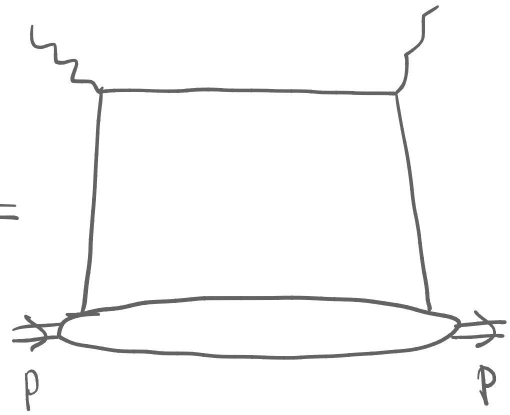
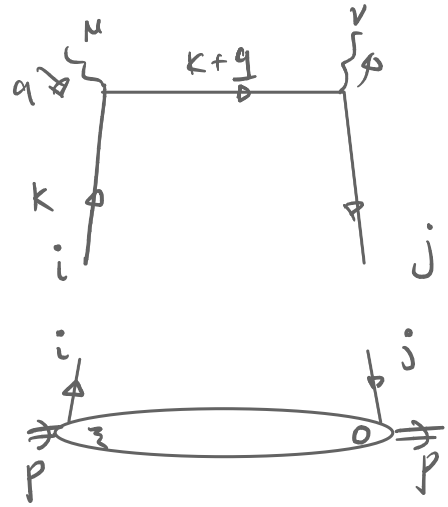
- Quick recap from last lecture

$W^{\mu\nu} =$



$=$



$H^{\mu\nu}_{ji}$

Φ_{ij}

$$H_{ji} = [\gamma^\nu (\not{k} + \not{q}) \gamma^\mu]_{ji}$$

$$\Phi_{ij} = \int \frac{d^4\xi}{(2\pi)^4} e^{ik\cdot\xi} \langle PS | \bar{\psi}_j(0)\, \psi_i(\xi) | PS \rangle$$

$$= \frac{1}{2}\left[f_1(x, k_T^2)\, \not{P} + h_1^\perp(x, k_T^2) \frac{\sigma^{\mu\nu} k_{T\mu} P_\nu}{M} \right]_{ij}$$

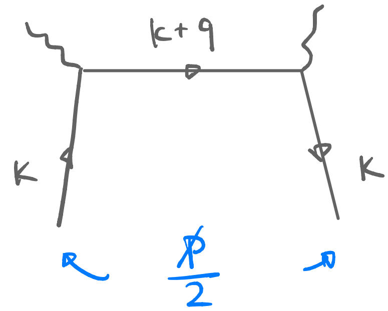
for our illustration below, let's work on unpolarized only

$$\Phi_{ij} = \frac{1}{2} f_1(x, k_T^2)\, (\not{P})_{ij}$$

$$W^{\mu\nu} = H^{\mu\nu}_{ji}\, \Phi_{ij} = H^{\mu\nu}_{ji}\, \frac{1}{2} f_1(x, k_T^2)\, (\not{P})_{ij}$$

$$= [\gamma^\nu (\not{k} + \not{q}) \gamma^\mu]_{ji} \left(\frac{\not{P}}{2}\right)_{ij} f_1(x, k_T^2)$$

$$W^{\mu\nu} = \mathrm{Tr}\left[\gamma^\nu (\not{k}+\not{q})\gamma^\mu \frac{\not{p}}{2}\right] \quad f_1(x, k_T^2)$$

$$=$$ 

$f_1(x, k_T^2)$

Hard partonic cross section

$\Rightarrow$ to be consistent with our power counting for hard function, we can drop small momentum k_T and set

$$k \approx x P \qquad \text{at leading power}$$

$\Rightarrow$ the k_T information is contained in the TMD PDF $f_1(x, k_T^2)$

$$W^{\mu\nu} \approx H^{\mu\nu}(Q) \quad f_1(x, k_T^2)$$

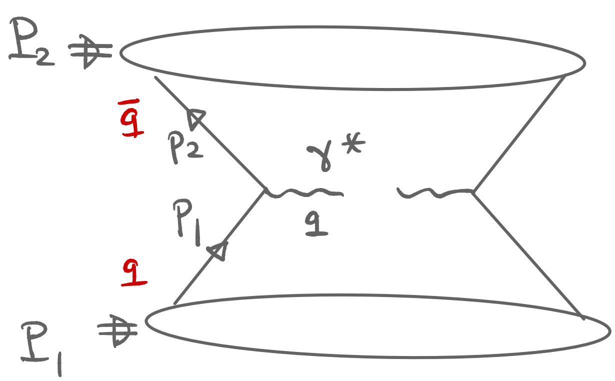
parton model result

- going beyond leading order

now let's work out this using Drell-Yan process as an example

[SIDIS also works of course. However, there're additional complication associated with the choice of frame (i.e. kinematics are more complicated)]

