Soft-Collinear Effective Theory (SCET)

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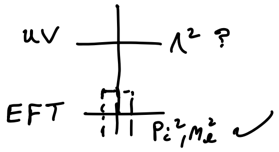
4 lectures, 1.5 hours each

EFT online course
[google "EFT mitxonline"]

EFT (& SCET) motivation

ⓐ simplest framework that captures essential physics ← IR d.o.f.

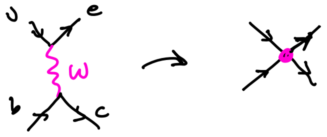
eg. Hydrogen Atom in NRQM not QED



ⓑ separate scales

- calculations easier
- do not need to know detailed dynamics at UV scales
- factorization in SCET/QCD

eg $m_\ell = m_b, m_c, \dots \ll M_W$

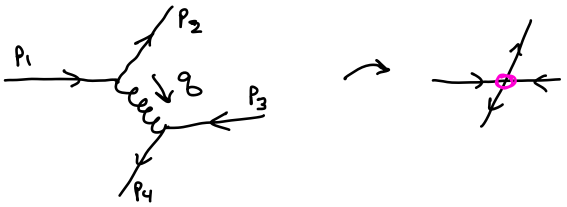


$\left[1 + \mathcal{O}\left(\frac{m_\ell^2}{M_W^2}\right)\right]$

"Fermi EFT"

eg. large momentum (hard scatter)

$q^2 \gg p_i^2$



SCET

$q = p_1 - p_2 = n_1 E_1 - n_2 E_2$, $p_i^2 \simeq 0$

$n_1 = (1, \hat z)$

$n_2 = (1, \hat n)$

$n_i^2 = 0$

$$q^2 = -2E_1E_2\, n_1\cdot n_2$$
$$= -2\underbrace{E_1E_2}_{}(1-\underbrace{\hat z\cdot\hat n}_{})$$

- E_i big
- deflection angle large

$\boxed{q^2 \sim Q^2 \text{ hard}}$

Key Principle To describe physics at small scale m^2, we do not need to know dynamics at scales $\Lambda^2 \gg m^2$

(Wilson nobel prize '82)

ⓒ Systematically improvable $\lambda \ll 1$

$$\mathcal{L} = \underset{\lambda^0}{\mathcal{L}^{(0)}} + \underset{\lambda}{\mathcal{L}^{(1)}} + \underset{\lambda^2}{\mathcal{L}^{(2)}} + \dots$$

- precision
- uncertainty estimates (know what we dropped)

• ∞ # operators, but only specific subset needed at a given order

ⓓ exploit symmetry — approximate or even emergent

eg. chiral symmetry (ChPT low energy, operator structure @ high energy)

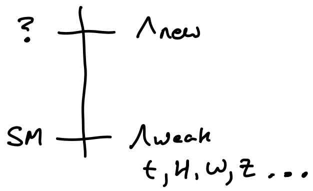
eg. heavy quark (spin-flavor) symmetry (Heavy Quark Effective Thy.)

ⓔ model independent — include all operators

— eg. characterize new physics in SMEFT

$$\mathcal{L} = \mathcal{L}^{(0)}_{SM} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \dots$$

$$\sim \frac{\mathcal{O}^{\text{dim-5}}}{\Lambda_{new}} \qquad \sim \frac{\mathcal{O}^{\text{dim-6}}}{\Lambda_{new}^2}$$



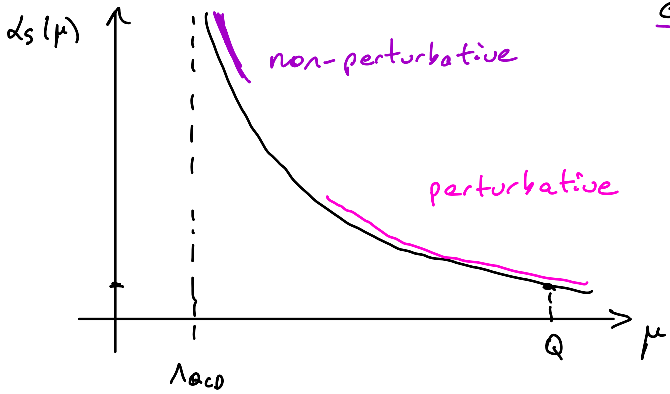
ⓕ sum large logs $\sum_k a_k (\alpha_s \ln)^k$, $\sum_k b_k (\alpha_s \ln^2)^k$, ...

$\ln\left(\frac{\Lambda_{high}}{\Lambda_{low}}\right)$ using Renormalization Group Equations (RGE) with anomalous dimensions

Ask, for EFT what else?

ⓖ include non-perturbative effects $Q \gg \Lambda_{QCD}$

- $\alpha_s(Q) \ll 1$ $\alpha_s(m_p) \sim 1$
- rigorous by considering quantum fields for $p^2 \sim \Lambda_{QCD}^2$
- universality in QCD/SCET, PDF, FF, ... $\sigma = \hat{\sigma} \otimes f$



confinement, α_s so strong q & g are confined in hadrons

$M_{proton} \propto \Lambda_{QCD}$ (size) $\propto \Lambda_{QCD}^{-1}$

Asymptotic freedom $\alpha_s(\mu \to \infty) = 0$

free quarks at short dist.

$$\alpha_s(\mu) = \frac{2\pi}{\beta_0 \ln \mu / \Lambda_{QCD}}$$ ~250 MeV

ⓗ have to think! construction of EFT not always obvious

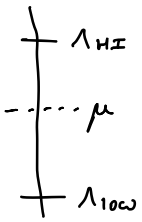
Examples?
- ChPT
- Non. Rel QCD & SCET w multiple gluon fields
- Expt. measurements can introduce new scales (→ need different EFT)

Any EFT
- degrees of freedom ← what fields [low energy / nearly on-shell modes]
- symmetries ← what interactions / operators [Lorentz, Gauge, Global, ...]
- power counting ← what expansions

- often expand in mass dimension of operators as in SMEFT or Fermi EFT, but not always (not in SCET)

Output: power counting handles powers $\Lambda_{Low}/\Lambda_{HI} \ll 1$

renormalization handles logs $\ln\left(\frac{\Lambda_{Low}}{\Lambda_{HI}}\right)$



Matching

$$\mathcal{L}_{EFT} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \dots$$

$$\mathcal{L}^{(k)}_{EFT} = \sum_i \underset{\uparrow}{C_i(\mu)}\ \underset{\uparrow}{O_i^{(k)}(\mu)}$$

short distance Wilson Coeff. (offshell)

long dist. operators (~ on-shell)

- $\mathcal{L}_{HI}$ & $\mathcal{L}_{EFT}$ have same IR physics, differ in UV
- $C_i(\mu)$ does not depend on IR scales (masses, Λ_{QCD}, IR regulators ...)

$$\begin{tikzcd}\boxed{\mathcal{L}_{HI}} \arrow[d, "?"] \\ \boxed{\mathcal{L}_{EFT}}\end{tikzcd}$$

"bottom-up EFT"

don't know or can't directly compute with $\mathcal{L}_{HI}$

form $\sum_i c_i O_i$ complete basis, exploit symmetry

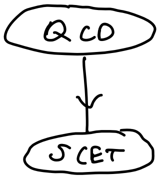
[eg. SMEFT, ChPT, ...]

" top-down EFT "

can use $\mathcal{L}_{HI}$ to construct $\mathcal{L}_{EFT}$ (perturbatively)

$$\mathcal{L}_{EFT}^{(k)} = \sum_i \underset{\substack{\uparrow \\ \text{calculable Wilson Coeffs}}}{C_i} \otimes \mathcal{O}_i^{(k)} \leftarrow \text{constructable}$$

eg. SCET (Hweak, HQET, NRQCD, ...)



hard Q^2

$\Lambda_{QCD}^2, p_{IR}^2 \ll Q^2$

SCET: EFT for soft & collinear IR physics in hard collisions in QCD [short review 2212.11107 pg 223]

$\Rightarrow$ jets, energetic hadrons, soft partons / hadrons

[collider physics EFT, also needed for heavy particle decays]

eg. $e^+e^- \to$ 2 jets, $e^-p \to e^- X$ (DIS), $pp \to$ Higgs $+X$
$e^-p \to e^-\pi X$ (SIDIS) $\to$ Higgs $+$ jet $+X$

$B \to \pi\pi$, ... many many more

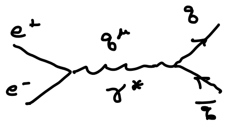
$pp \to J/\psi X$, ...

jet substructure, NGLs, Regge/High Energy limit, Heavy Ion,
electroweak logs, DM cross sections, gravity, ...

Concepts: Factorization, Wilson Lines, Sudakov double logs
(d.o.f A_n & A_s) (divergences $1/\epsilon^2$, $\frac{1}{\eta}$)
Power Corrections, Non-Pert. Effects

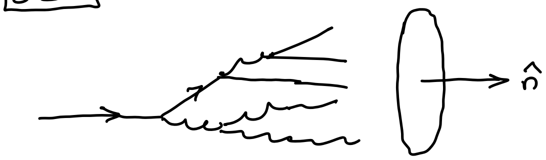
- Reproduce collinear & soft IR singularities

d.o.f $e^+e^- \to 2$ jets



$Q^2 = q^2$

Jet



collimated radiation in direction $\hat n$

$E_{Jet} \sim Q$

Let $n^\mu = (1, \hat n)$ direction

$\bar n = (1, -\hat n)$ aux.

$n^2 = \bar n^2 = 0, \quad n\cdot\bar n = 2$

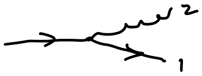
$$p^\mu = \underbrace{\bar n\cdot p}_{p^-}\frac{n^\mu}{2} + \underbrace{n\cdot p}_{p^+}\frac{\bar n^\mu}{2} + p_\perp^\mu$$

$$p^2 = \bar n\cdot p\, n\cdot p + \underbrace{p_\perp^2}_{-\vec p_\perp^{\,2}}$$

Collinear?

