Lecture 3:
Electromagnetic and gravitational form factors

Proton electromagnetic form factors

Elastic ep scattering amplitude

$$\langle p'|J^\mu_{em}|p\rangle = \bar{u}(p')\left[F_1\gamma^\mu + \frac{i\sigma^{\mu\rho}\Delta_\rho}{2m}(F_2 + i\gamma_5 F_3) + \frac{1}{m^2}(\not{\Delta}\Delta^\mu - \Delta^2\gamma^\mu)\gamma_5 F_a\right] u(p)$$

Dirac Pauli

Total electric charge $F_1(0) = 1$

Anomalous magnetic moment $F_2(0) = \frac{g-2}{2}$

Electric dipole moment (EDM) $\vec{d} = \frac{2F_3(0)}{2m}\vec{s}$ violates P, CP

Anapole moment $\vec{a} = \frac{2F_a}{m^2}\vec{s}$ violates P

Proton in external EM fields and currents

$$
\begin{aligned}
\mathcal{T} &= -\int d^4x \langle P'|J^{\mu}_{em}(x)A_\mu(x)|P\rangle = -\langle P'|J^{\mu}_{em}(0)|P\rangle \int d^4x A_\mu(x)e^{i\Delta\cdot x} \\
&\sim -\bar{u}(P')\left[\frac{i\sigma^{\mu\rho}\Delta_\rho}{2m}\left(F_1+F_2+i\gamma_5 F_3\right)+\frac{1}{m^2}(\not{\Delta}\Delta^\mu-\Delta^2\gamma^\mu)\gamma_5 F_a\right]u(P)\int d^4x A_\mu(x)e^{i\Delta\cdot x} \\
&= \frac{1}{2m}\bar{u}(P')\left[\sigma^{\mu\rho}\left(F_1+F_2+i\gamma_5F_3\right)\right]u(P)\int d^4x\partial_\rho A_\mu(x)e^{i\Delta\cdot x} \\
&\quad +\frac{1}{m^2}\bar{u}(P')(g^{\lambda\mu}\gamma^\rho-g^{\lambda\rho}\gamma^\mu)\gamma_5F_a u(P)\int d^4x\partial_\lambda\partial_\rho A_\mu(x)e^{i\Delta\cdot x} \\
&\approx -\frac{V_4}{m^2}\left[\epsilon^{\mu\rho\alpha\beta}P_\alpha S_\beta(F_1+F_2)+P^\mu S^\rho F_3\right]\partial_\rho A_\mu+\frac{2V_4}{m^2}F_aS^\rho\partial_\lambda F_\rho{}^\lambda
\end{aligned}
$$

Nonrelativistic approximation

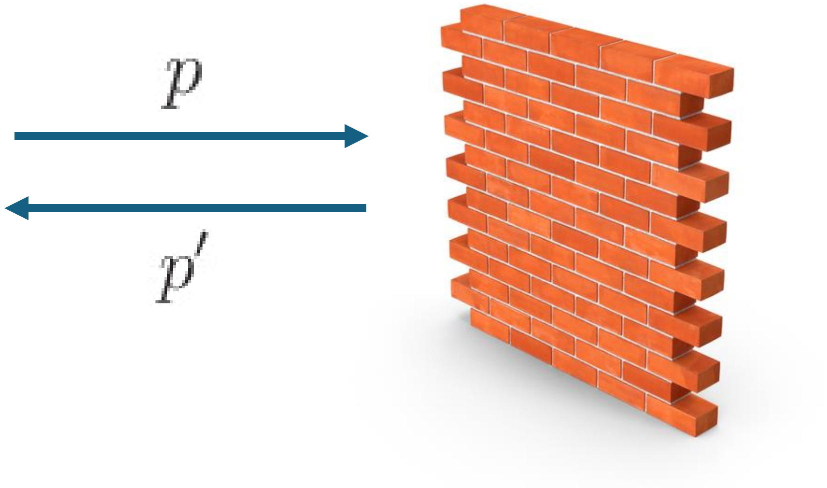
$$P^\mu = \delta_0^\mu m, \; S^{\rho'} = \delta_i^\rho S^i \qquad \vec{S} = 2m\vec{s} \text{ so that } |\vec{s}| = \frac{1}{2}$$

$$V = -\frac{\mathcal{T}}{2mV_4} = -\vec{B}\cdot\vec{\mu} - \vec{E}\cdot\vec{d} - \vec{a}\cdot\vec{J}$$

$$\vec{\mu} = \frac{2(F_1+F_2)}{2m}\vec{s}, \qquad \vec{d} = \frac{2F_3}{2m}\vec{s}, \qquad \vec{a} = \frac{2F_a}{m^2}\vec{s}$$

	B	E	s	J
P	+	-	+	-
T (CP)	-	+	-	-

Breit (brick wall) frame



$$p^\mu = \left(\sqrt{m^2 + \frac{\vec{\Delta}^2}{4}}, \frac{\vec{\Delta}}{2}\right) \qquad p'^\mu = \left(\sqrt{m^2 + \frac{\vec{\Delta}^2}{4}}, -\frac{\vec{\Delta}}{2}\right)$$

$$\begin{aligned}
\bar{u}(p's')\gamma^\mu u(ps) &= \frac{1}{2(p^0+m)}(\xi'^\dagger, \xi'^\dagger)\left(p^0\gamma^0 - \frac{\vec{\Delta}\cdot\vec{\gamma}}{2} + m\right)\gamma^\mu\left(p^0\gamma^0 + \frac{\vec{\Delta}\cdot\vec{\gamma}}{2} + m\right)\begin{pmatrix}\xi\\ \xi\end{pmatrix} \\
&= \left(2m\xi'^\dagger\xi, i\xi'^\dagger\vec{\sigma}\xi \times \vec{\Delta}\right)
\end{aligned}$$

$$\langle p's'|J^0(0)|ps\rangle = 2m\left(F_1 - \frac{\vec{\Delta}^2}{4m^2}F_2\right)\xi'\xi$$

