Fujita-Miyazawa-type three-body force in ultracold atoms

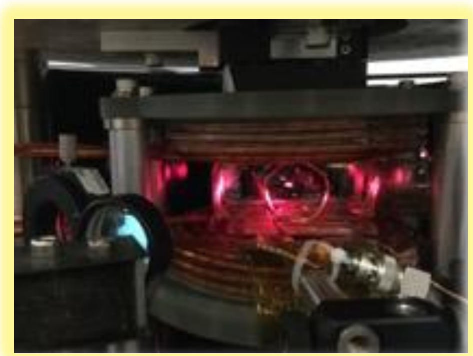
Hiroyuki Tajima

The University of Tokyo, Japan

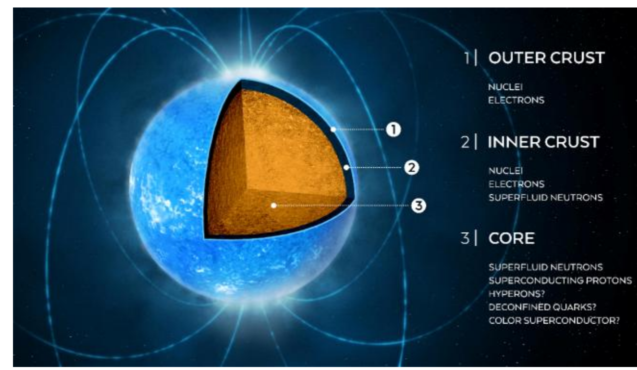
References: HT, E. Nakano, and K. Iida, PRA, **113**, L011305 (2026).

Toward analog quantum simulation of nuclear systems

Ultracold atoms



Neutron star matter



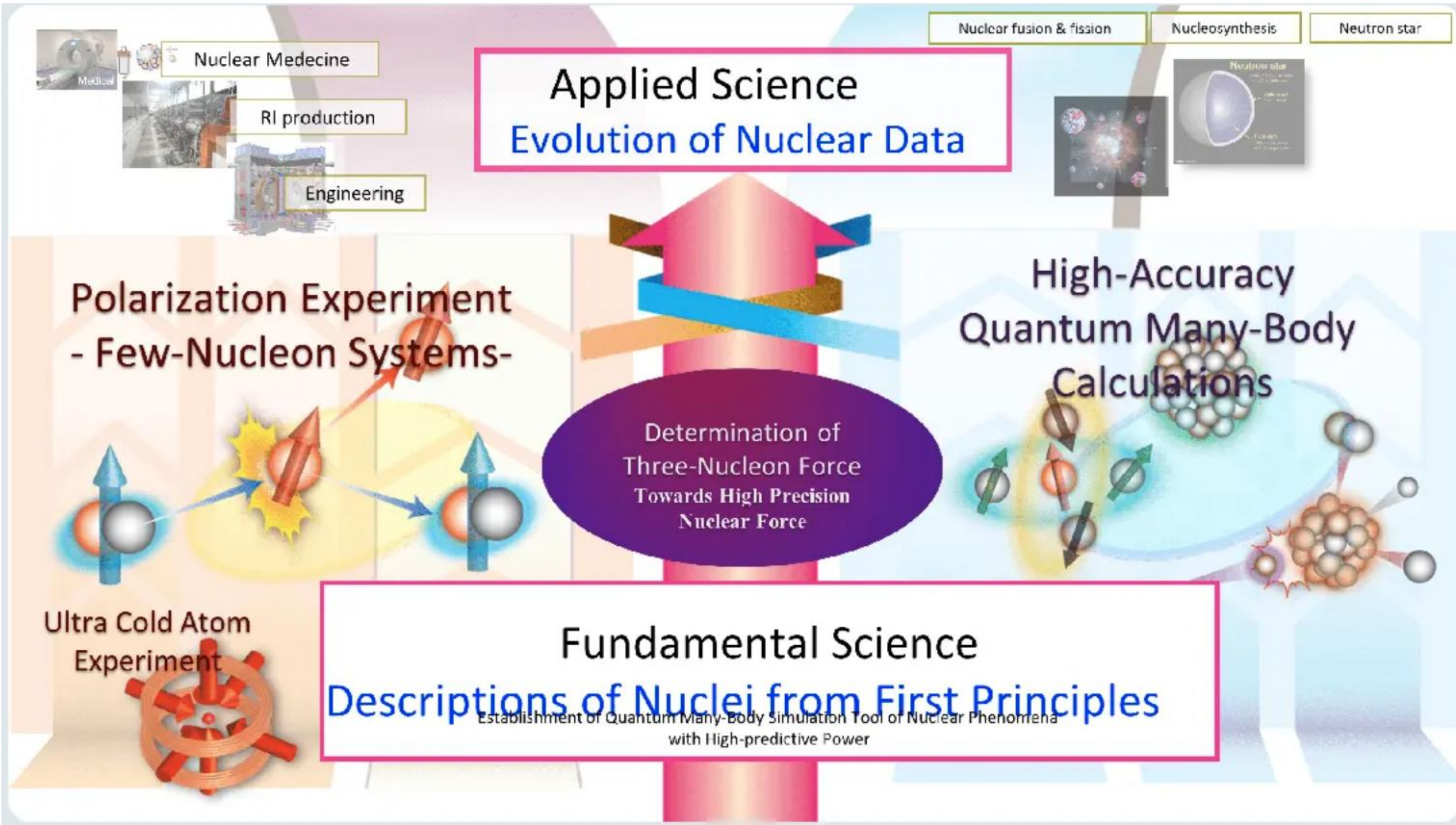
A. L. Watts, *et al.*, RMP **88**, 021001 (2016).

Tunable two-body force!

Tunable “three”-body force?

Is three-body force important?

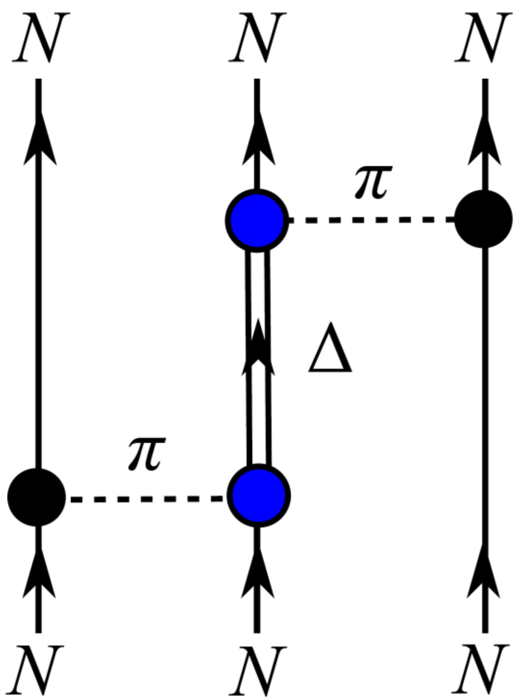
JST ERATO Sekiguchi Three-Nucleon Force Project



https://www.jst.go.jp/erato/sekiguchi/en/project/

Fujita-Miyazawa three-nucleon force

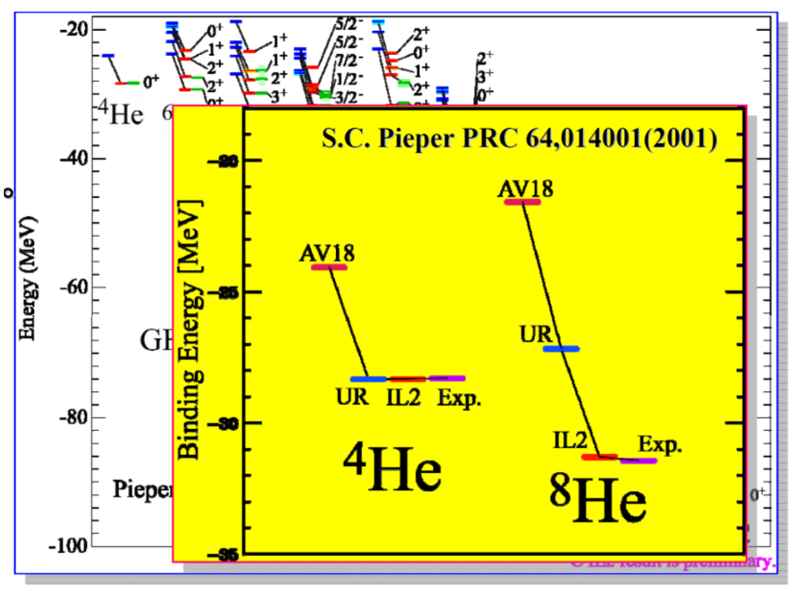
J. Fujita and H. Miyazawa PTEP **17**, 360 (1957).



Δ particle: excited state of nucleon

$M_{\Delta} = 1232$ MeV

$(J^{\pi}, \mathrm{T}) = (3/2^{+}, 3/2)$



From Sekiguchi-san's slide

Outline

- **Introduction**

  Tunable interactions in ultracold atoms

- **Yukawa interaction and beyond**

  Superfluid EFT $\simeq$ *Chiral EFT*

- **Fujita-Miyazawa-type three-body force**

  Demonstration in Bose-Einstein condensates

- **Summary**

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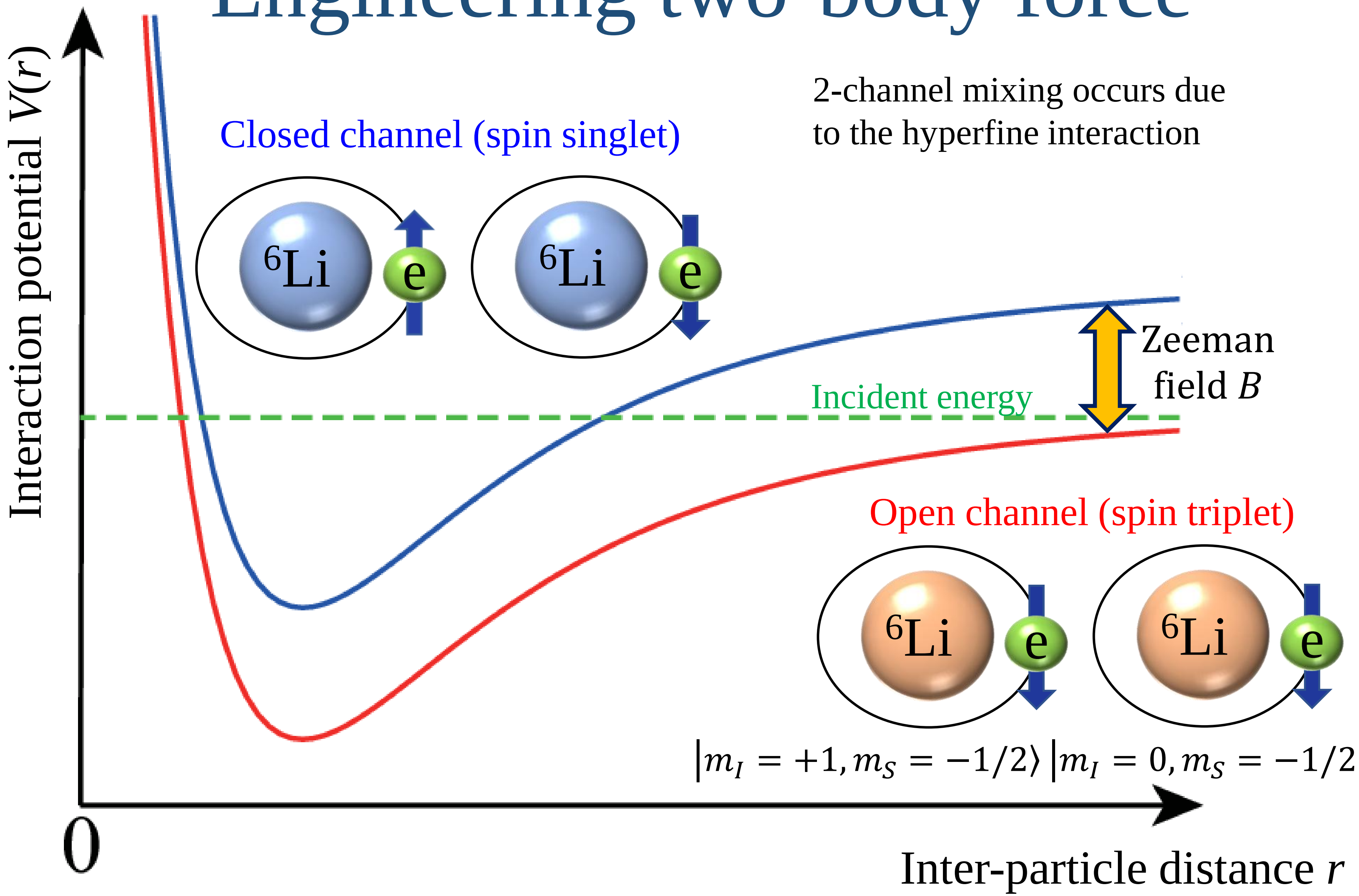
- **Summary**