1 massless: $p^\mu = \bar n\cdot p \frac{n^\mu}{2}, \quad \bar n\cdot p = Q, \quad p^2 = 0$

2 massless:



$$p_i^\mu = \bar n\cdot p_i \frac{n^\mu}{2} + p_{i\perp}^\mu + n\cdot p_i \frac{\bar n^\mu}{2}$$

$i = 1,2$

$\underbrace{\bar n\cdot p_i \sim Q}_{\text{large}}, \quad \underbrace{p_{i\perp}^\mu \ll Q}_{\text{collimated}}$ say $p_{i\perp} \sim \lambda Q$

$\boxed{\lambda \ll 1 \quad \text{power counting}}$

dimensionless

on-shell $n\cdot p_i = \dfrac{\vec p_{i\perp}^{\,2}}{\bar n\cdot p_i} \sim \lambda^2 Q$

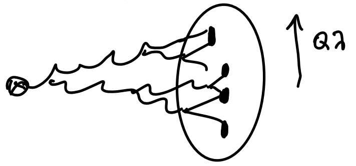
k massless: same

$\therefore$ n-collinear $p_n^\mu \sim Q(\overset{+}{\lambda^2}, \overset{-}{1}, \overset{\perp}{\lambda})$

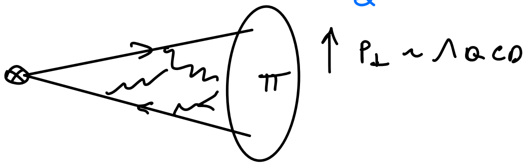
fields quark ξ_n

gluon A_n^μ

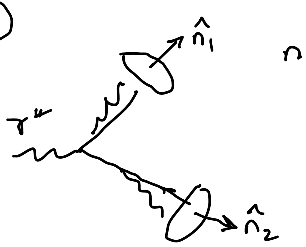
jet $\frac{\Lambda_{QCD}}{Q} \ll \lambda \ll 1$



energetic hadron $\lambda \sim \frac{\Lambda_{QCD}}{Q}$



2 jets



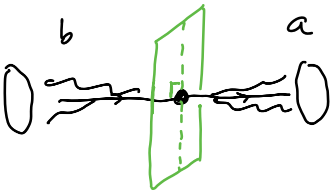
$n_1, \bar n_1$ $\xi_{n_1}, A^\mu_{n_1}$

$n_2, \bar n_2$ $\xi_{n_2}, A^\mu_{n_2}$

a, b = hemispheres

CM frame back-to-back

$n_2 = \bar n$
$\bar n_2 = n$



$n_1 = n$
$\bar n_1 = \bar n$

$\longrightarrow \hat t$ "thrust axis"

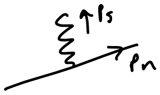
ξ_n, A_n $p_n \sim (\overset{+}{\lambda^2}, \overset{-}{1}, \overset{\perp}{\lambda})$

$\xi_{\bar n}, A_{\bar n}$ $p_{\bar n} \sim (1, \lambda^2, \lambda)$

$\hat t$ = direction with largest projection of final state $\vec p$.

$\boxed{\text{Soft}}$ $p_s^\mu \sim Q(\lambda^\alpha, \lambda^\alpha, \lambda^\alpha)$ homogeneous

soft + soft = soft
soft + hard = hard
collinear + hard = hard
n_1-collinear + n_2-collinear = hard ← hard interaction produces jets
suppressed
collinear + soft ?



$$(p_n + p_s)^2 = 2 p_n \cdot p_s = \underbrace{\bar n \cdot p_n \; n \cdot p_s}_{\lambda^0 * \lambda^\alpha} + \cdots \sim Q^2 \lambda^\alpha$$

Value of α depends on what we measure

eg1 Mass in (large enough) region a, $M_a^2 = \left(\sum_{i \in a} p_i^\mu\right)^2$

[mass of R = 1 jet, hemisphere mass, ---]

demand $M_a^2 \sim Q^2 \lambda^2 \ll Q^2$ [collimated jet has $E_J \gg M_J$]

n-collinear + n'-collinear $(p_n + p_{n'})^2 = 2\, p_n \cdot p_{n'} \sim Q^2 \lambda^2$

$$\begin{array}{cc} + & - \\ - & + \\ \perp & \perp \end{array}$$

collinear + soft $(p_n + p_s)^2 \sim Q^2 \lambda^\alpha$

$\therefore$ $\alpha = 2$ to contribute "ultrasoft"

eg2. Thrust $\tau = 1 - T \ll 1$ $\tau = \frac{M_a^2 + M_b^2}{Q^2} \ll 1$

$\alpha = 2$

$\uparrow L1$
$\downarrow L2$

eg3. Transverse Momenta, broadening $B_\perp = \sum_{i \in a} |\vec{p}_i^{\,\perp}| \ll Q$

$\sim \lambda$

Σ collinear ✓

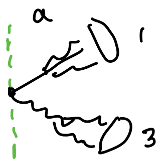
soft $\Rightarrow$ $\alpha = 1$ "soft"

$\alpha = 2$ "SCET$_I$" (simpler, and can derive SCET$_{II}$ from SCET$_I$)

$\alpha = 1$ "SCET$_{II}$"

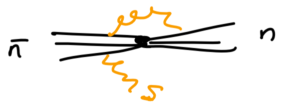
$\boxed{\geq 3\text{-jets?}}$ hemisphere $M_{a,b}^2 \ll Q^2 \Rightarrow$ 2-jets (not > 2 jets)

or $\tau \ll 1$



$(p_1 + p_3)^2 \sim 2 p_1 \cdot p_3 \sim Q^2$ $\therefore$ forbidden

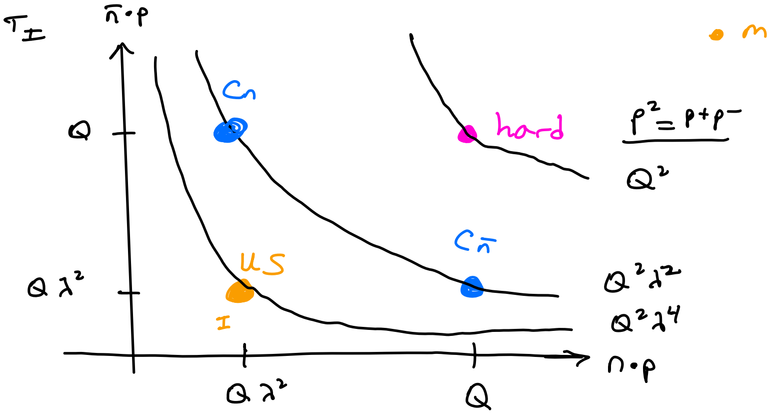
$\boxed{\text{DOF Picture}}$ e+e- → 2 jets (cm frame)



[virtual too]

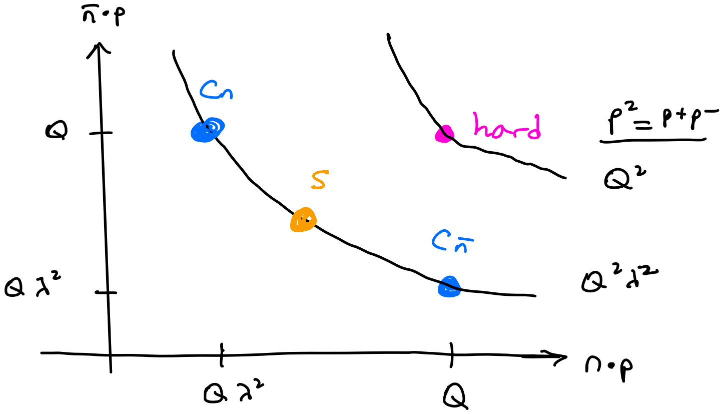
① SCET$_I$

$\alpha = 2$



- modes not classified by p^2 alone, rapidity $e^{-2Y} = \frac{p^+}{p^-}$
- modes cover regions of momentum space, extend into IR

② SCET$_{II}$ $\alpha = 1$



- power counting requires multiple fields for same particle
- relative scaling of modes is important [boost invariant, unlike absolute scaling]

- modes communicate through momenta with common scaling

Study SCET$_I$, come back to SCET$_{II}$

Field Power Counting Use free kinetic term

ξ_n propagator $p^2 = n\cdot p\ \bar{n}\cdot p + p_\perp^2$

$\lambda^2 * \lambda^0 + (\lambda)^2$ same size

$$\frac{i\not{p}}{p^2+i0} = \frac{i\not{n}}{2}\frac{\bar{n}\cdot p}{p^2+i0} + \cdots = \frac{i\not{n}}{2}\frac{1}{n\cdot p + \frac{p_\perp^2}{\bar{n}\cdot p} + i0\,\mathrm{sign}(\bar{n}\cdot p)} + \cdots$$

$$\int \underbrace{d^4x}_{\lambda^{-4}}\ \underbrace{e^{-ip\cdot x}}_{O(1)} \langle 0 |\, T\, \underset{\lambda}{\xi_n(x)}\ \underset{\lambda}{\bar{\xi}_n(0)}\, | 0 \rangle \checkmark = \frac{i\not{n}}{2}\frac{\bar{n}\cdot p}{\underbrace{p^2}_{\lambda^{-2}}}$$

$$\boxed{\therefore\ \xi_n \sim \lambda}$$

not the same as $\frac{3}{2}$ mass dimension, so p.c. ≠ operator dim.

Implies $\not{n}\,\xi_n = 0$ since $\not{n}^2 = n^2 = 0$

$$\xi_n = \underbrace{\frac{\not{n}\bar{\not{n}}}{4}}_{\text{proj. op.}}\psi \quad \text{for spin} \qquad u_n = \frac{\not{n}\bar{\not{n}}}{4}u(p)$$

$$\sum_s u_n^s \bar{u}_n^s = \cdots = \frac{\not{n}}{2}\bar{n}\cdot p \checkmark$$

"good fermion components" enhanced for high energy quark production

$$u_+(p) = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{p^-} \\ \cancel{\sqrt{p^+}\, e^{i\phi_p}} \\ \sqrt{p^-} \\ \cancel{\sqrt{p^+}\, e^{i\phi_p}} \end{pmatrix}$$

small comp. Killed by $\frac{\not{n}\not{\bar{n}}}{4} = \frac{1}{2}\begin{pmatrix} 1 & \sigma^3 \\ \sigma^3 & 1 \end{pmatrix}$

Dirac Rep. cf. Dixon hep-ph/9601359

etc. similar for $u_-(p)$, $v_+(p)$, $v_-(p)$

A_n^μ same propagator as QCD (incl. gauge fixing)

$$p^\mu \sim (\lambda^2, 1, \lambda) \sim i\partial_n^\mu \underset{\uparrow}{\sim} A_n^\mu \quad , \quad iD_n^\mu = i\partial_n^\mu + gA_n^\mu$$

makes sense for each comp.

$$\therefore \boxed{(n\cdot A_n, \bar{n}\cdot A_n, A_\perp^\mu) \sim (\lambda^2, 1, \lambda)}$$

any gauge

(could derive from prop.)