DY at LO: $q + \bar{q} \to \gamma^* \to e^+e^-$



$p(P_1) + p(P_2) \to \gamma^* + X$

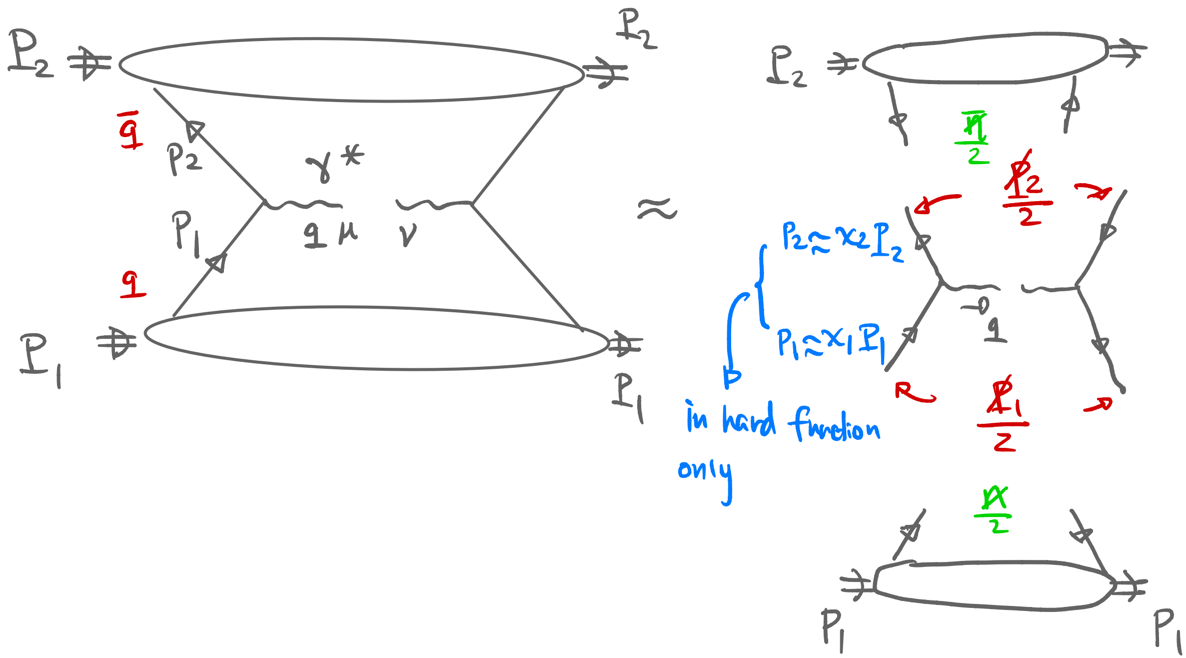
one can perform the same decomposition to separate partonic cross section from TMD correlators

$$P_1^\mu = (P_1^+, 0^-, 0_\perp) = P_1^+ \bar{n}^\mu \qquad \bar{n}^\mu = (1^+, 0^-, 0_\perp)$$

$$P_2^\mu = (0, P_2^-, 0_\perp) = P_2^- n^\mu \qquad n^\mu = (0^+, 1^-, 0_\perp)$$

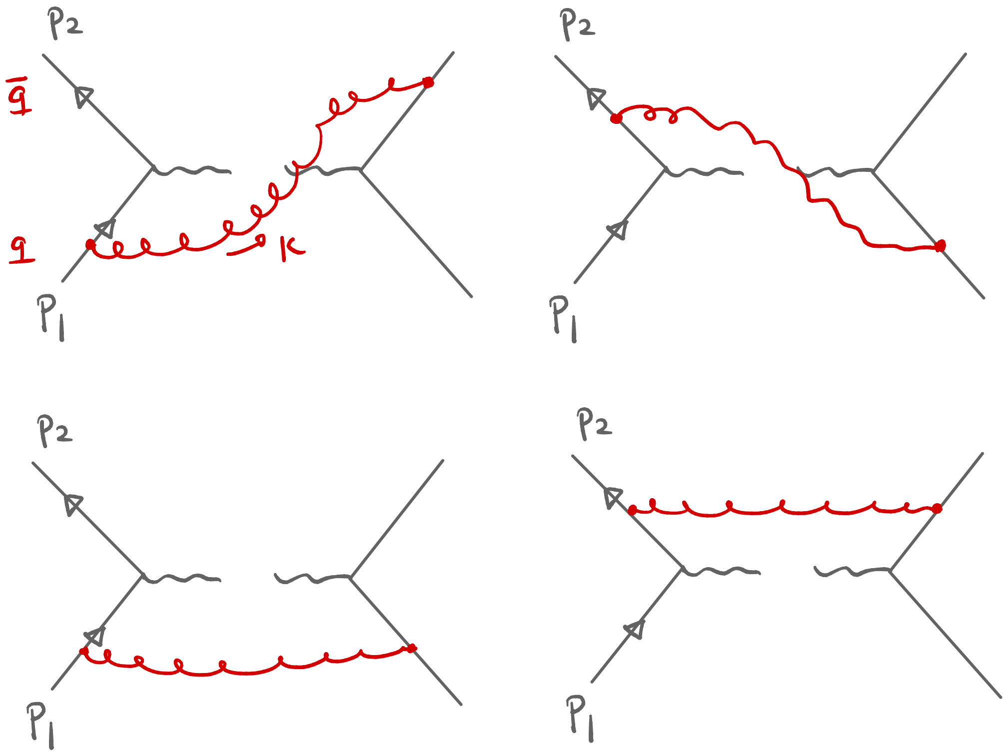
$$p_1^\mu \approx x_1 P_1^+ \bar{n}^\mu + k_{1T}^\mu$$

$$p_2^\mu \approx x_2 P_2^- n^\mu + k_{2T}^\mu$$

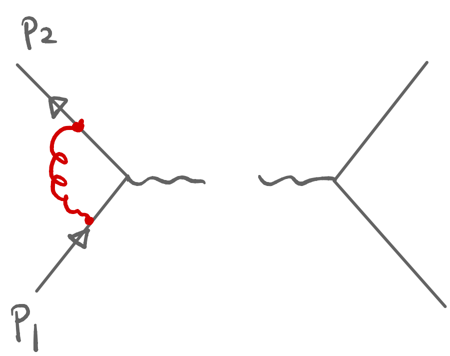


$$= \int d^2k_{1T}\, d^2k_{2T}\; f_1(x_1, k_{1T}^2)\, H^{\mu\nu}(Q)\, f_2(x_2, k_{2T})\; \delta^2(\vec{k}_{1T} + \vec{k}_{2T} - \vec{q}_T)$$

