Small x physics

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Outline

Lecture 1:
Introduction to Deep Inelastic Scattering
Parton model
Collinear framework: factorization
DGLAP evolution equations
Parton distribution functions from DGLAP
Nuclear structure functions
Nuclear PDFs
EIC prospects for inclusive DIS and nPDFs

Lecture 2:
Intro: Regge theory and Pomeron
Outline of BFKL construction:
- Effective Lipatov vertex
- Gluon reggeization: trajectory

BFKL equation
Eigenvalue. Collinear structure
Properties of the solution:
- Diffusion
- Increase with energy

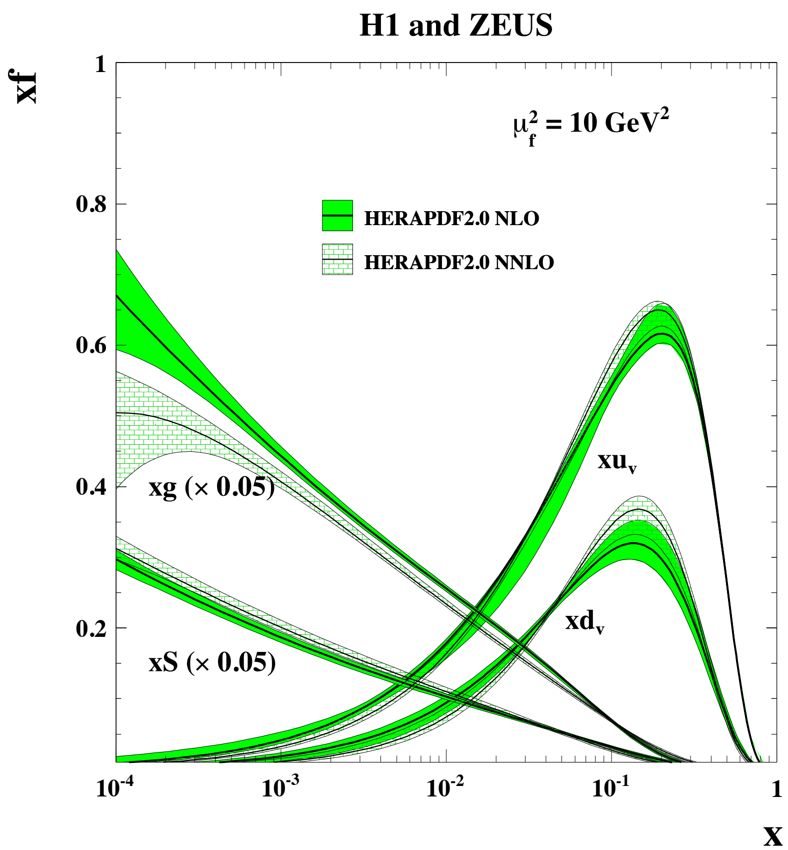
Small x anomalous dimension

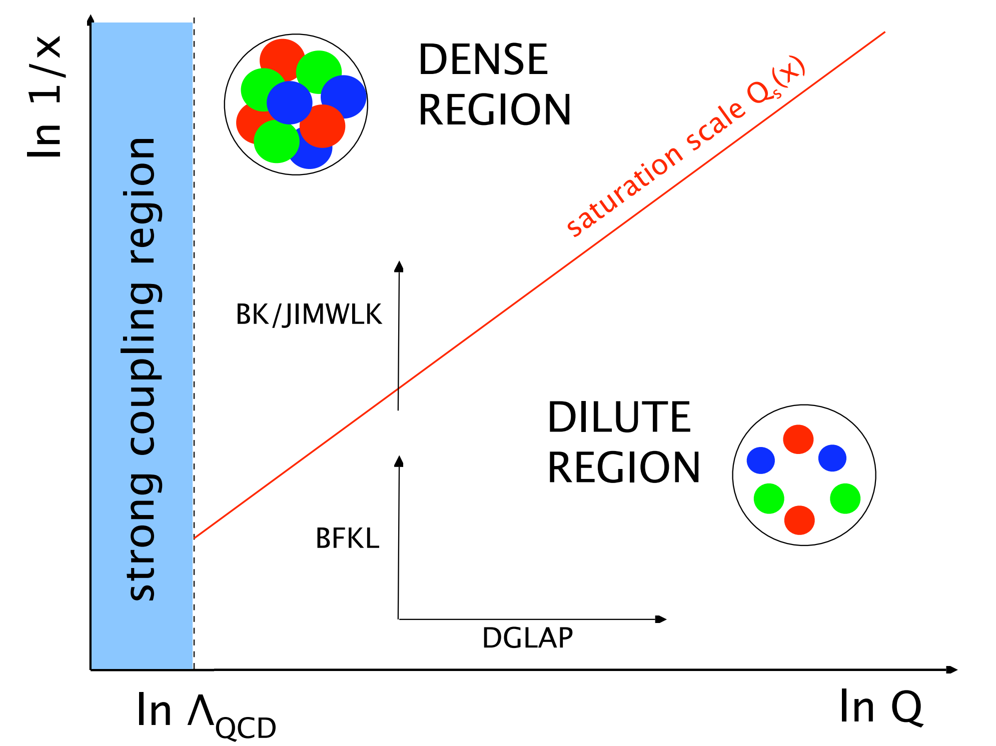
Lecture 3:
BFKL at NLO: large correction
Collinear limit of NLO BFKL
Resummation:
- Kinematical constraint, shifts of poles
- DGLAP anomalous dimension

Resummed result in Mellin space
Resummed result in momentum space
Improved small x splitting function
Phenomenology examples

Lecture 4:
Dipole model: GBW example
BFKL revisited: Mueller dipole evolution
Multiple rescattering: BK evolution
Properties of solution to BK equation
Saturation scale
Impact parameter dependence
Phenomenology examples: structure functions, diffraction, angular decorelations

Parton saturation





Gribov, Levin, Ryskin

QCD evolution leads to the **strong growth of the gluon density**

Parton saturation: additional modification due to the **gluon recombination**

Evolution will include **nonlinear** terms in density

Saturation scale: divide between dilute and dense region

$$\frac{A \times xg(x,Q_s^2)}{\pi A^{2/3}} \times \frac{\alpha_s(Q_s^2)}{Q_s^2} \sim 1 \qquad Q_s^2 \sim A^{1/3} Q_0^2 \left(\frac{1}{x}\right)^{\lambda}$$

Saturation scale

Theoretical frameworks for describing parton saturation

Nonlinear evolution equations in QCD:

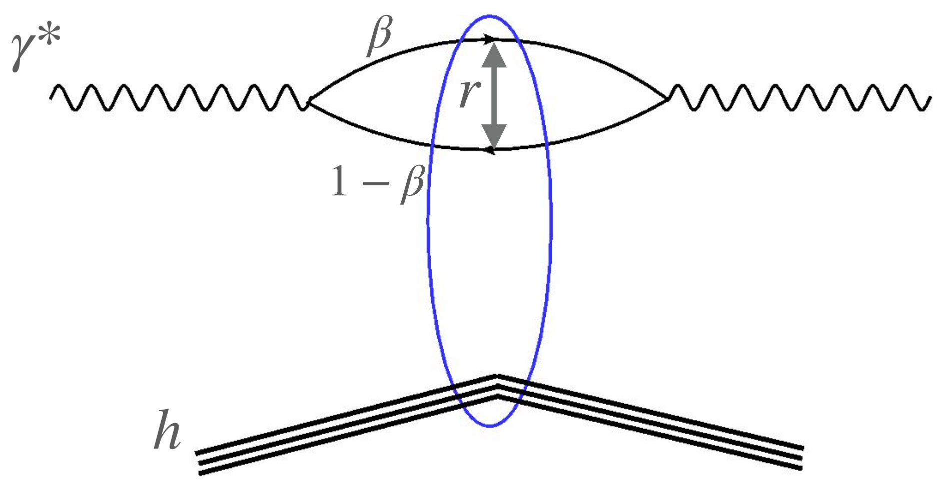
- *Gribov-Levin-Ryskin* nonlinear equation
- *Mueller-Qiu* equation
- *Kovchegov* nonlinear equation for dipoles
- *Balitsky* hierarchy for correlators of Wilson lines.
- Color Glass Condensate (*McLerran-Venugopalan*) with JIMWLK (*Jalilian-Marian,Iancu,McLerran,Weigert,Leonidov,Kovner*) renormalization group equation for high density QCD.

Models implementing saturation:

- Golec-Biernat and Wusthoff saturation model
- McLerran-Venugopalan
- …many others

Dipole representation

DIS at small x / high energy: **dipole** representation



The virtual photon fluctuates into quark-antiquark pair

The quark-antiquark pair interacts with the target

In the proton rest frame the Ioffe time

$$\tau_{q\bar{q}} \sim \frac{1}{\Delta E} \qquad \text{with} \qquad \Delta E \simeq x M_p$$

The transverse momentum of the quark $\boldsymbol{\kappa}$ is traded for the Fourier conjugate variable $\mathbf{r}$, the transverse separation of the $q\bar{q}$ pair

Recall: DIS cross section at small x for transverse polarized photons (k_T factorization):

$$\sigma_T(x, Q^2) = \frac{\alpha_{em}}{\pi} \sum_q e_q^2 \int \frac{d^2\mathbf{k}}{k^4} \int_0^1 d\beta \int d^2\boldsymbol{\kappa}\, \alpha_s \left\{ \left[\beta^2 + (1-\beta^2)\right] \left(\frac{\boldsymbol{\kappa}}{D_{1q}} - \frac{\boldsymbol{\kappa} - \boldsymbol{k}}{D_{2q}} \right)^2 + m_q^2 \left(\frac{1}{D_{1q}} - \frac{1}{D_{2q}} \right)^2 \right\} f\left(x, k^2\right)$$

In the strict high energy limit, argument of the gluon x is the same as Bjorken x

- $\boldsymbol{\kappa}$ transverse momentum of quark
- $\mathbf{k}$ transverse momentum of the gluon
- β fraction of the momentum of the photon carried by the quark

$$D_{1q} = \kappa^2 + \beta(1-\beta)Q^2 + m_q^2 \quad , D_{2q} = (\boldsymbol{\kappa} - \boldsymbol{k})^2 + \beta(1-\beta)Q^2 + m_q^2$$

Dipole representation

Using the Fourier transforms

$$\overline{Q}^2 = \beta(1-\beta)Q^2 + m_q^2$$

$$\left(\frac{\boldsymbol{\kappa}}{D_{1q}} - \frac{\boldsymbol{\kappa}-\boldsymbol{k}}{D_{2q}}\right) = \int \frac{d^2\mathbf{r}}{2\pi} e^{-i\boldsymbol{\kappa}\cdot\mathbf{r}}(1-e^{i\mathbf{k}\cdot\mathbf{r}}) i\frac{\mathbf{r}}{r}\overline{Q}K_1(\overline{Q}\mathbf{r})$$

$$\left(\frac{1}{D_{1q}} - \frac{1}{D_{2q}}\right) = \int \frac{d^2\mathbf{r}}{2\pi} e^{-i\boldsymbol{\kappa}\cdot\mathbf{r}}(1-e^{i\mathbf{k}\cdot\mathbf{r}})\, K_0(\overline{Q}\mathbf{r})$$

DIS cross section (k_T factorization transformed into coordinate space)

$$\sigma_T(x,Q^2) = \frac{\alpha_{em}}{\pi}\sum_q e_q^2 \int d^2\mathbf{r} \int_0^1 d\beta \left\{[\beta^2 + (1-\beta)^2]\overline{Q}^2 K_1^2(\overline{Q}r) + m_q^2 K_0^2(\overline{Q}r)\right\}$$

$$\times \int \frac{d^2\mathbf{k}}{k^4}\alpha_s\, f(x,k^2)\,(1-e^{-i\mathbf{k}\cdot\mathbf{r}})(1-e^{i\mathbf{k}\cdot\mathbf{r}})$$

photon wave function

$$|\Psi_T^q(\mathbf{r},\beta,Q^2)|^2 = \frac{3\alpha_{\rm em}}{2\pi^2} e_q^2 \left\{[\beta^2 + (1-\beta)^2]\overline{Q}^2 K_1^2(\overline{Q}r) + m_q^2 K_0^2(\overline{Q}r)\right\}$$

dipole cross section

$$\hat{\sigma}(x,\mathbf{r}) = \frac{2\pi}{3}\int \frac{d^2\mathbf{k}}{k^4}\,\alpha_s\, f(x,k^2)(1-e^{i\mathbf{k}\cdot\mathbf{r}})\,(1-e^{-i\mathbf{k}\cdot\mathbf{r}})$$

dipole representation for DIS

$$\sigma_{T,L}(x,Q^2) = \int d^2\mathbf{r}\int_0^1 d\beta \sum_q |\Psi_{T,L}^q(\mathbf{r},\beta,Q^2)|^2\, \hat{\sigma}(x,\mathbf{r})$$

Model with saturation

Golec-Biernat and Wusthoff (GBW) model

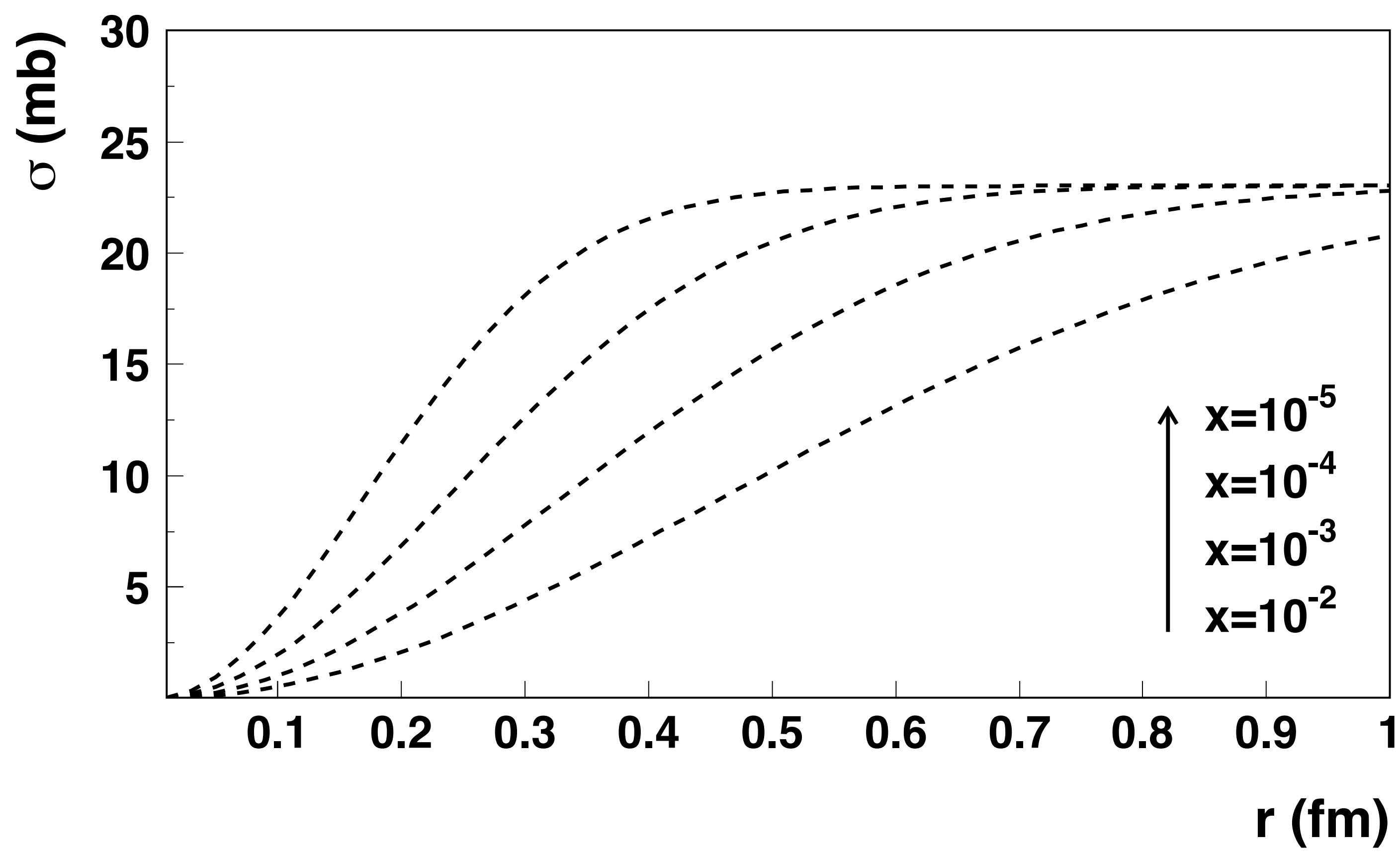
$$\hat{\sigma}(x,r) = \sigma_0 \left[1 - \exp(-Q_s^2(x) r^2/4)\right]$$

x dependent saturation scale $$Q_s^2(x) = Q_0^2 \left(\frac{x}{x_0}\right)^{-\lambda}$$

Fit parameters:

$\sigma_0, Q_0, x_0, \lambda$

(or saturation radius) $$R_0(x) = 1/Q_s(x)$$



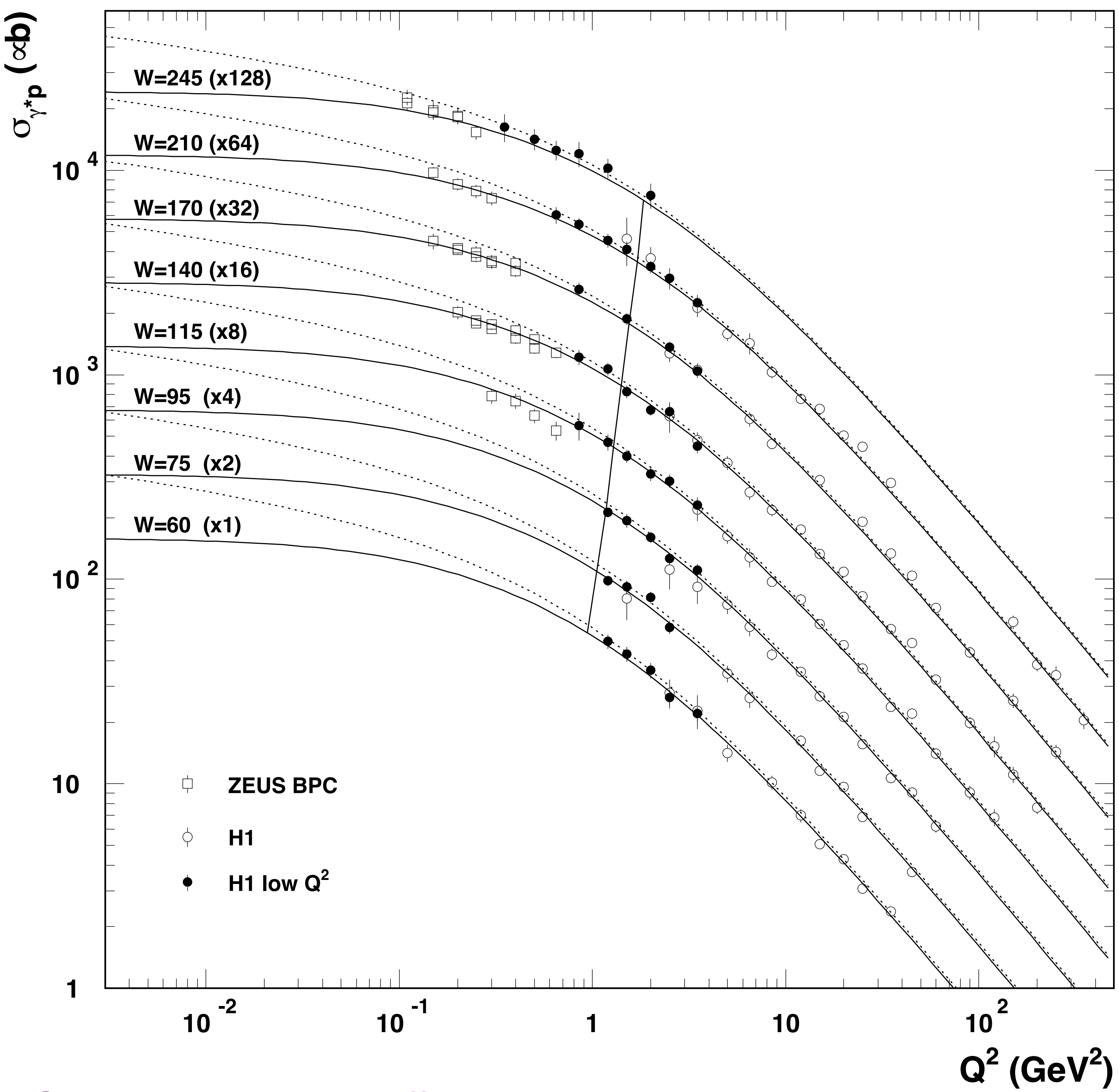
For large dipoles cross section goes to a constant

For small perturbative dipoles the cross section is $\sim r^2 Q_s^2(x)$

Saturation scale divides the dilute and dense regions

It is x-dependent and the transition occurs for smaller dipoles as x decreases

GBW model and HERA data



Golec-Biernat, Wusthoff

$\gamma^* p$ cross section $\sigma_T + \sigma_L$ from HERA for various energies as a function of Q^2

Dashed line: massless quarks

Solid line: $m_q = 140\,\mathrm{MeV}$

Masses important for low values of Q^2

Diagonal line: position of the saturation scale

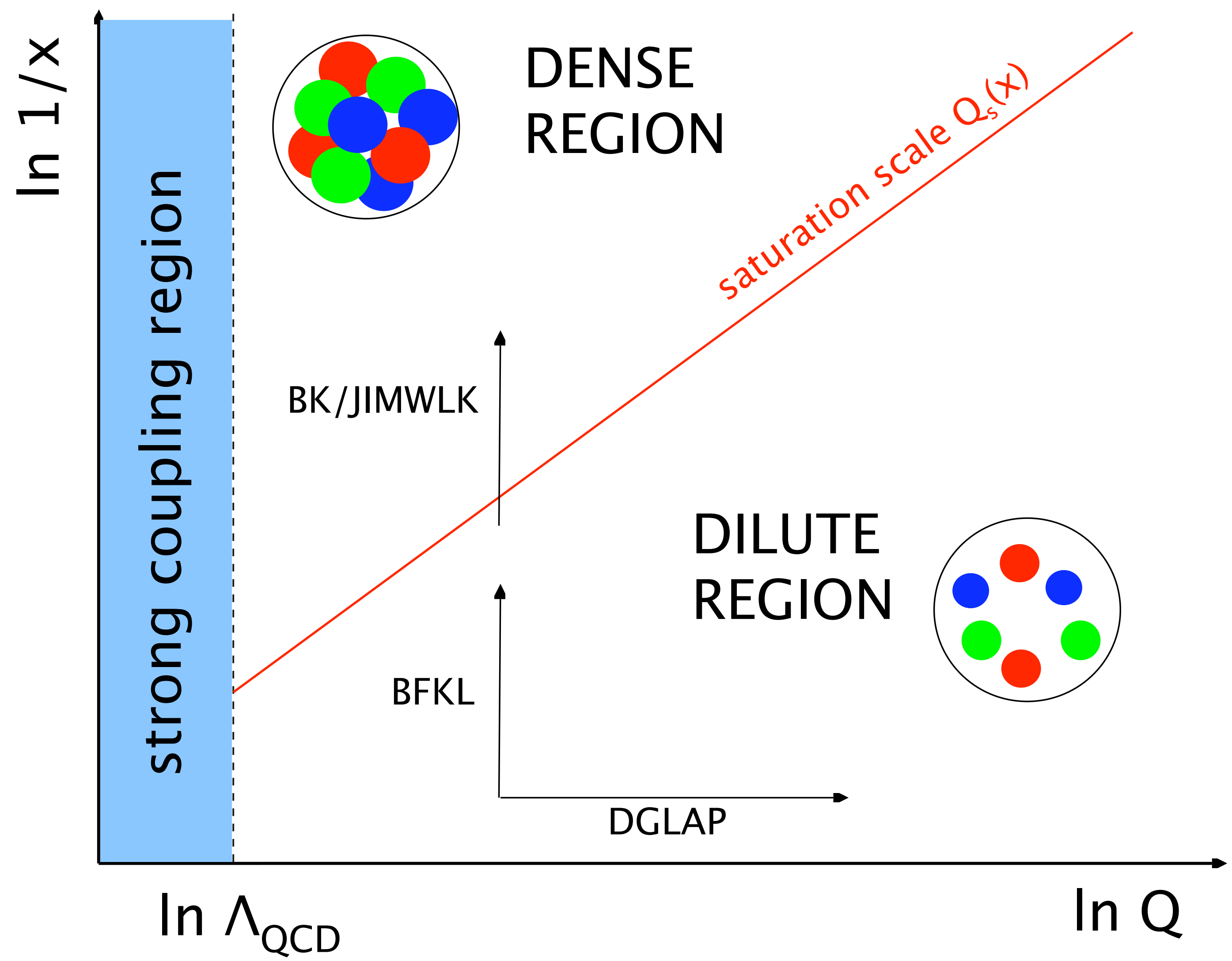
Successful description of the inclusive HERA data

Good description of the diffraction

Parton saturation and nonlinear evolution

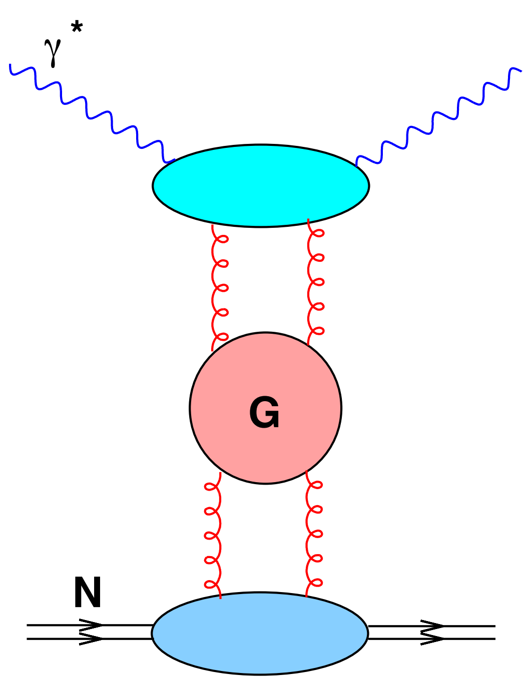
GBW model incorporates the **saturation** scale which divides dense and dilute regime

How such corrections can be included within the **parton evolution** ?

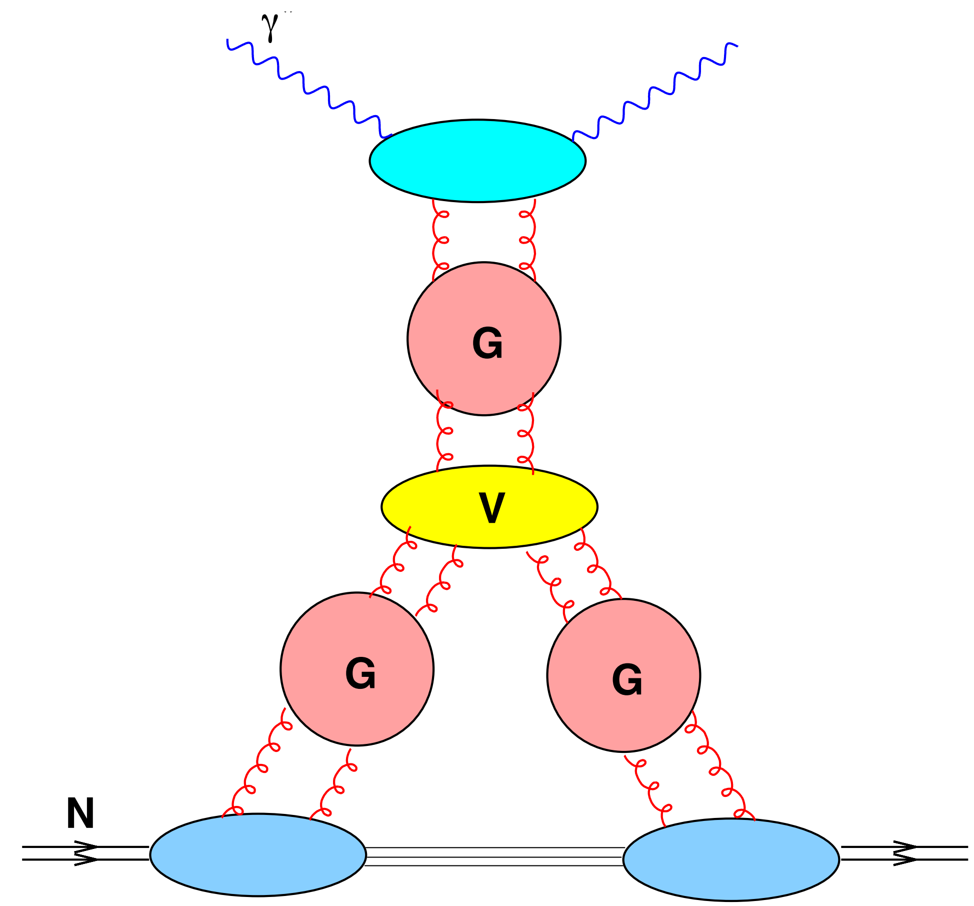


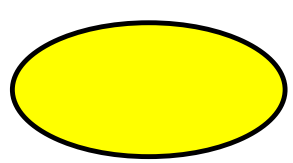
Nonlinear evolution for the gluon density

Linear evolution



'Fan' diagram structure





is a $2 \to 4$ gluon transition vertex.
'Fan' diagrams are contained in GLR equation

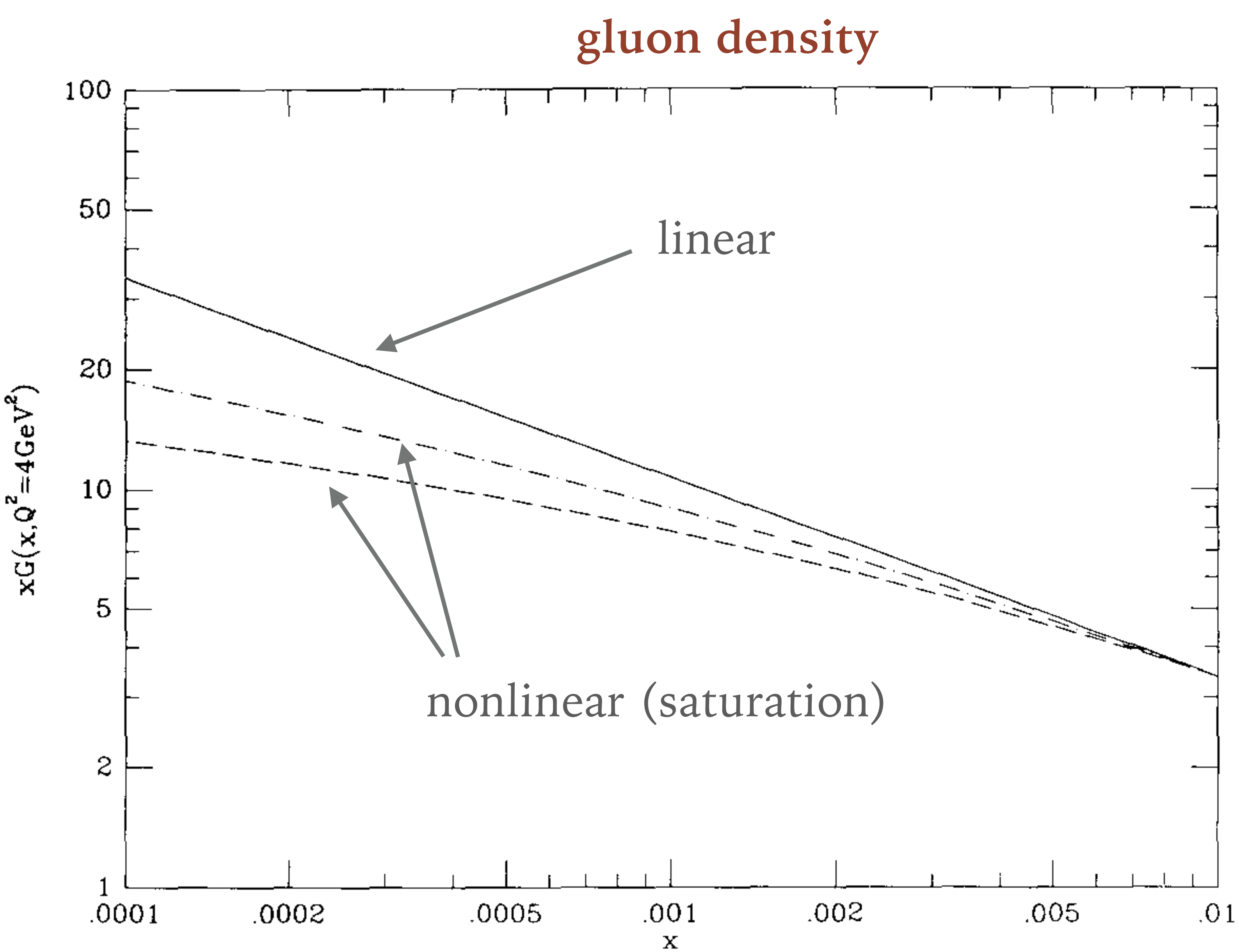
$$\frac{\partial^2 xg(x,t)}{dt\, d\ln 1/x} = \frac{\alpha_s N_c}{\pi} xg(x,t) - \frac{81\alpha_s^2}{16} \frac{1}{R^2 Q^2} [xg(x,t)]^2$$

Gribov, Levin, Ryskin

with $t = \ln Q^2/Q_0^2$. R target radius.