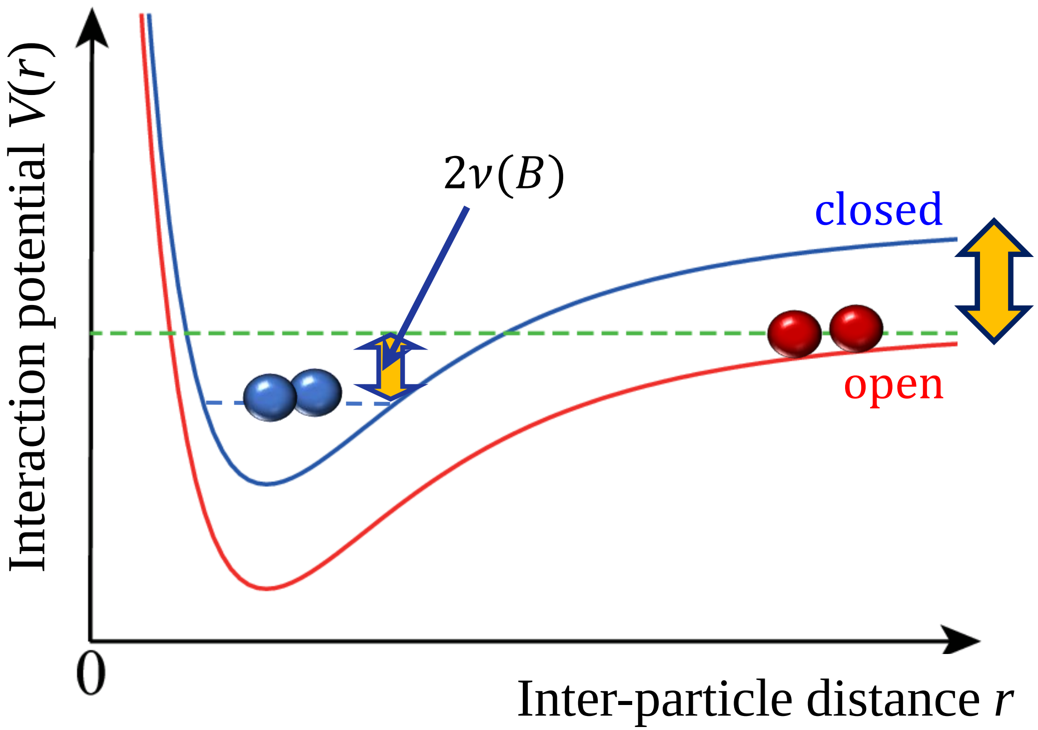
Feshbach resonance
Engineering two-body force

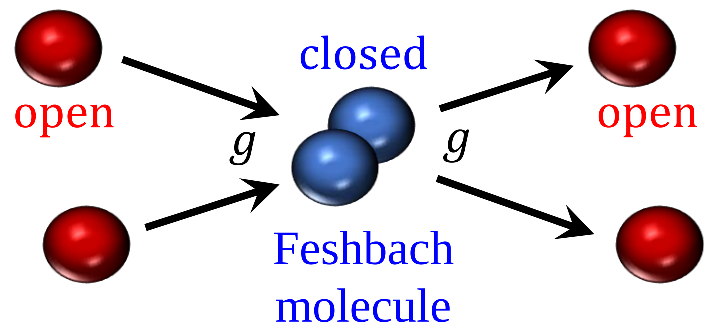


Feshbach resonance
Engineering two-body force

Atoms interacts with each other via an intermediate state

⇒attraction can be tuned by an external magnetic field





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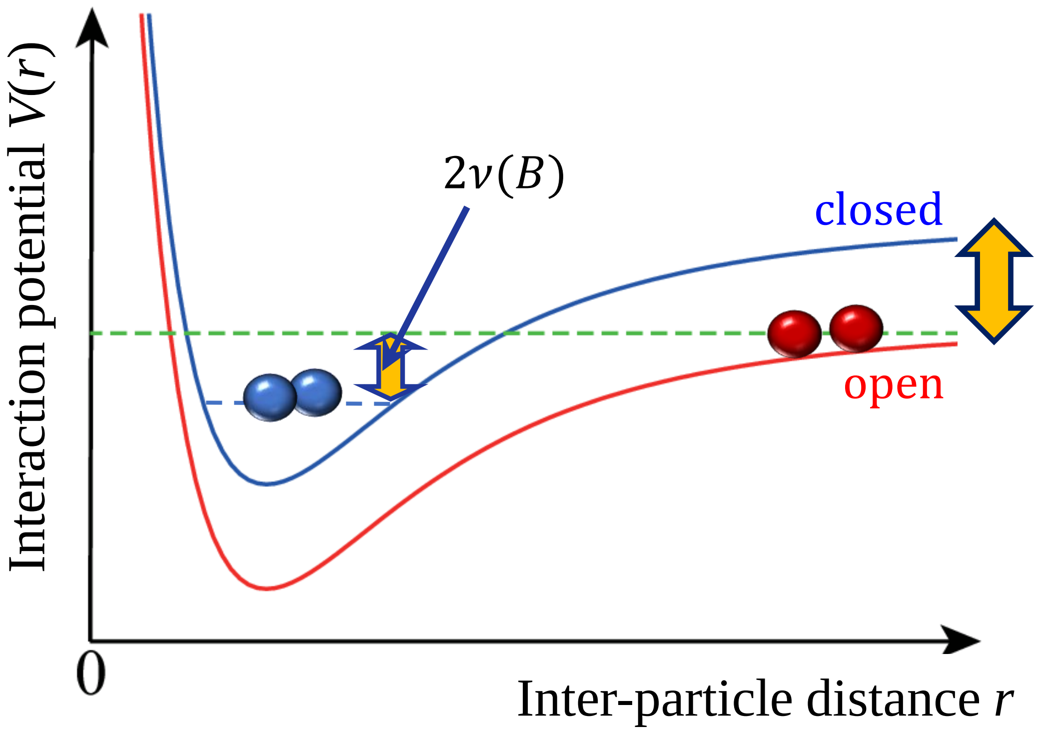
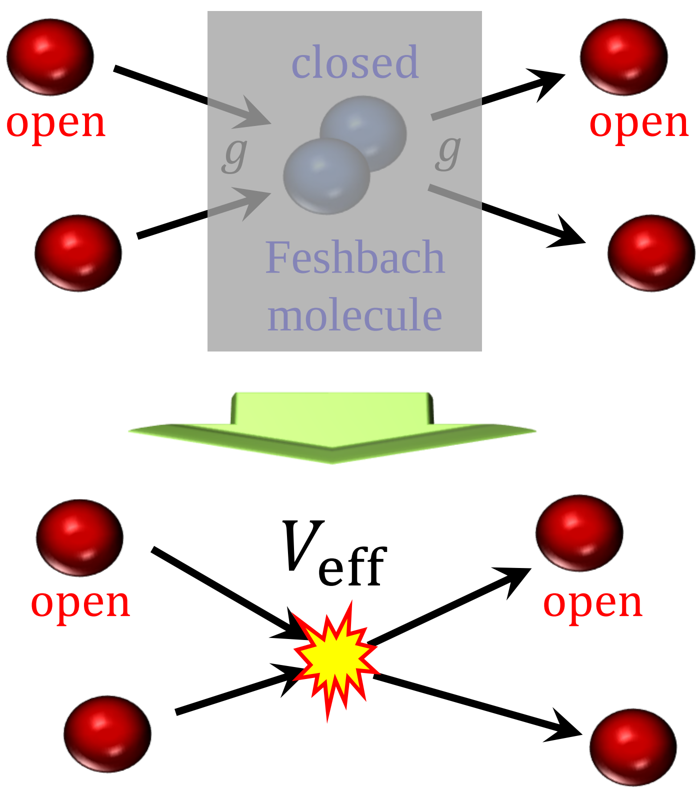


Effective attraction

$$V_{\mathrm{eff}} \simeq -\frac{g^2}{2\nu(B)}$$

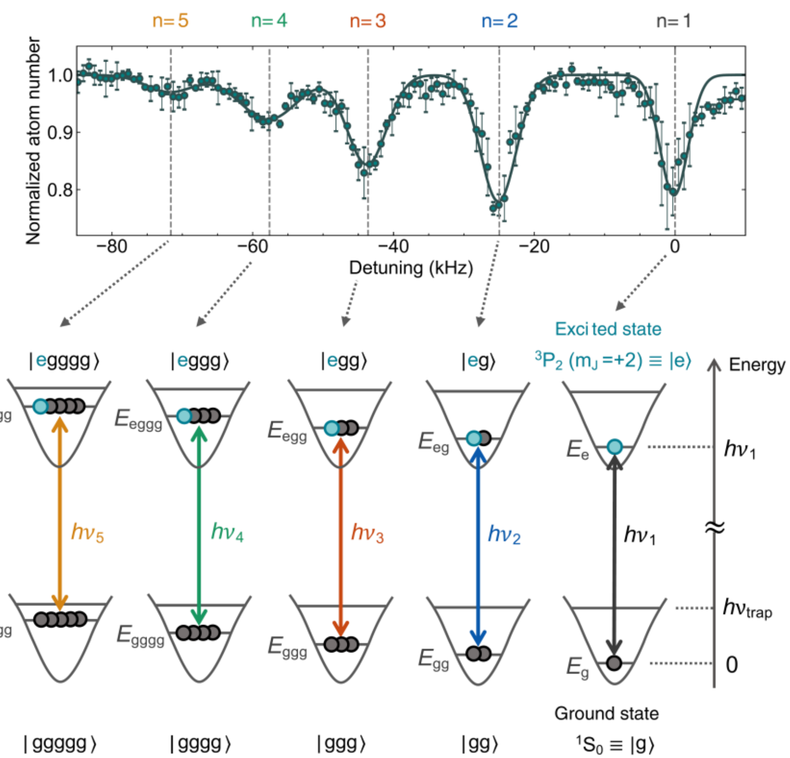
"Mediator" of effective interaction



Observation of three-body forces

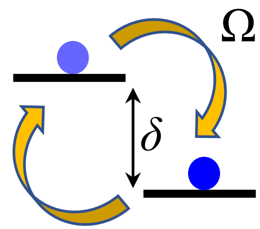
Multi-body force in "zero-dimensional" system

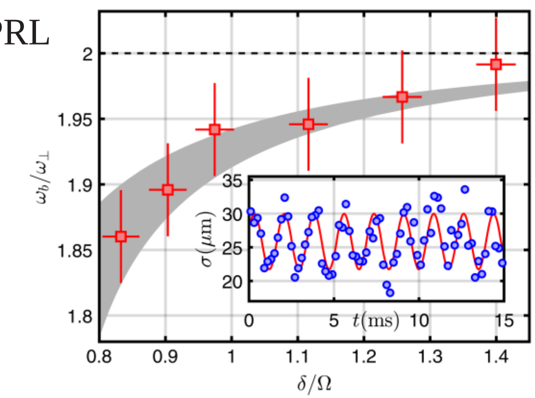
G. K. Campbell, et al., Science **313**, 649 (2006).
S. Will, et al., Nature **465**, 197 (2010).
L. Franchi,et al., New J. Phys. **19**, 103037 (2017).
A. Goban, et al., Nature **563**, 369 (2018).
K. Honda, et al., Phys. Rev. A **111**, 033303 (2025).



