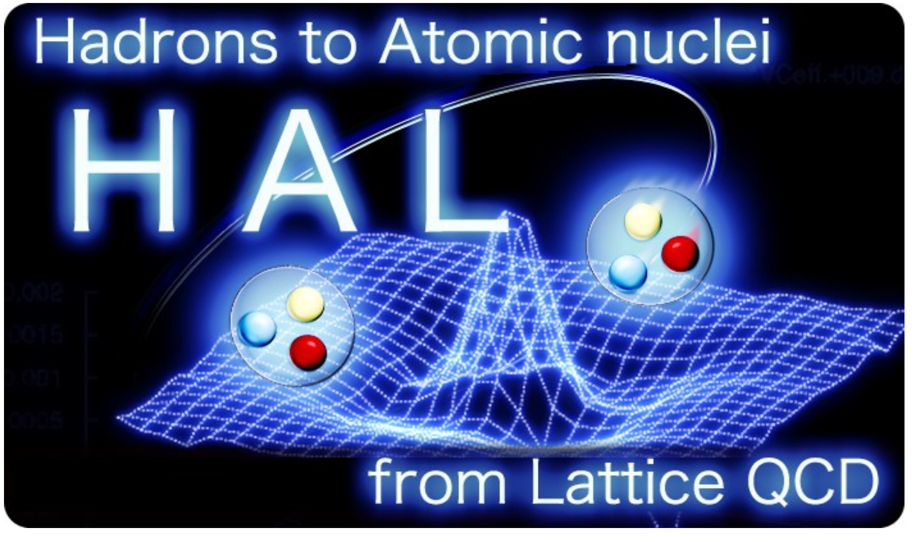
量子色力学に基づいたストレンジネス核物理

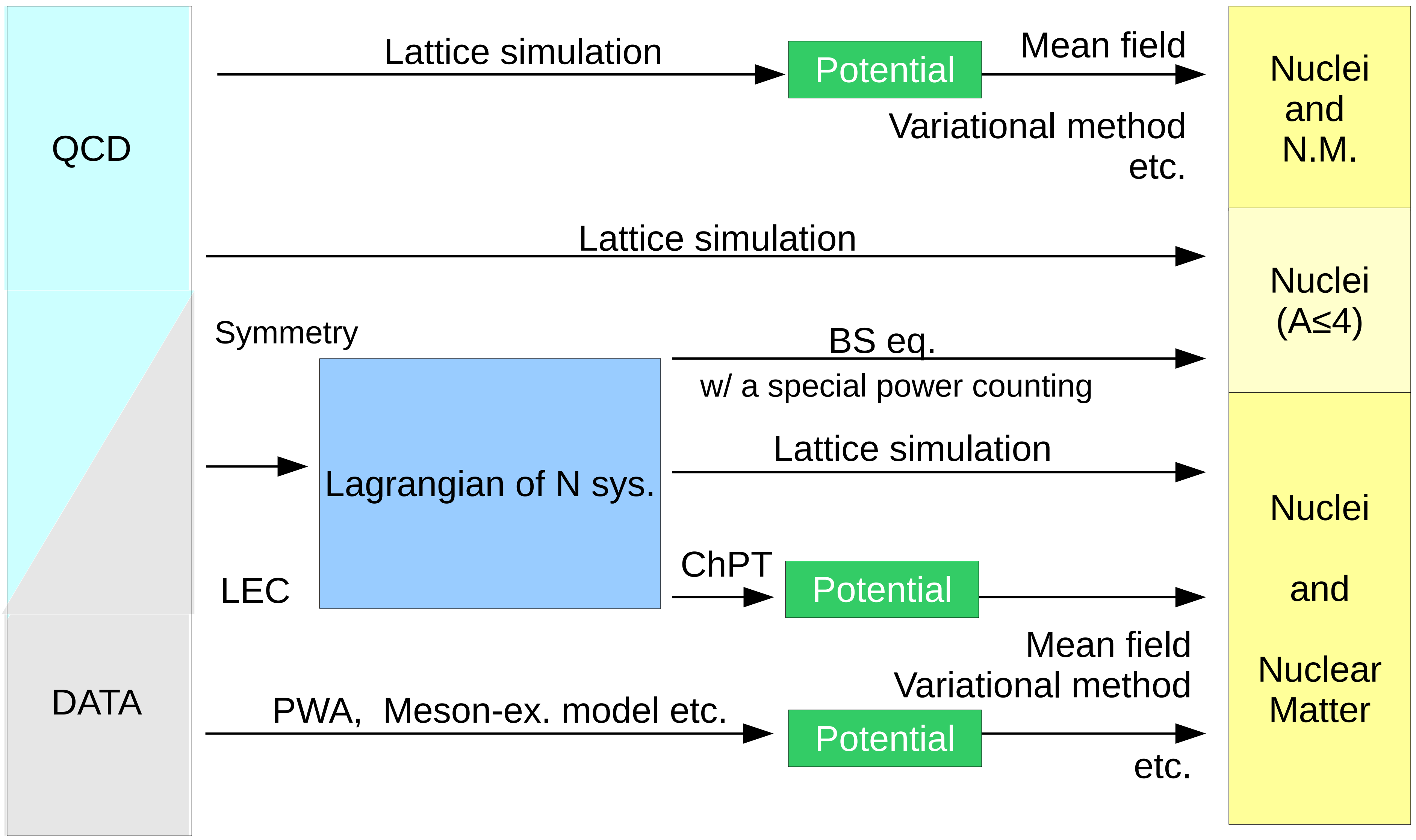
日本大学生物資源科学部　　井上貴史

HALQCD Collaboration



核力・ハドロン間相互作用と量子多体計算の発展, Feb 16-18, 2026, YITP

Various approaches in nuclear phys.



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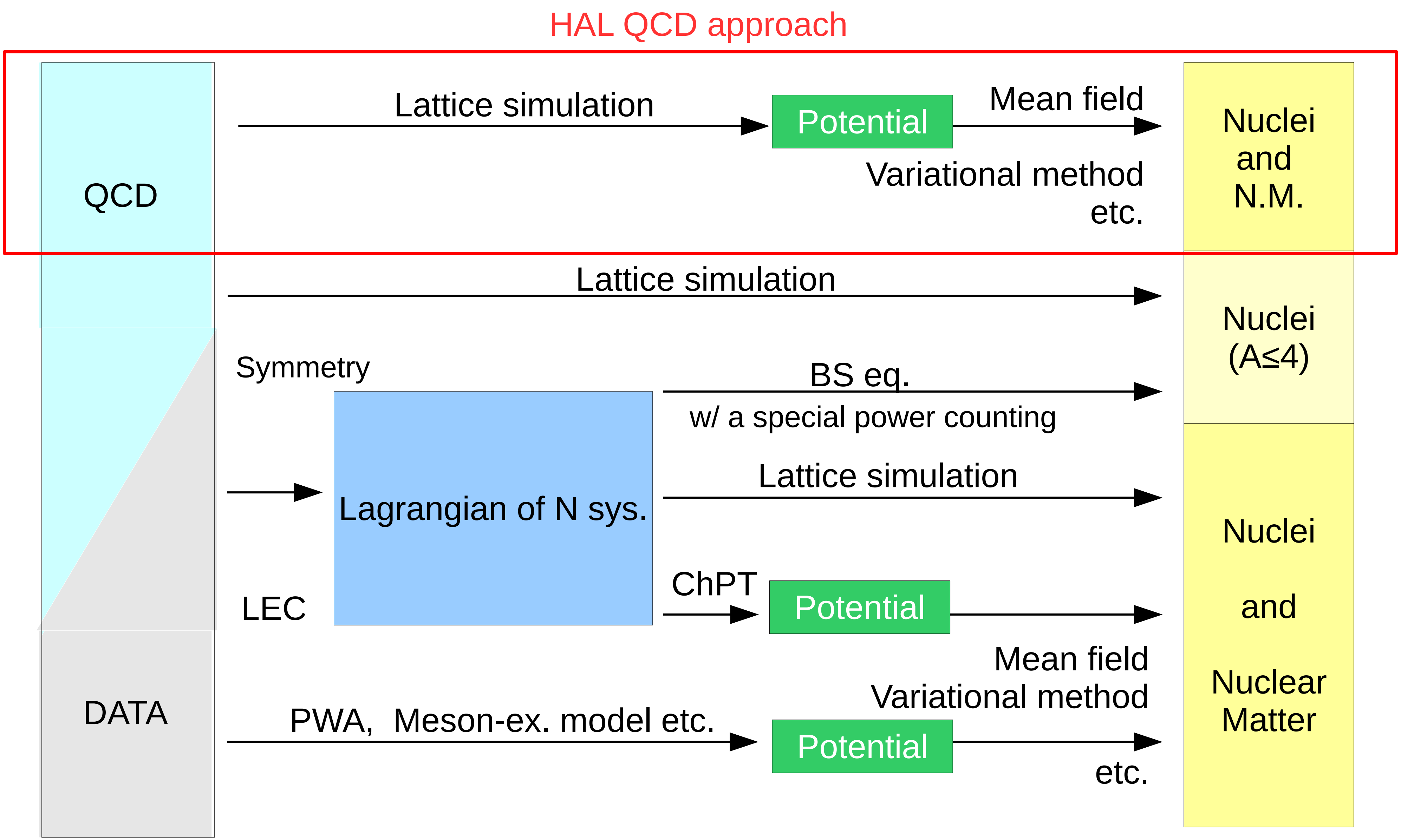


Our approach. Two step approach as traditional one.

HAL QCD Approach

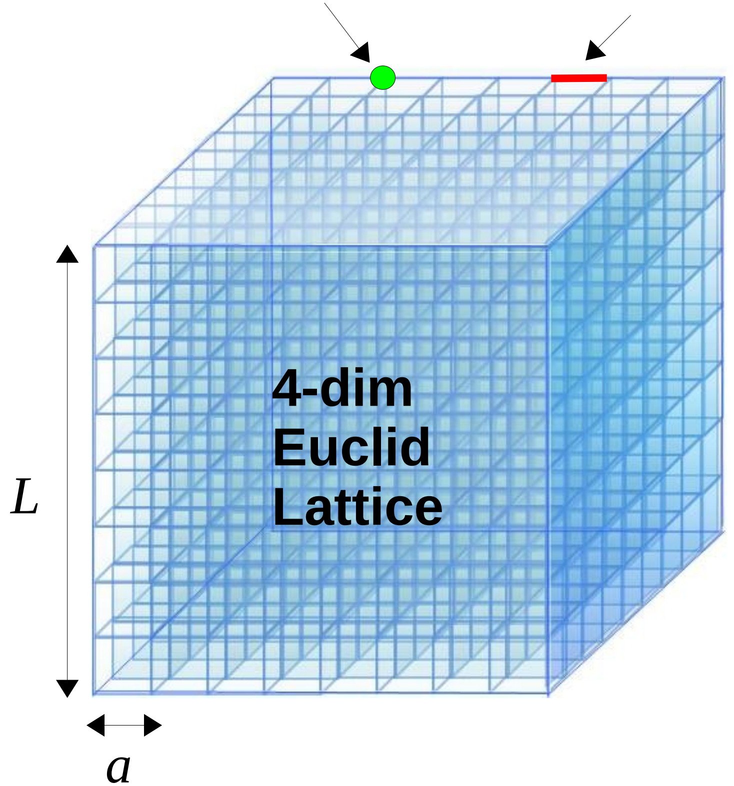
- Strong points
  - Based on the fundamental theory QCD, hence provides information independent of experiments and models.
  - Feasible. ↔ One step approach is very hard for (large) nuclei.
  - Can utilize established nuclear theories at the 2nd stage.
  - Easy to extend to strange sector, charm sector etc.

Lattice QCD

$$L = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \bar{q}\gamma^\mu \left(i\partial_\mu - g t^a A^a_\mu \right) q - m\bar{q}q$$

quarks q
on the sites

gluons $U = e^{iaA_\mu}$
on the links



Vacuum expectation value

$$\langle O(\bar{q},q,U) \rangle$$

path integral

$$= \int dU\, d\bar{q}\, dq\, e^{-S(\bar{q},q,U)}\, O(\bar{q},q,U)$$

$$= \int dU \det D(U)\, e^{-S_U(U)}\, O(D^{-1}(U))$$

quark propagator

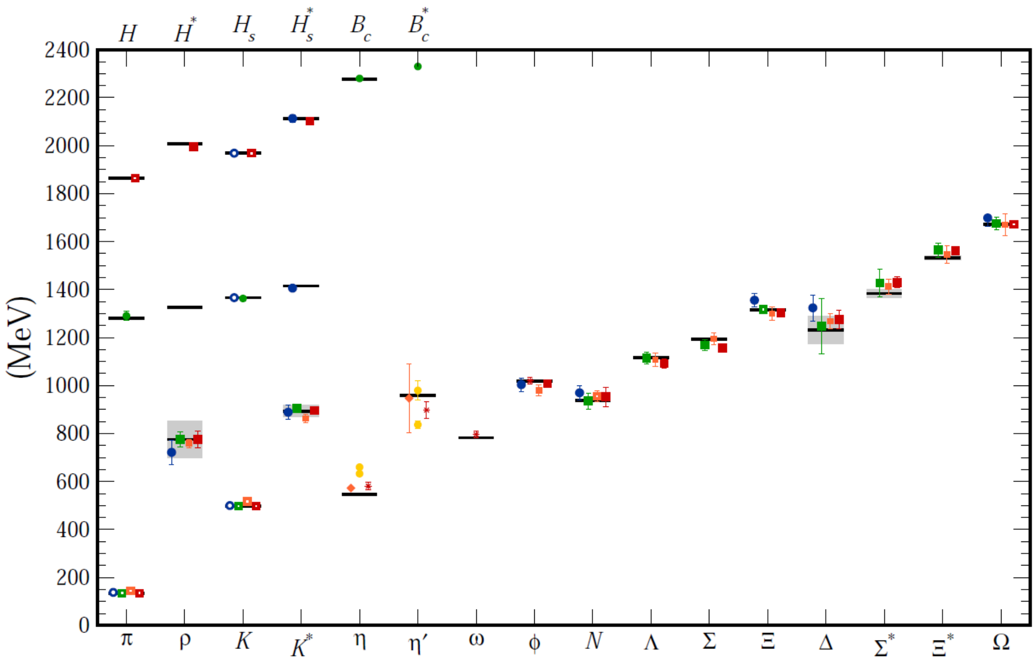
$$= \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} O(D^{-1}(U_i))$$

$\{ U_i \}$: ensemble of gauge conf. U
generated w/ probability of $\det D(U)\, e^{-S_U(U)}$

- Well defined (regularized)
- Manifest gauge invariance
- Fully non-perturbative
- Highly predictive

Lattice QCD

- LQCD simulations w/ the physical quark ware done.
  - PACS-CS, Phys. Rev. D81 (2010) 074503
  - BMW, JHEP 1108 (2011) 148



Summary by Kronfeld,
arXive 1203.1204

- Mass of (ground state) hadrons are well reproduced!