Soft $p_s \sim \lambda^\alpha$

$$\boxed{\begin{aligned} A_s^\mu &\sim p_s^\mu \sim \lambda^\alpha \\ \psi_s &\sim \lambda^{3\alpha/2} \end{aligned}}$$

$\alpha = 2$ SCET$_{\rm I}$

Structure of SCET $\mathcal{L}$

hard scattering

dynamics of IR fields

?!

$$\mathcal{L}_{SCET} = \mathcal{L}_{hard} + \mathcal{L}_{dyn}$$

$$= \Big(\underbrace{\mathcal{L}_{hard}^{(0)}}_{\text{leading}} + \underbrace{\sum_{i\geq 1} \mathcal{L}_{hard}^{(i)}}_{\text{Subleading}} \Big) + \Big(\underbrace{\mathcal{L}_{dyn}^{(0)}}_{\text{leading}} + \underbrace{\mathcal{L}_G^{(0)}}_{} + \underbrace{\sum_{i\geq 1} \mathcal{L}_{dyn}^{(i)}}_{\text{subleading}} \Big)$$

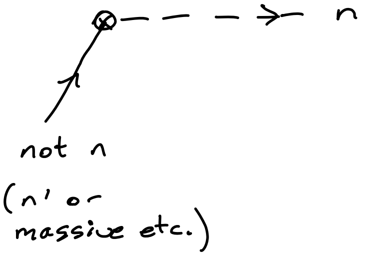
≥ 2 collinear or n-s interacting

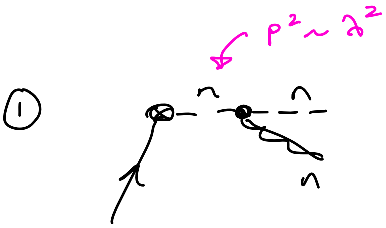
one n, s or n-us

"Glauber" has (potential) fact. violating terms (more later on)

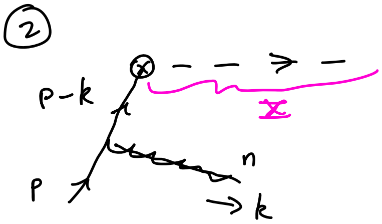
Collinear Wilson Lines

- $\bar{n}\cdot A_n \sim \lambda^0$? no suppression for building operators





interaction from $\mathcal{L}^{(0)}_{dyn}$



$$(p-k)^2 = \cancel{p^2} \cancel{- m^2} + \cancel{k^2} - 2p\cdot k = -\underbrace{\bar{n}\cdot k}_{\lambda^0}\, \underbrace{n\cdot p}_{\lambda^0 \text{ since not } n\text{-collinear}} + \dots$$

$\therefore$ offshell

integrate it out

$$= \overline{\underline{X}}\ \frac{i(\not{p}-\not{k}+m)}{(p-k)^2 - m^2 + i0}\,(igT^a \not{\epsilon}_n^a)\, u(p)$$

$$\not{\epsilon}_n^a \leftarrow \frac{\not{n}}{2}\,\bar{n}\cdot\epsilon_n^a + \dots$$

expand **Homework** since $\not{A} = \underbrace{\bar{n}\cdot A}_{\lambda^0}\frac{\not{n}}{2} + \dots$

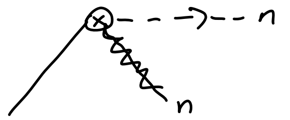
$$(\not{p}+m)\frac{\not{n}}{2} = \overset{\lambda^0}{n\cdot p} + \cancel{\frac{\not{n}}{2}(-\not{p}+m)} \quad \text{e.o.m.}$$

$$-\not{k}\frac{\not{n}}{2} = \mathcal{O}(\lambda)$$

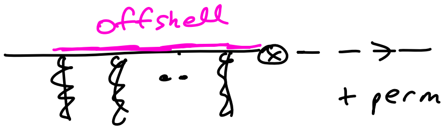
$$(p-k)^2 - m^2 = -2p\cdot k + k^2$$
$$= -n\cdot p\, \bar{n}\cdot k + \dots$$

$$= \overline{\underline{X}}\ \frac{(-g)\ \bar{n}\cdot A_n^a\ T^a\ u(p)}{-\bar{n}\cdot k + i0}$$

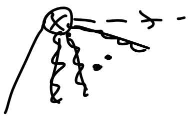
universal, independent of p, m, ...

$\Rightarrow$ 

keep going



+ perms $\Rightarrow$



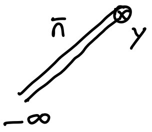
More HmWk

Gives Wilson line

$$\langle W_n \rangle = \sum_m (-g)^m \sum_{\substack{\text{perms}\\ \{1,\dots,m\}}} \frac{\bar{n}^{\mu_m} T^{A_m} \cdots \bar{n}^{\mu_1} T^{A_1}}{[\bar{n}\cdot q_1][\bar{n}\cdot(q_1+q_2)]\cdots[\bar{n}\cdot\sum_{i=1}^{m} q_i]}$$

in position space:

$$W_n(y,-\infty) = P\exp\left(ig\int_{-\infty}^{0} ds\, \bar{n}\cdot A_n(s\bar{n}+y)\right)$$



$$i\bar{n}\cdot D_n W_n = 0 \quad , \quad W_n^\dagger W_n = \mathbb{1}$$

$\boxed{W_n \sim \lambda^0}$ SCET operator $(\underbrace{\bar{\xi}_n W_n}_{\equiv \bar{\chi}_n})(\Gamma\psi)$ generic

Building Blocks for SCET operators

quark $\underline{\chi_n \equiv W_n^\dagger \xi_n}$

gluon $\underline{\mathcal{B}_{n\perp}^\mu \equiv \frac{1}{g}[W_n^\dagger iD_{n\perp}^\mu W_n]} = \left[\frac{1}{g\, i\bar{n}\cdot\partial_n} W_n^\dagger \underbrace{[i\bar{n}\cdot D_n, iD_\perp^\mu]}_{} W_n\right]$

field strength + adjoint Wilson line

$$= A_{n\perp}^\mu(k) - \frac{k_\perp^\mu}{\bar{n}\cdot k}\bar{n}\cdot A_n(k) + \cdots$$

[vanishes if $A^\mu \to k^\mu$, g. inv.]

$\{\chi_n, \mathcal{B}_{n\perp}^\mu, \hat{P}_\perp^\mu\}$ suffice to build <u>any</u> higher power collinear operator

(e.o.m., trade $i\bar{n}\cdot\partial_n \to W_n$)

<u>Physically</u>: • the color source "ψ" looks like a Wilson line to the fast moving n-collinear particles

- χ_n, $\mathcal{B}_{n\perp}$ "n-collinear gauge inv." $\chi_n \to (W_n^\dagger \cancel{U_n^\dagger})(\cancel{U_n}\xi_n)$

- nonshell ψ can't transform since doing so puts it for offshell, → encoded in W_n

More generally SCET has indep. gauge symmetry for each sector $\{ n, \bar{n}, s \}$

$\bar{n}\cdot A_n \to W_n$

singlet

$$\left[\begin{array}{l} i\bar{n}\cdot D_n W_n \Phi = W_n i\bar{n}\cdot\partial_n \Phi \\ W_n^\dagger i\bar{n}\cdot D_n W_n = i\bar{n}\cdot\partial_n \\ i\bar{n}\cdot D_n = W_n i\bar{n}\cdot\partial_n W_n^\dagger \end{array}\right.$$

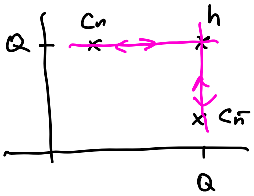
in building blocks

Hard-Collinear Factorization

$$\mathcal{L}^{(0)}_{hard} = \sum_i C_i \otimes O_i[\chi_n, \dots]$$

$$C(\underbrace{i\bar{n}\cdot\partial_n}_{\sim\lambda^0})\chi_n = \int d\omega\, \underbrace{C(\omega)}_{\text{functions}}\ \underbrace{\delta(\omega - i\bar{n}\cdot\partial_n)\chi_n}_{\equiv\ \chi_{n,\omega}}$$

definite large momentum ω

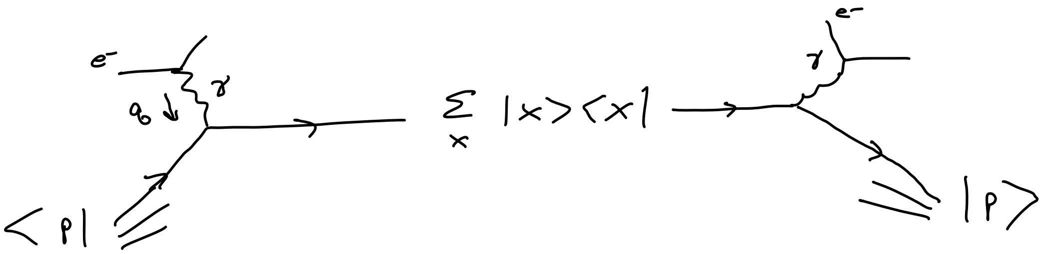


form constrained by gauge inv.
& momentum conservation

DIS $e^- p \to e^- X$ Inclusive Factorization

[full analysis requires more knowledge, eg $\mathcal{L}$, cover few key parts]

see hep-ph/0202088



Take $q = (0,0,0,Q) = \frac{Q}{2}(\bar{n} - n)$ $\quad q^2 = -Q^2$ spacelike

Bjorken $x = \frac{Q^2}{2p\cdot q}$

Breit frame, where proton is n-collinear

Proton $p_p^\mu = \frac{n^\mu}{2}\bar{n}\cdot p_p + \frac{\bar{n}^\mu}{2}\underbrace{\frac{M_p^2}{\bar{n}\cdot p_p}}_{\text{small}}$, big $\bar{n}\cdot p_p = \frac{Q}{x} \sim \lambda^0$

$p_X = p_p + q$,

$p_X^2 = Q^2(\frac{1}{x}-1) + M_p^2$

regions:

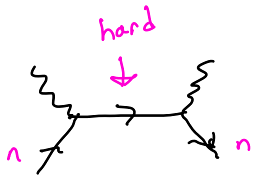
study this

p_X^2	$(\frac{1}{x}-1)$	
$\sim Q^2$	~ 1	inclusive
$\sim Q\Lambda$	$\sim \Lambda/Q$	threshold $x \to 1$ (jet)
$\sim \Lambda^2$	$\sim \Lambda^2/Q$	resonance ($e^-p \to e^-p'$)
$\gg Q^2$	$\gg 1$	small-x

$\underline{p_X = p + q = \text{hard}}$

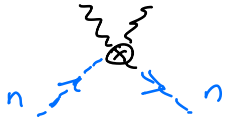
$\lambda = \frac{\Lambda_{QCD}}{Q} \ll 1$

QCD



$\longrightarrow$

SCET



$\mathcal{O}_q = \underset{\lambda}{\bar{\chi}_n} \frac{\bar{n}\!\!\!/}{2} \underset{\lambda}{\chi_n}$

$\mathcal{O} \sim \lambda^2$ twist-2

Add arbitrary pert. α_s^k corrections:

actually $C_{i=1,2}$, two structures for $W^{\mu\nu}$

also gluon $O_g = \mathcal{B}_{n\perp}^\mu \mathcal{B}_{n\perp\mu}$

$\mathcal{L}_{hard} = \int d\omega\, \cancel{d\omega'}\, C(\omega, \cancel{\omega'}, Q)\, \bar{\chi}_n \frac{\bar{n}\!\!\!/}{2} \cancel{\delta(\omega' + i\bar{n}\cdot\overleftarrow{\partial}_n)}\, \delta(\omega - i\bar{n}\cdot\partial_n)\, \chi_n$

forward $\langle p| \cdots |p\rangle$ matrix element fixes $\omega = \omega'$

$\sigma \sim \int d\omega\; \text{Im}\, C(\omega, Q) \langle p| \bar{\chi}_n \frac{\bar{n}\!\!\!/}{2} \delta(\omega - i\bar{n}\cdot\partial_n)\chi_n |p\rangle$

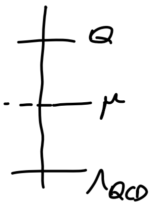
momentum of quark in proton

dimensionless

$\sim \int \frac{d\xi}{\xi} H(\frac{x}{\xi}, \frac{Q}{\mu}, \alpha_s(\mu))\; f_{q/p}(\xi, \frac{\mu}{\Lambda_{QCD}})$