Solution to the nonlinear GLR equation

Collins, Kwiecinski



Solution to the nonlinear evolution (with saturation corrections)

Compared to the pure DGLAP (linear evolution)

Reduction at smallest x

The reduction depends on the parameter R

Smaller R : stronger reduction ($5\,\mathrm{GeV}^{-1},\ 5/\sqrt{2}\,\mathrm{GeV}^{-1}$)

DLLA approximation

$$\frac{\partial^2 xg(x,t)}{dt\, d\ln 1/x} = \frac{\alpha_s N_c}{\pi} xg(x,t) - \frac{81\alpha_s^2}{16}\frac{1}{R^2Q^2}[xg(x,t)]^2$$

Nonlinear equation at small x: BK equation and CGC

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NUCLEAR PHYSICS B

Operator expansion for high-energy scattering

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Abstract

I demonstrate that the leading logarithms for high-energy scattering can be obtained as a result of evolution of the non-local operators—straight-line ordered gauge factors—with respect to the slope of the straight line.

PACS: 13.60.Hb; 12.38.Bx; 12.38.Cy
Keywords: small-*x*; pomeron; BFKL

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TPI-MINN-99/05

Small–x F_2 Structure Function of a Nucleus Including Multiple Pomeron Exchanges

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We derive an equation determining the small-x evolution of the F_2 structure function of a large nucleus which includes all multiple pomeron exchanges in the leading logarithmic approximation using Mueller's dipole model [1–4]. We show that in the double leading logarithmic limit this evolution equation reduces to the GLR equation [5,6].

Kovchegov approach based on the Mueller dipole framework to BFKL

Balitsky: Wilson line operators, arrives at hierarchy of equations, the first of which can be recast into Balitsky- Kovchegov (BK) equation

Color Glass Condensate (*McLerran-Venugopalan*) with corresponding JIMWLK equation (*Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner*)

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NUCLEAR PHYSICS B

Soft gluons in the infinite-momentum wave function and the BFKL pomeron *

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Accepted for publication 8 November 1993

We construct the infinite-momentum wave function for arbitrary numbers of soft gluons in a heavy quark–antiquark, onium, state. The soft gluon part of the wave function is constructed exactly within the leading logarithmic and large-N_c limits. The BFKL pomeron emerges when gluon number densities are evaluated.

Dipole picture: onium wavefunction

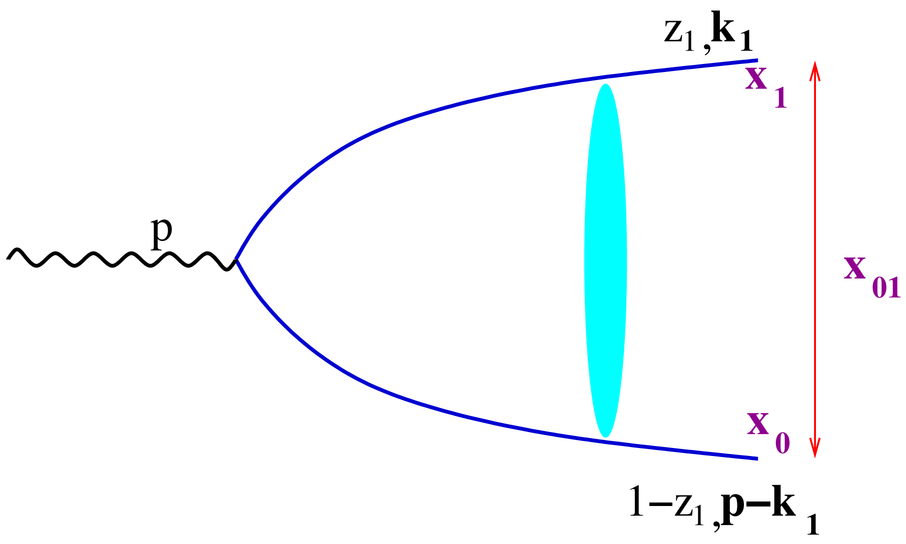
A.H.Mueller 94:

Heavy quark-antiquark (onium) wavefunction (without any gluons):

$$\psi^{(0)}_{\alpha\beta}(\mathbf{x}_0,\mathbf{x}_1,z_1) \;=\; \int \frac{d^2\mathbf{k}_1}{(2\pi)^2} e^{i\mathbf{x}_{01}\cdot\mathbf{k}_1}\psi^{(0)}_{\alpha\beta}(\mathbf{k}_1,z_1)$$

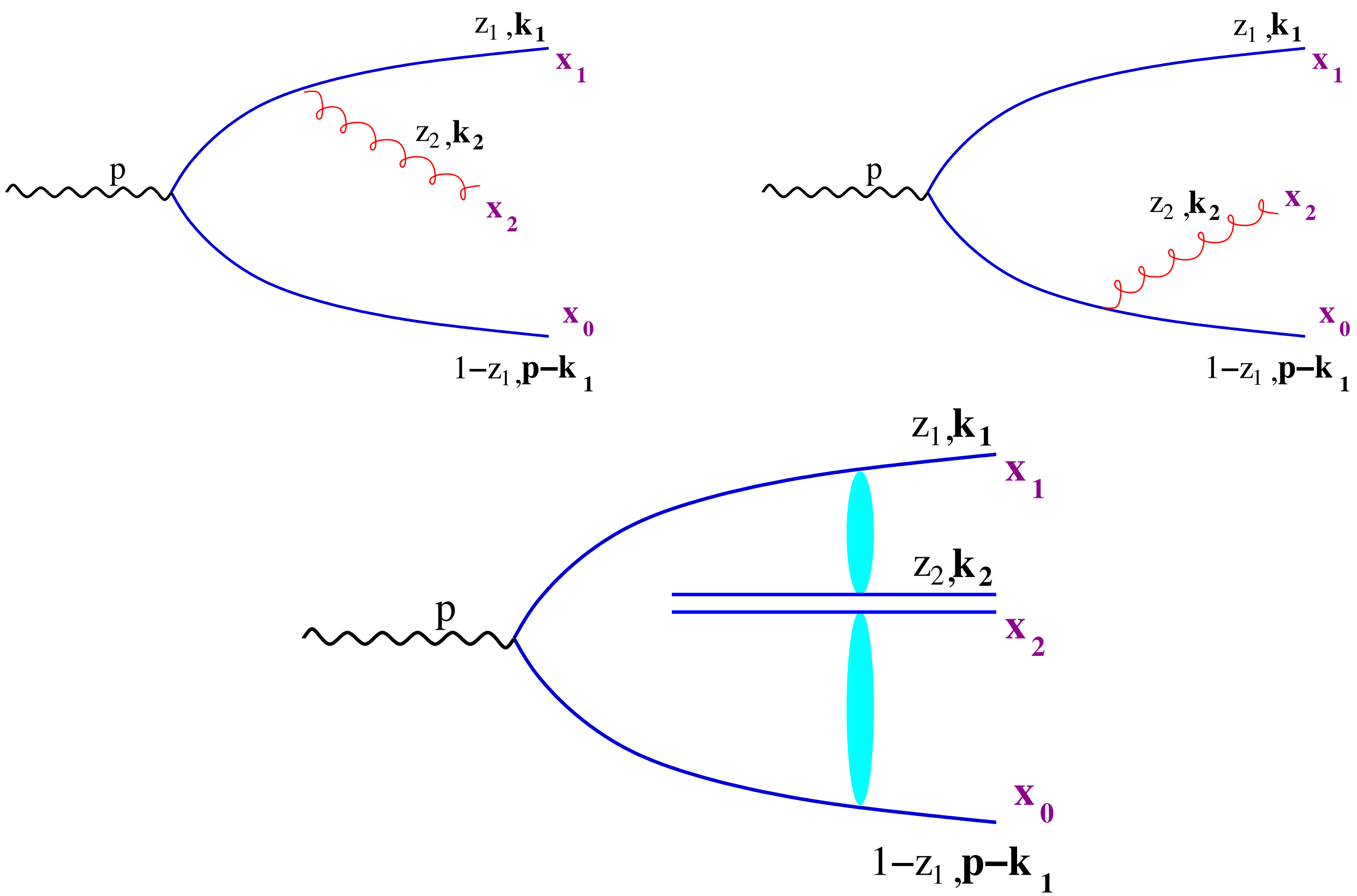
$$\Phi^{(0)}(\mathbf{x}_0,\mathbf{x}_1,z_1) \;=\; \sum_{\alpha,\beta}|\psi^{(0)}_{\alpha\beta}(\mathbf{x}_0,\mathbf{x}_1,z_1)|^2$$

$\mathbf{x}_{01}=\mathbf{x}_0-\mathbf{x}_1$ is a transverse size of the $q\bar{q}$ - onium pair, $z_1=\frac{k_{1+}}{p_+}$ (p photon momentum), $\mathbf{k}_1$ transverse momentum.



Dipole picture: onium wavefunction with one gluon

Onium wavefunction with one soft gluon. Soft means $z_2/z_1 \ll 1$:

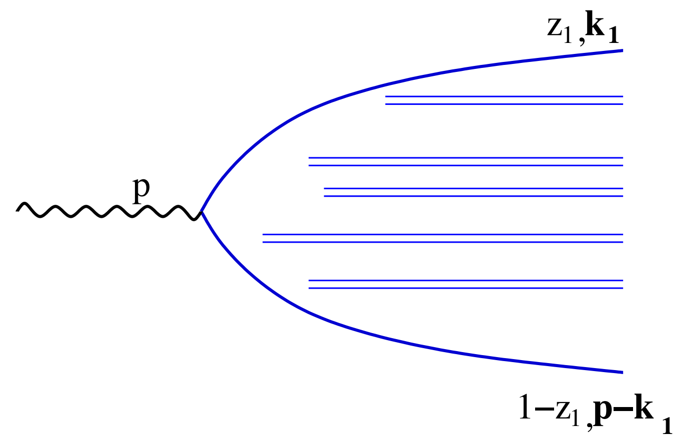


$$\Phi^{(1)}(\mathbf{x}_0, \mathbf{x}_1, z_1) = \frac{\alpha_s C_F}{\pi^2} \int_{z_0}^{z_1} \frac{dz_2}{z_2} \int d^2\mathbf{x}_2 \frac{x_{01}^2}{x_{20}^2 x_{12}^2} \Phi^{(0)}(\mathbf{x}_0, \mathbf{x}_1, z_1)$$

Branching of a single dipole $\mathbf{x}_{01}$ into two $\mathbf{x}_{20}, \ \mathbf{x}_{12}$

Generating functional for dipoles

Wavefunction with
n dipoles:



$$\Phi^{(n)}(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{n+1}) = \Phi^{(0)} \frac{\delta}{\delta u(\mathbf{x}_2)} \frac{\delta}{\delta u(\mathbf{x}_3)} \cdots \frac{\delta}{\delta u(\mathbf{x}_{n+1})} Z(\mathbf{x}_0, \mathbf{x}_1, z_1, u)|_{u=0}$$

probability of finding n dipoles at positions $\mathbf{x}_k, k = 2, \ldots, n$.
Generating functional: $Z(\mathbf{b}_{01}, \mathbf{x}_{01}, z_1, u = 1) = 1$

$$\frac{dZ(\mathbf{b}_{01}, \mathbf{x}_{01}, y, u)}{dy} =$$

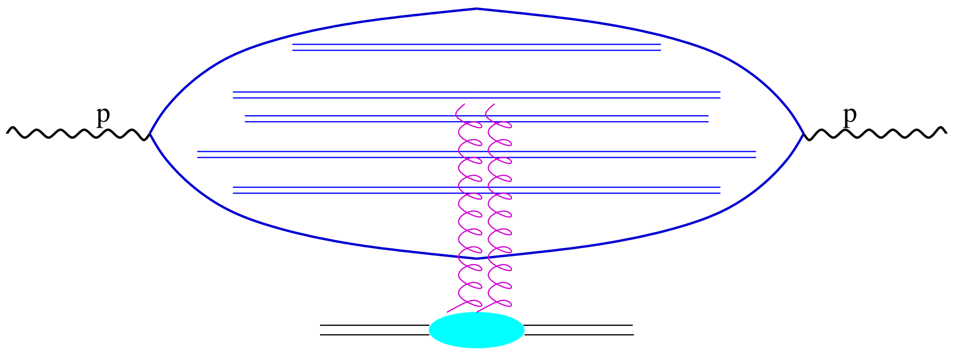
$$\int \frac{d^2\mathbf{x}_2 \mathbf{x}_{01}^2}{\mathbf{x}_{20}^2 \mathbf{x}_{12}^2} \left[Z(\mathbf{b}_{01} + \frac{\mathbf{x}_{12}}{2}, \mathbf{x}_{20}, y, u)\, Z(\mathbf{b}_{01} - \frac{\mathbf{x}_{20}}{2}, \mathbf{x}_{12}, y, u) \; - \; Z(\mathbf{b}_{01}, \mathbf{x}_{01}, y, u) \right]$$

$\mathbf{b}_{01} \equiv \frac{\mathbf{x}_0 + \mathbf{x}_1}{2}, \; \mathbf{x}_{01} \equiv \mathbf{x}_0 - \mathbf{x}_1$

Dipole scattering amplitude

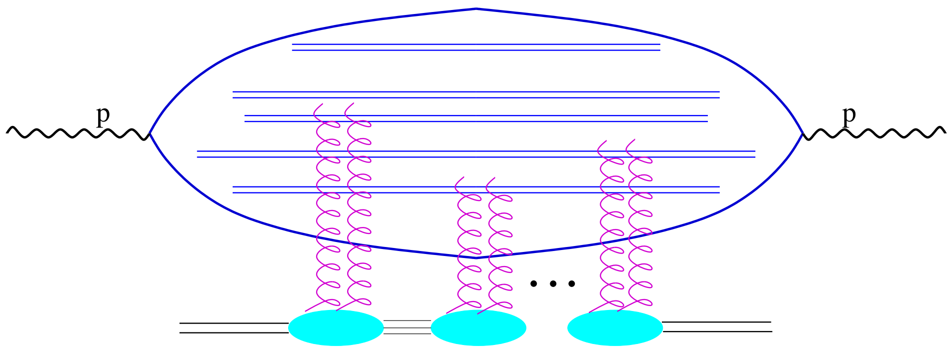
Need to construct the amplitude for scattering of dipoles on target.

One scattering



Linear evolution

Multiple scatterings



Nonlinear evolution

Dipole number densities:

$$n_1(x_{01},\mathbf{x},\mathbf{b}-\mathbf{b}_0,Y) = \frac{\delta}{\delta u(\mathbf{b},\mathbf{x})} Z(\mathbf{b}_{01},\mathbf{x}_{01},Y,u)|_{u=1}$$

Generally for k dipoles:

$$n_k = \Pi_{i=1}^{k} \frac{\delta}{\delta u(\mathbf{b}_i,\mathbf{x}_i)} Z|_{u=1}$$

Amplitude for dipole target scattering

One scattering:

$$N_1(\mathbf{x}_{01},\mathbf{b}_{01},Y) = \int d[\mathcal{P}_1] n_1\, \gamma_1$$

Multiple scatterings:

$$N(\mathbf{x}_{01},\mathbf{b}_{01},Y) \; = \sum_{k=1}^{\infty} \int d[\mathcal{P}_k]\, n_k\, \gamma_1 \ldots \gamma_k$$

- $d[\mathcal{P}]_k = \Pi_{i=1}^{i=k} \frac{d^2\mathbf{x}_i}{2\pi x_i^2} d^2\mathbf{b}_i$ phase space measure
- $\gamma_k \equiv \gamma_k(\mathbf{x}_k, \mathbf{b}_k)$ propagator of single dipole in the nucleus.

Equation for generating functional Z

↓

Set of equations for densities n_k

↓

Closed equation for amplitude N

Dipole cross section vs dipole scattering amplitude: $\hat{\sigma}(x,r) = 2 \int d^2\mathbf{b}\, N(\mathbf{r},\mathbf{b},Y=\ln 1/x)$

Equations for the dipole amplitude

Linear equation for 'one-scattering' amplitude N_1 *(A.H.Mueller 94)*:

$$\frac{dN_1(\mathbf{b}_{01},\mathbf{x}_{01},Y)}{dY}=\bar{\alpha}_s\int\frac{d^2\mathbf{x}_2\,\mathbf{x}_{01}^2}{\mathbf{x}_{20}^2\,\mathbf{x}_{12}^2}\Big[N_1(\mathbf{b}_{01}+\frac{\mathbf{x}_{12}}{2},\mathbf{x}_{20},Y)+N_1(\mathbf{b}_{01}-\frac{\mathbf{x}_{20}}{2},\mathbf{x}_{12},Y)-N_1(\mathbf{b}_{01},\mathbf{x}_{01},Y)\Big]$$

Dipole version of the BFKL equation

Non-linear equation for 'mutliple-scattering' amplitude N *(Yu.Kovchegov 99)*:

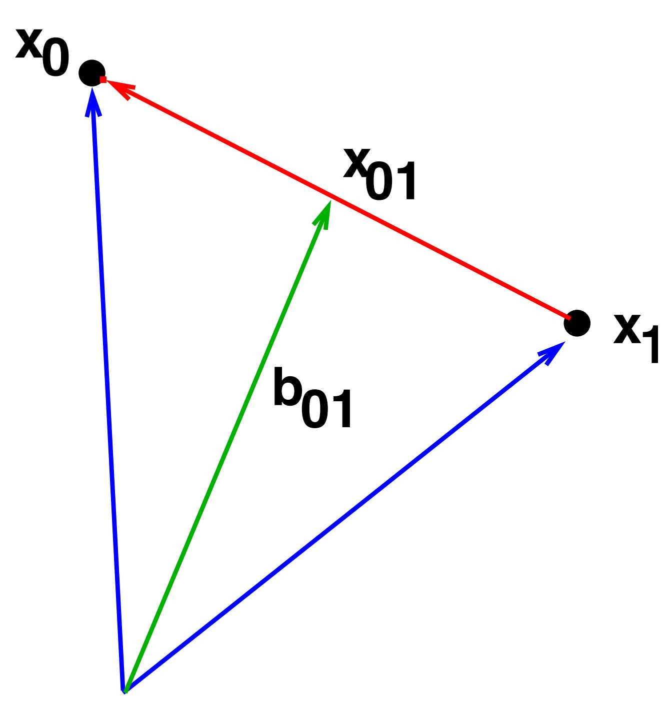
$$\frac{dN(\mathbf{b}_{01},\mathbf{x}_{01},Y)}{dY}=\bar{\alpha}_s\int\frac{d^2\mathbf{x}_2\,\mathbf{x}_{01}^2}{\mathbf{x}_{20}^2\,\mathbf{x}_{12}^2}\Big[N(\mathbf{b}_{01}+\frac{\mathbf{x}_{12}}{2},\mathbf{x}_{20},Y)+N(\mathbf{b}_{01}-\frac{\mathbf{x}_{20}}{2},\mathbf{x}_{12},Y)-N(\mathbf{b}_{01},\mathbf{x}_{01},Y)-N(\mathbf{b}_{01}+\frac{\mathbf{x}_{12}}{2},\mathbf{x}_{20},Y)\,N(\mathbf{b}_{01}-\frac{\mathbf{x}_{20}}{2},\mathbf{x}_{12},Y)\Big]$$

Solution to BK

$$\frac{dN(\mathbf{b}_{01},\mathbf{x}_{01},Y)}{dY} = \bar{\alpha}_s \int \frac{d^2\mathbf{x}_2\,\mathbf{x}_{01}^2}{\mathbf{x}_{20}^2\,\mathbf{x}_{12}^2}\Big[N(\mathbf{b}_{01}+\frac{\mathbf{x}_{12}}{2},\mathbf{x}_{20},Y)+N(\mathbf{b}_{01}-\frac{\mathbf{x}_{20}}{2},\mathbf{x}_{12},Y)$$
$$- N(\mathbf{b}_{01},\mathbf{x}_{01},Y) - N(\mathbf{b}_{01}+\frac{\mathbf{x}_{12}}{2},\mathbf{x}_{20},Y)\,N(\mathbf{b}_{01}-\frac{\mathbf{x}_{20}}{2},\mathbf{x}_{12},Y)\Big]$$

$N(\mathbf{b}_{01},\mathbf{x}_{01},Y)$ - amplitude for dipole-hadron scattering

- Evolution in $Y = \ln 1/x$ rapidity
- Need to specify initial conditions $N^{(0)}(\mathbf{b}_{01},\mathbf{x}_{01},Y=0)$ which depend on the target.
- α_s fixed $\longrightarrow$ LLx approximation
- $\mathbf{b}_{01}$ impact parameter of dipole, $\mathbf{x}_{01}$ size of the dipole
  $\rightarrow (4+1)$ dimensions



$$\mathbf{r}_{01} \equiv \mathbf{x}_{01}$$

Solution to BK

$$\frac{dN(\mathbf{b}_{01},\mathbf{x}_{01},Y)}{dY} = \bar{\alpha}_s \int \frac{d^2\mathbf{x}_2\,\mathbf{x}_{01}^2}{\mathbf{x}_{20}^2\,\mathbf{x}_{12}^2} \Big[N(\mathbf{b}_{01}+\frac{\mathbf{x}_{12}}{2},\mathbf{x}_{20},Y) + N(\mathbf{b}_{01}-\frac{\mathbf{x}_{20}}{2},\mathbf{x}_{12},Y) - N(\mathbf{b}_{01},\mathbf{x}_{01},Y) - N(\mathbf{b}_{01}+\frac{\mathbf{x}_{12}}{2},\mathbf{x}_{20},Y)\,N(\mathbf{b}_{01}-\frac{\mathbf{x}_{20}}{2},\mathbf{x}_{12},Y)\Big]$$

Search for fixed points:

$$\frac{dN(\mathbf{b}_{01},\mathbf{x}_{01},Y)}{dY} = 0$$

- $N=0$
- $N=1$

Toy model in $(0+1)$ dimensions, $N \equiv N(Y)$:

$$\frac{dN}{dY} = \omega\,(N - N^2),\ \omega > 0$$

Verhulst or logistic equation used as a model for population growth

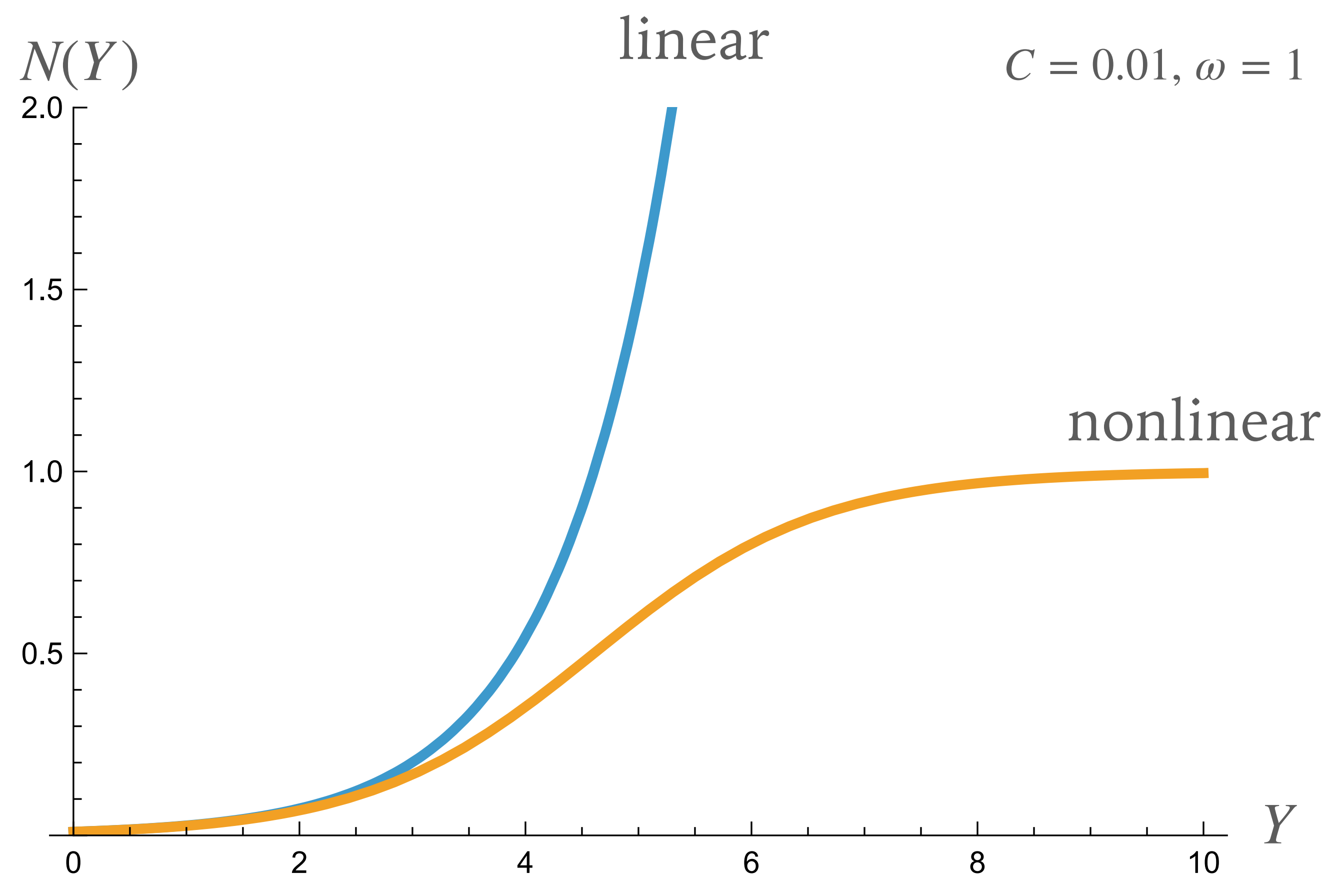
Toy model: 0 transverse dimensions

- Solution to linear part:

$$N_1 = Ce^{\omega Y}$$

- Solution to nonlinear eqn.:

$$N = \frac{Ce^{\omega Y}}{1 + Ce^{\omega Y}}$$



Nonlinear solution saturates to 1 at large Y

$$\forall_{C\neq 0} \;\; N(Y) \stackrel{Y\to\infty}{\longrightarrow} 1$$

Solution to BK: 1 transverse dimension

$$\frac{dN(\mathbf{b}_{01},\mathbf{x}_{01},Y)}{dY} = \bar{\alpha}_s \int \frac{d^2\mathbf{x}_2\,\mathbf{x}_{01}^2}{\mathbf{x}_{20}^2\,\mathbf{x}_{12}^2}\Big[N(\mathbf{b}_{01}+\frac{\mathbf{x}_{12}}{2},\mathbf{x}_{20},Y)+N(\mathbf{b}_{01}-\frac{\mathbf{x}_{20}}{2},\mathbf{x}_{12},Y) - N(\mathbf{b}_{01},\mathbf{x}_{01},Y) - N(\mathbf{b}_{01}+\frac{\mathbf{x}_{12}}{2},\mathbf{x}_{20},Y)\,N(\mathbf{b}_{01}-\frac{\mathbf{x}_{20}}{2},\mathbf{x}_{12},Y)\Big]$$

- Kernel depends only on sizes x_{01}, x_{02}, x_{12}.
- Assume solution has translational invariance:

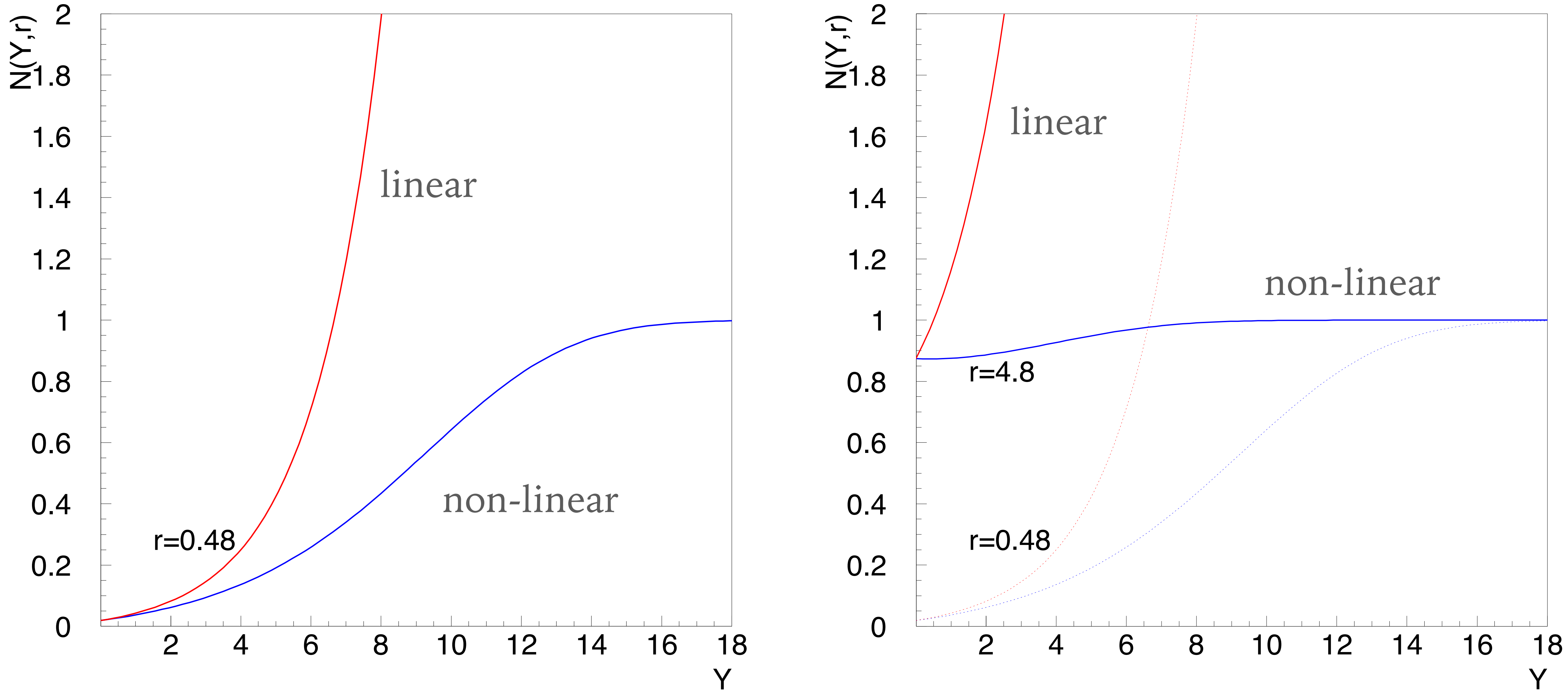
$$N(\mathbf{b}_{01},\mathbf{x}_{01},Y) \rightarrow N(|\mathbf{x}_{01}|,Y)$$

No impact parameter b_{01} dependence

- Physically corresponds to uniform and infinite nucleus.