Exercise: show this

Electric form factor $G_E(t) = F_1(t) + \frac{t}{4M^2}F_2(t)$

Proton charge radius

$$G_E(t=-\vec{\Delta}^2) = \int d^3\vec{r}\rho(r)e^{i\vec{\Delta}\cdot\vec{r}} \quad = \quad \int d^3\vec{r}\rho(r)\left(1+i\vec{\Delta}\cdot\vec{r}-\frac{1}{2}r^2\vec{\Delta}^2\cos^2\theta+\cdots\right)$$

$$\approx \quad C - \frac{\vec{\Delta}^2}{6}\langle r^2\rangle$$

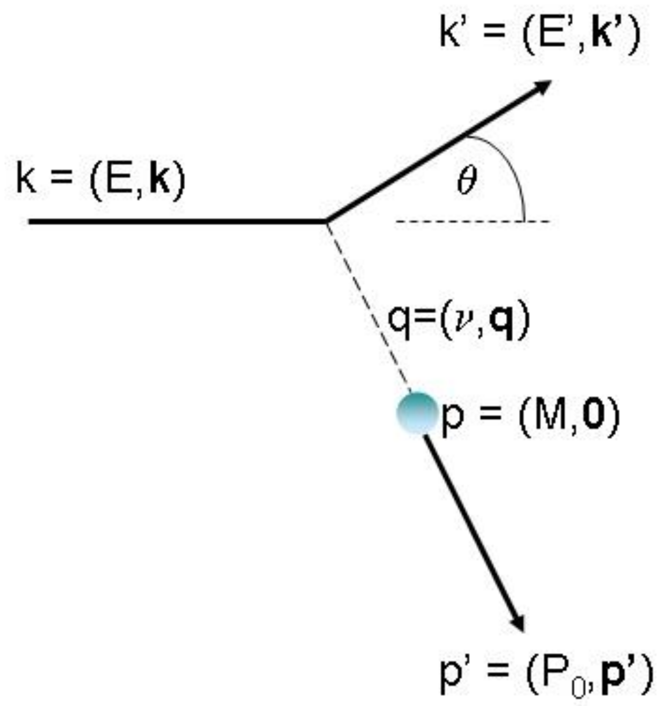
Electric charge density

Proton charge radius $$\langle r^2\rangle = 6\frac{dG_E(t)}{dt}\bigg|_{t=0}$$

Caveat 1: In a relativistic system, interpretation of `coordinates' and `radius' should be taken with care.

Caveat 2 : This is just one definition of radius. For the neutron, $\langle r^2\rangle < 0$

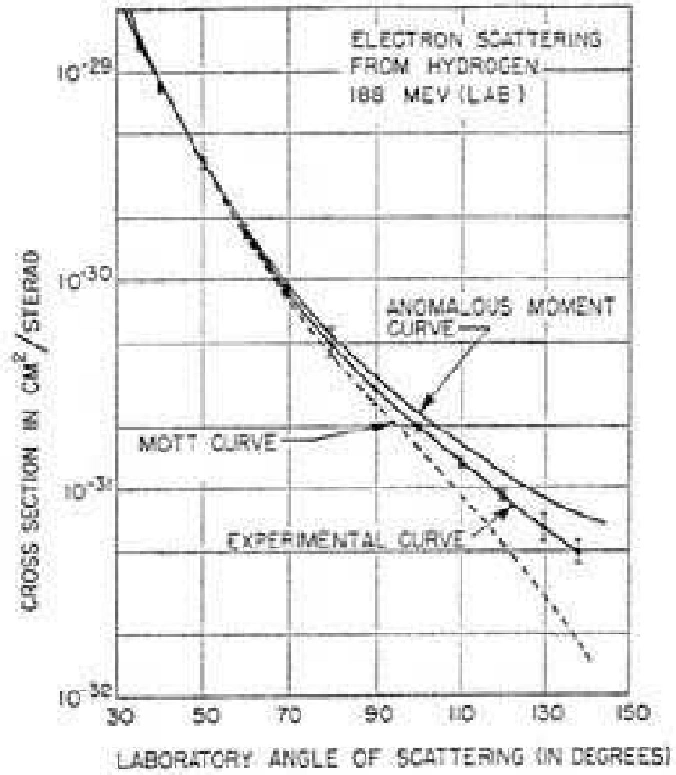
Charge radius from electron scattering (1950s~)



