# イオントラップを用いたDM、重力波探索

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from Kobe University

Refs: AI, Ryuichiro Kitano, Wakutaka Nakano, Ryoto Takai (JHEP 02 (2024) 124, arXiv: 2311.11632 [hep-ph]).

# 量子センシングに関連したこれまでの研究

- **マグノンの量子非破壊測定を用いたアクシオンDM探索**

  T.Ikeda, AI, K.Miuchi, J.Soda, H.Kurashige, Y.Shikano, Phys.Rev.D 105 (2022) 10, 102004.

- **マグノンを用いた高周波重力波観測** $(\mathrm{GHz} \sim 10\mathrm{GHz})$

  AI, T.Ikeda, K.Miuchi, J.Soda, Eur.Phys.J.C 80 (2020) 3, 179,
  AI, J.Soda, Eur.Phys.J.C 80 (2020) 6, 545,
  AI, J.Soda, Eur.Phys.J.C 83 (2023) 8, 766.

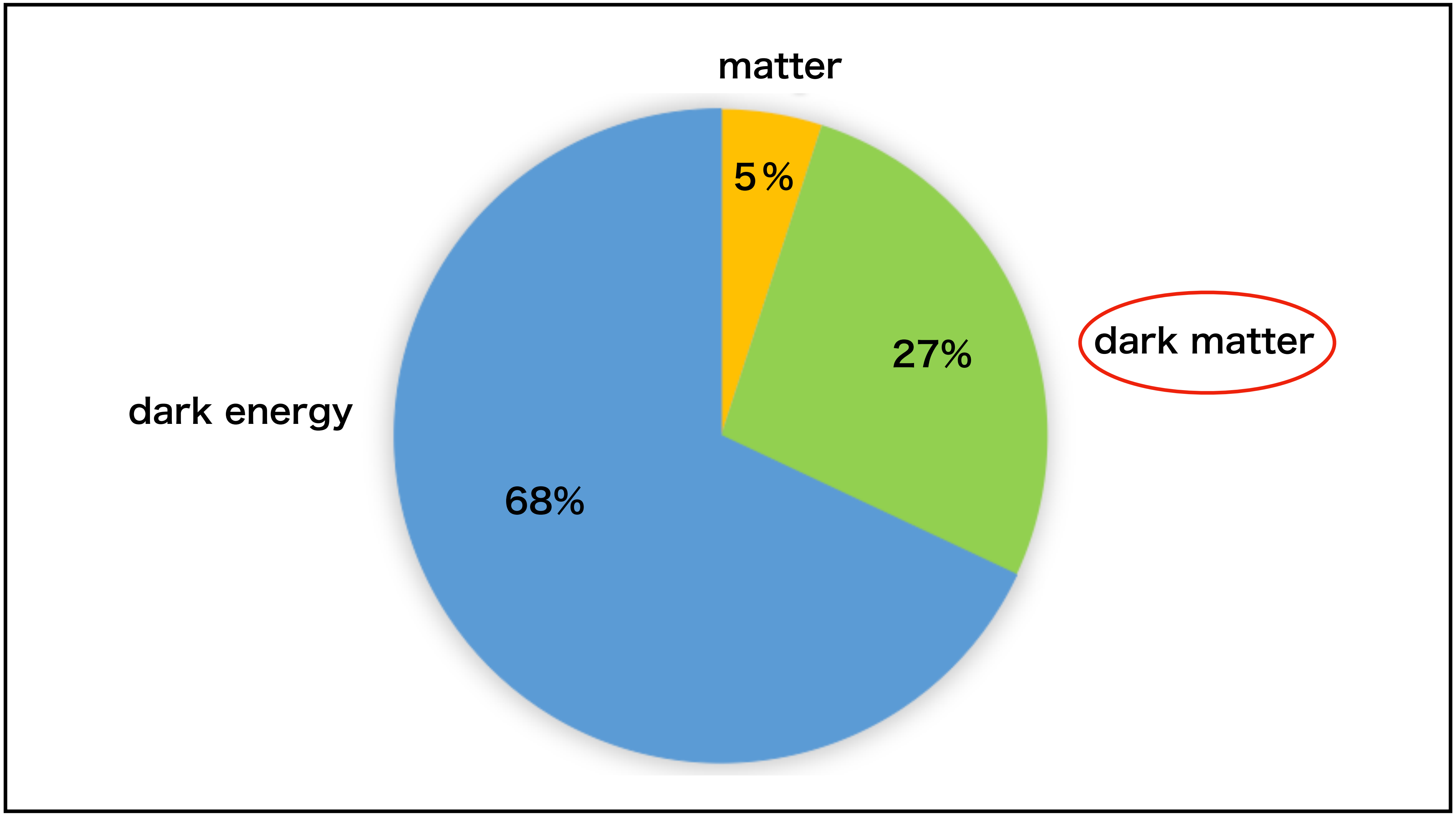
- **One-electron quantum cyclotron を用いた高周波重力波観測** $(10\mathrm{GHz} \sim 100\mathrm{GHz})$

  AI, R.Kitano, JCAP 04 (2024) 068.

- **イオントラップにおける、量子エンタングルメントを利用したDM(アクシオン、ダークフォトン)探索**

  AI, Ryuichiro Kitano, Wakutaka Nakano, Ryoto Takai, JHEP 02 (2024) 124.

# Introduction




what is the dark matter?

- WIMP ⟶ Particle DM (> eV)
- Light DM such as axion and dark photon (< eV) ⟶ Wave DM

# Talk plan

1. Light DM - Axion DM and dark photon DM

2. Ion traps as light DM detectors

3. Entanglement of ions for light DM detection

4. Application to high-frequency GWs

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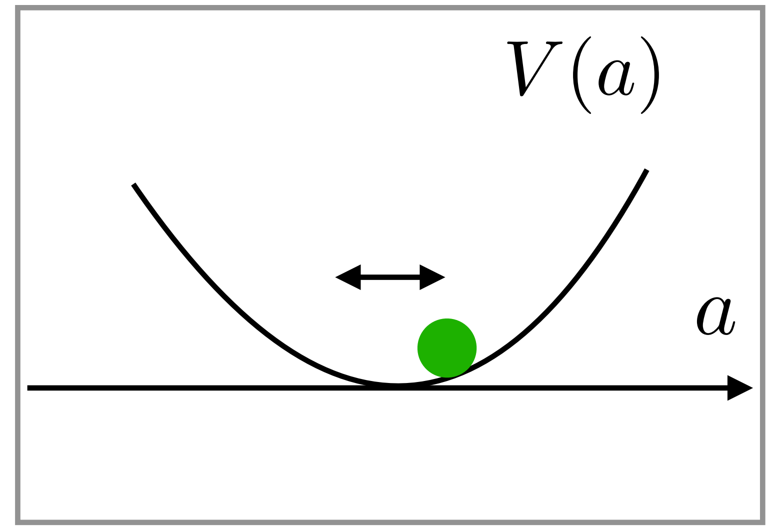
# Axion DM