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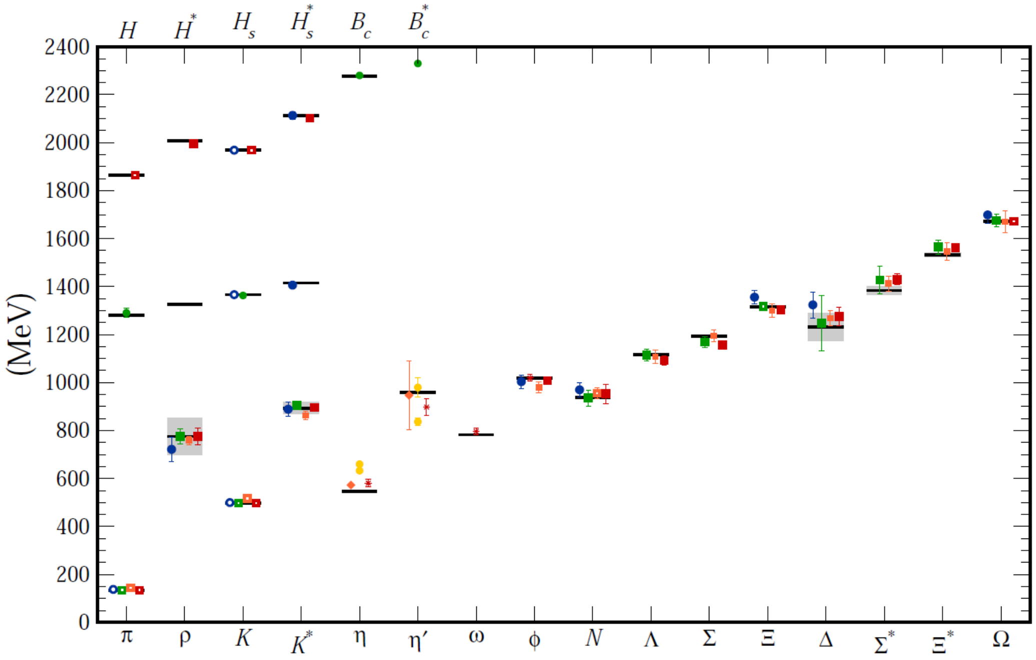

Summary by Kronfeld,
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- Mass of (ground state) hadrons are well reproduced!
- Now, most important challenge is hadron interactions.

HAL QCD method

Multi-hadron in LQCD

Few-hadron system

- Direct : utilize temporal correlator and eigen-energy
  - Lüscher's finite volume method for phase-shifts
  - Infinite volume extrapolation method for bound states
- HAL : utilize spatial correlation and "potential" $V(r)$ + ...

1 chann., S-wave, leading

$$V(\vec{r}) = \frac{1}{2\mu}\frac{\nabla^2\psi(\vec{r},t)}{\psi(\vec{r},t)} - \frac{\frac{\partial}{\partial t}\psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B$$

$\psi(\vec{r},t)$: 4-point function contains NBS w.f.

  - Advantages
    - No need to separate E eigenstate. Just need to measure
    - Then, potential can be extracted.
    - Demand a minimal lattice volume. No need to extrapolate to V=∞.
    - Can output many observables.

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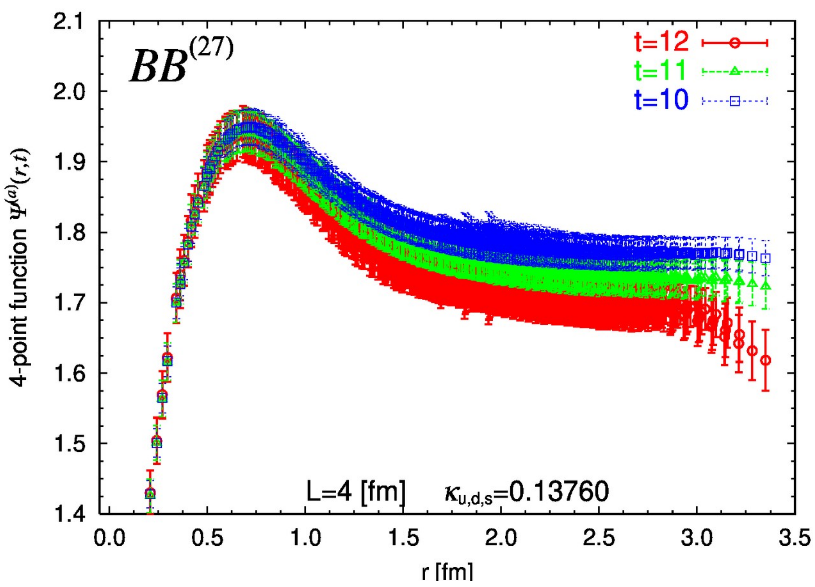

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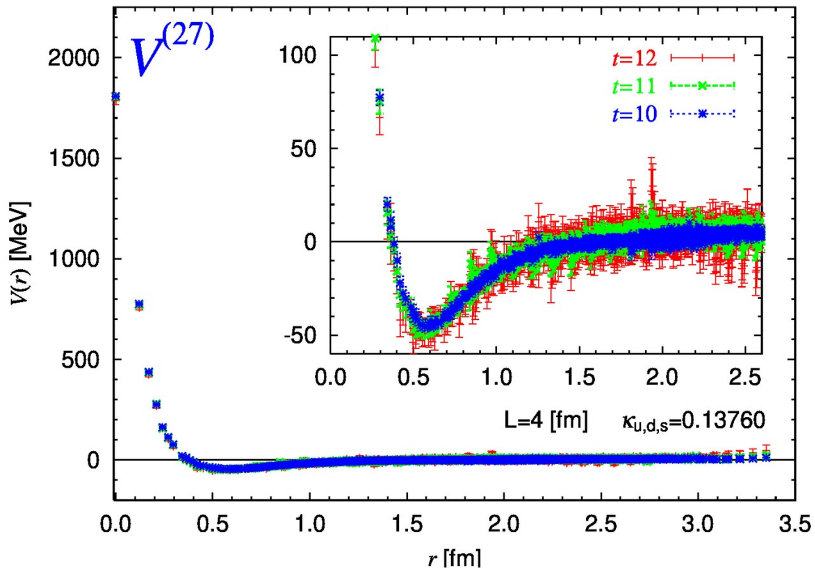

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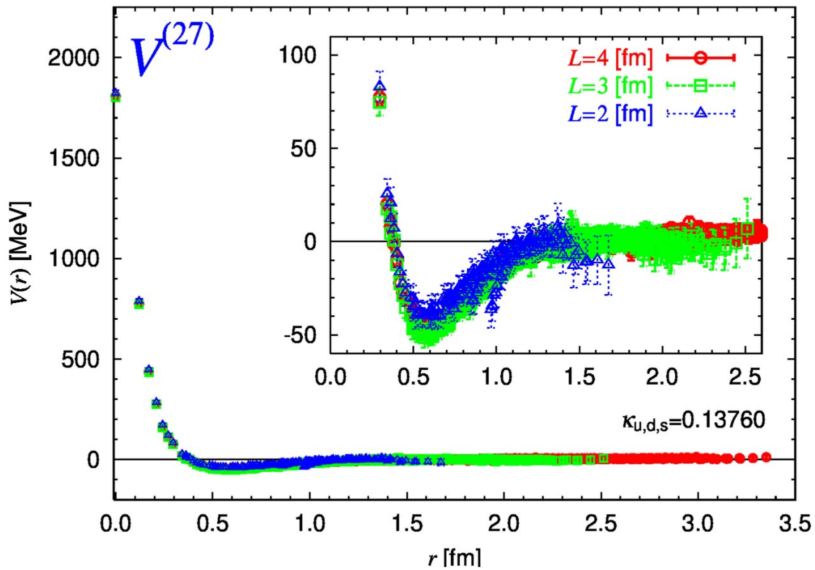

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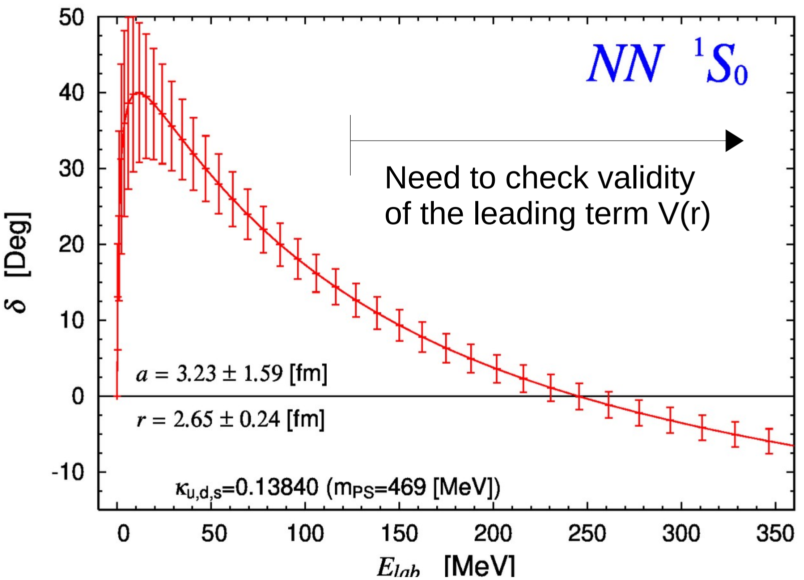

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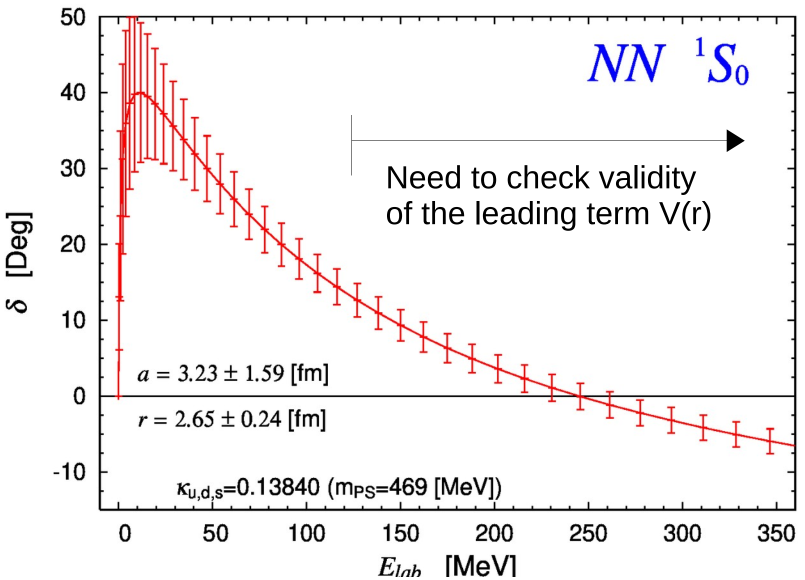


★ We can attack nuclei, hyper-nuclei, and NS from QCD

HAL method

S. Aoki, T. Hatsuda, N. Ishii, Prog. Theo. Phys. 123 89 (2010)
N. Ishii etal. [HAL QCD coll.] Phys. Lett. B712 , 437 (2012)

NBS wave function $\phi_{\vec{k}}(\vec{r}) = \sum_{\vec{x}} \langle 0 | B_i(\vec{x}+\vec{r},t) B_j(\vec{x},t) | B=2, \vec{k} \rangle$

Define a common "potential" U for all E eigenstates via "Schrödinger" eq.

$$\left[-\frac{\nabla^2}{2\mu} \right] \phi_{\vec{k}}(\vec{r}) + \int d^3\vec{r}\,' U(\vec{r},\vec{r}\,') \phi_{\vec{k}}(\vec{r}\,') = E_{\vec{k}} \phi_{\vec{k}}(\vec{r})$$

Non-local but
energy independent
below inelastic threshold

Measure 4-point function in LQCD

$$\psi(\vec{r},t) = \sum_{\vec{x}} \langle 0 | B_i(\vec{x}+\vec{r},t) B_j(\vec{x},t) \; J(t_0) | 0 \rangle = \sum_{\vec{k}} A_{\vec{k}} \phi_{\vec{k}}(\vec{r}) e^{-W_{\vec{k}}(t-t_0)} + \cdots$$

$$\left[2M_B - \frac{\nabla^2}{2\mu} \right] \psi(\vec{r},t) + \int d^3\vec{r}\,' U(\vec{r},\vec{r}\,') \psi(\vec{r}\,',t) = -\frac{\partial}{\partial t} \psi(\vec{r},t)$$

∇ expansion & truncation

$$U(\vec{r},\vec{r}\,') = \delta(\vec{r}-\vec{r}\,') V(\vec{r},\nabla) = \delta(\vec{r}-\vec{r}\,') [V(\vec{r}) + \sout{\nabla + \nabla^2} \ldots]$$

Therefor, in the leading

$$V(\vec{r}) = \frac{1}{2\mu} \frac{\nabla^2 \psi(\vec{r},t)}{\psi(\vec{r},t)} - \frac{\frac{\partial}{\partial t}\psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B$$

Source and sink operator

- NBS wave function and 4-point function

$$\phi_{\vec{k}}(\vec{r}) = \sum_{\vec{x}} \langle 0 | B_i(\vec{x}+\vec{r}, \overbrace{t) B_j(\vec{x}, t}^{\text{equal}}) | B=2, \vec{k} \rangle \quad \text{QCD eigenstate}$$

$$\psi(\vec{r}, t) = \sum_{\vec{x}} \langle 0 | \underbrace{B_i(\vec{x}+\vec{r}, t) B_j(\vec{x}, t)}_{\text{sink}} \underbrace{J(t_0)}_{\text{source}} | 0 \rangle = \sum_{\vec{k}} A_{\vec{k}} \phi_{\vec{k}}(\vec{r}) e^{-W_{\vec{k}}(t-t_0)} + \cdots$$

- Point type octet baryon field operator at sink

$$p_\alpha(\underline{x}) = \epsilon_{c_1 c_2 c_3} (C\gamma_5)_{\beta_1\beta_2} \delta_{\beta_3\alpha}\, u(\xi_1) d(\xi_2) u(\xi_3) \quad \text{with } \xi_i = \{c_i, \beta_i, \underline{x}\}$$

$$\Lambda_\alpha(x) = -\epsilon_{c_1 c_2 c_3} (C\gamma_5)_{\beta_1\beta_2} \delta_{\beta_3\alpha} \sqrt{\frac{1}{6}} \left[d(\xi_1) s(\xi_2) u(\xi_3) + s(\xi_1) u(\xi_2) d(\xi_3) - 2u(\xi_1) d(\xi_2) s(\xi_3) \right]$$

- Wall type quaruk source of two-baryon state

e.g. $$\overline{BB}^{(1)} = -\sqrt{\frac{1}{8}}\, \overline{\Lambda}\,\overline{\Lambda} + \sqrt{\frac{3}{8}}\, \overline{\Sigma}\,\overline{\Sigma} + \sqrt{\frac{4}{8}}\, \overline{N}\,\overline{\Xi} \quad \text{for flavor-singlet}$$

Baryon interactions from QCD

LQCD simulation

- We did LQCD simulation on K-computer

Previous !