Three-body force in Rabi-coupled BEC

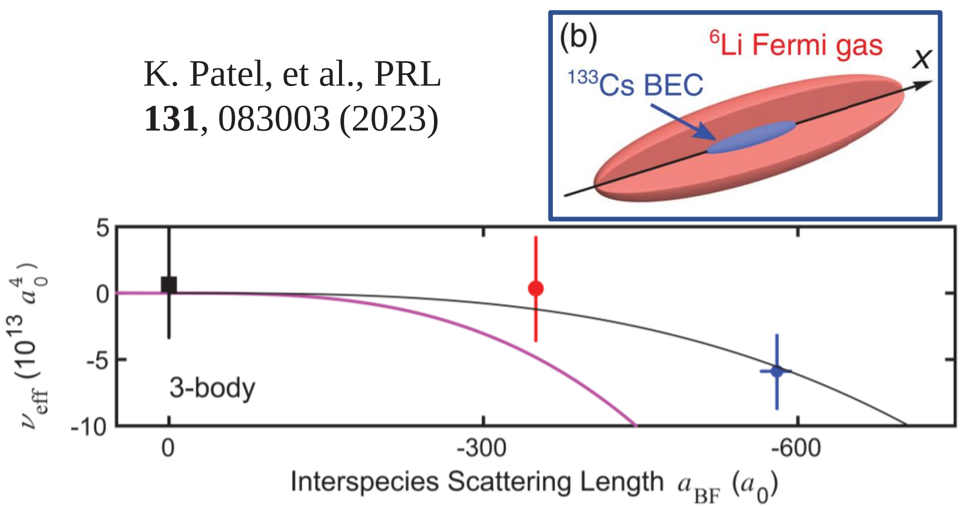
A. Hammond, et al., PRL **128**, 083401 (2022)





Three-body force with fermion exchange

K. Patel, et al., PRL **131**, 083003 (2023)

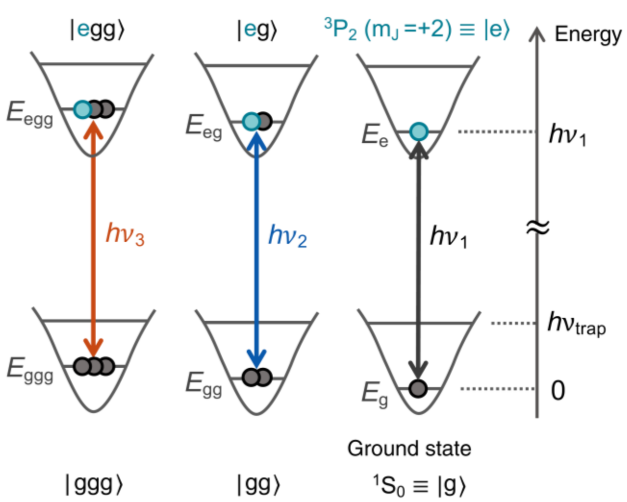


Emergent three-body forces in low-energy effective model

“Effective interaction” appears when the excited states and medium are “traced out”

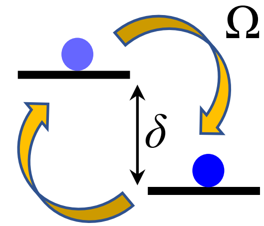
➡ Consider excited states as mediator of effective multi-body interaction V_{eff}

Multi-body force in “zero-dimensional” system



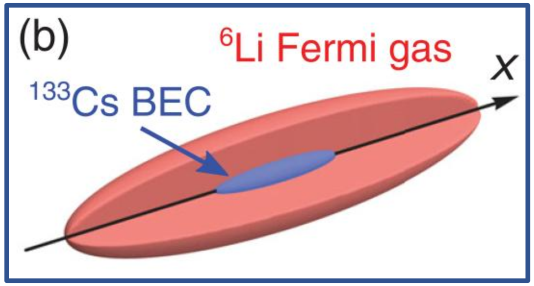
Focus: Vibrational ground state
Vibrational excited state

Three-body force in Rabi-coupled BEC



Focus: Rabi-splitted ground state
Rabi-splitted excited state

Three-body force with fermion exchange



Focus: BEC
Excitation of fermions

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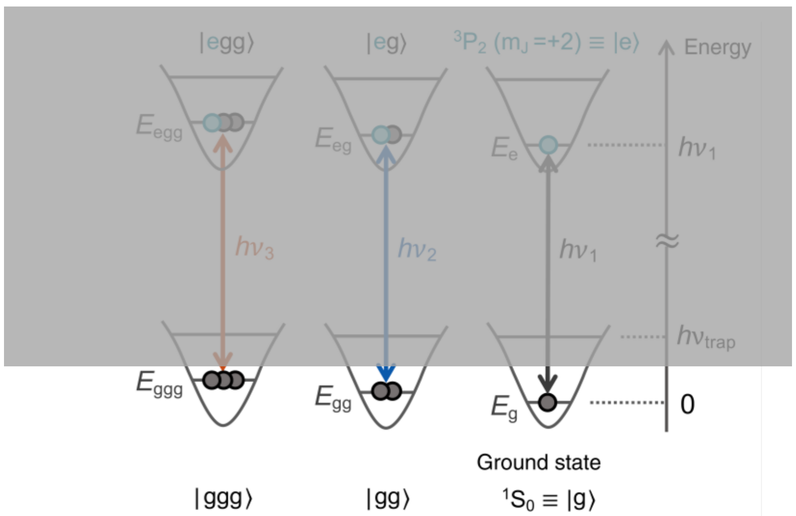
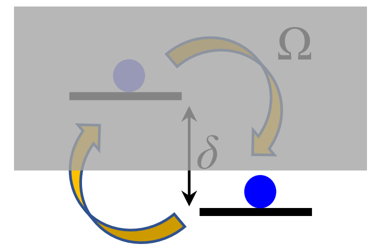


Focus: Vibrational ground state + V_{eff}

Vibrational excited state

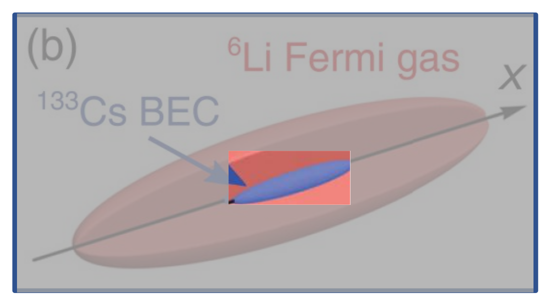
Three-body force in Rabi-coupled BEC



Focus: Rabi-splitted ground state + V_{eff}

Rabi-splitted excited state

Three-body force with fermion exchange



Focus: BEC + V_{eff}

Excitation of fermions

Simulating nuclear force in cold atoms?

Nuclear force: “Effective” interactions among nucleons

Focus: Nucleons (*N*)

Excited states (e.g., Δ particle)

Medium (e.g., pion clouds)

Fujita-Miyazawa three-body force

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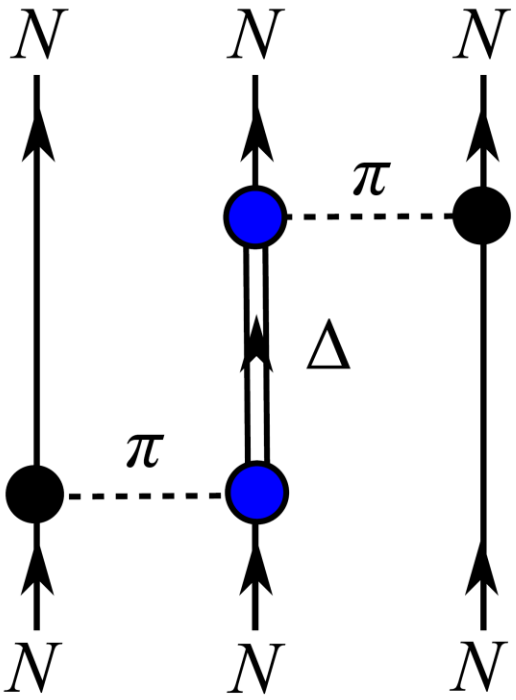


Simulating nuclear force in cold atoms?

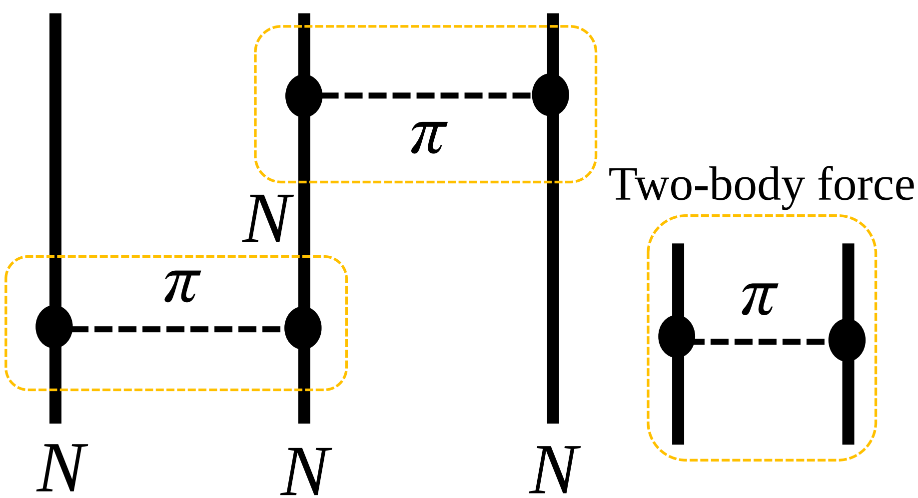
Nuclear force: "Effective" interactions among nucleons

Focus: Nucleons (*N*)

Excited states (e.g., Δ particle)

Medium (e.g., pion clouds)

Distinct from 2nd-order perturbation of two-body force



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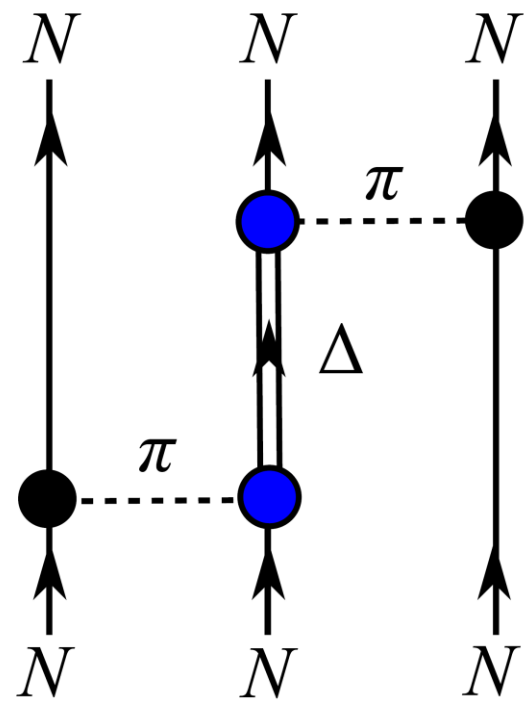

Simulating nuclear force in cold atoms?

Nuclear force: "Effective" interactions among nucleons

Focus: Nucleons (N) + V_{eff}

Excited states (e.g., Δ particle)

Medium (e.g., pion clouds)

What do we need?

✓ Excited states mimicking Δ particle

✓ Medium mimicking pion clouds

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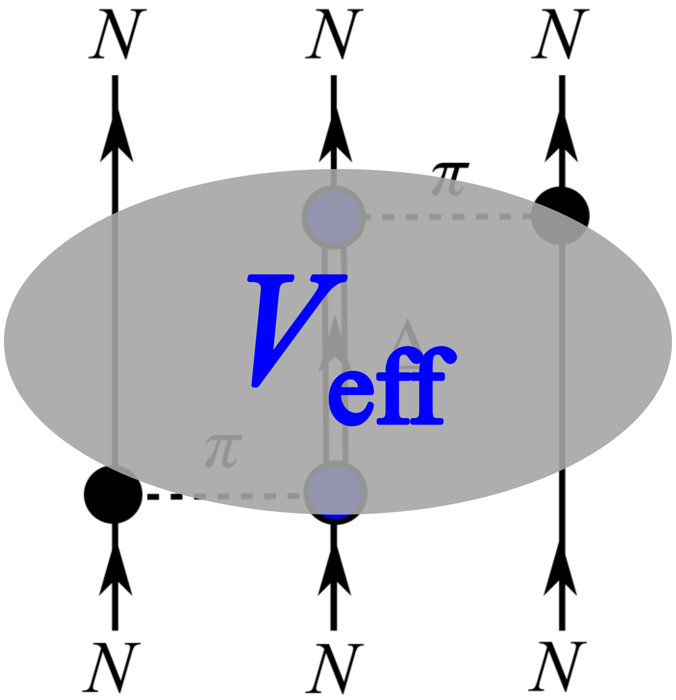


Simulating nuclear force in cold atoms?