Hard

Collinear PDF



$\frac{Q}{\omega} = \frac{Q}{\xi\,\bar{n}\cdot p} = \frac{x}{\xi}$

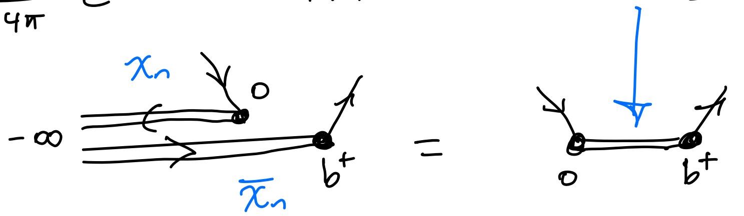
$\xi = \frac{\omega}{\bar{n}\cdot p}$

- all orders in α_s (no use of pert. theory), $\mathcal{O}\left(\frac{\Lambda^2_{QCD}}{Q^2}\right)$ corrections
- universal $f_{q/p}$
- H dimensionless → $\alpha_s \ln \mu/Q$ dependence on Q, Bjorken scaling
- $f_{q/p}(\xi,\mu)$ encodes ∞ set of "twist-2" operators

In position space

$$f_{q/p}(\xi) = \int \frac{db^+}{4\pi}\, e^{-i(\xi p^-)b^+} \langle p|\, \bar{\xi}_n(b^+)\, W_n(b^+,0)\, \frac{\bar{n}\!\!\!/}{2}\, \xi_n(0)\,|p\rangle$$

same in QCD & SCET



↑L22
↓L23

Comments on Renormalization

Ask: Operators with same Quantum numbers mix under ren.

⇒ Loops can change ξ, $f_q(\xi)$ mixes with $f_q(\xi')$. Also mix parton types $i = q, g$.

$$f_i^{bare}(\xi) = \sum_j \int d\xi'\, Z_{ij}(\xi,\xi')\, f_j(\xi',\mu)$$

↑ μ indep. ↑ $\frac{1}{\epsilon_{UV}}$ ↑ renormalized

$$\Rightarrow \mu\, d/d\mu\, f_i(\xi,\mu) = \sum_j \int d\xi'\, \underbrace{\gamma_{ij}(\xi,\xi')}_{\propto\, P_{ij}(\xi/\xi')}\, f_j(\xi',\mu)$$

splitting functions

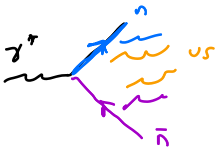
$$\text{with}\quad \gamma_{ij} = -\sum_{i'} \int d\xi''\, Z^{-1}_{ii'}(\xi,\xi'')\, \mu\frac{d}{d\mu} Z_{i'j}(\xi'',\xi')$$

More Hard Operators

$\{\chi_n, \mathcal{B}^\mu_{n\perp}, q^\mu_{\perp}, \text{soft fields}\}$

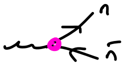
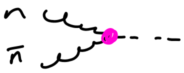
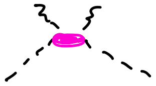
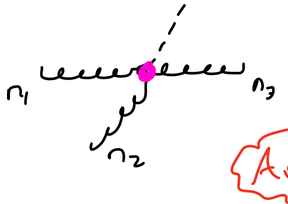
- If hard region is purely virtual then match at Amplitude level. Else match for σ (eg. DIS).

eg. $e^+e^- \to$ dijets, $\tau \ll 1$



only $n, \bar{n}$, us radiation in final state since $\tau = \frac{M_a^2 + M_b^2}{Q^2} \ll 1$

no hard radiation ⇒ Amplitude Matching

Example	Operators		matching for
$e^+e^- \to 2$ jets	$\bar\chi_n \gamma_\perp^\mu \chi_{\bar n}$		Amplitude
$gg \to H$ + 0-jets	$\mathcal{B}_{n\perp}^{\mu}\, \mathcal{B}_{\bar n \perp \mu}\, H$		Ampl.
DIS, DY, incl. H	$\bar\chi_n \frac{\bar{n}\!\!\!/}{2}\, \delta(\omega - i\bar n\cdot\partial)\, \chi_n$ etc.		$\sigma \sim \text{Ampl.}^2$
$pp \to H$ + 1-jet	(remove top)		Ampl

Ask how many operators ?

no $d^{a_1a_2a_3}$ by charge conj.

spin/helicity

- $H\, \mathcal{B}_{n_1\perp}^{a_1\mu_1}\, \mathcal{B}_{n_2\perp}^{a_2\mu_2}\, \mathcal{B}_{n_3\perp}^{a_3\mu_3}\, (if^{a_1a_2a_3})$ $\quad T_{\mu_1\mu_2\mu_3}$ $\quad 4 \leftarrow$ 2 indep Coeff. (due to parity)
- $H\, \mathcal{B}_{n_1\perp}^{a\mu}\, \bar\chi_{n_2}^{\bar\alpha}\, T^a\, \chi_{n_3}^{\beta}$ $\quad T_{\mu_1}^{\alpha\bar\beta}$ $\quad 4 \leftarrow$ 2 indep Coeff. (chg. conj.)

Helicity basis: natural in SCET since we have direction to use $\hat n$

$$\mathcal{B}^a_{n\pm} \equiv -\epsilon_\mp^\mu(n,\bar n)\, \mathcal{B}^a_{n\perp\mu}\,, \qquad \epsilon_\mp = \frac{1}{\sqrt2}(0,1,\pm i,0)$$

$$J_{n_1n_2\pm} \propto \epsilon_\mp^\mu(n_1,n_2)\, \bar\chi_{n_1\pm}\gamma_\mu \underbrace{\chi_{n_2\pm}} \longrightarrow \left(\frac{1\pm\gamma_5}{2}\right)\chi_n$$

Allowed

H	$\mathcal{B}$	$\mathcal{B}$	$\mathcal{B}$	
	+	+	+	
	+	+	−	
	−	−	+	Wilson Coeff fixed by Parity
	−	−	−	

H	$\mathcal{B}$	J	
	+	+	
	−	+	
	+	−	fixed by charge Conj.
	−	−	

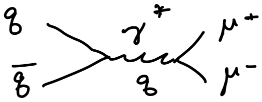
4 non-trivial coefficients

[note: no evanescent operators in leading power SCET due to helicity conservation]

Easy to exploit modern spinor-helicity results.

[see 1508.02397 for more on helicity operators in SCET.]

eg. Drell-Yan (Higgs) factorization $pp \to \mu^+\mu^- X$



Q^2 (M_H^2) $\gg \Lambda_{QCD}^2$ (in pictures)

$$\frac{d\sigma}{dQ^2dY} = \int_{x_a}^{1}\frac{d\xi_a}{\xi_a}\int_{x_b}^{1}\frac{d\xi_b}{\xi_b}\, f_{i/p}(\xi_a,\mu)\, f_{j/p}(\xi_b,\mu)\, H_{ij}\left(\frac{x_a}{\xi_a},\frac{x_b}{\xi_b},Q,\mu\right) * \left[1+\mathcal{O}\left(\frac{\Lambda_{QCD}^2}{Q^2}\right)\right]$$

Fact. Scale = Ren. Scale for PDF

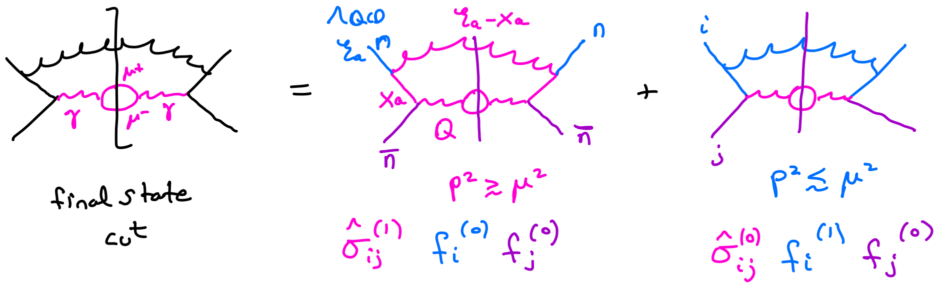
$\frac{d\hat\sigma_{ij}}{dQ^2dY}$ "partonic $\hat\sigma$" (removing IR)

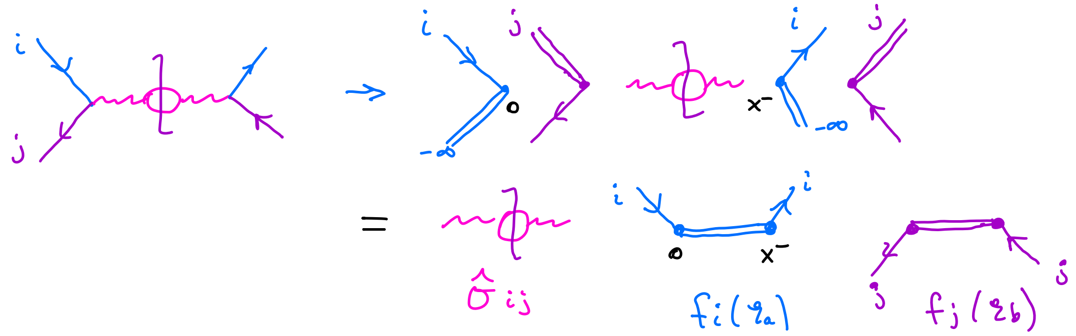
$x_a = \frac{Q e^{Y}}{\sqrt{s}} \simeq \frac{q^+}{P_A^+}$, $x_b = \frac{Q e^{-Y}}{\sqrt{s}}$

$\xi_a = x_a$ at tree level

$\xi_a > x_a$ due to gluon radiation, two scales Q, Λ_{QCD}

"hard" "collinear"





[SCET building block fields]

More subtle: $\mathcal{L}_G^{(0)}$ cancels

softs cancel

Lagrangian $\quad \mathcal{L}^{(0)}_{SCET_I} = \mathcal{L}^{(0)}_{dyn} + \mathcal{L}^{(0)}_{hard}$

$$\mathcal{L}^{(0)}_{dyn} = \underbrace{\left(\mathcal{L}^{(0)}_n + \mathcal{L}^{(0)}_{\bar n} + \mathcal{L}^{(0)}_s\right)}_{\text{factorizable}} + \mathcal{L}^{(0)}_G$$

ultrasoft, denote with "s"

$$\mathcal{L}^{(0)}_s = \mathcal{L}_{QCD}[q_s, A_s] = \underset{\lambda^3\cdot\lambda^2\cdot\lambda^3}{\bar\psi_s\, i\not{D}_s\, \psi_s} - \frac14 \underset{\lambda^4\cdot\lambda^4}{G^{\mu\nu}_{sA} G^A_{s\mu\nu}} \quad \text{+ gauge fixing}, \quad d^4x \sim \lambda^{-8}$$

$$\mathcal{L}^{(0)}_n = \mathcal{L}^{(0)}_n\big[\xi_n, A_n, \underbrace{\underset{\lambda^2}{in\cdot\partial} + \underset{\lambda^2}{gn\cdot A_s} + \underset{\lambda^2}{gn\cdot A_n}}_{= in\cdot D}, \underbrace{\underset{\lambda}{i\partial_\perp} + \underset{\lambda}{gA_{n\perp}}}_{= iD_{n\perp}}, \underbrace{\underset{\lambda}{i\bar n\cdot\partial} + \underset{\lambda}{g\bar n\cdot A_n}}_{= i\bar n\cdot D_n}\big]$$

$$\mathcal{L}^{(0)}_n \sim \lambda^4 \quad \& \quad d^4x \sim \lambda^{-4}$$

the above D's

$$= \bar\xi_n \frac{\bar{\not{n}}}{2}\left[in\cdot D + i\not{D}_{n\perp} \frac{1}{i\bar n\cdot D_n} i\not{D}_{n\perp}\right]\xi_n + \frac{1}{2g^2}\mathrm{tr}\left([iD_\mu, iD_\nu]\right)^2$$