Rosenbluth formula

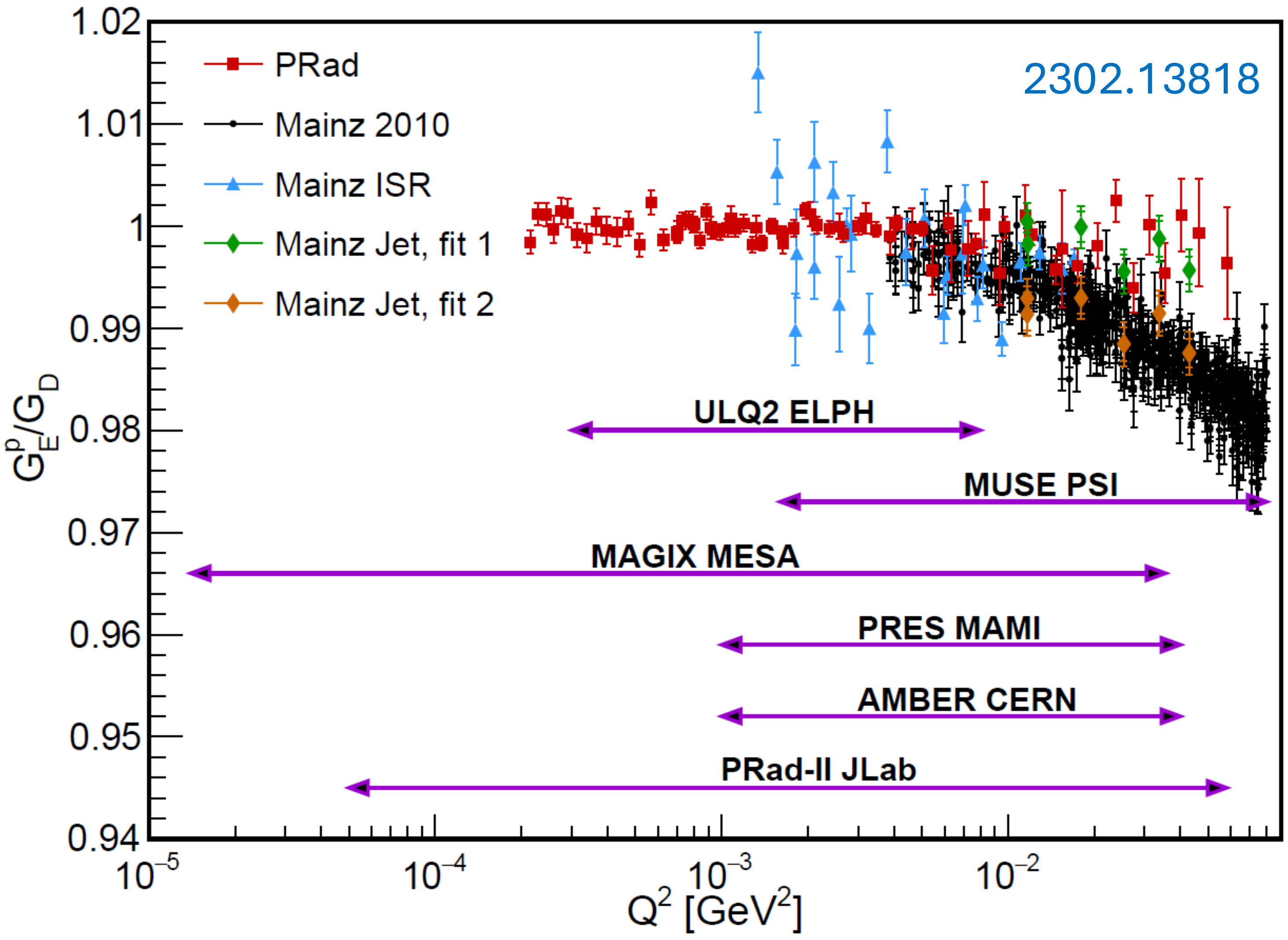
$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2\left[\left(F_1^2 - \frac{q^2}{4m^2}F_2^2\right)\cos^2\frac{\theta}{2} - \frac{q^2}{2m^2}(F_1+F_2)^2\sin^2\frac{\theta}{2}\right]}{2E^2\left[1+\frac{2E}{m}\sin^2\frac{\theta}{2}\right]\sin^4\frac{\theta}{2}}$$

$$\langle r^2 \rangle = 6\frac{dG_E(t)}{dt}\bigg|_{t=0} \sim (0.8\ \mathrm{fm})^2$$



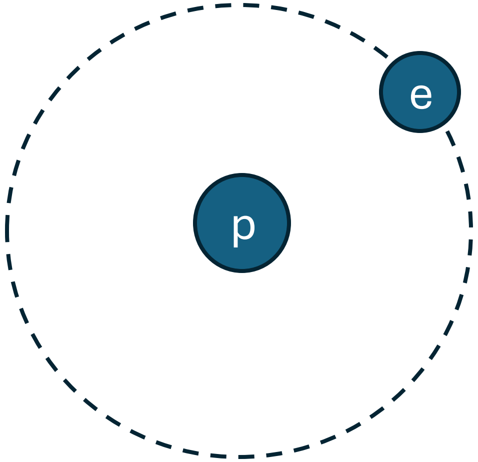
R. Hofstadter

Elastic scattering 70 years later



Charge radius from hydrogen atom spectrum

Coulomb potential modified in a hydrogen atom



$$V(r) = \frac{-e^2}{4\pi r} \Rightarrow -e^2 \int \frac{d^3k}{(2\pi)^3} \frac{G_E(k^2) e^{-i\vec{k}\cdot\vec{r}}}{k^2}$$

$$\delta V(r) = -e^2 \int \frac{d^3k}{(2\pi)^3} \frac{(G_E(k^2) - 1) e^{-i\vec{k}\cdot\vec{r}}}{k^2} \approx \frac{4\pi\alpha}{6} \langle r^2 \rangle \delta^{(3)}(\vec{r})$$

Energy shift

$$\Delta E = \int d^3r \psi^*(r)\delta V(r)\psi(r) = \frac{4\pi\alpha}{6}\langle r^2\rangle|\psi(0)|^2$$

$$\Delta E_{l=0} = \frac{2\alpha^4 m^3}{3n^3}\langle r^2\rangle$$

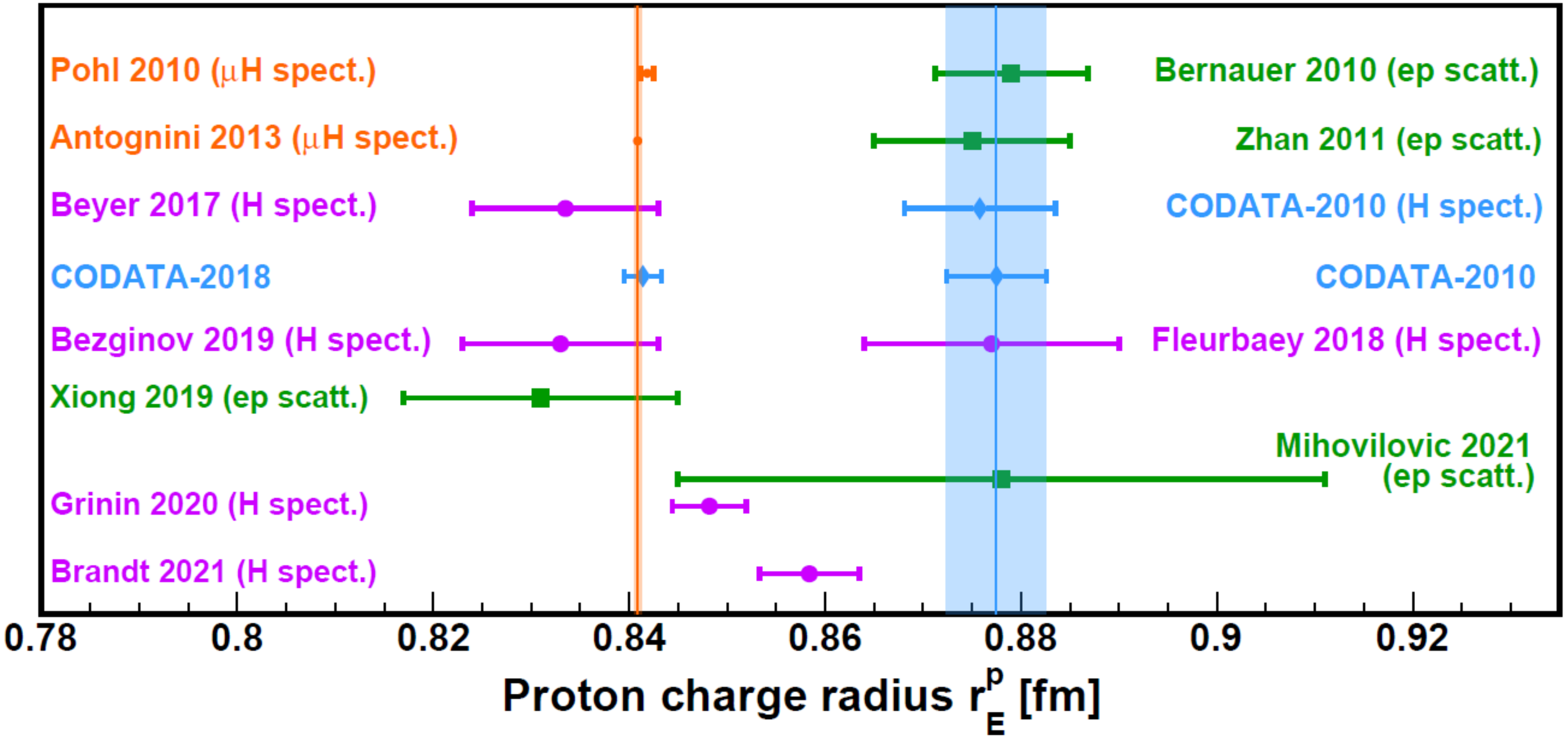
Proton charge radius!