- Nf = 2+1 full QCD
  - Clover fermion + Iwasaki gauge w/ stout smearing
  - Volume $96^4 \simeq (8\text{ fm})^4$ large enough to accommodate *BB* interaction
  - $1/a = 2333$ MeV, $a = 0.0845$ fm

K-configuration

  - $M_\pi \simeq 146$, $M_K \simeq 525$ MeV almost physical point
    $M_N \simeq 956$, $M_\Lambda \simeq 1121$, $M_\Sigma \simeq 1201$, $M_\Xi \simeq 1328$ MeV
  - Collaboration in HPCI Strategic Program Field 5 Project 1

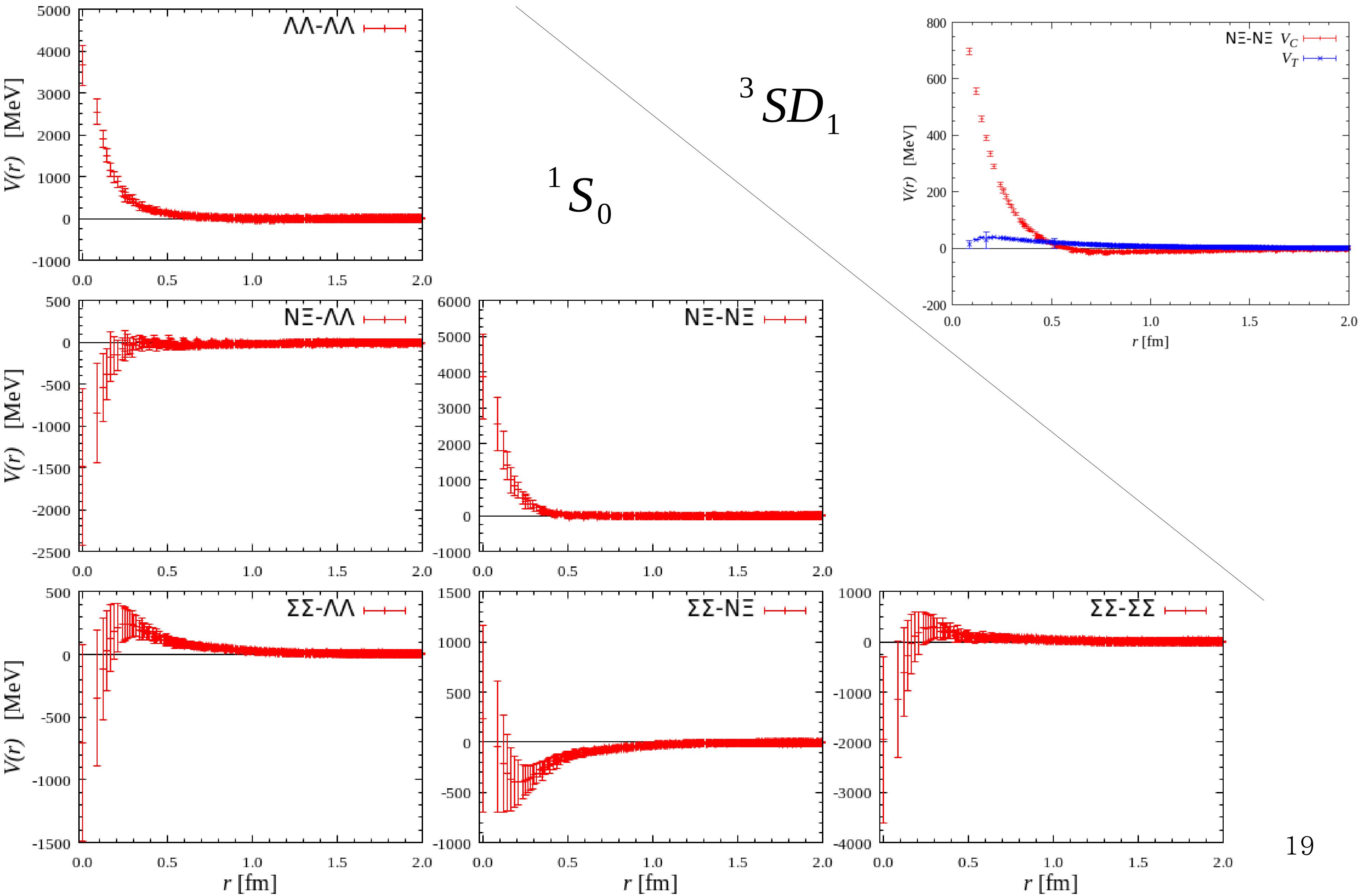
- Extracted

S/N better →

  - All S-wave *BB* interactoin potentials in S=0,-1,-2,-3,-4 ...
  - S= 0, NN pot, T. Doi, EPJ Web Conf. 175 (2018) 05009
  - S=-1, YN pot, H. Nemura, AIP Conf. Proc. 2130, 040005 (2019)
  - S=-2, XN,YY pot, K. Sasaki, Nucl.Phys.A 998 (2020) 121737
  - S=-3, XY pot, N. Ishii, EPJ Web Conf. 175 (2018) 05013
  - S=-4, XX pot, T. Doi, EPJ Web Conf. 175 (2018) 05009

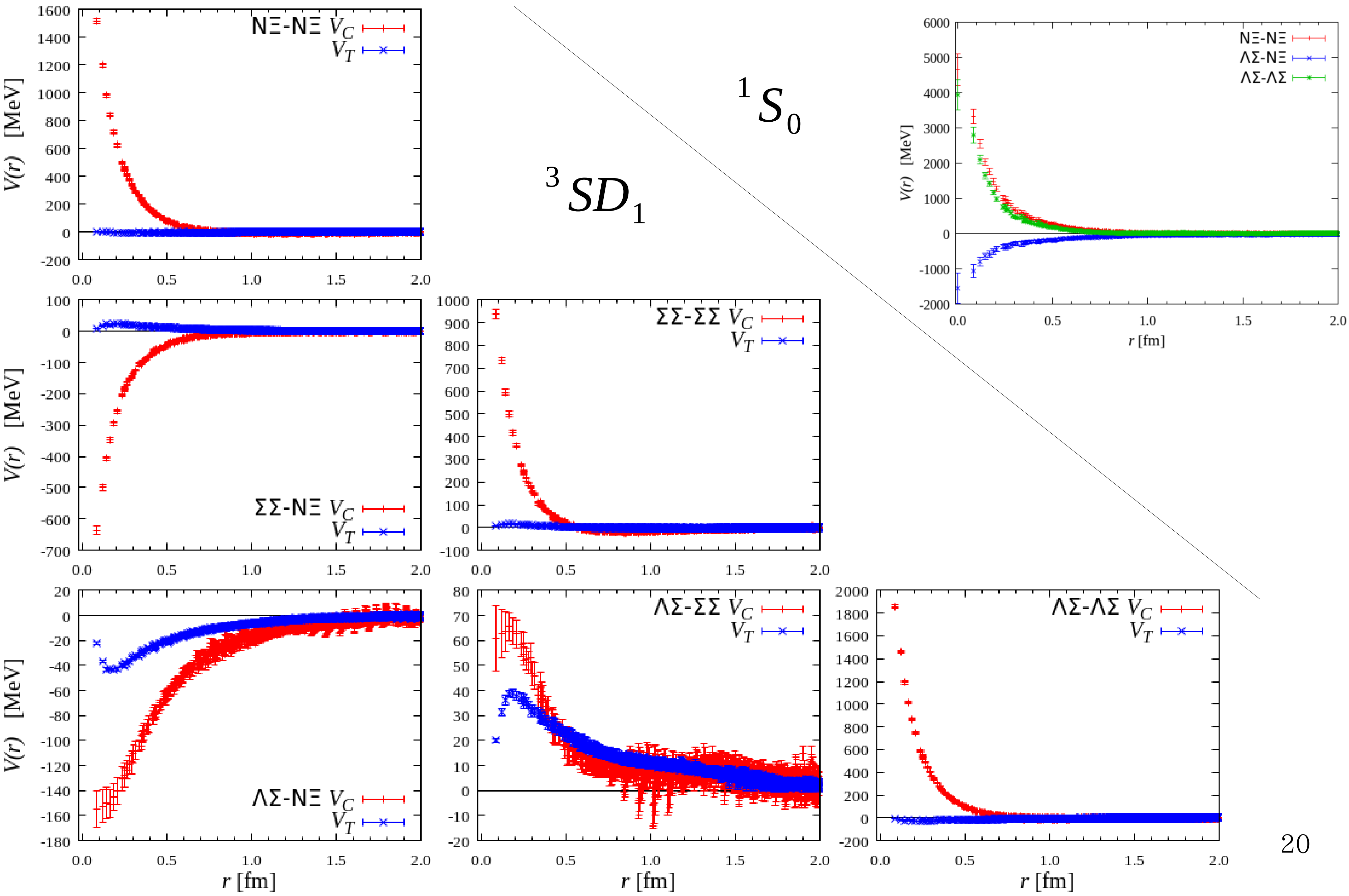
S=−2, I=0, BB potentials

(96,96) src
$t{-}t_0$ = 12



S=−2, I=1, BB potentials

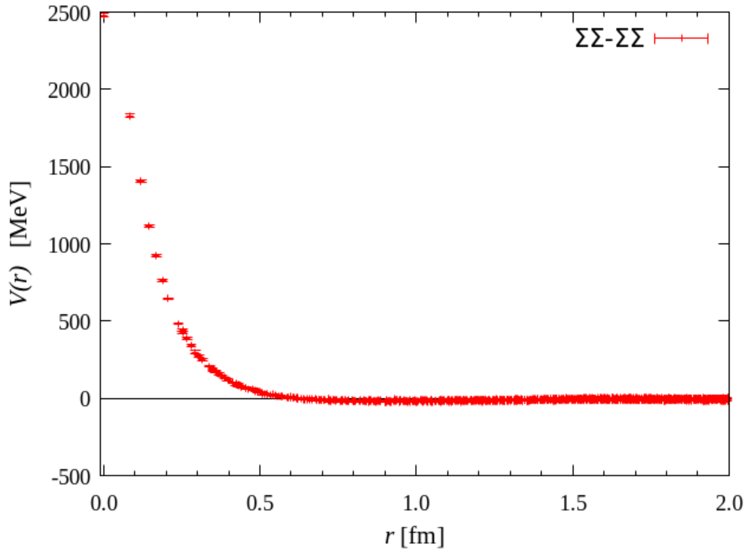
(96,96) src
t−t0 = 12



S=−2, I=2, BB potential

(96,96) src
$t-t_0$ = 12

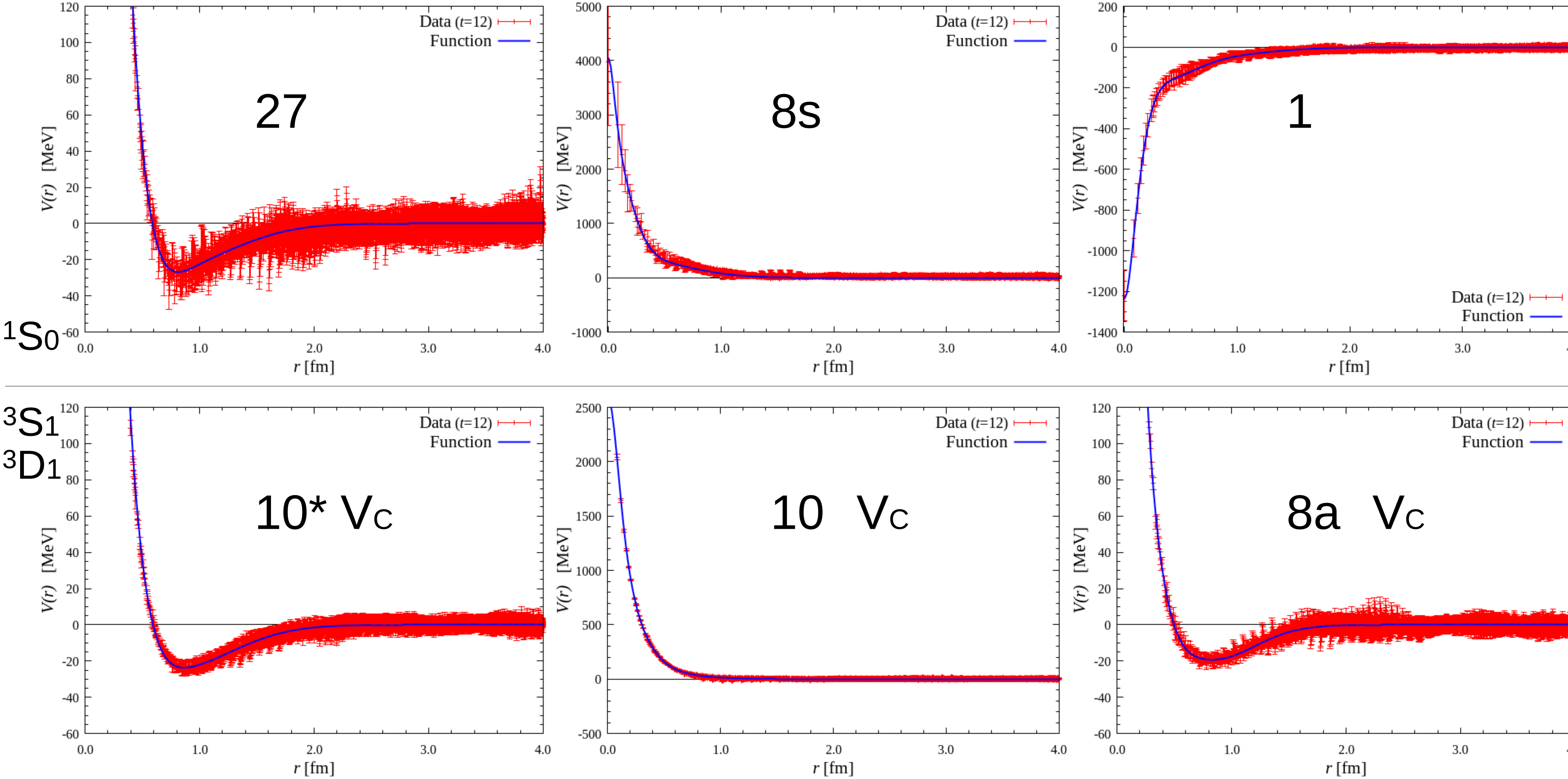
$^1S_0$



- Thus, we can extract BB interactions from QCD on lattice which are difficult to access experimentally.
- We can construct flavor SU(3) version of BB interaction by rotating these data into the flavor irr.-rep. basis once and then rotating back, with the CG coefficient of SU(3).
- The SU(3)$_F$ symmetric version is more covenient to apply since number of independent potential is small.