- Now work at NLO



real



etc

virtual

Now because we measure γ^* q_T transverse momentum

Q invariant mass

$q_T \ll Q$

Soft gluon + collinear gluon radiations can both contribute to the cross section

Soft gluon: $k^\mu = (k^+, k^-, \vec{k}_T) \sim Q(\lambda, \lambda, \lambda)$

where $\lambda = \dfrac{q_T}{Q}$

invariant mass: $k^2 = 2k^+k^- - k_T^2 \sim Q^2\lambda^2$

$\bar{n}$-collinear gluon: k collinear to $p_1 = (p_1^+, 0^-, 0_\perp)$

$$k^\mu \sim Q(1, \lambda^2, \lambda)$$

$$k^2 = 2\underset{1}{k^+}\underset{\lambda^2}{k^-} - \underset{\lambda^2}{k_T^2} = Q^2\lambda^2$$

n-collinear gluon: k collinear to $p_2 = (0^+, p_2^-, 0_\perp)$

$$k^\mu \sim Q(\lambda^2, 1, \lambda)$$

$$k^2 \sim Q^2\lambda^2$$

They have the same invariant mass $\sim Q^2\lambda^2 \sim q_T^2$

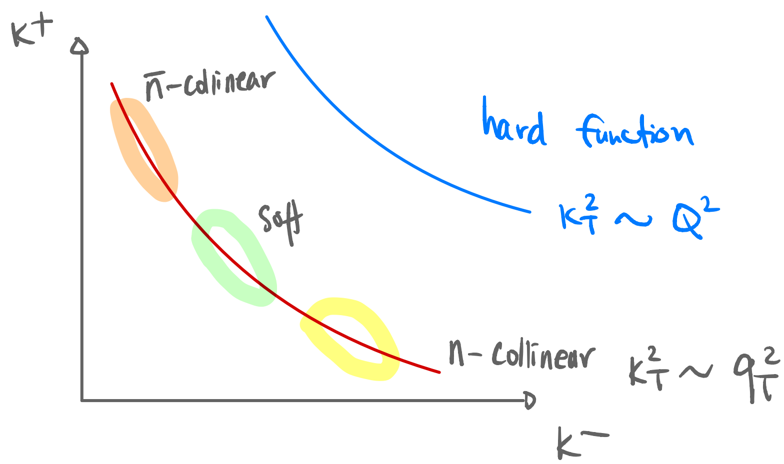
However, their rapidities are different

$$y = \frac{1}{2}\ln\frac{k^+}{k^-}$$

soft: $\quad y_s \sim O(1)$

$\bar{n}$-collinear: $\quad y_{\bar{n}} \sim \frac{1}{2}\ln\frac{1}{\lambda} \to +\infty \quad$ for small λ

n-collinear: $\quad y_n \sim \frac{1}{2}\ln\frac{\lambda}{1} \to -\infty$



The key point is: these modes have the same invariant mass, so dimensional regularization cannot separate them!

- The core message for TMD factorization

TMD factorization requires two separations

(1) invariant mass separation

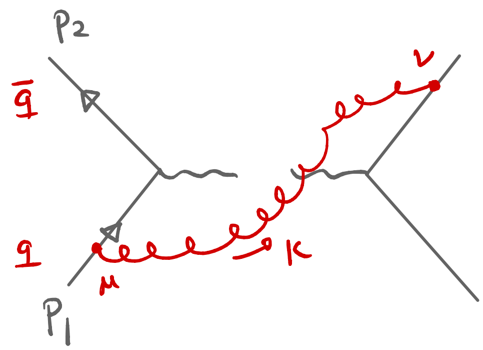
[hard vs collinear/soft]

(2) rapidity separation

[collinear vs soft]

$$\boxed{\begin{aligned}\frac{d\sigma^{DY}}{dQ^2\,dy\,d^2q_T} \sim\ & H_{q\bar{q}}(Q,\mu) \\ & * \int d^2b_T\, e^{i\vec{q}_T\cdot\vec{b}_T}\ f_q(x_1, b_T; \mu, \zeta_1/\nu^2) \\ & \qquad * f_{\bar{q}}(x_2, b_T; \mu, \zeta_2/\nu^2) \\ & \qquad * S(b_T; \mu, \nu)\end{aligned}}$$

- Soft function



In the soft limit, the gluon does not change the direction of fast parton, so the propagator will become eikonal

k is soft

$$\text{left} = \frac{i(\not{p}_1 - \not{k})}{(p_1-k)^2 + i\epsilon}(-ig\gamma^\mu)\quad u(p_1)$$