Part of the Lamb shift. Same order as fine structure, but suppressed by

$$m^2\langle r^2\rangle \sim (0.5\mathrm{MeV}\cdot 1\mathrm{fm})^2 \sim \left(\tfrac{1}{400}\right)^2$$

Enhanced in the muonic hydrogen! $m_\mu \approx 200 m_e$

Proton radius puzzle?



PRad (2019) $$r_p = 0.831 \pm 0.007_{\text{stat}} \pm 0.012_{\text{syst}}$$

Both CODATA and PDG now recommend the smaller value ~0.84fm.

Radius zoo

2312.12984

Charge radius

$$\langle r^2\rangle_c = \frac{\int d\boldsymbol{x} x^2 \rho_c(\boldsymbol{x})}{\int d\boldsymbol{x} \rho_c(\boldsymbol{x})} = \frac{6}{G_E(0)} \frac{dG_E(t)}{dt}\bigg|_{t=0}$$

Magnetic radius

$$\langle r^2\rangle_M = \frac{6}{G_M(0)} \frac{dG_M(t)}{dt}\bigg|_{t=0}$$

Baryon number radius

$$\langle r^2\rangle_B = \frac{\int d\boldsymbol{x} x^2 \rho_B(\boldsymbol{x})}{\int d\boldsymbol{x} \rho_B(\boldsymbol{x})}$$

Mass radius

$$\langle r^2\rangle_m = \frac{\int d\boldsymbol{x}\, x^2 T^{00}(\boldsymbol{x})}{\int d\boldsymbol{x} T^{00}(\boldsymbol{x})} = 6\frac{dA(t)}{dt}\bigg|_{t=0} - \frac{3D(0)}{2M^2}$$

Scalar radius

$$\langle r^2\rangle_s = \frac{\int d\boldsymbol{x}\, x^2 T^{\mu}_{\mu}(\boldsymbol{x})}{\int d\boldsymbol{x} T^{\mu}_{\mu}(\boldsymbol{x})} = 6\frac{dA(t)}{dt}\bigg|_{t=0} - \frac{9D(0)}{2M^2}$$

Tensor radius

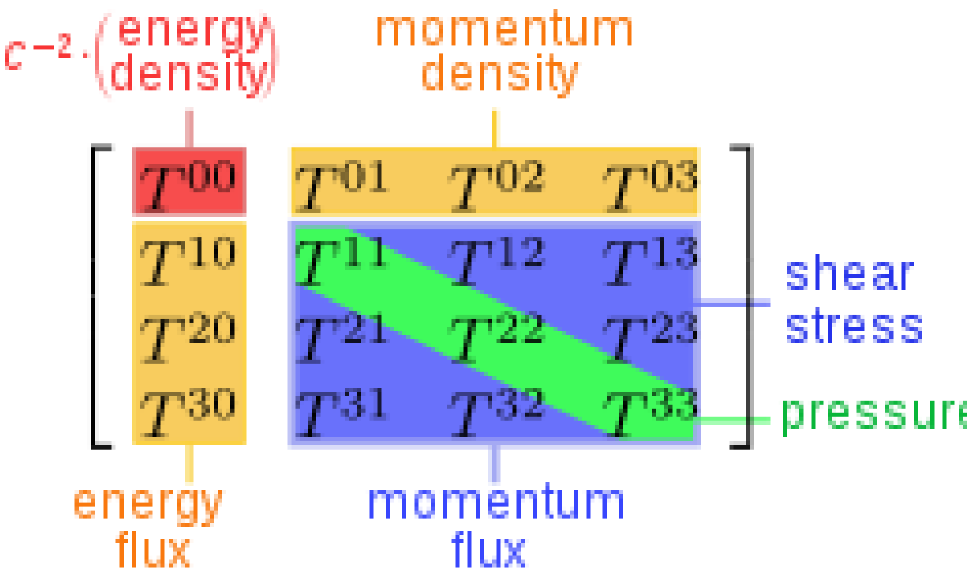
$$\langle r^2\rangle_t \equiv \frac{\int d\boldsymbol{x}\, x^2 \left(T^{00}(\boldsymbol{x}) + \frac{1}{2}T_{ii}(\boldsymbol{x})\right)}{\int d\boldsymbol{x} \left(T^{00} + \frac{1}{2}T_{ii}\right)} = 6\frac{dA(t)}{dt}\bigg|_{t=0}$$

Mechanical radius

$$\langle r^2\rangle_{mech} = \frac{\int d\boldsymbol{x} x^2 \frac{x_i x_j}{x^2} T_{ij}(\boldsymbol{x})}{\int d\boldsymbol{x} \frac{x_i x_j}{x^2} T_{ij}(\boldsymbol{x})} = \frac{6D(0)}{\int_{-\infty}^{0} dt D(t)}$$

...

