV runs	$\log_{10}(\zeta_{\text{AC}}^{\text{max}}(4\rho_{\text{sat}}))$	f_2 [kHz]	δf_2 [Hz]
zero BV	28.808	3.036	—
low BV	30.466	3.056	20.260
medium BV	30.893	3.087	51.520
high BV	31.274	3.126	90.960

From microphysics to GW (post merger)



Strange quark matter

assuming ungapped phase



$$n = \frac{1}{3} (n_u + n_d + n_s)$$

u, d, s quarks - need of e for charge neutrality

$$\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s - n_e = 0$$

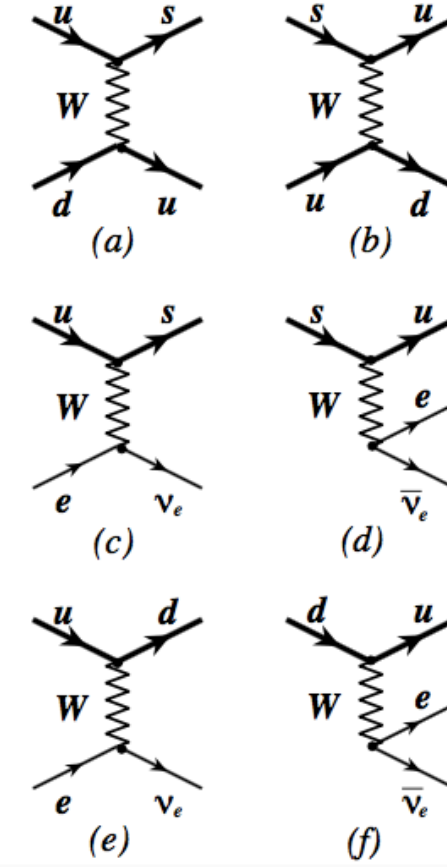
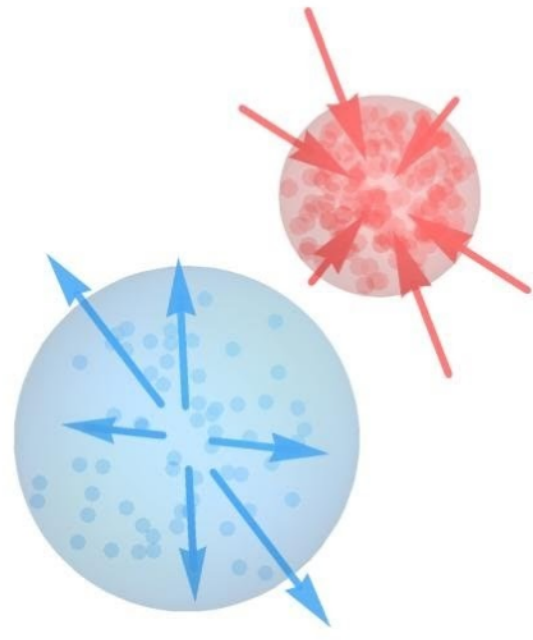
Beta equilibrium conditions

$$\mu_s = \mu_d = \mu_u + \mu_e$$

At very low temperatures, system characterised by different (constrained)

quark chemical potential -assumed massless, except for the strange quark

Out-of equilibrium: electroweak interactions restore the equilibrium



$$\mu_s \neq \mu_d, \quad \mu_s \neq \mu_u + \mu_e$$

$$\mu_1 \equiv \mu_s - \mu_d,$$

$$\mu_2 \equiv \mu_u - \mu_d + \mu_e$$

Nonleptonic

$$u + s \leftrightarrow u + d$$

$$\mu_1 \lambda_1 = \Gamma_{u+s \rightarrow u+d} - \Gamma_{u+d \rightarrow u+s}$$

$$(\mu_2 - \mu_1) \lambda_3 = \Gamma_{s \rightarrow u+e+\bar{\nu}_e} - \Gamma_{u+e \rightarrow s+\nu_e}$$

$$-\mu_2 \lambda_2 = \Gamma_{d \rightarrow u+e+\bar{\nu}_e} - \Gamma_{u+e \rightarrow d+\nu_e}$$

SemiLeptonic

$$u + e^- \rightarrow d + \nu_e$$

$$d \rightarrow u + e^- + \bar{\nu}_e$$

$$u + e^- \rightarrow s + \nu_e$$

$$s \rightarrow u + e^- + \bar{\nu}_e$$

$$\lambda_1 \approx \frac{64}{5\pi^3} G_F^2 \sin^2 \Theta_C \cos^2 \Theta_C \mu_d^5 T^2$$

$$\lambda_2 \approx \frac{17}{40\pi} G_F^2 \sin^2 \Theta_C \mu_s m_s^2 T^4$$

$$\lambda_3 \approx \frac{17}{15\pi^2} G_F^2 \cos^2 \Theta_C \alpha_s \mu_d \mu_u \mu_e T^4$$

Iwamoto, '80

Bulk viscosity in strange quark matter

Frequency dependent oscillations

$$\Pi \propto e^{-i\omega t}$$

$$\frac{dn_s}{dt} \approx -\mu_1 \lambda_1 - (\mu_2 - \mu_1) \lambda_3$$

$$\frac{dn_e}{dt} \approx -\mu_2 \lambda_2 + (\mu_2 - \mu_1) \lambda_3$$

$$\Pi \equiv P - P_{\text{eq}}$$

$$\zeta = \frac{\kappa_1 + \kappa_2 \omega^2}{\kappa_3 + \kappa_4 \omega^2 + \omega^4}$$

$$\kappa_i = \kappa_i \left(\mu_a, n_B, \lambda_1(T), \lambda_2(T), \lambda_3(T), (\chi^{-1})_a^a \right)$$

$$(\chi^{-1})_a^b = (\partial \mu_a / \partial n_b)_{n_{c \neq b}, T}$$

Burgers equation for the quark viscous pressure

$$\Pi \equiv P - P_{\text{eq}}$$

In nuclear matter the bulk viscous pressure obeys a IS equation

$$\tau D_u \Pi + \Pi = -\zeta_0 \vartheta \qquad D_u \equiv u^\mu \nabla_\mu$$

Quark matter obeys a Burgers type equation

$$\Pi = \Pi_1 \mu_1 + \Pi_2 \mu_2 \qquad \mu_1 \equiv \mu_s - \mu_d, \qquad \mu_2 \equiv \mu_u - \mu_d + \mu_e$$

$$\det \mathcal{T} D_u^2 \Pi + \text{Tr} \mathcal{T} D_u \Pi + \Pi = -\zeta \theta - \xi D_u \theta$$

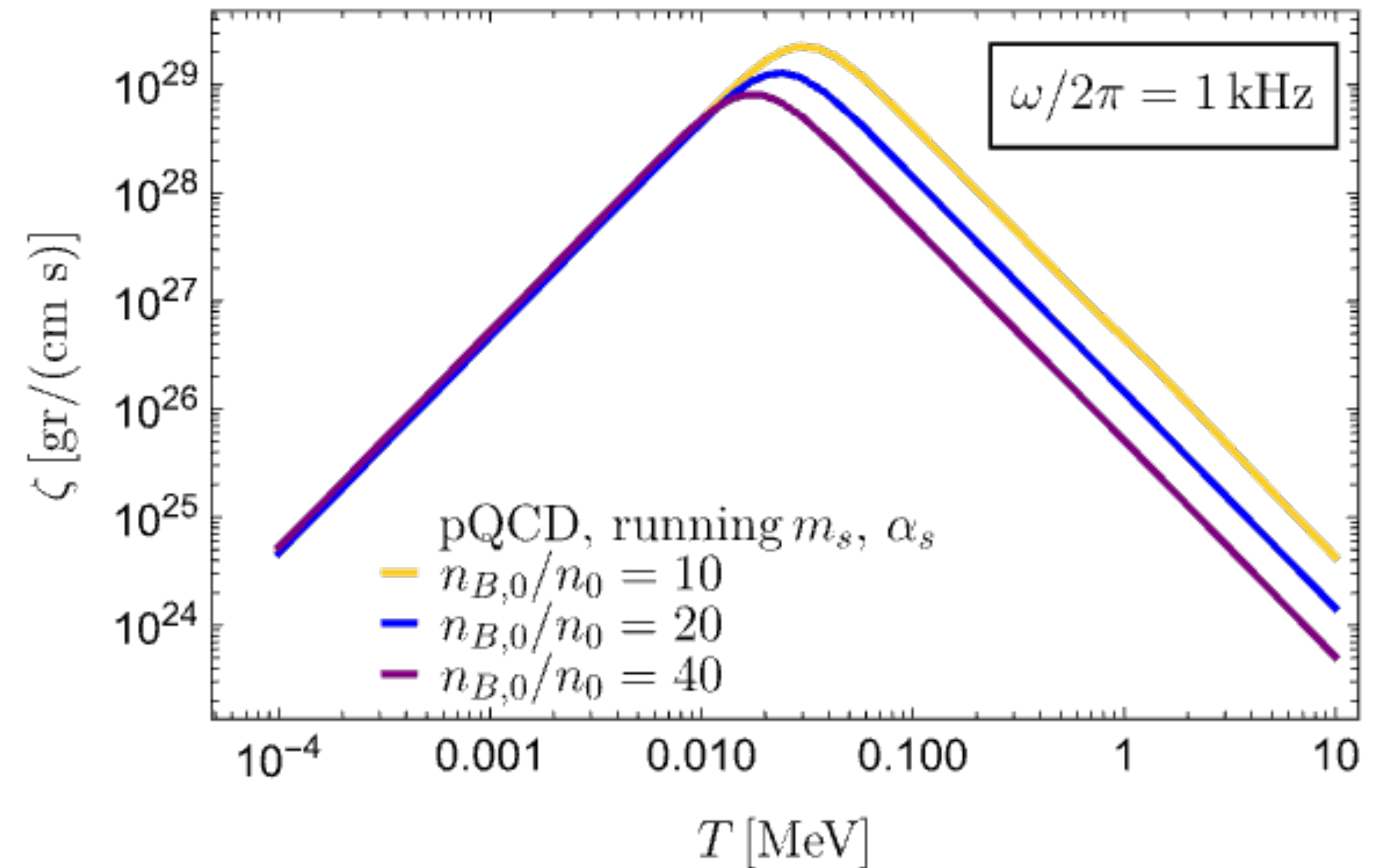
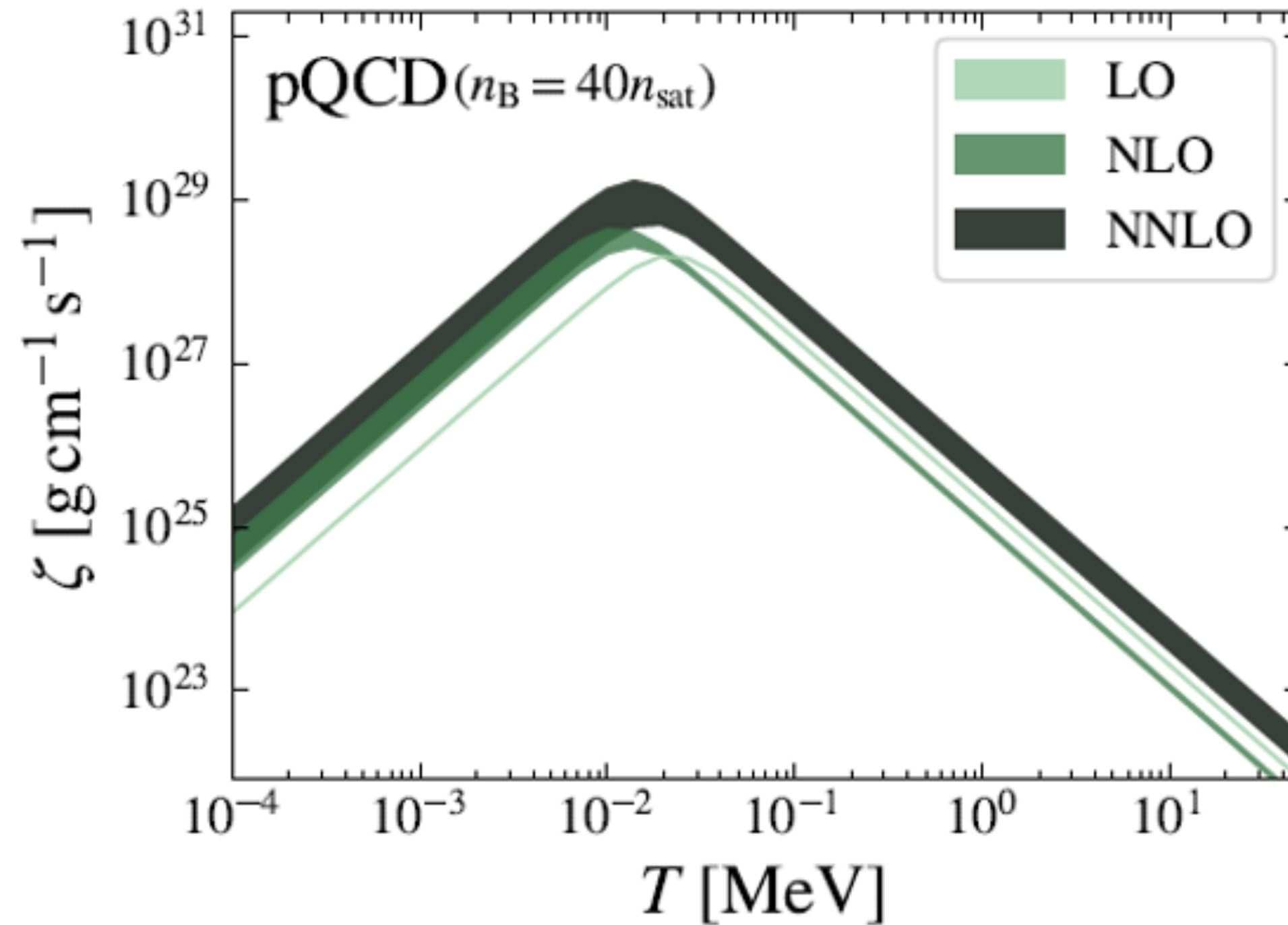
It is equivalent to two IS equations with partial viscosities

$$(\tau_+, \zeta_+, \tau_-, \zeta_-) \qquad \zeta_+ = \frac{\zeta \tau_+ - \xi}{\tau_+ - \tau_-}, \qquad \zeta_- = \frac{\zeta \tau_- - \xi}{\tau_- - \tau_+}.$$

$$\zeta(\omega) = \frac{\zeta_+}{1 + \tau_+^2 \omega^2} + \frac{\zeta_-}{1 + \tau_-^2 \omega^2}$$