+ gauge fixing

$$\mathcal{L}^{(0)}_{\bar n} = \left(\mathcal{L}^{(0)}_n \text{ with } n \leftrightarrow \bar n\right)$$

Comments ① $\quad \dfrac{\not{n}/2}{n\cdot p + \frac{p_\perp^2}{\bar n\cdot p} + i0\,\mathrm{sign}(\bar n\cdot p)} = \dfrac{\bar n\cdot p\,\frac{\not{n}}{2}}{p^2 + i0}$ ✓

② multipole expanded $\quad \bar n\cdot k_{us} \ll \bar n\cdot k_n, \quad k^\perp_{us} \ll k^\perp_n$

③ $\mathbb{1}\psi = \left(\dfrac{\not{n}\bar{\not{n}}}{4} + \dfrac{\bar{\not{n}}\not{n}}{4}\right)\psi = \underset{\text{good}}{\xi_n} + \underset{\text{bad}}{\varphi_n}$

$$\mathcal{L}_{QCD} = \bar\psi i\not{D}\psi = \bar\xi_n \frac{\bar{\not{n}}}{2} in\cdot D\,\xi_n + \bar\varphi_n \frac{\not{n}}{2} i\bar n\cdot D\,\varphi_n + \bar\xi_n i\not{D}_\perp \varphi_n + \bar\varphi_n i\not{D}_\perp \xi_n$$

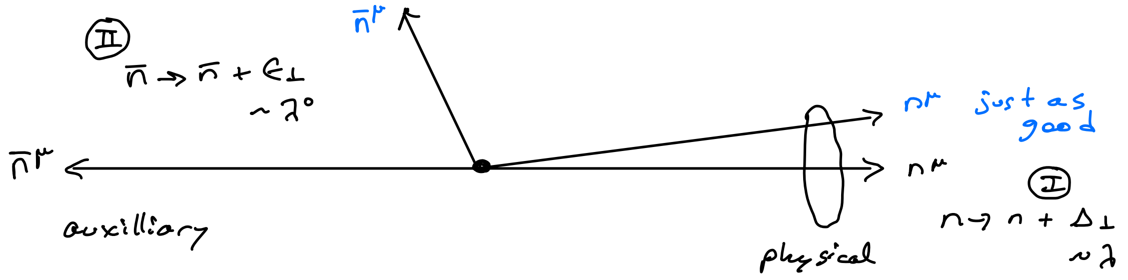
e.o.m. $\quad \delta/\delta\bar\varphi_n \Rightarrow \quad \varphi_n = \dfrac{1}{i\bar n\cdot D}\, i\not{D}_\perp \dfrac{\bar{\not{n}}}{2}\xi_n$

plug in to get $\mathcal{L}^{(0)}_n$ quark term above

→ int. out exactly. Useful because hard ops produce ξ_n

④ Satisfies Reparameterization Inv (RPI)

freedom to choose n & $\bar{n}$ satisfying $n^2 = \bar{n}^2 = 0$, $n\cdot\bar{n} = 2$



Ⅲ $n \to \kappa\, n$

$\bar{n} \to \dfrac{\bar{n}}{\kappa}$ $\Rightarrow$ Numerator (# n's − # $\bar{n}$'s)

= Denominator (# n's − # $\bar{n}$'s)

Each collinear sector has its own RPI symmetry

⑤ $\mathcal{L}_n^{(0)}$ only couples to $n\cdot A_s$ (eikonal) [more soon]

RG Evolution & Matching

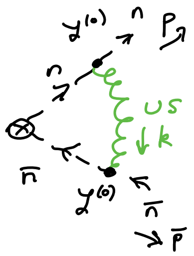
UV renormalization in SCET [now]

compare renormalized QCD S-matrix to " SCET S-matrix & extract C's [later]

eg. $e^+e^- \to$ dijets

$$J^\mu = \bar{\psi}\gamma^\mu\psi = C \otimes \underbrace{\bar{\chi}_n\, \gamma_\perp^\mu\, \chi_{\bar{n}}}_{(\bar{\xi}_n W_n)\, \gamma_\perp^\mu\, (W_{\bar{n}}^\dagger \xi_{\bar{n}})} + \text{higher power}$$

(use Feyn. Gauge, offshell IR regulator $p^2, \bar{p}^2 \neq 0$)



finite terms

$$= \frac{\alpha_s C_F}{4\pi}\left[-\frac{2}{\epsilon^2} + \frac{2}{\epsilon}\ln\left(\frac{\cancel{(-p^2)(-\bar{p}^2)}}{\mu^2(-Q^2)}\right) + \cdots \right][\bar{u}_n \Gamma u_{\bar{n}}]$$

$$\int \frac{d^dk \quad n\cdot\bar{n}}{\left(n\cdot k + \frac{p^2}{Q}\right)\left(\bar{n}\cdot k + \frac{\bar{p}^2}{Q}\right)k^2}$$

$C_F = \frac{4}{3}$, $\bar{n}\cdot p = n\cdot\bar{p} = Q$

from W_n

$$= \frac{\alpha_s C_F}{4\pi}\left[\frac{2}{\epsilon^2} + \frac{2}{\epsilon} - \frac{2}{\epsilon}\ln\left(\frac{\cancel{(-p^2)}}{\mu^2}\right) + \cdots\right][\bar{u}\Gamma u]$$

$$\uparrow \quad \int đ^d k \left[\frac{\bar{n}\cdot(k+p)}{\bar{n}\cdot k\,(k+p)^2\,k^2} - \frac{\bar{n}\cdot p}{\bar{n}\cdot k\,(\bar{n}\cdot p\, n\cdot k + p^2)\,k^2}\right]$$

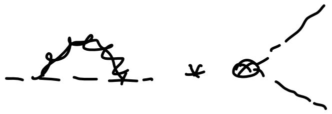
naive collinear integrand

0-bin subtraction (scaleless)

0-bin: collinear modes in $SCET_I$ have 0-bin subtractions from region $k^\mu \sim Q\lambda^2$ to avoid double counting IR region described by usoft mode. [part of proper multipole expansion]

from $W_{\bar{n}}^\dagger$

$$= \frac{\alpha_s C_F}{4\pi}\left[\frac{2}{\epsilon^2} + \frac{2}{\epsilon} - \frac{2}{\epsilon}\ln\left(\frac{\cancel{(-\bar{p}^2)}}{\mu^2}\right) + \cdots\right][\bar{u}\Gamma u]$$



$$= -\frac{\alpha_s C_F}{4\pi}\left[\frac{1}{\epsilon} + \cdots\right][\bar{u}\Gamma u]$$

- in sum $\frac{\ln(-p^2)}{\epsilon}$ & $\frac{\ln(-\bar{p}^2)}{\epsilon}$ cancel [mixed UV * IR] [crossed out above]

$$\text{sum} = \frac{\alpha_s C_F}{4\pi}\left[\frac{2}{\epsilon^2} + \frac{2}{\epsilon}\ln\frac{\mu^2}{-Q^2-i0} + \frac{3}{\epsilon} + \cdots\right][\bar{u}\Gamma u]$$

$C^{bare} = Z_c\, C$ $\qquad$ $\overline{MS}$ counter term

$(Z_c - 1)$

$$= -\frac{\alpha_s C_F}{4\pi}\left[\frac{2}{\epsilon^2} + \frac{2}{\epsilon}\ln\frac{\mu^2}{-Q^2-i0} + \frac{3}{\epsilon}\right][\bar{u}\Gamma u]$$

$$0 = \mu\frac{d}{d\mu}C^{bare} = \mu\frac{d}{d\mu}\left[Z_c(\mu,\epsilon)\,C(\mu)\right]$$

$$= \left[\mu\, d/d\mu\, Z_c\right] C + Z_c\left[\mu\, d/d\mu\, C\right]$$

$$\mu \frac{d}{d\mu} C(\mu) = \underbrace{\left[- Z_c^{-1}\, \mu\, d/d\mu\, Z_c \right]}_{\gamma_c} C(\mu)$$

$\underline{O(\alpha_s)}$ $\quad Z_c^{-1} \to 1 \qquad \mu\, d/d\mu\, \alpha_s = -2\epsilon\, \alpha_s + O(\beta_0 \alpha_s^2)$

{recall $\alpha_s^{bare} = \mu^{2\epsilon} \alpha_s(\mu) Z_\alpha$ implies this]

$$\mu \frac{d}{d\mu} Z_c = \frac{C_F}{4\pi} \alpha_s (-2\epsilon) \left(-\cancel{\frac{2}{\epsilon^2}} - \frac{2}{\epsilon} \ln \frac{\mu^2}{-Q^2} - \frac{3}{\epsilon} \right)$$

$$+ \frac{C_F \alpha_s}{4\pi} \left(-\cancel{4/\epsilon} \right)$$ ⟵ from $\mu\, d/d\mu\, \ln \mu^2 = 2$

$$\gamma_c = -\frac{\alpha_s(\mu)}{4\pi} \left[\underbrace{4 C_F} \ln \frac{\mu^2}{-Q^2} + 6 C_F \right]$$ finite

$$= - \overset{\text{cusp anomalous dimension}}{\Gamma_{cusp}[\alpha_s]} \ln \frac{\mu^2}{-Q^2} + \gamma_H[\alpha_s]$$

form at all orders in α_s

when we square the amplitude we get hard function $\quad H = |C(Q,\mu)|^2$

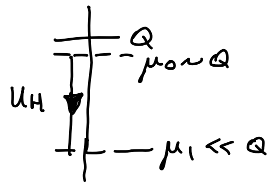
$$\mu \frac{d}{d\mu} H(Q,\mu) = (\gamma_c + \gamma_c^*) H = -\frac{\alpha_s(\mu)}{2\pi} \left[\underbrace{8 C_F \ln \frac{\mu}{Q}}_{\substack{\text{leading dble logs} \\ \alpha_s \ln \sim 1}} + \underbrace{6 C_F}_{\substack{\text{part of NLL,} \\ \text{also need} \\ \text{2-loop cusp } \alpha_s^2 \ln \frac{\mu}{Q} \\ \text{term}}} \right] H(Q,\mu)$$

$\Rightarrow$

$$H(Q,\mu_1) = H(Q,\mu_0)\, U_H(Q,\mu_0,\mu_1)$$

$$= H(Q,\mu_0) \exp\left[- \#\, \alpha_s \ln^2\left(\frac{\mu_1}{Q}\right) \overset{\text{boundary condition}}{+ \cdots} \right]$$ ⟵ frozen coupling result

$$= H(Q,\mu_0) \exp\left[-\frac{\#}{\alpha_s(\mu_0)} f\left(\frac{\alpha_s(\mu_1)}{\alpha_s(\mu_0)} \right) \right]$$ ⟵ running coupling result



Details in Hmwk

Ask about scales

Sudakov Form Factor

no emission until μ_1

$\bar\chi_n \Gamma \chi_{\bar n}$ SCET operator restricts radiation (collinear & soft emissions only below μ_1)

- To discuss the order we're working look at series in

$$\ln C(\omega,\mu) \sim \underbrace{\alpha_s^k \ln^{k+1}}_{LL} + \underbrace{\alpha_s^k \ln^k}_{NLL} + \underbrace{\alpha_s^k \ln^{k-1}}_{NNLL} + \cdots$$

- What do we need to compute?

	tree-level	1-loop	2-loop	3-loop
LL	matching	$1/\epsilon^2$	—	—
NLL	matching	$1/\epsilon$	$1/\epsilon^2$	—
NNLL		matching	$1/\epsilon$	$1/\epsilon^2$ ← cusp. anom. dim.