Gravitational form factors



QCD energy momentum tensor

$$T^{\mu\nu} = \sum_f \bar{\psi}_f \gamma^{(\mu} i D^{\nu)} \psi_f - F^{\mu\rho} F^{\nu}{}_{\rho} + \frac{g^{\mu\nu}}{4} F^{\alpha\beta} F_{\alpha\beta}$$

Forward matrix element $\langle P | T^{\mu\nu} | P \rangle = 2 P^\mu P^\nu$

Frequently asked question: Isn't there a term proportional to $g^{\mu\nu}$?

Nonforward matrix element

$$\langle P' | T^{\mu\nu} | P \rangle = \bar{u}(P') \left[A(t) \gamma^{(\mu} \bar{P}^{\nu)} + B(t) \frac{\bar{P}^{(\mu} i \sigma^{\nu)\alpha} \Delta_\alpha}{2M} + D(t) \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{4M} \right] u(P)$$

$A(0) = 1, B(0) = 0$ but $D(0)$ is unconstrained (and unknown).

Quark and gluon components

Energy momentum tensor consists of quark and gluon parts

$$T^{\mu\nu} = \underbrace{-F^{\mu\lambda}F^{\nu}{}_{\lambda} + \frac{\eta^{\mu\nu}}{4}F^2}_{T_g^{\mu\nu}} + \underbrace{i\bar{q}\gamma^{(\mu}D^{\nu)}q}_{T_q^{\mu\nu}}$$

$$\langle P'|T_{q,g}^{\mu\nu}|P\rangle = \bar{u}(P')\Big[A_{q,g}\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + D_{q,g}\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4M} + \bar{C}_{q,g}M\eta^{\mu\nu}\Big]u(P)$$

4th form factor

$$\sum_q \bar{C}_q(t) + \bar{C}_g(t) = 0$$

because the total energy momentum tensor is conserved.

Relation between $\bar{C}_{q,g}$ and $\langle P|F^{\mu\nu}F_{\mu\nu}|P\rangle$

Scheme dependent!
4-loop calculation in MSbar

1 loop, 2 loop: YH, Rajan, Tanaka (2018)
3 loop: Tanaka (2019)
4 loop: Ahmed, Chen, Czakon (2022)

$$\left\langle \mathrm{Tr}\left([\Theta_g]_R^{\overline{\mathrm{MS}}}\right)\right\rangle_{\mathrm{P}} = \langle [O_F]_R\rangle_{\mathrm{P}}\,\left(-0.437676\,\alpha_s - 0.261512\,\alpha_s^2 - 0.183827\,\alpha_s^3 - 0.256096\,\alpha_s^4\right)$$
$$+ \langle [O_m]_R\rangle_{\mathrm{P}}\,\left(0.495149\,\alpha_s + 0.776587\,\alpha_s^2 + 0.865492\,\alpha_s^3 + 0.974674\,\alpha_s^4\right)\;,$$

$$\left\langle \mathrm{Tr}\left([\Theta_q]_R^{\overline{\mathrm{MS}}}\right)\right\rangle_{\mathrm{P}} = \langle [O_F]_R\rangle_{\mathrm{P}}\,\left(0.079578\,\alpha_s + 0.058870\,\alpha_s^2 + 0.021604\,\alpha_s^3 + 0.013675\,\alpha_s^4\right)$$
$$+ \langle [O_m]_R\rangle_{\mathrm{P}}\,\left(1\,+\,0.141471\,\alpha_s - 0.008235\,\alpha_s^2 - 0.064351\,\alpha_s^3 - 0.065869\,\alpha_s^4\right)$$

D-term—the last global unknown

$$\langle P'|T^{ij}|P\rangle \sim (\Delta^i\Delta^k - \delta^{ik}\Delta^2)D(t)$$

$D(t=0)$ is a conserved charge of the nucleon, similar to the magnetic moment

Fourier transform $\vec{\Delta} \to \vec{r}$ can be interpreted as `pressure' inside a nucleon

Polyakov (2003)

$$T^{ij}(r) = \left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right)s(r) + \delta^{ij}p(r)$$

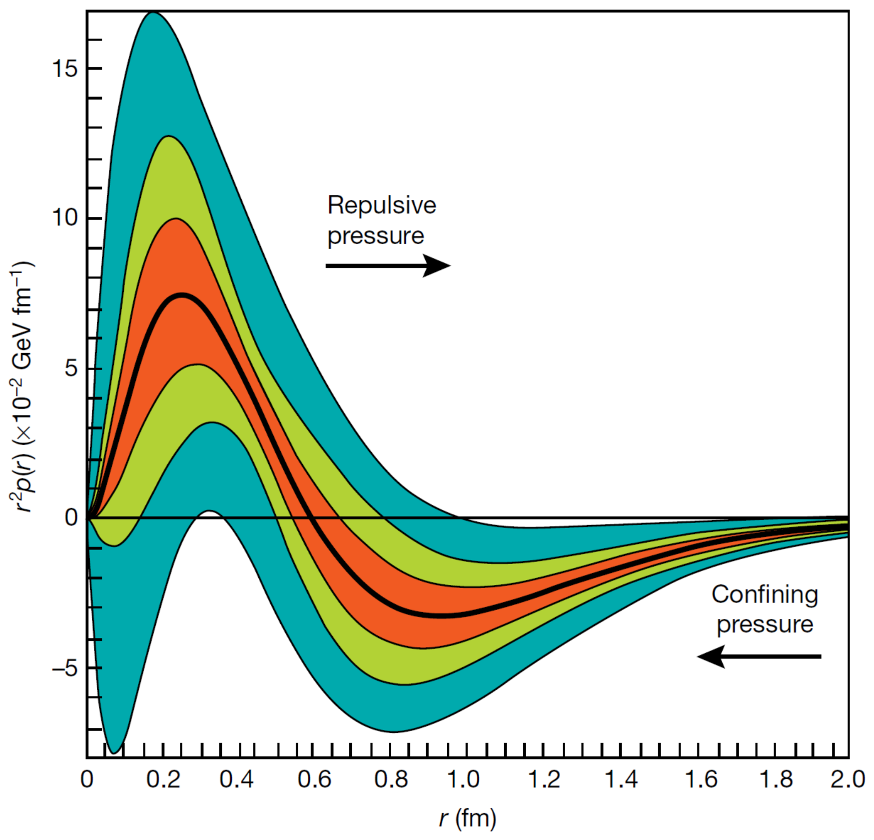
$$p(r) = \frac{1}{6M}\int \frac{d\mathbf{\Delta}}{(2\pi)^3} e^{i\mathbf{\Delta}\cdot\mathbf{r}} tD(t) \qquad D = M\int d^3r r^2 p(r)$$