Nuclear force: "Effective" interactions among nucleons

Focus: Nucleons (N) + V_{eff}

Excited states (e.g., Δ particle)

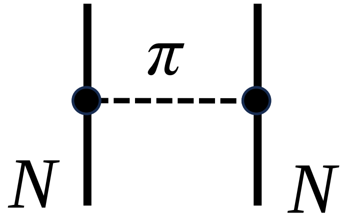
Medium (e.g., pion clouds)

What do we need?

✓ Excited states mimicking Δ particle

✓ Medium mimicking pion clouds

➡ In the first place, in cold atoms,
can we simulate **Yukawa interaction**…?



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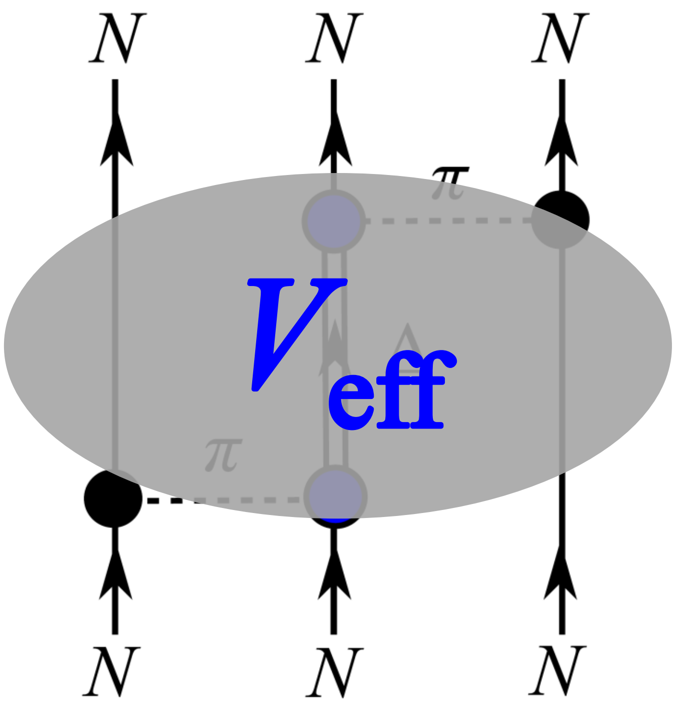

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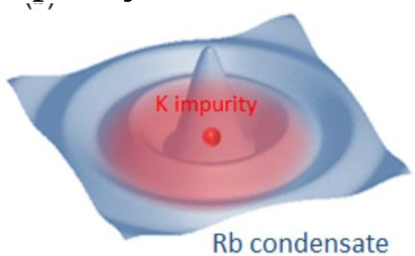
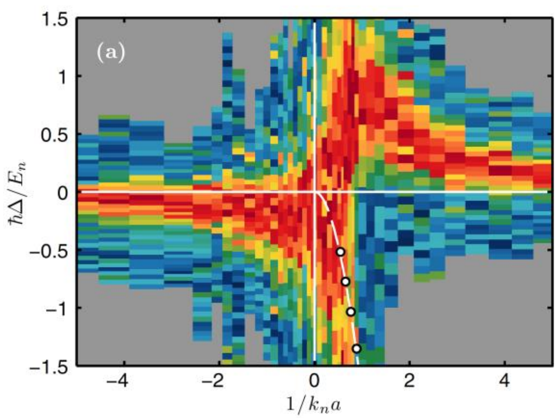
Yukawa interaction in cold atoms

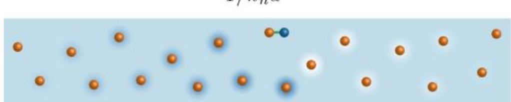
Two-polaron interaction in BEC is induced by exchange of superfluid phonons (analogous to pion exchange)

“Bose” polaron

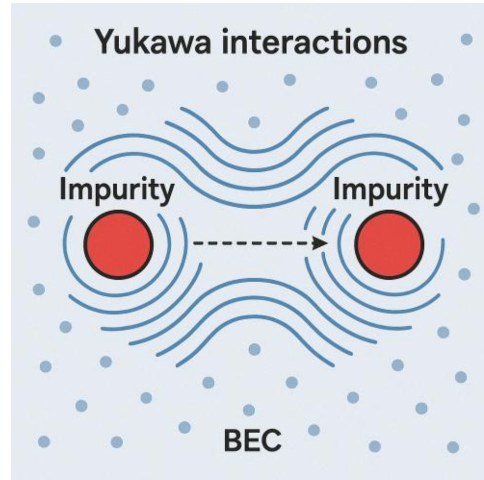
Impurity immersed in BEC







Nils B. Jørgensen, et al., PRL **117**, 055302 (2016).



by Chat GPT画伯

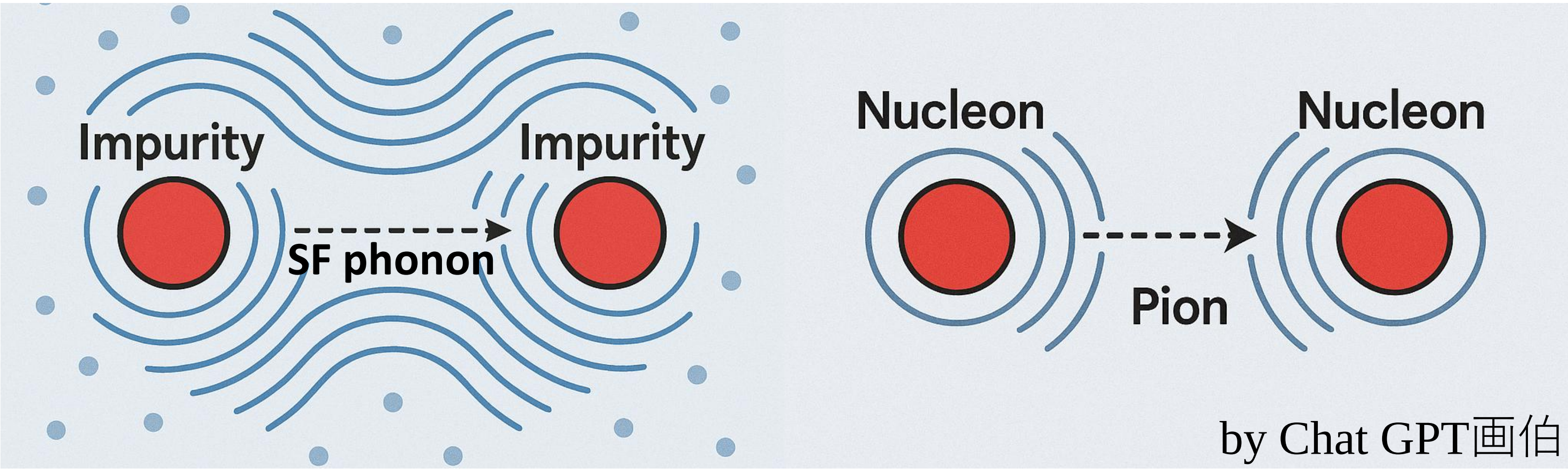
$$V_{2\mathrm{b}}(r) = -\frac{\alpha}{r} e^{-r/\xi}$$

ξ: BEC healing length

M. J. Mijslma, et al., PRA **61**, 053601 (2000).

Analogy between polaron and nucleon

Inter-polaron force **Nuclear force**



$$V_{2b}(r) = -\frac{\alpha}{r} e^{-r/\xi}$$

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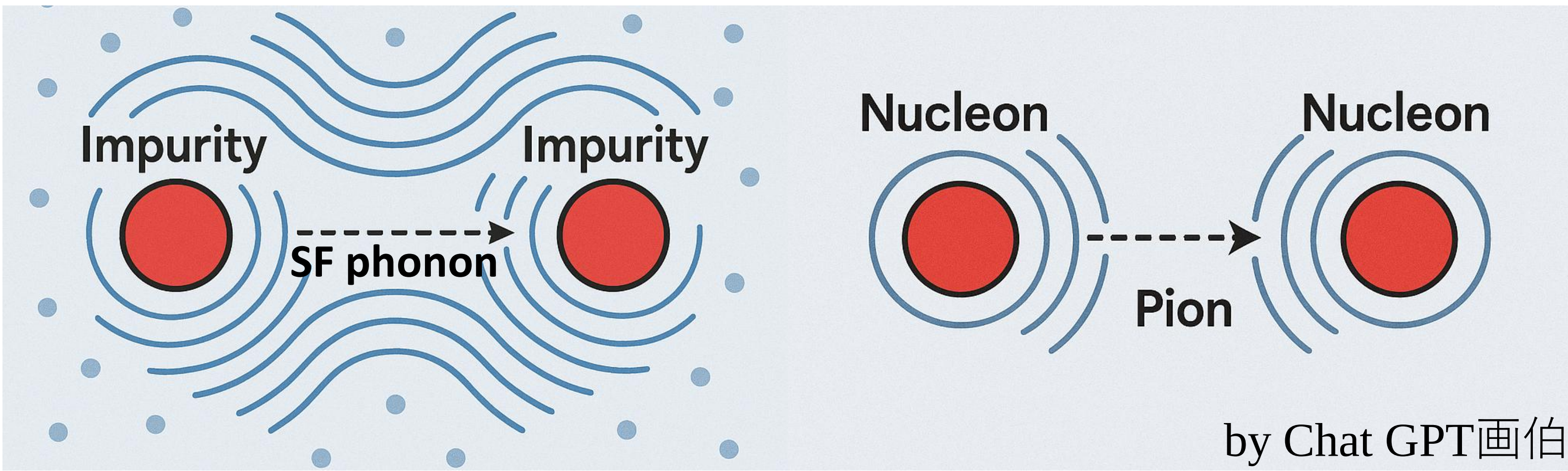
$$V_{2b}(r) = -\frac{\alpha'}{r} e^{-m_\pi r}$$

m_π^{-1}: inverse pion mass

Analogy between polaron and nucleon

Inter-polaron force **Nuclear force**



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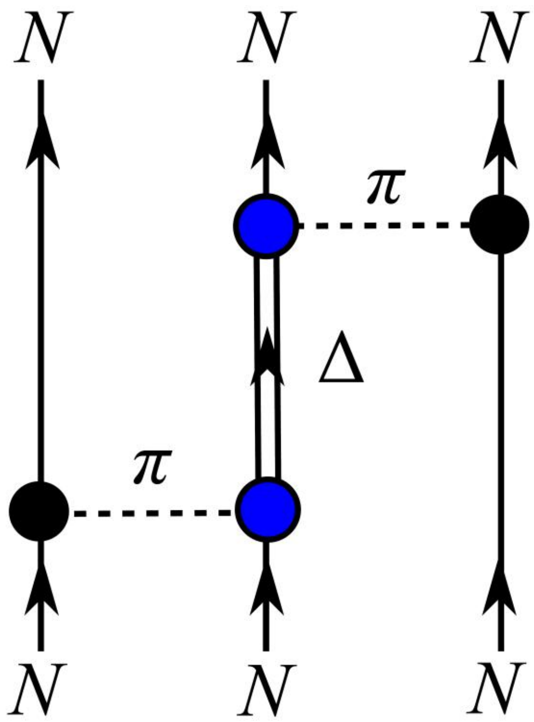
m_π^{-1}: inverse pion mass

⇨ **Fujita-Miyazawa counterpart in three-polaron force?**

Fujita-Miyazawa-type three-body force among polarons

HT, E. Nakano, and K. Iida, PRA **113**, L011305 (2026).