1-loop Matching Example

$e^+e^- \to$ dijets

[Feyn. Gauge again]

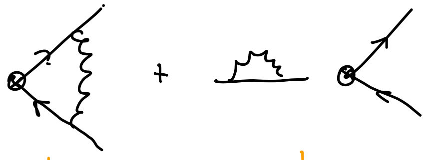
$$\begin{tikzcd} \boxed{QCD} \arrow[d] & \mathcal{L}_{QCD} + J^\mu = \bar\psi \gamma^\mu \psi \arrow[d] \\ \boxed{SCET} & \mathcal{L}^{(0)}_{SCET} + \mathcal{L}^{(0)}_{hard} = C\, \bar\chi_n \gamma_\perp^\mu \chi_{\bar n} \end{tikzcd}$$

↑ find C at $\mathcal{O}(\alpha_s)$

$$(\text{1-loop ren. QCD}) - (\text{1-loop ren. SCET}) = C^{(1\text{-loop})} \langle O^{(0)}_{SCET} \rangle$$

- Must use some IR regulator in QCD & SCET $\quad p^2 = \bar p^2 \neq 0$

QCD



$$= \frac{\alpha_s(\mu) C_F}{4\pi}\left[-2\ln^2\left(\frac{p^2}{Q^2}\right) - 3\ln\left(\frac{p^2}{Q^2}\right) - \frac{2\pi^2}{3} - 1\right]$$

$\frac{1}{\epsilon_{UV}}$ $\quad -\frac{1}{\epsilon_{UV}}$ ← cancel since cons. current

$$\text{QCD} - \text{SCET} = \frac{\alpha_s(\mu)C_F}{4\pi}\left[-\ln^2\left(\frac{\mu^2}{-Q^2-i0}\right) - 3\ln^2\left(\frac{\mu^2}{-Q^2-i0}\right) - 8 + \frac{\pi^2}{6}\right]$$

matching:

$$C(Q,\mu) = 1 + \frac{\alpha_s(\mu)C_F}{4\pi}\left[\quad '' \quad\quad '' \quad\quad '' \quad\quad '' \quad\right]$$

- no dependence on IR regulator
- indep of IR choices {regulator, states, ...}

Ultrasoft - Collinear Factorization

put $n\cdot A_{us}$ into usoft Wilson lines

$$Y_n(x) = P\exp\left(ig\int_{-\infty}^{0} ds\; n\cdot A_{us}(x+ns)\right)$$

$$[n\cdot D_{us}\, Y_n] = 0\,, \qquad Y_n^\dagger Y_n = \mathbb{1} = Y_n Y_n^\dagger$$

Field Redefinition: $\xi_n(x) = Y_n(x)\,\xi_n'(x)$

$$A_n^\mu(x) = Y_n(x)\,A_n'^\mu(x)\,Y_n^\dagger(x) \qquad \left[\begin{matrix}\text{same for}\\ \text{ghost } c_n\end{matrix}\right]$$

$$W_n = \sum_{\text{perms}} \exp\left(\frac{-g}{i\bar{n}\cdot\partial_n}\,\bar{n}\cdot A_n\right) \xrightarrow[\substack{\text{use}\\ \text{multipole}\\ \text{expn}}]{} Y_n\, W_n'\, Y_n^\dagger$$

Also: $\chi_n \to Y_n\,\chi_n'$, $\mathcal{B}_{n\perp} \to Y_n\,\mathcal{B}_{n\perp}'\,Y_n^\dagger$

$$\mathcal{L}_{n\xi}^{(0)} = \bar{\xi}_n'\,\frac{\bar{n}\!\!\!/}{2}\left[Y_n^\dagger\, i n\cdot D_{us}\, Y_n + Y_n^\dagger\left(Y_n g\, n\cdot A_n'\, Y_n^\dagger\right)Y_n + \dots\right]\xi_n'$$

$$= \bar{\xi}_n'\,\frac{\bar{n}\!\!\!/}{2}\left[i n\cdot\partial + g\, n\cdot A_n' + i D\!\!\!/_{n\perp}'\,\frac{1}{i\bar{n}\cdot D_n'}\,i D\!\!\!/_{n\perp}'\right]\xi_n'$$

$$\mathcal{L}_{n\xi}^{(0)}(\xi_n, A_n, n\cdot A_{us}) = \mathcal{L}_{n\xi}^{(0)}(\xi_n', A_n', 0)$$

same for $\mathcal{L}_{ng}^{(0)}$

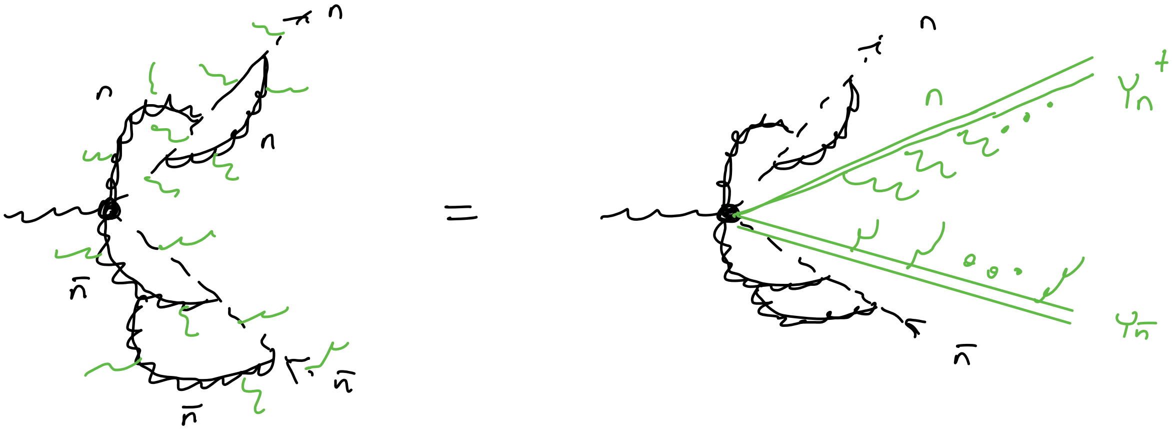
so decoupled in $\mathcal{L}^{(0)}$

Reappear in currents:

eg $(\bar{\chi}_n \Gamma \chi_{\bar{n}}) \longrightarrow \underbrace{\bar{\chi}_n'}_{(n\text{-collin})} \underbrace{(Y_n^\dagger Y_{\bar{n}})}_{(\text{usoft})} \Gamma \underbrace{\chi_{\bar{n}}'}_{(\bar{n}\text{-collin})}$

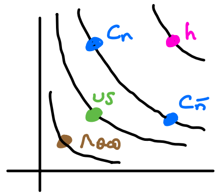
factorized up to global color & spin indicies

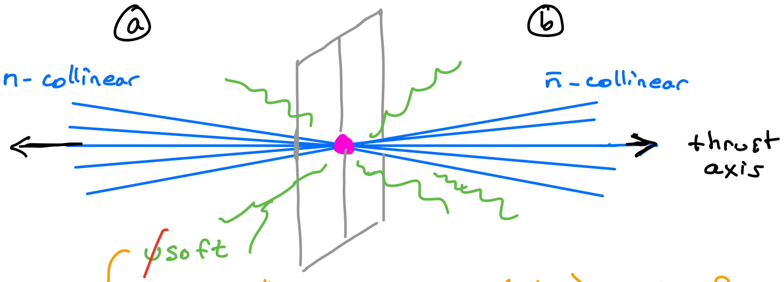
Sums up ∞ class of diagrams



e+e- → dijets $\cancel{\alpha_s}$

SCET$_{\text{I}}$





$e^+e^- \to \underset{q^\mu}{\gamma^* \text{ or } Z^*} \to X_n X_{\bar{n}} X_s$

~~u~~soft "soft" (drop "u" for simplicity)

(e^+e^-) cm frame

all orders in α_s

expand $\lambda \ll 1$

good up to $\mathcal{O}(\lambda^2)$

measure thrust

$$\tau = \frac{M_a^2 + M_b^2}{Q^2} \ll 1 \quad , \quad M_{a,b} \sim Q\sqrt{\tau}$$

2 Jets: $M_a^2 \equiv (p_{X_a}^\mu)^2 = \Big(\sum_{i\in a} p_i^\mu\Big)^2 \ll Q^2$

$M_b^2 = \Big(\sum_{i\in b} p_i^\mu\Big)^2 \ll Q^2$

Hemisphere invariant masses

Scales

- hard scale $\mu_h \sim Q = \sqrt{q^2}$
- jet $\mu_J \sim Q\sqrt{\tau}$

n - collinear $Q(\lambda^2, 1, \lambda)$

$\bar{n}$ - collinear $Q(1, \lambda^2, \lambda)$

$$\lambda \sim \frac{M_{a,b}}{Q} \sim \sqrt{\tau}$$

- soft radiation - uniform - eikonal - jets communicate

$$p_{us}^{\mu} \sim Q\lambda^2 \sim \frac{M_{a,b}^2}{Q} \sim Q\tau$$

$\mu_s \sim Q\tau$

$\nearrow$ $Q\tau \gg \Lambda_{QCD}$ soft perturbative

$\mu_h \gg \mu_J \gg \mu_s \gg \Lambda_{QCD}$ "tail" region

$$\text{Perturbative} * \left[1 + \mathcal{O}\left(\frac{\Lambda_{QCD}}{Q\tau}\right) + \dots\right]$$

constant parameter

$\searrow$ $Q\tau \sim \Lambda_{QCD}$ non-perturbative soft fn.