Conjecture: All stable hadrons must have $D < 0$

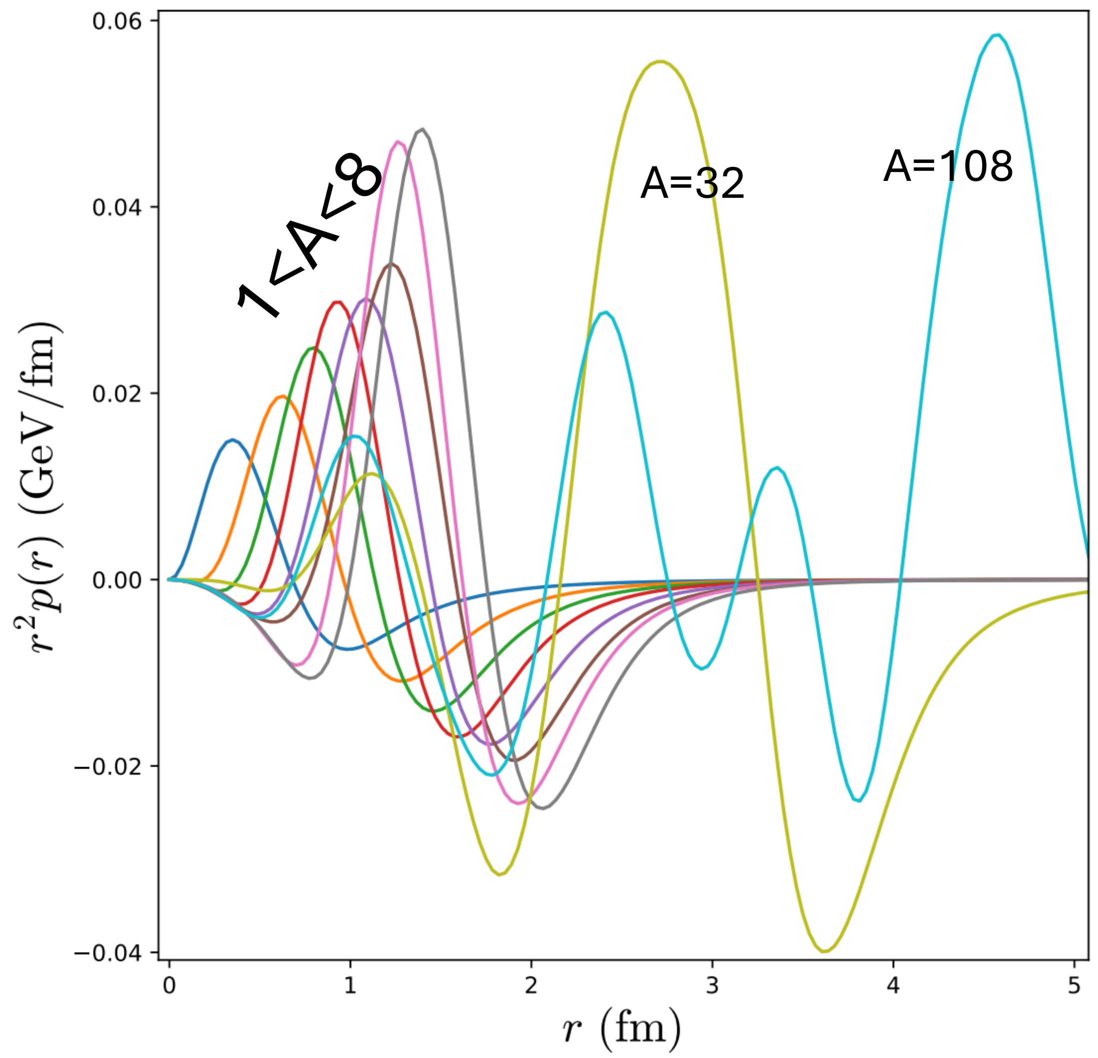
Analogy with continuous medium should be taken with a grain of salt.

` Pressure’ inside nucleon and nuclei

Burkert, Elouadrhiri, Girod (2018)



Martin-Caro, Huidobro, YH (2023),



Pion GFFs

Spin-0 hadron → 2 GFFs $\langle p'|T^{\mu\nu}|p\rangle = 2A(t)P^\mu P^\nu + \frac{D(t)}{2}(\Delta^\mu\Delta^\nu - \Delta^2 g^{\mu\nu})$

In the chiral limit of QCD, $D(0) = -1$

$$P^\mu = \frac{p^\mu + p'^\mu}{2}, \quad \Delta^\mu = p'^\mu - p^\mu$$

Proof: take the limit $p'^\mu \to 0$.