Two possible resonant peaks

Bulk viscosity in perturbative QCD

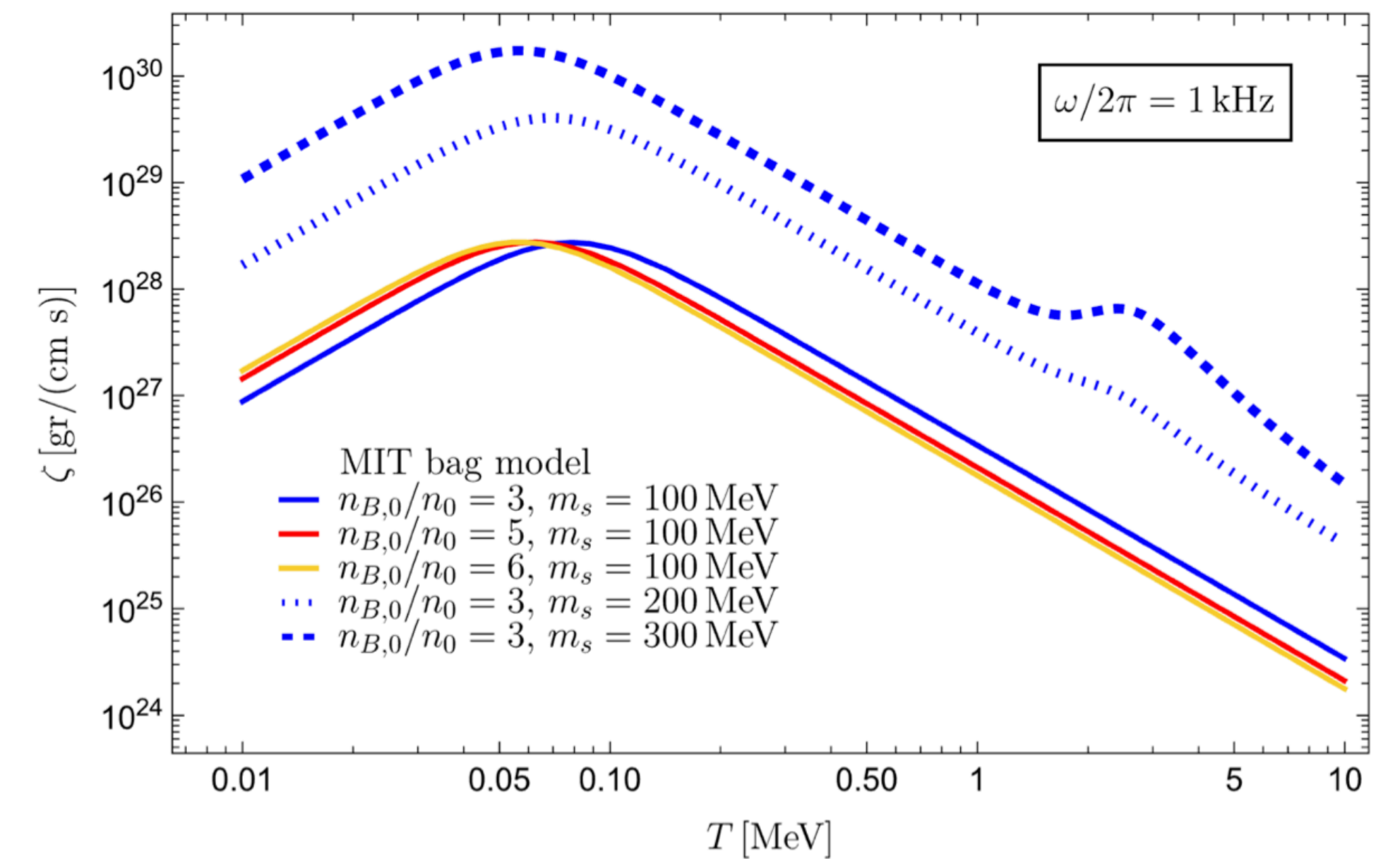
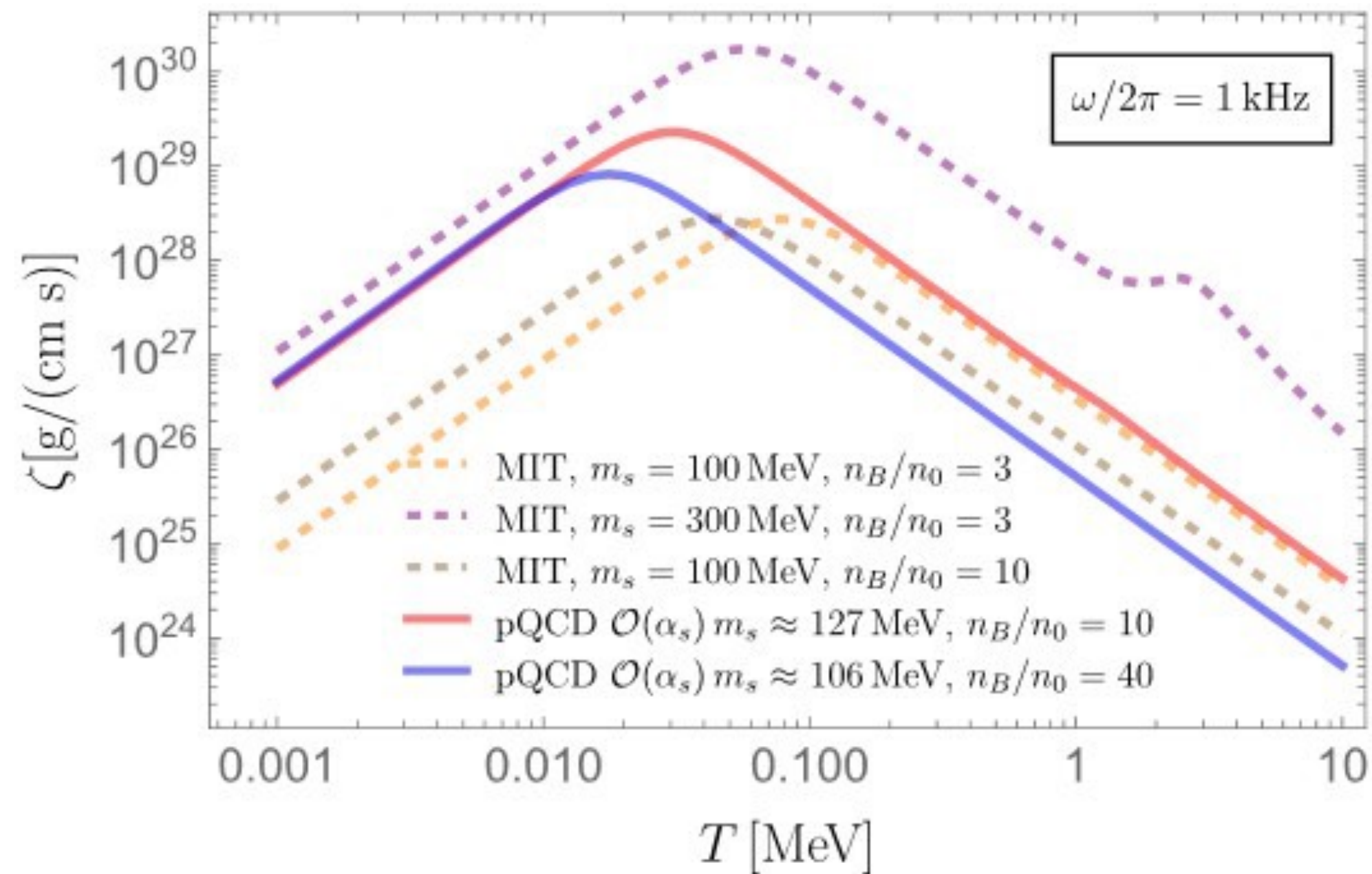


¹Rojas, Gorda, Hoyos, Jokela, Järvinen, Kurkela, Paatelainen, Säppi, Vuorinen, 2024, arXiv:2402.00621
Hernandez, Manuel, Tolos, arXiv: 2402.06595

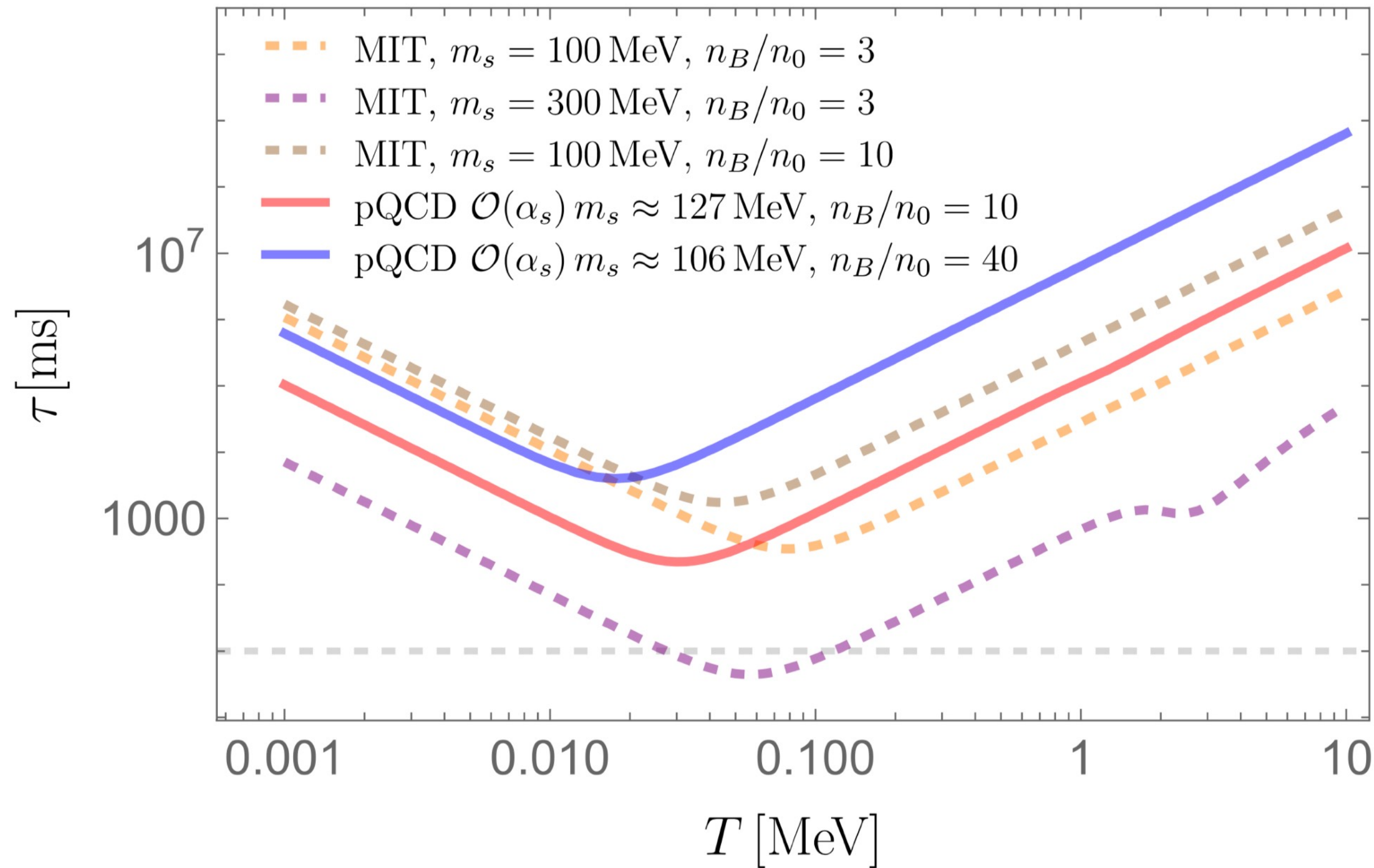
Strange quark mass is not a free parameter