$\mu_h \gg \mu_J \gg \mu_s \sim \Lambda_{QCD}$ "peak" region

$$\text{Perturbative} * \underbrace{\left(\Lambda_{QCD}/Q\tau\right)^k \sim 1}_{}$$ any k

Λ_{QCD} function

Current $J^{\mu} = \bar{\psi}\Gamma^{\mu}\psi \rightarrow \underbrace{\int d\omega\, d\bar{\omega}\; C(\omega\bar{\omega})}_{C(Q^2)} (\bar{\xi}_n W_n)_{\omega} \Gamma^{\mu} (Y_n^{\dagger} Y_{\bar{n}}) (W_{\bar{n}}^{\dagger} \xi_{\bar{n}})_{\bar{\omega}}$

after field redefinition

skipped

Kinematics $q^{\mu} = p_{X_n}^{\mu} + p_{X_{\bar{n}}}^{\mu} + p_{X_s}^{\mu} = Q\left(\frac{n^{\mu} + \bar{n}^{\mu}}{2}\right)$

$\bar{n}\cdot q = Q = \bar{n}\cdot p_{X_n} + \cdots$ $\quad \omega = Q$

$n\cdot q = Q = n\cdot p_{X_n} + \cdots$ $\quad \bar{\omega} = Q$

small

momentum conservation strong enough that no convolutions in $\omega, \bar{\omega}$

Factorize the Cross-Section

$e^+e^- \to \gamma^*$

$$\text{QCD} \quad \sigma = \sum_{\substack{X \\ \text{dijet}}} (2\pi)^4 \delta^4(q - p_X) L_{\mu\nu} \langle 0 | J^{\nu\dagger}(0) | X \rangle \langle X | J^{\mu}(0) | 0 \rangle$$

restrict to dijet X states. SCET allows us to move restrictions into operators

- $|X\rangle = |X_n\rangle\,|X_{\bar n}\rangle\,|X_{us}\rangle$ $\qquad \mathcal{L} = \mathcal{L}_n + \mathcal{L}_{\bar n} + \mathcal{L}_{us}$

so Hilbert Space factorizes

$$\sigma = N_0 \sum_{\vec n} \sum_{X_n, X_{\bar n}, X_{us}} (2\pi)^4\, \delta^4(q - p_{X_n} - p_{X_{\bar n}} - p_{X_{us}}) \langle 0| Y_n^{\dagger} Y_{\bar n} |X_{us}\rangle\langle X_{us}| Y_{\bar n}^{\dagger} Y_n |0\rangle$$

$$* \; |C(Q)|^2 \; \langle 0|\, \bar{n}\!\!\!/ \, \chi_{n,Q} |X_n\rangle\langle X_n| \bar\chi_n |0\rangle$$

$$* \; \langle 0| \bar\chi_{\bar n, Q} |X_{\bar n}\rangle\langle X_{\bar n}|\, n\!\!\!/ \, \chi_{\bar n} |0\rangle$$

insert 1,
measurement
we want

$$* \int d\tau \int dM_a^2\, dM_b^2\; \delta\Big(\tau - \frac{M_a^2 + M_b^2}{Q^2}\Big) \quad \delta\big(M_a^2 - (p_{X_n} + p_{X_s}^a)^2\big)$$
$$\delta\big(M_b^2 - (p_{X_{\bar n}} + p_{X_s}^b)^2\big)$$

$$= \int d\tau \quad \delta\Big(\tau - \frac{(p_{X_n} + p_{X_s}^a)^2 + (p_{X_{\bar n}} + p_{X_s}^b)^2}{Q^2}\Big)$$

Factorize Measurement , Simplify, - - -

$$\frac{d\sigma}{d\tau} = \sigma_0\, |C(Q)|^2 \int dk^+ d\ell^+ dk^- d\ell^- \;\delta\Big(\tau - \frac{(k^+ + \ell^+) + (k^- + \ell^-)}{Q}\Big)$$

$$* \sum_{X_n} \frac{1}{2\pi} \int d^4x\; e^{ik^+x^-/2}\; \mathrm{tr}\, \langle 0| \frac{\bar{n}\!\!\!/}{4N_c} \chi_{n,Q}(x) |X_n\rangle\langle X_n| \bar\chi_n(0) |0\rangle$$

$$* \sum_{X_{\bar n}} \frac{1}{2\pi} \int d^4y\; e^{ik^-y^+/2}\; \mathrm{tr}\, \langle 0| \bar\chi_{\bar n, Q}(y) |X_{\bar n}\rangle\langle X_{\bar n}| \frac{n\!\!\!/}{4N_c} \chi_{\bar n}(0) |0\rangle$$

$$* \sum_{X_s} \frac{1}{N_c}\, \delta(\ell^+ - p_{X_s}^{a+})\, \delta(\ell^- - p_{X_s}^{b-})\; \mathrm{tr} \langle 0| Y_n^{\dagger} Y_{\bar n} |X_s\rangle\langle X_s| Y_{\bar n}^{\dagger} Y_n |0\rangle$$

$$= \sigma_0\; Q\; H(Q,\mu) \int dk\, d\ell\;\; \delta\Big(\tau - \frac{k+\ell}{Q}\Big)\; J_\tau(Qk, \mu)\quad S_\tau(\ell, \mu)$$

where $\quad J_\tau(s) = \int ds'\; J(s - s')\; J(s')$

$\qquad\quad S_\tau(\ell) = \int d\ell'\; S(\ell - \ell', \ell')$

$$= \sigma_0\; Q^2\; H(Q,\mu) \int d\ell \quad J_\tau(Q^2\tau - Q\ell, \mu)\; S_\tau(\ell, \mu)$$

- soft function encodes both $\ell \sim Q\tau$ and $\ell \sim \Lambda_{QCD}$

{dijet factorization theorem for hemisphere masses}

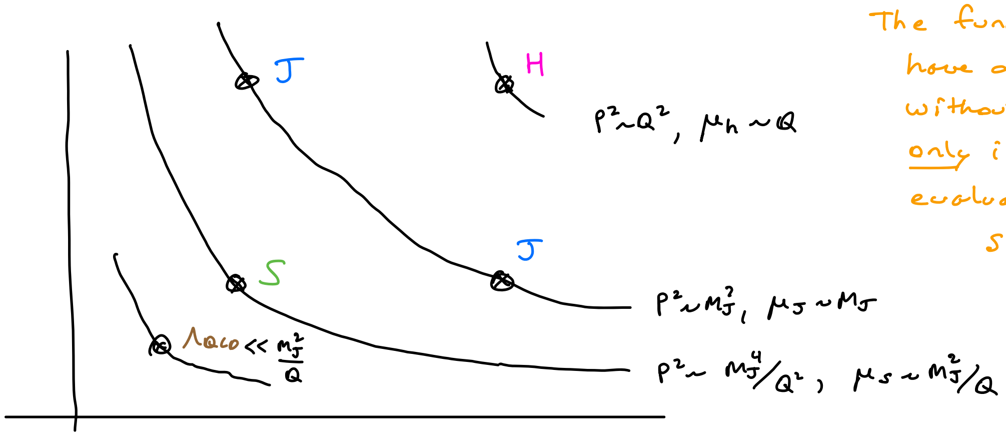
Bare → Renormalized

$$H(Q) = Z_H \; H(Q,\mu)$$
$$\underset{\tau}{J}(M^2) = Z_J \otimes \underset{\tau}{J}(M'^2, \mu^2)$$
$$\underset{\tau}{S}(\ell') = Z_S \otimes \underset{\tau}{S}(\ell, \mu)$$

← integrals, like for PDF example

$$H(Q,\epsilon) \int d\ell' \; \underset{\tau}{J}(M_J^2 - Q\ell', \epsilon) \; \underset{\tau}{S}(\ell', \epsilon) \quad \leftarrow \text{bare}$$

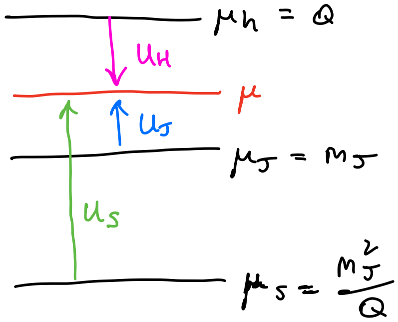
$$= H(Q,\mu) \int d\ell \; \underset{\tau}{J}(M_a^2 - Q\ell, \mu) \; \underset{\tau}{S}(\ell, \mu) \quad \leftarrow \text{renormalized}$$



The functions H, J, S have α_s expansions without large logs only if each is evaluated at different scale μ!

RGE Coefficient Ren. = $\begin{pmatrix}\text{Operator} \\ \text{Ren.}\end{pmatrix}^{-1}$ "consistency conditions"

↪ relates $\gamma_{H,J,S}$ anomalous dimensions



- large logs all in evolution factors U_H, U_J, U_S
- can pick any μ

Sudakov Form Factor

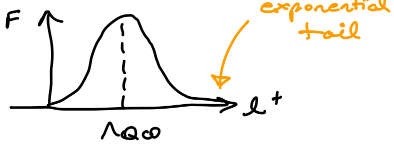
⊗ = integral

$$\frac{1}{\sigma_0} \frac{d\sigma}{dM_J^2} = H(Q,\mu_h) \; U_H(Q, \mu_h, \mu) \; J_\tau(M_J^2 - s', \mu_J) \overset{s'}{\otimes} U_J(s' - Q\ell, \mu_J, \mu)$$
$$\overset{\ell}{\otimes} \; S_\tau(\ell - \ell', \mu_S) \overset{\ell'}{\otimes} U_S(\ell', \mu_S, \mu)$$

pick $\mu = \mu_S$: $U_S(\ell', \mu_S, \mu_S) = \delta(\ell')$ → only need U_J, U_H ✓

Soft Fn OPE

$$S_\tau(\ell,\mu) = \int d\ell' \, \underset{\substack{\uparrow \\ \text{power law terms} \\ \sim \frac{(\ln \ell/\mu)^k}{\ell}}}{\hat{S}(\ell-\ell',\mu)} \; \underset{\substack{\uparrow \\ \sim \Lambda_{QCD} \text{ effects}}}{F(\ell')}$$



$\Lambda_{QCD} \ll \ell$ expansion:

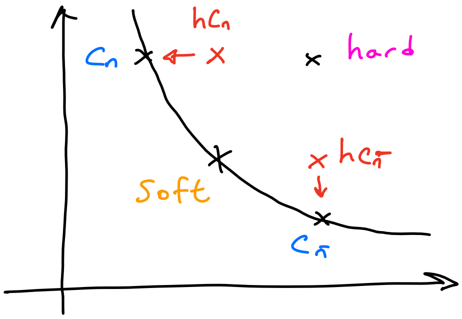
$$S_\tau(\ell) = \hat{S}(\ell)\, 1 - \underbrace{\left[\frac{\partial}{\partial \ell}\hat{S}(\ell)\right]}_{\substack{\text{Wilson Coeff. for} \\ \text{power correction}}} \Omega_1 + \dots$$

$$\int_0^\infty dk\, F(k) = 1 \qquad\qquad \int_0^\infty dk\, k\, F(k)$$

$$\Omega_1 = \langle 0 | \bar{Y}_{\bar{n}}^\dagger Y_n^\dagger \hat{\mathcal{E}}_T Y_n \bar{Y}_{\bar{n}} | 0 \rangle \sim \Lambda_{QCD}$$