Then $\Delta^\mu = -p^\mu, \quad P^\mu = \frac{p^\mu}{2}, \quad t = \Delta^2 = 0$

Right hand side becomes $\frac{p^\mu p^\nu}{2}(A(0) + D(0))$

Left hand side vanishes due to soft pion theorem

$$\lim_{p'\to 0}\langle\pi_a(p')|T^{\mu\nu}|p\rangle = \frac{i}{f_\pi}\langle 0|[Q_5^a, T^{\mu\nu}]|p\rangle = 0$$

$$Q_5^a = \int d^3x \bar{\psi}\gamma^0\gamma_5\frac{\tau^a}{2}\psi$$ generator of chiral rotation

$$D(0) = -A(0) = -1$$

In real QCD with massive pion $D = -1 + \mathcal{O}\left(\frac{m_\pi^2}{f_\pi^2}\right)$

Soft pion theorem: A quick derivation

Pion decay constant definition $\langle 0|J^{\mu}_{5a}(x)|\pi_b(p)\rangle = -if_\pi p^\mu e^{-ip\cdot x}\delta_{ab}$

Take the divergence $\langle 0|\partial_\mu J^{\mu}_{5a}|\pi_b(p)\rangle = -f_\pi m_\pi^2 \delta_{ab}$

Pion interpolating field $\pi_a \sim -\frac{1}{f_\pi m_\pi^2}\partial_\mu J^{\mu}_{5a}$

LSZ reduction formula

$$\begin{aligned}\langle f, \pi_a(p)|\mathcal{O}(0)|i\rangle &= i\int d^4x e^{ip\cdot x}(\Box_x + m_\pi^2)\langle f|\mathrm{T}\{\pi_a(x)\mathcal{O}(0)\}|i\rangle \\ &= \frac{-i}{f_\pi m_\pi^2}\int d^4x e^{ip\cdot x}(\Box_x + m_\pi^2)\Big(\partial^x_\mu\langle f|\mathrm{T}\{J^{\mu}_{5a}(x)\mathcal{O}(0)\}|i\rangle - \delta(x^0)\langle f|[J^{0}_{5a}(x),\mathcal{O}(0)]|i\rangle\Big)\end{aligned}$$

Take the limit $p^\mu \to 0$

Ji decomposition (form factor version)

$$\langle P|J^z_{q,g}|P\rangle = \frac{1}{V}\langle P|\epsilon^{ij}\int d^3x x^i T^{0j}_{q,g}(x)|P\rangle$$

Belinfante energy momentum tensor (Poynting vector components)

$$= \frac{1}{V}\lim_{P'\to P}\langle P'|\epsilon^{ij}\int d^3x x^i T^{0j}_{q,g}(x)|P\rangle$$

$$= -i\lim_{\Delta\to 0}\epsilon^{ij}\frac{\partial}{\partial\Delta^i}\langle P'|T^{0j}_{q,g}(0)|P\rangle$$

$$\hat{O}(x) = e^{i\hat{P}x}\hat{O}(0)e^{-i\hat{P}x}$$

$$\Delta = P' - P$$

$$\bar{P} = \frac{P+P'}{2}$$

Parametrization (gravitational form factors)

$$\langle P'|T^{\alpha\beta}_q|P\rangle = \bar{u}(P')\left[A_q(t)\gamma^{(\alpha}\bar{P}^{\beta)} + B_q(t)\frac{\bar{P}^{(\alpha}i\sigma^{\beta)\lambda}\Delta_\lambda}{2m_N} + D_q(t)\frac{\Delta^\alpha\Delta^\beta - g^{\alpha\beta}\Delta^2}{4m_N} + \bar{C}_q(t)m_N g^{\alpha\beta}\right]u(P)$$

Use $\bar{u}(P+\Delta)\gamma^i u(P) \approx -i\epsilon^{ijk}\Delta^j\xi'\sigma^k\xi$

$$\Rightarrow \quad \frac{1}{2} = \sum_q J_q + J_g \qquad J_{q,g} = \frac{1}{2}(A_{q,g}(0) + B_{q,g}(0))$$

$A_q(0), A_g(0)$ Momentum fraction of proton carried by quarks/gluons

$$\langle P|T_q^{++}|P\rangle = 2A_q(0)(P^+)^2 \qquad T_q^{++} = \bar{\psi}\gamma^+ D^+ \psi$$

$$\int_1^1 dx x q(x) = A_q(0) \qquad q(x) = \int \frac{d\xi^-}{4\pi} e^{ixP^+\xi^-} \langle P|\bar{\psi}(0)\gamma^+\psi(\xi^-)|P\rangle$$

$$\sum_q A_q(0) + A_g(0) = 1 \quad \Rightarrow \quad \sum_q B_q(0) + B_g(0) = 0$$

Further decomposition in the quark part

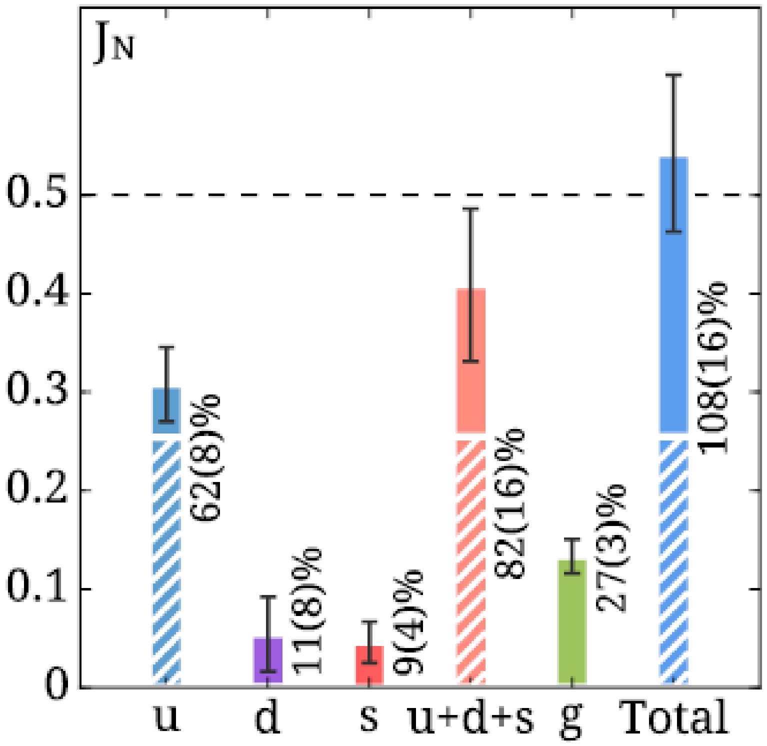
$$\bar{\psi} i\gamma^{(\mu} \overleftrightarrow{D}^{\nu)} \psi = \bar{\psi} i\gamma^{\mu} \overleftrightarrow{D}^{\nu} \psi - \frac{1}{4}\epsilon^{\mu\nu\rho\sigma}\partial_\rho(\bar{\psi}\gamma_\sigma\gamma_5\psi)$$

$$J_q = \frac{1}{2}\Delta\Sigma + L^q_{kin}$$

kinetic OAM (features covariant derivative)

Gluon part J_g cannot be decomposed into spin and OAM in terms of gauge invariant, local operators.