Bulk viscosity MIT bag model

Strange quark mass as a parameter



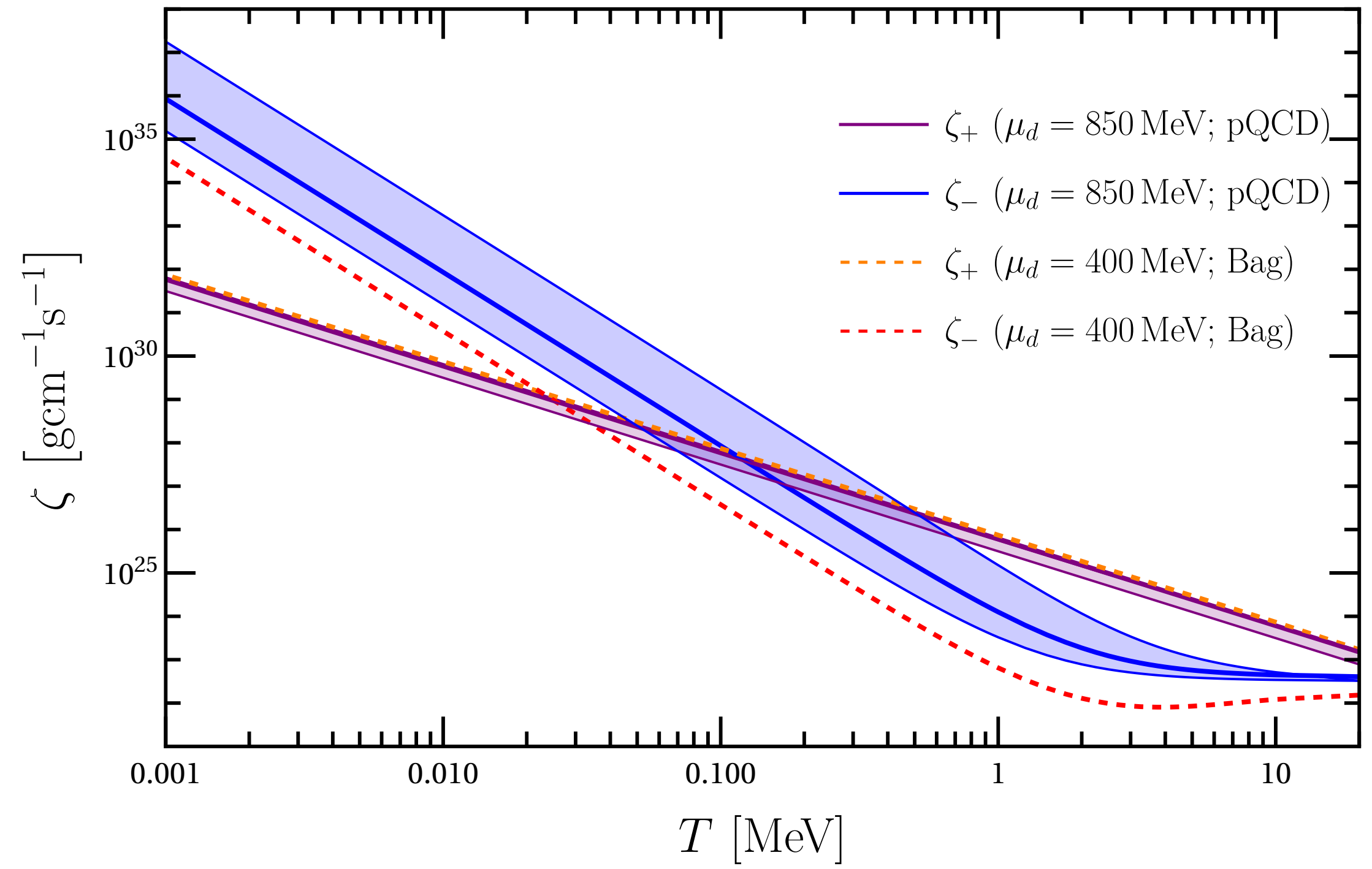
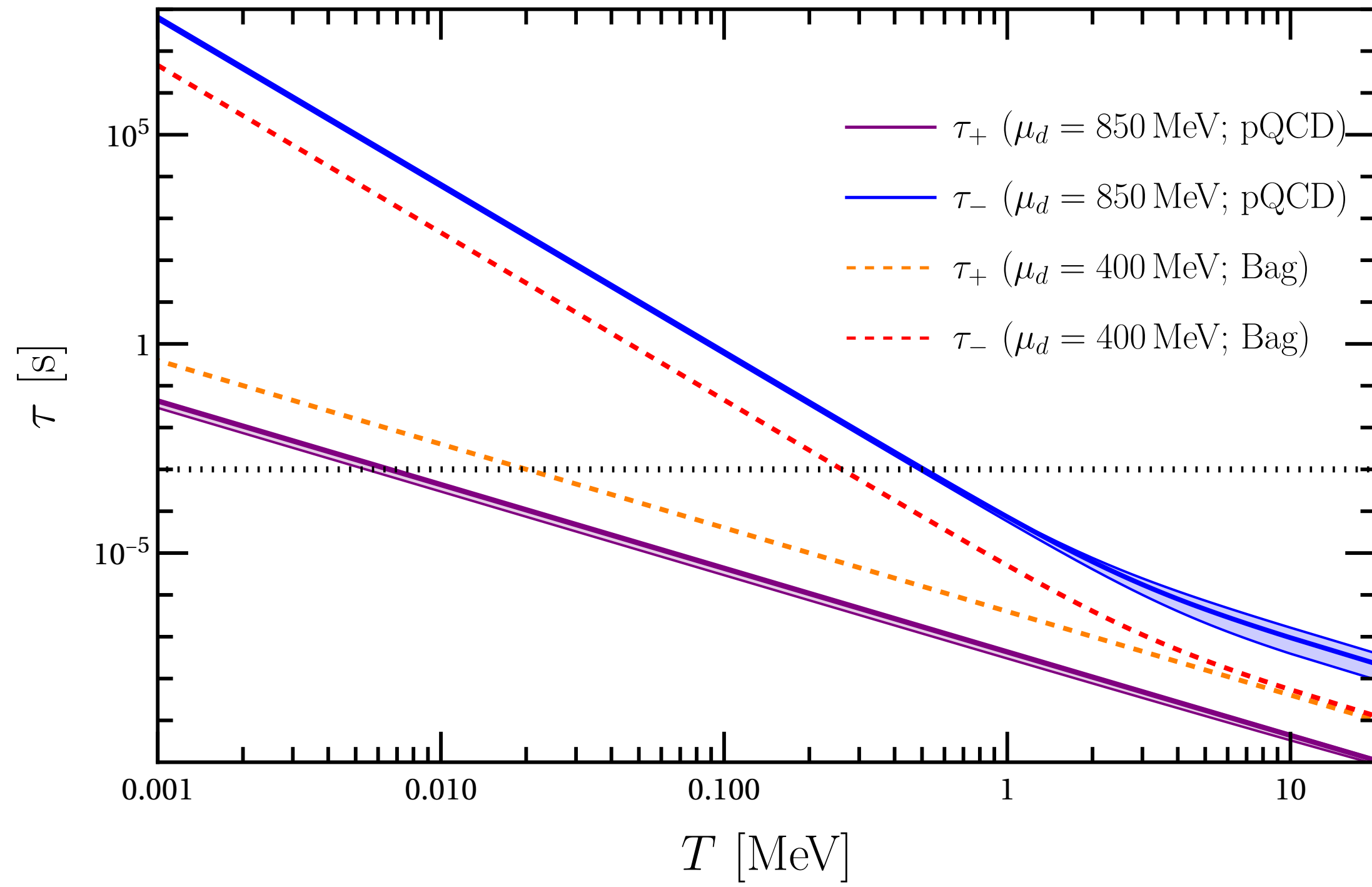
Damping of density oscillations



$$\tau = \frac{n_B^2}{\omega^2 \zeta} \left. \frac{\partial^2 \varepsilon}{\partial n_B^2} \right|_{n_s, n_e}$$

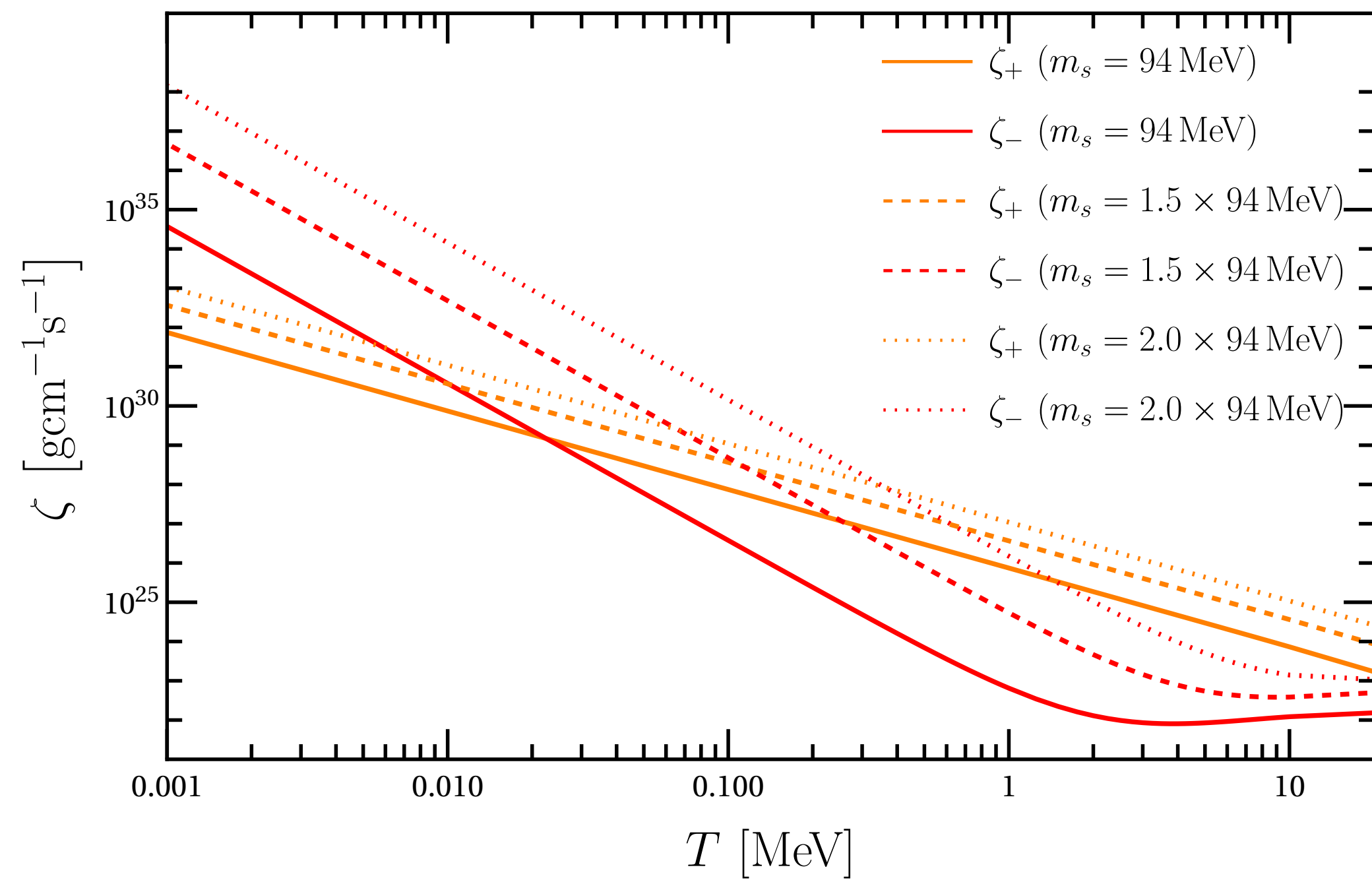
$\tau_{\min}$ reaching few ms at $T \in [0.01, 0.1]$ MeV

$n_{B,0}/n_0$	T_m [MeV]	$\zeta_{\max}$ [gr/(cm·s)]	$\tau_{\min}$ [ms]	m_s [MeV]
3	7.9×10^{-2}	2.73×10^{27}	213.24	100
3	6.9×10^{-2}	4.05×10^{29}	13.93	200
3	5.7×10^{-2}	1.73×10^{30}	3.19	300
5	6.2×10^{-2}	2.75×10^{28}	420.14	100
6	5.7×10^{-2}	2.76×10^{28}	535.37	100



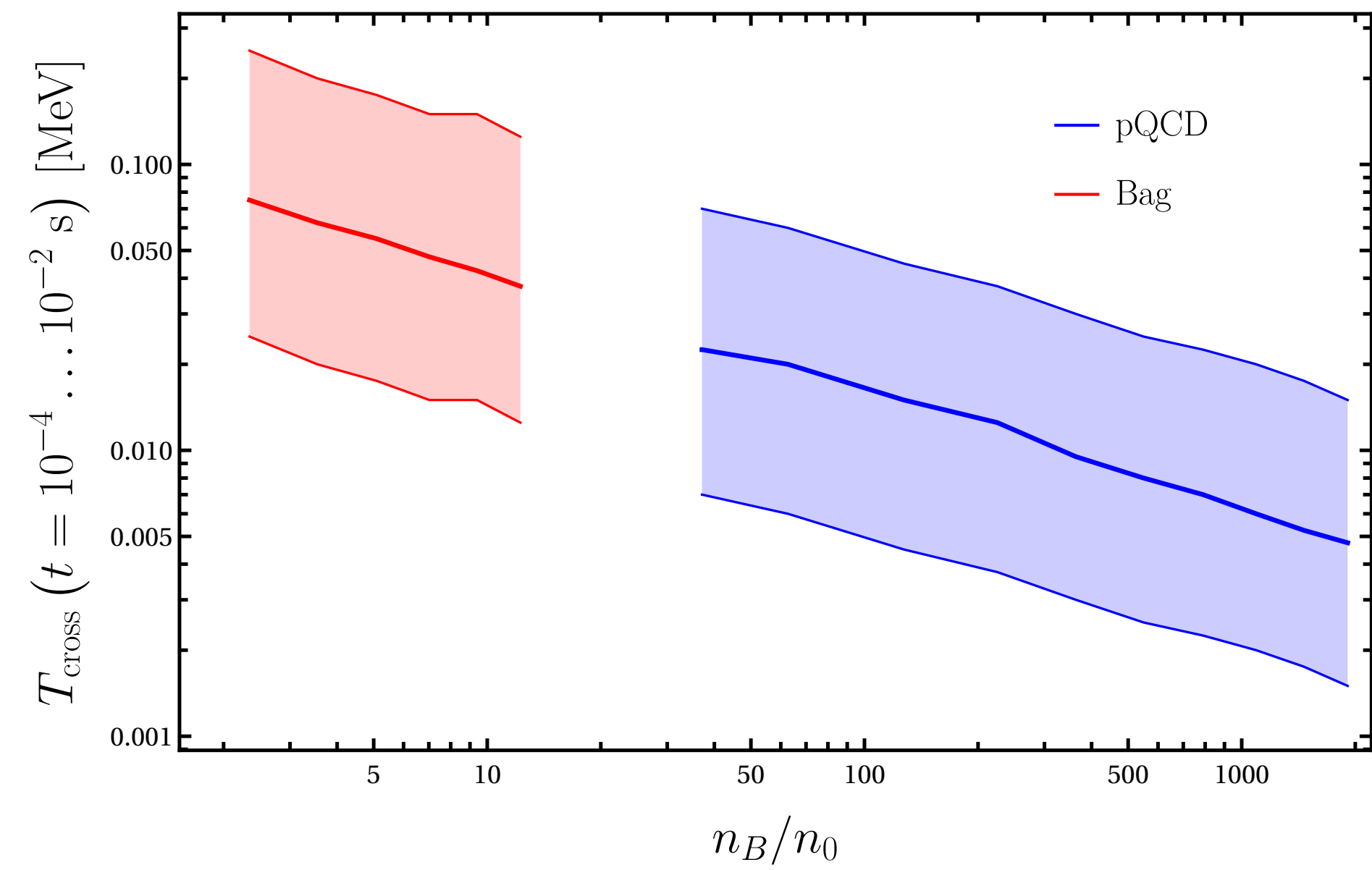
Our analysis shows that for $T \sim 1$ MeV and above a Burgers equation should be used