$\Big\Downarrow$ k is soft, we can drop comparing to p_1

$$\not{p}_1 - \not{k} \approx \not{p}_1$$

$$(p_1-k)^2 = \underbrace{p_1^2}_{0} + \underbrace{k^2}_{Q^2\lambda^2} - 2p_1\cdot k = -2p_1^+ k^-$$

$$= g\frac{\not{p}_1}{-2p_1^+ k^-}\gamma^\mu \quad u(p_1)$$

$\Big\Downarrow$ $\not{p}_1\gamma^\mu = 2p_1^\mu - \gamma^\mu \not{p}_1$

$$\not{p}_1 u(p_1) = 0$$

$$= g\frac{2p_1^\mu}{-2p_1^+ k^- + i\epsilon}\quad u(p_1)$$

$$\Downarrow \quad p_1^\mu = p_1^+ \bar{n}^\mu$$

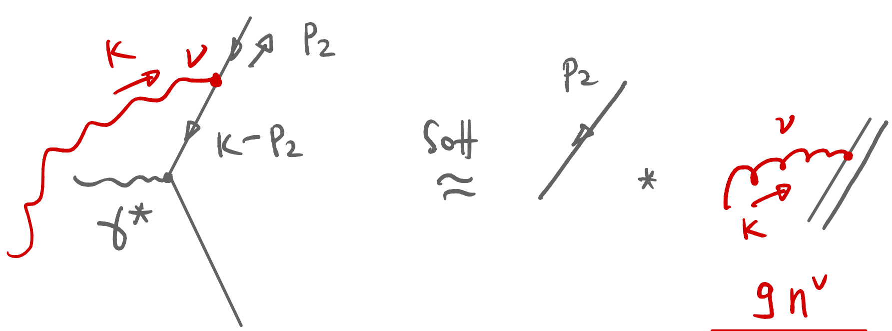
$$= -g \frac{\bar{n}^\mu}{k^- - i\epsilon} u(p_1)$$

$$\Downarrow \quad k^- = \bar{n} \cdot k$$

$$= -g \frac{\bar{n}^\mu}{\bar{n} \cdot k} u(p_1)$$

$$\frac{-g\bar{n}^\mu}{\bar{n} \cdot k - i\epsilon}$$

likewise, for right-side gluon attachment



$$\frac{g n^\nu}{n \cdot k + i\epsilon}$$

$$\text{right} = \frac{-i(\not{k} - \not{p}_2)}{(k - p_2)^2 - i\epsilon} (i g \gamma^\nu) \, v(p_2)$$

drop k like before

$$\text{right} = -g \, \frac{\not{p}_2 \gamma^\nu}{-2p_2^- k^+ - i\epsilon} \, v(p_2)$$

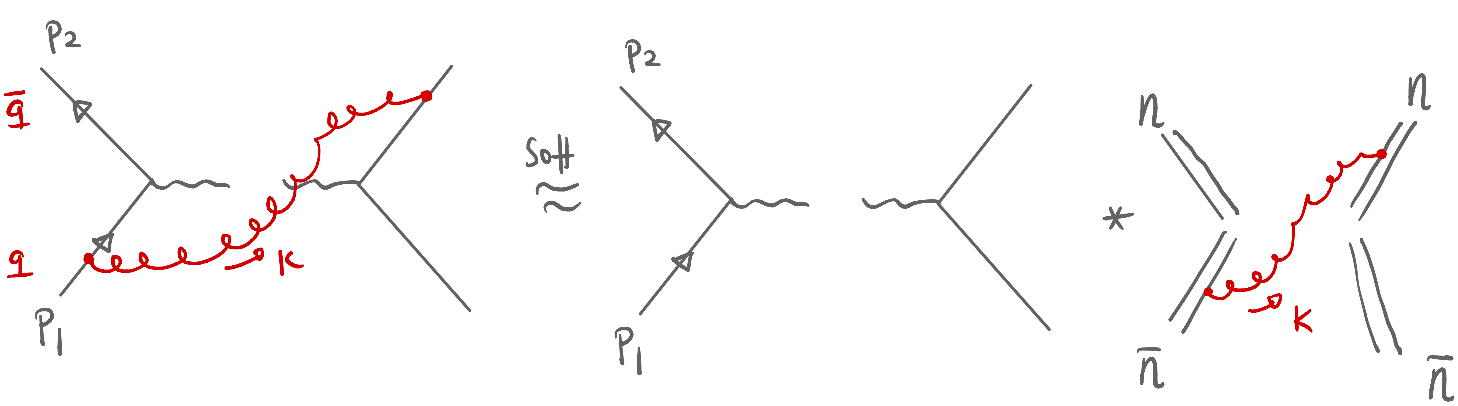
$$\Downarrow \quad \not{p}_2 \gamma^\nu = 2p_2^\nu - \gamma^\nu \not{p}_2$$