Axions can behave as cold DM in the universe
if it oscillates around the bottom of the potential




$$a(t) = a_0 \cos(m_a t - \phi_a)$$

corresponding to the abundance of the axion DM

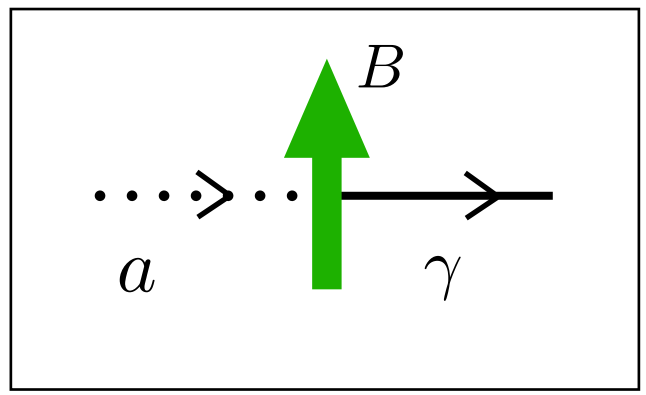
determined by the axion mass $(\omega = m_a)$

# Axion DM

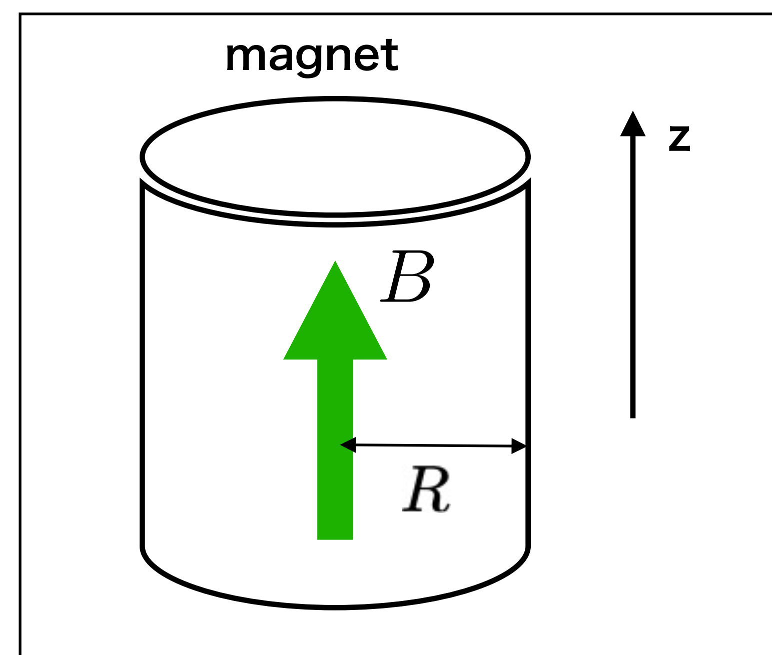
Axions can couple to photons through the Chern-Simon coupling

$$\boxed{-\frac{1}{4} g_{a\gamma} a F^{\mu\nu} \tilde{F}_{\mu\nu} = g_{a\gamma} a \vec{E} \cdot \vec{B}}$$

In the presence of background magnetic field, axions can convert into photons




In a cylindrical magnet, the axion DM induced electric field is




$$\boxed{E_{a,z} = \epsilon_a \sqrt{2\rho_{\mathrm{DM}}} \sin\left(m_a t - \phi_a\right)}$$

where $$\epsilon_a = \frac{B_z}{2} \frac{g_{a\gamma}}{m_a} (m_a R)^2 \left[ \left( \log \frac{m_a R}{2} + \gamma - \frac{1}{2} \right)^2 + \left( \frac{\pi}{2} \right)^2 \right]^{1/2}$$

# Dark photon DM

Dark photons also can behave as cold DM as well as the axion DM.

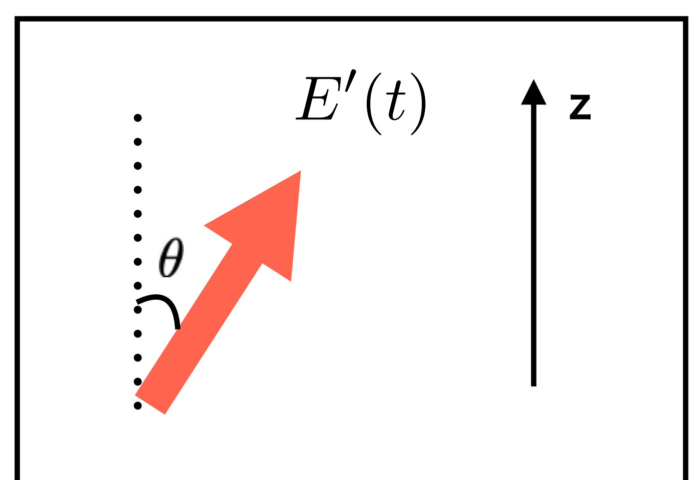
| | |
|---|---|
| axion DM | oscillating scalar field $a(t) = a_0 \cos(m_a t - \phi_a)$ |
| dark photon DM | oscillating dark electric field<br>$\boxed{\vec{E}'(t) = \vec{E}'_0 \sin\left(m_{\mathrm{DP}} t - \phi_{\mathrm{DP}}\right)}$<br>corresponding to the abundance of the dark photon DM<br>frequency is determined by the dark photon mass |

Dark photons can couple to photons through gauge kinetic mixing $\boxed{\frac{1}{2}\epsilon F^{\mu\nu} F'_{\mu\nu}}$

Induced electric field by the dark electric field is

$$\boxed{E_{\mathrm{DP},z} = \epsilon_{\mathrm{DP}} \sqrt{2\rho_{\mathrm{DM}}} \sin\left(m_{\mathrm{DP}} t - \phi_{\mathrm{DP}}\right)}$$

where $\epsilon_{\mathrm{DP}} = \epsilon \cos\theta$



# Light DM induced electric fields

light (wave) DM such as axion DM and dark photon DM can induces weak electric fields

$$E_{a,z} = \epsilon_a \sqrt{2\rho_{\rm DM}} \sin\left(m_a t - \phi_a\right) \;\; \propto g_{a\gamma}$$

$$E_{{\rm DP},z} = \epsilon_{\rm DP} \sqrt{2\rho_{\rm DM}} \sin\left(m_{\rm DP} t - \phi_{\rm DP}\right) \;\; \propto \epsilon$$

DM induced very weak electric fields

→ It is possible to search for light DM by detecting induced weak electric fields
ex.) Josephson parametric amplifier

→ In this talk, we demonstrate that ion traps can be utilized for light DM search