Nucleon
Pion
Δ resonance



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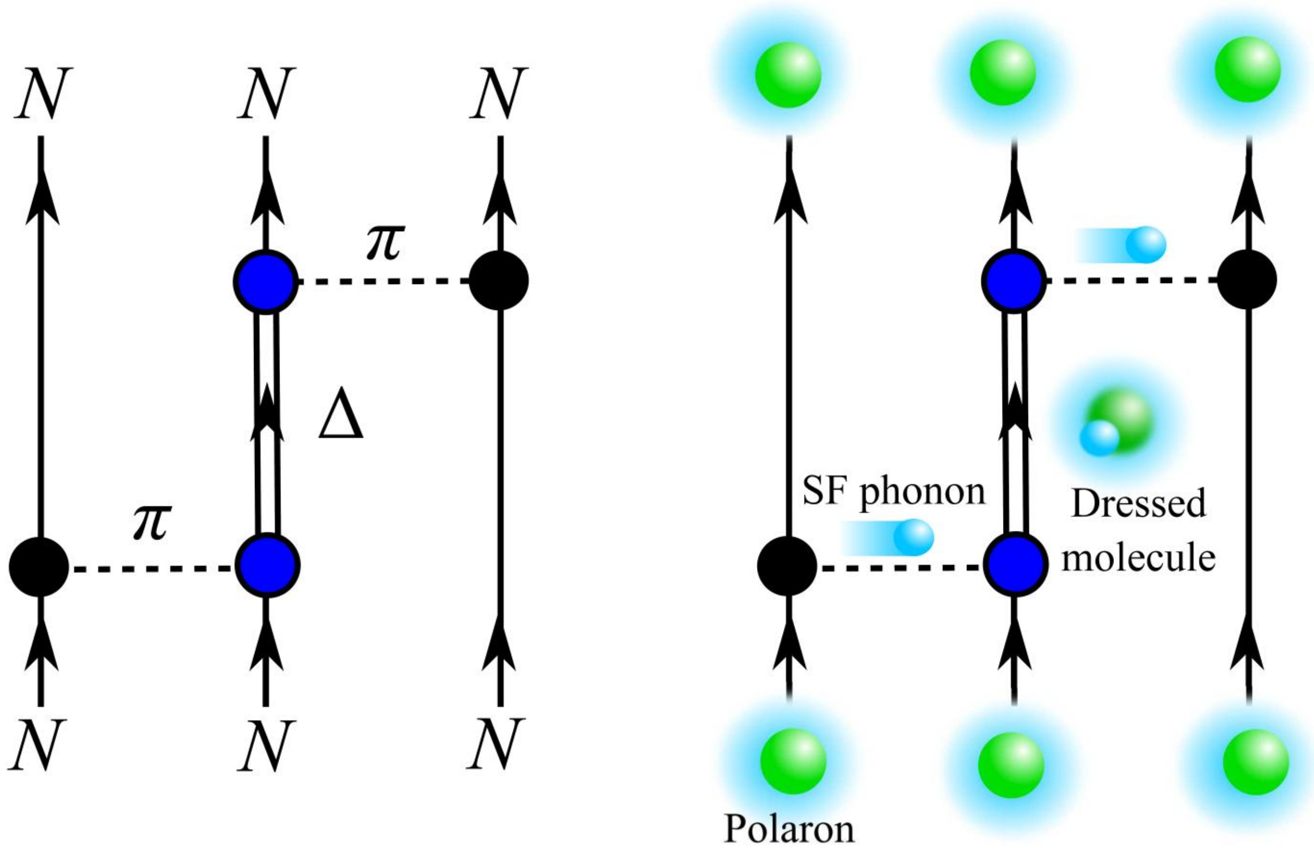
HT, E. Nakano, and K. Iida, PRA **113**, L011305 (2026).

Nucleon ⇔ polaron
Pion ⇔ superfluid phonon
Δ resonance ⇔ Feshbach molecule (closed-channel)



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Two-channel model of cold atoms near the Feshbach resonance

$$\hat{H} = \sum_{\boldsymbol{k}} \left[\xi_{\boldsymbol{k},b} \hat{b}^\dagger_{\boldsymbol{k}} \hat{b}_{\boldsymbol{k}} + \xi_{\boldsymbol{k},c} \hat{c}^\dagger_{\boldsymbol{k}} \hat{c}_{\boldsymbol{k}} \right] + \sum_{\boldsymbol{P}} \xi_{\boldsymbol{P},A} \hat{A}^\dagger_{\boldsymbol{P}} \hat{A}_{\boldsymbol{P}}$$

$$+ \frac{U_{bb}}{2} \sum_{\boldsymbol{k},\boldsymbol{k}',\boldsymbol{P}} \hat{b}^\dagger_{\boldsymbol{k}+\frac{\boldsymbol{P}}{2}} \hat{b}^\dagger_{-\boldsymbol{k}+\frac{\boldsymbol{P}}{2}} \hat{b}_{-\boldsymbol{k}'+\frac{\boldsymbol{P}}{2}} \hat{b}_{\boldsymbol{k}'+\frac{\boldsymbol{P}}{2}}$$

$$+ U_{bc} \sum_{\boldsymbol{k},\boldsymbol{k}',\boldsymbol{P}} \hat{b}^\dagger_{\boldsymbol{k}+\frac{M_b}{M_A}\boldsymbol{P}} \hat{c}^\dagger_{-\boldsymbol{k}+\frac{M_c}{M_A}\boldsymbol{P}} \hat{c}_{-\boldsymbol{k}'+\frac{M_c}{M_A}\boldsymbol{P}} \hat{b}_{\boldsymbol{k}'+\frac{M_b}{M_A}\boldsymbol{P}}$$

$$+ g \sum_{\boldsymbol{P},\boldsymbol{k}} \left[\hat{A}^\dagger_{\boldsymbol{P}} \hat{b}_{-\boldsymbol{k}+\frac{M_b}{M_A}\boldsymbol{P}} \hat{c}_{\boldsymbol{k}+\frac{M_c}{M_A}\boldsymbol{P}} + \text{h.c.} \right]$$

Kinetic energies:

$$\xi_{\boldsymbol{k},b} = k^2/(2M_b) - \mu_b$$
$$\xi_{\boldsymbol{k},c} = k^2/(2M_c) - \mu_c$$
$$\xi_{\boldsymbol{P},A} = P^2/(2M_A) - \mu_b - \mu_c + \nu$$

$\mu_{b,c}$: chemical potential
$M_{b,c,A}$: mass
$\nu(B)$: closed-channel energy

Two-channel model of cold atoms near the Feshbach resonance

Medium boson **Impurity** **Closed-channel molecule**

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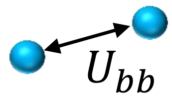
Boson-boson interaction



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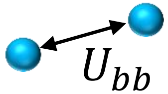
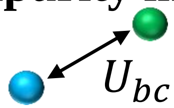
Boson-boson interaction



$$+ U_{bc} \sum_{\boldsymbol{k},\boldsymbol{k}',\boldsymbol{P}} \hat{b}^\dagger_{\boldsymbol{k}+\frac{M_b}{M_A}\boldsymbol{P}} \hat{c}^\dagger_{-\boldsymbol{k}+\frac{M_c}{M_A}\boldsymbol{P}} \hat{c}_{-\boldsymbol{k}'+\frac{M_c}{M_A}\boldsymbol{P}} \hat{b}_{\boldsymbol{k}'+\frac{M_b}{M_A}\boldsymbol{P}}$$

Boson-impurity interaction



$$+ g \sum_{\boldsymbol{P},\boldsymbol{k}} \left[\hat{A}^\dagger_{\boldsymbol{P}} \hat{b}_{-\boldsymbol{k}+\frac{M_b}{M_A}\boldsymbol{P}} \hat{c}_{\boldsymbol{k}+\frac{M_c}{M_A}\boldsymbol{P}} + \text{h.c.} \right]$$

Kinetic energies:

$$\begin{aligned} \xi_{\boldsymbol{k},b} &= k^2/(2M_b) - \mu_b \\ \xi_{\boldsymbol{k},c} &= k^2/(2M_c) - \mu_c \\ \xi_{\boldsymbol{P},A} &= P^2/(2M_A) - \mu_b - \mu_c + \nu \end{aligned}$$

$\mu_{b,c}$: chemical potential
$M_{b,c,A}$: mass
$\nu(B)$: closed-channel energy

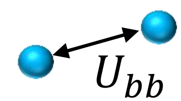
Two-channel model of cold atoms near the Feshbach resonance

Medium boson **Impurity** **Closed-channel molecule**

$$\hat{H} = \sum_{\boldsymbol{k}} \left[\xi_{\boldsymbol{k},b} \hat{b}_{\boldsymbol{k}}^{\dagger} \hat{b}_{\boldsymbol{k}} + \xi_{\boldsymbol{k},c} \hat{c}_{\boldsymbol{k}}^{\dagger} \hat{c}_{\boldsymbol{k}} \right] + \sum_{\boldsymbol{P}} \xi_{\boldsymbol{P},A} \hat{A}_{\boldsymbol{P}}^{\dagger} \hat{A}_{\boldsymbol{P}}$$

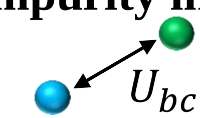
$$+ \frac{U_{bb}}{2} \sum_{\boldsymbol{k},\boldsymbol{k}',\boldsymbol{P}} \hat{b}^{\dagger}_{\boldsymbol{k}+\frac{\boldsymbol{P}}{2}} \hat{b}^{\dagger}_{-\boldsymbol{k}+\frac{\boldsymbol{P}}{2}} \hat{b}_{-\boldsymbol{k}'+\frac{\boldsymbol{P}}{2}} \hat{b}_{\boldsymbol{k}'+\frac{\boldsymbol{P}}{2}}$$

Boson-boson interaction



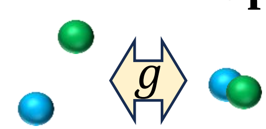
$$+ U_{bc} \sum_{\boldsymbol{k},\boldsymbol{k}',\boldsymbol{P}} \hat{b}^{\dagger}_{\boldsymbol{k}+\frac{M_b}{M_A}\boldsymbol{P}} \hat{c}^{\dagger}_{-\boldsymbol{k}+\frac{M_c}{M_A}\boldsymbol{P}} \hat{c}_{-\boldsymbol{k}'+\frac{M_c}{M_A}\boldsymbol{P}} \hat{b}_{\boldsymbol{k}'+\frac{M_b}{M_A}\boldsymbol{P}}$$

Boson-impurity interaction



$$+ g \sum_{\boldsymbol{P},\boldsymbol{k}} \left[\hat{A}_{\boldsymbol{P}}^{\dagger} \hat{b}_{-\boldsymbol{k}+\frac{M_b}{M_A}\boldsymbol{P}} \hat{c}_{\boldsymbol{k}+\frac{M_c}{M_A}\boldsymbol{P}} + \mathrm{h.c.} \right]$$

Feshbach coupling



Kinetic energies:

$$\xi_{\boldsymbol{k},b} = k^2/(2M_b) - \mu_b$$
$$\xi_{\boldsymbol{k},c} = k^2/(2M_c) - \mu_c$$
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$\mu_{b,c}$: chemical potential

$M_{b,c,A}$: mass

$\nu(B)$: closed-channel energy

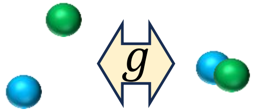
Polaron in Bose-Einstein condensate

$$b_{\boldsymbol{k}} = \sqrt{n_0}\delta_{\boldsymbol{k},\mathbf{0}} + \hat{\pi}_{\boldsymbol{k}}(1 - \delta_{\boldsymbol{k},\mathbf{0}}),$$

n_0: BEC condensate density

Feshbach coupling

$$g\sum_{\boldsymbol{P},\boldsymbol{k}}\left[\hat{A}^{\dagger}_{\boldsymbol{P}}\hat{b}_{-\boldsymbol{k}+\frac{M_b}{M_A}\boldsymbol{P}}\hat{c}_{\boldsymbol{k}+\frac{M_c}{M_A}\boldsymbol{P}} + \text{h.c.}\right]$$