$$= -g \, \frac{2p_2^\nu}{-2p_2^- k^+ - i\epsilon} \, v(p_2)$$

$$\Downarrow \quad p_2^\nu = p_2^- n^\nu$$

$$= g \, \frac{n^\nu}{k^+ + i\epsilon} \, v(p_2)$$

$$= g \, \frac{n^\nu}{n \cdot k + i\epsilon} \, v(p_2)$$



This particular soft function diagram is given by

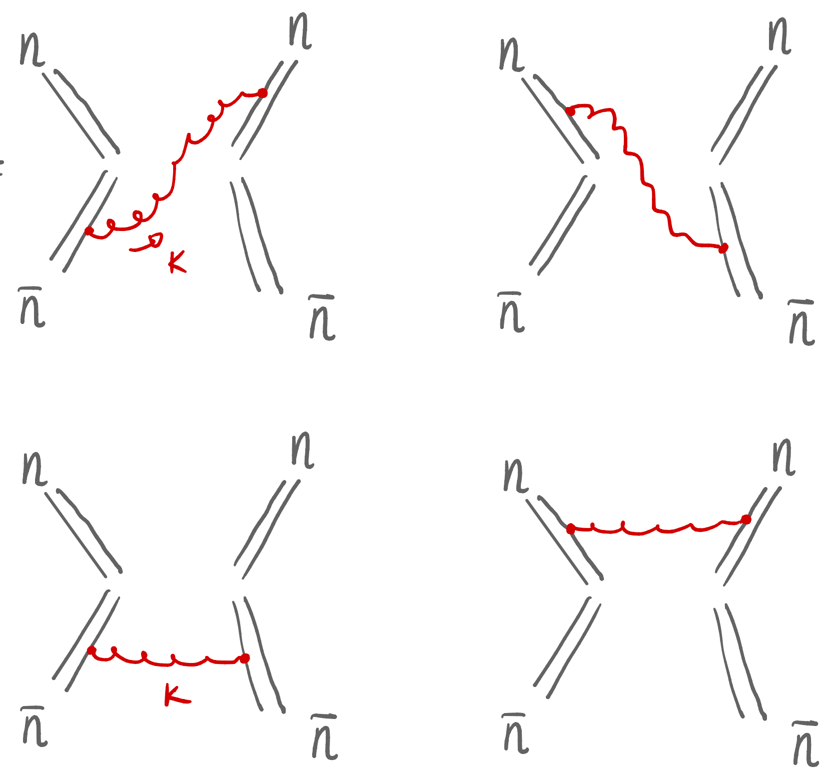
$$S_{fig1}(k_T) = \int \frac{d^d \ell}{(2\pi)^d} \, 2\pi \, \delta(\ell^2) \, \delta^2(\vec{\ell}_\perp - \vec{k}_\perp)$$

$$* \left[-g \, \frac{\bar{n}^\mu}{\bar{n} \cdot k} \; g \, \frac{n^\nu}{n \cdot k} \right] (-g_{\mu\nu}) \; * \text{ color}$$

Apply the same procedure to all

We end up with the following soft function

$S(k_T) =$



$$\Big\Downarrow \; \bar{n}\cdot n = 1$$

$$= 2 g^2 C_F \int \frac{d^d \ell}{(2\pi)^d} \; 2\pi \, \delta(\ell^2) \; \delta^{d-2}(\vec{\ell}_\perp - \vec{k}_\perp) \; \frac{1}{\ell^+ \ell^-}$$

$$\delta(\ell^2) = \delta\left(2\ell^+\ell^- - \vec{\ell}_\perp^{\,2}\right)$$

$$S(b_T) = \int d^{d-2}\vec{k}_T \; e^{i \vec{k}_T \cdot \vec{b}_T} \; S(\vec{k}_T)$$

$$S(b_T) = 2g^2 C_F \int d^{d-2}\vec{k}_T \, e^{i\vec{k}_T\cdot\vec{b}_T}$$

$$* \int \frac{dl^+ \, dl^- \, d^{2-2\epsilon} l_\perp}{(2\pi)^d} \, 2\pi\delta(2l^+l^- - \vec{l}_\perp^{\,2}) \, \delta^{d-2}(\vec{l}_\perp - \vec{k}_\perp) \frac{1}{l^+l^-}$$

$$= 2g^2 C_F \int d^{d-2}\vec{k}_T \, e^{i\vec{k}_T\cdot\vec{b}_T} \int \frac{dl^+ dl^-}{(2\pi)^{d-1}} \frac{1}{2l^-} \delta\left(l^- - \frac{\vec{k}_\perp^{\,2}}{2l^+}\right) \frac{1}{l^+l^-}$$

$$= \frac{g^2 C_F}{\pi} \int \frac{d^{d-2}\vec{k}_T}{(2\pi)^{d-2}} e^{i\vec{k}_T\cdot\vec{b}_T} \frac{1}{\vec{k}_T^{\,2}} \boxed{\int_0^\infty \frac{dl^-}{l^-}}$$

or just use our momentu

$$= \frac{g^2 C_F}{\pi} \int \frac{d^{d-2}\vec{k}_T}{(2\pi)^{d-2}} e^{i\vec{k}_T\cdot\vec{b}_T} \frac{1}{k_T^2} \int_0^\infty \frac{dk^-}{k^-}$$

When $k^- \to 0$ or $k^- \to \infty$, we have divergence!

for $k^- \to 0$, we have $k^+ = \frac{k_T^2}{2k^-} \to \infty$

$\Rightarrow$ rapidity of gluon $y = \frac{1}{2} \ln \frac{k^+}{k^-} \to +\infty$

for $k^- \to \infty$, $k^+ \to 0 \Rightarrow y \to -\infty$

$\Rightarrow$ rapidity divergence!

However, these $y \to +\infty$ region should be going to $\bar{n}$- collinear region, while $y \to -\infty$ region should be going to n- collinear region

$\to$ absorb these rapidity divergence to the corresponding TMD PDF $f_q(x_1, k_{1T})$ or $f_{\bar{q}}(x_2, k_{2T})$

would make properly-defined TMD PDF

$\Rightarrow$ the story is very similar to collinear factorization

one performs a "regularization" procedure to

[dimensional regularization]

isolate the pole, then you could proceed further with the TMD definiton

$\Rightarrow$ immediately one realizes the dimensional regularization does not work

[above, we already applied DR, but still divergent]

dimensional regularization: (μ, ϵ)
rapidity regularization: (ν, η)

- rapidity regulator

$$\int_0^\infty \frac{dk^-}{k^-} \to \int_0^\infty \frac{dk^-}{k^-} \left|\frac{2k_z}{\nu}\right|^{-\eta}$$

$$= \int_0^\infty \frac{dk^-}{k^-} \left|\frac{k^+ - k^-}{\nu/\sqrt{2}}\right|^{-\eta}$$

$$= \left(\frac{\nu}{\sqrt{2}}\right)^{\eta} \int_0^\infty \frac{dk^-}{k^-} \left|\frac{\vec{k}_T^{\,2}}{2k^-} - k^-\right|^{-\eta}$$

$$= \frac{\nu^\eta k_T^{-\eta}}{2^\eta \sqrt{\pi}} \Gamma\left(\frac{1-\eta}{2}\right) \Gamma\left(\frac{\eta}{2}\right)$$

The regulator effectively cuts off extreme rapidity regions!