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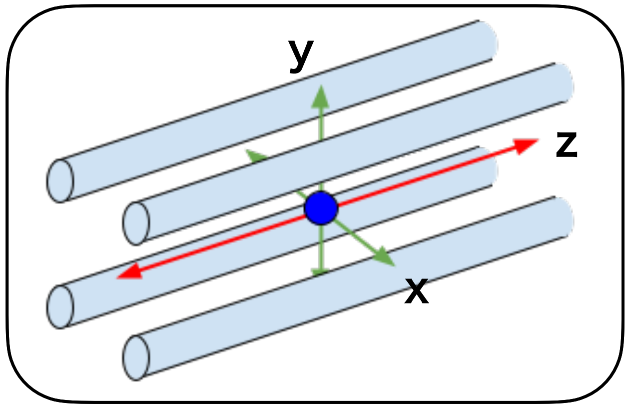
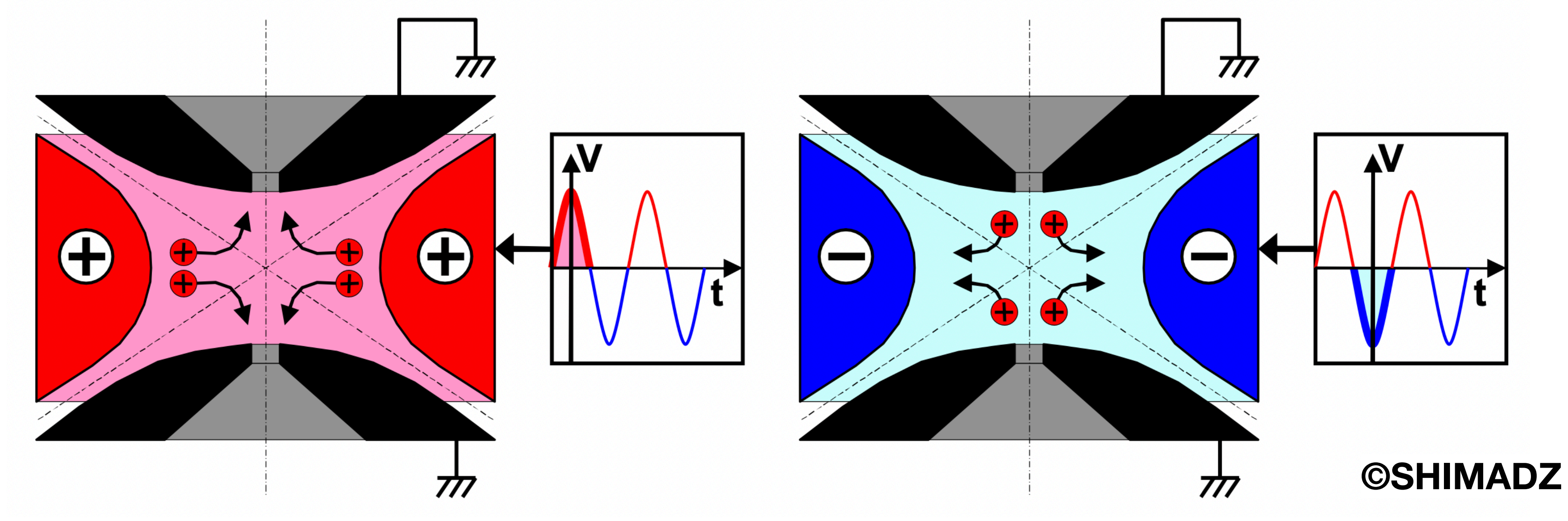
# Paul ion trap

Ion traps are expected to work as qubits, in particular, for implementation in quantum computers.
ex.) Paul traps, Penning traps

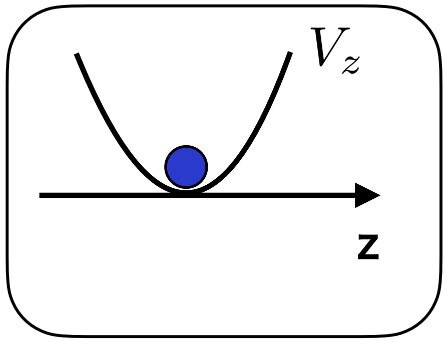
## Paul traps




In x-y plane, ions are trapped by quadrupole electric potential V(t)



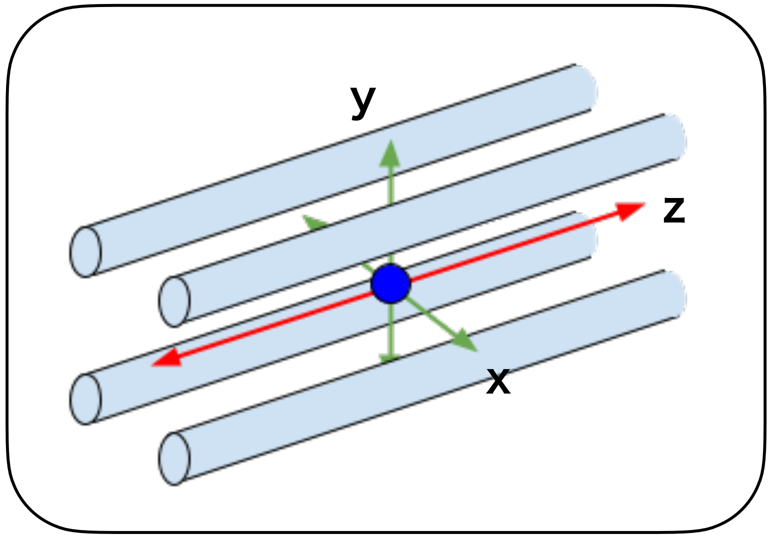

Along z-axis, a static electric field is applied and vibration modes (harmonic oscillator) appear, the Hamiltonian is $H_0 = \omega_z a^\dagger a$




→ The ground $|0\rangle$ and first excited $|1\rangle$ states behave as a qubit

# Effect of DM on a single ion

An ion has a vibration mode along z-axis.
The Hamiltonian is given by $H_0 = \omega_z a^\dagger a$




If there exists an electric field, it couples with the vibration mode as

$$\begin{aligned} H_X &= ezE_{X,z} \\ &\simeq \frac{eE_{X,z}}{\sqrt{2m_{\mathrm{ION}}\omega_z}}(a^\dagger e^{i\omega_z t} + a e^{-i\omega_z t}) \\ &= e\epsilon_X \sin(m_X t - \phi_X)\sqrt{\frac{\rho_{\mathrm{DM}}}{m_{\mathrm{ION}}\omega_z}}(a^\dagger e^{i\omega_z t} + a e^{-i\omega_z t}) \qquad \left(X = a \text{ or DP}\right) \end{aligned}$$

This system is just a forced harmonic oscillator and exactly solvable.
The solution is given by the displacement operator, $D(\beta) = \exp\left(\beta a^\dagger - \beta^* a\right)$,

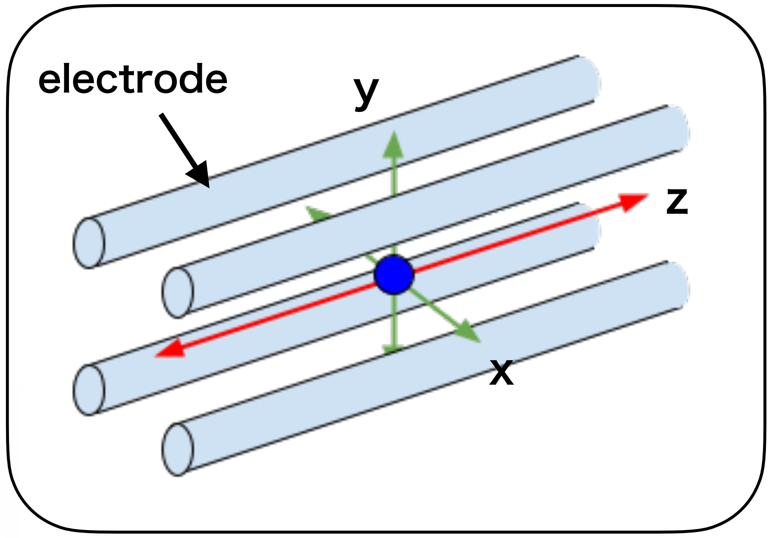
$$|\Phi(T)\rangle = D(\beta)\,|0\rangle \quad \text{where} \quad \beta = \alpha_X T, \quad \alpha_X = \frac{e\epsilon_X e^{i\phi_X}}{2}\sqrt{\frac{\rho_{\mathrm{DM}}}{m_{\mathrm{ION}} m_X}}.$$

However, in a realistic situation,
we need to take into account of noises which also excite the vibrational qubit.