Solution to BK: 1 transverse dimension

Rapidity dependence of $N(Y, r = |x_{01}|)$:

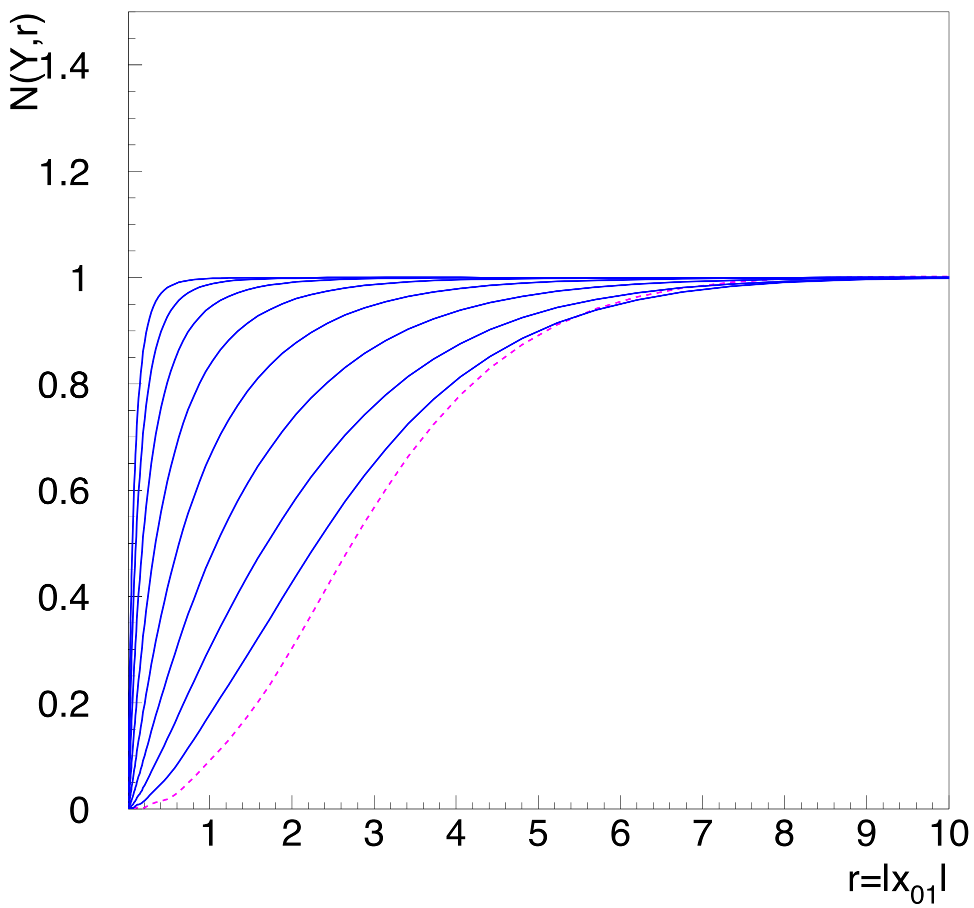


System saturates earlier (i.e. at smaller $Y's$) for larger dipole sizes.

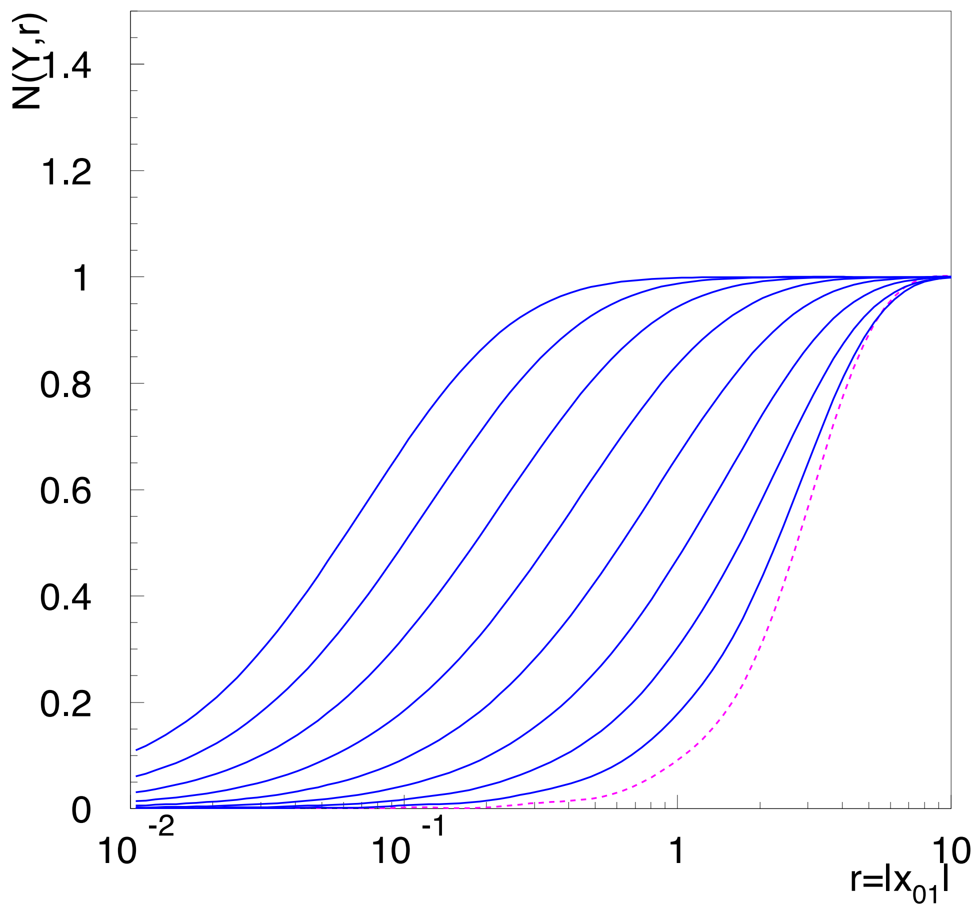
Solution to BK: 1 transverse dimension

Dipole size dependence at different $Y: 0, 1.95, \ldots, 15.6$

Linear scale



Log scale

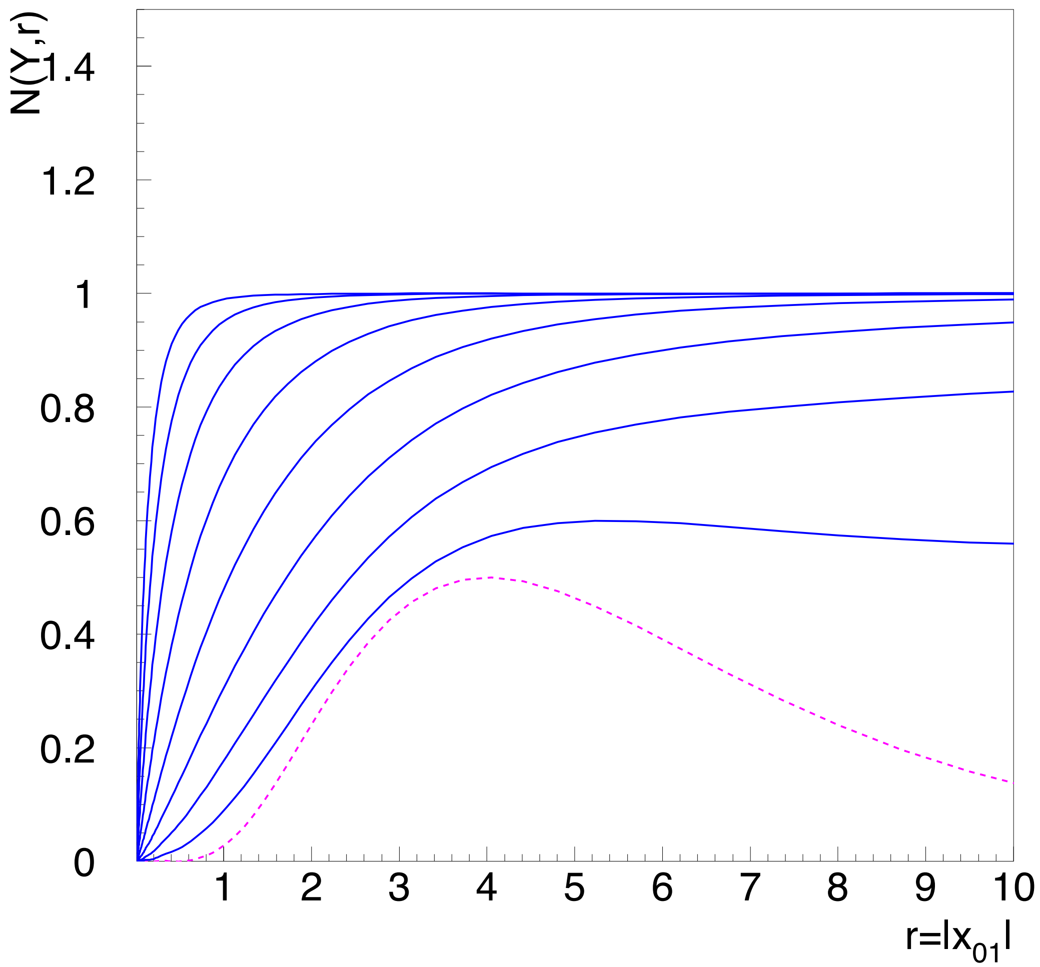


N becomes $\mathcal{O}(1)$ for smaller dipoles as Y increases.

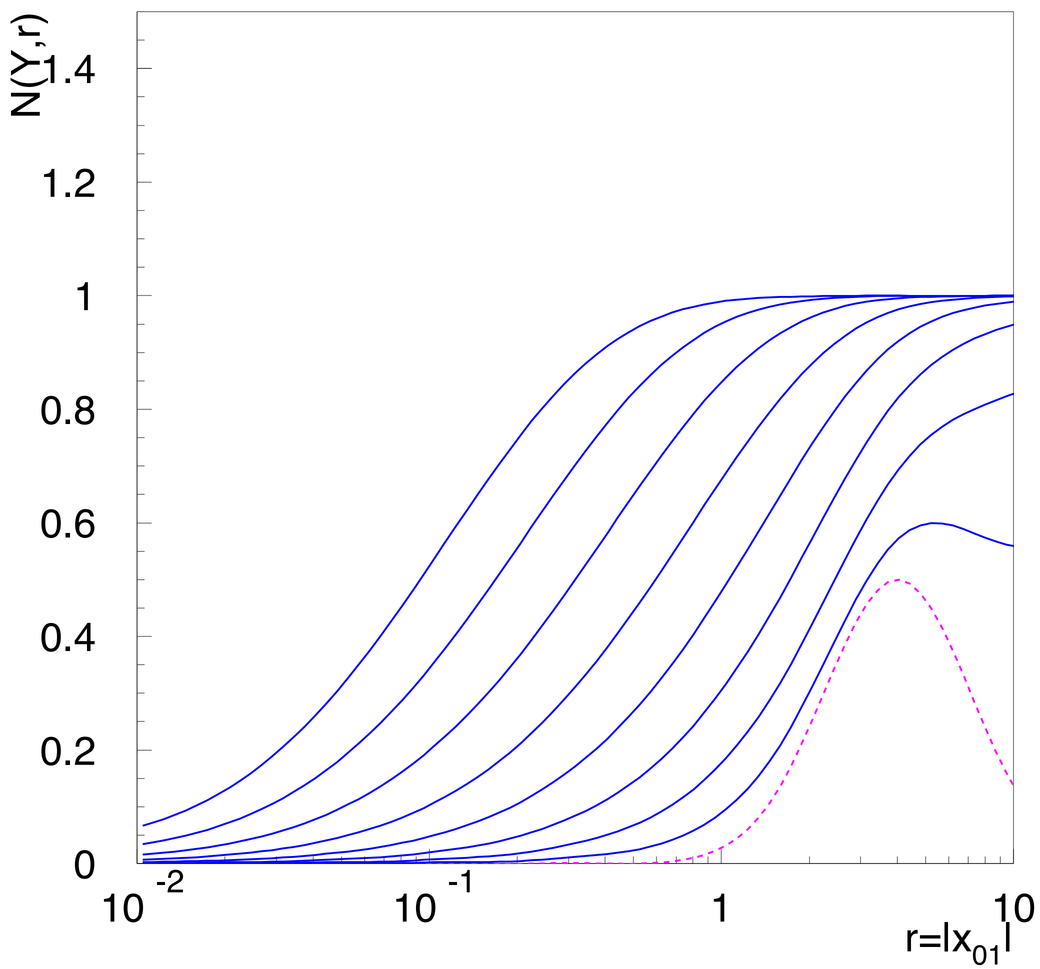
Solution to BK: 1 transverse dimension

Dependence on initial conditions

Linear scale



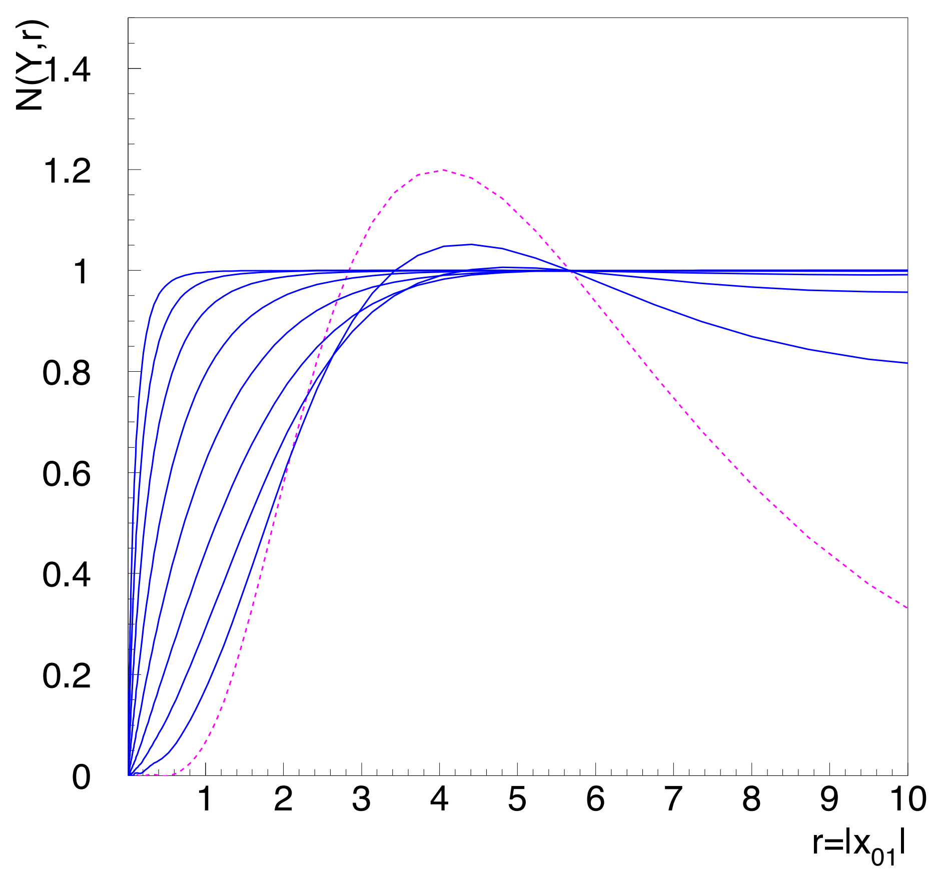
Log scale



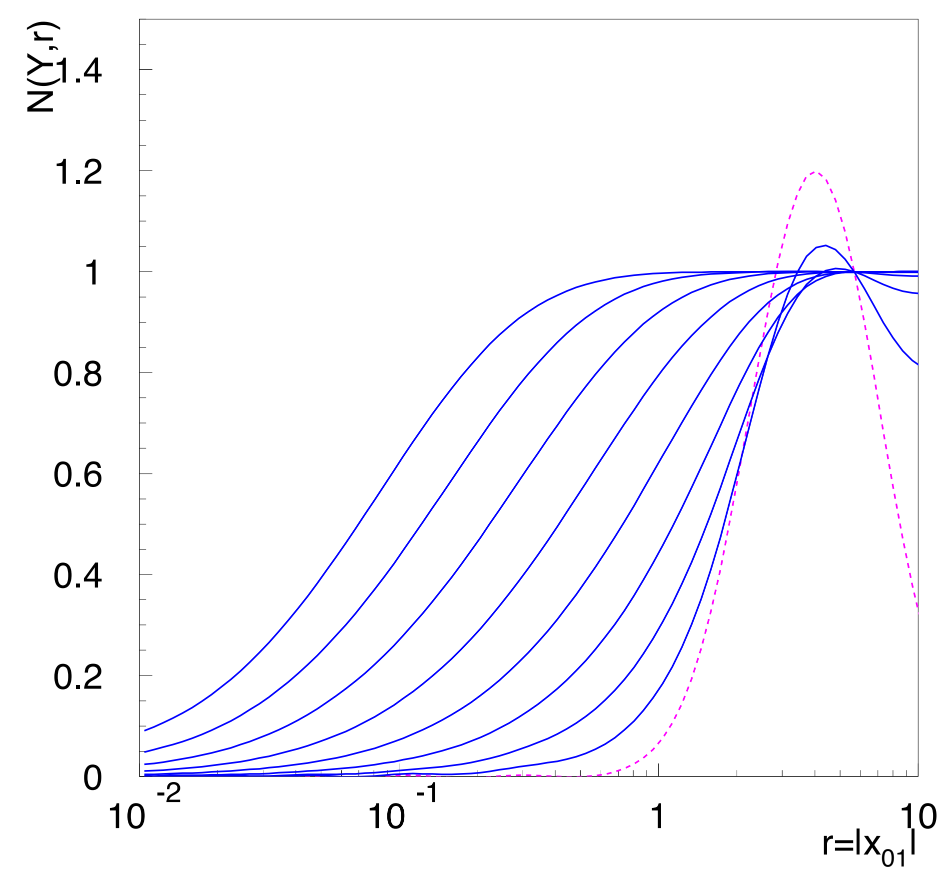
Solution to BK: 1 transverse dimension

Dependence on initial conditions

Linear scale



Log scale



Universal behaviour of solution for different initial condition.

$N = 0$ unstable fixed point

$N = 1$ stable fixed point

Saturation scale

Introduce saturation scale $Q_s(Y)$ such that:

$$r < \frac{1}{Q_s(Y)} \quad \longrightarrow \quad N \ll 1 \quad \longrightarrow \quad \text{dilute system}$$
$$r > \frac{1}{Q_s(Y)} \quad \longrightarrow \quad N \sim 1 \quad \longrightarrow \quad \text{dense system}$$

Saturation scale is rapidity dependent:

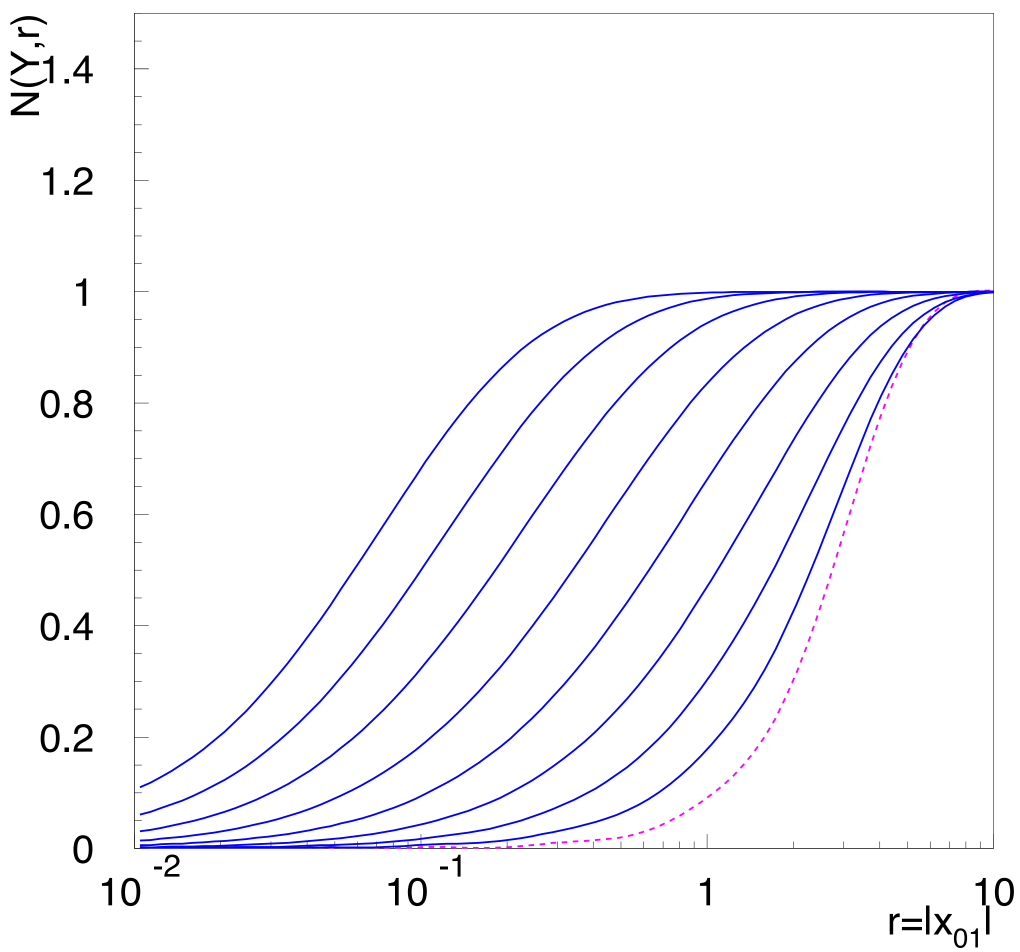
$$Q_s(Y) \; = \; Q_0 \exp(\bar{\alpha}_s \, \lambda \, Y) \, Y^{-\beta}, \qquad \lambda \simeq 2.4$$

Properties of BK solution are similar to the dipole cross section from Golec-Biernat & Wüsthoff saturation model *(K.Golec-Biernat,M.Wusthoff 98)*:

$$\hat{\sigma}(r,x) \equiv 2 \int d^2\mathbf{b} \, N(\mathbf{b},r,Y) \; = \; \sigma_0 \left[1 - \exp(-\frac{r^2 Q_s^2(x)}{4}) \right]$$

$$r < \frac{1}{Q_s(Y)} \quad \longrightarrow \quad \sigma(Y,r)/\sigma_0 \; \simeq \; r^2 \, Q_s^2(Y)/4$$
$$r > \frac{1}{Q_s(Y)} \quad \longrightarrow \quad \sigma(Y,r)/\sigma_0 \; \simeq \; 1$$

Scaling properties of BK solution



Universal shape of solution.
Solution has property of
(geometrical) scaling

$$
\begin{aligned}
N(r,Y) &= N(r\,Q_s(Y)) \\
Q_s(Y) &= Q_0 \exp(\bar{\alpha}_s\,\lambda\,Y)\,Y^{-\beta} \\
\ln r + \ln Q_s(Y) &= \ln r + \bar{\alpha}_s \lambda Y + \mathcal{O}(\ln Y)
\end{aligned}
$$

Travelling wave solution
Soliton *(M. Braun 00)*

Scaling region $r \gtrsim 1/Q_s(Y)$. For small r, scaling violations.

GBW model also has scaling property.

Geometrical scaling

Interestingly if the dipole cross section approximately has geometrical scaling, i.e. depends on the combination of dipole size and saturation scale

$$\hat{\sigma}(x,r) = \sigma_0 \left[1 - \exp(-Q_s^2(x) r^2/4)\right]$$
$$= \sigma_0 g(r Q_s(x))$$

The cross section should also depend on one combined variable in the small x region

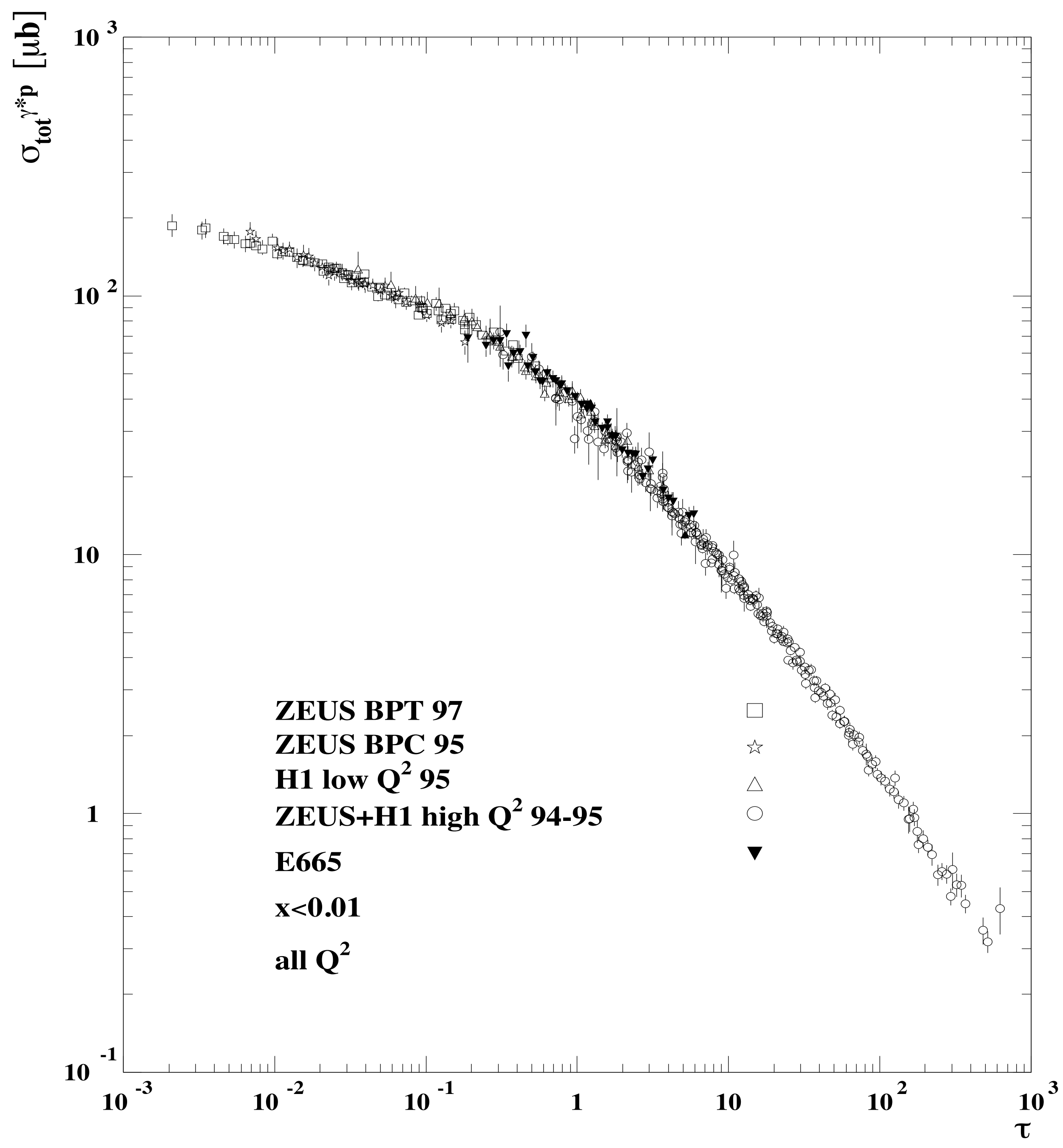
$$\sigma^{\gamma^* p}(x, Q^2) = \sigma^{\gamma^* p}(\tau) \qquad \tau = Q^2/Q_s^2(x)$$

Interestingly, data seem to support this trend

Many questions: the same data can be described by DGLAP linear evolution

Further analysis showed that DGLAP could preserve scaling in the evolution if there is scaling in the initial conditions : extended geometrical scaling

Small x<0.01 data for DIS



BK in momentum space

$$\frac{dN(\mathbf{b}_{01},\mathbf{x}_{01},Y)}{dY} = \bar{\alpha}_s \int \frac{d^2\mathbf{x}_2\,\mathbf{x}_{01}^2}{\mathbf{x}_{20}^2\,\mathbf{x}_{12}^2}\Big[N(\mathbf{b}_{01}+\frac{\mathbf{x}_{12}}{2},\mathbf{x}_{20},Y)+N(\mathbf{b}_{01}-\frac{\mathbf{x}_{20}}{2},\mathbf{x}_{12},Y) - N(\mathbf{b}_{01},\mathbf{x}_{01},Y) - N(\mathbf{b}_{01}+\frac{\mathbf{x}_{12}}{2},\mathbf{x}_{20},Y)\,N(\mathbf{b}_{01}-\frac{\mathbf{x}_{20}}{2},\mathbf{x}_{12},Y)\Big]$$

Fourier transform to momentum space:

$$\phi(k,Y) := \int_0^\infty \frac{dr}{r} J_0(k\,r)\,N(r,Y)$$

BK equation ($(1+1)$ dim.) in momentum space:

$$\frac{d\phi(k,Y)}{dY} = \bar{\alpha}_s \int \frac{dk'}{k'}\mathcal{K}(k,k')\,\phi(k',Y) - \bar{\alpha}_s\phi^2(k,Y)$$