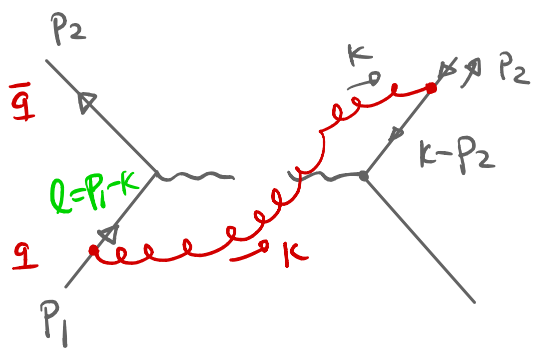
$$S_q(b_T;\epsilon,\eta) = \frac{g^2 C_F}{\pi}\,\frac{\nu^\eta}{2^\eta\sqrt{\pi}}\,\Gamma\!\left(\frac{1-\eta}{2}\right)\Gamma\!\left(\frac{\eta}{2}\right)$$

$$* \,\frac{\pi^\epsilon\,\Gamma\!\left(-\epsilon-\frac{\eta}{2}\right)}{4\pi\,2^\eta\,\Gamma\!\left(1+\frac{\eta}{2}\right)}\; b_T^{2\epsilon+\eta}$$

$$\boxed{\underset{\substack{\eta\to 0\\ \epsilon\to 0}}{=}\ \frac{\alpha_s C_F}{2\pi}\left[\frac{2}{\epsilon^2} + 4\left(\frac{1}{\epsilon}+L_b\right)\left(-\frac{1}{\eta}+\ln\frac{\mu}{\nu}\right) - L_b^2 - \frac{\pi^2}{6}\right] + O(\eta) + O(\epsilon)}$$

$$L_b = \ln\left(\frac{\mu^2}{\mu_b^2}\right) \qquad \mu_b = \frac{2e^{-\gamma_E}}{b_T}$$

• Now let's look at collinear region



$l^+ = p_1^+ - k^+ = x\, p_1^+$

k is collinear to $p_1^\mu = (p_1^+, 0^-, 0_\perp)$

$$k^\mu = Q\,(1, \lambda^2, \lambda)$$

set $k^+ = (1-x)\, p_1^+$

Note since $k \parallel p_1$, there's not much approximation one can do on the propagator $l = p_1 - k$ except that one only keeps l^+ into hard function

$$\text{right} = \frac{-i(\not{k} - \not{p}_2)}{(k-p_2)^2 - i\epsilon} \, (ig\gamma^\nu) \, v(p_2) \, [\, g_{\mu\nu} \,]$$

$g_{\mu\nu}$ is the gluon propagator

Grammer–Yennie decomposition (1973)

longitudinal — transverse pieces

$$g_{\mu\nu} = K_{\mu\nu} + G_{\mu\nu}$$

$$= \frac{n_\mu k_\nu}{n \cdot k} + \left(g_{\mu\nu} - \frac{n_\mu k_\nu}{n \cdot k} \right)$$

- K term (longitudinal gluons)

since it contains k_ν

$$\text{right}-K = \frac{-i(\not{k}-\not{p}_2)}{(k-p_2)^2-i\epsilon}(ig\gamma^\nu)\, v(p_2)\, K_{\mu\nu}$$

$$= \frac{-i(\not{k}-\not{p}_2)}{(k-p_2)^2-i\epsilon}(ig\gamma^\nu)\frac{n_\mu k_\nu}{n\cdot k}\, v(p_2)$$

$$\gamma^\nu k_\nu = \not{k}$$

$$= \frac{g(\not{k}-\not{p}_2)\not{k}}{(k-p_2)^2-i\epsilon}\frac{n_\mu}{n\cdot k}\, v(p_2)$$

$$\Downarrow \quad \not{k} = (\not{k}-\not{p}_2)+\not{p}_2$$

$$(\not{k}-\not{p}_2)\not{k} = (k-p_2)^2 + (\not{k}-\not{p}_2)\not{p}_2$$

$$\not{p}_2\, v(p_2) = 0$$

$$= \frac{g(k-p_2)^2}{(k-p_2)^2-i\epsilon}\frac{n_\mu}{n\cdot k}\, v(p_2)$$

$$= \frac{g\, n_\mu}{n\cdot k}\, v(p_2) = \frac{g\, n^\nu}{n\cdot k}\, v(p_2) \quad * \; g_{\mu\nu}$$

- G-term is suppressed!