All the operators involved are local and gauge invariant
→ calculable on a lattice



Alexandrou et al.

Can we measure GFFs in experiments?

- Introduced theoretically in the 60s.
- Received far less attention than EM form factors, not because they are less interesting/important.
- The obvious reason: We cannot measure them directly!

One-graviton exchange cross section $\frac{d\sigma}{dt} \sim G_N^2 \frac{s^2}{t^2}$

Newton constant $G_N \sim 1/M_P^2$ Planck mass $M_P \sim 10^{19}\ \mathrm{GeV}$

- There are, however, indirect ways to measure them.

Appendix 1: Gordon identities

For any matrix Γ in Dirac space

$$\bar{u}(p')\Gamma u(p) = \frac{1}{2m}\bar{u}'\left(\{\not{P},\Gamma\} + \frac{1}{2}[\not{\Delta},\Gamma]\right)u(p)$$

$$P = \frac{p+p'}{2}$$

$$\Delta = p' - p$$

$$\bar{u}(p')\gamma^\mu u(p) = \bar{u}(p')\left[\frac{P^\mu}{m} + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m}\right]u(p)$$

$$\bar{u}(p')\gamma^\mu\gamma_5 u(p) = \bar{u}(p')\left[\frac{\Delta^\mu\gamma_5}{2m} + \frac{i\sigma^{\mu\nu}P_\nu\gamma_5}{m}\right]u(p)$$

$$\frac{\Delta^\mu}{2}\bar{u}(p')u(p) = -\bar{u}(p')i\sigma^{\mu\nu}P_\nu u(p)$$

$$\bar{u}(p')P^\mu\gamma_5 u(p) = -\bar{u}(p')\frac{i\sigma^{\mu\nu}\Delta_\nu}{2}\gamma_5 u(p)$$

Exercise: derive these

Appendix 2: PT transformation

Parity & time-reversal symmetry: powerful method to constrain the matrix element of local and nonlocal operators.

$$\langle P'S'|\hat{O}|PS\rangle = \langle P, -S|(PT\hat{O}T^{-1}P^{-1})^\dagger|P', -S'\rangle$$

Quark bilinear $\mathcal{O} = \bar{\psi}\Gamma\psi$ $\qquad PT\psi(x)T^{-1}P^{-1} = \gamma^0\gamma^1\gamma^3\psi(-x)$

Peskin, (3.126), (3.139)

$$\bar{\psi}\Gamma\psi \to \left(\bar{\psi}\gamma^3\gamma^1\gamma^0\Gamma^*\gamma^0\gamma^1\gamma^3\psi\right)^\dagger$$

$$\Gamma = \begin{pmatrix} \gamma^\mu \\ \gamma^\mu\gamma_5 \\ i\sigma^{\mu\nu} \\ \gamma_5 \\ i\sigma^{\mu\nu}\gamma_5 \end{pmatrix} \Rightarrow \begin{pmatrix} \gamma^\mu \\ -\gamma^\mu\gamma_5 \\ -i\sigma^{\mu\nu} \\ \gamma_5 \\ -i\sigma^{\mu\nu}\gamma_5 \end{pmatrix}$$

$$\langle P'|\bar{\psi}\gamma^\mu\psi|P\rangle = \bar{u}(P')\left[\gamma^\mu A + (P^\mu + P'^\mu)B + (P^\mu - P'^\mu)\cancel{C}\right]u(P)$$

$$||$$

$$\langle P|\bar{\psi}\gamma^\mu\psi|P'\rangle$$

$$\Gamma = \begin{pmatrix} \gamma^\mu \\ \gamma^\mu\gamma_5 \\ i\sigma^{\mu\nu} \\ \gamma_5 \\ i\sigma^{\mu\nu}\gamma_5 \end{pmatrix} \Rightarrow \begin{pmatrix} \gamma^\mu \\ -\gamma^\mu\gamma_5 \\ -i\sigma^{\mu\nu} \\ \gamma_5 \\ -i\sigma^{\mu\nu}\gamma_5 \end{pmatrix}$$

$$\langle P'|\bar{\psi}\gamma^\mu\gamma_5\psi|P\rangle = \bar{u}(P')\left[\gamma^\mu\gamma_5\tilde{A} + (P^\mu + P'^\mu)\gamma_5\cancel{\tilde{B}} + (P^\mu - P'^\mu)\gamma_5\tilde{C}\right]u(P)$$

$$||$$

$$-\langle P|\bar{\psi}\gamma^\mu\gamma_5\psi|P'\rangle$$

PT transformation of nonlocal quark bilinear

$$\langle p', s'|\bar{\psi}(0) \begin{pmatrix} \gamma^\mu \\ \gamma^\mu\gamma_5 \\ i\sigma^{\mu\nu} \\ \gamma_5 \\ i\sigma^{\mu\nu}\gamma_5 \end{pmatrix} W_{0x}\psi(x)|p, s\rangle = \langle p, -s|\bar{\psi}(-x) \begin{pmatrix} \gamma^\mu \\ -\gamma^\mu\gamma_5 \\ -i\sigma^{\mu\nu} \\ \gamma_5 \\ -i\sigma^{\mu\nu}\gamma_5 \end{pmatrix} W_{-x0}\psi(0)|p', -s'\rangle$$

PT transformation of spinor bilinear

$$\bar{u}(p's') \begin{pmatrix} \gamma^\mu \\ \gamma^\mu\gamma_5 \\ i\sigma^{\mu\nu} \\ \gamma_5 \\ i\sigma^{\mu\nu}\gamma_5 \end{pmatrix} u(p, s) = \bar{u}(p, -s) \begin{pmatrix} \gamma^\mu \\ -\gamma^\mu\gamma_5 \\ -i\sigma^{\mu\nu} \\ \gamma_5 \\ -i\sigma^{\mu\nu}\gamma_5 \end{pmatrix} u(p', -s')$$