Neutrinos trapped around $T \sim 10$ MeV



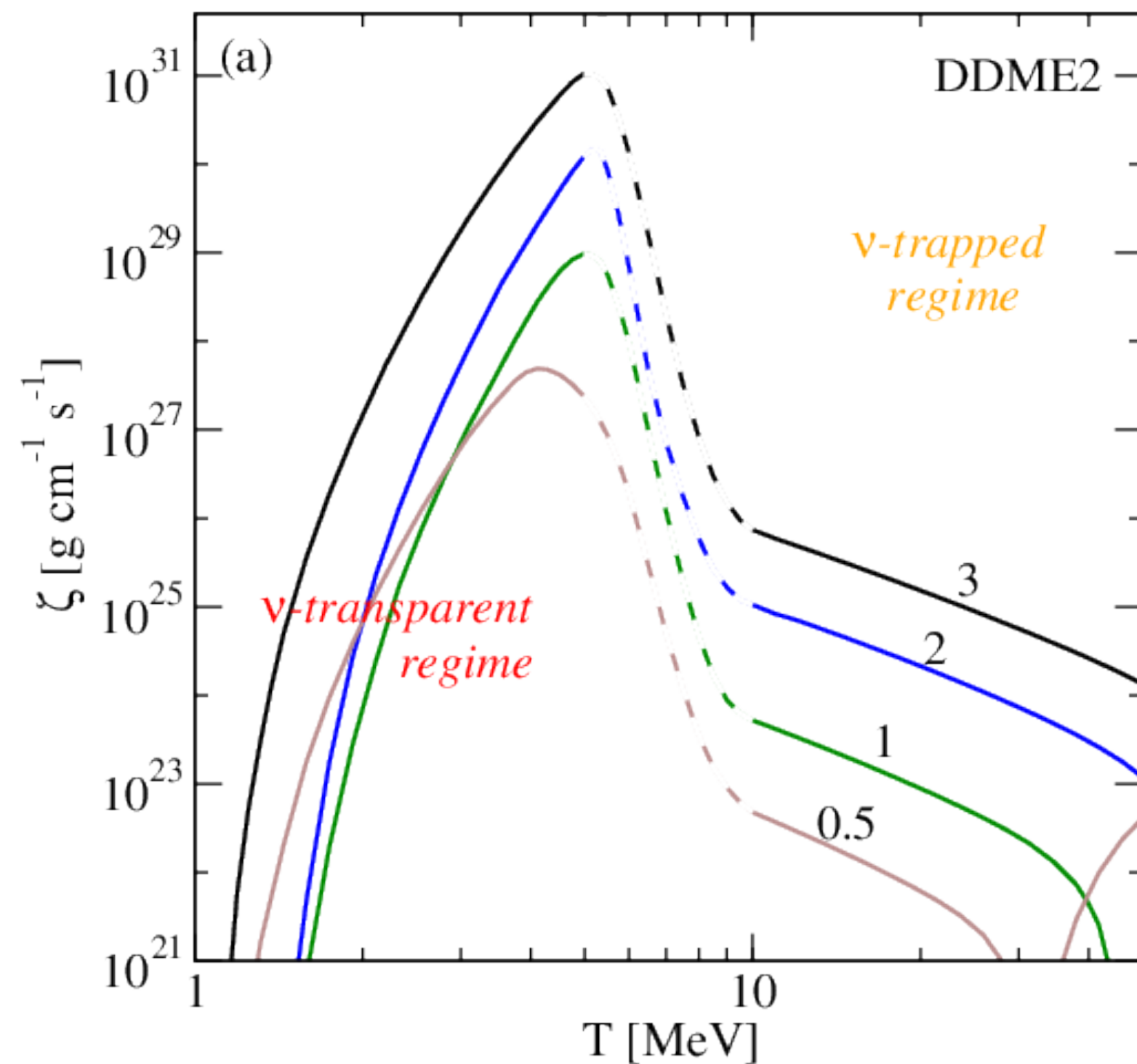
$$\zeta_+ \propto m_s^2$$

$$\zeta_- \propto m_s^5$$

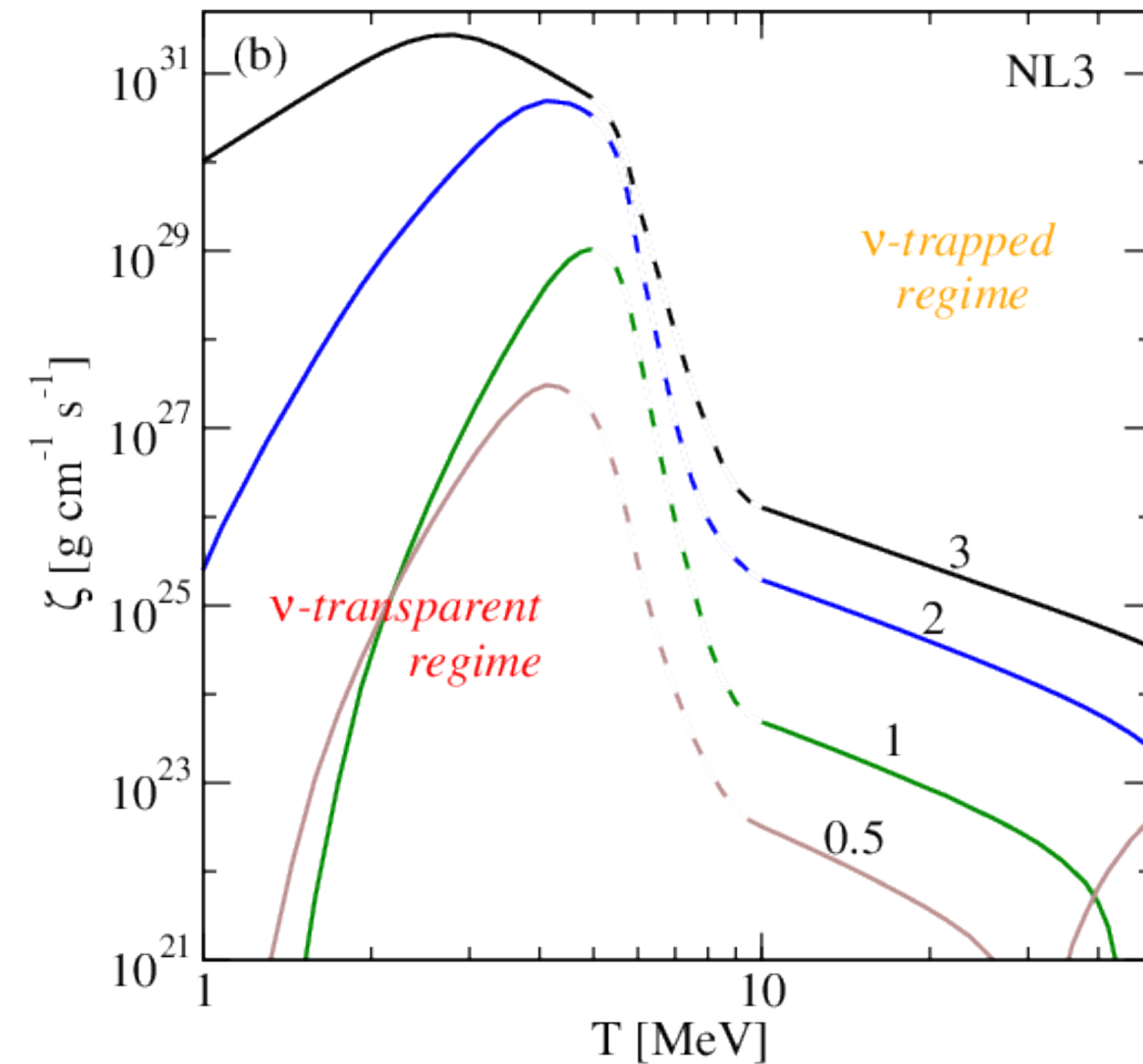


Bulk viscosity in nuclear matter

For two different models



Frequency 1 kHz



Alford, Harutyunyan, and Sedrakian, 2006.07975

Differences critical values of density for the opening of dURCA (5 n₀ vs 2.5 n₀)

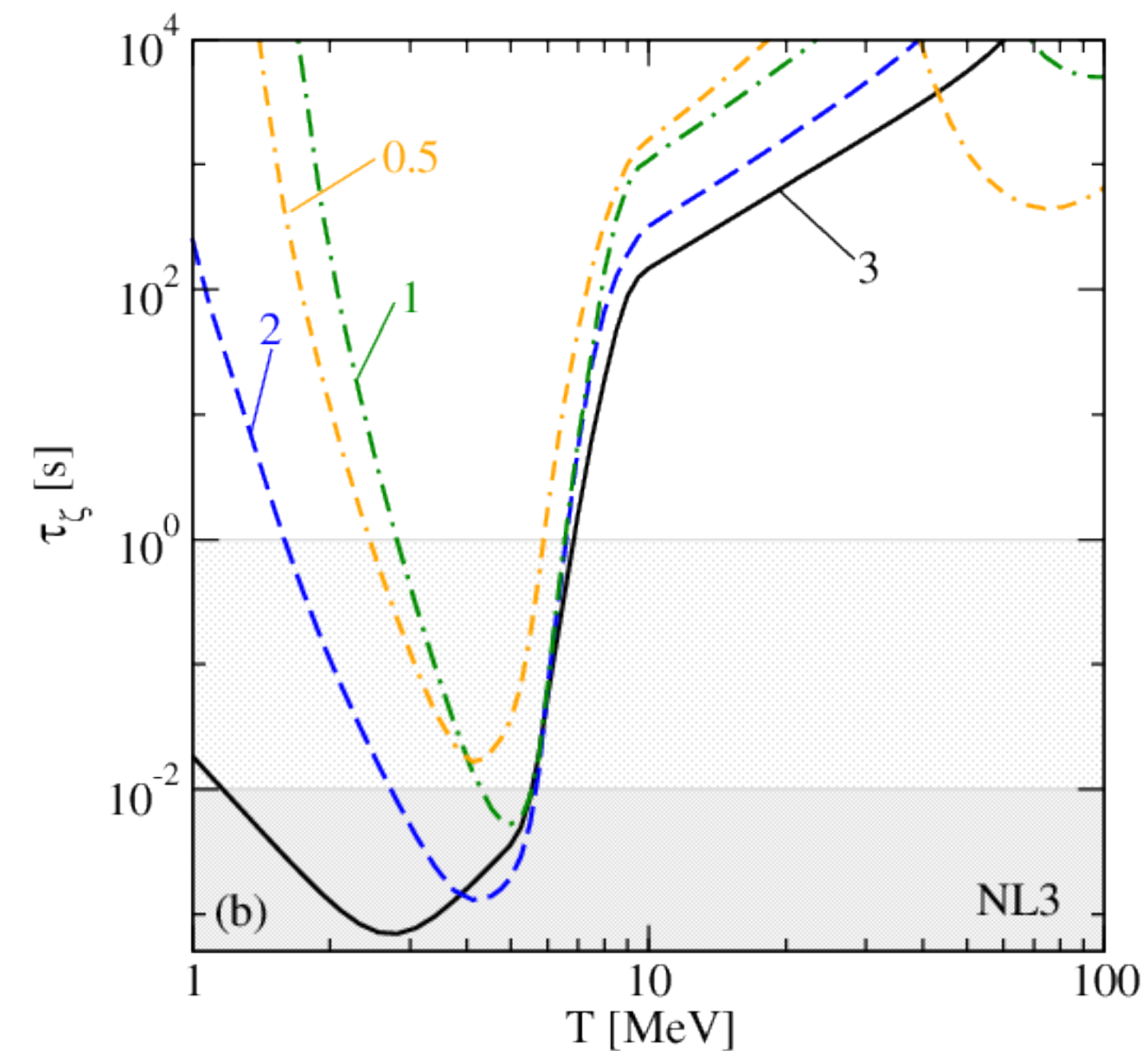
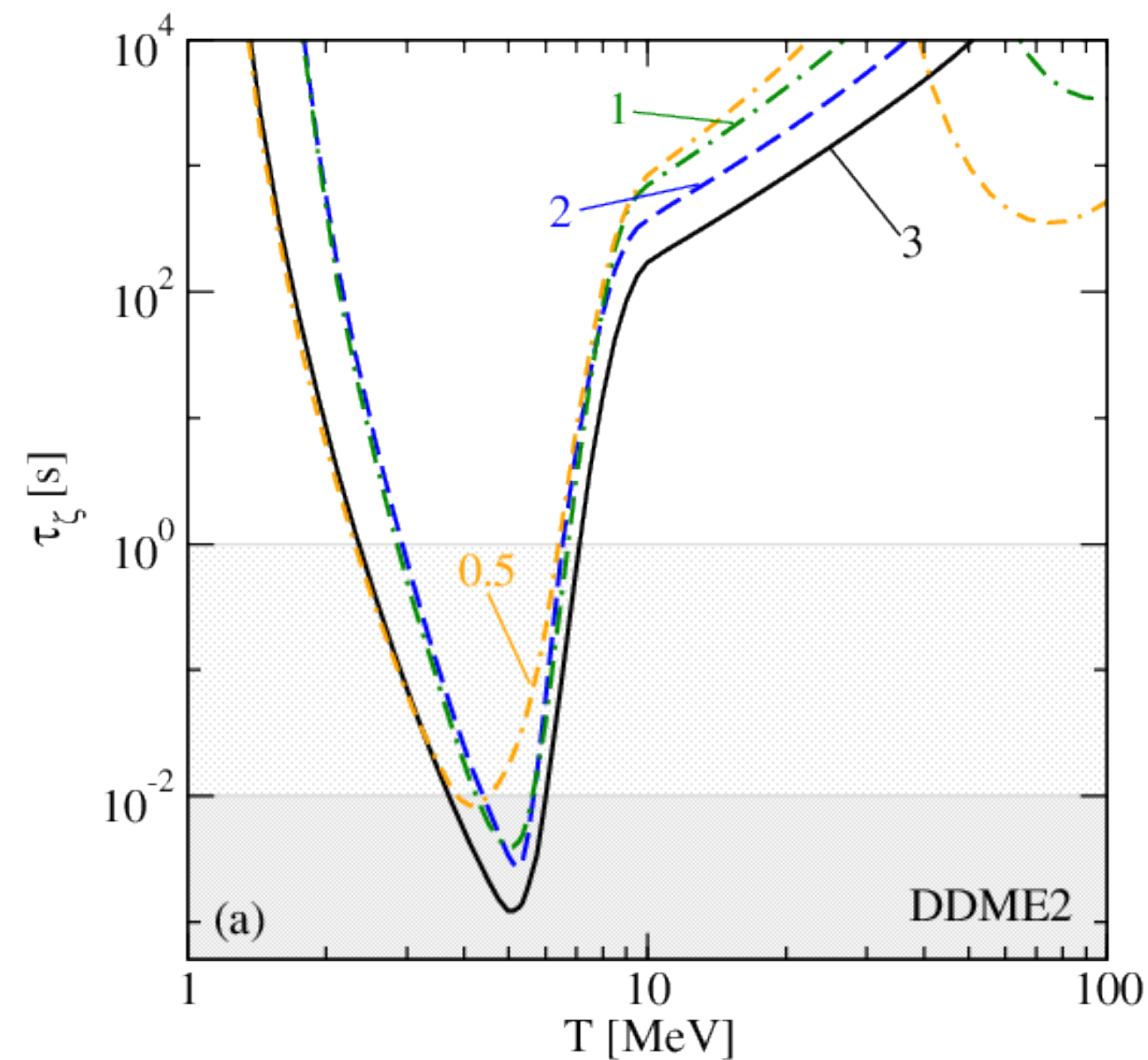
Damping of density oscillations