↳ hadronization parameter, universal across dijet event shapes once properly accounting for hadron mass equivalence classes

$SCET_{II}$ from $SCET_I$



$$q = p_n + p_s \sim Q(\overset{+}{\lambda}, \overset{-}{1}, \overset{\perp}{\lambda})$$

$$\underset{\text{offshell}}{q^2 = Q^2\lambda} \gg Q^2\lambda^2 \sim p_n^2 \;!$$

$q \sim Q(\lambda, 1, \sqrt{\lambda})$ on-shell scaling

hard-collinear mode

match $hc_n \to c_n$

$hc_{\bar{n}} \to c_{\bar{n}}$

Constructing $\mathrm{SCET_{II}}$ operators using $\mathrm{SCET_I}$:

1) Match QCD → $\mathrm{SCET_I}$ (hc_n, $hc_{\bar n}$, soft)
2) Factorize (field redefinition)
3) Match $\mathrm{SCET_I}$ → $\mathrm{SCET_{II}}$ (c_n, $c_{\bar n}$, soft)

eg. $e^+e^- \to$ dijet $p_\perp$

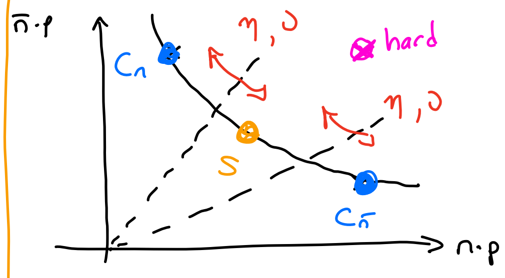
$$J_{\mathrm{SCET_I}} = \bar\chi_n^{hc}\,(Y_n^\dagger Y_{\bar n})\,\Gamma\,\chi_{\bar n}^{hc}$$
$$\Downarrow$$
$$J_{\mathrm{SCET_{II}}} = \bar\chi_n\,(S_n^\dagger S_{\bar n})\,\Gamma\,\chi_{\bar n}$$

soft Wilson lines

- can also be obtained by matching QCD → $\mathrm{SCET_{II}}$, but more work
- with ≥ 2 $\mathrm{SCET_I}$ operators having usoft & collinear field
  can get $\int dp^-\,dk^+\,J(p^-,k^+)\,C_n(p^-)\,S(k^+)$, J = h.c_n matchin

$$\mathcal{L}^{(0)}_{\mathrm{SCET_{II}}} = \underbrace{\mathcal{L}^{(0)}_{\mathrm{soft}} + \sum_n \left(\mathcal{L}^{(0)}_{\xi n} + \mathcal{L}^{(0)}_{gn}\right)}_{\text{already decoupled}} + \mathcal{L}^{(0)}_{G}$$

same $\mathrm{SCET_I} = \mathrm{SCET_{II}}$



- $\mathrm{SCET_{II}}$ also has 0-bin subt.
- modes distinguished by rapidity

$$e^{2Y} = \frac{p^-}{p^+} \sim \underset{C_n}{\lambda^{-2}},\ \underset{S}{\lambda^0},\ \underset{C_{\bar n}}{\lambda^2}$$

- And can have "rapidity divergences" not regulated by ϵ

$\epsilon \Longleftrightarrow$ regulates k^2 offshellness

$$\int \frac{dk^+}{k^+}\, R(k,\eta,\nu)$$

$k^- = \frac{k_\perp^2}{k^+}$, along hyperbola

reg. like ϵ

scale like μ

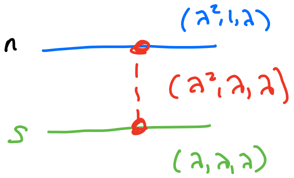
Glauber Exchange $\mathcal{L}_G^{(0)}$ see arXiv: 1601.04695

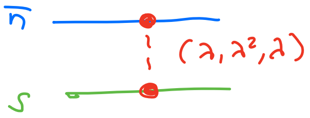
- offshell modes with $p^+p^- \ll \vec{p}_\perp^{\,2} \sim \lambda^2$
- mediates Forward scattering $s \gg -t$
  small-x phenomena (Regge, BFKL, ...)
- violates factorization in Hard Scattering unless it cancels out

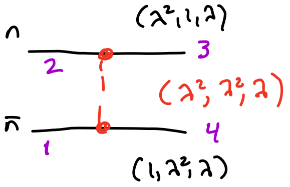
Match from QCD, integrating Glauber out:

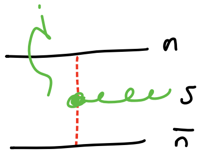
$$\mathcal{L}_G^{(0)} = \sum_{n} \sum_{i,j=q,g} O_n^{iB} \frac{1}{P_\perp^2} O_s^{j_n B} + \sum_{n,n'} \sum_{i,j=q,g} O_n^{iB} \frac{1}{P_\perp^2} O_s^{BC} \frac{1}{P_\perp^2} O_{n'}^{jC}$$

(2-rapidities) (3-rapidities)









Lipatov etc.

- $\frac{1}{\vec{P}_\perp^{\,2}}$ potentials, instantaneous in z & t like Coulomb

$$O_n^{qB} = \bar{\chi}_n\, T^B \frac{\bar{n}\!\!\!/}{2} \chi_n \,, \qquad O_n^{gB} = \frac{i}{2} f^{BCD} \mathcal{B}_{n\perp\mu}^{C} \frac{\bar{n}}{2}\cdot(i\partial_n - i\overleftarrow{\partial}_n) \mathcal{B}_{n\perp}^{D\mu}$$

similar $O_{\bar{n}}^{qB}$, $O_{\bar{n}}^{gB}$

$O_s^{q_n B}$, $O_s^{g_n B}$

$$O_s^{BC} = 8\pi\alpha_s \Big\{ \mathcal{P}_\perp^\mu \mathcal{S}_n^T \mathcal{S}_{\bar n} \mathcal{P}_{\perp\mu} - \mathcal{P}_{\perp\mu}\, g\, \tilde{\mathcal{B}}_{S\perp}^{n\mu} \mathcal{S}_n^T \mathcal{S}_{\bar n} - \mathcal{S}_n^T \mathcal{S}_{\bar n}\, g\, \tilde{\mathcal{B}}_{S\perp}^{\bar n\mu} \mathcal{P}_{\perp\mu}$$

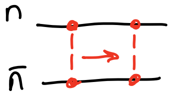
$$- g\, \tilde{\mathcal{B}}_{S\perp}^{n\mu} \mathcal{S}_n^T \mathcal{S}_{\bar n} \tilde{\mathcal{B}}_{S\perp\mu}^{\bar n} - \frac{n_\mu \bar n_\nu}{2} \mathcal{S}_n^T\, i g\, \tilde{G}_S^{\mu\nu} \mathcal{S}_{\bar n} \Big\}^{BC}$$

$$\tilde{\mathcal{B}}_{S\perp}^{n\,AB} = -i f^{ABC} \mathcal{B}_{S\perp}^{nC}$$

<u>Note</u>

- universal for $i,j = q, g$
- no hard coefficient (loop corrections) for $\mathcal{L}_G^{(0)}$
- 1 or 2 collinear directions in $\mathcal{L}_G^{(0)}$
  others are T-products
- breaks factorization $\mathcal{L}_G^{(0)}(\{\xi_{n_i}, A_{n_i}\}, q_s, A_s)$
  couples $n, \bar n, s$ modes at $\mathcal{O}(\lambda^0)$
- encodes known examples of fact. violation
  (Wilson line directions, $i\pi$'s, ...)

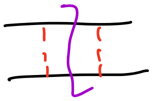
Collapse

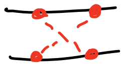


$$= \left(\frac{-i}{4\pi}\right) \int \frac{d^{d-2}k_\perp}{k_\perp^2 (k_\perp - q_\perp)^2} \quad [-i\pi]$$

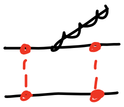
(• needs rapidity regulator)

• Pure unitarity cut result



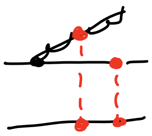


$= 0$

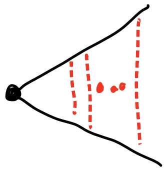
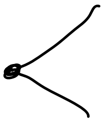
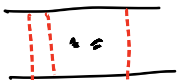


$= 0$ can't collapse to equal t & z

Collapse ⟷ shockwave picture for high energy scattering



$\neq 0$

Σ  $\sim e^{i\phi(0)}$  , Σ  $\sim e^{i\phi(b_T)} - 1$

- $\cancel{\mathcal{L}_G^{(0)}}$ cancels in $e^+e^- \to$ dijets, Drell-Yan, ...

  due to $e^{i\phi}$, collapse & inclusive unitarity

- $\mathcal{L}_G^{(0)}$ determines Wilson line directions $W(-\infty, y)$ $W(y, +\infty)$

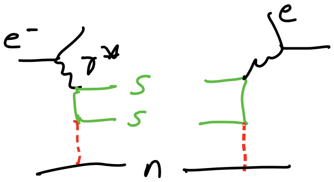
  (re absorbed into collinear/soft)

  which matter in some processes

  eg. SIDIS vs. Drell-Yan for $\frac{d\sigma}{dq_T^2\, d\phi \cdots}$

Examples $\mathcal{L}_G^{(0)}$ gives dominant contributions

- $\mathcal{L}_G^{(0)}$ important for small-x resummation



$$W^{\mu\nu} = \int d^2k_\perp \, S^{\mu\nu}\left(q, k_\perp, \frac{\nu}{xP^-}\right) C\left(k_\perp, \frac{\nu}{P^-}\right)$$

$$\nu\frac{\partial}{\partial\nu} S^{\mu\nu} = \gamma^{BFKL} \otimes_\perp S^{\mu\nu} , \qquad \nu\frac{\partial}{\partial\nu} C = \cdots$$

  - rapidity RGE is BFKL equation

  → small-x resummation

- $\mathcal{L}_G^{(0)}$ describes diffractive scattering (my Friday talk)

- $\mathcal{L}_G^{(0)}$ can be used to study fact. violation

Conclusions

SCET provides powerful framework for analyzing hard scattering σ

⇒ Factorization : universal non-pert. functions
universal perturbative functions

⇒ Resummation : large double & single logs via RGE
both μ (inv. mass) & ν (rapidity)

⇒ Manifest Power Counting $\mathcal{L}^{(0)} \sim \lambda^0$
$T[\mathcal{L}^{(0)}, \mathcal{L}^{(2)}] \sim \lambda^2$

can also handle multi-scale problems
$Q \gg Q\sqrt{\tau} \gg Q\tau \gg \Lambda_{QCD}$

can study power corrections, same method

⇒ Provides universal description of factorization violation in hard-scattering : $\mathcal{L}_G^{(0)}$

Interestingly the same $\mathcal{L}_G^{(0)}$ mediates phenomena in Forward scattering kinematics $s \gg -t$, small-x

⇒ Fun mathematical structure W_n, Y_n

⇒ Multi IR-Mode EFT, prototype for other EFTs with more complicated kinematics

The End