# Heating noises

The main noise is the thermal photons from electrodes.

The total Hamiltonian including the thermal noise is given by




$$\boxed{H_{\mathrm{total}} = \omega_z a^\dagger a + \sum_j \omega(k_j) b_j^\dagger b_j + H_X + H_{\mathrm{heat}}}$$

where

$$\left( \begin{aligned} H_X &= e\epsilon_X \sin(m_X t - \phi_X) \sqrt{\frac{\rho_{\mathrm{DM}}}{m_{\mathrm{ION}}\omega_z}} (a^\dagger e^{i\omega_z t} + a e^{-i\omega_z t}) \\ H_{\mathrm{heat}} &= \sum_j g_j \left( a^\dagger b_j e^{-i(\omega_z - \omega(k_j))t} + a b_j^\dagger e^{i(\omega_z - \omega(k_j))t} \right) \end{aligned} \right.$$

From the total Hamiltonian, by tracing out the noise, one can calculate the excitation rate from $|0\rangle$ to $|1\rangle$.

$$\boxed{\text{(Excitation rate)} \simeq \underbrace{\dot{n} t}_{\text{Noise}} + \underbrace{|\alpha_X|^2 t^2}_{\text{DM signal}}}$$

$$\left( \alpha_X = \frac{e\epsilon_X}{2} \sqrt{\frac{\rho_{\mathrm{DM}}}{m_{\mathrm{ION}} m_X}} \right)$$

$\dot{n}$ : heating rate by the noise

# Sensitivity of a single ion

For a measurement during $T$ , SNR (Signal to Noise Ration) is

$$\boxed{\mathrm{SNR} = \frac{|\alpha_X|^2 T^2}{\sqrt{\dot{\bar{n}} T}} = \frac{|\alpha_X|^2 T^{3/2}}{\sqrt{\dot{\bar{n}}}}} \qquad \left( \alpha_X = \frac{e\epsilon_X}{2}\sqrt{\frac{\rho_{\mathrm{DM}}}{m_{\mathrm{ION}} m_X}} \right)$$

After $T_{\mathrm{total}}/T$ times measurements, the SNR is improved as

$$\boxed{\mathrm{SNR} = \sqrt{\frac{T_{\mathrm{total}}}{T}} \times \frac{|\alpha_X|^2 T^{3/2}}{\sqrt{\dot{\bar{n}}}} = \frac{|\alpha_X|^2 T T_{\mathrm{total}}^{1/2}}{\sqrt{\dot{\bar{n}}}}}$$

Then, the 95% C.L. sensitivity of a single ion in a Paul trap to the axion-photon coupling $g_{a\gamma}$ is

$$\begin{aligned} g_{a\gamma} = 4.4 \times 10^{-11}\ \mathrm{GeV}^{-1} \times \left(\frac{\dot{\bar{n}}}{0.1\ \mathrm{s}^{-1}}\right)^{1/4} \left(\frac{T_{\mathrm{total}}}{1\ \mathrm{day}}\right)^{-1/4} \left(\frac{T}{0.4\ \mathrm{s}}\right)^{-1/2} \left(\frac{R}{3\ \mathrm{m}}\right)^{-2} \\ \times \left(\frac{B_z}{100\ \mathrm{mT}}\right)^{-1} \left(\frac{m_{\mathrm{ION}}}{37\ \mathrm{GeV}}\right)^{1/2} \left(\frac{m_a}{10\ \mathrm{neV}}\right)^{-1/2} \left(\frac{\rho_{\mathrm{DM}}}{0.45\ \mathrm{GeV\ cm}^{-3}}\right)^{-1/2} \end{aligned}$$

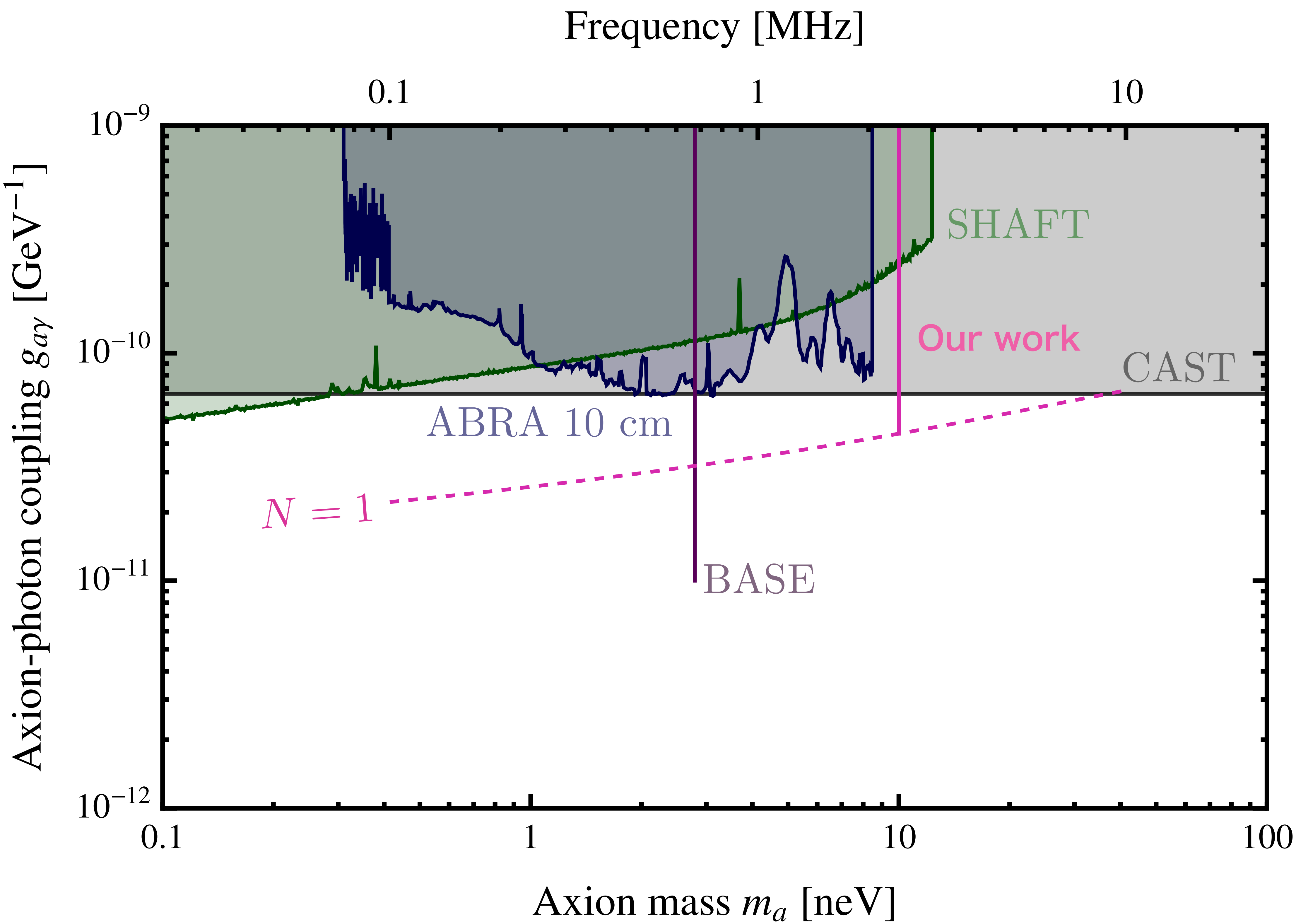
and to the gauge kinetic mixing $\epsilon$ is

$$\begin{aligned} \epsilon = 6.4 \times 10^{-12} \times \left(\frac{\dot{\bar{n}}}{0.1\ \mathrm{s}^{-1}}\right)^{1/4} \left(\frac{T_{\mathrm{total}}}{1\ \mathrm{day}}\right)^{-1/4} \left(\frac{T}{0.4\ \mathrm{s}}\right)^{-1/2} \\ \times \left(\frac{m_{\mathrm{ION}}}{37\ \mathrm{GeV}}\right)^{1/2} \left(\frac{m_{\mathrm{DP}}}{10\ \mathrm{neV}}\right)^{1/2} \left(\frac{\rho_{\mathrm{DM}}}{0.45\ \mathrm{GeV\ cm}^{-3}}\right)^{-1/2} \end{aligned}$$