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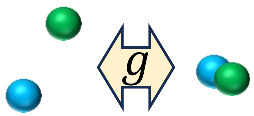
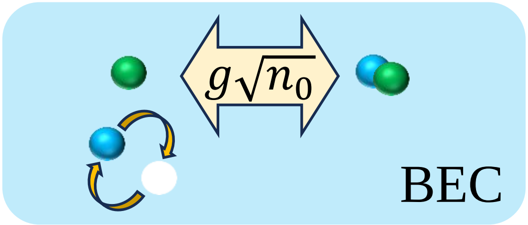


Coherent atom-molecule mixing

$$g\sqrt{n_0}\sum_{\boldsymbol{k}}\left(c^{\dagger}_{\boldsymbol{k}}A_{\boldsymbol{k}} + A^{\dagger}_{\boldsymbol{k}}c_{\boldsymbol{k}}\right)$$



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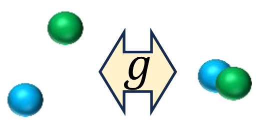

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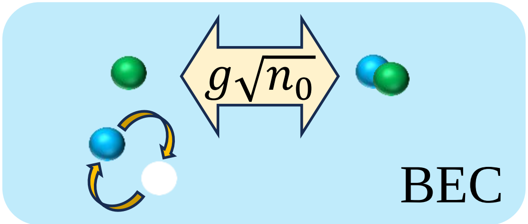


Nucleon-like and Δ-like polarons as diagonalized eigenstates

$$\hat{H} = \hat{H}_N + \hat{H}_\Delta + \hat{H}_\pi + \hat{V}$$

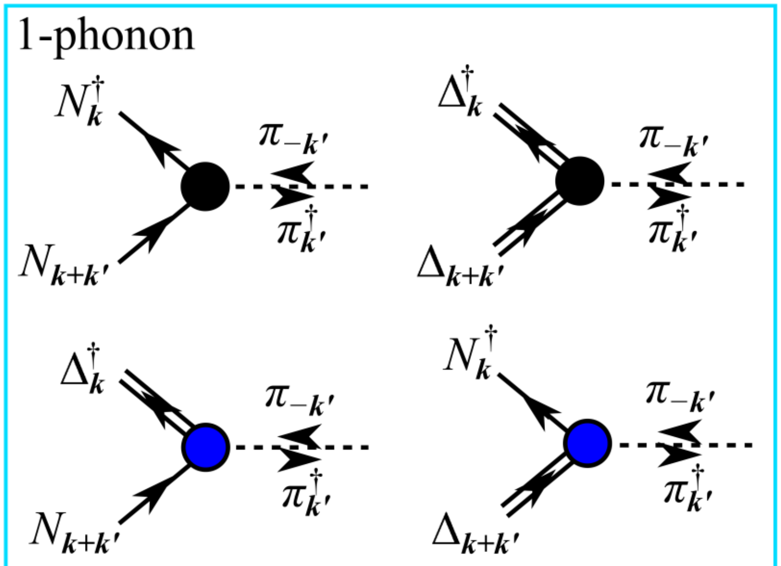
Ground state: $\hat{H}_N = \sum_{\boldsymbol{k}}\xi_{\boldsymbol{k},N}\hat{N}^\dagger_{\boldsymbol{k}}\hat{N}_{\boldsymbol{k}}$

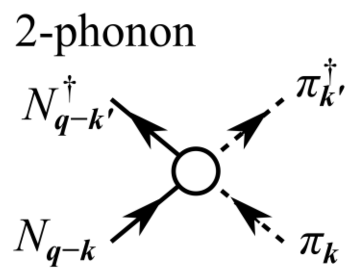
Excited state: $\hat{H}_\Delta = \sum_{\boldsymbol{k}}\xi_{\boldsymbol{k},\Delta}\hat{\Delta}^\dagger_{\boldsymbol{k}}\hat{\Delta}_{\boldsymbol{k}}$

Bogoliubov Hamiltonian for pion-like boson excitation

$$\hat{H}_\pi = \sum_{\boldsymbol{k}}(\xi_{\boldsymbol{k},b} + 2U_{bb}n_0)\hat{\pi}^\dagger_{\boldsymbol{k}}\hat{\pi}_{\boldsymbol{k}} + \frac{U_{bb}n_0}{2}\sum_{\boldsymbol{k}}\left[\hat{\pi}^\dagger_{\boldsymbol{k}}\hat{\pi}^\dagger_{-\boldsymbol{k}} + \hat{\pi}_{-\boldsymbol{k}}\hat{\pi}_{\boldsymbol{k}}\right]$$

Absorption and emission of pion-like boson excitations





*Higher order at $n_0 \gg 1$

$$\hat{V} = \sum_{\boldsymbol{k},\boldsymbol{k}'} \Big[f^{NN\pi}_{\boldsymbol{k},\boldsymbol{k}'} \hat{N}^{\dagger}_{\boldsymbol{k}} \hat{N}_{\boldsymbol{k}+\boldsymbol{k}'} \hat{\pi}^{\dagger}_{\boldsymbol{k}'} + f^{\Delta\Delta\pi}_{\boldsymbol{k},\boldsymbol{k}'} \hat{\Delta}^{\dagger}_{\boldsymbol{k}} \hat{\Delta}_{\boldsymbol{k}+\boldsymbol{k}'} \hat{\pi}^{\dagger}_{\boldsymbol{k}'} + f^{\Delta N\pi}_{\boldsymbol{k},\boldsymbol{k}'} \hat{\Delta}^{\dagger}_{\boldsymbol{k}} \hat{N}_{\boldsymbol{k}+\boldsymbol{k}'} \hat{\pi}^{\dagger}_{\boldsymbol{k}'} + f^{N\Delta\pi}_{\boldsymbol{k},\boldsymbol{k}'} \hat{N}^{\dagger}_{\boldsymbol{k}} \hat{\Delta}_{\boldsymbol{k}+\boldsymbol{k}'} \hat{\pi}^{\dagger}_{\boldsymbol{k}'} \Big] + \mathrm{h.c.},$$

Hamiltonian effective field theory based on the open-system description

We do not have to resort to path integral formalism

Grand-canonical partition function

$$Z = \mathrm{Tr}\left[e^{-\beta(\hat{H}_N+\hat{H}_\Delta+\hat{H}_\pi+\hat{V})}\right]$$

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"Trace out"

"Effective nucleon system"

$$Z = \mathrm{Tr}_N\left[e^{-\beta(\hat{H}_N + \hat{V}_{\mathrm{eff}})}\right]$$

$\hat{V}_{\mathrm{eff}}$: effective interaction

$\mathrm{Tr}_N[\ldots]$: partial trace of N state

Hamiltonian effective field theory based on the open-system description

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$\mathrm{Tr}_N[\dots]$: partial trace of N state

Equation for effective interaction

$$e^{-\beta \hat{V}_{\mathrm{eff}}} = \mathrm{Tr}_{\Delta\pi}\left[e^{-\beta(\hat{H}_\Delta + \hat{H}_\pi)}\hat{S}(\beta)\right]$$

S-matrix operator

$$\hat{S}(\beta) = T_\tau \exp\left[-\int_0^\beta d\tau\, \hat{V}(\tau)\right]$$

Interaction representation in the imaginary time formalism

$$\hat{V}(\tau) = e^{\tau(\hat{H}_N + \hat{H}_\Delta + \hat{H}_\pi)}\hat{V} e^{-\tau(\hat{H}_N + \hat{H}_\Delta + \hat{H}_\pi)}$$

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$$\hat{V}(\tau) = e^{\tau(\hat{H}_N+\hat{H}_\Delta+\hat{H}_\pi)}\hat{V}e^{-\tau(\hat{H}_N+\hat{H}_\Delta+\hat{H}_\pi)}$$

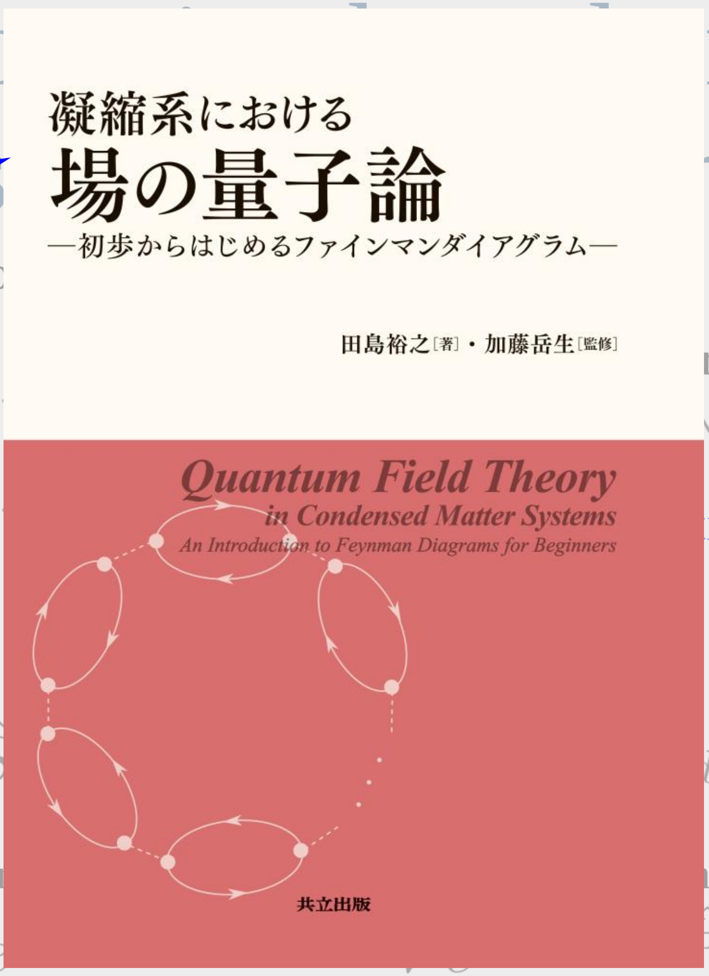
Perturbative expression of the effective interaction

$$\hat{V}_{\mathrm{eff}} = \sum_{\ell=1}^{\infty}\frac{(-1)^{\ell-1}}{\ell!\beta}\int_0^\beta d\tau_1\cdots\int_0^\beta d\tau_\ell\langle T_\tau[\hat{V}(\tau_1)\cdots\hat{V}(\tau_\ell)]\rangle$$

$$\langle\cdots\rangle = \mathrm{Tr}_{\pi\Delta}[e^{-\beta(\hat{H}_\Delta+\hat{H}_\pi)}\ldots]/\mathrm{Tr}_{\pi\Delta}[e^{-\beta(\hat{H}_\Delta+\hat{H}_\pi)}]$$

Effective inter... n

More details about effective two- and three-body interactions can be found in our new book



We do not have to reso...