BB S-wave potentials

(96,96) src
$t-t_0 = 12$



$$8 \times 8 = \underbrace{27 + 8s + 1}_{^1S_0} + \underbrace{10^* + 10 + 8a}_{^3S_1,\ ^3D_1}$$

BB S-wave potentials

(96,96) src
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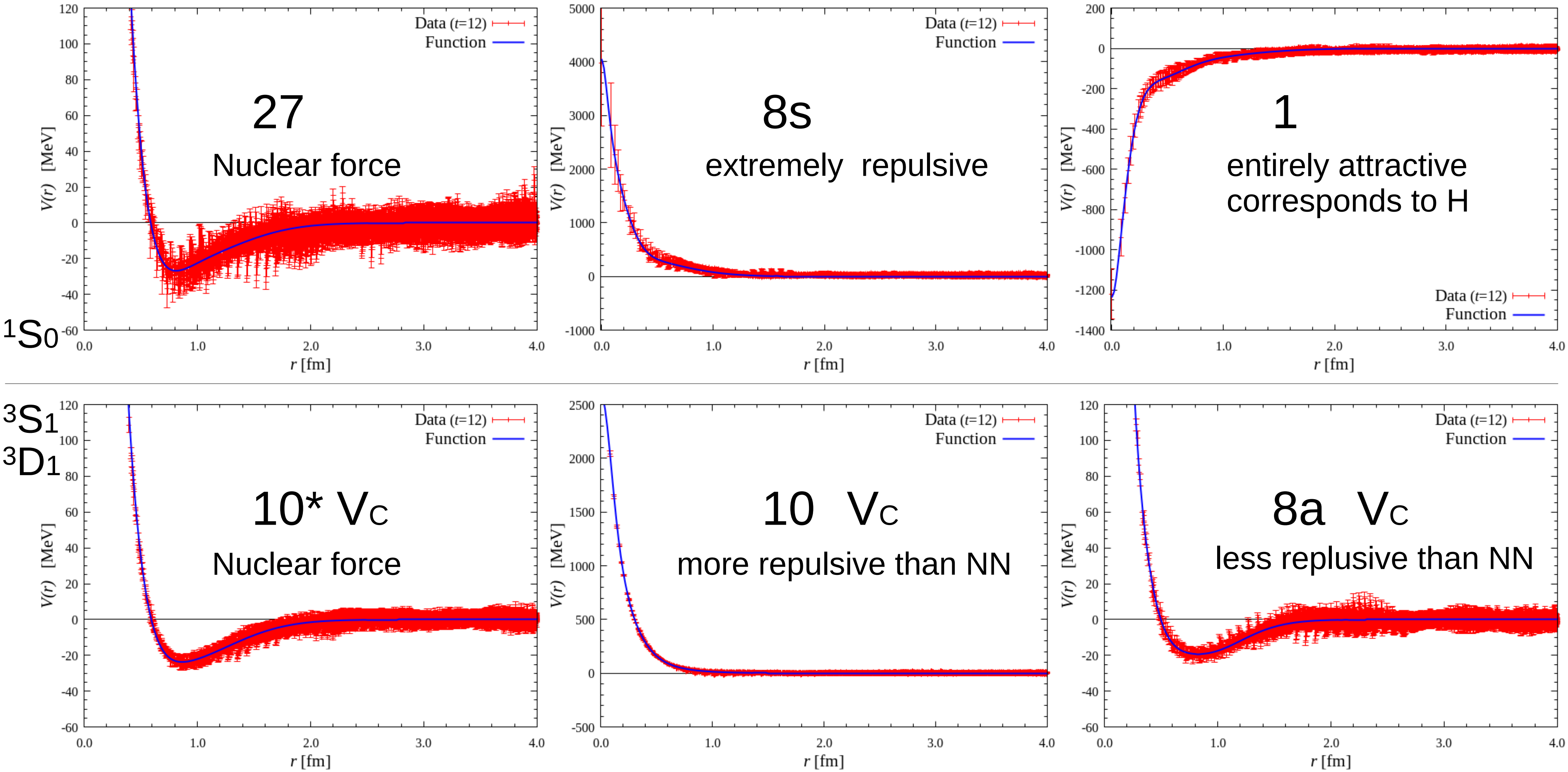
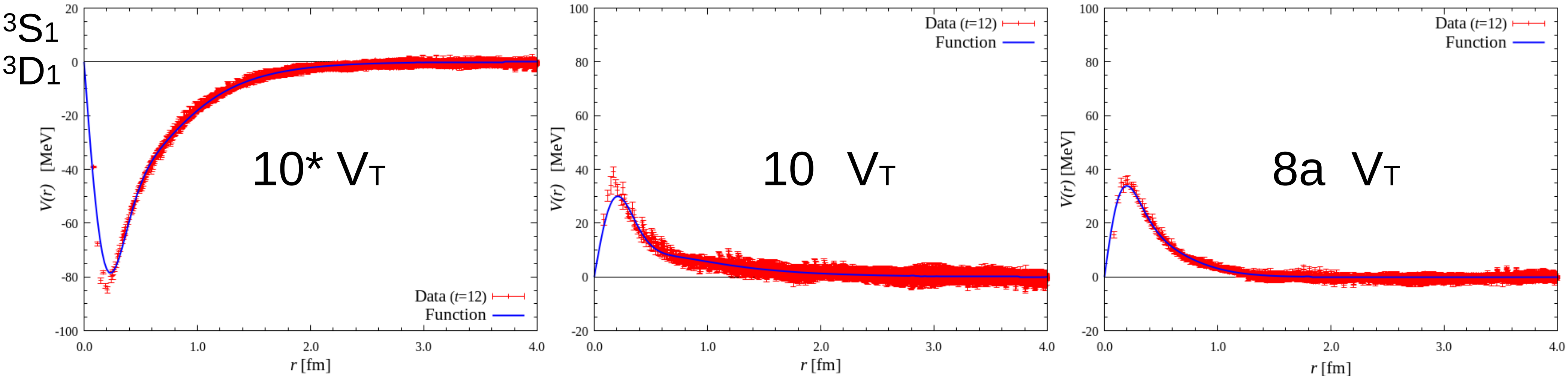


- Qualitatively reasonable NN forces are obtained from QCD.
- Features can be understood by the quark Pauli + OGE.

e.g. Oka, Shimizu, Yazaki, Nucl. Phys. A464 (1987)

BB S-wave potentials

(96,96) src
$t - t_0 = 12$



- Functions fitted to data

$$V_C(r) = a_1\, e^{-a_2 r^2} + a_3\, e^{-a_4 r^2} + a_5 \left[\left(1 - e^{-a_6 r^2}\right) \frac{e^{-a_7 r}}{r} \right]^2$$

$$V_T(r) = a_1 \left(1 - e^{-a_2 r^2}\right)^2 \left(1 + \frac{3}{a_3 r} + \frac{3}{(a_3 r)^2}\right) \frac{e^{-a_3 r}}{r} + a_4 \left(1 - e^{-a_5 r^2}\right)^2 \left(1 + \frac{3}{a_6 r} + \frac{3}{(a_6 r)^2}\right) \frac{e^{-a_6 r}}{r}$$

- Since $SU(3)_F$ is broken at the physical point (K-conf.), there exist weak flavor-base off-diagonal potentials.
- But, to begin with, let's ignore them, and apply V_{YN}, V_{YY} constructed with these diagonal potentials only.

Application of hyperon forces

Hyperon single particle potential

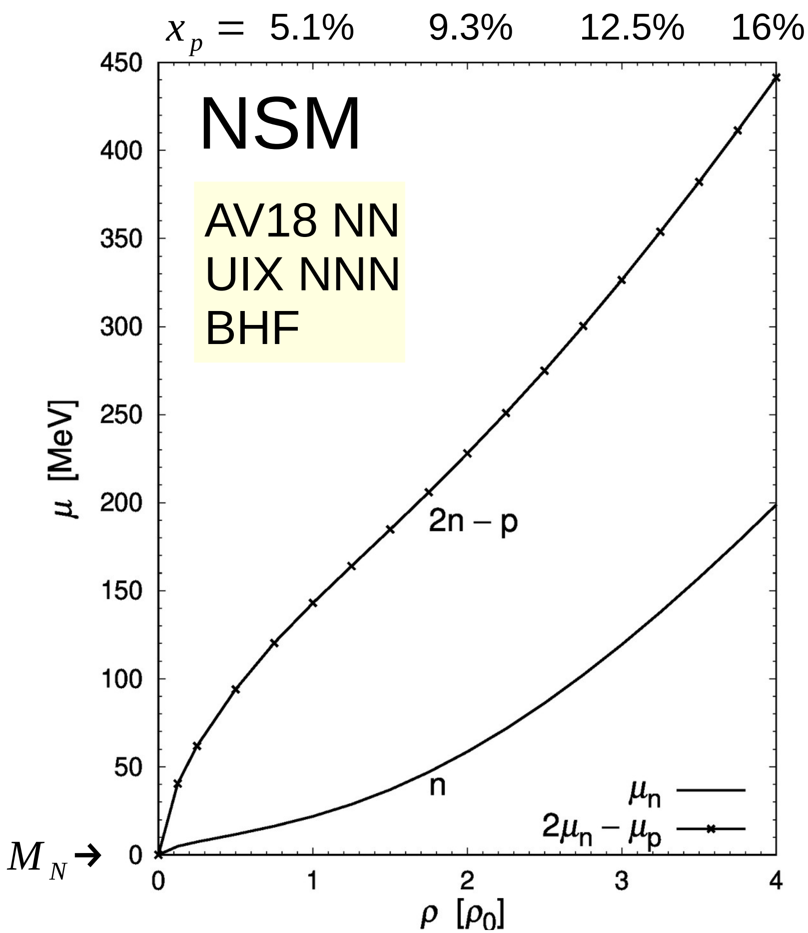
- As I expalined already, we have BB inteactions extraced from QCD, including hyperon forces.
  - measure h-h 4pt func. in lattice QCD simulation.
  - define & extract interaction “potential” from 4pt.

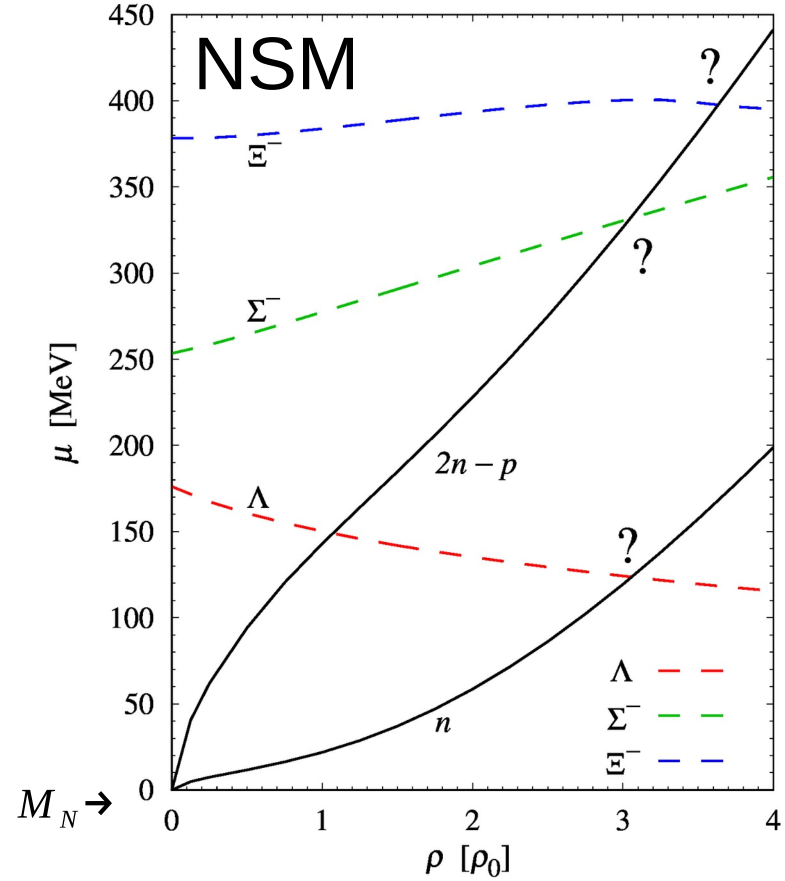
HALQCD method

- So, let us study hyperons in nuclear matter on the bassis of the hyperon forces extracted from QCD.
  - We calculate hyperon single-particle potential $U_Y(k;\rho)$
  - defined by $$e_Y(k;\rho) = \frac{k^2}{2M_Y} + U_Y(k;\rho)$$

    $e_Y(k;\rho)$: sepectrum in matter
  - U_Y is crucial for hyperon chemical potential.