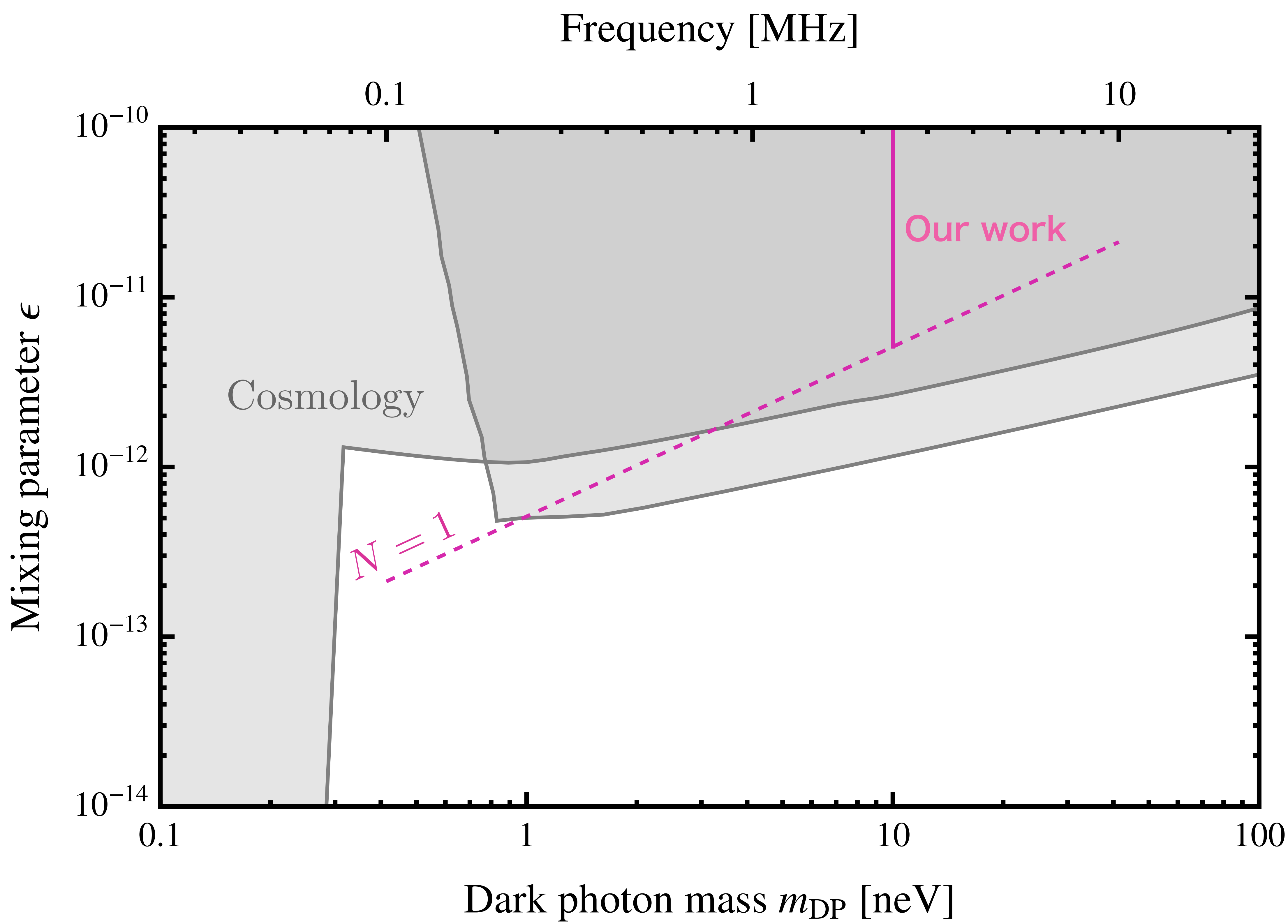
( ※ The coherence time of DM is

$$T_{\mathrm{coh}} = \frac{2\pi}{m_{\mathrm{DM}} v_{\mathrm{DM}}^2} \sim 0.4\ \mathrm{s} \times \left(\frac{10\ \mathrm{neV}}{m_{\mathrm{DM}}}\right)$$
)

# Sensitivity of a single ion



# Sensitivity of a single ion



# Talk plan

1. Light DM - Axion DM and dark photon DM

2. Ion traps as light DM detectors

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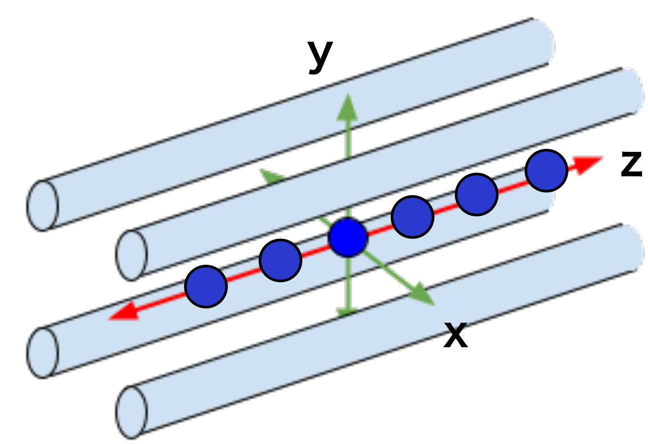
# Quantum enhancement with entangled ions

- Using multiple $N$ ions, the excitation rate is enhanced by $N$
- On the other hand, noise is enhanced by $\sqrt{N}$

$N$ ions are trapped




SNR is enhanced by $$\mathrm{SNR} = \underline{\sqrt{N}} \times \mathrm{SNR}_1$$

statistical enhancement

It is possible that quantum entangled ions allow enhancement of the signal by $N^2$