$$n(t) = \bar{n} + \Delta n \cos(\omega t)$$

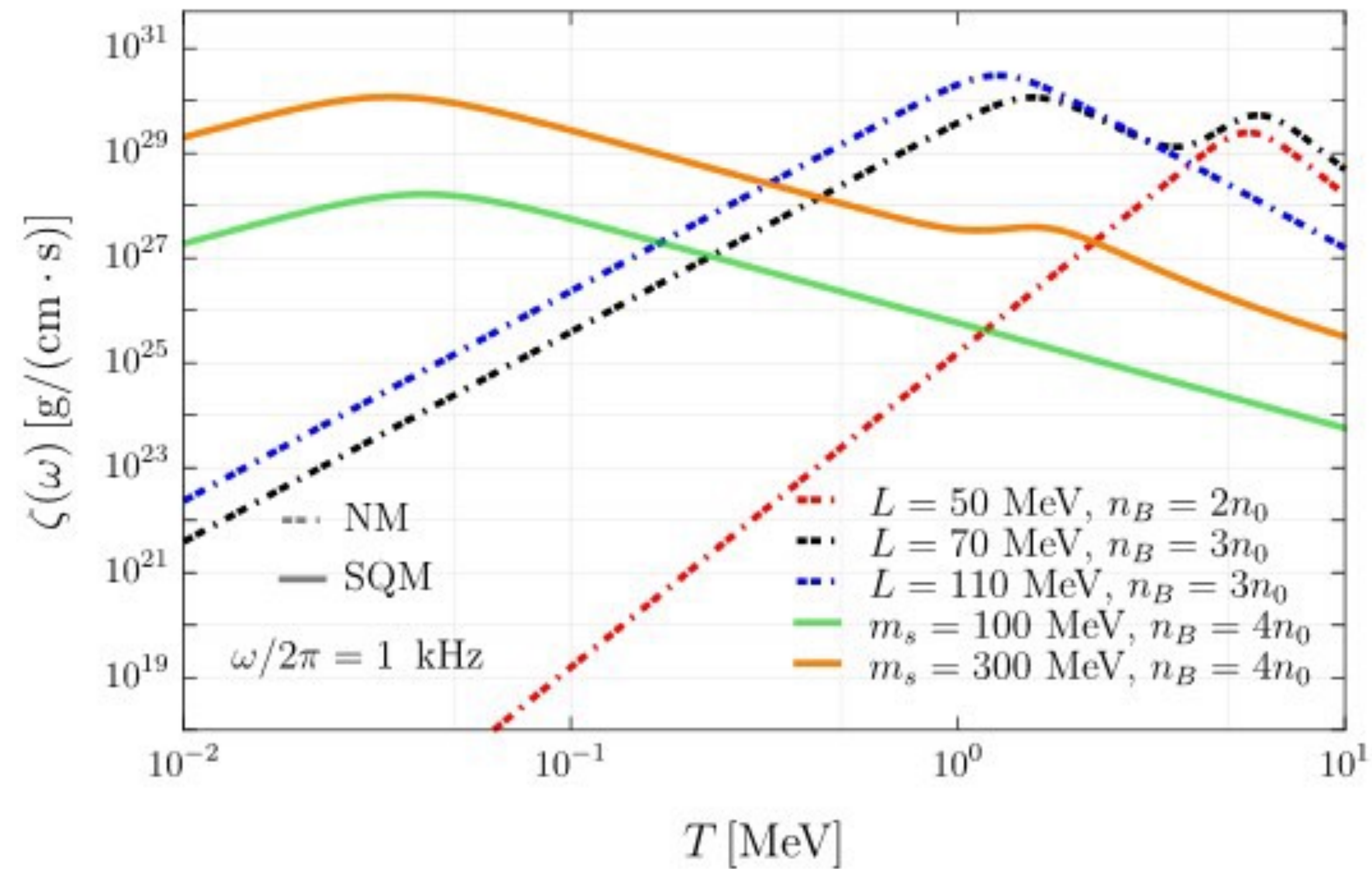
Damping time of density oscillations

$$\tau_\zeta = \frac{E}{dE/dt} = \frac{K\bar{n}}{9\omega^2\zeta(\omega)}$$

K nuclear compressibility



Bulk viscosity: quark matter vs nuclear matter



A constraint on the dissipative tidal deformability of neutron stars

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Accepted: 21 June 2024

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 Check for updates

Justin L. Ripley  , Abhishek Hegade K R  , Rohit S. Chandramouli  & Nicolás Yunes 

The gravitational waves emitted by neutron star binaries probe the physics of matter at supranuclear densities. During the late inspiral, tidal deformations raised on each star by the gravitational field of its companion depend crucially on the star’s internal properties. The misalignment of a star’s tidal bulge with its companion’s gravitational field encodes the strength of internal dissipative processes, which imprint onto the phase of the gravitational waves emitted. Here, we analyse gravitational wave data from the GW170817 (binary neutron star) event detected by LIGO and Virgo and find a constraint on the dissipative tidal deformability of a neutron star. From this constraint, assuming a temperature profile for each star in the binary, we obtain an order of magnitude bound on the averaged bulk (ζ) and shear (η) viscosity of each star during the inspiral: $\zeta \lesssim 10^{31} \text{ g cm}^{-1} \text{ s}^{-1}$ and $\eta \lesssim 10^{28} \text{ g cm}^{-1} \text{ s}^{-1}$. We forecast that these bounds could be improved by two orders of magnitude with third-generation detectors, like Cosmic Explorer, using inspiral data. These constraints already inform nuclear physics models and motivate further theoretical work to better understand the interplay between viscosity and temperature in the late inspiral of neutron stars.

Neutron stars are the densest material objects in the Universe. Their interiors reach densities many times that of atomic nuclei at temperatures well below those probed by heavy-ion collisions. Determining the physical properties of neutron star matter, therefore, has remained an outstanding problem in astrophysics, gravitational physics and nuclear physics for almost a century¹. Gravitational waves (GWs) from neutron star binaries encode the tidal deformations that neutron stars experience during their late inspiral before merger. These tidal deformations are in turn affected by the material properties of the stars. The prompt, conservative, relativistic, tidal response is described by the tidal deformability λ (refs. 2–4), which encodes aspects of the equilibrium properties of neutron star matter through the neutron star equation of state (EOS)¹. The GWs from the binary neutron star event GW170817 have been used to constrain the tidal deformability of these two neutron stars and, thus, their EOS^{5,6}.

The tidal response of a star can be visualized as a ‘tidal bulge’ that is sourced but not aligned with the time-varying, externally imposed, gravitational field of its companion (Fig. 1). Nonequilibrium dissipative

effects within the star force the bulge to trail the orbit, inducing a tidal lag angle between the direction of the bulge and the orbital separation. The extent to which the tidal multipolar moments are misaligned with the external gravitational multipolar moments is, to a first approximation, described by the tidal lag time τ_d . The tidal lag time is a universal feature for self-gravitating astrophysical objects, from planets⁷ to black holes^{8,9}, and it has been observed in many planetary systems^{7,10}. This tidal misalignment torques the two stars and heats them up through tidal, viscous heating (for example, refs. 11,12).

Since the 1990s, it had been thought that the magnitude of these effects on the dynamics of neutron star binaries is too small to measure with GWs. Indeed, unphysically large values of viscosity are required to tidally lock the spins of the two stars to their orbit before merger¹³. Recent work in nuclear physics, however, suggests that weak-force processes can induce an effective bulk viscosity^{14–16}, which, although not large enough to tidally lock the spins of the two stars, may still have a measurable effect in the GWs emitted during the late inspiral¹⁷. This has motivated different numerical groups to model out-of-equilibrium

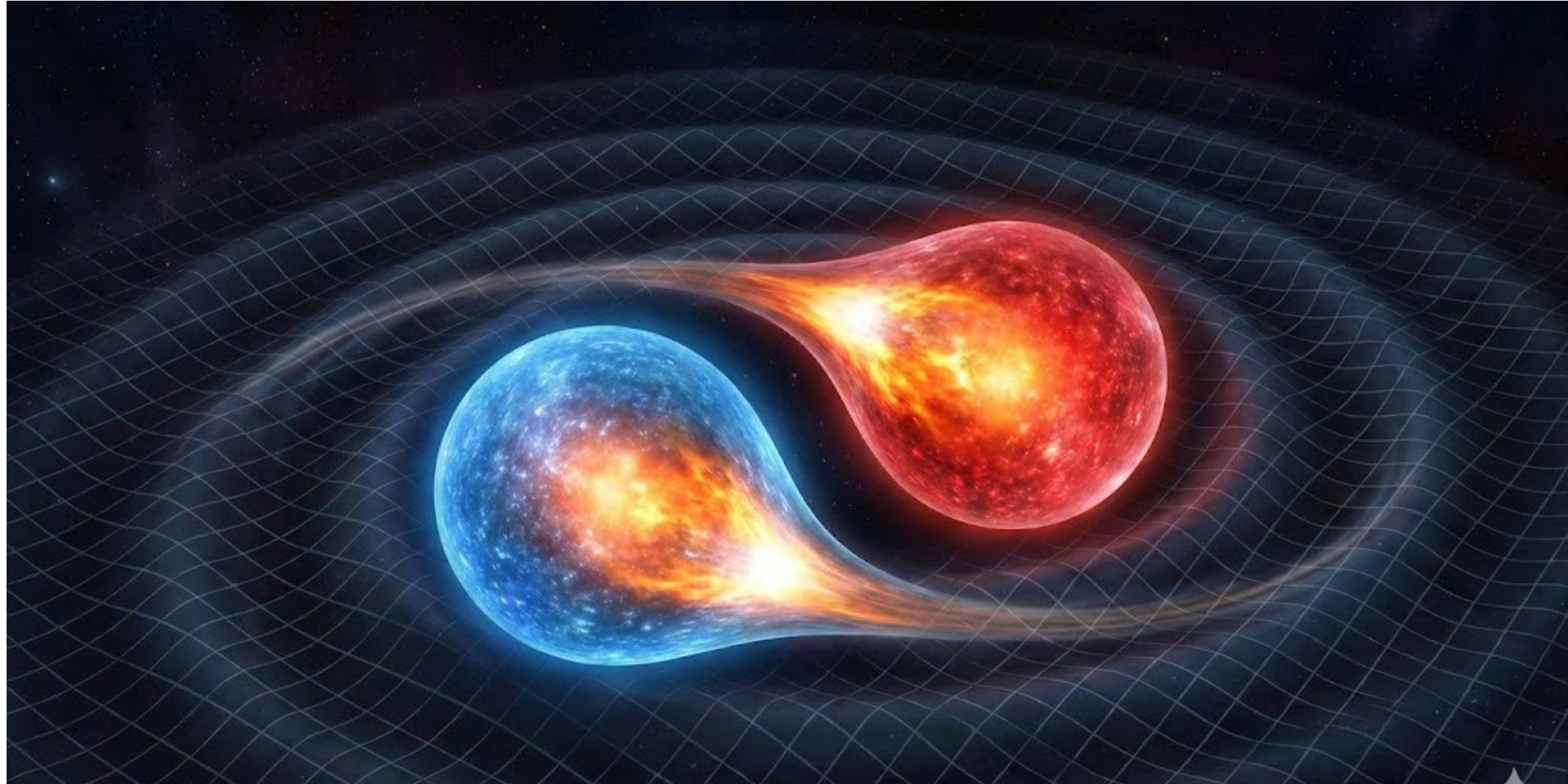
$$\eta \leq 10^{28} \text{ g cm}^{-1} \text{ s}^{-1}$$
$$\zeta \leq 10^{31} \text{ g cm}^{-1} \text{ s}^{-1}$$

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TIDAL HEATING IN THE INSPIRAL PHASE



Assume two (quark) stars in a circular orbit

$$M_1 \quad qM_1$$

Tidal forces drives the system out of equilibrium and hydrodynamical f-modes are excited

Energy dissipation due to viscosities

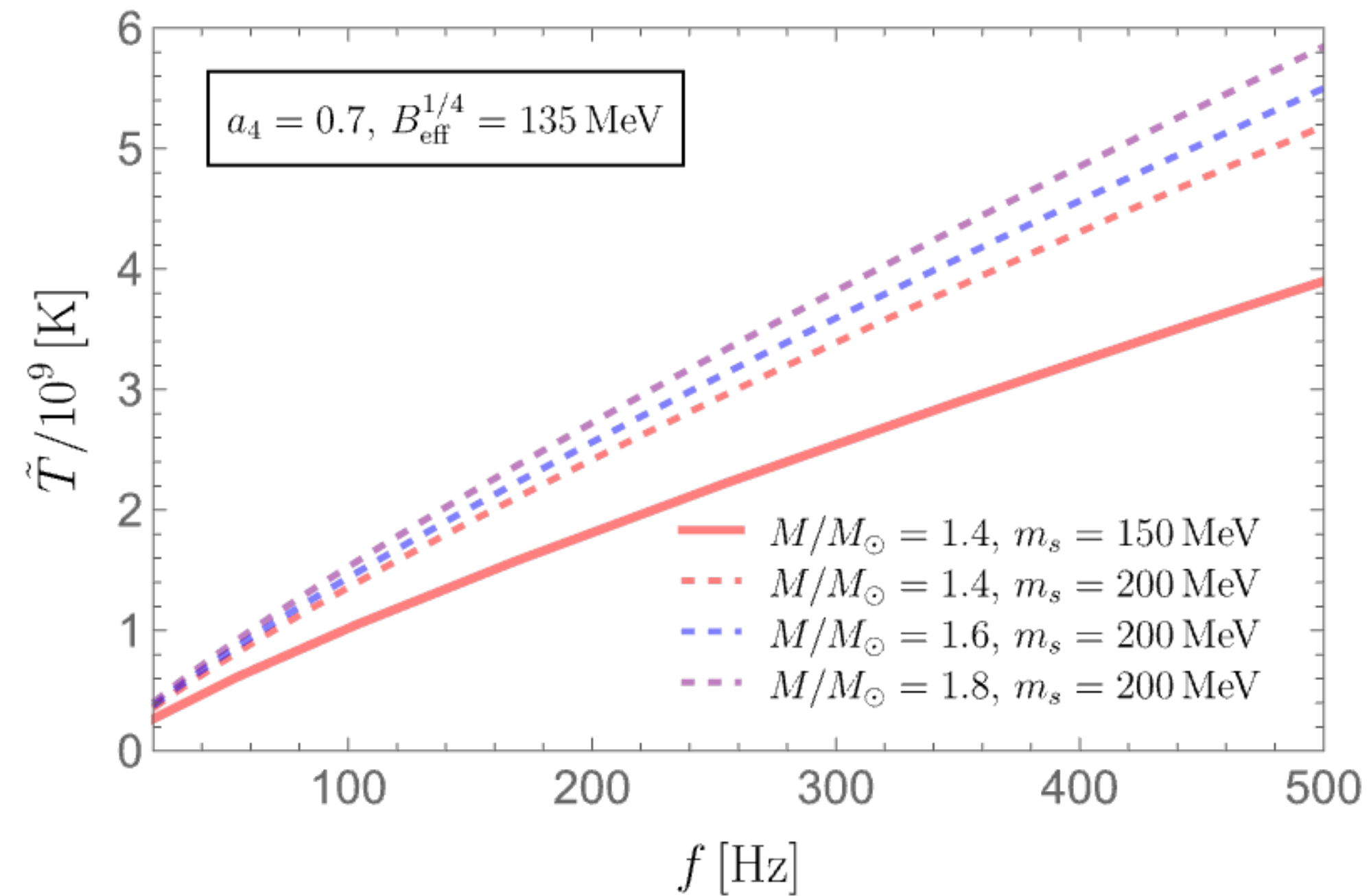
Dissipation heats up the star

$$\dot{E}_{\text{visc}} = \frac{24\pi}{5} q^2 (1 + q) \frac{M_1^2}{R} \frac{Q^2}{\bar{\omega}^4} \left(\frac{R}{D} \right)^9 \gamma$$

$$\frac{dD}{dt} = -\frac{64}{5} \frac{M_1^3 q (1 + q)}{D^3}$$

$$\gamma = \frac{A \tilde{T}^2}{B + \tilde{T}^4}$$

The dissipated energy heats up the system: thermal evolution



$$C_V \frac{d\tilde{T}}{dt} = \dot{E}_{\text{visc}}$$

$$C_V = \sigma \tilde{T}$$

(neutrino cooling is negligible)