$$\text{right} = \frac{-i(\not{k} - \not{p}_2)}{(k-p_2)^2 - i\epsilon} (ig\gamma^\nu) \, v(p_2) \, [G_{\mu\nu}]$$

$$\gamma^\nu G_{\mu\nu} = \left(\gamma^+ \bar{n}^\nu + \gamma^- n^\nu + \gamma_\perp^\nu \right) G_{\mu\nu}$$

$$\not{p}_2 \, v(p_2) = 0$$

$$\Downarrow$$

$$p_2^- \gamma^+ v(p_2) = 0$$

$\Rightarrow \quad \gamma^+ v(p_2) = 0$

on the other hand $n^\nu G_{\mu\nu} = n^\nu \left(g_{\mu\nu} - \dfrac{n_\mu k_\nu}{n\cdot k} \right)$

$$= n_\mu - n_\mu = 0$$

$\Rightarrow \quad \gamma^- n^\nu$ term vanish

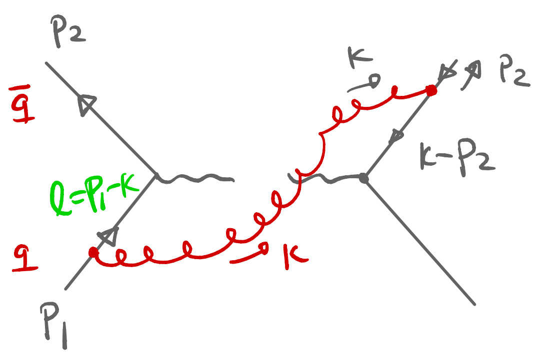
we're left with only $\gamma_\perp^\nu G_{\mu\nu}$

Since $\gamma_\perp^\nu$ will have to contract with a transverse momentum, but all transverse component $\sim \lambda Q$

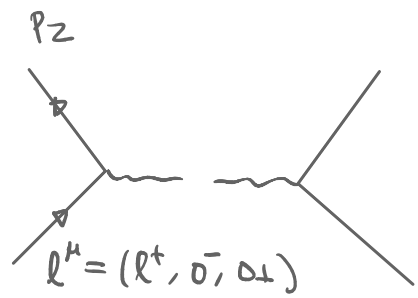
Grammer-Yennie decomposition splits gluon propagator into a longitudinal piece $K^{\mu\nu}$ that triggers a ward identity and produces the eikonal wilson line

+

a transverse piece $G^{\mu\nu}$ that would be power suppressed !



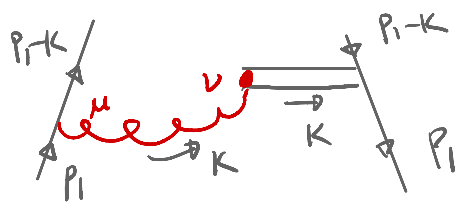
$k \parallel p_1$ $\approx$



$$\ell^\mu = (\ell^+, 0^-, 0_\perp)$$
$$= (x p_1^+, 0^-, 0_\perp)$$

$\otimes$

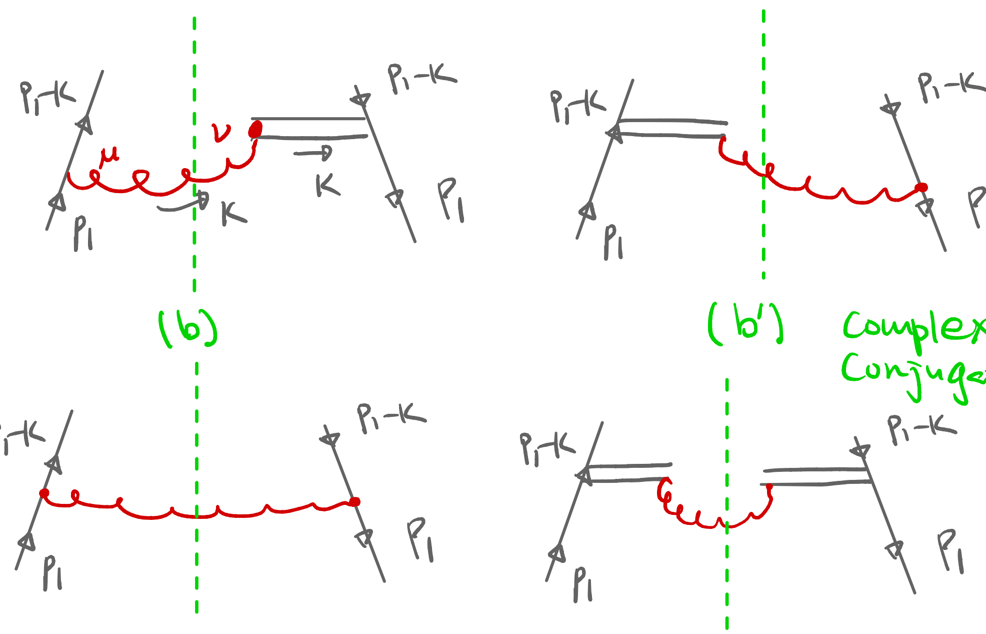
NLO contribution to TMD PDF $\longleftarrow$



add such contributions together, we have

TMDPDF

$$f_q(x, k_T^2) =$$



(b) (b') Complex Conjugate of (b)

(a) Vanish! why?

TMD Handbook

$$f_q(x, b_T) = \frac{g^2 C_F}{2\pi}\left[\frac{1+x^2}{1-x} - \epsilon(1-x)\right] \int \frac{d^{d-2}k_T}{(2\pi)^{d-2}} \frac{e^{i\vec{k}_T\cdot\vec{b}_T}}{\vec{k}_T^2}$$

$$\Big\downarrow \quad g^2 \to g^2\left(\mu^2 e^{\gamma_E}/4\pi\right)^{\epsilon}$$

$$= \frac{\alpha_s}{2\pi} C_F \left[\frac{1+x^2}{1-x} - \epsilon(1-x)\right] \Gamma(-\epsilon) \left(\frac{\mu^2}{\mu_b^2}\right)^{\epsilon}$$

- The above result still has one problem

$x \to 1$ the result is divergent

What does it mean?

recall gluon $k^\mu = [(1-x)p_1^+, k^-, \vec{k}_\perp]$

thus $x \to 1 \Rightarrow k^+ \to 0$

This $\bar{n}$-collinear gluon goes outside its correct collinear region, goes into soft region