Hyperon onset in Neutron Stars





- Neutron Star Matter
  ANM + e^-, μ^-
  at $Q=0$, β-eq.

- Hyperon chemical pot. in NSM

$$\mu_Y(\rho) \simeq M_Y - M_N + U_Y^{ANM}(0;\rho)$$

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- Hypernuclear experiment suggest that

$@\rho = 0.17\,[\mathrm{fm}^{-3}]$
$x = 0.5$

$$U_\Lambda^{\mathrm{Exp}}(0) \simeq -30, \quad U_\Sigma^{\mathrm{Exp}}(0) \geq +20\ ? \quad U_\Xi(0)^{\mathrm{Exp}} \simeq -10\ ??, \quad [\mathrm{MeV}]$$

attraction | repulsion | small attractionl

Theory for nuclear matter

- N.M. = uniform matter consisting an infinite number of nucleons interacting each other via nuclear force

excluding coulomb

- Brueckner Hartree Fock
  - K.A. Brueckner and J.L.Gammel Phys. Rev. 109 (1958) 1023
- Relativistic Mean Field
  - J. D. Walecka, Ann. Phys. 83 (1974) 491
- Fermi Hyper-Netted Chain
  - A. Akmal, V.R. Phandharipande, D.G. Ravenhall Phys. Rev. C 58 (1998) 1804
- Quantum Monte Carlo
  - J. Carlson, J. Morales, V.R. Pandharipande, D.G. Ravenhall, Phys. Rev. C68(2003) 025802
- Self-consistent Green's function
  - W. H. Dickhoff, C. Barbieri, Prog. Part. Nucl. Phys. (2004),377
- Cupled Cluster
  - G.Baardsen, A. Ekstrom, G.Hagen, M.Hjorth-Jensen, Phys. Rev. C88(2013)

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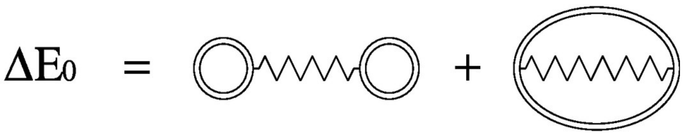
Brueckner-Hartree-Fock LOBT

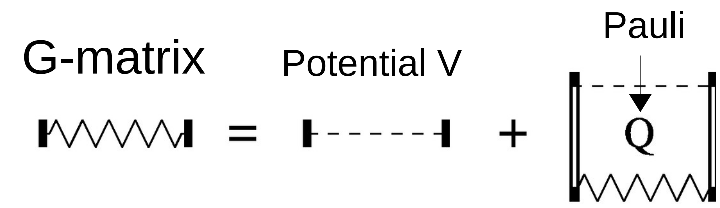
M.I. Haftel and F. Tabakin, Nucl. Phys. A158(1970) 1-42

- Ground state energy in BHF framework

$$E_0 = \gamma \sum_{k}^{k_F} \frac{k^2}{2M} + \frac{1}{2} \sum_{i}^{N_{ch}} \sum_{k,k'}^{k_F} \mathrm{Re} \langle G_i(e(k)+e(k')) \rangle_A$$

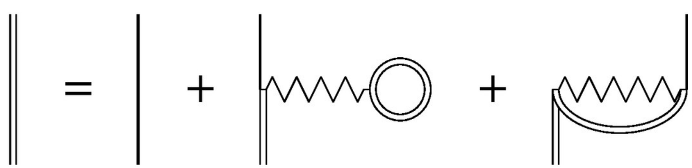


- Bethe Goldston eq.



$$\langle k_1 k_2 | G(\omega) | k_3 k_4 \rangle = \langle k_1 k_2 | V | k_3 k_4 \rangle + \sum_{k_5,k_6} \frac{\langle k_1 k_2 | V | k_5 k_6 \rangle Q(k_5,k_6) \langle k_5 k_6 | G(\omega) | k_3 k_4 \rangle}{\omega - e(k_5) - e(k_6)}$$

- Single particle spectrum & potential



$$e(k) = \frac{k^2}{2M_\mathrm{N}} + U(k)$$

$$U(k) = \sum_{i} \sum_{k' \le k_F} \mathrm{Re} \langle k k' | G_i(e(k)+e(k')) | k k' \rangle_A$$

- Partial wave decomposition $^{2S+1}L_J = {}^1S_0\,,\ {}^3S_1\,,\ {}^3D_1\,,\ {}^1P_1\,,\ {}^3P_J \cdots$
- Continuous choice w/ effective mass approx. Angle averaged Q-operator

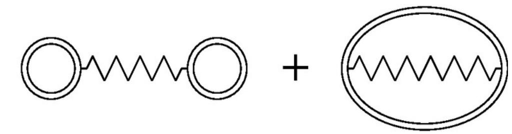
Brueckner-Hartree-Fock LOBT

M.I. Haftel and F. Tabakin, Nucl. Phys. A158(1970) 1-42

- Ground state energy in BHF framework

$$E_0 = \gamma \sum_{k}^{k_F} \frac{k^2}{2M} + \frac{1}{2} \sum_{i}^{N_{ch}} \sum_{k,k'}^{k_F} \mathrm{Re} \langle G_i(e(k)+e(k')) \rangle_A$$

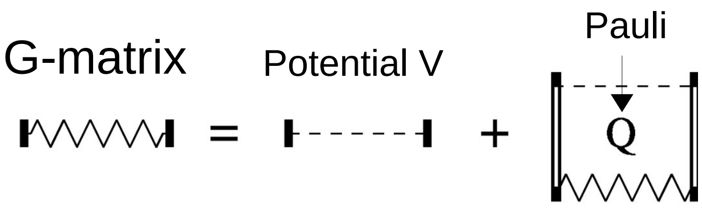
ΔE_0 =



- Bethe Goldston eq.

AV18 + “UIX”

$$\langle k_1 k_2 | G(\omega) | k_3 k_4 \rangle = \langle k_1 k_2 | V | k_3 k_4 \rangle + \sum_{k_5, k_6} \frac{\langle k_1 k_2 | V | k_5 k_6 \rangle Q(k_5, k_6) \langle k_5 k_6 | G(\omega) | k_3 k_4 \rangle}{\omega - e(k_5) - e(k_6)}$$

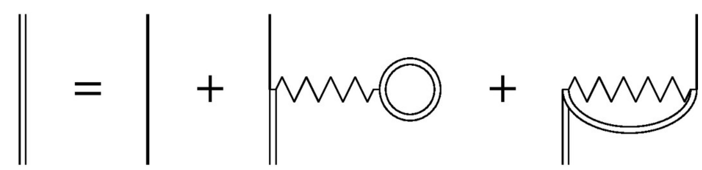


- Single particle spectrum & potential

$$e(k) = \frac{k^2}{2M_N} + U(k)$$

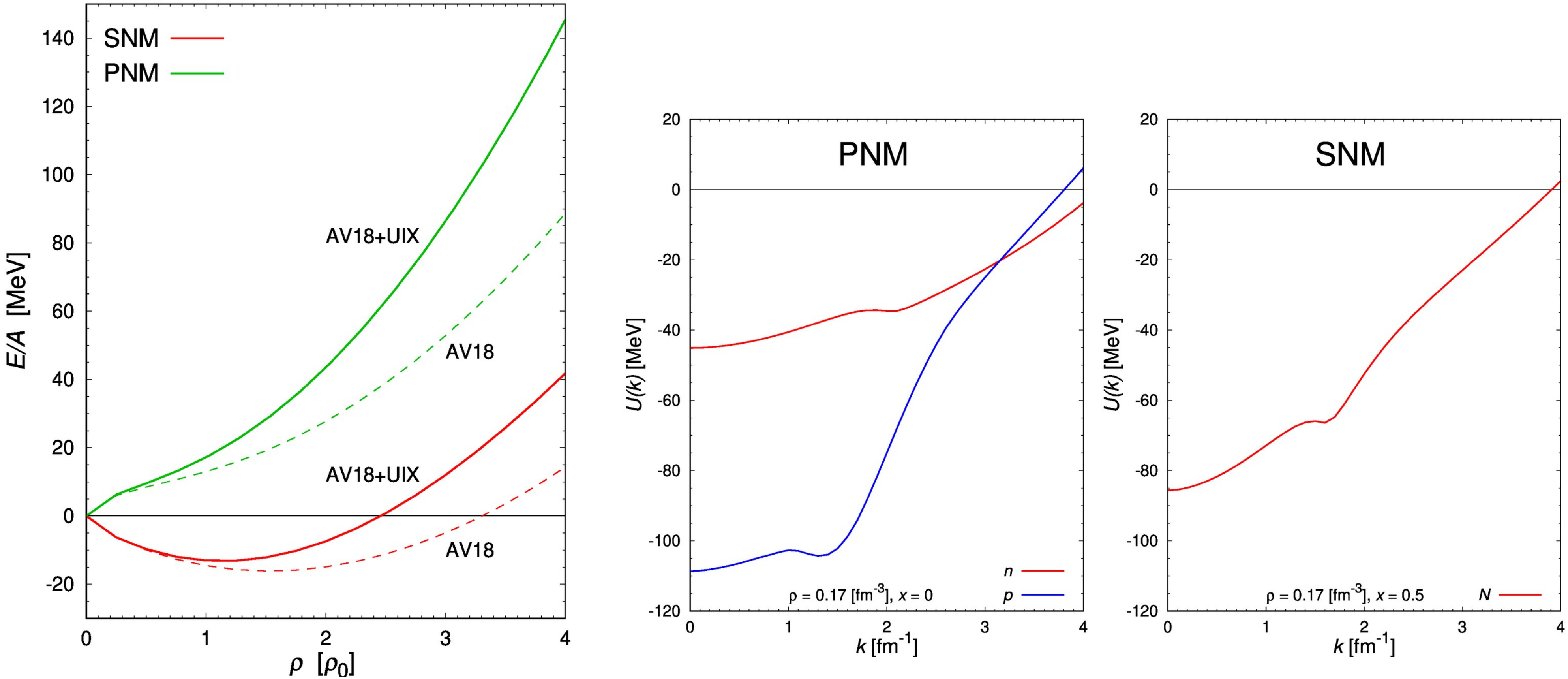
Physical

$$U(k) = \sum_{i} \sum_{k' \leq k_F} \mathrm{Re} \langle k k' | G_i(e(k)+e(k')) | k k' \rangle_A$$



- Partial wave decomposition $^{2S+1}L_J = {}^1S_0, {}^3S_1, {}^3D_1, {}^1P_1, {}^3P_J \cdots$
- Continuous choice w/ effective mass approx. Angle averaged Q-operator

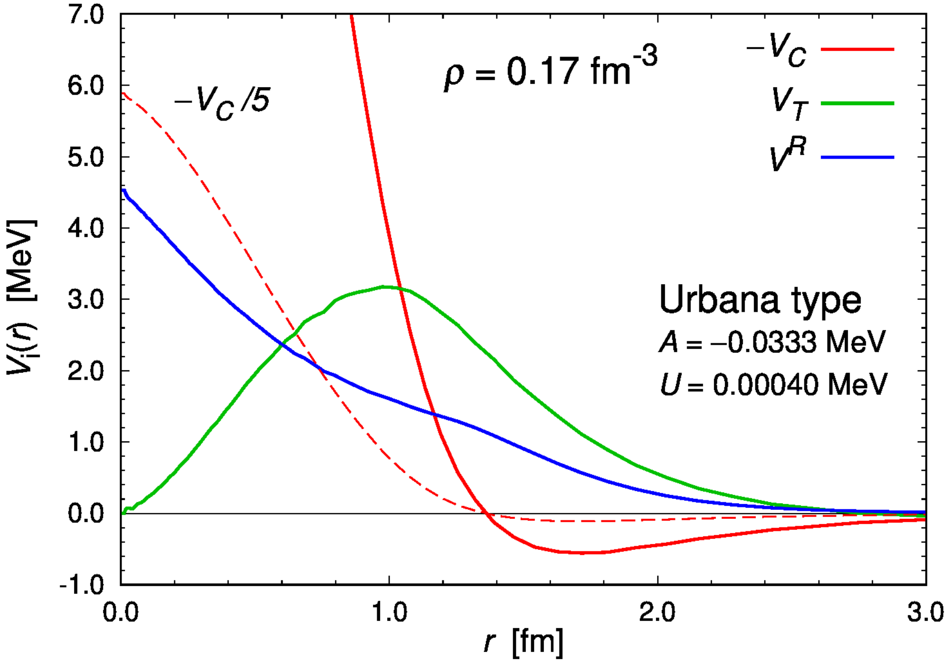
Nuclear matter

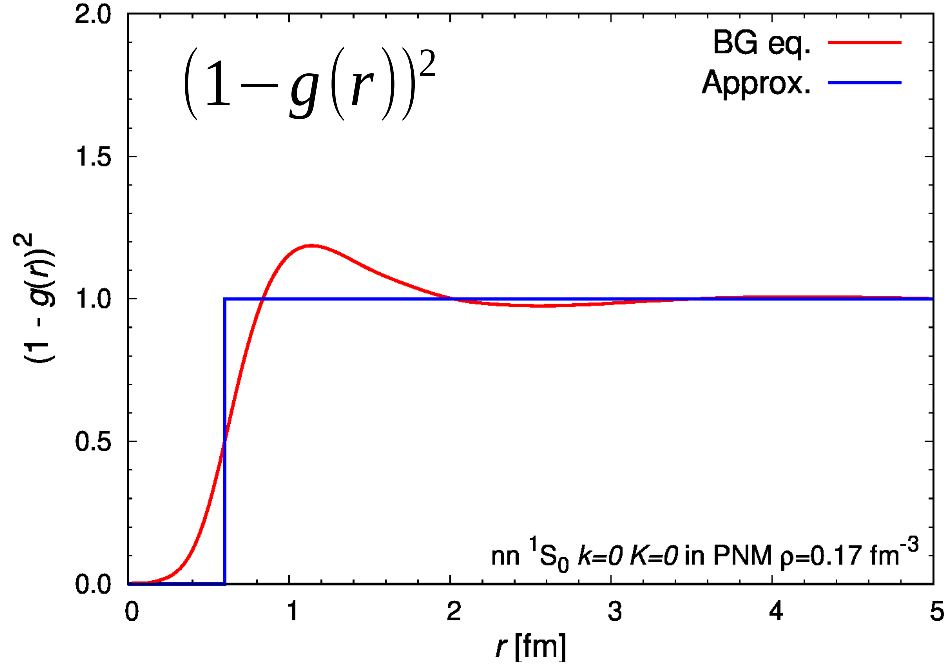


- Urbana NNN force is adjusted so that AV18 + Urbana reproduce the "emprical" saturation property of SNM.

roughly

NNN force





- Effective two-body potential

$$\bar{V}(\rho;\boldsymbol{r}) = \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\left[\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 \underline{V_C}(\rho;r) + S_{12}(\hat{\boldsymbol{r}})\underline{V_T}(\rho;r)\right] + \underline{V^R}(\rho;r)$$

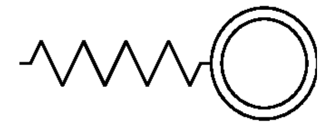
- obtained by integrating out positon of 3rd nucleon.
- Here, $\bar{V}(\rho,\boldsymbol{r})$ is ρ-propotional due to a fixed defect.