Grand-canonical partition function ... m”

$$Z = \mathrm{Tr}\left[e^{-\beta(\hat{H}_N+\hat{H}_\Delta+\hat{H}_\pi+\hat{V})} \cdots {}_N+\hat{V}_{\mathrm{eff}})\right]$$

“Trace out” ... ion

... N state

Equation for effective interaction

$$e^{-\beta\hat{V}_{\mathrm{eff}}} = \mathrm{Tr}_{\Delta\pi}\left[e^{-\beta(\hat{H}_\Delta+\hat{H}_\pi)}\hat{S} \cdots d\tau\,\hat{V}(\tau)\right]$$

Interaction r... ...alism

$$\hat{V}(\tau) = e \cdots \hat{H}_\Delta+\hat{H}_\pi)$$

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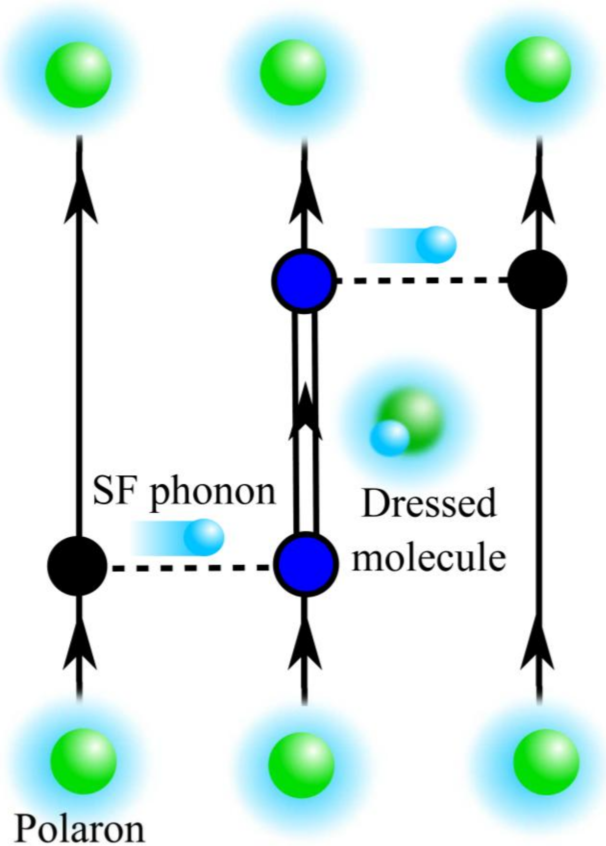
$$\hat{V}_{\mathrm{eff}} = \sum_{\ell=1}^{\infty}\frac{(-1)^{\ell-1}}{\ell!\beta}\int_0^\beta d\tau_1\cdots\int_0^\beta d\tau_\ell\langle T_\tau[\hat{V}(\tau_1)\cdots\hat{V}(\tau_\ell)]\rangle$$

$$\langle\cdots\rangle = \mathrm{Tr}_{\pi\Delta}[e^{-\beta(\hat{H}_\Delta+\hat{H}_\pi)}\cdots]/\mathrm{Tr}_{\pi\Delta}[e^{-\beta(\hat{H}_\Delta+\hat{H}_\pi)}]$$

Fujita-Miyazawa three-body force



$$\hat{V}_{\rm FM} = \frac{1}{6}\sum_{\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{q}_1,\boldsymbol{q}_2} U_{\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3}(\boldsymbol{q}_1,\boldsymbol{q}_2)$$

$$\times \hat{N}^\dagger_{\boldsymbol{k}_1}\hat{N}^\dagger_{\boldsymbol{k}_2}\hat{N}^\dagger_{\boldsymbol{k}_3}\hat{N}_{\boldsymbol{k}_3-\boldsymbol{q}_1}\hat{N}_{\boldsymbol{k}_2+\boldsymbol{q}_1-\boldsymbol{q}_2}\hat{N}_{\boldsymbol{k}_1+\boldsymbol{q}_2}$$

2$\boldsymbol{\pi}$-exchange-like form of coupling strength

$$U_{\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3}(\boldsymbol{q}_1,\boldsymbol{q}_2) = -6\mathcal{G}^{\Delta}_{\boldsymbol{k}_2+\boldsymbol{q}_1}\mathcal{G}^{N\pi\Delta}_{\boldsymbol{k}_1,\boldsymbol{q}_2,\boldsymbol{k}_2+\boldsymbol{q}_1}\mathcal{G}^{\Delta\pi N}_{\boldsymbol{k}_2,\boldsymbol{q}_1,\boldsymbol{k}_3}$$

Δ prop.

π-like SF phonon prop.
with form factors

At $g \ll U_{bc}\sqrt{n_0}$

$$U_{\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3}(\boldsymbol{q}_1,\boldsymbol{q}_2) \propto \frac{1}{(\boldsymbol{q}_1^2+\xi^{-2})(\boldsymbol{q}_2^2+\xi^{-2})}$$

ξ: BEC healing length

How to measure?

Interaction energy in the impurity equation of state

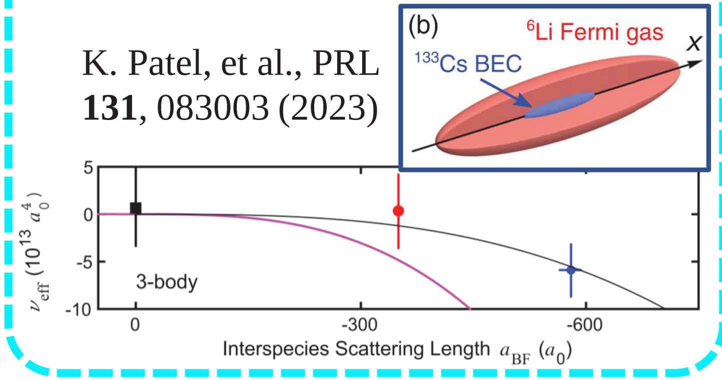
$$\delta E_3 \propto U_{\mathbf{0},\mathbf{0},\mathbf{0}}(\mathbf{0},\mathbf{0}) n_N^3$$

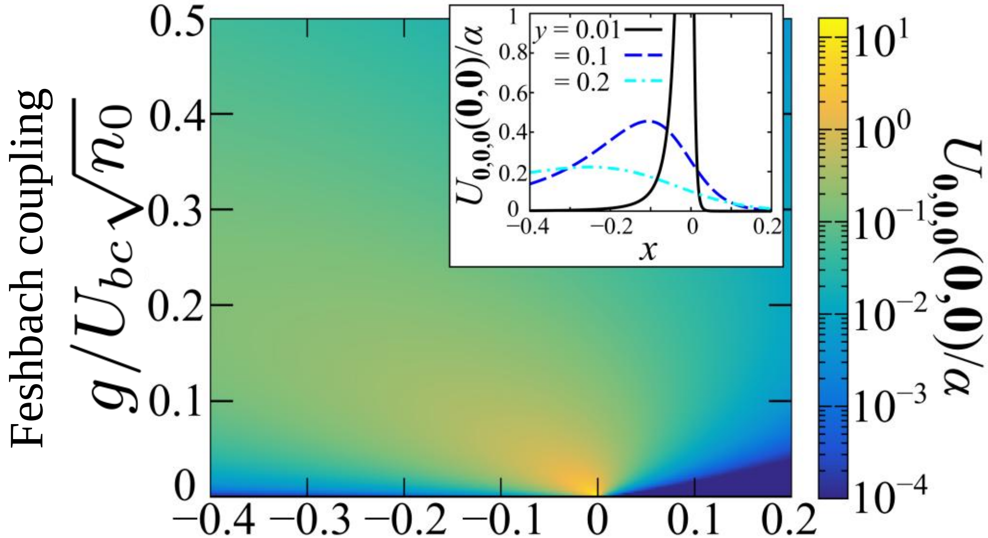
n_N: ground-state polaron density

$$U_{\mathbf{0},\mathbf{0},\mathbf{0}}(\mathbf{0},\mathbf{0}) = \frac{\alpha y^2 (1 - x/2)^2}{(x^2 + 4y^2)^{3/2}} \left(\frac{1}{2} - \frac{y^2 + x/2}{2\sqrt{x^2 + 4y^2}} \right)^2$$

Observation of fermion-mediated three-body force

K. Patel, et al., PRL **131**, 083003 (2023)

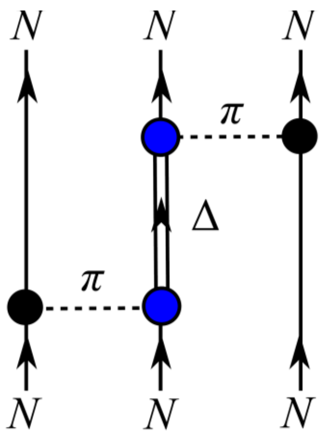




Closed-channel energy level: $\{\nu - (U_{bc} + U_{bb})n_0\}/U_{bc}n_0$

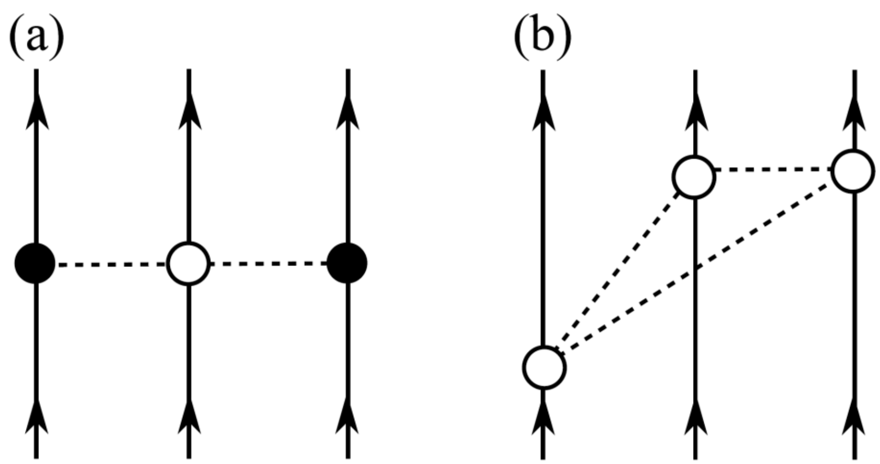
Tunable via $\nu(B)$!

Other three-body forces



$\sim O(U_{bc}^{2}\, n_0 \mathcal{G}_{\Delta})$ $\mathcal{G}_{\Delta} \simeq -\nu^{-1}$

Depending on the energy of Δ resonance $\boldsymbol{\nu}$



$\sim O(U_{bc}^{2}\, n_0)$ $\sim O(U_{bc}^{3})$ $\sim O(U_{bc}^{2} \mathcal{G}_{\Delta})$

Independent of $\boldsymbol{\nu}$ **Negilgible when $\boldsymbol{n_0 \gg 1}$**

Outline

- **Introduction**

  Tunable interactions in ultracold atoms

- **Yukawa interaction and beyond**

  Superfluid EFT $\simeq$ Chiral EFT

- **Fujita-Miyazawa-type three-body force**

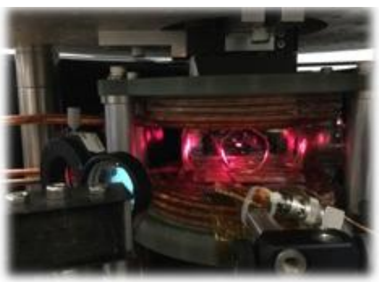
  Demonstration in Bose-Einstein condensates

- **Summary**

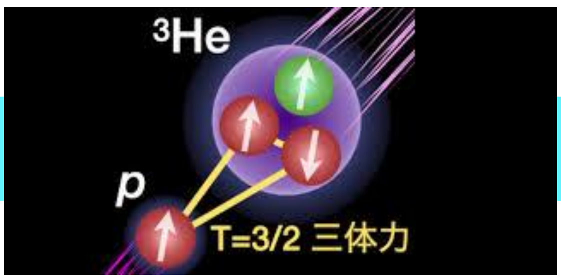
Summary

HT, E. Nakano, and K. Iida, PRA **113**, L011305 (2026).

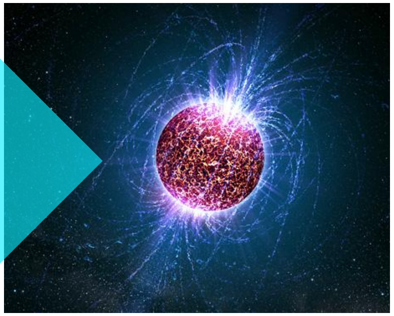
- Cold atoms are ideal platform for simulating quantum many-body systems with multi-body interactions including superconductors, nuclei, and neutron stars.
- Polarons in BEC and their interactions can mimic nuclear forces.
- Fujita-Miyazawa-type three-body force among polarons is proposed theoretically.

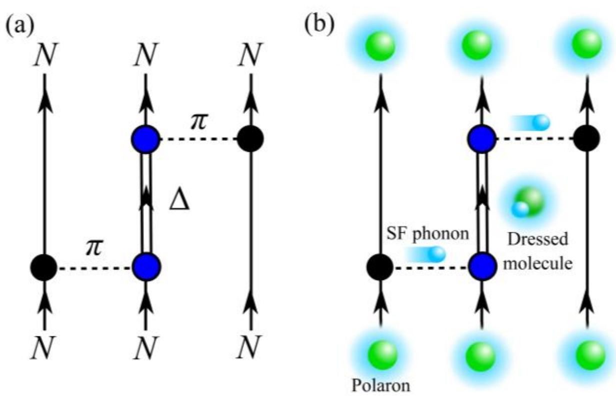


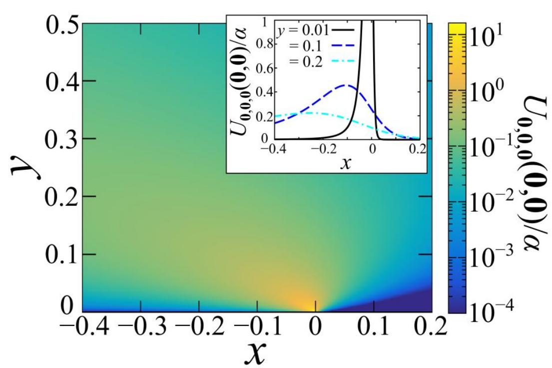
Cold atoms@OMU



Phys. Rev. C **103**, 044001 (2021).