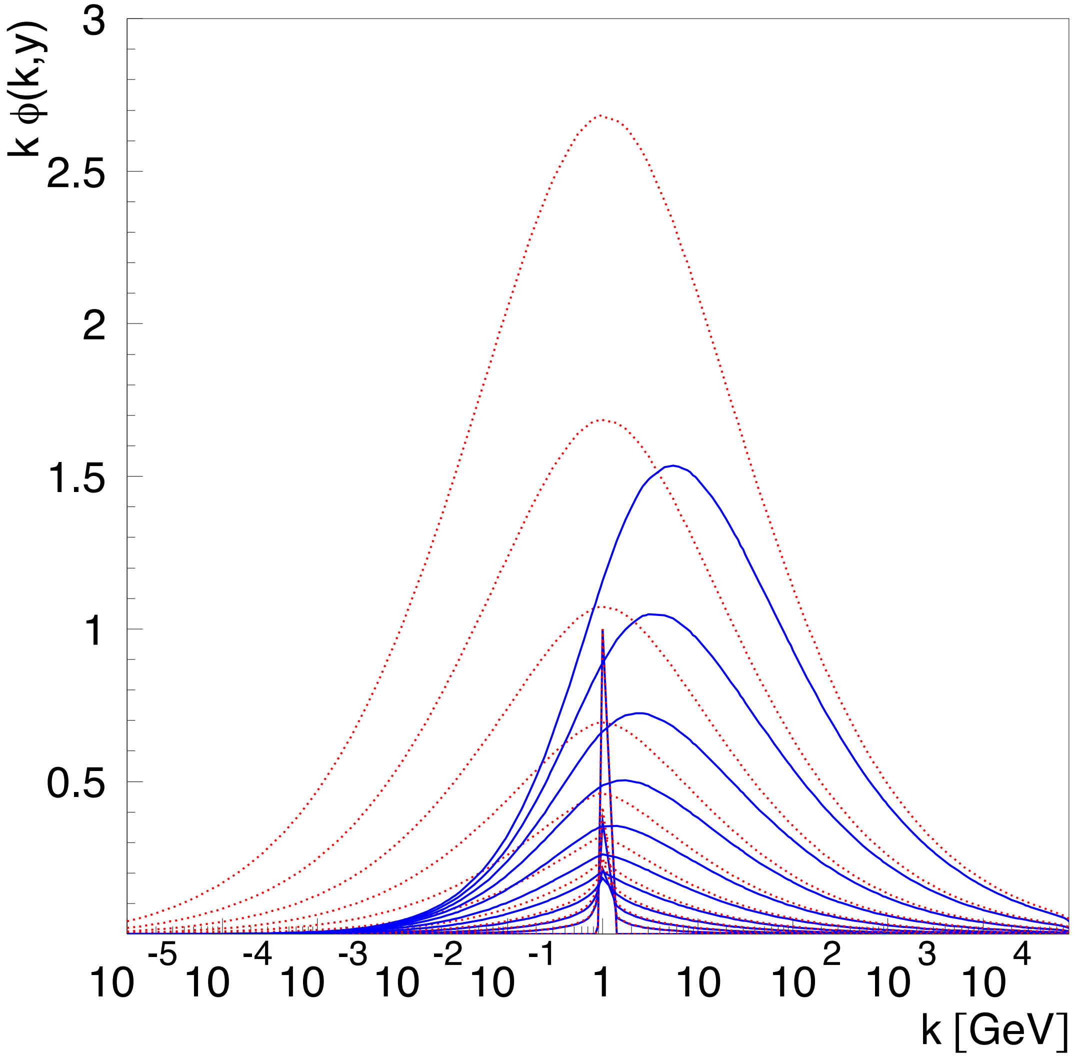
where $\mathcal{K}$ is usual BFKL kernel in momentum space.
Solution of the linear equation in saddle point approximation:

$$k\phi(k,Y) = \frac{1}{\sqrt{\pi\bar{\alpha}_s\chi''(0)Y}}\exp(\bar{\alpha}_s\chi(0)Y)\,\exp\left(-\frac{\ln^2(k^2/k_0^2)}{2\bar{\alpha}_s\chi''(0)Y}\right)$$

Diffusion into infrared region of momenta k

BFKL vs BK solution

Distribution in momentum $\ln k$ for increasing rapidities



- Suppresion of diffusion into infrared for nonlinear solution *(K.Golec-Biernat, L.Motyka, A.S., 01)*
- Peak moves from k_0 towards larger k for increasing Y
- Define saturation scale as position of maximum

$$Q_s(Y) \equiv k_{\max}(Y)$$

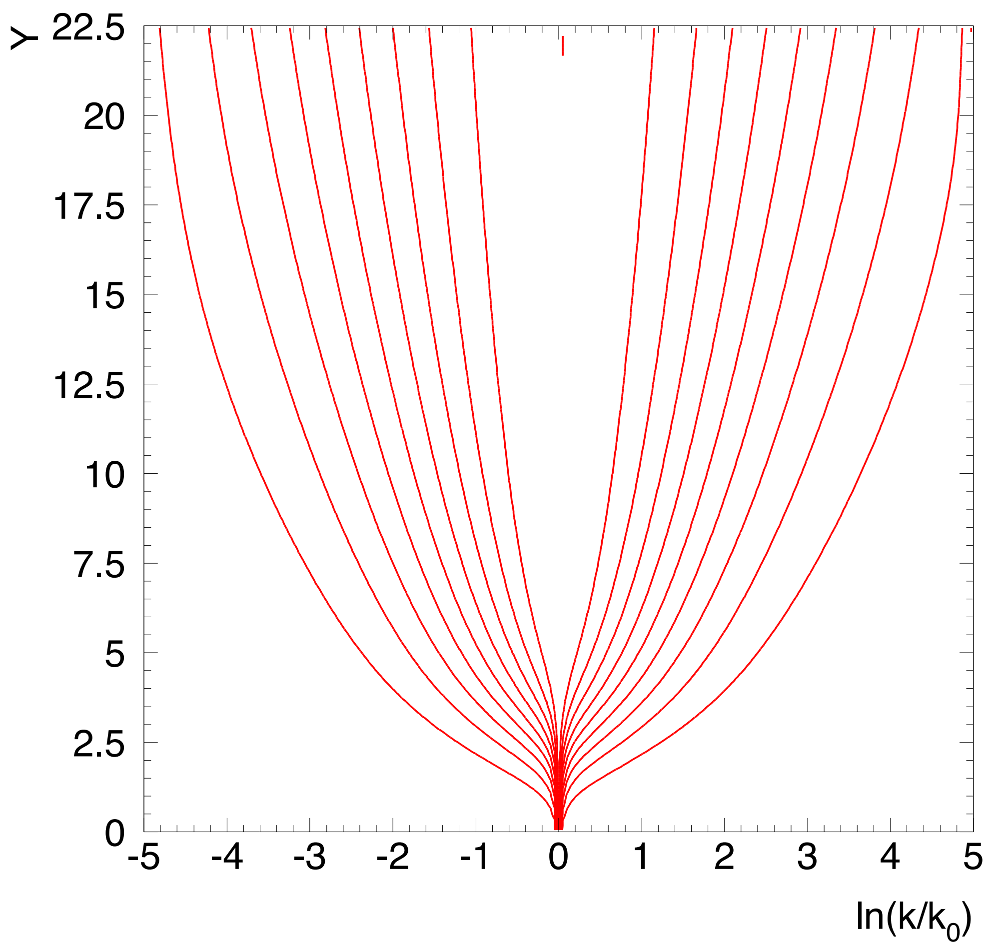
$$k\phi^{(\mathrm{lin})}(k,Y) \sim e^{\bar{\alpha}_s \chi(0) Y} \, e^{\left(-\frac{\ln^2(k^2/k_0^2)}{2\bar{\alpha}_s \chi''(0) Y}\right)}$$

Diffusion properties of BK

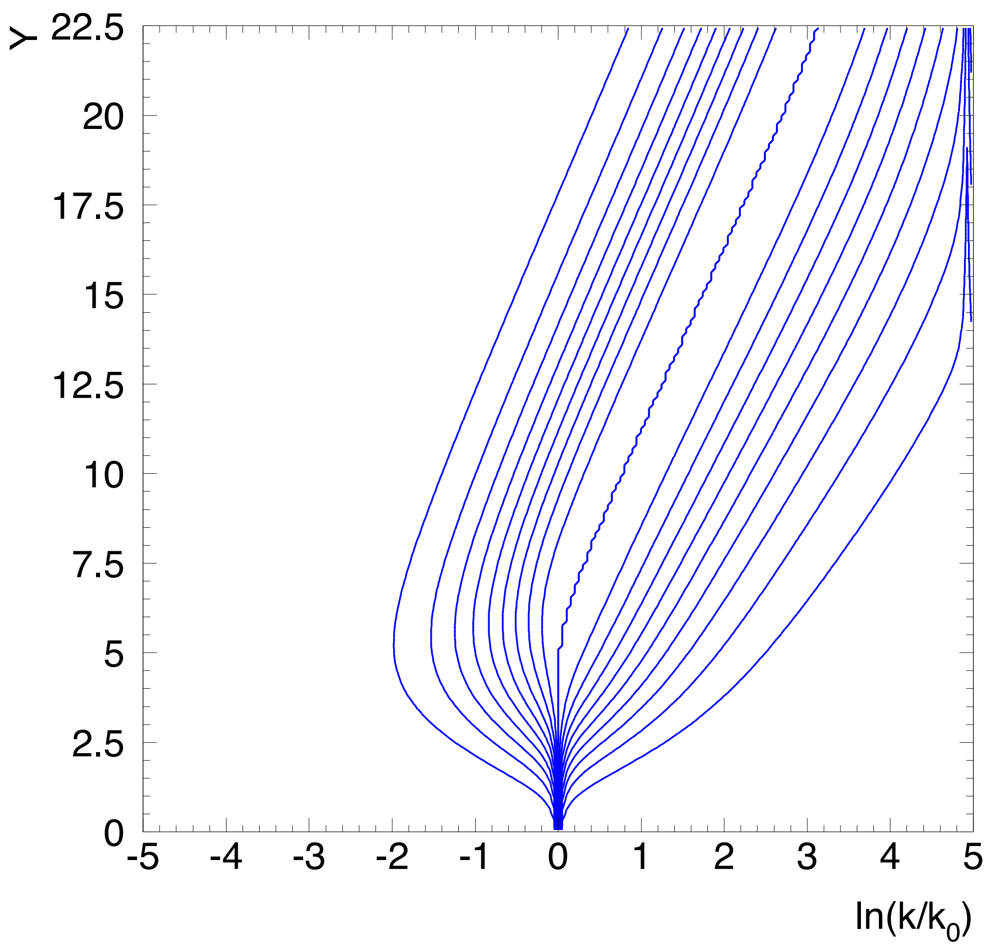
Renormalised distribution:

$$\Psi(k,Y) \;=\; \frac{k\phi(k,Y)}{k_{\max}(Y)\phi(k_{\max}(Y),Y)}$$

Linear

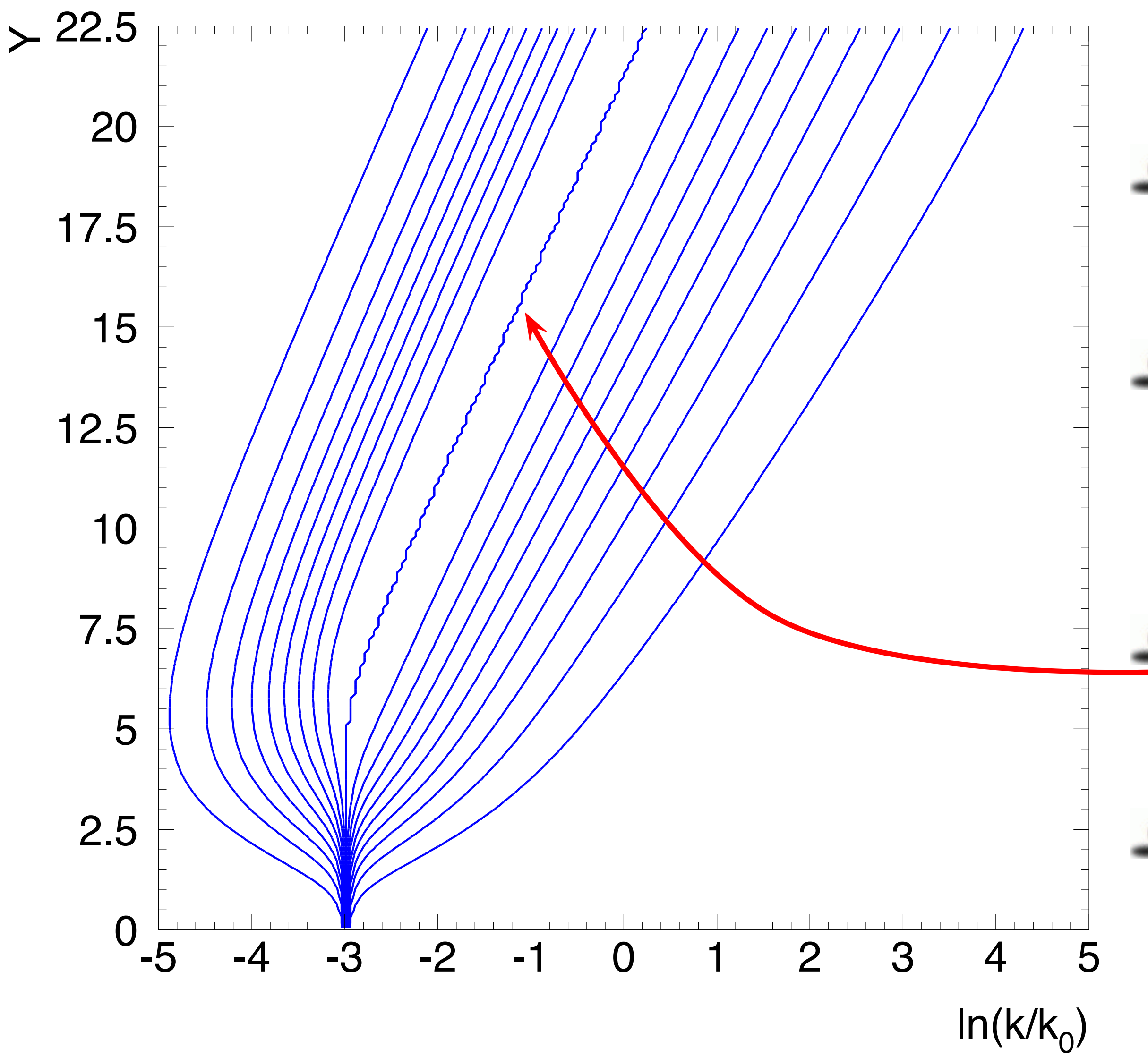


Nonlinear



Suppression of diffusion and scaling

Nonlinear: different cuts



- Stright lines $\xi = \ln k/k_0 - \lambda Y$
- Scaling since solution depends only on ξ (when $\xi < \xi_s$)
- Saturation scale $Q_s(Y)$ defined by critical line ξ_s
- Diffusion to the right of the critical line

Nonlinear equation might be replaced by the linear diffusion equation with the absorptive boundary *(A.H. Mueller, D. Triantafyllopoulos, 02)*

Relation of BK to diffusion equation with nonlinear term

BK equation can be approximated as a diffusion equation with nonlinear term *(S. Munier, R. Peschanski 03,04)*:

$$\frac{d\phi(k,Y)}{dY} = \bar{\alpha}_s \int \frac{dk'}{k'} \mathcal{K}(k,k')\,\phi(k',Y) - \bar{\alpha}_s \phi^2(k,Y)$$

Fisher and Kolmogorov-Petrovsky-Piscounov (FKPP) equation:

$$\partial_t u(t,x) = \partial_x^2(t,x) + u(t,x)[1-u(t,x)]$$

change of variables $(Y, \ln k) \longrightarrow (t,x)$ and $\phi \longrightarrow u$

FKPP equation has travelling wave solutions at large t:

$$u(t,x) \overset{t\to+\infty}{\sim} w[x - m_\beta(t)],$$

where initial condition satisfies $u(t_0,x) \overset{x\to+\infty}{\sim} \exp(-\beta x)$.

Travelling wave $\leftrightarrow$ Geometrical Scaling

Running coupling

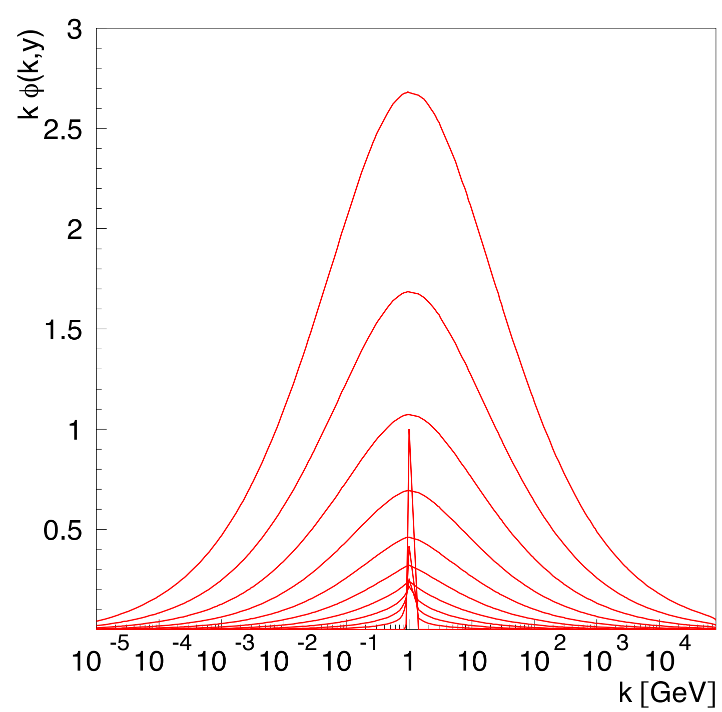
LLx approximation $\rightarrow \bar{\alpha}_s$ fixed

Phenomenological way of introducing NLLx effect $\rightarrow \bar{\alpha}_s$ running:

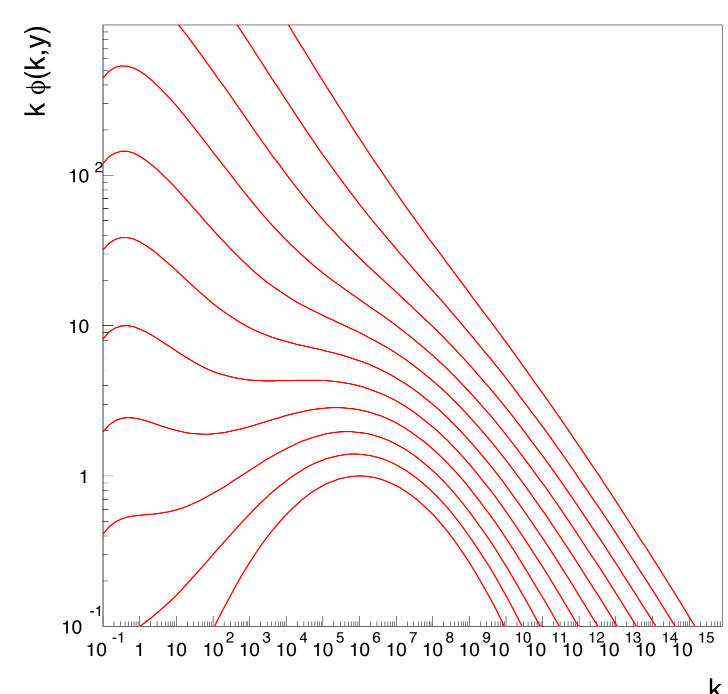
$$\frac{d\phi(k,Y)}{dY} = \bar{\alpha}_s(k) \int \frac{dk'}{k'} \mathcal{K}(k,k')\,\phi(k',Y) - \bar{\alpha}_s(k)\phi^2(k,Y)$$

It is well known that linear equation becomes very unstable $\rightarrow$ large sensitivity to the regularisation of $\bar{\alpha}_s(k)$ in the infrared.

Fixed $\bar{\alpha}_s$:

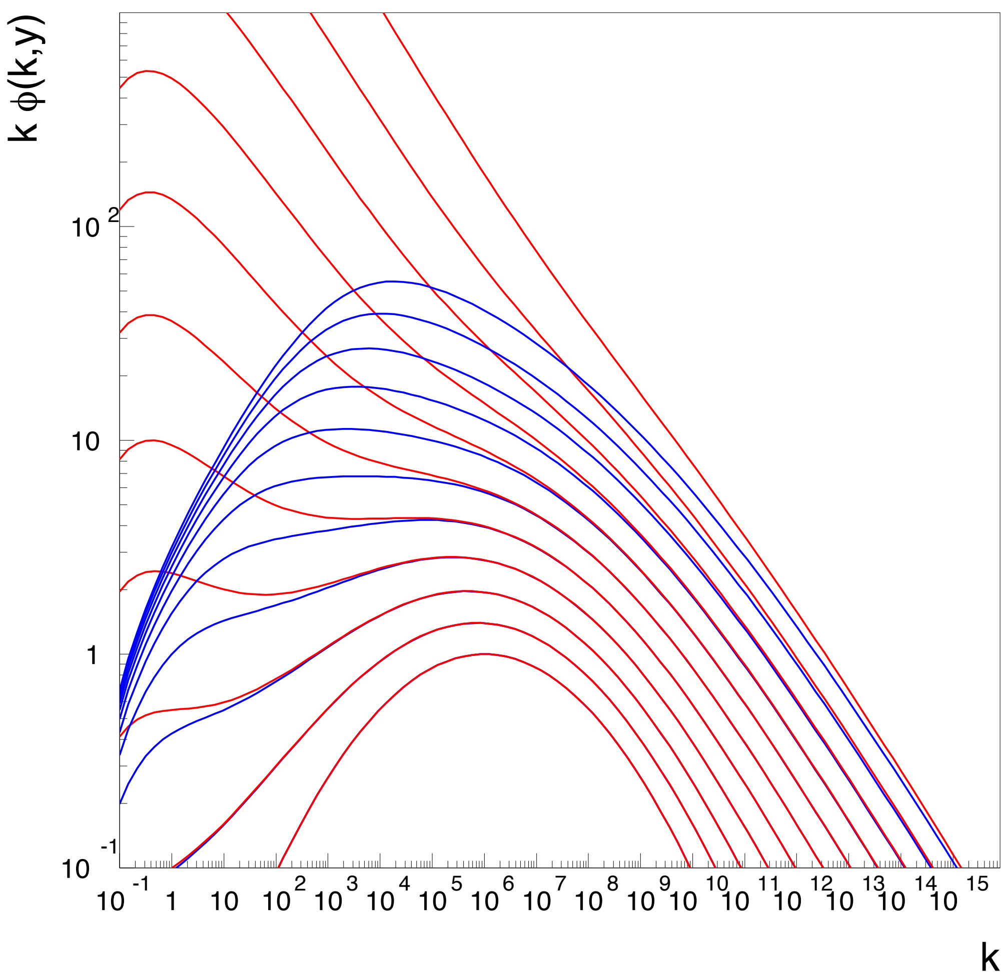


Running $\bar{\alpha}_s$:



Running coupling in BK with 1 spatial dimension

Nonlinear equation has much more stable behaviour:

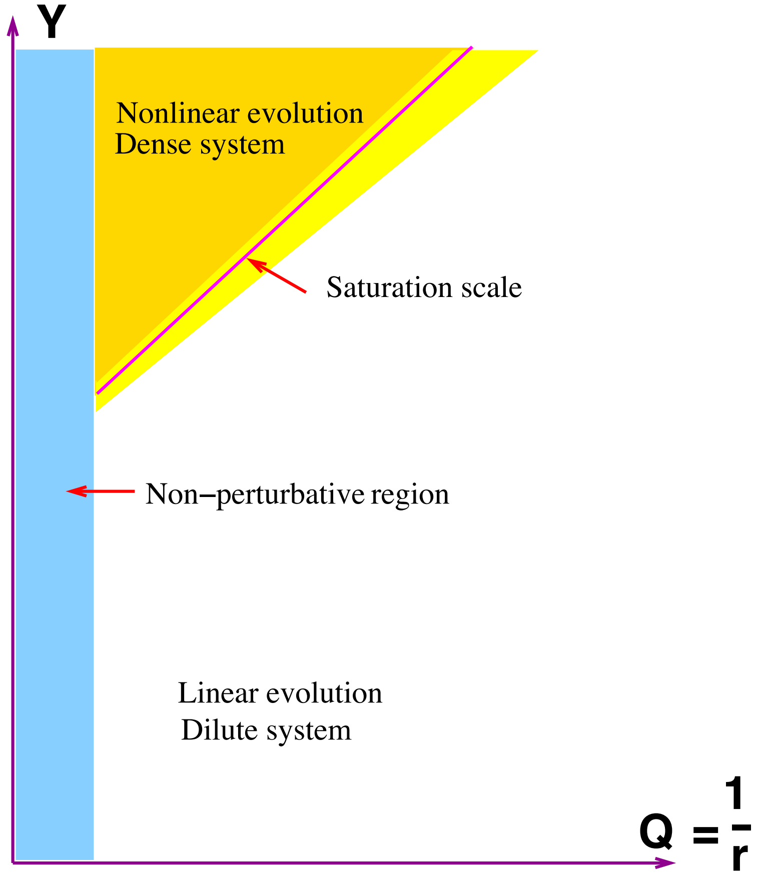


- Nonlinear term damps diffusion into infrared
- Saturation scale $Q_s(Y)$ provides a natural cutoff for momenta
- No dependence on the regularisation of $\bar{\alpha}_s(k)$ in the infrared
- Geometrical scaling still holds

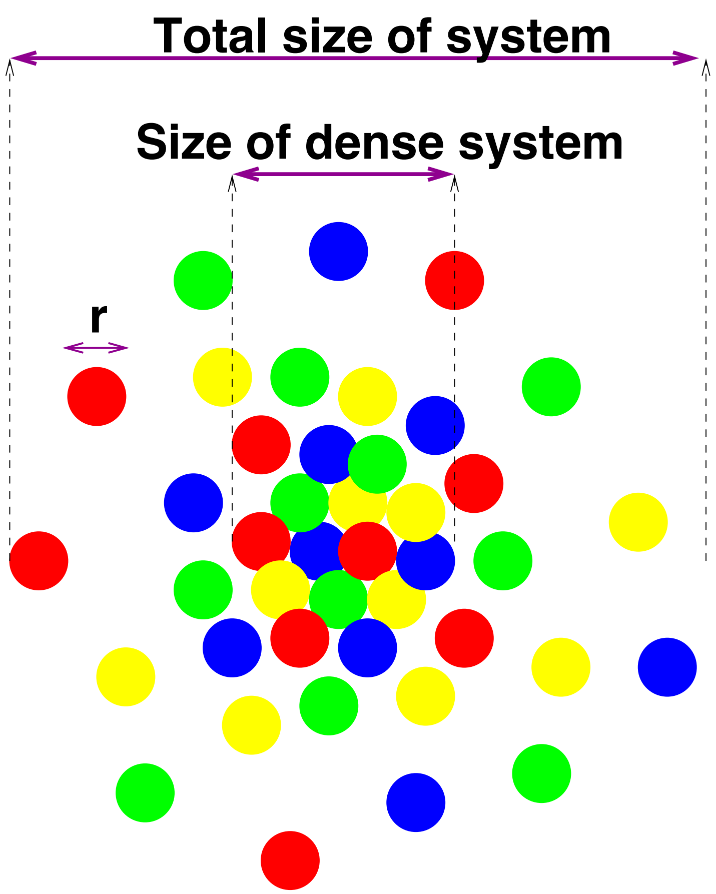
Different rapidity dependence of saturation scale:

$$Q_s(Y) \;=\; \Lambda \exp\left(\sqrt{\frac{24}{b_0}(Y-Y_0)+\ln^2 Q_0/\Lambda}\right)$$

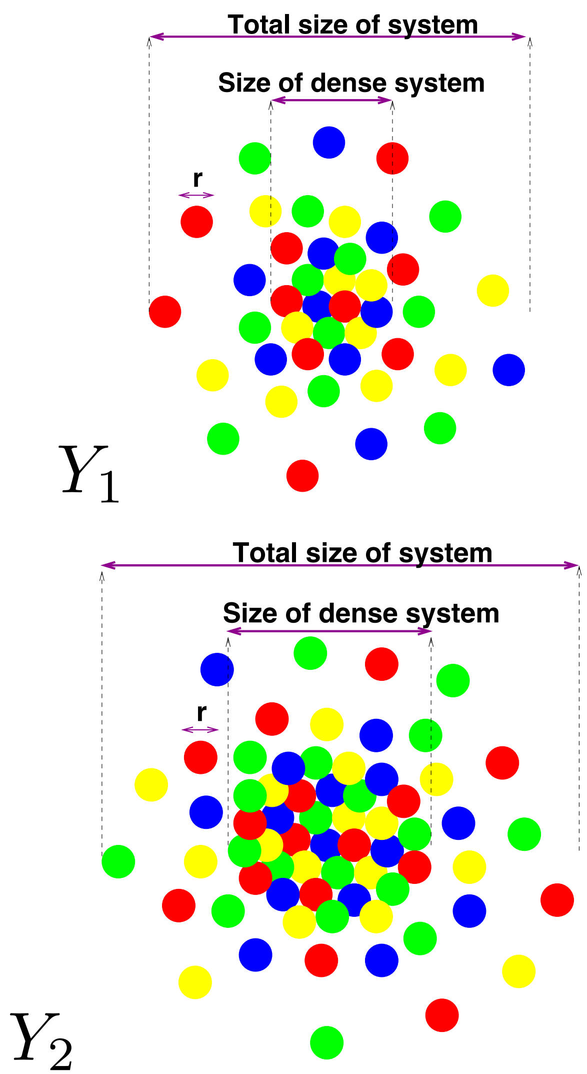
Spatial distribution: impact parameter dependence



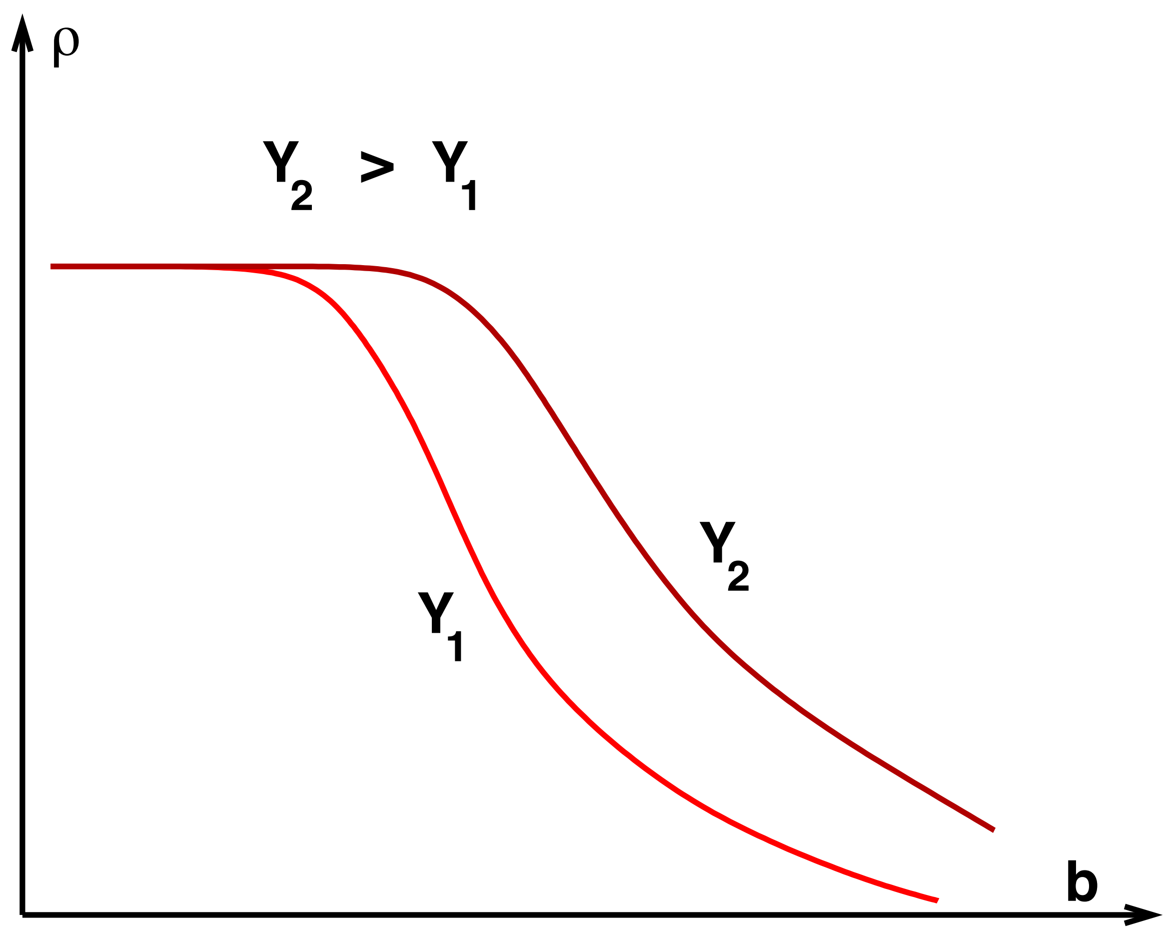
- Approximated picture, densities are averaged
- BK equation only in $(1+1)$ dimension, infinite size of target.
- What about spatial distribution?