SNR is enhanced by $$\mathrm{SNR} = \underline{N^{3/2}} \times \mathrm{SNR}_1$$

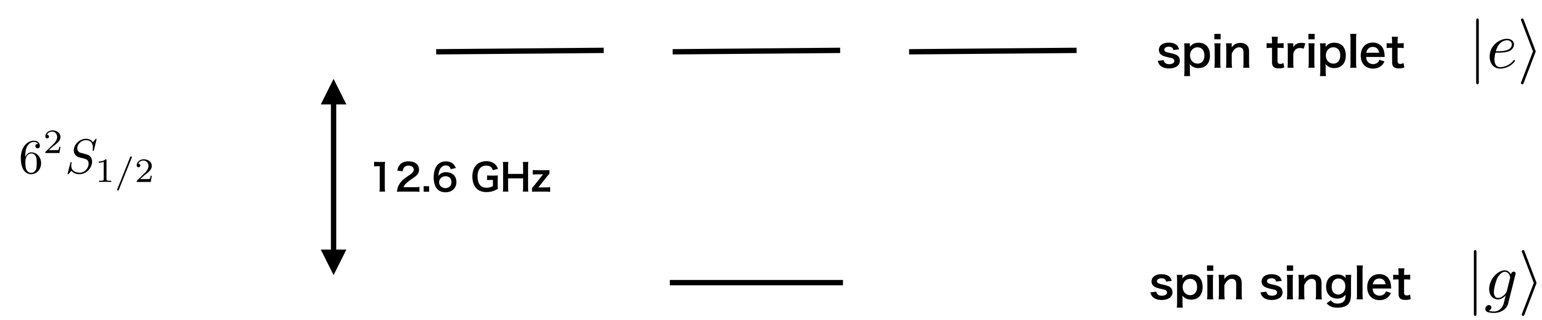
quantum enhancement

# Spin qubits

In a Paul trap, there is another type of qubit called a spin qubit in addition to the vibrational qubit.
A spin qubit is composed by internal states of an ion such as $^{171}\mathbf{Yb}^+$ and $^{40}\mathrm{Ca}^+$.

For example, $^{171}\mathbf{Yb}^+$ has hyperfine structure due to spin-spin interaction

The spin qubit of $^{171}\mathbf{Yb}^+$




In general, states of an ion is described as $|g,0\rangle$ , $|e,0\rangle$ , $|g,1\rangle$ , $|e,1\rangle$ .

Thanks to the existence of spin qubits, through sideband resonances, one can control the qubit states.

→ We can create entangled vibrational qubit states in a Paul trap

# Quantum enhancement with entangled ions

1. We first prepare a GHZ (Greenberger- Horne-Zeilinger) state where spin qubits are entangled

$$\boxed{|g,g,\ldots,g,0\rangle \to |\Psi_{\rm GHZ}\rangle = \frac{1}{\sqrt{2}}\left(|g,g,\ldots,g,0\rangle + |e,e,\ldots,e,0\rangle\right)}$$

2. We create an entanglement among vibrational qubits by the Hadamard gate and red sideband resonance

$$\boxed{|\Psi_2\rangle = \frac{1}{\sqrt{2}}\left[\left(\frac{|g,0\rangle+|g,1\rangle}{\sqrt{2}}\right)^{\otimes N} + \left(\frac{|g,0\rangle-|g,1\rangle}{\sqrt{2}}\right)^{\otimes N}\right]}$$

$$D(\beta) = e^{\beta a^\dagger - \beta^* a} = e^{i\beta_{\rm i}\sigma^1_{\rm vib} - i\beta_{\rm i}\sigma^2_{\rm vib}} \simeq \left(1 - i\beta_{\rm r}\sigma^2_{\rm vib}\right) e^{i\beta_{\rm i}\sigma^1_{\rm vib}} \qquad \begin{pmatrix} \sigma^1_{\rm vib} = a^\dagger + a \\ \sigma^2_{\rm vib} = ia^\dagger - ia \end{pmatrix}$$

3. DM acts on the state

$$\boxed{\begin{aligned}|\Psi_3\rangle = D(\beta)\,|\Psi_2\rangle \simeq \frac{1}{\sqrt{2}}\Bigg[& e^{iN\beta_{\rm i}}\left(\frac{|g,0\rangle+|g,1\rangle}{\sqrt{2}} + \beta_{\rm r}\frac{|g,0\rangle-|g,1\rangle}{\sqrt{2}}\right)^{\otimes N} \\ & + e^{-iN\beta_{\rm i}}\left(\frac{|g,0\rangle-|g,1\rangle}{\sqrt{2}} + \beta_{\rm r}\frac{|g,0\rangle+|g,1\rangle}{\sqrt{2}}\right)^{\otimes N}\Bigg]\end{aligned}}$$

( where we have used $\sigma^1_{\rm vib}\frac{(|g,0\rangle \pm |g,1\rangle)}{\sqrt{2}} = \pm\frac{(|g,0\rangle \pm |g,1\rangle)}{\sqrt{2}}$ and $\sigma^2_{\rm vib}\frac{(|g,0\rangle \pm |g,1\rangle)}{\sqrt{2}} = i\frac{(|g,0\rangle \mp |g,1\rangle)}{\sqrt{2}}$ )

# Quantum enhancement with entangled ions

4. By applying the inverse operation of the red side band resonance and the Hadamard gate, we obtain

$$|\Psi_4\rangle = \frac{1}{\sqrt{2}}\left[e^{iN\beta_i}\left(|g,0\rangle + \beta_r|e,0\rangle\right)^{\otimes N} + e^{-iN\beta_i}\left(|e,0\rangle + \beta_r|g,0\rangle\right)^{\otimes N}\right]$$

5. Operation of the CNOT gate and the Hadamard gate only for the first spin qubit yields

$$|\Psi_5\rangle \approx |g,g,g,\ldots,g,0\rangle - iN\beta_i\,|e,g,g,\ldots,g,0\rangle + \cdots$$

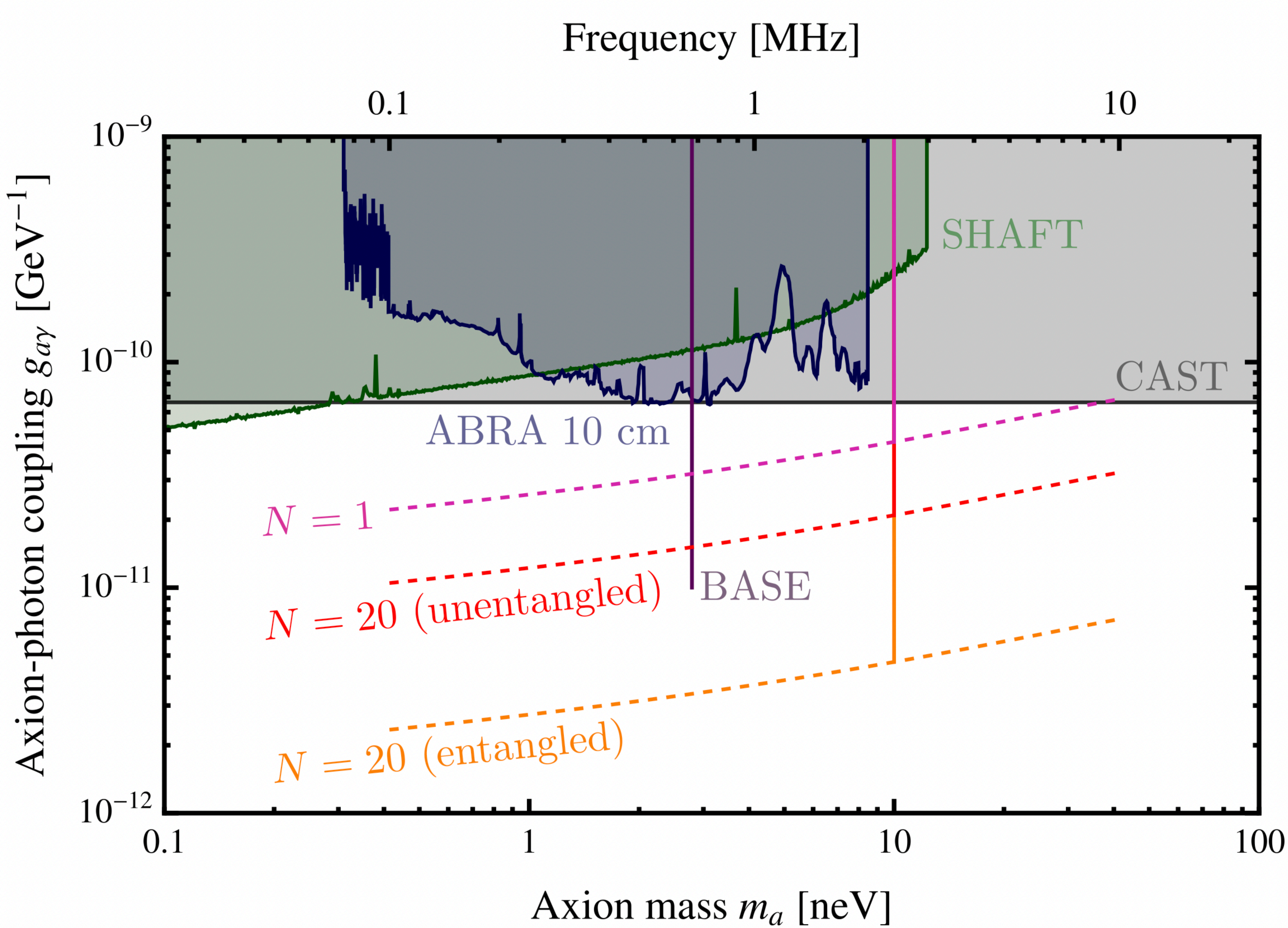
6. We observe the $|e,g,g,\ldots,g,0\rangle$ state, then the DM signal is enhanced by $N^2$