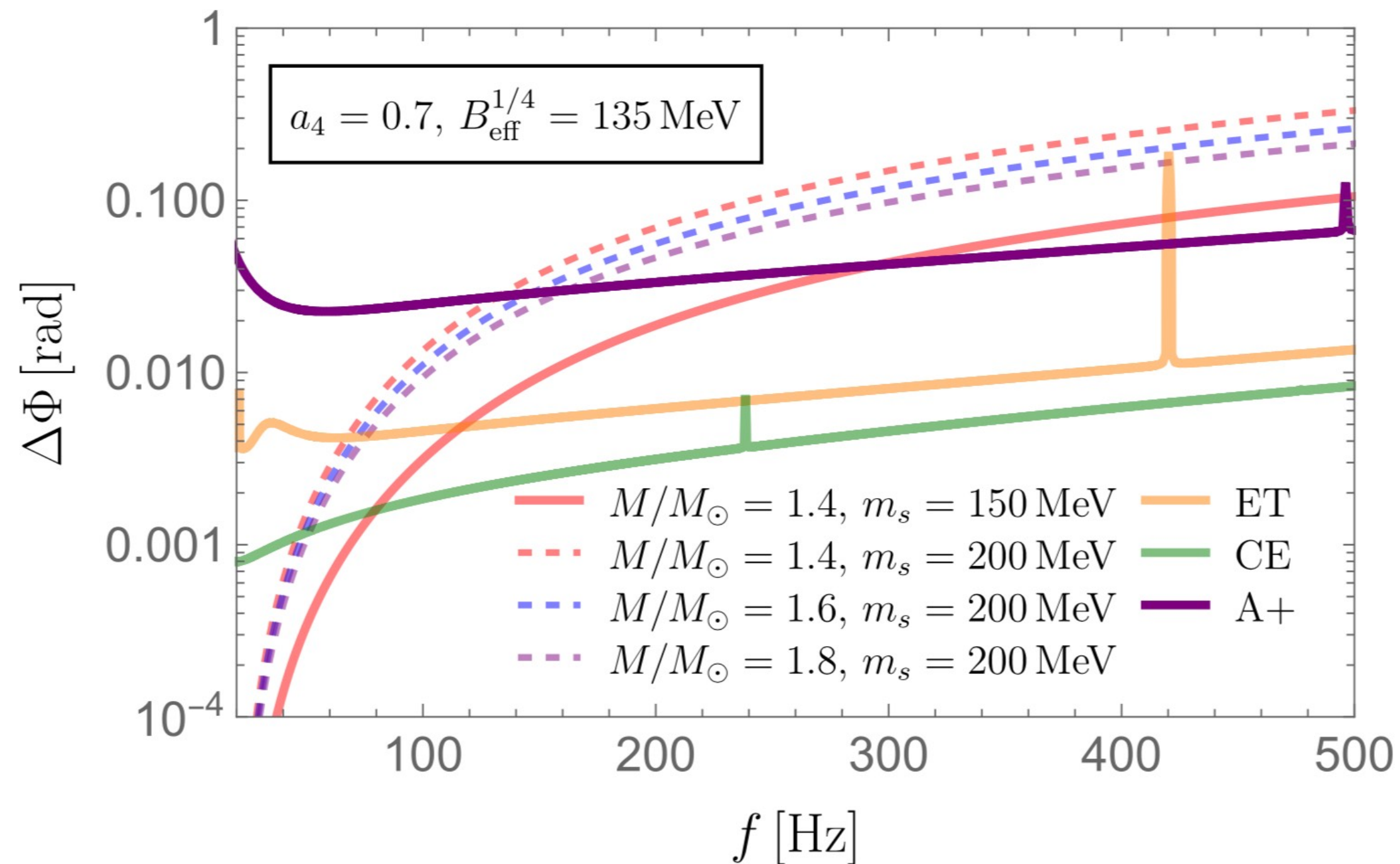
$$\frac{\tilde{T}_f^4}{4} + B \ln \left(\frac{\tilde{T}_f}{\tilde{T}_0} \right) = \frac{\tilde{T}_0^4}{4} + \frac{\pi}{3240} \frac{Q^2 q}{\bar{\omega}^4} \frac{A}{\sigma} \frac{R^3}{M_1} \left(\frac{3R}{D} \right)^5.$$

Phase shift to the gravitational waveform

Bulk-viscous dissipation might induce a phase shift in the gravitational waveform

$$\tilde{h}(v) = \tilde{A}(v)e^{-i\Phi(v)}, \quad v = (\pi M f)^{1/3}$$

M the total mass of the binary and f the GW frequency

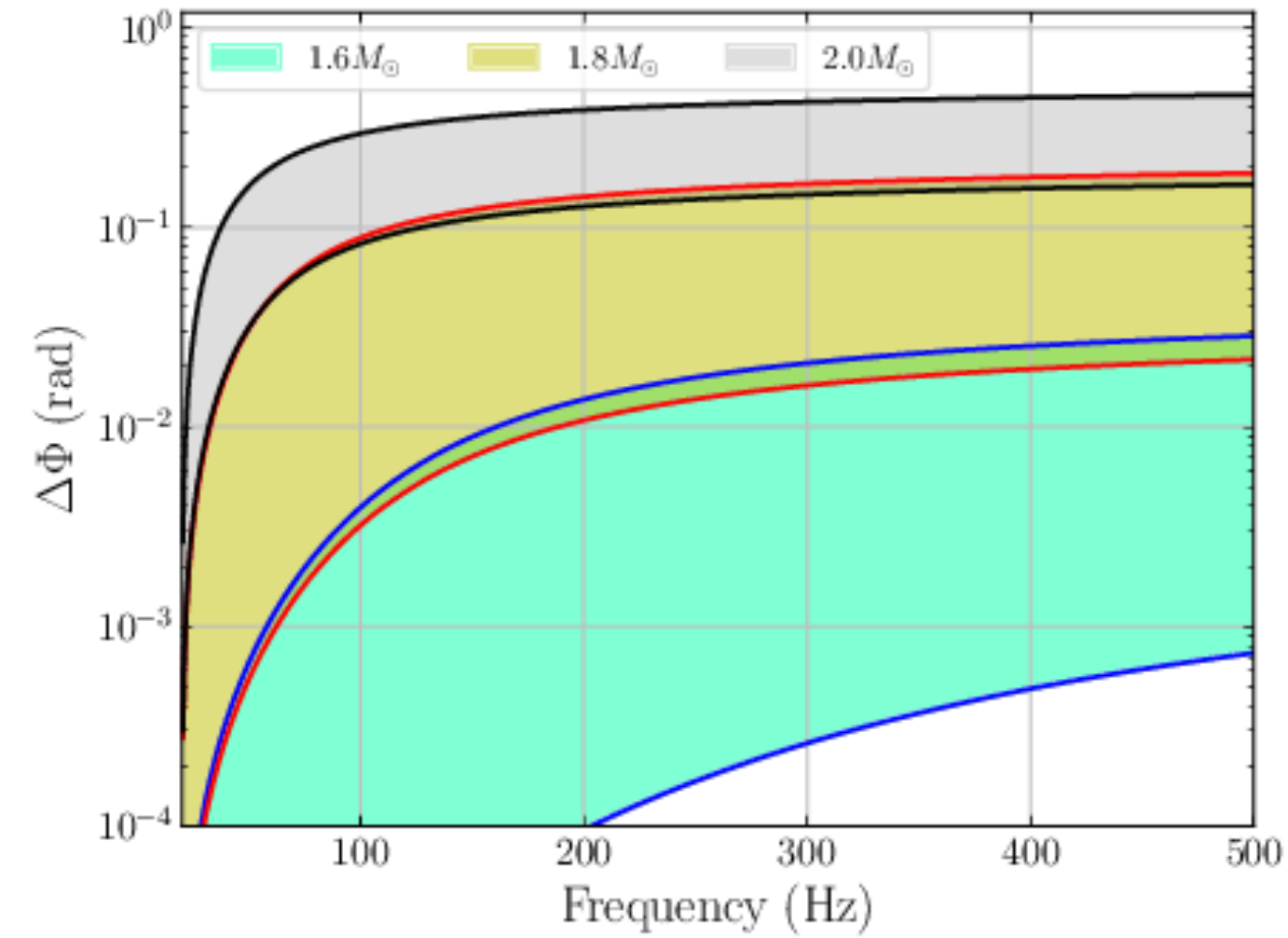
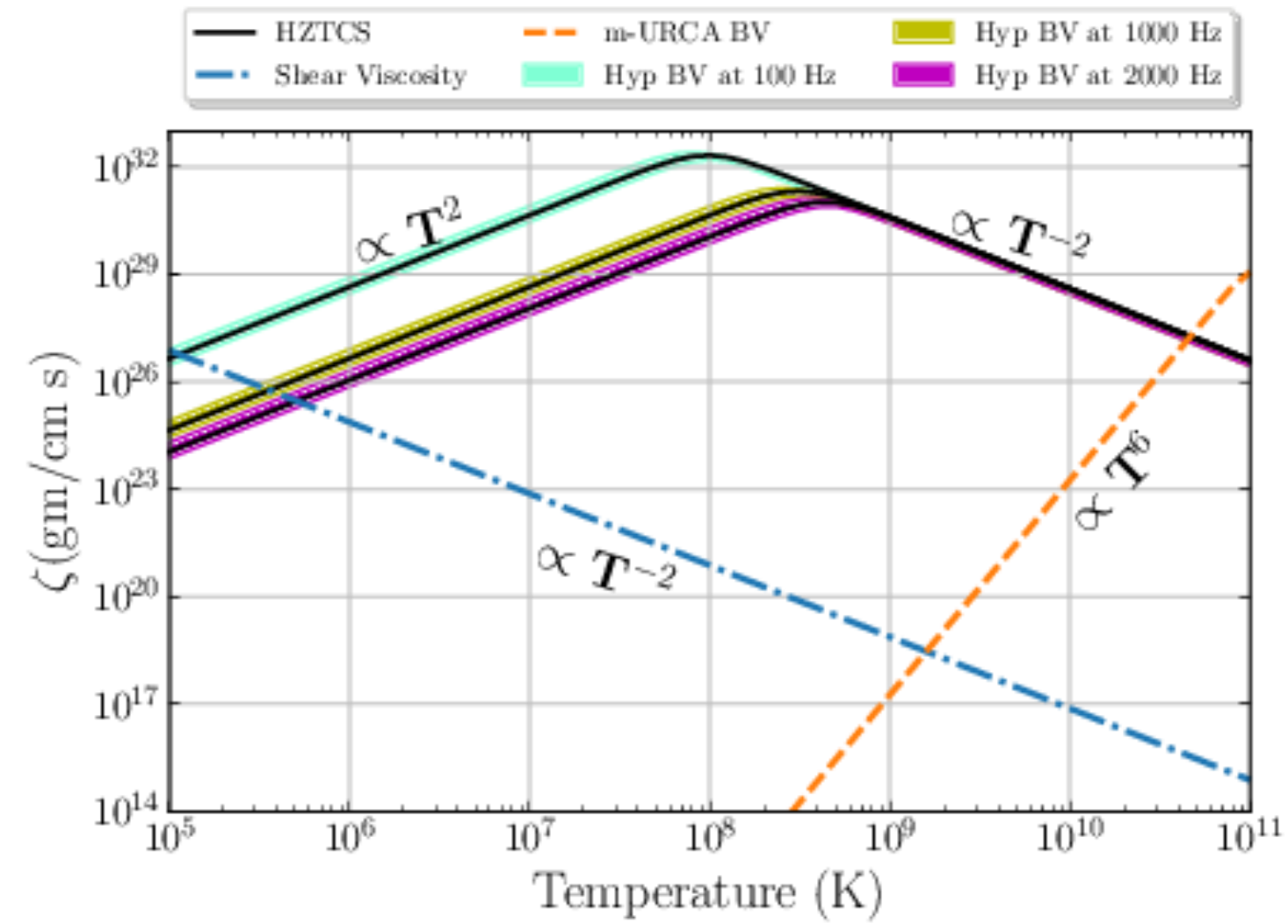


$\Delta\Phi$ difference between the phase with and w/o energy dissipation

For nuclear matter, the effect on the phase is not significant [1]

In the presence of hyperons, the bulk viscosity dominated by

$$n + p \leftrightarrow p + \Lambda$$



NUCLEAR MATTER

Matter composed by neutrons, protons and electrons - only neutrinos if $T > 5 \text{ MeV}$
(muons can as well appear if the density is high enough, depends on the EoS)

In beta equilibrium

$$\mu_n = \mu_p + \mu_e$$

Direct Urca processes

$$n \rightarrow p + e + \bar{\nu}_e$$

$$p + e \rightarrow n + \nu_e$$

Most available EoS don't allow these processes unless the density is very high

Modified Urca processes

$$n + N \rightarrow p + N + e + \bar{\nu}_e$$

$$p + N + e \rightarrow n + N + \nu_e,$$

The bulk viscosity of nuclear matter may change by several orders of magnitude if the parameter that controls the so called symmetry energy