Spatial distribution: impact parameter dependence



Impact parameter profile:



System expands in space as energy grows.

What is the spatial distribution generated from full (4+1) BK equation

BK with impact parameter dependence

$$\frac{dN(\mathbf{b}_{01},\mathbf{x}_{01},Y)}{dY} = \bar{\alpha}_s \int \frac{d^2\mathbf{x}_2\,\mathbf{x}_{01}^2}{\mathbf{x}_{20}^2\,\mathbf{x}_{12}^2}\left[N(\mathbf{b}_{01}+\frac{\mathbf{x}_{12}}{2},\mathbf{x}_{20},Y)+N(\mathbf{b}_{01}-\frac{\mathbf{x}_{20}}{2},\mathbf{x}_{12},Y)\right.$$
$$\left. - N(\mathbf{b}_{01},\mathbf{x}_{01},Y) - N(\mathbf{b}_{01}+\frac{\mathbf{x}_{12}}{2},\mathbf{x}_{20},Y)\,N(\mathbf{b}_{01}-\frac{\mathbf{x}_{20}}{2},\mathbf{x}_{12},Y)\right]$$

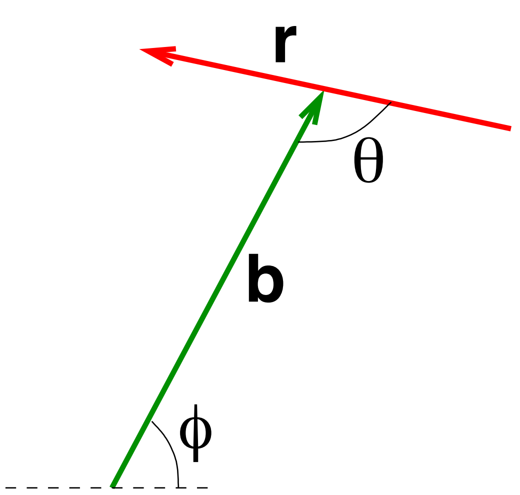
Difficult problem $\rightarrow (4+1)$ dimensions.
Integral measure

$$\frac{d^2\mathbf{x}_2\,\mathbf{x}_{01}^2}{\mathbf{x}_{20}^2\,\mathbf{x}_{12}^2}$$

is invariant under rotations in transverse space

$$\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2 \longrightarrow \mathcal{O}(\phi)\mathbf{x}_0,\mathcal{O}(\phi)\mathbf{x}_1,\mathcal{O}(\phi)\mathbf{x}_2$$

Assume that $N(|\mathbf{b}|,|\mathbf{r}|,\theta;Y)$ cyllindricaly symmetric $\rightarrow (3+1)$.



Solving BK with impact parameter

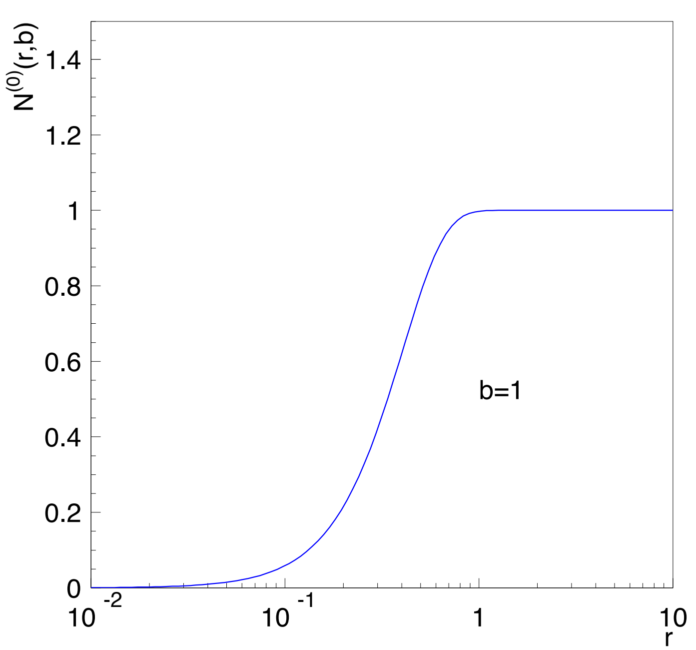
Initial conditions: Glauber-Mueller form

$$N^{(0)}(r,b,\theta;Y=0) \;=\; 1-\exp[-r^2 S(b)]$$

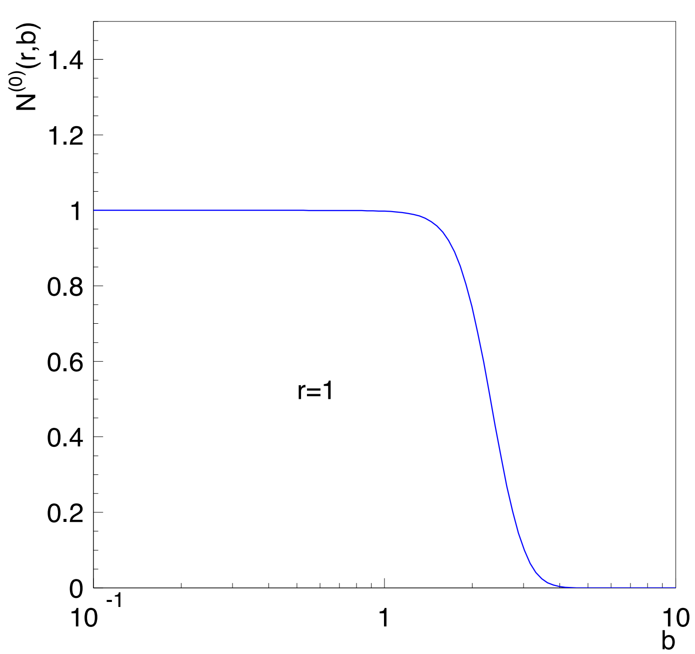
where $S(b)$ is impact parameter profile

$$S(b) \;=\; S_0\,\exp\left(-\frac{b^2}{R_0^2}\right)$$

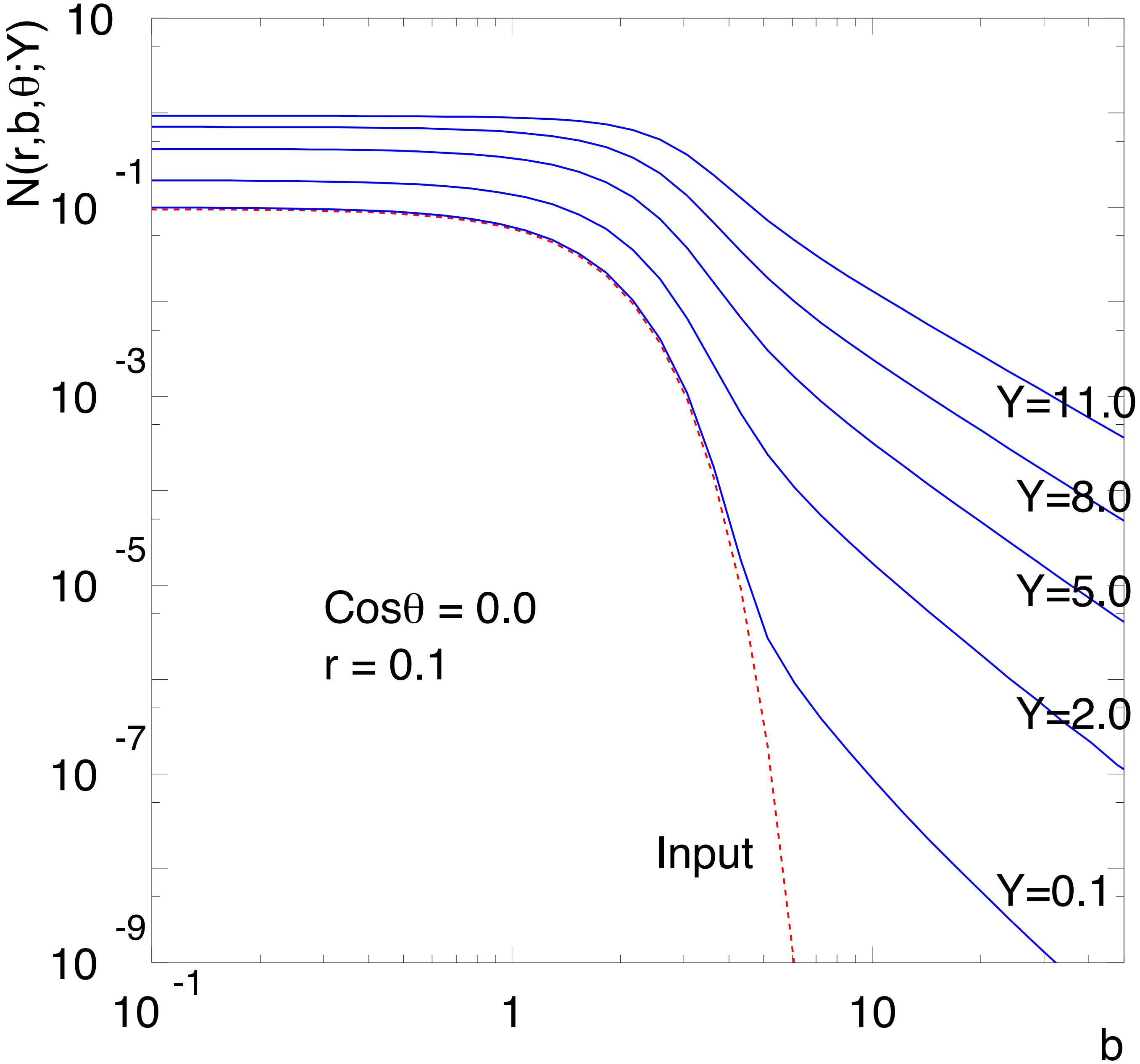
Dipole size profile:



Impact parameter profile:



BK with impact parameter



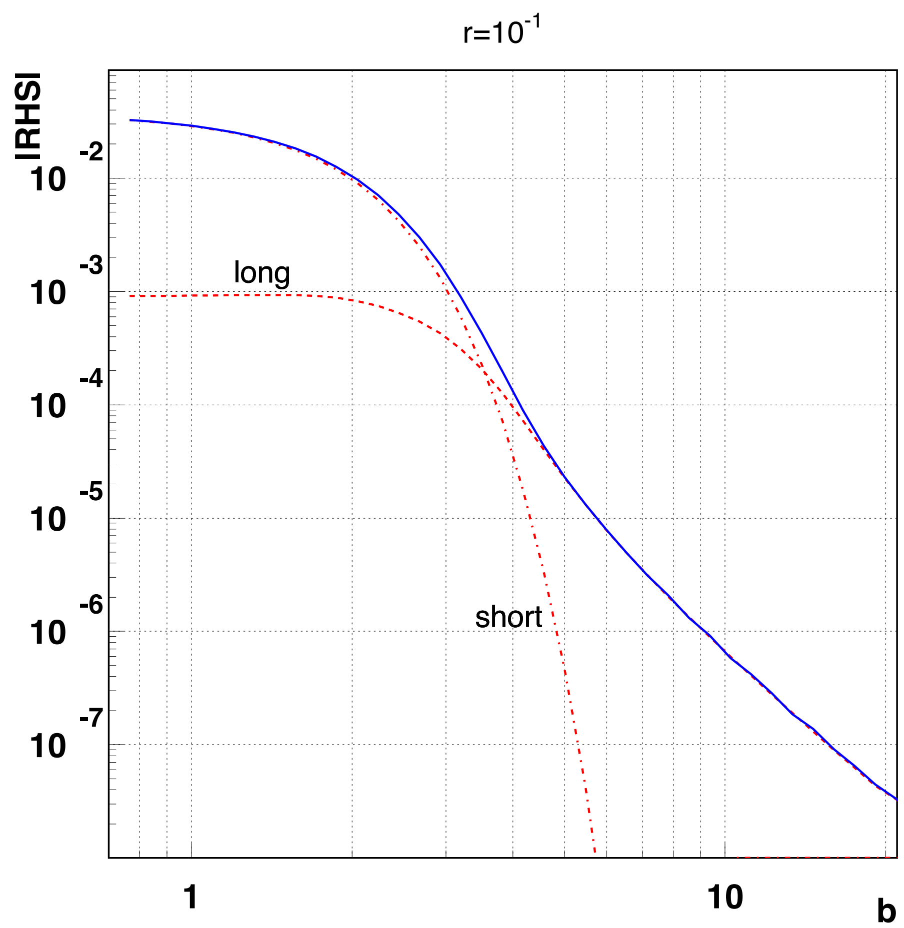
- Saturation for small b's, fast growth at larger b's
- Expansion of satured region with increasing rapidity
- Initial impact parameter profile is not preserved
- Power tail $\sim 1/b^4$ is immediately generated

Golec-Biernat, AS; Berger, AS

Short vs long range contribution

$$\Bigg[\overbrace{\int \Theta(r_0 - |\mathbf{x}_2 - \mathbf{b}|)}^{short} + \overbrace{\int \Theta(|\mathbf{x}_2 - \mathbf{b}| - r_0)}^{long}\Bigg] \frac{d^2\mathbf{x}_2(\mathbf{x}_0 - \mathbf{x}_1)^2}{(\mathbf{x}_0 - \mathbf{x}_2)^2(\mathbf{x}_1 - \mathbf{x}_2)^2}$$

$$\cdot \left(N_{02}^{(0)} + N_{12}^{(0)} - N_{01}^{(0)} - N_{02}^{(0)} N_{12}^{(0)}\right)$$



- short $\rightarrow$ exponential behaviour, factorisation of initial profile at small b
- long $\rightarrow$ power behaviour, $\sim 1/b^4$ at large b

Violation of Froissart bound

Power tails in $b \leftarrow$ long range interaction

At large b there are contributions from large dipoles

$$\frac{d^2\mathbf{x}_2\,\mathbf{x}_{01}^2}{\mathbf{x}_{20}^2\,\mathbf{x}_{12}^2} \simeq d^2\mathbf{x}_2\,\frac{r^2}{b^4}$$

Fast expansion of the system leads of the violation of Froissart bound.
Instead of :

$$\sigma \leq \frac{\pi}{m_\pi^2}(\ln 1/x)^2$$

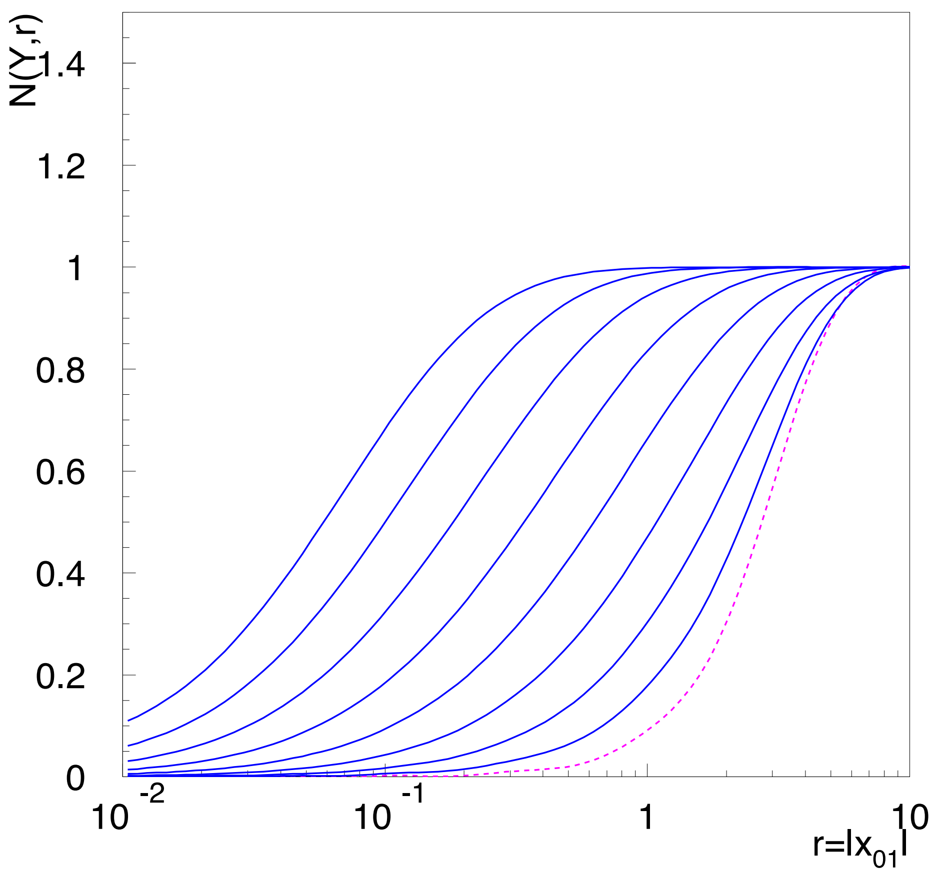
we have :

$$\sigma \;=\; \int d^2\mathbf{b}\, N(\mathbf{r},\mathbf{b};Y=\ln 1/x) \;\sim\; x^{-\lambda}$$

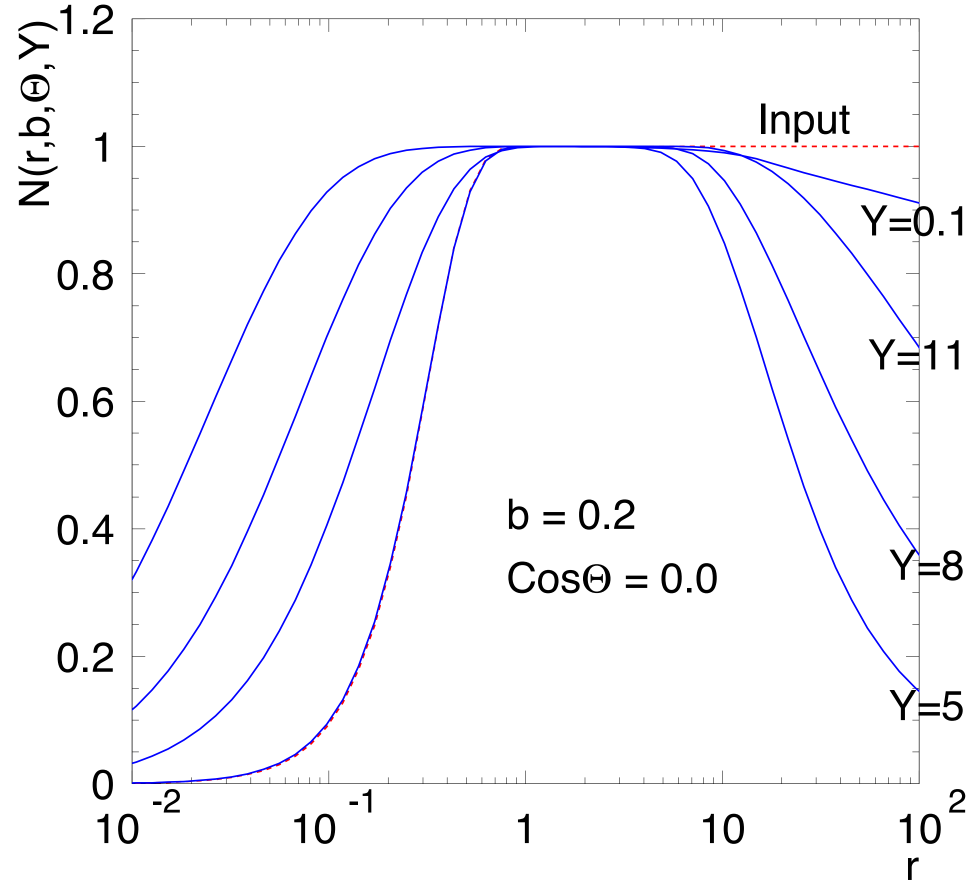
Equation is conformally invariant at LLx: no mass scale

Dipole size dependence

Without b dependence:



With b dependence:

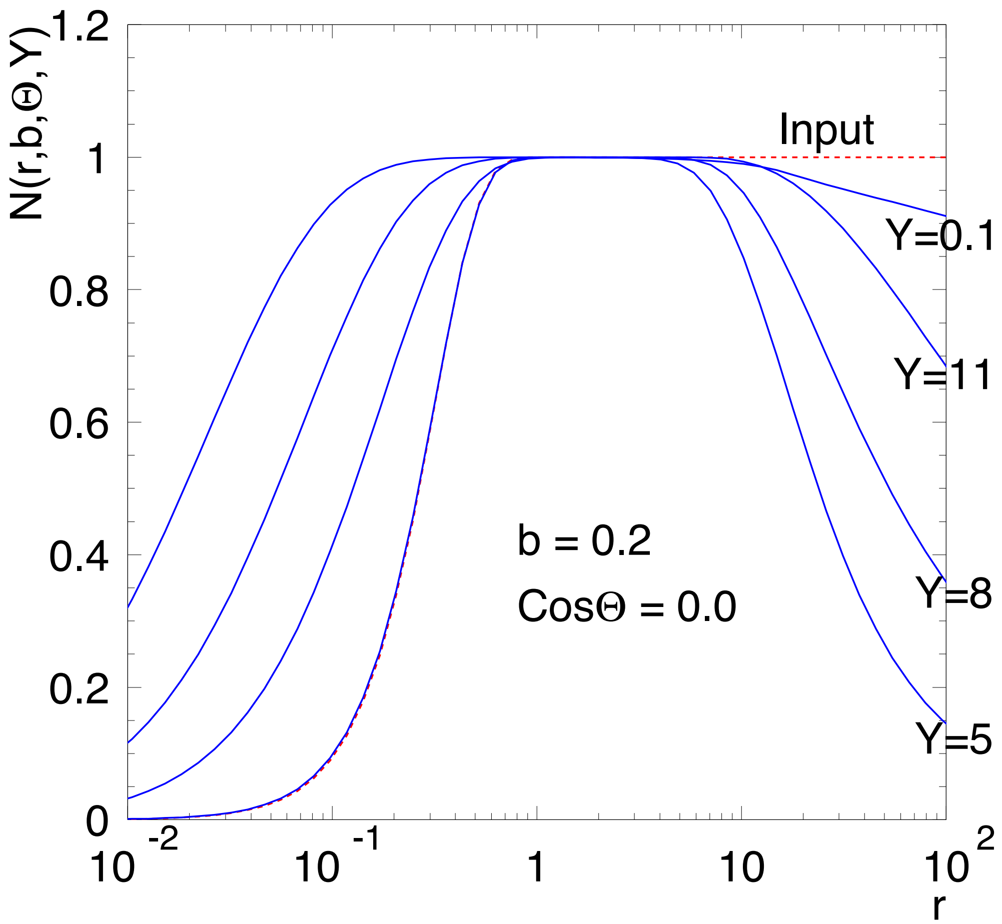


- At small values of r shape similar to previous analysis.
- Fall-off at large values of r.
- Dipole is larger than the target $\rightarrow$ it misses the target.

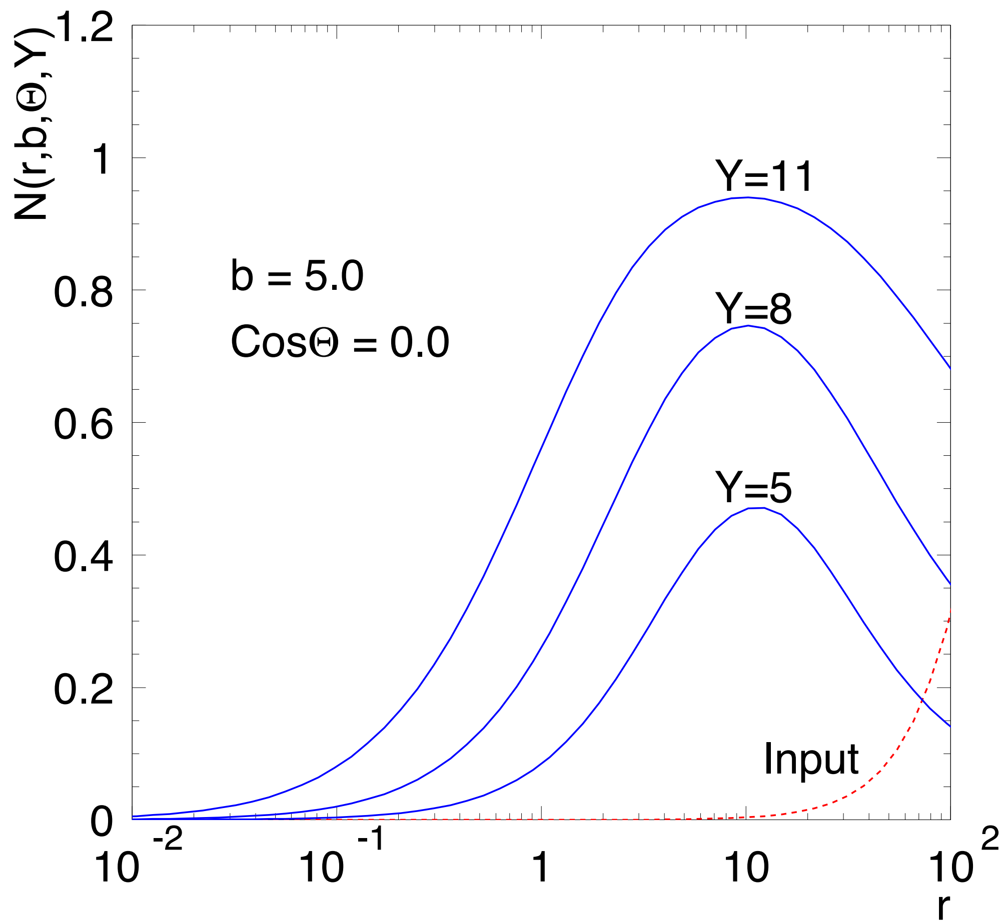
Dipole size dependence

Study different b:

Small b



Large b



- Peak around $b = \frac{r}{2}$
- At large values of $r \gg b$ amplitude independent of b.