This rapidity divergence will only cancel once combined with soft function

$$\frac{1+x^2}{1-x} \to \frac{1+x^2}{1-x} \left| \frac{k^+}{\nu/\sqrt{2}} \right|^{-\eta} = \frac{1+x^2}{1-x} \left[\frac{(1-x)p_1^+}{\nu/\sqrt{2}} \right]^{-\eta}$$

$$\frac{1+x^2}{1-x}(1-x)^{-\eta} = -\left(\frac{2}{\eta} + \frac{3}{2}\right)\delta(1-x) + \left[\frac{1+x^2}{1-x}\right]_+ + O(\eta)$$

at the end of day

$$f_q(x, b_T, \epsilon, \eta) = \frac{\alpha_s}{2\pi} C_F \Big\{ -\left(\frac{1}{\epsilon} + L_b\right) \left[P_{qq}(x)\right]_+ + (1-x)$$

$$+ \delta(1-x) \left(\frac{1}{\epsilon} + L_b\right)\left(\frac{3}{2} + \frac{2}{\eta} - 2\ln\left(\frac{x p_1^+}{\nu/\sqrt{2}}\right)\right) \Big\}$$

$$+ O(\eta) + O(\epsilon)$$

- Collins - Soper / rapidity scale ζ

a scale corresponding to the light-cone momentum carried by the struck quark

$$\zeta \propto (n \cdot P_1)^2 = 2(x p_1^+)^2$$

$$\ln\left(\frac{\sqrt{2}\, x p_1^+}{\nu}\right) = \frac{1}{2}\ln\left(\frac{\zeta}{\nu^2}\right)$$

since ζ & ν would appear in pair ζ/ν^2

one could use ζ - evolution instead of ν evolution

- combine naive TMD PDF computed

$$f_q^{(u)}(x, b_T; \epsilon, \eta)$$

and the soft function

$$\sqrt{S(b_T; \epsilon, \eta)}$$

Then the rapidity divergence would cancel out

one would only need to take care of

the standard UV divergence

$$\Rightarrow \quad f_q^{bare}(x, b_T; \epsilon, \zeta) = \frac{\alpha_s}{2\pi} C_F \, \delta(1-x) \left[\frac{1}{\epsilon_{UV}^2} - \frac{L_b^2}{2} + \left(\frac{1}{\epsilon_{UV}} + L_b \right) \left(\frac{3}{2} + \ln\left(\frac{\mu^2}{\zeta}\right) \right) - \frac{\pi^2}{12} \right]$$

$$+ \frac{\alpha_s}{2\pi} C_F \left[- \left(\frac{1}{\epsilon_{IR}} + L_b \right) \left(P_{qq} \right)_+ + (1-x) \right]$$

$$f_q(x, b_T; \mu, \zeta) = Z_{UV}(\mu, \zeta, \epsilon) \; f_q^{bare}(x, b_T, \epsilon, \zeta)$$

$$= \frac{\alpha_s}{2\pi} C_F \Big[-\Big(\frac{1}{\epsilon_{IR}} + L_b\Big) \big(P_{qq}(x) \big)_+$$

$$+ (1-x) - \frac{L_b^2}{2} + L_b\Big(\frac{3}{2} + \ln\frac{\mu^2}{\zeta}\Big) - \frac{\pi^2}{12} \Big]$$

$$\frac{d \ln f_q(x, b_T; \mu, \zeta)}{d \ln \mu} = \gamma_\mu^q(\mu, \zeta)$$

$$\frac{d \ln f_q(x, b_T; \mu, \zeta)}{d \ln \sqrt{\zeta}} = \gamma_\zeta^q(\mu, b_T)$$

$$\frac{d \gamma_\zeta^q(\mu, b_T)}{d \ln \mu} = -2\, \Gamma_{cusp}^q(\mu, b_T)$$

$$\Rightarrow \boxed{\begin{aligned} f_q(x, b_T; \mu, \zeta) = \; & f_q(x, b_T; \mu_0, \zeta_0) \\ & * \exp\left[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_\mu^q(\mu', \zeta_0)\right] \\ & * \exp\left[\frac{1}{2} \gamma_\zeta^q(\mu, b) \ln\left(\frac{\zeta}{\zeta_0}\right)\right] \end{aligned}}$$

Eventually, we have TMD evolution (μ, ζ) for TMDPDF, and standard RG evolution (μ) for hard function, thus the resummation of $\log\left(\frac{Q^2}{q_T^2}\right)$ is more transparent than "Sudakov resummation"

It also allows you to take advantage of universality of TMDPDF (or sign change) across different processes!

$$\frac{d\sigma^{DY}}{dQ^2\,dy\,d^2q_T} \sim H_{q\bar{q}}(Q,\mu)$$

$$* \int d^2b_T\, e^{i\vec{q}_T\cdot\vec{b}_T}\, f_q(x_1, b_T; \mu, \zeta_1)$$

$$* f_{\bar{q}}(x_2, b_T; \mu, \zeta_2)$$

Similarly for SIDIS

$$e + p \to e' + h + X$$

$$\frac{d\sigma^{SIDIS}}{dx\,dy\,dz_h\,d^2P_{hT}} \sim H^{DIS}(Q,\mu)$$

$$* \int d^2b_T\, e^{i\vec{b}_T\cdot\vec{P}_{hT}/z_h}\, f_q(x, b_T; \mu, \zeta_1)$$

$$* D_{q\to h}(z_h, b_T; \mu, \zeta_2)$$