$$|\langle e,g,g,\ldots,g,0|\Psi_5\rangle|^2 = N^2\beta_i^2$$

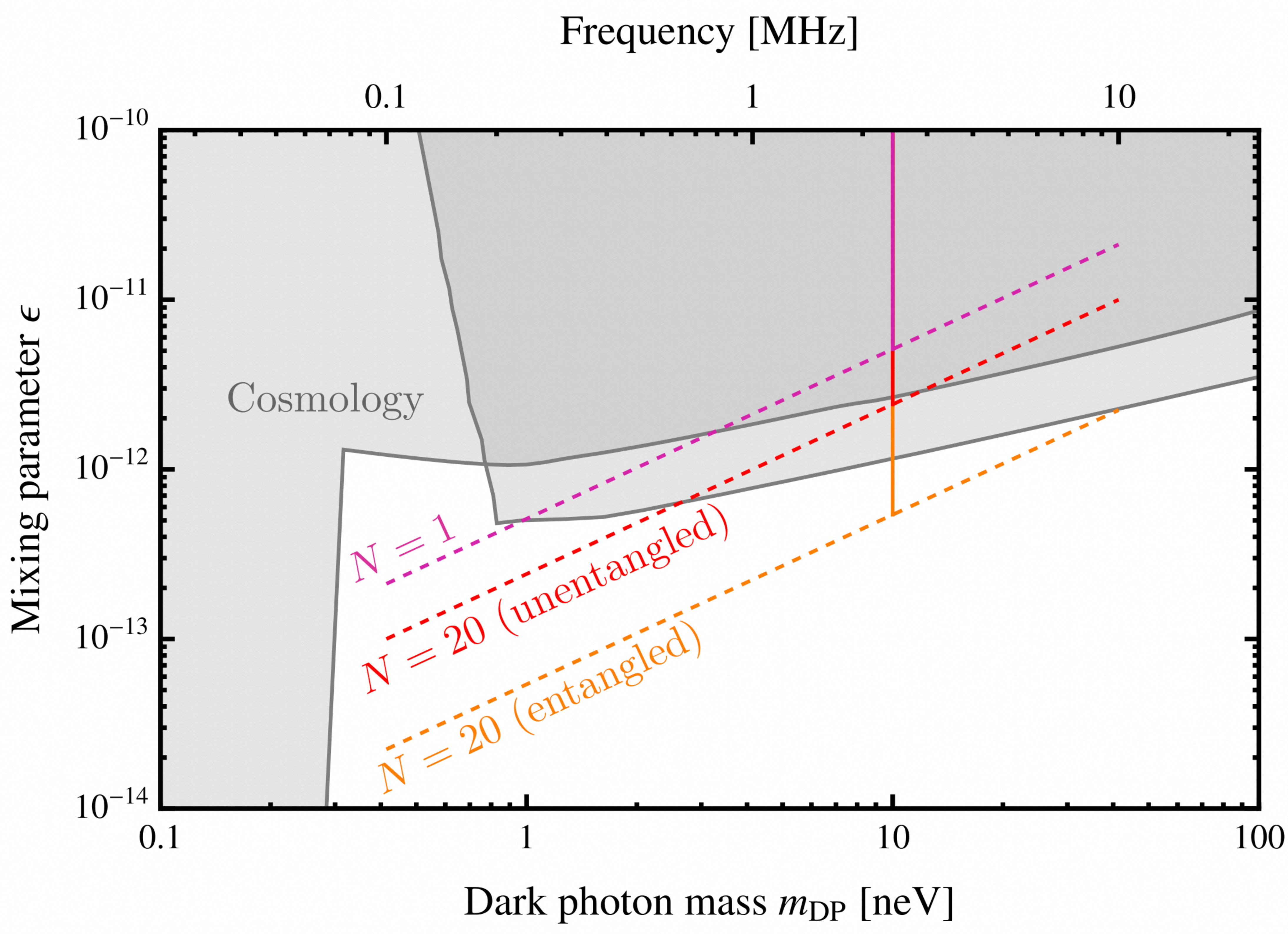
On the other hand, the excitation by noises is only enhanced by $\sqrt{N}$ , since the noises are expected to be incoherent.

SNR is enhanced by $\mathrm{SNR} = N^{3/2} \times \mathrm{SNR}_1$

# Quantum enhancement with entangled ions



# Quantum enhancement with entangled ions



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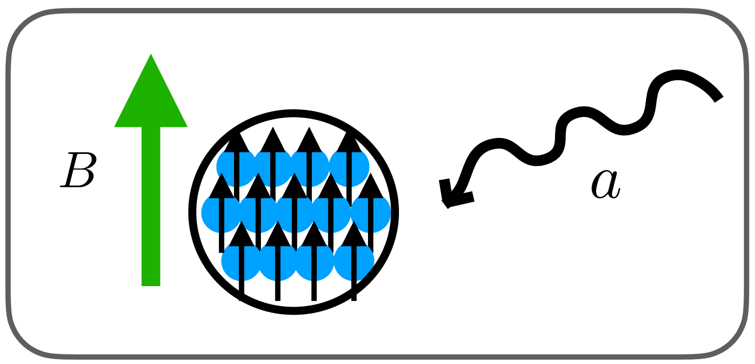
# High-frequency GW detection

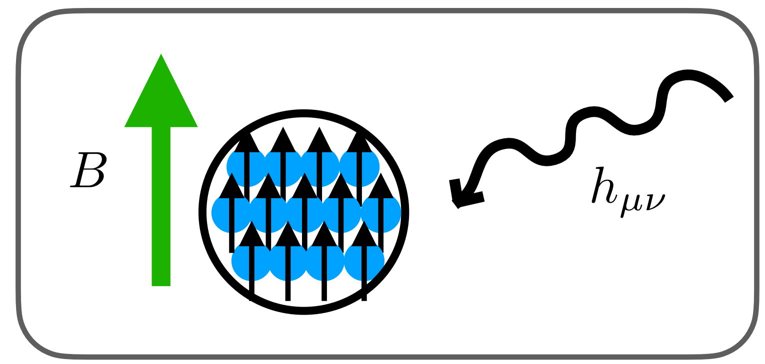
**Recently, detection methods for high frequency GWs are well studied theoretically and experimentally.**