Conformal invariance at LLx

Integral kernel:

$$\frac{d^2\mathbf{z}\,(\mathbf{x}-\mathbf{y})^2}{(\mathbf{y}-\mathbf{z})^2\,(\mathbf{x}-\mathbf{z})^2}$$

is invariant under Möbius transformation:

$$x \rightarrow \frac{a\,x+b}{c\,x+d}$$

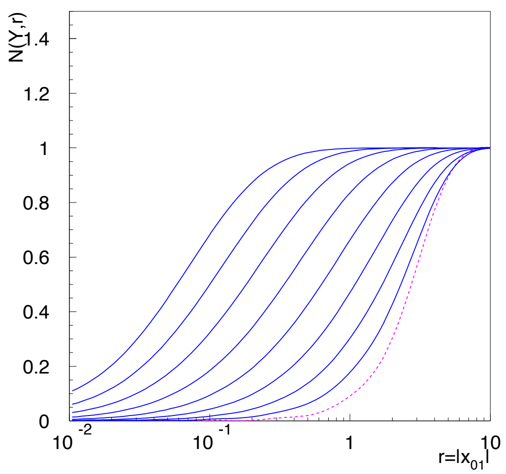
where $x = x_1 + ix_2, \quad \mathbf{x} = (x_1, x_2)$ (the same for $\mathbf{y}, \mathbf{z}$).
In the linear case it was shown that the solution depends on one variable *anharmonic ratio* *L.N. Lipatov 86*.
This means that:

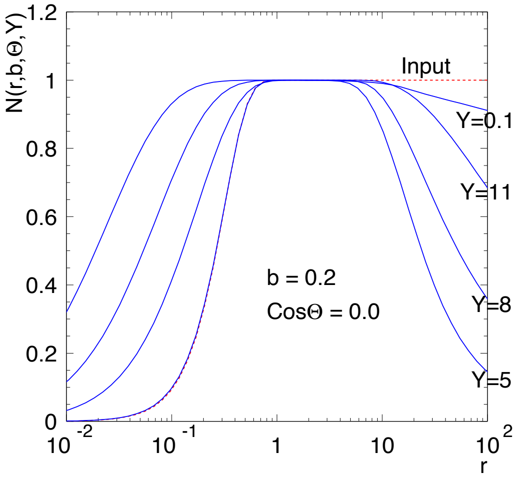
$$N\left(\frac{r^2 r_0^2}{b^4}\right) \quad \text{when} \quad r, r_0 \ll b$$

$$N\left(\frac{r_0^2}{r^2}\right) \quad \text{when} \quad r \gg b, r_0$$

r_0 is given by initial conditions

Saturation scale



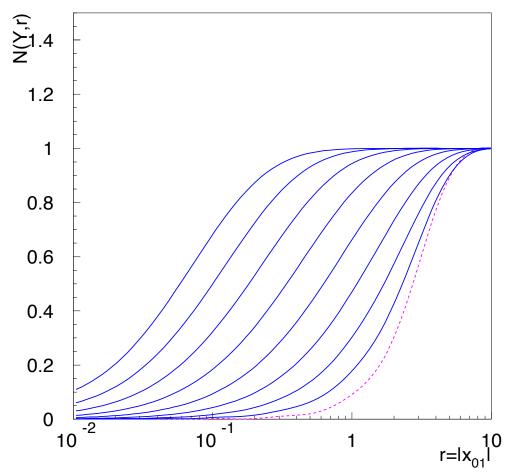


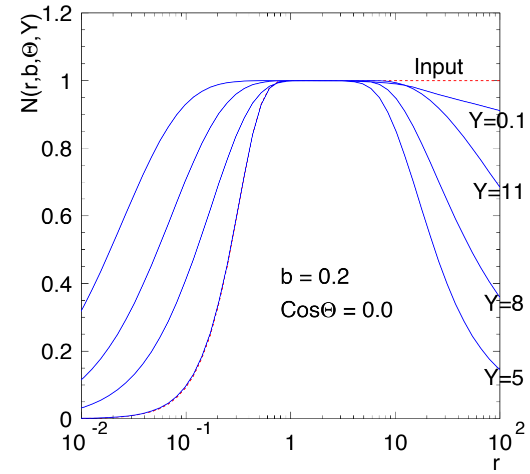
Saturation scale: $\langle N(r = 1/Q_s, b, \theta; Y)\rangle_\theta \; = \; \kappa, \quad \kappa \sim 0.5$
Two solutions. Saturation, $N \sim 1$ when:

$$\frac{1}{Q_s(b,Y)} \; < \; r \; < \; R_H(b,Y), \qquad R_H(b,Y) \sim R_H(Y)$$

$Q_s(b,Y) \rightarrow b$-dependent saturation scale

Saturation scale from BK with b impact parameter





Saturation scale: $\langle N(r = 1/Q_s, b, \theta; Y)\rangle_\theta = \kappa, \quad \kappa \sim 0.5$

Two solutions. Saturation, $N \sim 1$ when:

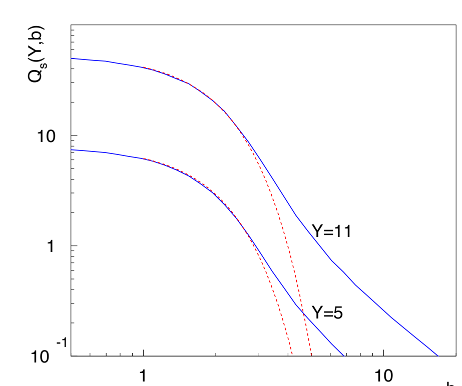
$$\frac{1}{Q_s(b,Y)} < r < R_H(b,Y), \qquad R_H(b,Y) \sim R_H(Y)$$

$$Q_s^2(b,Y) \simeq g(b)\exp(\bar{\alpha}_s 2\lambda_s Y), \qquad \lambda_s \simeq 2$$

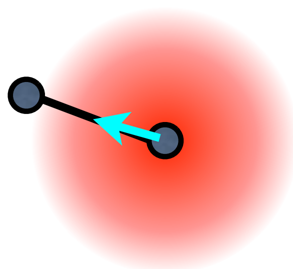
where

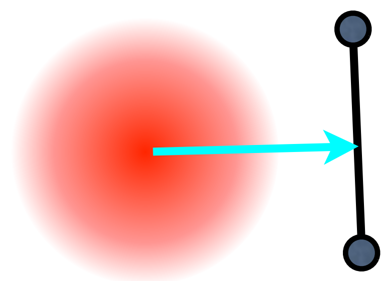
$$g(b) = \exp(-b^2/2) \leftarrow b \text{ small},$$
$$g(b) = 1/b^4 \leftarrow b \text{ large}$$

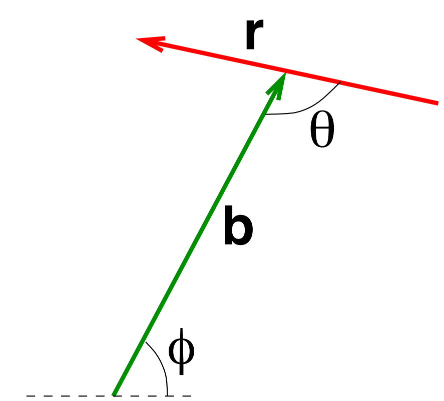
Saturation scale:

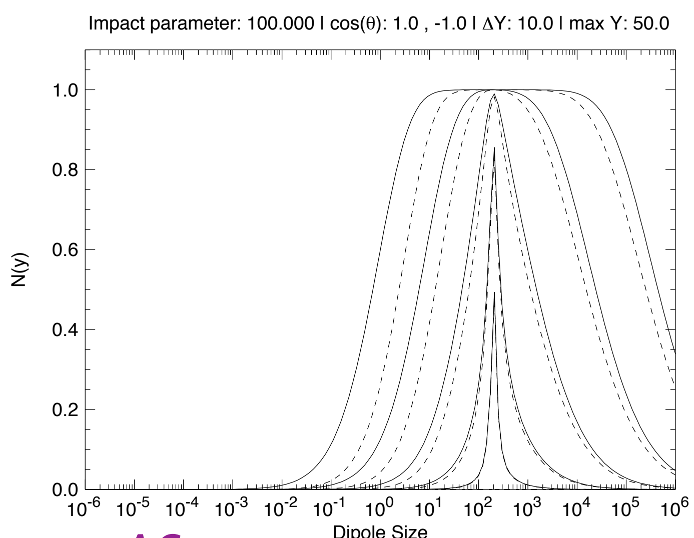


Angular correlations in BK with impact parameter



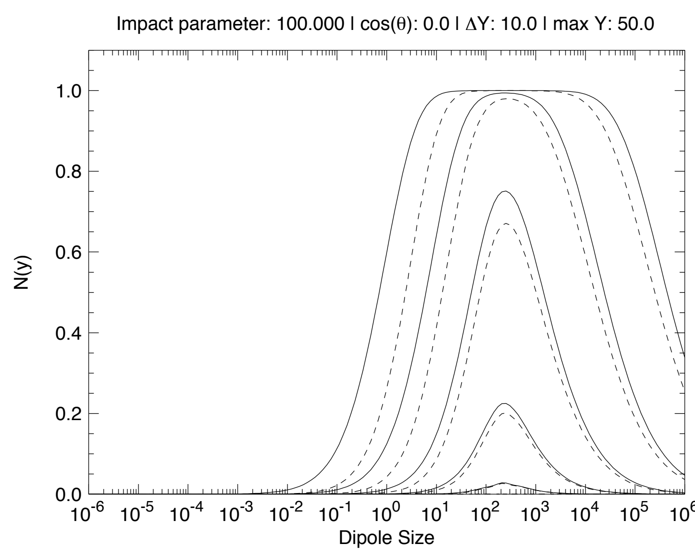




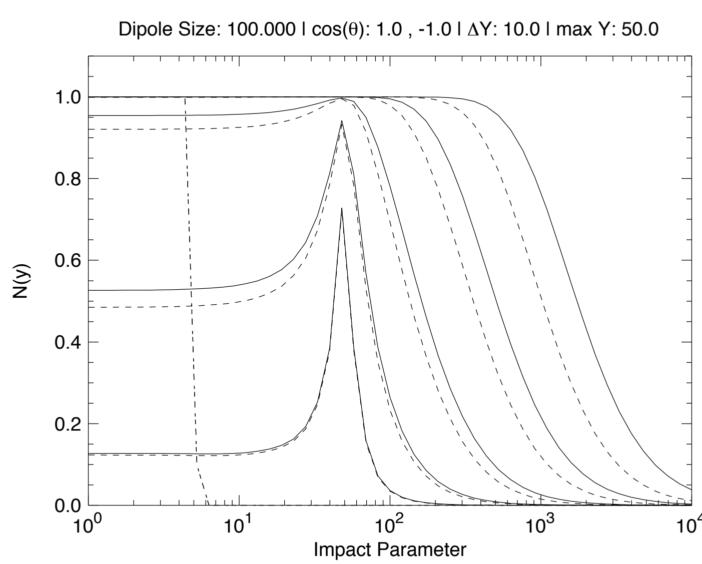


Berger,AS

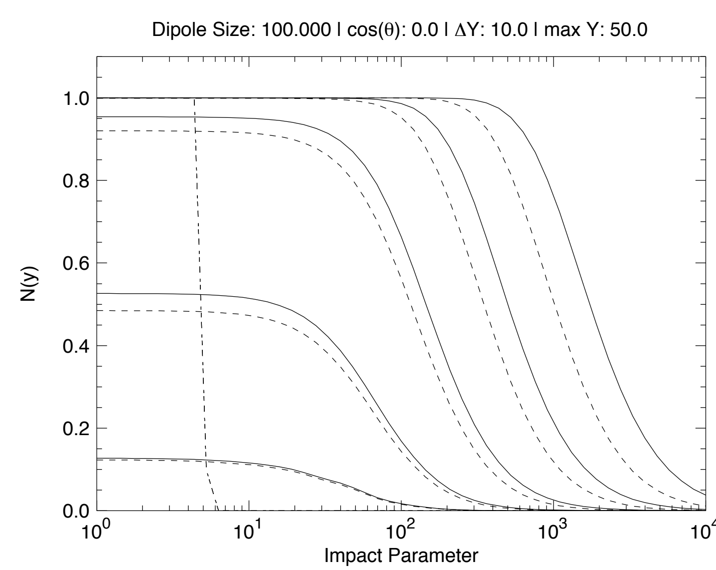
(a) $\cos(\theta) = 1.0, -1.0$



(b) $\cos(\theta) = 0.0$



(a)$cos(\theta) = 1.0, -1.0$



(b)$cos(\theta) = 0.0$

Angular correlations present in the solution

Amplitude larger for aligned configurations of the dipole

Could be relevant for the angular sensitive observables

Possible sensitivity through diffractive dijet in photoproduction/DIS

Hatta,Xiao,Yuan;
Altinoluk,Armesto,Beuf,Rezaeian;

Next to leading order calculations at small x

First NLL calculations in small x

NLL BFKL equation: *Fadin,Lipatov; Camici,Ciafaloni*

NLL calculation for the nonlinear evolution

NLL calculation of BK equation: *Balitsky-Chirilli*

NLL calculation of JIMWLK equation: *Kovner, Lublinsky, Mulian*

Impressive progress has been achieved in calculations of hard factors at NLO e.g.:

Photon-gluon impact factors: *Balitsky-Chirilli;*
Total DIS cross section in dipole framework: *Beuf; Hanninen et al*
Heavy quarks in DIS: *Beuf, Lappi, Paateleinen*
Vector mesons in DIS: *Boussarie et al, Mantysaari,Penttala*
Dihadrons/jets in DIS: *Caucal et al, Bergabo, Jalilian-Marian, Taels et al*
Diffractive DIS: *Beuf et al*
Diffractive dijet: *Boussarie et al, Iancu et al*
Photon+dijet in DIS: *Roy, Venugopalan*
inclusive hadron production in pA : *Chirilli et al;*
single jet production in pA: *Liu et al*
…

Fits to DIS structure function from rcBK

Successful fits to DIS structure functions. Example running coupling(rc) LL BK (no b dependence)
Albacete, Armesto, Guilherme Milhano, Salgado

Impact parameter is integrated out and enters as a parameter

$$\hat{\sigma}(r,Y) = \sigma_0 \, N(r,Y)$$

Initial conditions:

GBW-like :

$$N(r,Y=0) \; = \; 1 - \exp\left[- \left(\frac{r^2 Q_{s0}^2}{4} \right)^{\gamma} \right]$$

McLerran-Venugopalan (MV)-like

$$N(r,Y=0) \; = \; 1 - \exp\left[- \left(\frac{r^2 Q_{s0}^2}{4} \right)^{\gamma} \ln\left(\frac{1}{r\Lambda_{\rm QCD}} + e \right) \right]$$

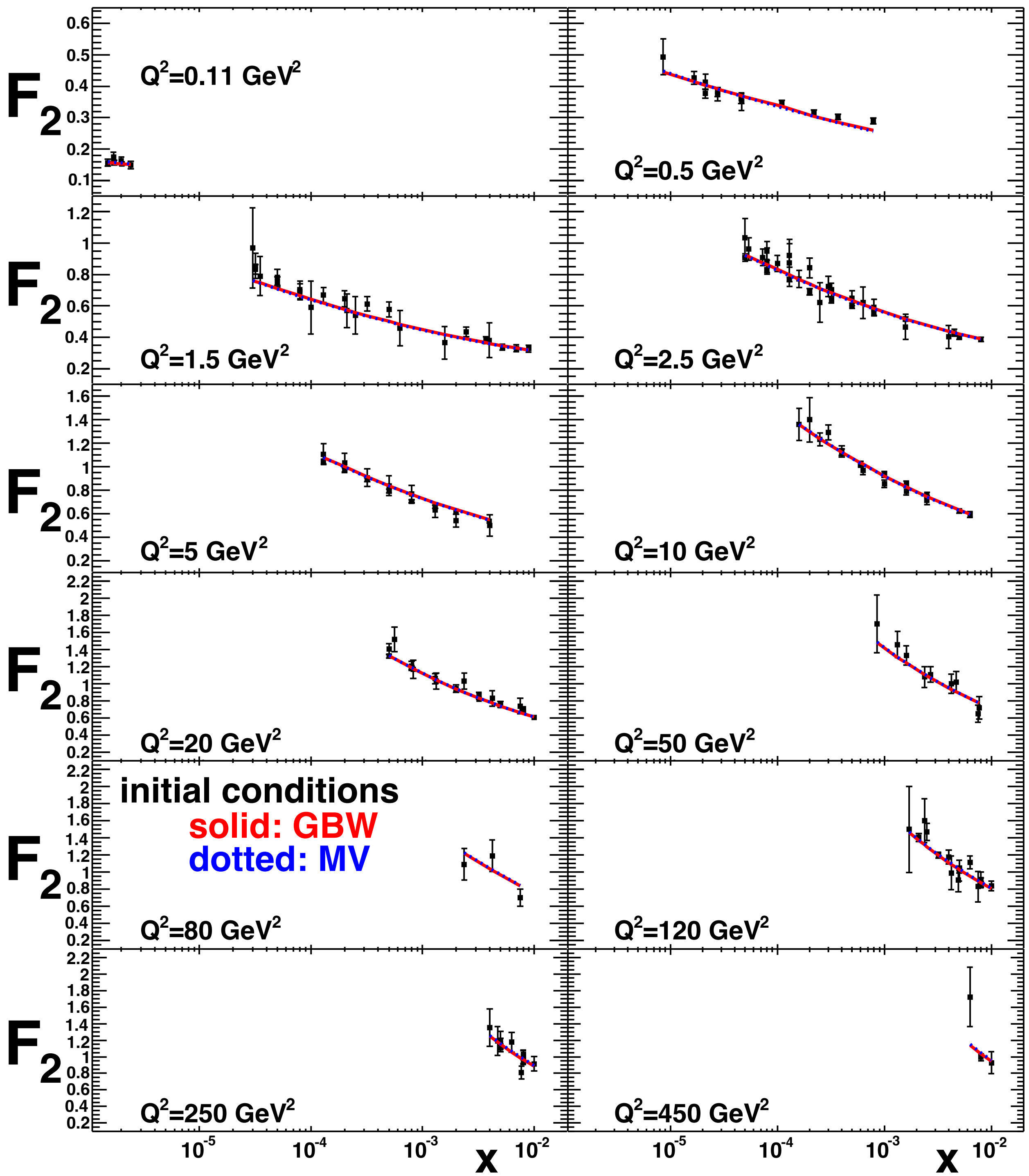
fit parameters:

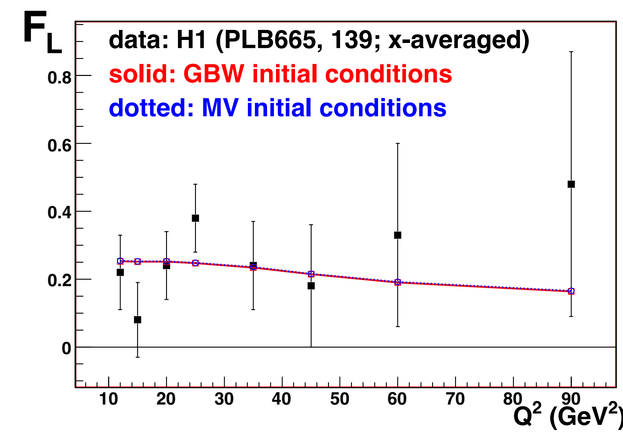
- σ_0 : total normalization
- Q_{s0}^2 saturation scale at the initial scale
- γ : anomalous dimension in the initial condition
- C: parameter relating running coupling in coordinate space to momentum space
- Additionally: freezing parameter for the coupling at large dipoles

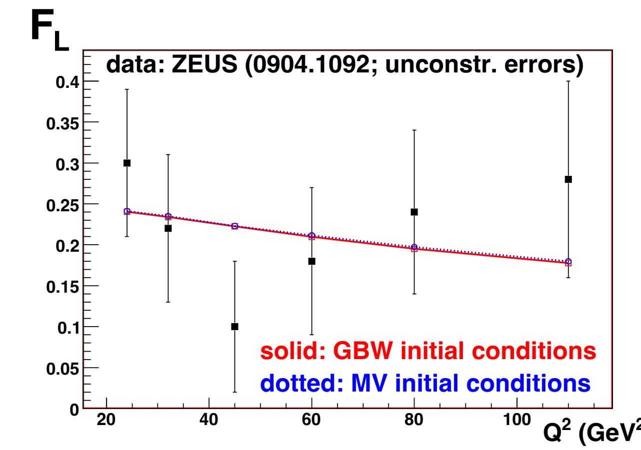
$$\alpha_s(r^2) = \frac{12\pi}{(11N_c - 2N_f)\ln(\frac{4C^2}{r^2\Lambda_{QCD}^2})}$$

Fits to DIS structure function from rcBK

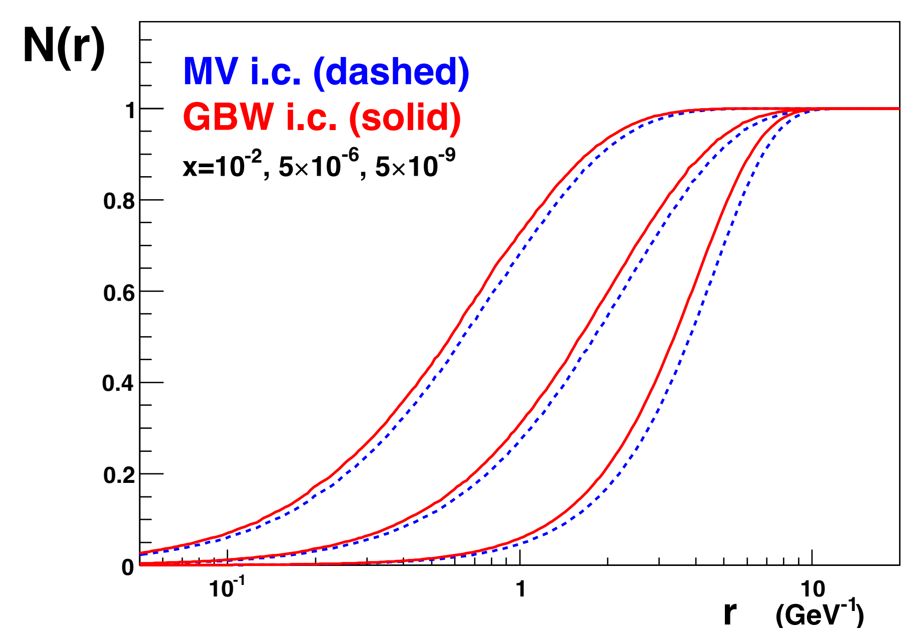
Albacete, Armesto, Guilherme Milhano, Salgado







Initial condition	σ_0 (mb)	Q^2_{s0} (GeV^2)	C^2	γ	χ^2/d.o.f.
GBW	31.59	0.24	5.3	1 (fixed)	916.3/844=1.086
MV	32.77	0.15	6.5	1.13	906.0/843=1.075

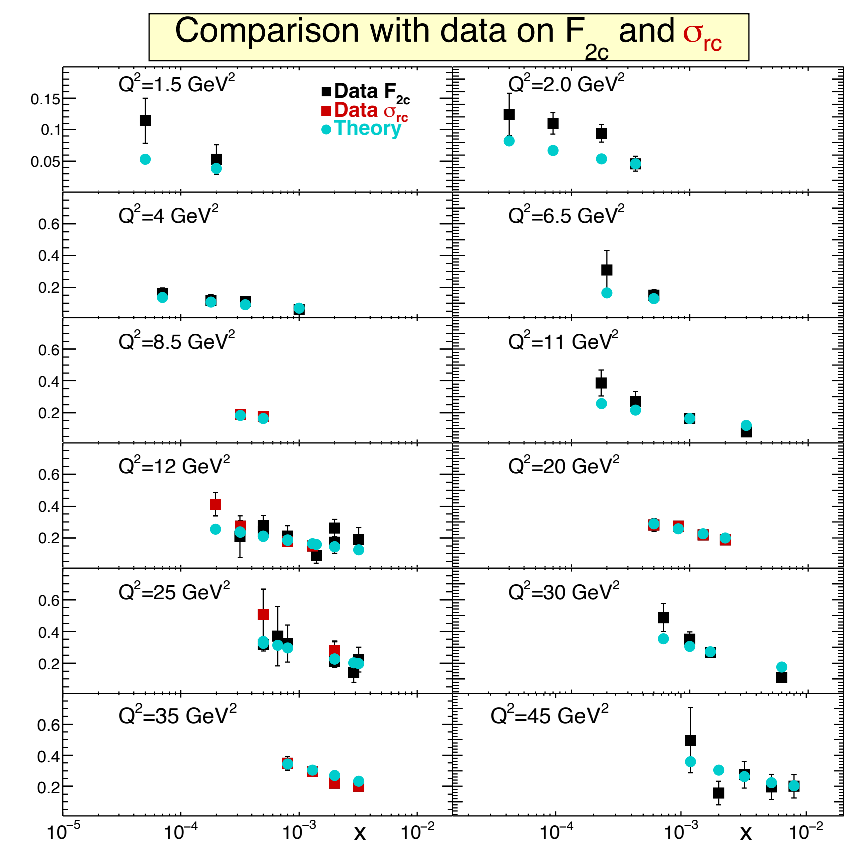


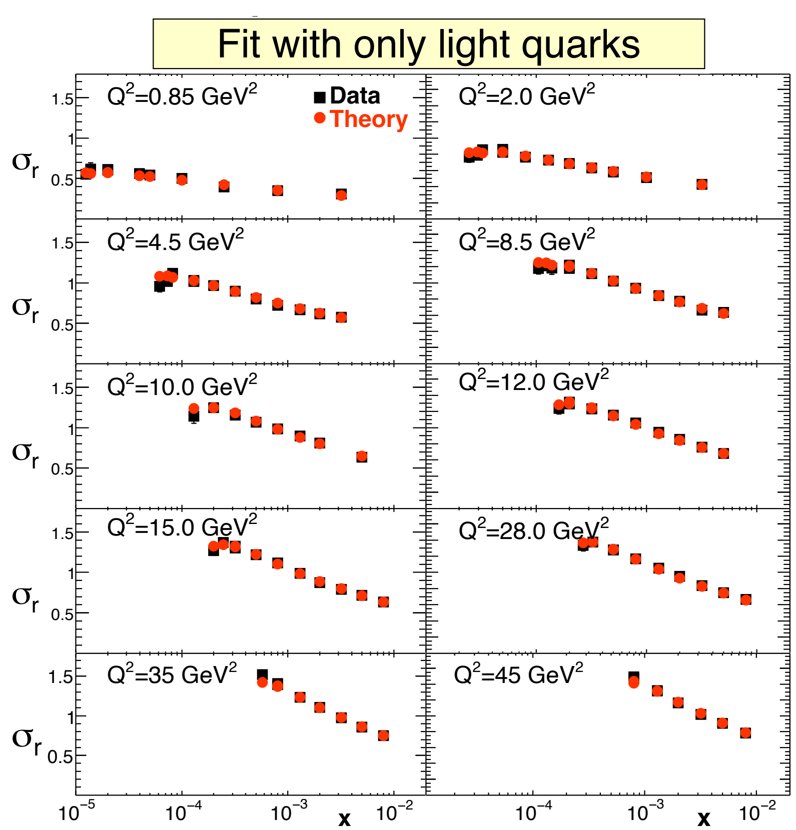
Fits to DIS structure function from rcBK

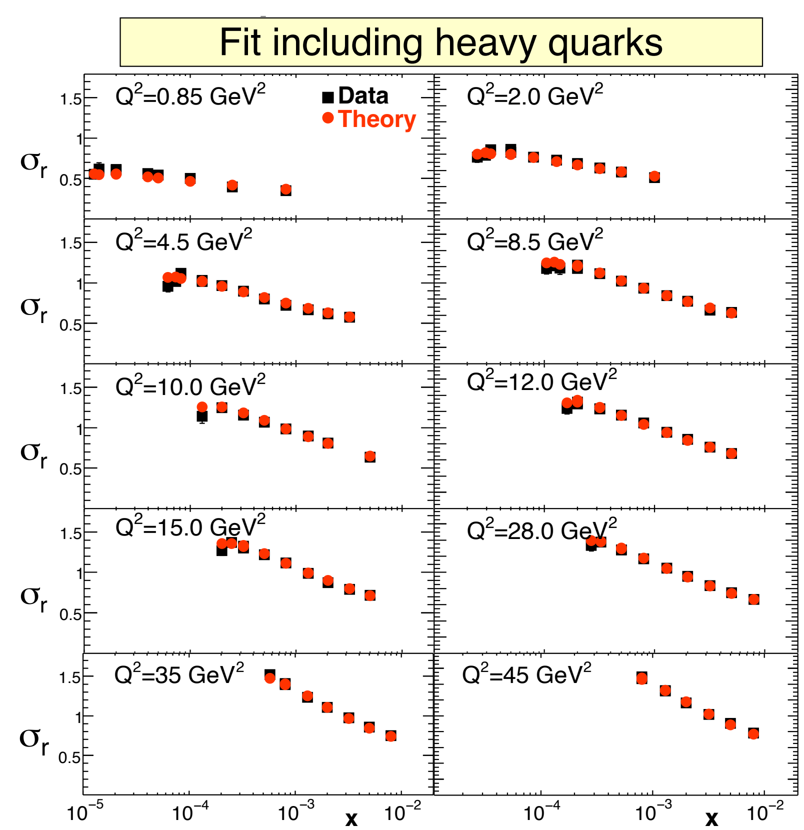
Albacete, Armesto, Guilherme Milhano, Quiroga, Salgado

	fit	$\chi^2_{d.o.f}$	Q^2_{s0}	σ_0	γ	Q^2_{s0c}	σ_{0c}	γ_c	C	m_l^2
	GBW									
a	$\alpha_{fr}=0.7$	1.269	0.2294	36.953	1.259	0.2289	18.962	0.881	4.363	fixed
a’	$\alpha_{fr}=0.7$ (Λ_{m_τ})	1.302	0.2341	36.362	1.241	0.2249	20.380	0.919	7.858	fixed
b	$\alpha_{fr}=0.7$	1.231	0.2386	35.465	1.263	0.2329	18.430	0.883	3.902	1.458E-2
c	$\alpha_{fr}=1$	1.356	0.2373	35.861	1.270	0.2360	13.717	0.789	2.442	fixed
d	$\alpha_{fr}=1$	1.221	0.2295	35.037	1.195	0.2274	20.262	0.924	3.725	1.351E-2
	MV									
e	$\alpha_{fr}=0.7$	1.395	0.1673	36.032	1.355	0.1650	18.740	1.099	3.813	fixed
f	$\alpha_{fr}=0.7$	1.244	0.1687	35.449	1.369	0.1417	19.066	1.035	4.079	1.445E-2
g	$\alpha_{fr}=1$	1.325	0.1481	40.216	1.362	0.1378	13.577	0.914	4.850	fixed
h	$\alpha_{fr}=1$	1.298	0.156	37.003	1.319	0.147	19.774	1.074	4.355	1.692E-2

Fits with heavy quarks with LL rcBK required separate parameters for heavy quarks







Fits to DIS structure function from rcBK at NLL

Beuf, Hanninen, Lappi, Mantysaari; Hanninen, Mantysaari, Paatelainen, Penttala

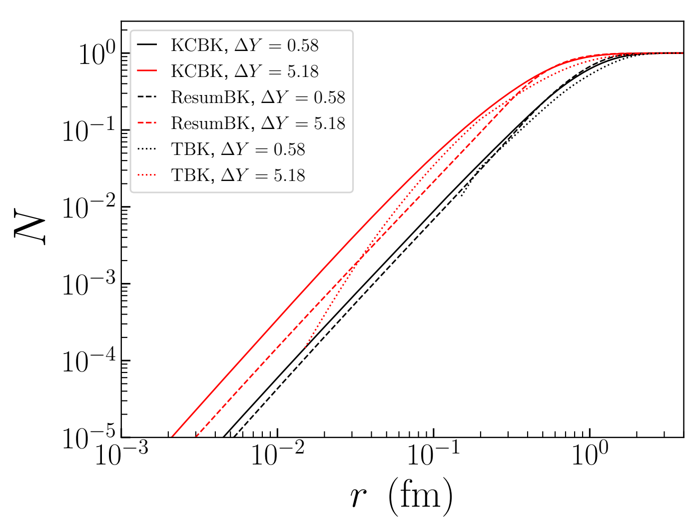
Beyond rcBK, including kinematically improved BK and NLL impact factors

Including NLO impact factors for heavy quarks

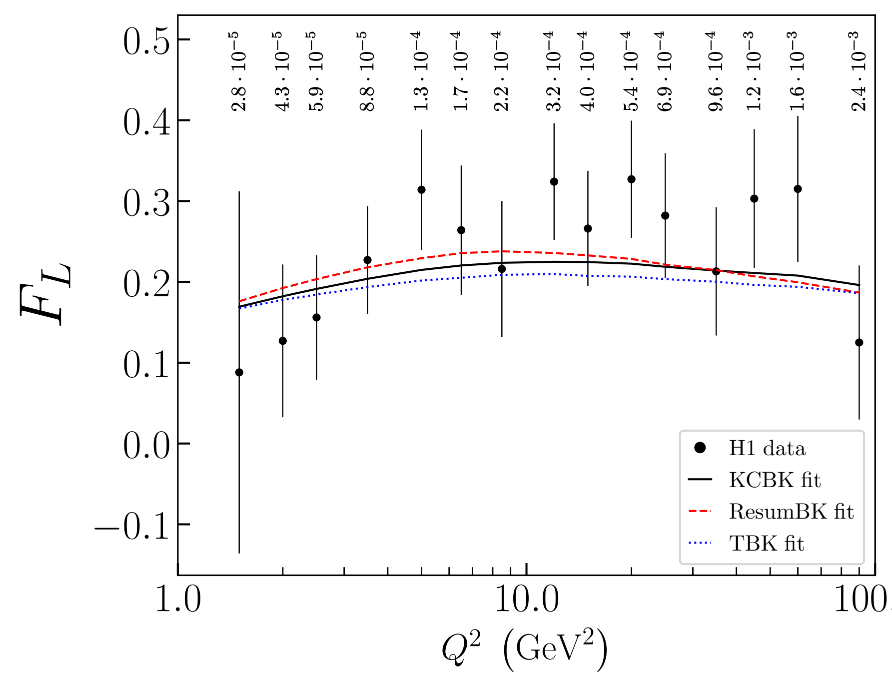
Consistent description of charm within the dipole+rcBK framework

Data	α_s	$Y_{0,\mathrm{BK}}$	χ^2/N	$Q^2_{s,0}$ [GeV^2]	C^2	γ	$\sigma_0/2$ [mb]	$Q^2_s(Y=\ln\frac{1}{0.01})$ [GeV^2]
HERA	parent	$\ln\frac{1}{0.01}$	1.85	0.0833	3.49	0.98	9.74	0.11
light-q	parent	$\ln\frac{1}{0.01}$	1.58	0.0753	37.7	1.25	18.41	0.11
HERA	parent	0	1.24	0.0680	79.9	1.21	18.39	0.20
light-q	parent	0	1.18	0.0664	1340	1.47	27.12	0.14
HERA	Bal + SD	$\ln\frac{1}{0.01}$	1.89	0.0905	0.846	1.21	8.68	0.13
light-q	Bal + SD	$\ln\frac{1}{0.01}$	2.63	0.0720	1.91	1.55	12.44	0.11
HERA	Bal + SD	0	1.49	0.1114	0.846	1.94	8.53	0.26
light-q	Bal + SD	0	1.69	0.1040	2.87	7.70	12.09	0.14