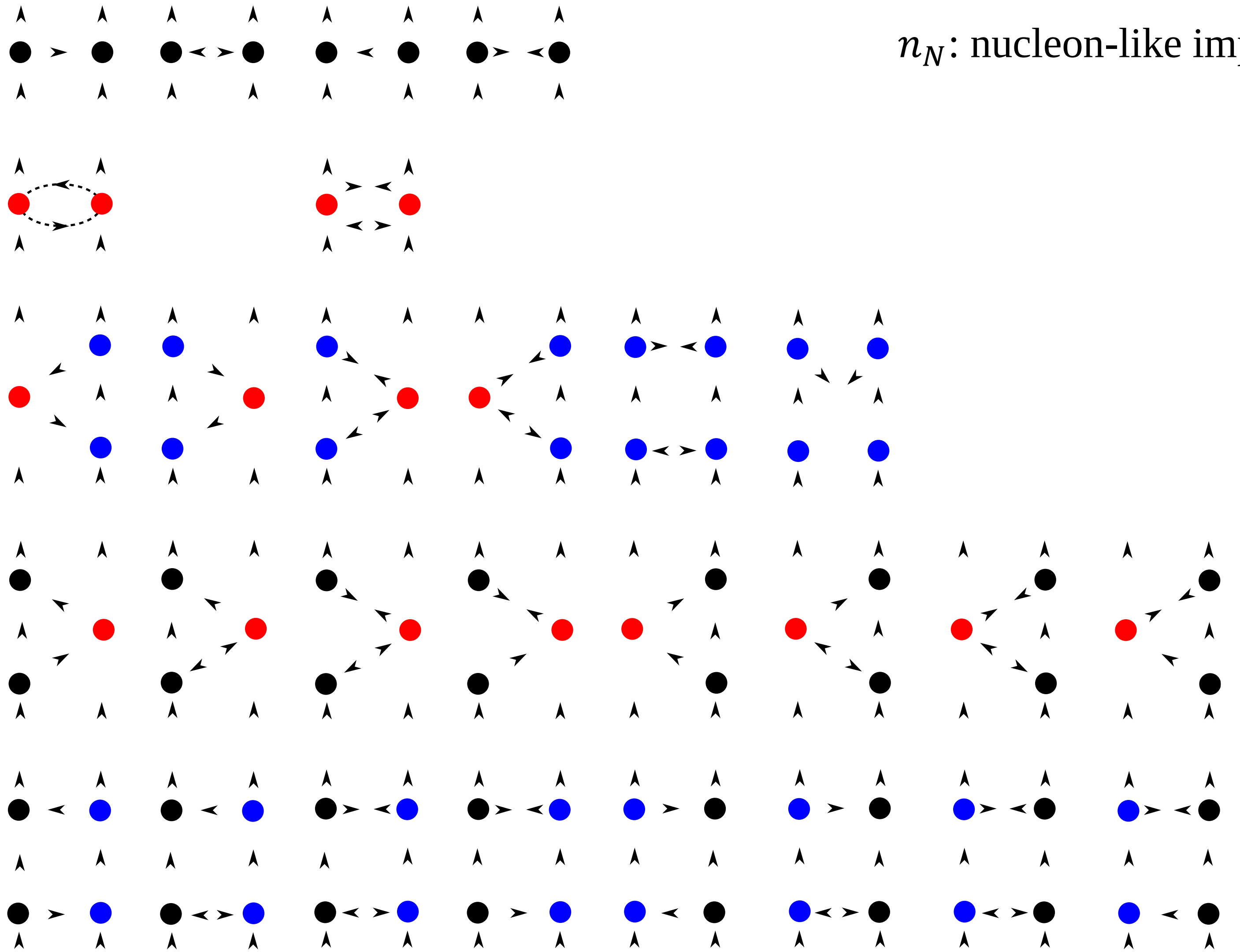


Future perspectives: exploring observables sensitive to three-body force, analog of two-body current, including spin and isospin-like degrees of freedom

Appendix

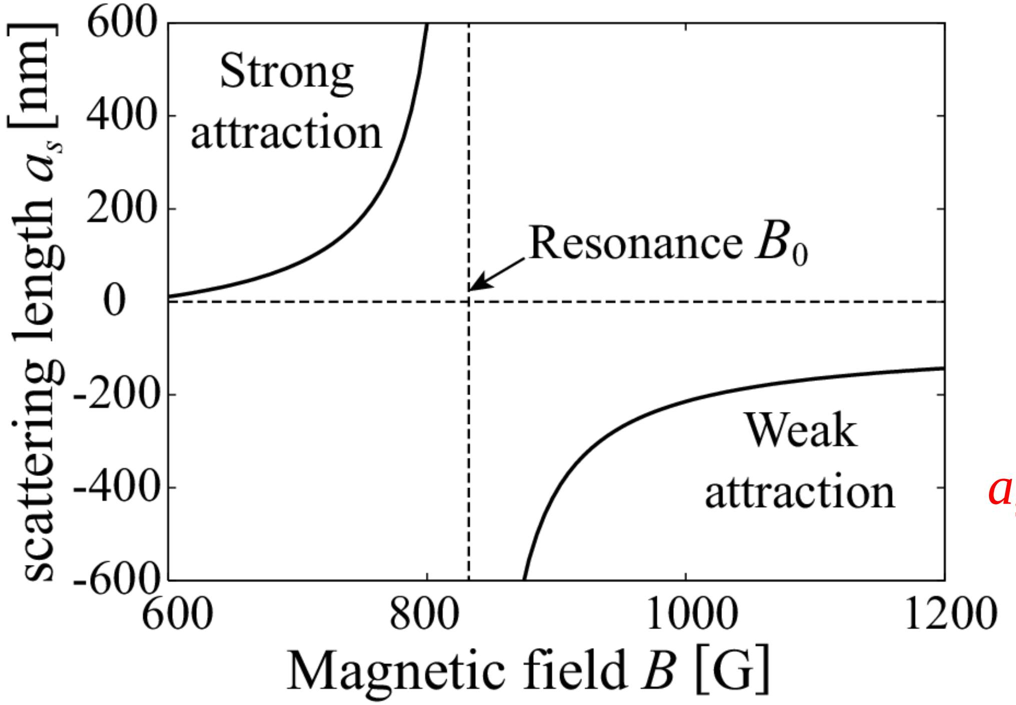
Two-body forces $\delta E \propto n_N^2$

n_N: nucleon-like impurity density



Feshbach resonance
Engineering two-body force

Feshbach resonance in a ^{6}Li Fermi gas



***B*-dependence near Feshbach resonance**

$$a_s(B) = a_{\rm bg}\left(1 + \frac{W_{\rm res}}{B - B_0}\right)$$

Resonance width $W_{\rm res} = 262.3\mathrm{G}$

Background scattering length $a_{\rm bg} = -1582 a_0$

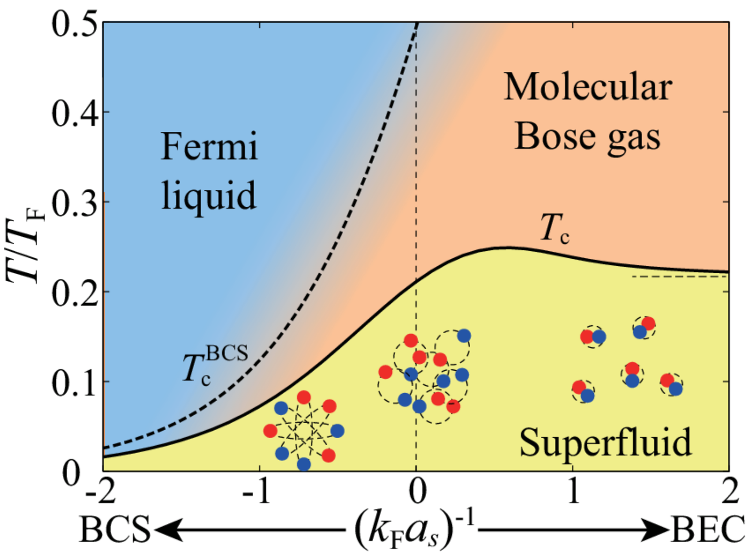
a_s can be controlled precisely by tuning B

G. Zürn, *et al.*, PRL **110**, 135301 (2013).

Strongly-interacting fermions: BCS-BEC crossover

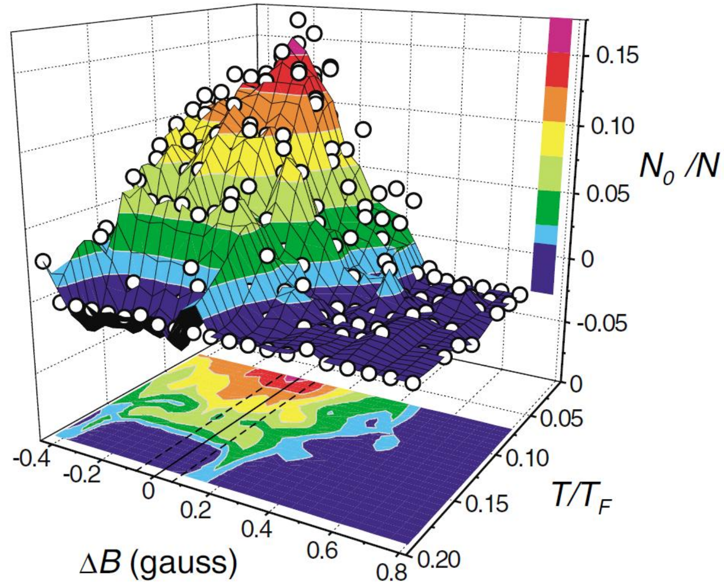
Crossover from Bardeen-Cooper-Schrieffer (BCS) superfluid to Bose-Einstein condensates (BEC) of bound molecules is realized by tuning the interaction.

Phase diagram of the BCS-BEC crossover



Y. Ohashi, HT, and P. van Wyk, Prog. Part. Nucl. Phys. **111**, 103739 (2020).

Observation of BCS-BEC crossover in a ^{40}K Fermi gas



C. Regal, *et al.*, PRL **92**, 040403 (2004).

Promising system for simulating strongly-interacting fermions such as nuclear matter and high-T_c superconductors

Realization of tunable three-body interaction in cold atoms

A. Hammond, *et al.*, Phys. Rev. Lett. **128**, 083401 (2022)

EOS in Rabi-coupled 2-com. 1D BEC

$$\frac{E_{\rm MF}}{V} = -\frac{\hbar\Omega}{2}(\phi_\uparrow^*\phi_\downarrow + \phi_\downarrow^*\phi_\uparrow) + \frac{\hbar\delta}{2}(|\phi_\uparrow|^2 - |\phi_\downarrow|^2) + \sum_{\sigma\sigma'} \frac{g_{\sigma\sigma'}}{2}|\phi_\sigma|^2|\phi_{\sigma'}|^2.$$

Low-energy EFT

$$\frac{E_{\rm MF}}{N} \approx \epsilon_- + g_2\frac{n}{2} + g_3\frac{n^2}{3}$$

with $$g_2 = g - \frac{\bar{g}}{1+\delta^2/\Omega^2}$$

and $$g_3 = -\frac{3\bar{g}^2}{\hbar\Omega}\frac{\delta^2/\Omega^2}{(1+\delta^2/\Omega^2)^{5/2}}$$

Breezing mode frequency

$$\omega_b = 2\omega_\perp\sqrt{1 + E_3/E_{\rm pot}}$$