Brueckner-Hartree-Fock LOBT

- Hyperon single-particle potential

M. Baldo, G.F. Burgio, H.-J. Schulze, Phys. Rev. C58, 3688 (1998)

$$\underset{\Lambda \text{ or } \Sigma}{U_Y(k)} = \sum_{N=n,p} \sum_{SLJ} \sum_{k' \leq k_F} \langle k k' | G^{SLJ}_{(YN)(YN)}\left(e_Y(k)+e_N(k')\right) | k k' \rangle$$



$$^{2S+1}L_J = {}^1S_0 ,\ {}^3S_1 ,\ {}^3D_1 ,\ {}^1P_1 ,\ {}^3P_J \ \cdots$$

in our study ($^1S_0$, $^3S_1$, $^3D_1$) | limitation ($^1P_1$, ^3P_J, …)

- YN G-matrix using $M^{\text{Phys}}_{N,Y}$, $U^{\text{AV18+UIX}}_{n,p}$, $V^{\text{LQCD}}_{S=-1}$ (SU(3)$_F$ version) and, U^{LQCD}_{Y}

$$Q=0 \quad \begin{pmatrix} G^{SLJ}_{(\Lambda n)(\Lambda n)} & G_{(\Lambda n)(\Sigma^0 n)} & G_{(\Lambda n)(\Sigma^- p)} \\ G_{(\Sigma^0 n)(\Lambda n)} & G_{(\Sigma^0 n)(\Sigma^0 n)} & G_{(\Sigma^0 n)(\Sigma^- p)} \\ G_{(\Sigma^- p)(\Lambda n)} & G_{(\Sigma^- p)(\Sigma^0 n)} & G_{(\Sigma^- p)(\Sigma^- p)} \end{pmatrix}$$

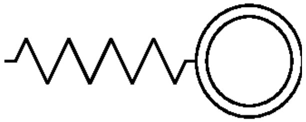
$$Q=+1 \quad \begin{pmatrix} G^{SLJ}_{(\Lambda p)(\Lambda p)} & G_{(\Lambda p)(\Sigma^0 p)} & G_{(\Lambda p)(\Sigma^+ n)} \\ G_{(\Sigma^0 p)(\Lambda p)} & G_{(\Sigma^0 p)(\Sigma^0 p)} & G_{(\Sigma^0 p)(\Sigma^+ n)} \\ G_{(\Sigma^+ n)(\Lambda p)} & G_{(\Sigma^+ n)(\Sigma^0 p)} & G_{(\Sigma^+ n)(\Sigma^+ n)} \end{pmatrix}$$

$$Q=-1 \quad G^{SLJ}_{(\Sigma^- n)(\Sigma^- n)} \qquad\qquad Q=+2 \quad G^{SLJ}_{(\Sigma^+ p)(\Sigma^+ p)}$$

Brueckner-Hartree-Fock

- Hyperon single-particle potential

$$U_{\Xi}(k) = \sum_{N=n,p} \sum_{SLJ} \sum_{k' \le k_F} \langle k k' | G^{SLJ}_{(\Xi N)(\Xi N)} \left(e_{\Xi}(k) + e_N(k') \right) | k k' \rangle$$



- ΞN G-matrix using $M^{\mathrm{Phys}}_{N,Y}$, $U^{\mathrm{AV18+UIX}}_{n,p}$, $U^{\mathrm{LQCD}}_{\Lambda,\Sigma}$, $V^{\mathrm{LQCD}}_{S=-2}$ (SU(3)$_F$ version), U^{LQCD}_{Ξ}

Flavor symmetric $^1S_0$ sectors

$$Q=0 \quad \begin{pmatrix}
G^{SLJ}_{(\Xi^0 n)(\Xi^0 n)} & G_{(\Xi^0 n)(\Xi^- p)} & G_{(\Xi^0 n)(\Sigma^+ \Sigma^-)} & G_{(\Xi^0 n)(\Sigma^0 \Sigma^0)} & G_{(\Xi^0 n)(\Sigma^0 \Lambda)} & G_{(\Xi^0 n)(\Lambda \Lambda)} \\
G_{(\Xi^- p)(\Xi^0 n)} & G_{(\Xi^- p)(\Xi^- p)} & G_{(\Xi^- p)(\Sigma^+ \Sigma^-)} & G_{(\Xi^- p)(\Sigma^0 \Sigma^0)} & G_{(\Xi^- p)(\Sigma^0 \Lambda)} & G_{(\Xi^- p)(\Lambda \Lambda)} \\
G_{(\Sigma^+ \Sigma^-)(\Xi^0 n)} & G_{(\Sigma^+ \Sigma^-)(\Xi^- p)} & G_{(\Sigma^+ \Sigma^-)(\Sigma^+ \Sigma^-)} & G_{(\Sigma^+ \Sigma^-)(\Sigma^0 \Sigma^0)} & G_{(\Sigma^+ \Sigma^-)(\Sigma^0 \Lambda)} & G_{(\Sigma^+ \Sigma^-)(\Lambda \Lambda)} \\
G_{(\Sigma^0 \Sigma^0)(\Xi^0 n)} & G_{(\Sigma^0 \Sigma^0)(\Xi^- p)} & G_{(\Sigma^0 \Sigma^0)(\Sigma^+ \Sigma^-)} & G_{(\Sigma^0 \Sigma^0)(\Sigma^0 \Sigma^0)} & G_{(\Sigma^0 \Sigma^0)(\Sigma^0 \Lambda)} & G_{(\Sigma^0 \Sigma^0)(\Lambda \Lambda)} \\
G_{(\Sigma^0 \Lambda)(\Xi^0 n)} & G_{(\Sigma^0 \Lambda)(\Xi^- p)} & G_{(\Sigma^0 \Lambda)(\Sigma^+ \Sigma^-)} & G_{(\Sigma^0 \Lambda)(\Sigma^0 \Sigma^0)} & G_{(\Sigma^0 \Lambda)(\Sigma^0 \Lambda)} & G_{(\Sigma^0 \Lambda)(\Lambda \Lambda)} \\
G_{(\Lambda \Lambda)(\Xi^0 n)} & G_{(\Lambda \Lambda)(\Xi^- p)} & G_{(\Lambda \Lambda)(\Sigma^+ \Sigma^-)} & G_{(\Lambda \Lambda)(\Sigma^0 \Sigma^0)} & G_{(\Lambda \Lambda)(\Sigma^0 \Lambda)} & G_{(\Lambda \Lambda)(\Lambda \Lambda)}
\end{pmatrix}$$

$$Q=+1 \quad \begin{pmatrix}
G^{SLJ}_{(\Xi^0 p)(\Xi^0 p)} & G_{(\Xi^0 p)(\Sigma^+ \Lambda)} \\
G_{(\Sigma^+ \Lambda)(\Xi^0 p)} & G_{(\Sigma^+ \Lambda)(\Sigma^+ \Lambda)}
\end{pmatrix} \qquad Q=-1 \quad \begin{pmatrix}
G^{SLJ}_{(\Xi^- n)(\Xi^- n)} & G_{(\Xi^- n)(\Sigma^- \Lambda)} \\
G_{(\Sigma^- \Lambda)(\Xi^- n)} & G_{(\Sigma^- \Lambda)(\Sigma^- \Lambda)}
\end{pmatrix}$$

Brueckner-Hartree-Fock

(SU(3)F version)

- ΞN G-matrix using $M^{\rm Phys}_{N,Y}$, $U^{\rm AV18+UIX}_{n,p}$, $U^{\rm LQCD}_{\Lambda,\Sigma}$, $V^{\rm LQCD}_{S=-2}$, $U^{\rm LQCD}_{\Xi}$

Flavor anti-symmetric $^3S_1$, $^3D_1$ sectors

$Q=0$

$$\begin{pmatrix} G^{SLJ}_{(\Xi^0 n)(\Xi^0 n)} & G_{(\Xi^0 n)(\Xi^- p)} & G_{(\Xi^0 n)(\Sigma^+\Sigma^-)} & G_{(\Xi^0 n)(\Sigma^0\Lambda)} \\ G_{(\Xi^- p)(\Xi^0 n)} & G_{(\Xi^- p)(\Xi^- p)} & G_{(\Xi^- p)(\Sigma^+\Sigma^-)} & G_{(\Xi^- p)(\Sigma^0\Lambda)} \\ G_{(\Sigma^+\Sigma^-)(\Xi^0 n)} & G_{(\Sigma^+\Sigma^-)(\Xi^- p)} & G_{(\Sigma^+\Sigma^-)(\Sigma^+\Sigma^-)} & G_{(\Sigma^+\Sigma^-)(\Sigma^0\Lambda)} \\ G_{(\Sigma^0\Lambda)(\Xi^0 n)} & G_{(\Sigma^0\Lambda)(\Xi^- p)} & G_{(\Sigma^0\Lambda)(\Sigma^+\Sigma^-)} & G_{(\Sigma^0\Lambda)(\Sigma^0\Lambda)} \end{pmatrix}$$

$Q=+1$

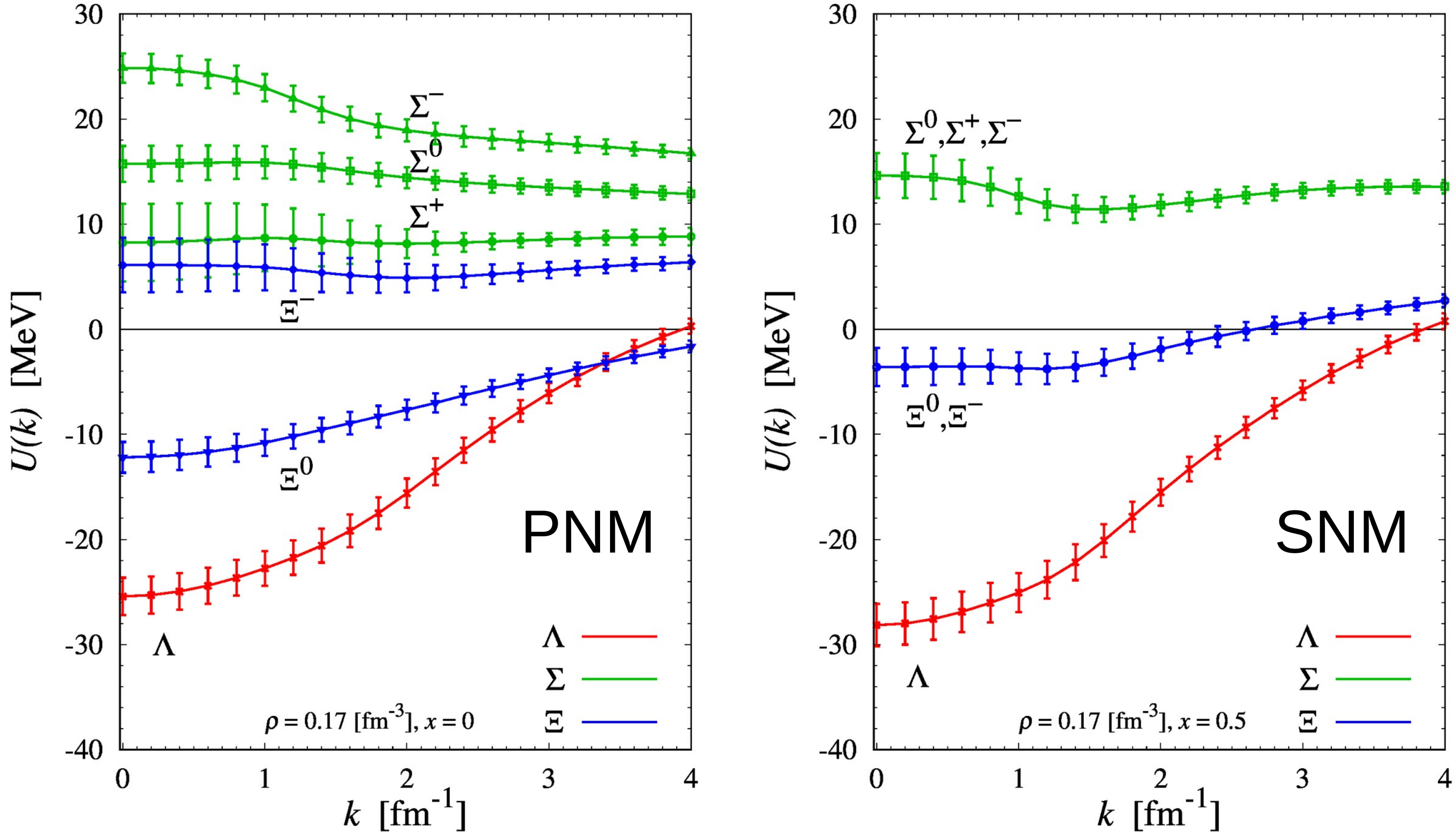
$$\begin{pmatrix} G^{SLJ}_{(\Xi^0 p)(\Xi^0 p)} & G_{(\Xi^0 p)(\Sigma^+\Sigma^0)} & G_{(\Xi^0 p)(\Sigma^+\Lambda)} \\ G_{(\Sigma^+\Sigma^0)(\Xi^0 p)} & G_{(\Sigma^+\Sigma^0)(\Sigma^+\Sigma^0)} & G_{(\Sigma^+\Sigma^0)(\Sigma^+\Lambda)} \\ G_{(\Sigma^+\Lambda)(\Xi^0 p)} & G_{(\Sigma^+\Lambda)(\Sigma^+\Sigma^0)} & G_{(\Sigma^+\Lambda)(\Sigma^+\Lambda)} \end{pmatrix}$$