Yang, Hippert, Speranza, Noronha, 2504.07895

Equation of state of nuclear matter



A metamodel encodes the thermodynamics of nuclear matter in terms of nuclear parameters [1]

$$\varepsilon(n_B, X_p, n_e, n_\mu) = n_B \left(m - B_{\text{sat}} + \frac{K}{2}x^2 + \frac{Q}{6}x^3 \right) + n_B S(n_B) \beta^2 + \frac{\mu_e^4}{4\pi^2} \\ + \frac{1}{8\pi^2} \left\{ \mu_\mu \sqrt{\mu_\mu^2 - m_\mu^2} (2\mu_\mu^2 - m_\mu^2) - m_\mu^4 \ln \left[\left(\mu_\mu + \sqrt{\mu_\mu^2 - m_\mu^2} \right) / m_\mu \right] \right\}$$

$$\text{with } x \equiv \frac{n_B - n_0}{3n_0}, \quad \beta \equiv 1 - 2X_p,$$

$$S(n_B) \equiv J + Lx + \frac{K_{\text{sym}}}{2}x^2 + \frac{Q_{\text{sym}}}{6}x^3$$

Constraints from nuclear experiments at nuclear saturation density

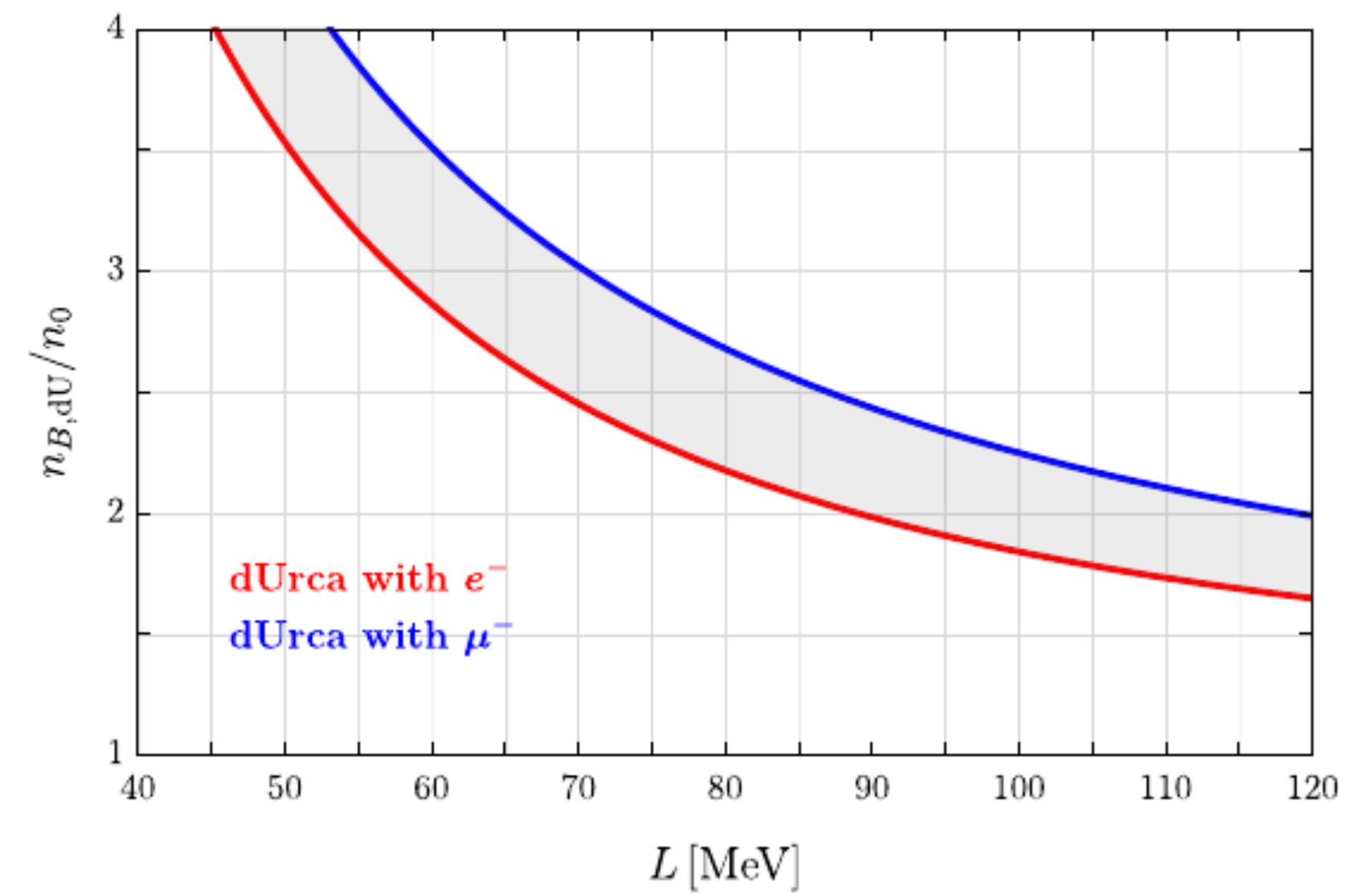
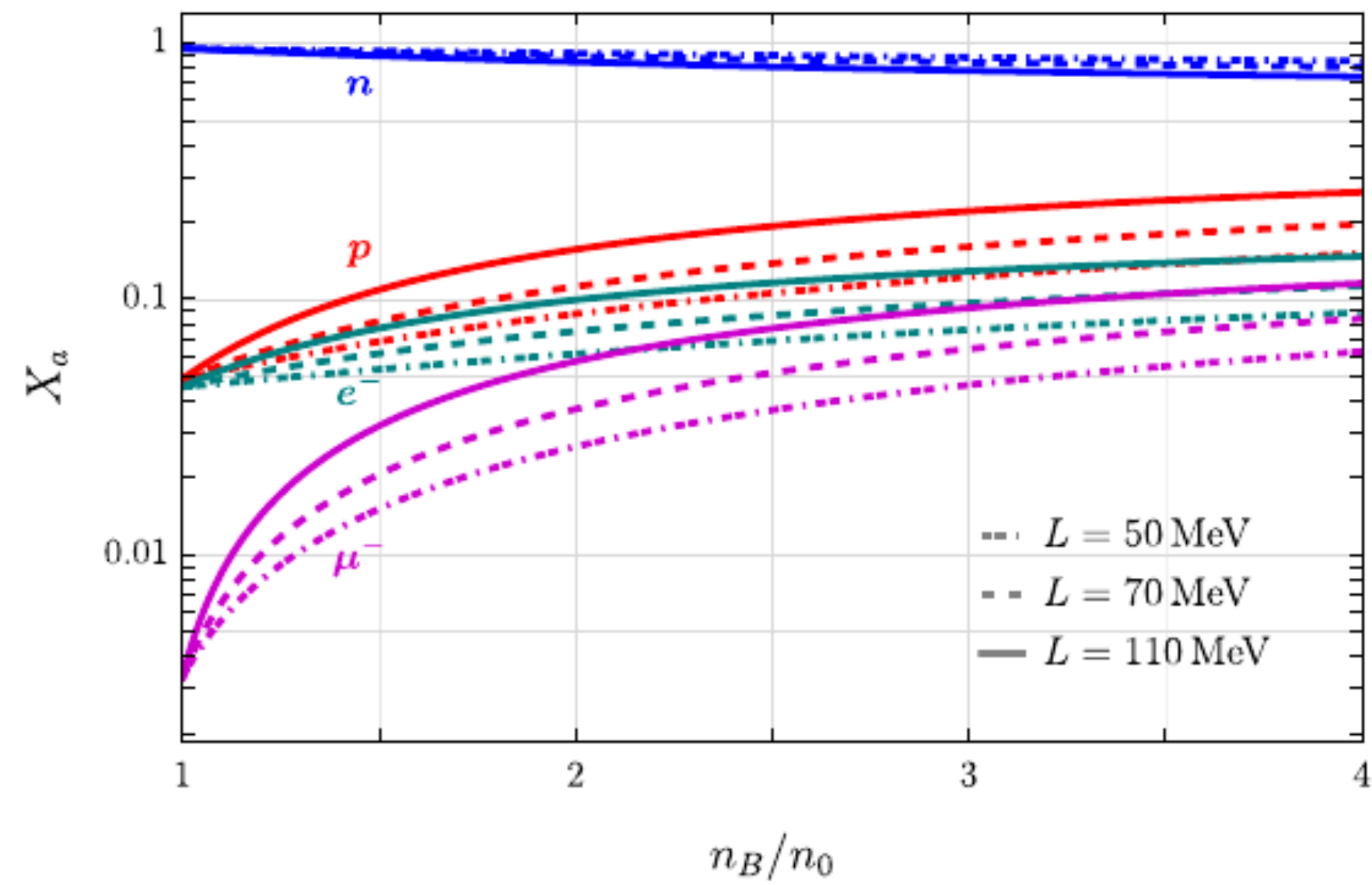
Particle Fractions