**In particular, "Axion-graviton correspondence" draws much attention.**

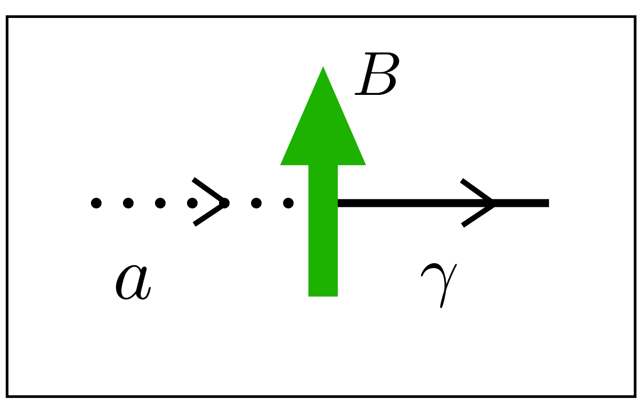
- **magnon excitation by axion DM** ⟷ **magnon excitation by GWs**

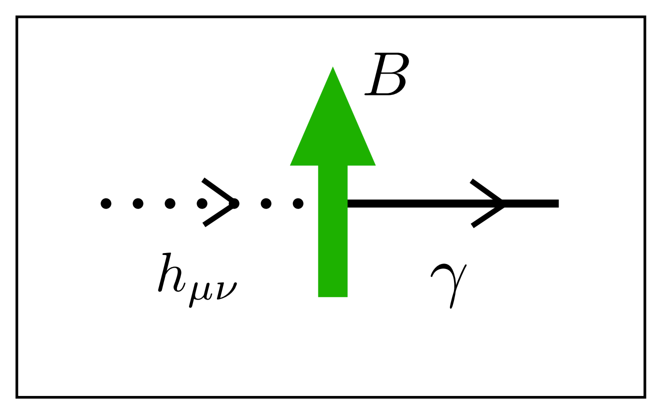
(AI, T.Ikeda, K.Miuchi, J.Soda (2020), AI, J.Soda (2020))




- **axion to photon (resonant) conversion** ⟷ **graviton to photon (resonant) conversion**




$$\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \to -\frac{1}{4}F_{\mu\nu}F_{\alpha\beta}g^{\mu\alpha}h^{\nu\beta}\right)$$

→ **We can utilize axion DM experiments for detecting high-frequency GWs**

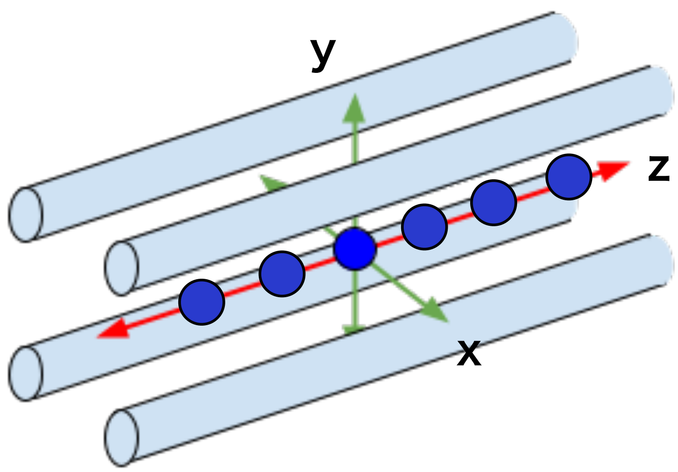
$\sim \mu\mathrm{eV}$ $\sim \mathrm{GHz}$

# GW detection with ion traps

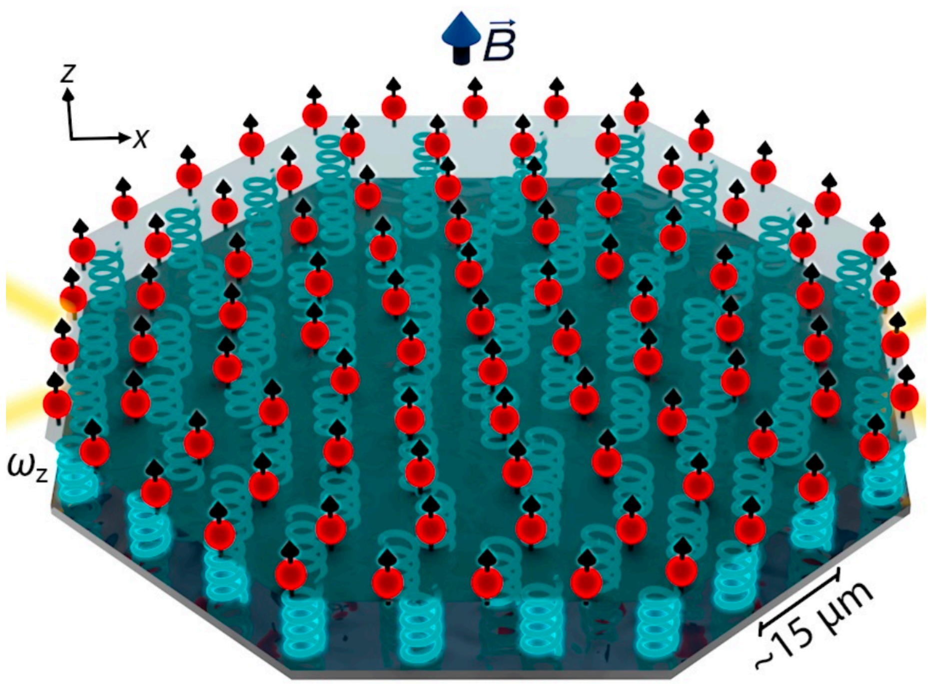
$$H_{\rm int} = \frac{1}{2}T^{\mu\nu}\delta g_{\mu\nu} \simeq \frac{1}{2}T^{00}\delta g_{00} = \frac{1}{2}m_{\rm ion}R_{0k0l}x^k x^l = \frac{1}{4}m_{\rm ion}h_{kl}\omega_g^2 x^k x^l$$

heavier and bigger is better

## Paul ion trap




## Penning ion trap




| | Paul ion trap | Penning ion trap |
|---|---|---|
| size: | $1\mu{\rm m} \sim 100\mu{\rm m}$ | $10\mu{\rm m} \sim 100{\rm mm}$ ? |
| the number of ions: | $1 \sim 20$ | $100 \sim 10^8$ |

# Summary

- Light DM such as axions and dark photons induce weak electric fields

- We showed that Paul ion traps can be used as sensors for weak electric fields

  → 
  - Paul traps work as light DM detectors.
  - Sensitivities of Paul ion traps to axion-photon coupling and gauge kinetic mixing can reach previously unexplored parameter space.

- We also demonstrated a scheme to enhance the signal by using entangled ions

  → 
  - The signal can be enhanced by $N^2$ rather than $N$

Currently, entanglement of ions in a Paul trap with high fidelity is realized for $N \lesssim 20$ .
The rapid development of ion traps or other qubit systems for quantum computers may lead to the realization of a much larger number of entanglements with high fidelity in the near future.

- Application to high-frequency GWs (in preparation)