$Q=-1$

$$\begin{pmatrix} G^{SLJ}_{(\Xi^- n)(\Xi^- n)} & G_{(\Xi^- n)(\Sigma^-\Sigma^0)} & G_{(\Xi^- n)(\Sigma^-\Lambda)} \\ G_{(\Sigma^-\Sigma^0)(\Xi^- n)} & G_{(\Sigma^-\Sigma^0(\Sigma^-\Sigma^0))} & G_{(\Sigma^-\Sigma^0)(\Sigma^-\Lambda)} \\ G_{(\Sigma^-\Lambda)(\Xi^- n)} & G_{(\Sigma^-\Lambda)(\Sigma^-\Sigma^0)} & G_{(\Sigma^-\Lambda)(\Sigma^-\Lambda)} \end{pmatrix}$$

Results

Hyperon single-particle potentials



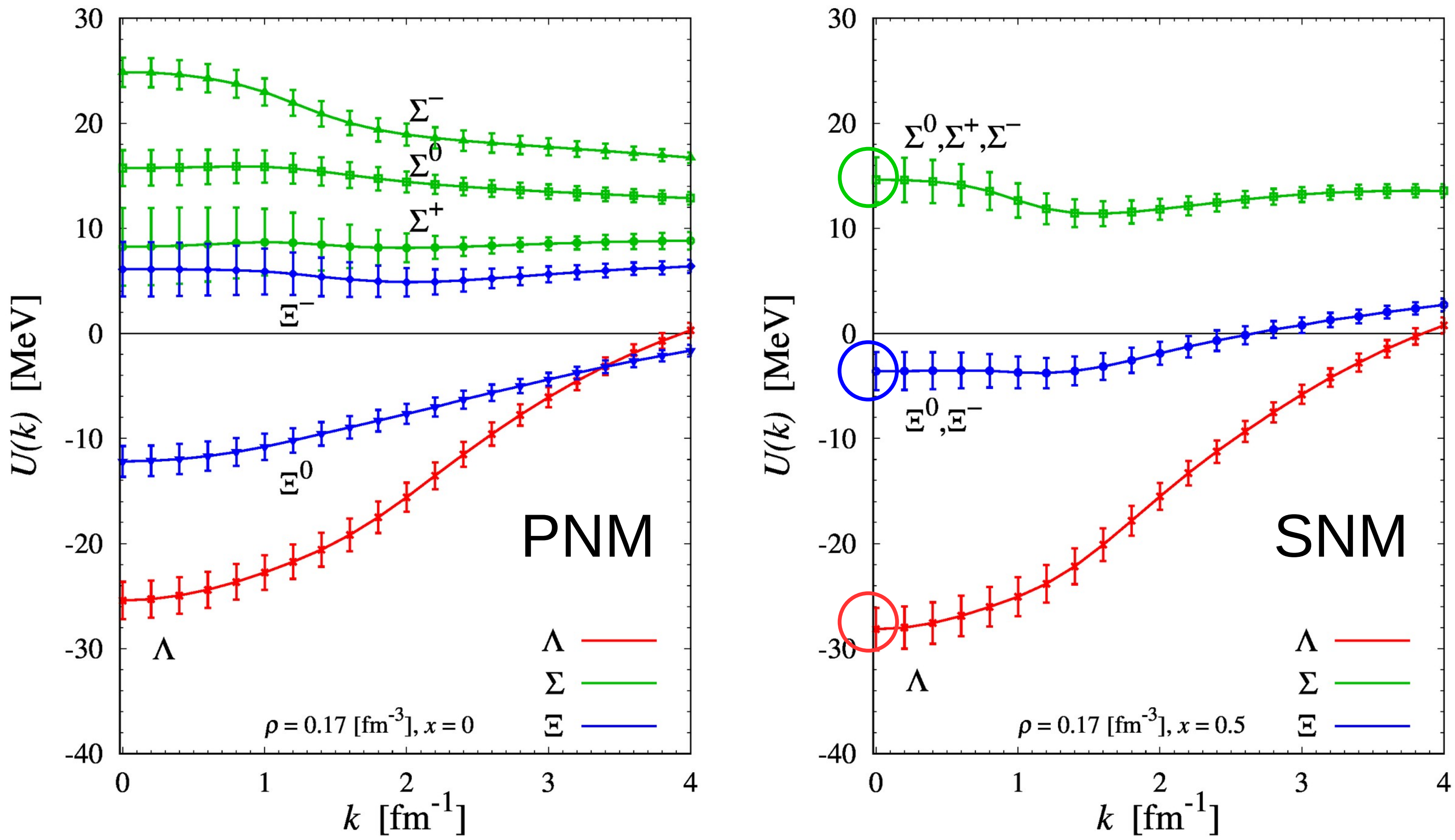
AIP Conf. Proc. 2130(2019)

@ $\rho = 0.17\,[\mathrm{fm}^{-3}]$

Vertical vars show statistical error only

- obtained by using *YN*,*YY* S-wave forces form QCD.

Hyperon single-particle potentials



AIP Conf. Proc. 2130(2019)

$@\rho = 0.17\,[\mathrm{fm}^{-3}]$

Remarkable. Encouraging.

- obtained by using *YN*,*YY* S-wave forces form QCD.
- Results are compatible with experimental suggestion.

$$U_\Lambda^{\mathrm{Exp}}(0) \simeq -30, \quad U_\Sigma^{\mathrm{Exp}}(0) \geq +20\,? \quad U_\Xi(0)^{\mathrm{Exp}} \simeq -10\,??, \quad [\mathrm{MeV}]$$

attraction — repulsion — small attraction

Hyperon single-particle potentials

- Breakdown of $U_Y(0\,;\rho_0)$ in SNM including spin,iso-spin multiplicity

	I=1/2						total
Λ	$^1S_0$	$^3S_1$	$^3D_1$				
	−3.49	−24.84	0.18				−28.16

	I=1/2			I=3/2			total
Σ	$^1S_0$	$^3S_1$	$^3D_1$	$^1S_0$	$^3S_1$	$^3D_1$	
	7.43	−9.28	0.07	−4.97	21.80	−0.43	14.62

	I=0			I=1			total
Ξ	$^1S_0$	$^3S_1$	$^3D_1$	$^1S_0$	$^3S_1$	$^3D_1$	
	−4.48	−4.37	−0.01	9.08	−3.74	−0.08	−3.60

Hyperon single-particle potentials

- Breakdown of $U_Y(0\,;\rho_0)$ in SNM including spin,iso-spin multiplicity

	I=1/2						total
	$^1S_0$	$^3S_1$	$^3D_1$				
Λ	−3.49	−24.84	0.18				−28.16

	I=1/2			I=3/2			total
	$^1S_0$	$^3S_1$	$^3D_1$	$^1S_0$	$^3S_1$	$^3D_1$	
Σ	7.43	−9.28	0.07	−4.97	21.80	−0.43	14.62

	I=0			I=1			total
	$^1S_0$	$^3S_1$	$^3D_1$	$^1S_0$	$^3S_1$	$^3D_1$	
Ξ	−4.48	−4.37	−0.01	9.08	−3.74	−0.08	−3.60

All ΛN S-wave interactions provide attraction in SNM.
Compatible with existance of Λ-hypernulcei.

$^1S_0$ small attraction may be affected by syst. error in 8_s

Hyperon single-particle potentials

- Breakdown of $U_Y(0\,;\rho_0)$ in SNM including spin,iso-spin multiplicity

	I=1/2						total
Λ	$^1S_0$	$^3S_1$	$^3D_1$				
	−3.49	−24.84	0.18				−28.16

	I=1/2			I=3/2			total
Σ	$^1S_0$	$^3S_1$	$^3D_1$	$^1S_0$	$^3S_1$	$^3D_1$	
	7.43	−9.28	0.07	−4.97	21.80	−0.43	14.62

	I=0			I=1			total
Ξ	$^1S_0$	$^3S_1$	$^3D_1$	$^1S_0$	$^3S_1$	$^3D_1$	
	−4.48	−4.37	−0.01	9.08	−3.74	−0.08	−3.60

I=1/2 $^3S_1$ and I=3/2 $^1S_0$ ΣN interactions provide attraction in SNM

Some few-body system w/ Σ can be bound by the selection.

Hyperon single-particle potentials

- Breakdown of $U_Y(0\,;\rho_0)$ in SNM including spin,iso-spin multiplicity

	I=1/2						total
Λ	$^1S_0$	$^3S_1$	$^3D_1$				
	−3.49	−24.84	0.18				−28.16

	I=1/2			I=3/2			total
Σ	$^1S_0$	$^3S_1$	$^3D_1$	$^1S_0$	$^3S_1$	$^3D_1$	
	7.43	−9.28	0.07	−4.97	21.80	−0.43	14.62

	I=0			I=1			total
Ξ	$^1S_0$	$^3S_1$	$^3D_1$	$^1S_0$	$^3S_1$	$^3D_1$	
	−4.48	−4.37	−0.01	9.08	−3.74	−0.08	−3.60

All ΞN S-wave interactions provide attraction in SNM except for I=1, $^1S_0$ chanel

Pure theoretical prediction based on QCD

Summary and Outlook

Summary and Outlook

★ Summary

previous
K-computer

- Review of HALQCD approach and method
- $S=-2$ BB int. potentials from QCD at almost the physical point
- Application of YN, YY pot. to hyperon s.p. pot. in N.M.
  - $SU(3)_F$ version(especially $S=-1$ sector), S-wave only
  - "compatible" with experiments, remarkable

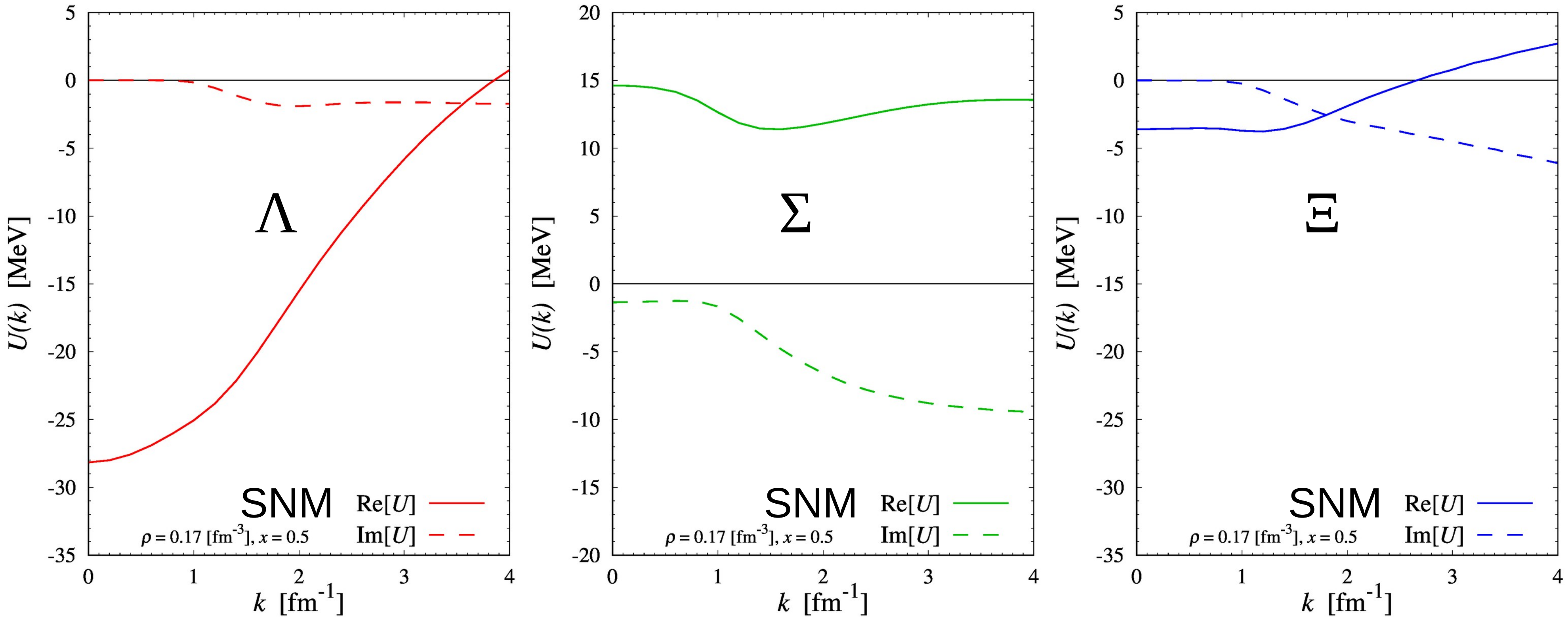
★ Outlook

- Application of new potentials from QCD at the physical point obtaind recently on the Fugaku, w/ explicit $SU(3)_F$ breaking
- Inculde BB interactions in higher partial waves (P-wave)
- Nuclear matter based on nulclear force from QCD.
- We will need to have NNN force in the HALQCD method.
- and so on

Thank you !!

Backup

Hyperon single-particle potentials



- $\mathrm{Im}[U_Y]$ are obtained by summing up $\mathrm{Im}[G_{YN,YN}]$.
- But, $\mathrm{Im}[U_B]$ are not taken into the Bethe-Goldsone eq.