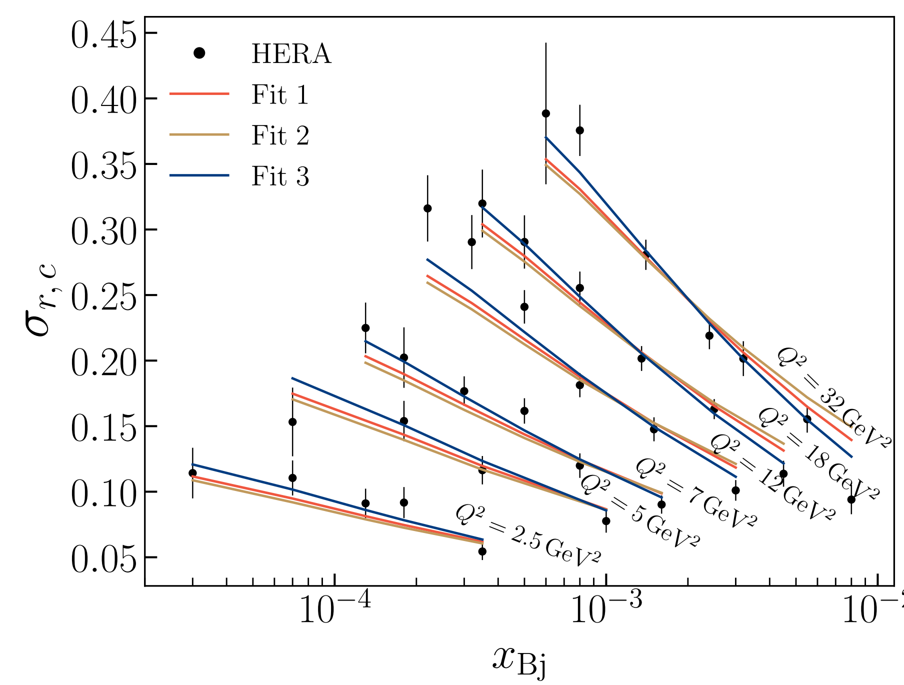
TABLE I: Fits to HERA and light quark data with the Kinematically Constrained BK evolution (KCBK).



dipole amplitude

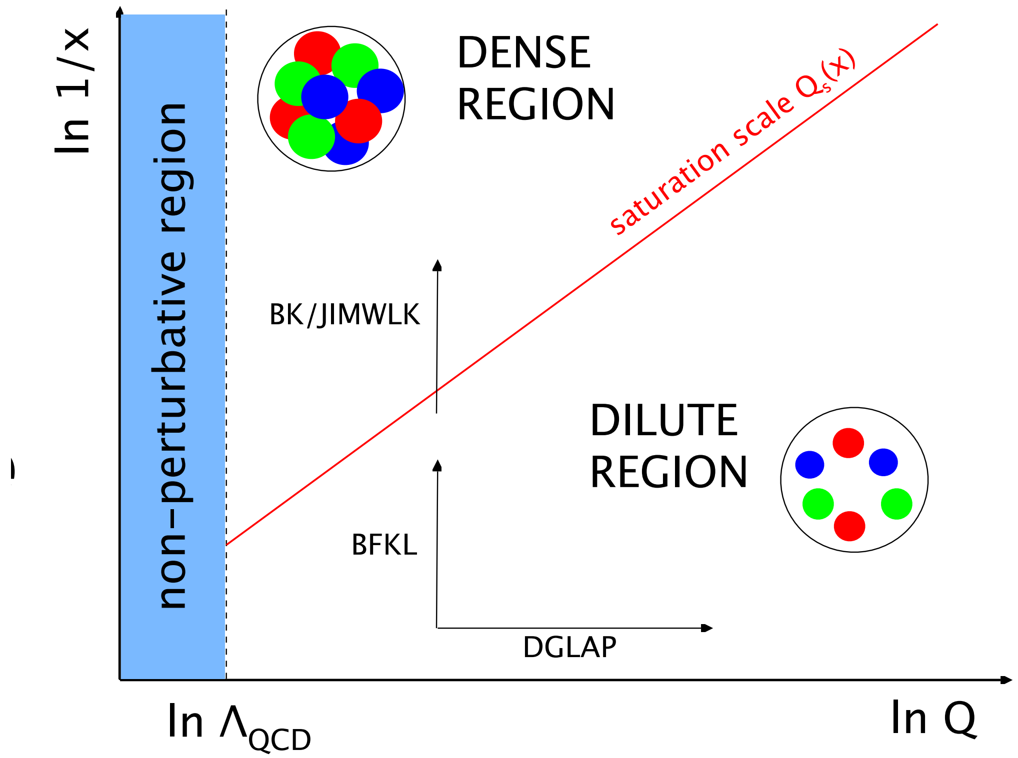


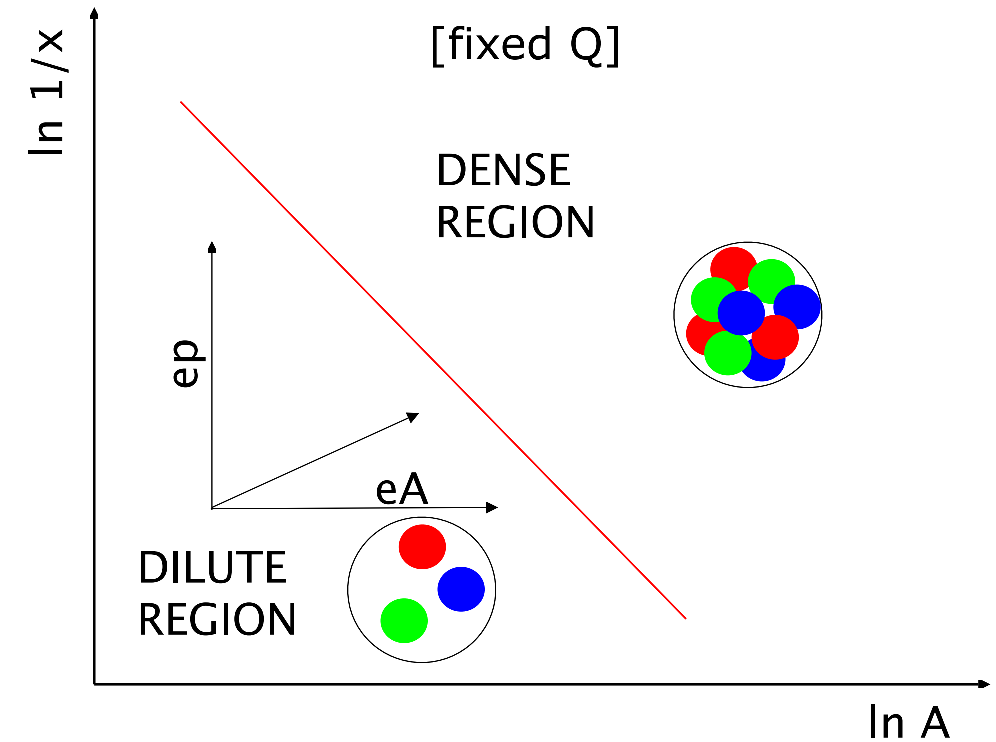
longitudinal structure function



charm structure function

Studying saturation at EIC with nuclei

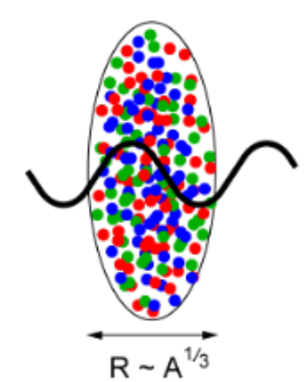




Can we study explore the saturation with EIC ?

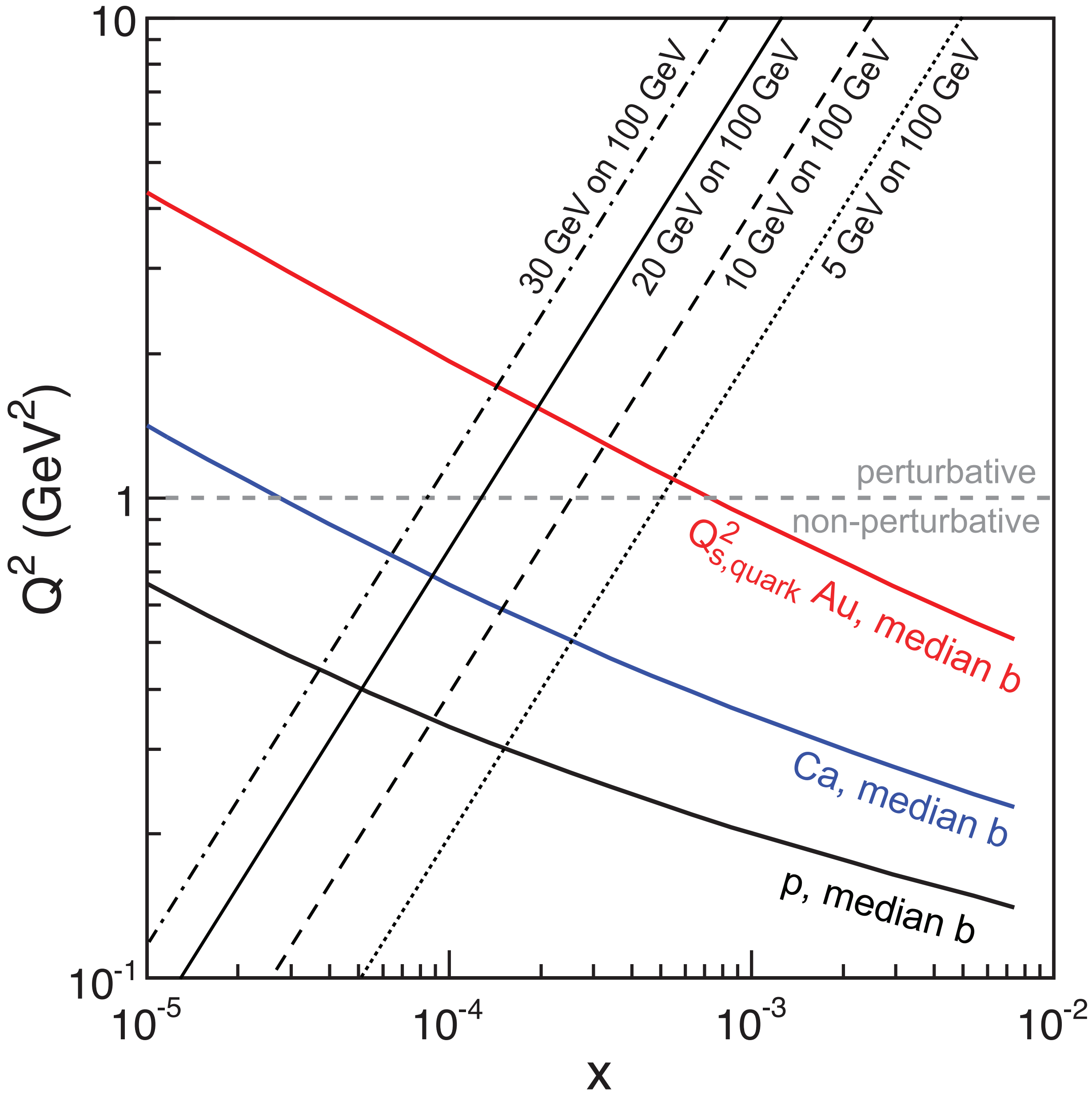
The dense regime can be reached in two ways: **small x** and/or **large A**

$$\frac{A \times xg(x,Q_s^2)}{\pi A^{2/3}} \times \frac{\alpha_s(Q_s^2)}{Q_s^2} \sim 1 \qquad\qquad Q_s^2 \sim A^{1/3} Q_0^2 \left(\frac{1}{x}\right)^{\lambda}$$



Nuclei provide enhancement of the density : opportunities to test saturation at EIC

EIC sensitivity to saturation scale



$$Q_s^2 \sim A^{1/3} Q_0^2 \left(\frac{1}{x}\right)^\lambda$$

EIC sensitive to perturbative saturation region in scattering with heavy nuclei.

Shown: median b-impact parameter.

Exclusive processes can be sensitive to different b.

BK for the nucleus

One can apply the BK to the case of nucleus, simply by modifying the initial conditions

$$\hat{\sigma}^p(r) = \sigma_0 N^p(r)$$

relation between the dipole cross section and the dipole amplitude for the proton (impact parameter integrated over in the prefactor)

In the dilute limit, small dipole sizes (corresponding to large scales) dipole nucleus cross section should go into incoherent sum of dipole-proton cross sections

$$\hat{\sigma}^A(r) = A\hat{\sigma}^p(r)$$

(impulse-approximation)

On the other hand the dipole-nucleus amplitude should be limited

$$N^A(r,b) \leq 1$$

(b:impact parameter for nucleus)

For example one can take the following model

$$N^A(r,b) = \left[1 - \exp(-\frac{AT_A(b)}{2}\hat{\sigma}^p)\right]$$

with $T_A(b)$ nuclear profile

$$\hat{\sigma}^A(r) = 2\int d^2\mathbf{b}\, N^A(r,b)$$

Testing saturation through inclusive structure functions at EIC

Study differences in evolution between **linear DGLAP** evolution and **nonlinear** evolution with **saturation**

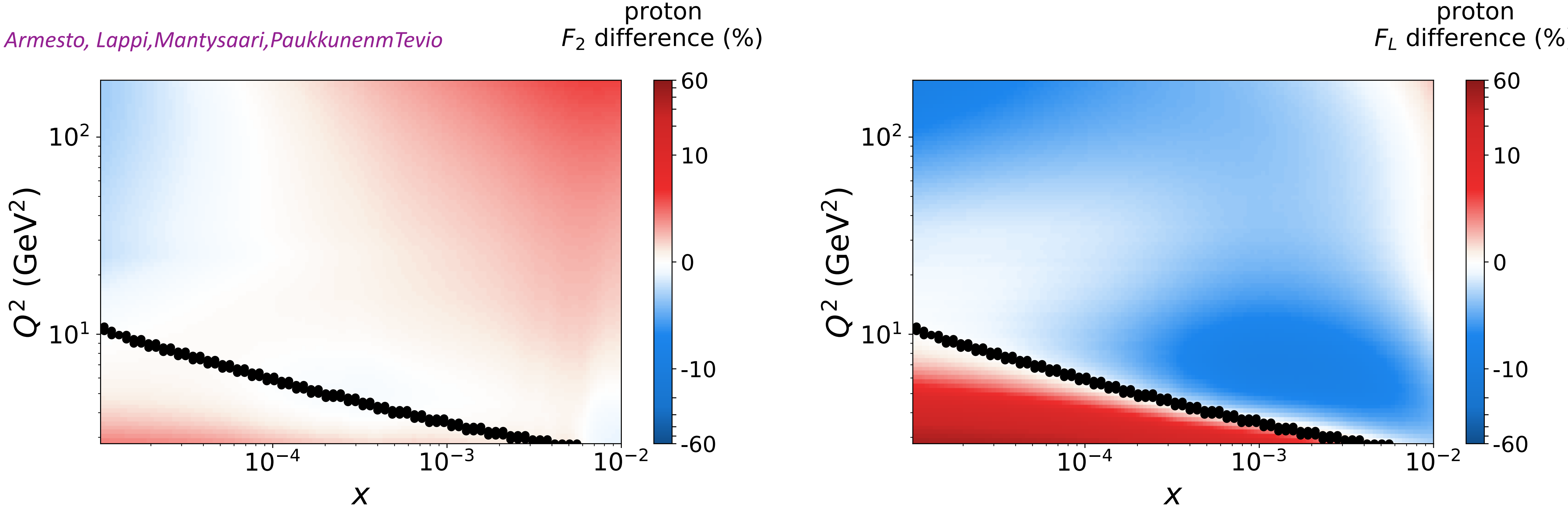
Matching of both approaches in the region where saturation effects expected to be small

Bayesian reweighting method using NNPDF replicas. Matching in the region: $Q^2 \simeq 10\, Q_s(x)^2$

Quantify differences away from the matching region: **differences in evolution dynamics**

$$\frac{F_{2,L}^{\mathrm{BK}} - F_{2,L}^{\mathrm{Rew}}}{F_{2.L}^{\mathrm{BK}}}$$

Armesto, Lappi,Mantysaari,PaukkunenmTevio



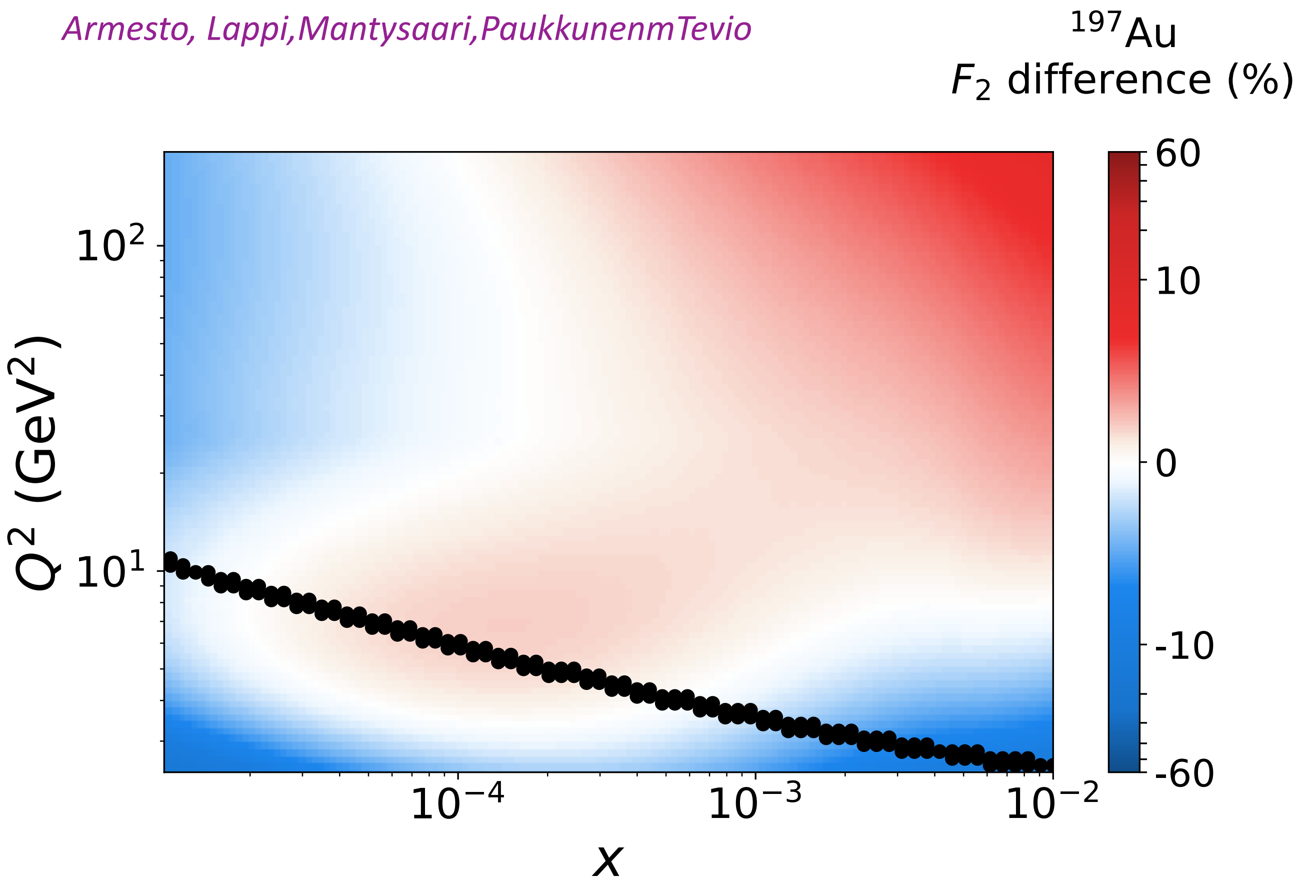
Dark line indicates the matching between DGLAP and BK evolution

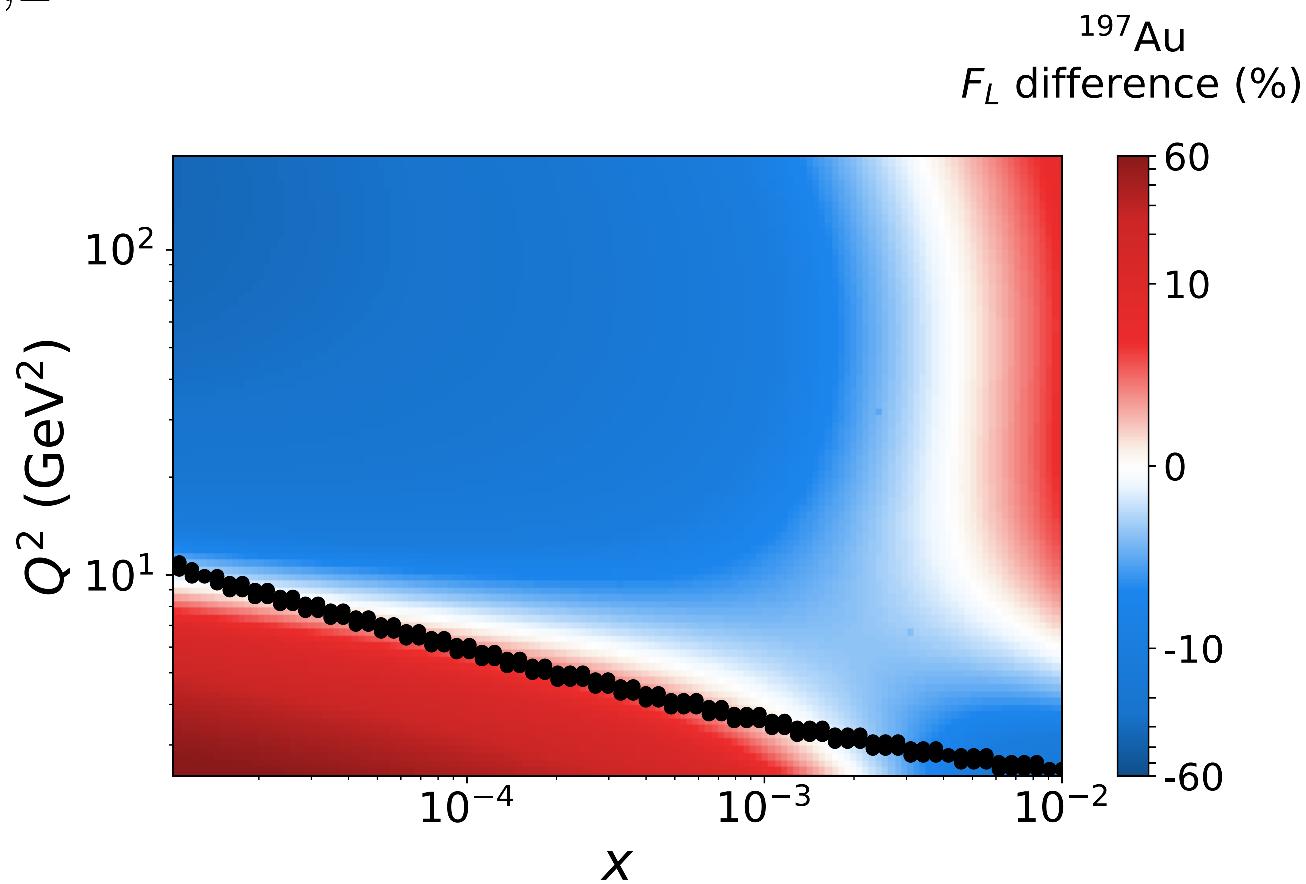
Proton: difference between DGLAP and nonlinear are small % for F_2^p and up to 10% for F_L^p.

Testing saturation through inclusive structure functions at EIC

$$\frac{F_{2,L}^{\mathrm{BK}} - F_{2,L}^{\mathrm{Rew}}}{F_{2,L}^{\mathrm{BK}}}$$

Armesto, Lappi,Mantysaari,PaukkunenmTevio



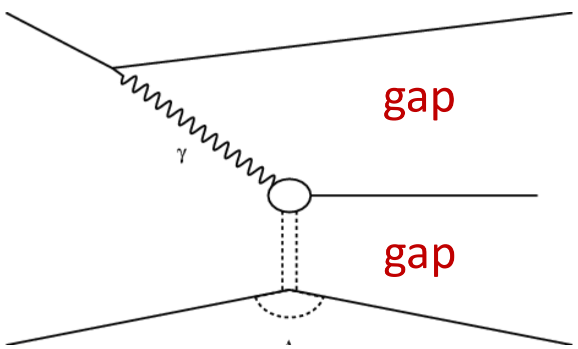


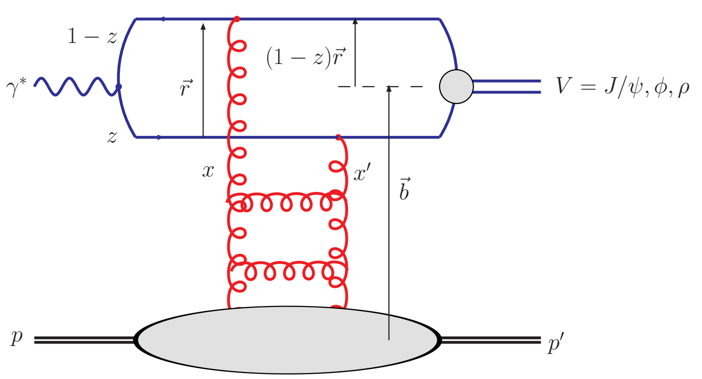
Heavy nucleus: difference between DGLAP and nonlinear are few % for F_2^A and up to 20% for F_L^A.
Second observable will significantly help constrain the model(s) for the evolution
Longitudinal structure function can provide good sensitivity at EIC

Exclusive vector meson photo and electro-production

Weiler; Ryskin; Jung, Schuler, Terron; Nemchik, Nikolaev Zakharov…

$$\gamma^{(*)} p \to V p$$





Exclusive (photo)production of vector mesons
Mass of the heavy vector meson provides a hard scale
Process sensitive to **square of gluon density**
Explored at HERA collider

$$x = (Q^2 + M^2_{J/\psi})/(W^2 + M^2_{J/\psi})$$

$$\bar{Q}^2 = (Q^2 + M^2_{J/\psi})/4,$$

Lowest order perturbative formula for electroproduction

Ryskin

$$\frac{d\sigma}{dt}(\gamma^* p \to J/\psi\ p)\Big|_{t=0} = \frac{\Gamma_{ee} M^3_{J/\psi} \pi^3}{48\alpha} \left[\frac{\alpha_s(\bar{Q}^2)}{\bar{Q}^4} xg(x,\bar{Q}^2)\right]^2 \left(1 + \frac{Q^2}{M^2_{J/\psi}}\right)$$

Can test dynamics at low x and different A if nuclear targets are involved

Exclusive vector meson photoproduction

Measurements in ep

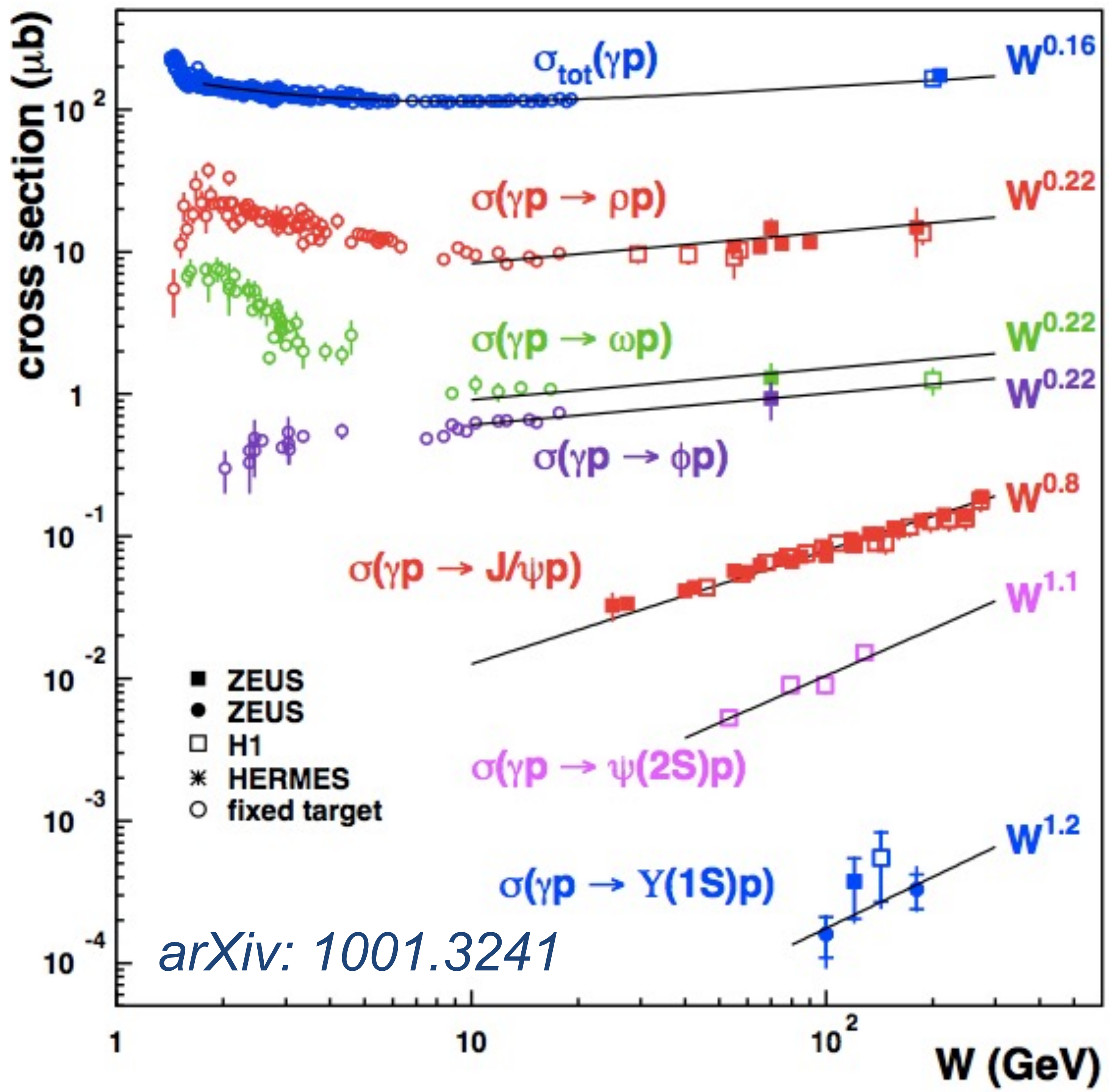
Rise in the slope of the energy
with the scale of the process

$$\sigma \sim W^{\delta}$$

$$\delta \sim 4(\alpha_p(t) - 1)$$

$$\alpha_p(t) = \alpha_p(0) + \alpha' t$$

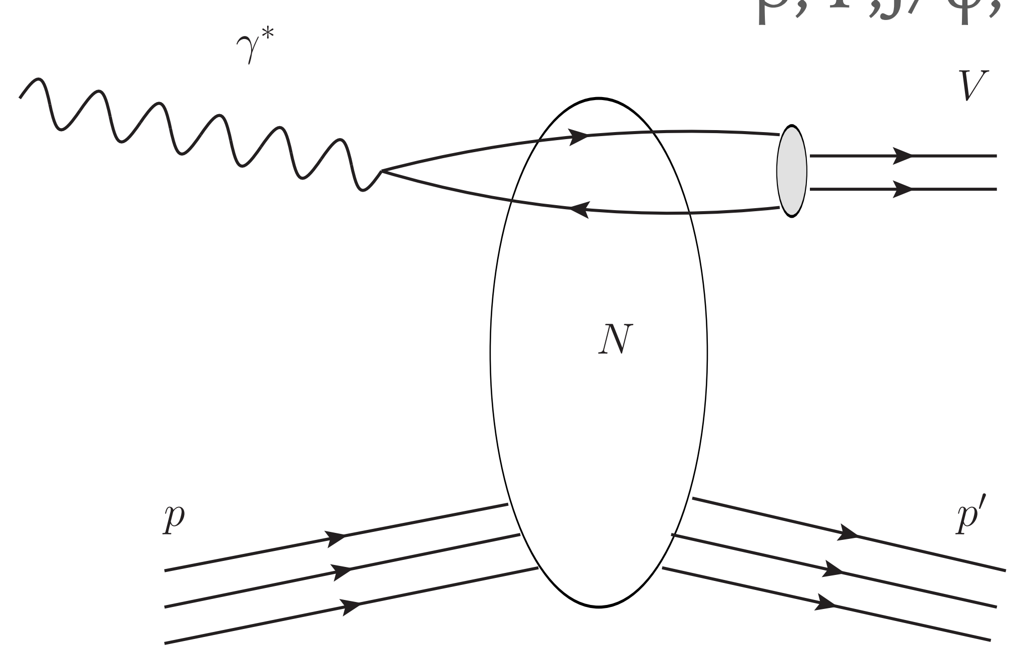
Transition from soft to hard phenomena



Exclusive diffractive production of vector mesons

Nikolaev,Zkharov;
Strikman,Frankfurt,Rogers;
Levin et al;
Munier,Mueller,AS;
Motyka,Kowalski,Watt;
Berger,AS;
Armesto,Rezeaian;
Lappi,Mantysaari,Schenke;...