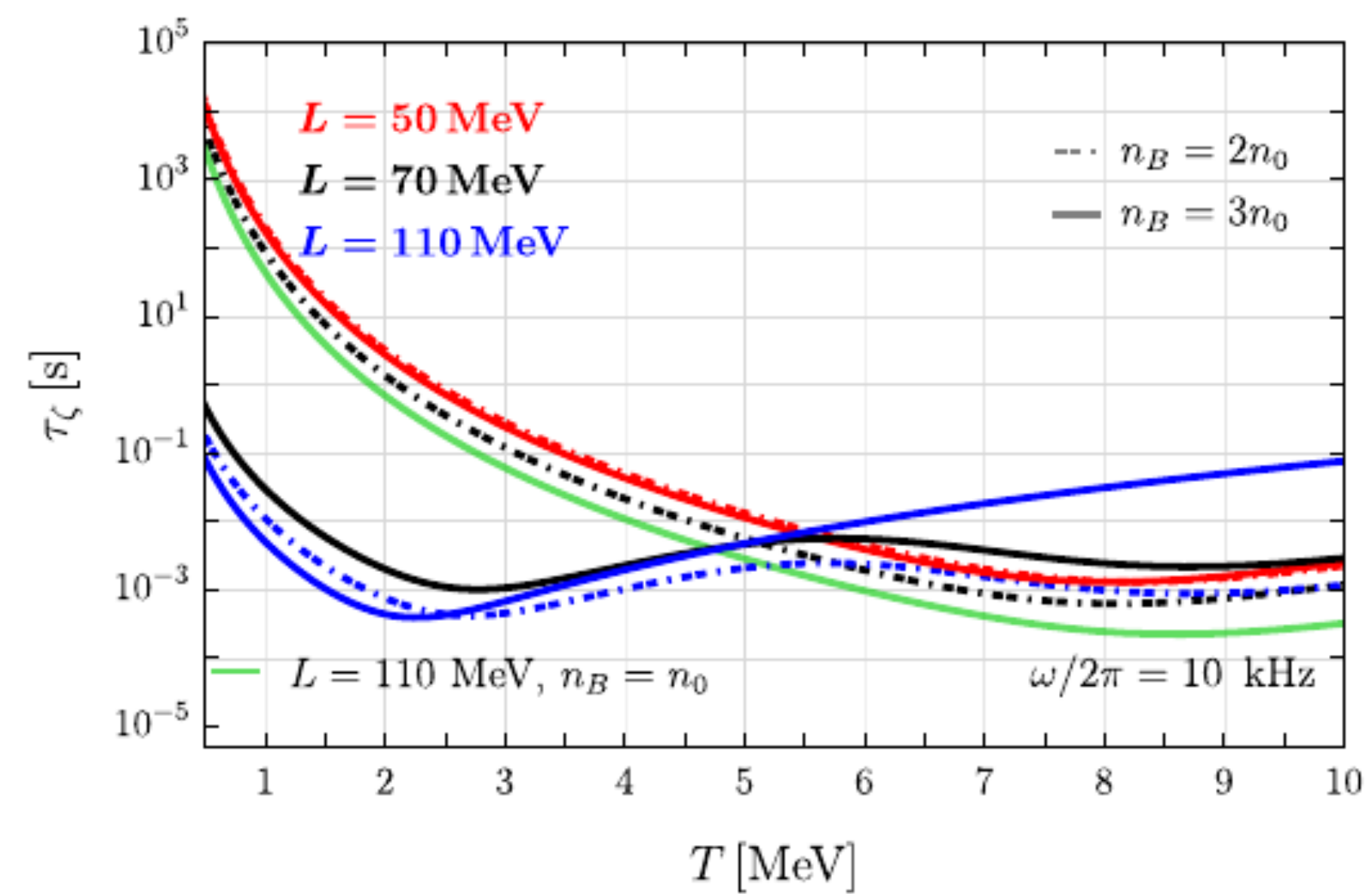
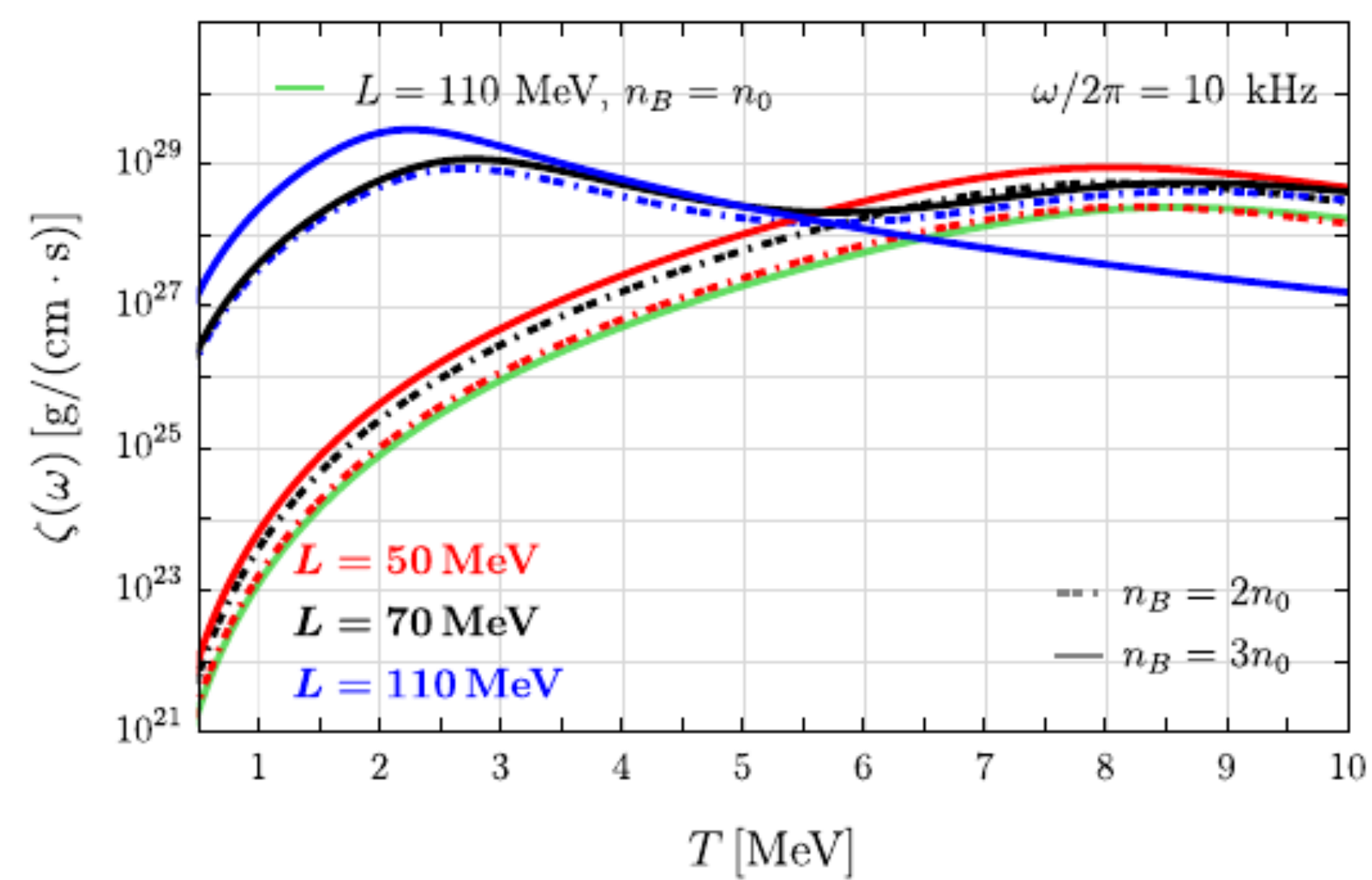
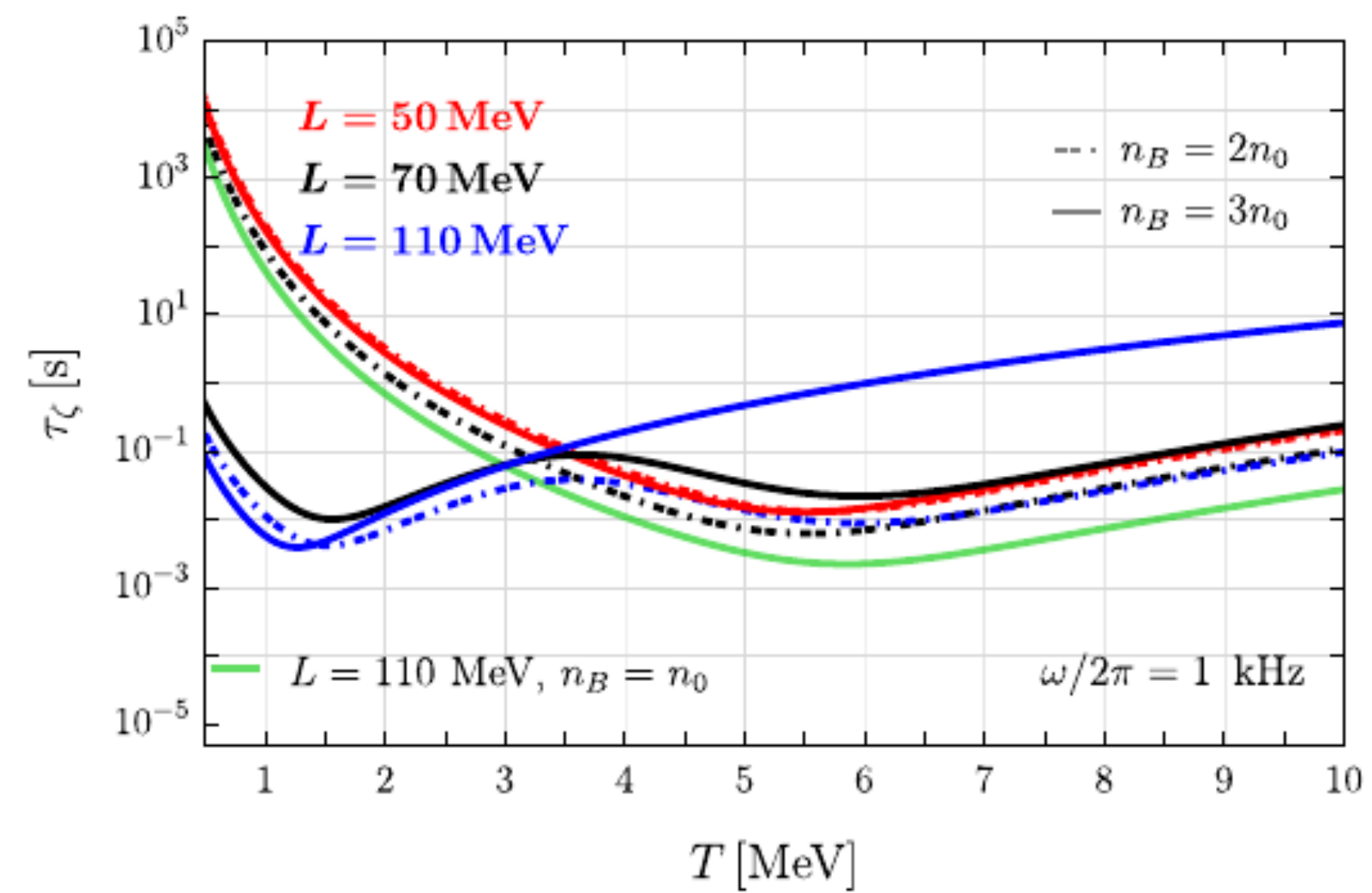
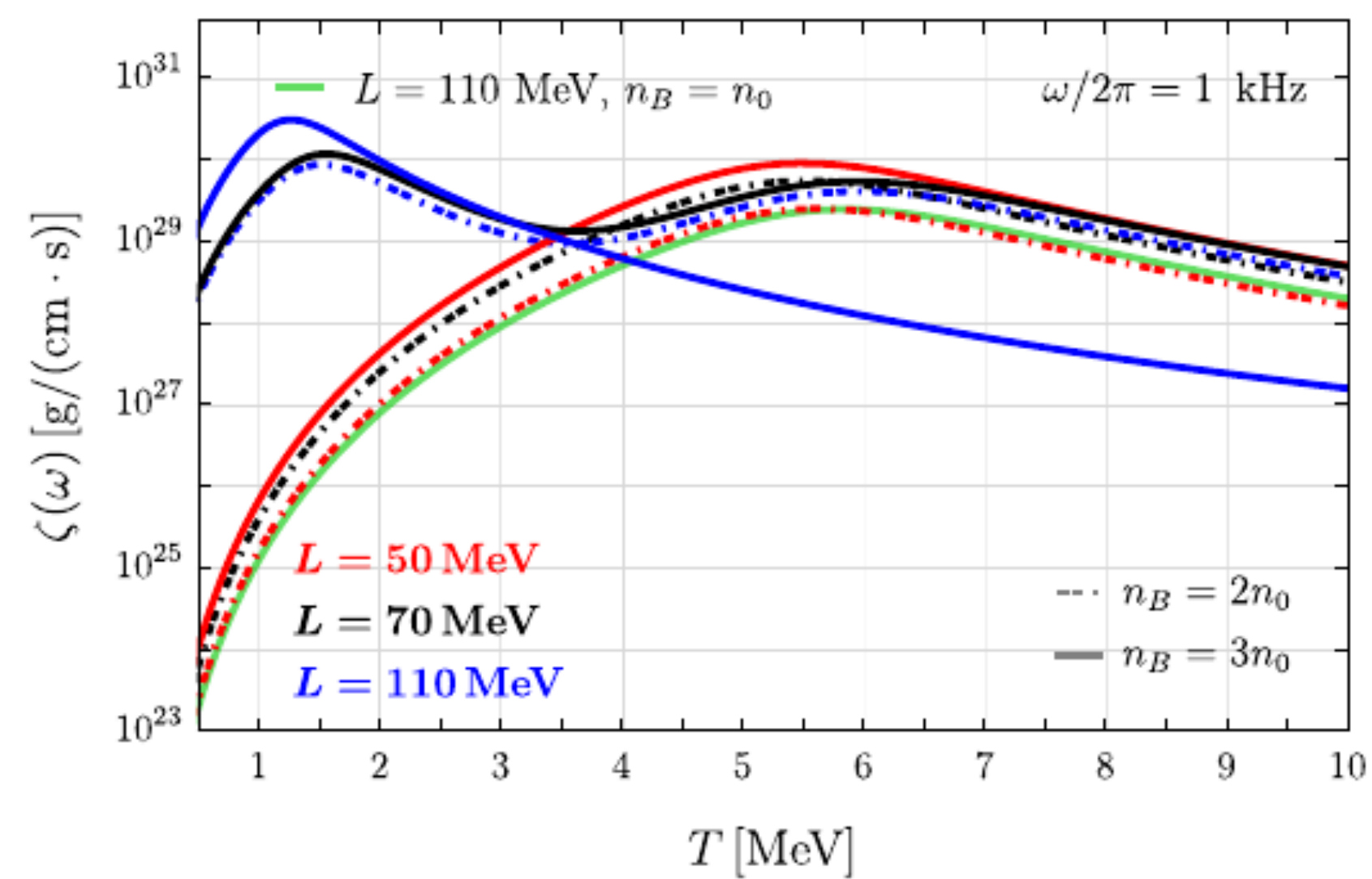
19

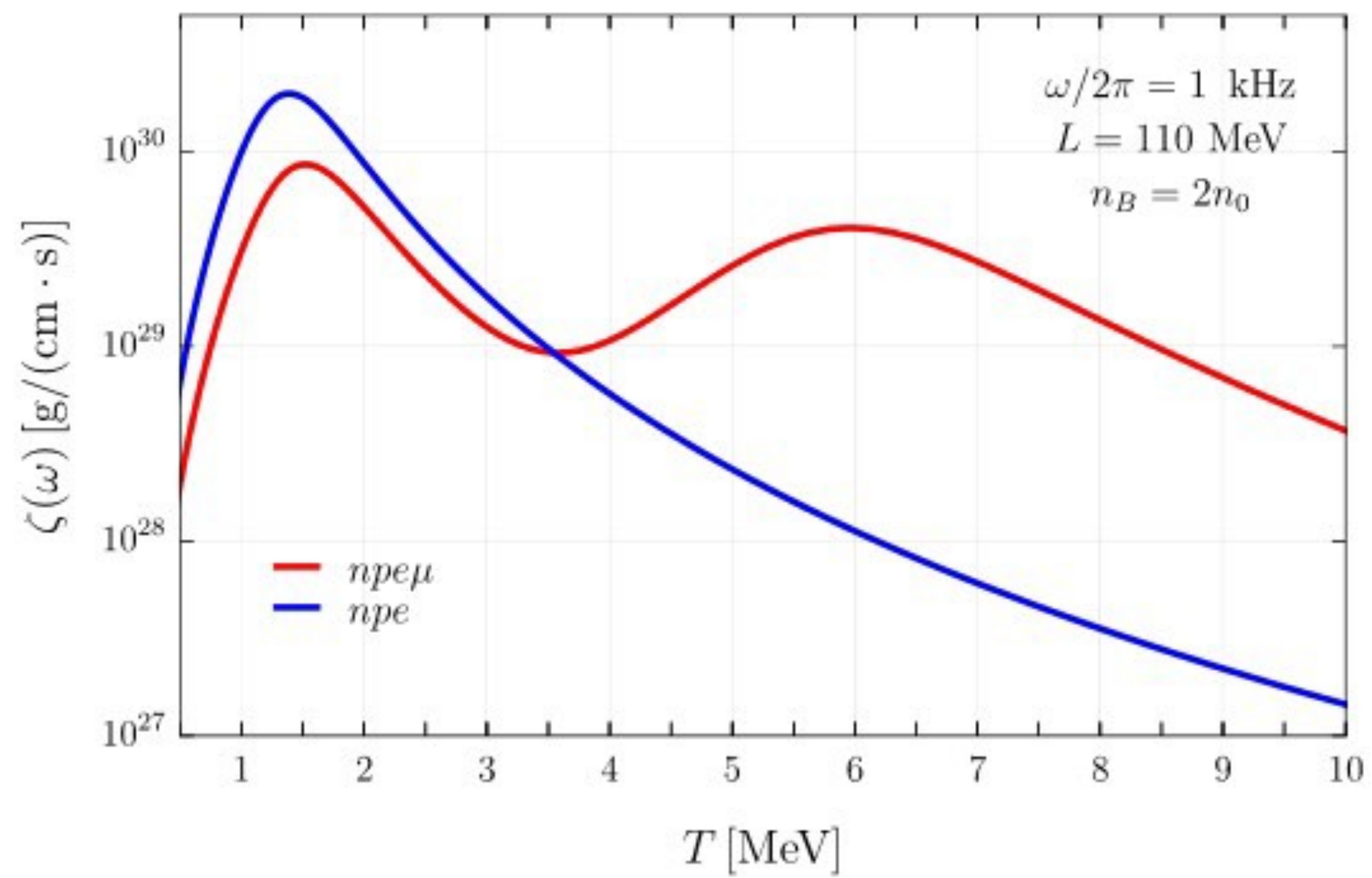
Particle population is obtained for neutral matter in chemical equilibrium and fixed nuclear parameters

From now on we vary $L = 50, 70, 110$ MeV and fix:

$$B_{\text{sat}} = -16 \text{ MeV}, \quad J = 32 \text{ MeV}, \quad K = 240 \text{ MeV}, \quad Q = K_{\text{sym}} = Q_{\text{sym}} = 0$$







CONCLUSIONS

- Exciting times ahead for nuclear physics at high density thanks to GW Astronomy
- Realistic possibilities of fixing not only the M and R (and then the EOS) of dense nuclear matter, but also of transport properties
- Quark matter: chances that the effect of the bulk viscosity might have an effect on GW, the strength of the effect clearly sensible to the value of the strange quark mass

Burgers equation in viscoelastic material

$$\sigma + \left(\frac{\eta_1}{R_1} + \frac{\eta_1}{R_2} + \frac{\eta_2}{R_2} \right) \dot{\sigma} + \frac{\eta_1 \eta_s}{R_1 R_2} \ddot{\sigma} = \eta_1 \dot{\varepsilon} + \frac{\eta_1 \eta_2}{R_2} \ddot{\varepsilon}$$