ρ,Φ,J/ψ,Υ production



cross section

$$\frac{d\sigma}{dt} = \frac{1}{16\pi}|A(x,\Delta,Q)|^2$$

amplitude

$$A(x,\Delta,Q) = \sum_{h,\bar{h}} \int d^2\mathbf{r} \int dz\, \Psi_{h,h^*}(\mathbf{r},z,Q^2)\, \mathcal{N}(x,\mathbf{r},\Delta)\, \Psi^V_{h,h^*}(\mathbf{r},z)$$

dipole cross section

$$\sigma_{\rm dip}(x,\mathbf{r}) = \mathrm{Im}\, i\mathcal{N}(x,\mathbf{r},\Delta=0)$$

dipole amplitude

$$\mathcal{N}(x,\mathbf{r},\Delta) = 2\int d^2\mathbf{b}\, N(x,\mathbf{r},\mathbf{b})\, e^{i\Delta\cdot\mathbf{b}}$$

$\mathbf{r}$ dipole size

$\mathbf{b}$ impact parameter

Δ momentum transfer

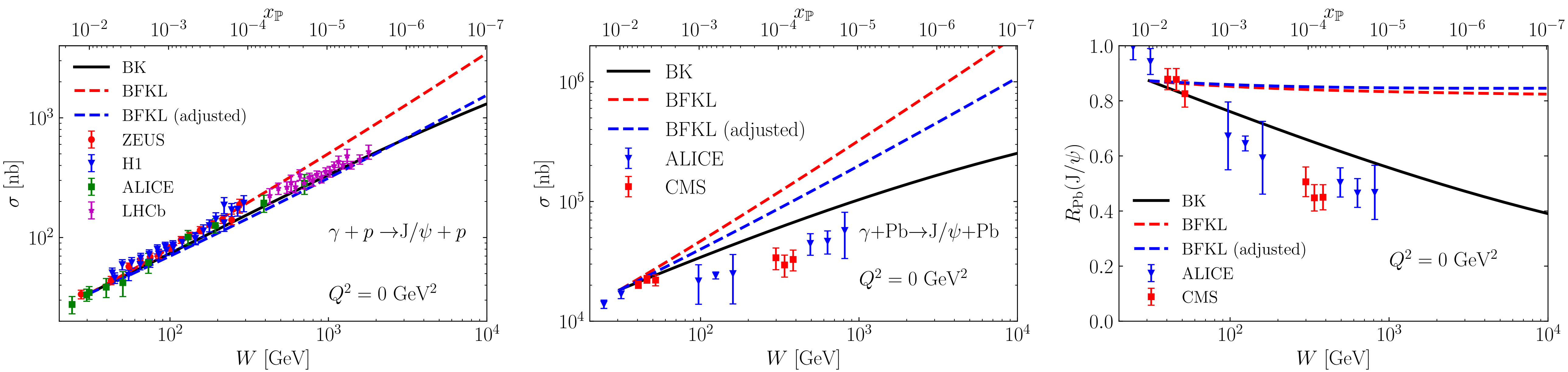
$z, (1-z)$ fraction of the longitudinal momentum of the photon carried by the quark(anti-quark)

Exclusive photoproduction of J/ψ

Penttala, Royon

Calculation of the processes $\gamma + p \to J/\psi + p$ and $\gamma + Pb \to J/\psi + Pb$

using linear BFKL and non-linear BK evolution



(a) Proton. (b) Lead. (c) Nuclear suppression factor.

Proton target: non-linear effects are negligible, linear evolution describes the data

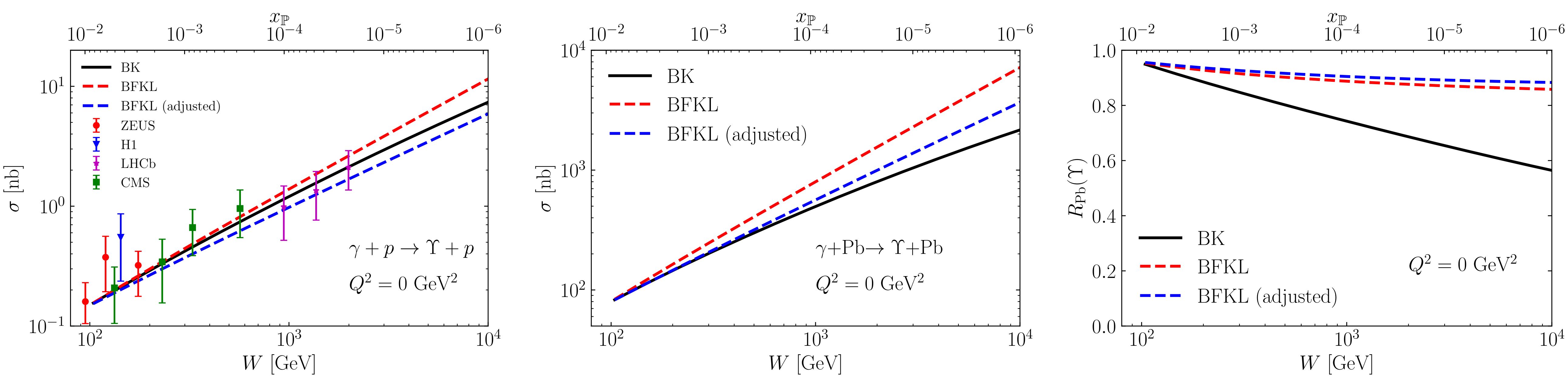
Lead target: non-linear evolution closer to the experimental data

Exclusive photoproduction of Υ

Penttala, Royon

Calculation of the processes $\gamma + p \to \Upsilon + p$ and $\gamma + Pb \to \Upsilon + Pb$

using linear BFKL and non-linear BK evolution



(a) Proton. (b) Lead. (c) Nuclear suppression factor.

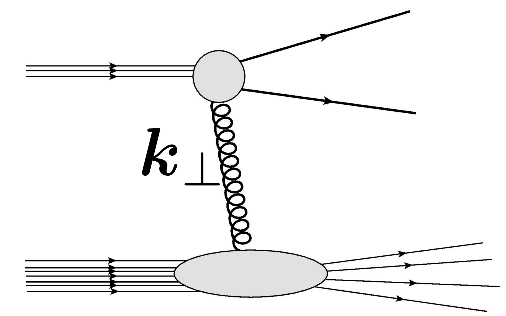
Proton target: non-linear effects are negligible, linear evolution describes the data

Lead target: predictions from BFKL and BK are closer together, nonlinear effects small

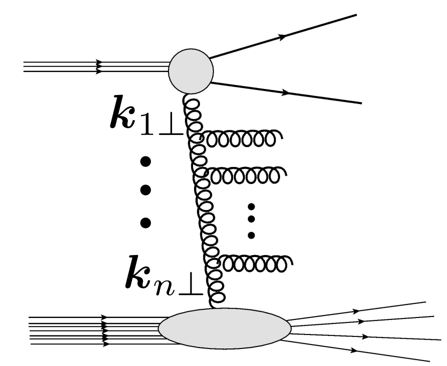
Higher scale: smaller nonlinear effects

Angular correlations in small x/CGC physics

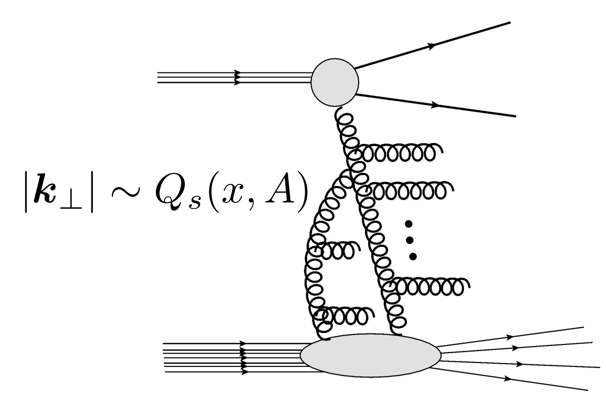
- **Transverse momentum** from the target can lead to the modification of **angular correlation** between the two produced hadrons/jets (at forward rapidities)



- **Broadening** of angular correlation should be present even in linear BFKL due to the **diffusion** of the transverse momenta at low x



- In the dense regime **decorrelation** due to multiple scatterings and the **saturation scale** being dependent on x and A



Marquet; Albacete, Marquet

Angular (de)correlations in saturation

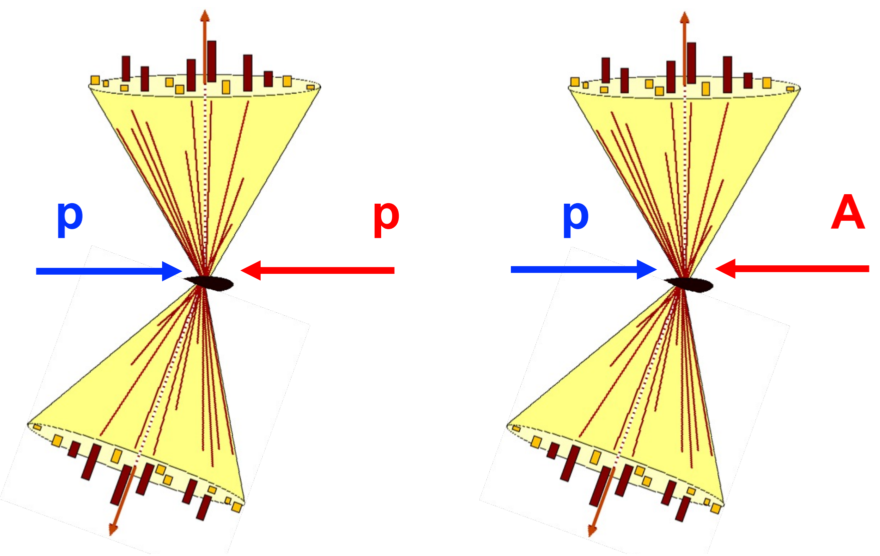
single inclusive cross section (at LO)

$$\frac{d\sigma^{pA\to hX}}{d^2b\, d^2p_\perp\, dy_h} = \int_{z_h}^{1} \frac{dz_1}{z_1^2} \Big[D_{h/q}(z_1) x_p q_f(x_p) F_{x_g}(k_\perp) + x_p g_f(x_p) \tilde{F}_{x_g}(k_\perp) D_{h/g}(z_1)\Big] \ ,$$

Test in dA or pA collisions

differential cross section for two-particle production

$$\frac{d\sigma^{(pA\to h_1 h_2)}_{\text{corr.}}}{dy_{h_1} dy_{h_2} d^2p_{1\perp} d^2p_{2\perp}} = \int \frac{dz_1}{z_1^2}\frac{dz_2}{z_2^2}\frac{\alpha_s^2}{\hat{s}^2} \Big[x_p q(x_p) \mathcal{F}^{(i)}_{qg} \times H^{(i)}_{qg} \big(D_{h_1/q}(z_1) D_{h_2/g}(z_2) + D_{h_2/q}(z_1) D_{h_1/g}(z_2)\big) + x_p g(x_p) \mathcal{F}^{(i)}_{gg} H^{(i)}_{gg} D_{h_1/g}(z_1) D_{h_2/g}(z_2)\Big] \ ,$$



picture from STAR collaboration

$g(x), q(x)$	collinear PDFs in the projectile
$F_{x_g}(k_T), \tilde{F}_{x_g}(k_T)$	gluon distributions in the target
$D_{h/q}(z), D_{h/g}(z)$	fragmentation functions

di-hadron correlation

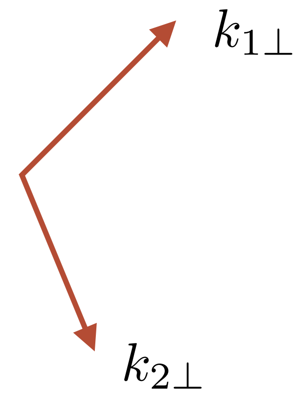
$$C(\Delta\phi) = \frac{\int_{|p_{1\perp}|,|p_{2\perp}|} \frac{d\sigma^{pA\to h_1 h_2}}{dy_1\, dy_2\, d^2p_{1\perp}\, d^2p_{2\perp}}}{\int_{|p_{1\perp}|} \frac{d\sigma^{pA\to h_1}}{dy_1\, d^2p_{1\perp}}}$$

Dihadron correlations: saturation and Sudakov included

Including Sudakov effects in CGC calculations:

Mueller, Xiao, Yuan

$$\frac{d\sigma_{\text{corr.}}^{(pA\to h_1h_2)}}{dy_{h_1}dy_{h_2}d^2p_{1\perp}d^2p_{2\perp}} = \int \frac{dz_1}{z_1^2}\frac{dz_2}{z_2^2}\frac{\alpha_s^2}{\hat{s}^2}\left[x_p q(x_p)\mathcal{F}_{qg}^{(i)}\right.$$
$$\times H_{qg}^{(i)}\left(D_{h_1/q}(z_1)D_{h_2/g}(z_2) + D_{h_2/q}(z_1)D_{h_1/g}(z_2)\right)$$
$$\left. + x_p g(x_p)\mathcal{F}_{gg}^{(i)} H_{gg}^{(i)} D_{h_1/g}(z_1)D_{h_2/g}(z_2)\right] ,$$



Transverse mom. imbalance

$$k_\perp = k_{1\perp} + k_{2\perp}$$

Relative transverse mom.

$$P_\perp = \frac{1}{2}|k_{1\perp} - k_{2\perp}|$$

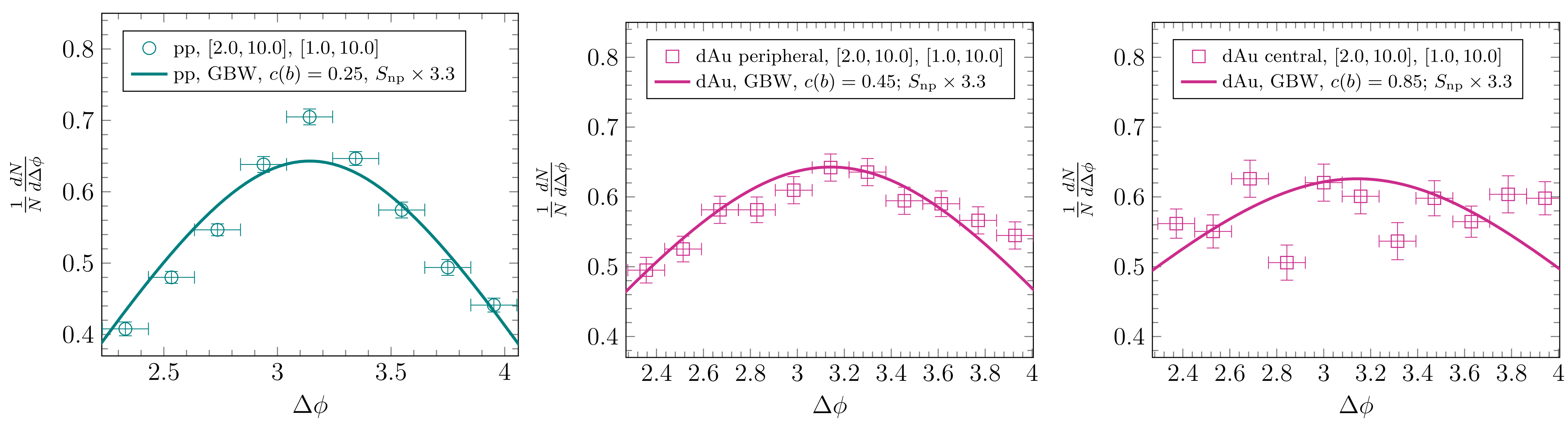
Large logarithms:

$$\alpha_s \ln^2(P_\perp^2/k_\perp^2)$$

Sudakov effects factorize

Modify functions $\mathcal{F}_{\alpha\beta}^{(i)}$

Normalized forward dihadron angular correlation compared with the STAR data (also normalized)



In pp Sudakov effects are very important

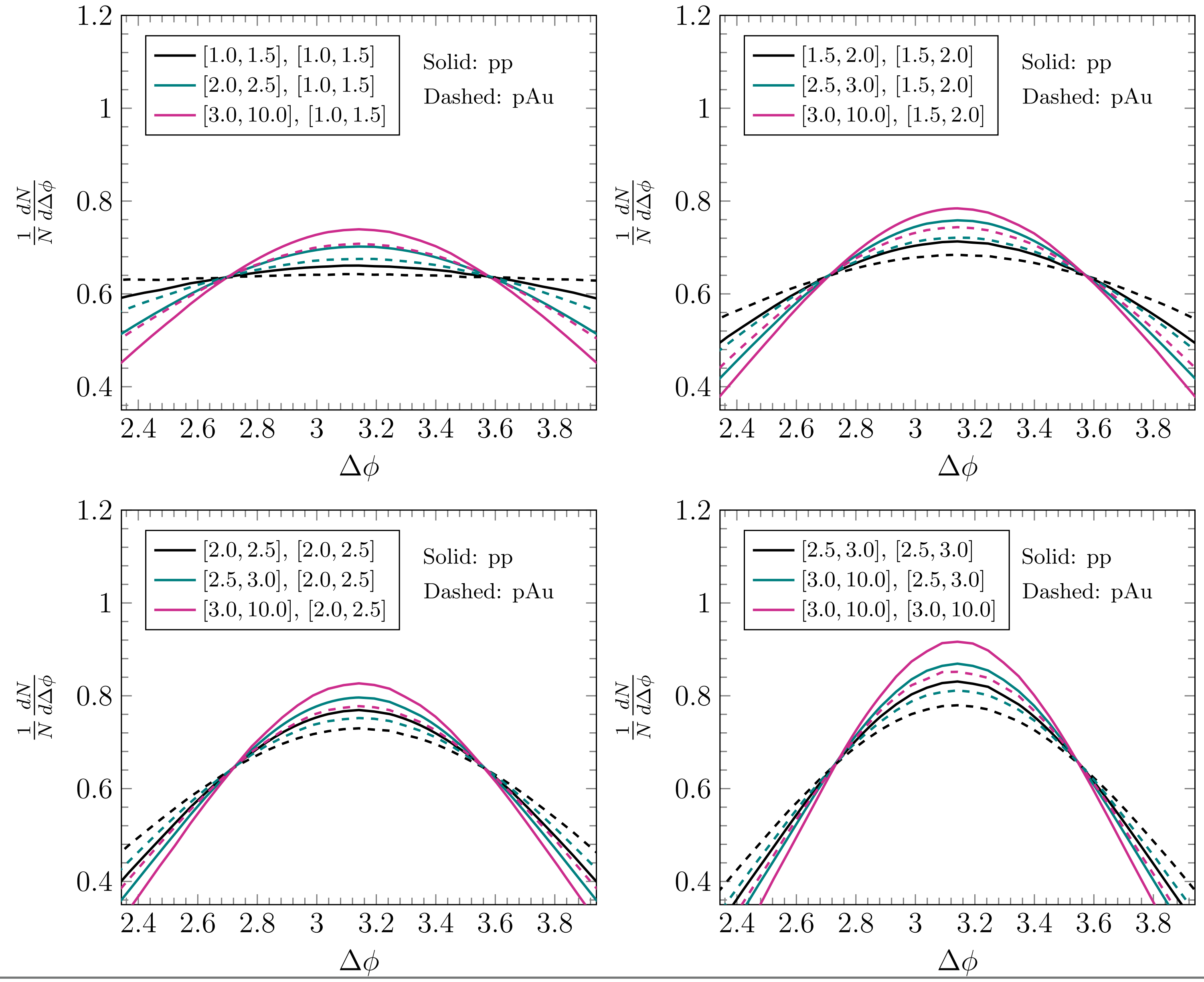
AS, Wei, Xiao, Yuan

Predictions for pA

Normalized forward dihadron angular correlation in pA

Distributions become flat for lower p_T

Still pA always flatter than pp



AS, Wei, Xiao, Yuan

Di-hadron correlations in pp,pAl,pAu

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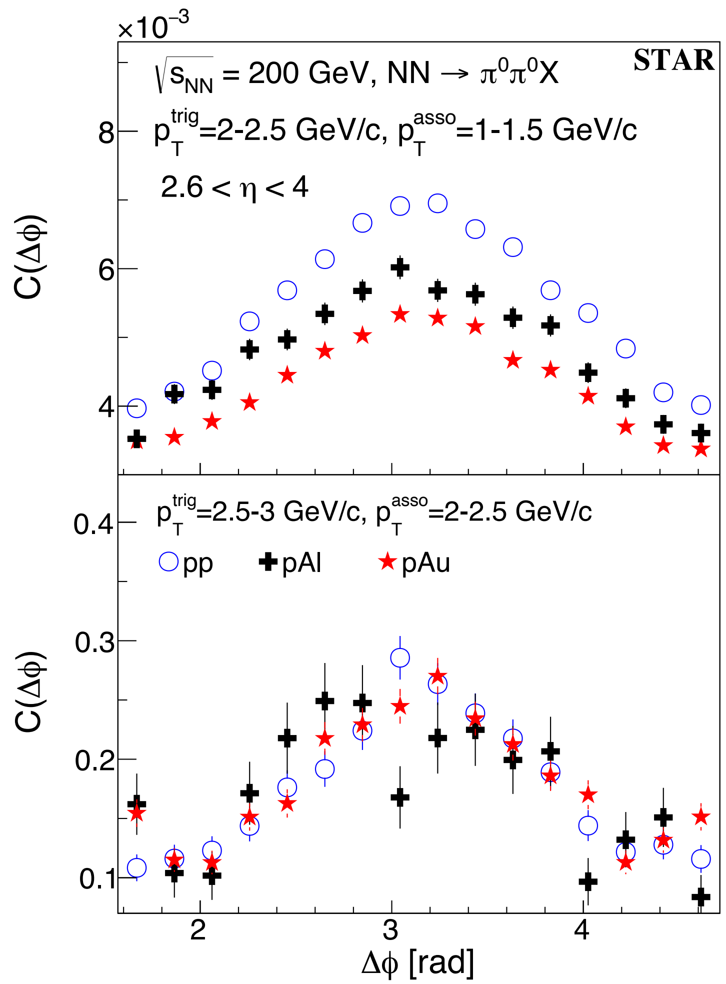
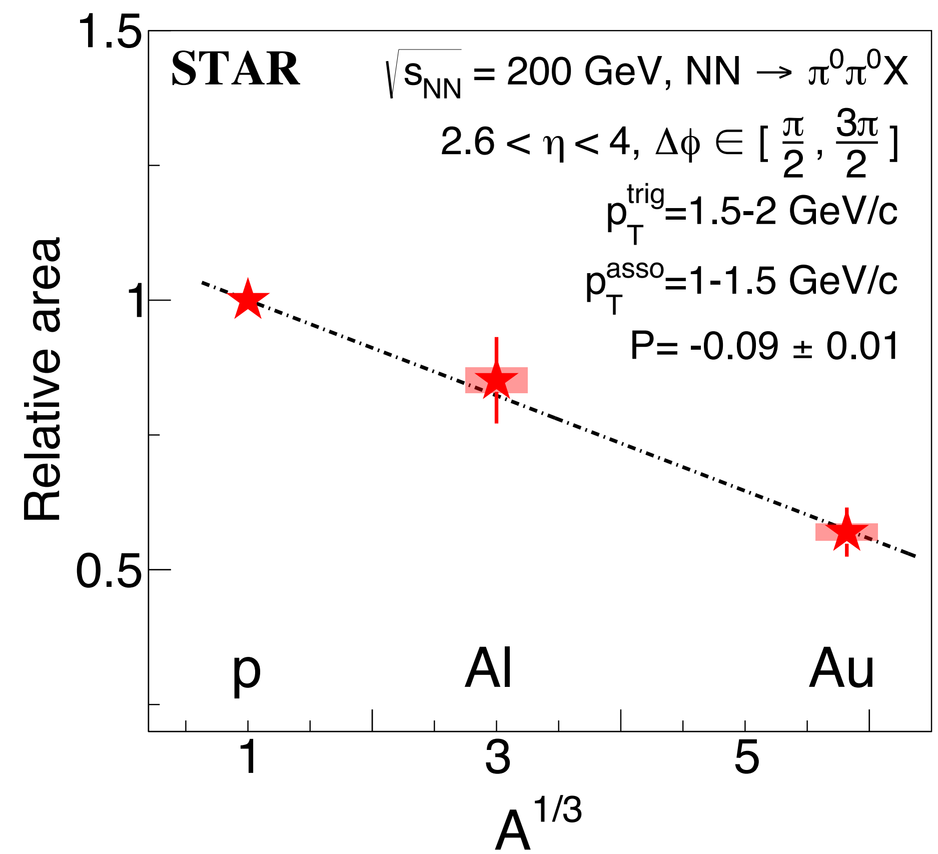
Evidence for Nonlinear Gluon Effects in QCD and Their Mass Number Dependence at STAR

The STAR Collaboration reports measurements of back-to-back azimuthal correlations of di-π^0s produced at forward pseudorapidities ($2.6 < \eta < 4.0$) in $p+p$, $p+\mathrm{Al}$, and $p+\mathrm{Au}$ collisions at a center-of-mass energy of 200 GeV. We observe a clear suppression of the correlated yields of back-to-back π^0 pairs in $p+\mathrm{Al}$ and $p+\mathrm{Au}$ collisions compared to the $p+p$ data. The observed suppression of back-to-back pairs as a function of transverse momentum suggests nonlinear gluon dynamics arising at high parton densities. The larger suppression found in $p+\mathrm{Au}$ relative to $p+\mathrm{Al}$ collisions exhibits a dependence of the saturation scale Q_s^2 on the mass number A. A linear scaling of the suppression with $A^{1/3}$ is observed with a slope of -0.09 ± 0.01.

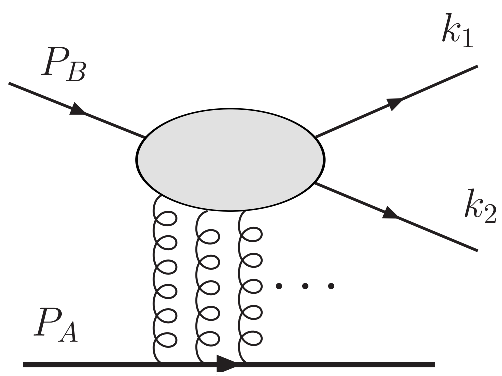




Clear suppression observed in pA, scales with A

Testing saturation through (de)correlations of hadrons in eA

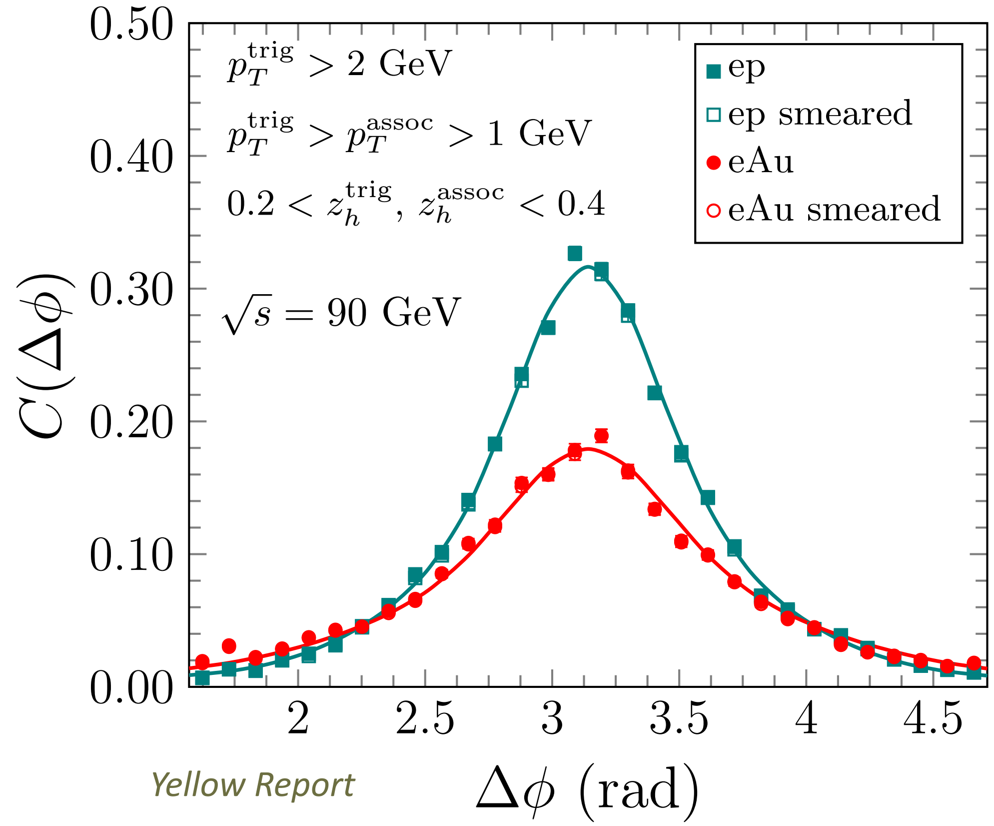
Azimuthal (**de)correlations of two hadrons** (dijets) in DIS in eA: direct test of the **Weizsacker -Williams unintegrated gluon distribution**



$$C(\Delta\phi) = \frac{1}{\frac{d\sigma_{\rm SIDIS}^{\gamma^*+A\to h_1+X}}{dz_{h1}}} \frac{d\sigma_{\rm tot}^{\gamma^*+A\to h_1+h_2+X}}{dz_{h1}dz_{h2}d\Delta\phi}$$

$$\frac{d\sigma^{\gamma^*+A\to h_1+h_2+X}}{dz_{h1}dz_{h2}d^2p_{h1T}d^2p_{h2T}} \sim \mathcal{F}(x_g,q_T)\otimes\mathcal{H}(z_q,k_{1T},k_{2T})\otimes D_q(z_{h1}/z_q,p_{1T})\otimes D_q(z_{h2}/z_q,p_{2T})$$

- Clear differences between the ep and eA: **suppression** of the correlation peak in **eA** due to **saturation** effects (including the **Sudakov resummation)**
- Further observables: azimuthal correlations of dihadrons/dijets in diffraction, photon+jet/dijet.
- Possibility to test various **CGC correlators**



Yellow Report