Nijmegen

Partial wave contributions to $U_\Lambda(\rho_0)^{(a)}$

	$^1S_0$	$^3S_1$	$^1P_1$	$^3P_0$	$^3P_1$	$^3P_2$	D	sum
ESC08c1	–14.3	–29.9	2.7	0.2	1.6	–3.1	–1.6	–44.3
ESC08c1$^+$	–13.2	–26.8	2.9	0.3	1.8	–2.6	–1.5	–39.1
ESC08c2	–13.9	–34.1	2.8	0.2	1.6	–3.2	–1.6	–48.4
ESC08c2$^+$	–12.0	–28.9	3.2	0.3	1.9	–2.4	–1.5	–39.3

Partial wave contributions to $U_\Sigma(\rho_0)$

model	T	$^1S_0$	$^3S_1$	$^1P_1$	$^3P_0$	$^3P_1$	$^3P_2$	D	U_Σ	Γ_Σ
ESC08c1	$1/2$	10.5	–22.6	2.2	1.9	–5.5	–1.1	–0.7		
	$3/2$	–14.1	29.9	–4.6	–1.8	5.6	–1.8	–0.3	–2.3	
ESC08c1$^+$	$1/2$	10.7	–21.5	2.3	1.9	–5.4	–1.0	–0.6		
	$3/2$	–13.3	31.4	–4.4	–1.7	5.8	–1.5	–0.2	+2.4	
ESC08c2	$1/2$	14.6	–22.0	3.1	1.9	–5.5	–1.1	–0.6		
	$3/2$	–15.5	35.2	–4.7	–1.7	5.9	–1.0	–0.2	+8.3	
ESC08c2$^+$	$1/2$	14.8	–20.8	3.2	1.9	–5.3	–0.8	–0.5		
	$3/2$	–14.1	37.6	–4.3	–1.6	6.1	–0.5	–0.1	+15.4	

Nijmegen

Partial wave contributions to $U_\Xi(\rho_0)$

model		$^1S_0$	$^3S_1$	$^1P_1$	$^3P_0$	$^3P_1$	$^3P_2$	U_Ξ	Γ^c_Ξ
ESC08c1	$T=0$	3.1	–9.8	–0.1	0.5	1.7	–1.5		
	$T=1$	9.1	–7.6	1.3	1.0	–2.4	0.0	–4.7	6.4
ESC08c1$^+$	$T=0$	2.9	–8.8	–0.1	0.5	1.8	–1.4		
	$T=1$	9.7	–5.3	1.5	1.0	–2.2	0.4	+0.1	6.3
ESC08c2	$T=0$	3.6	–11.1	–0.1	0.2	1.8	–1.4		
	$T=1$	8.7	–10.1	1.2	0.9	–2.7	–0.5	–9.6	5.1
ESC08c2$^{+\prime\prime}$	$T=0$	3.4	–9.5	–0.0	0.2	1.9	–1.1		
	$T=1$	9.8	–6.2	1.6	1.0	–2.4	0.1	–1.3	4.8

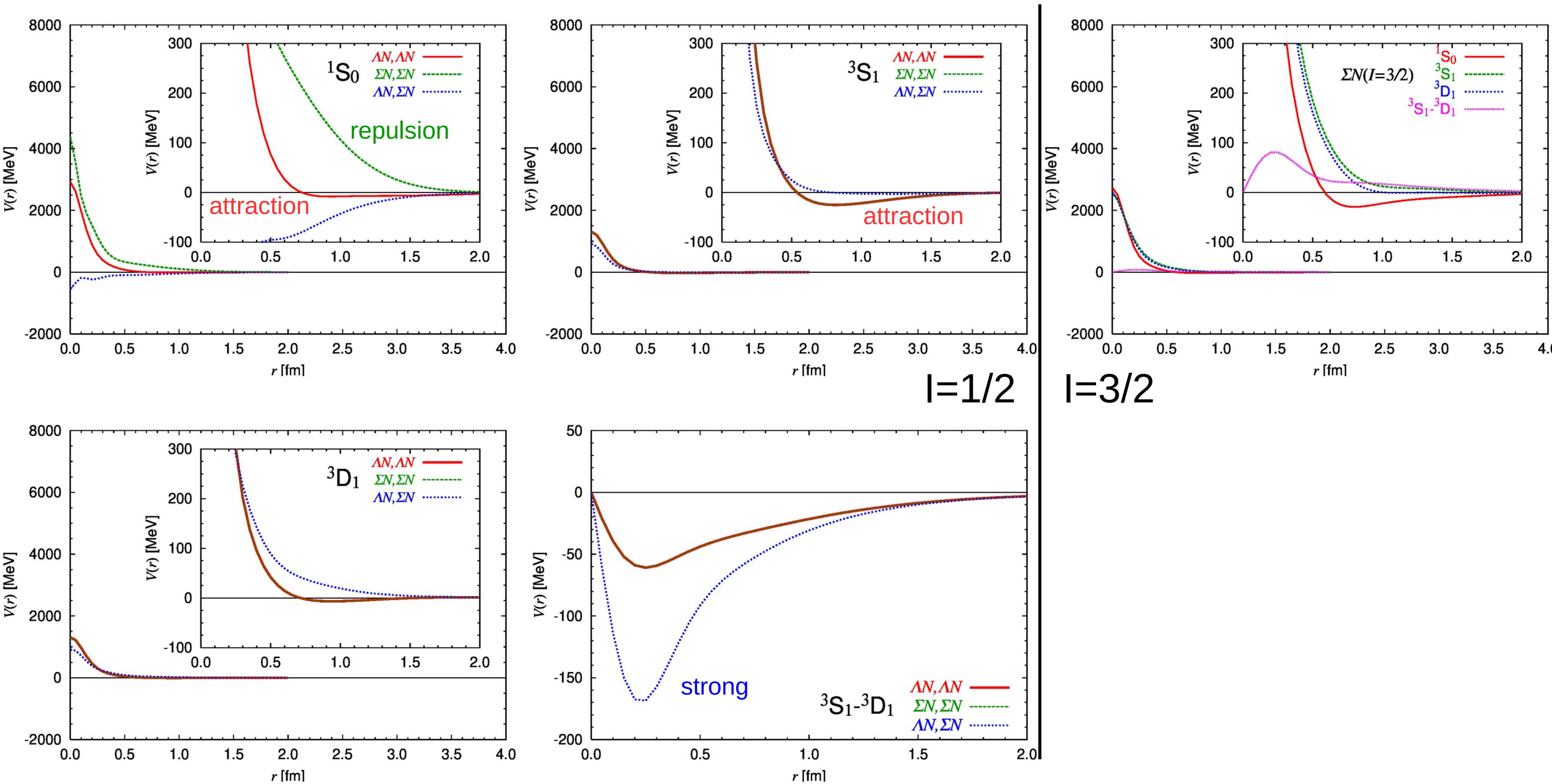
Quark model

Taken from M. Kohno etal.
Prog. Part. Nucl. Phys. 58, 439-520 (2007)

	$U_\Lambda(0)$ [MeV]		$U_\Sigma(0)$ [MeV]			
	fss2 (FSS)	NSC89	fss2 (FSS)		NSC89	
I	$1/2$	$1/2$	$1/2$	$3/2$	$1/2$	$3/2$
$^1S_0$	-14.8 (-20.1)	-15.3	6.7 (6.1)	-9.2 (-8.8)	6.7	-12.0
$^3S_1 + {}^3D_1$	-28.4 (-21.2)	-13.0	-23.9 (-20.2)	41.2 (48.2)	-14.9	6.7
$^1P_1 + {}^3P_1$	2.1 (0.4)	3.6	-6.5 (-7.0)	3.3 (4.0)	-3.5	3.9
$^3P_0$	-0.4 (0.5)	0.2	2.9 (3.0)	-2.2 (-2.3)	2.6	-2.0
$^3P_2 + {}^3F_2$	-5.7 (-4.6)	-4.0	-1.6 (-1.3)	-2.5 (-1.2)	-0.5	-1.9
subtotal			-23.8 (-21.0)	31.3 (40.8)	-9.8	-5.5
total	-48.2 (-46.0)	-29.8	7.5 (19.8)		-15.3	

LQCD ΛN-ΣN

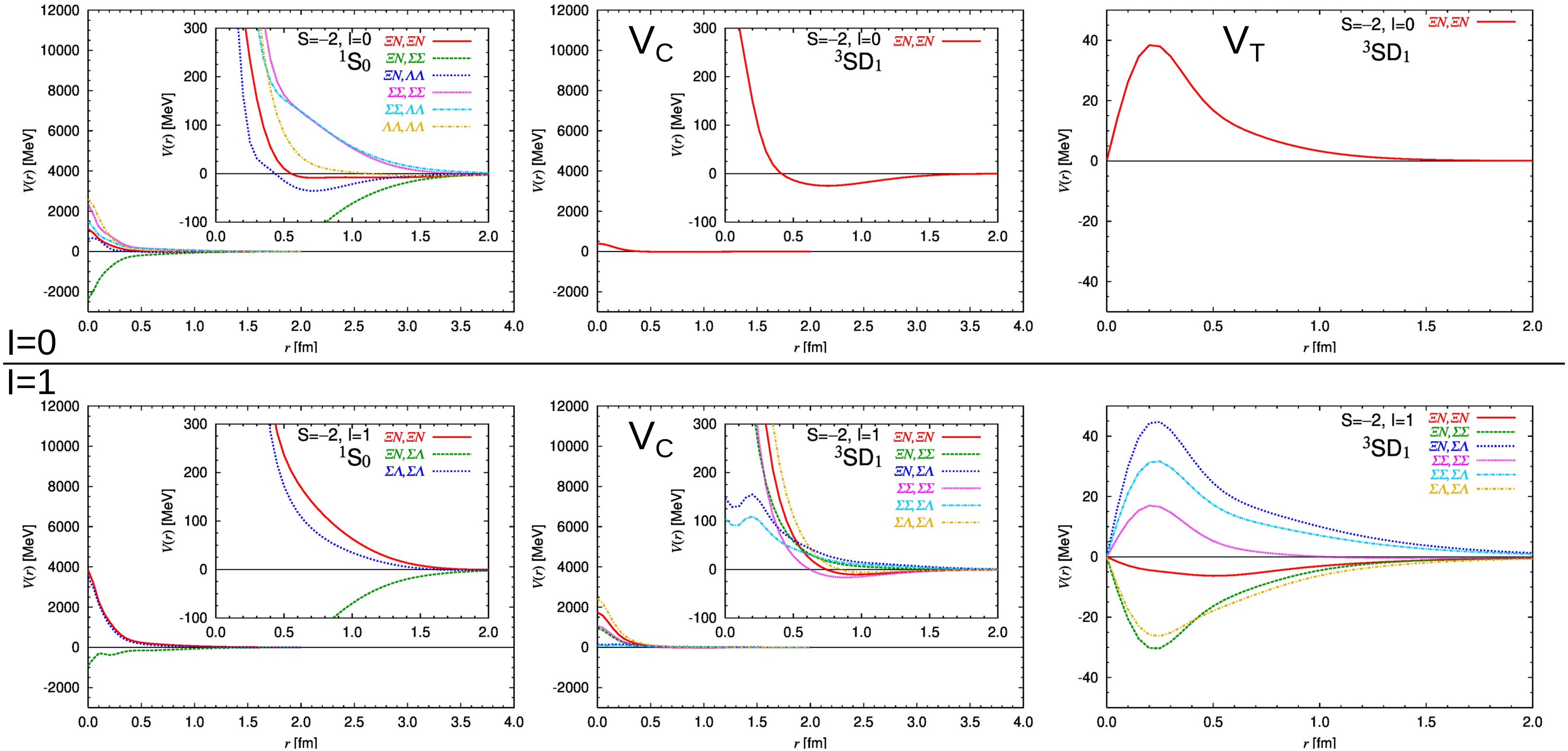
From K-conf. but rotated from the irr.-rep. base diagonal potentials.



- In I=1/2, $^1S_0$ channel, ΛN has an attraction, while ΣN is repulsive.
- In I=1/2, $^3S_1$ channel, both ΛN and ΣN have an attraction.
- In I=1/2, strong tensor coupling in flavor off-diagonal.

LQCD ΞN-YY

From K-conf. but rotated from the irr.-rep. base diagonal potentials.



- Many experimentally unknown coupled-channel potentials.
- One can see predictive power of the HALQCD method.

FAQ

1. Does your potential depend on the choice of source?

2. Does your potential depend on choice of operator at sink?

3. Does your potential $U(r,r')$ or $V(r)$ depends on energy?

FAQ

1. Does your potential depend on the choice of source?

→ No. Some sources may enhance excited states in 4-point func. However, it is no longer a problem in our new method.

2. Does your potential depend on choice of operator at sink?

→ Yes. It can be regarded as the "scheme" to define a potential. Note that a potential itself is not physical observable. We will obtain unique result for physical observables irrespective to the choice, as long as the potential $U(r,r')$ is deduced exactly.

FAQ

3. Does your potential $U(r,r')$ or $V(r)$ depends on energy?

- ➔ By definition, $U(r,r')$ is non-local but energy independent. While, determination and validity of its leading term $V(r)$ depend on energy because of the truncation.

  However, we know that the dependence in NN case is very small (thanks to our choice of sink operator = point) and negligible at least at $E_{lab.} = 0 - 90$ MeV. We rely on this in our study.

  If we find some dependence, we will determine the next leading term of the expansion from the dependence.

FAQ

in $SU(3)_F$ limit, ie. heavy u,d quark world

4. Is the H a compact six-quark object or a tight BB bound state?

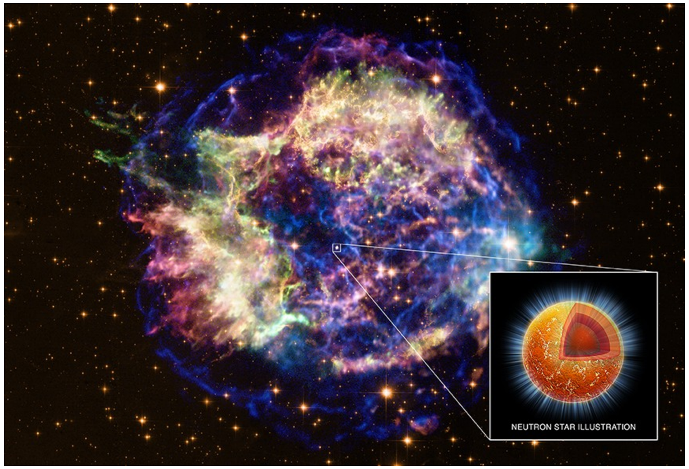
FAQ

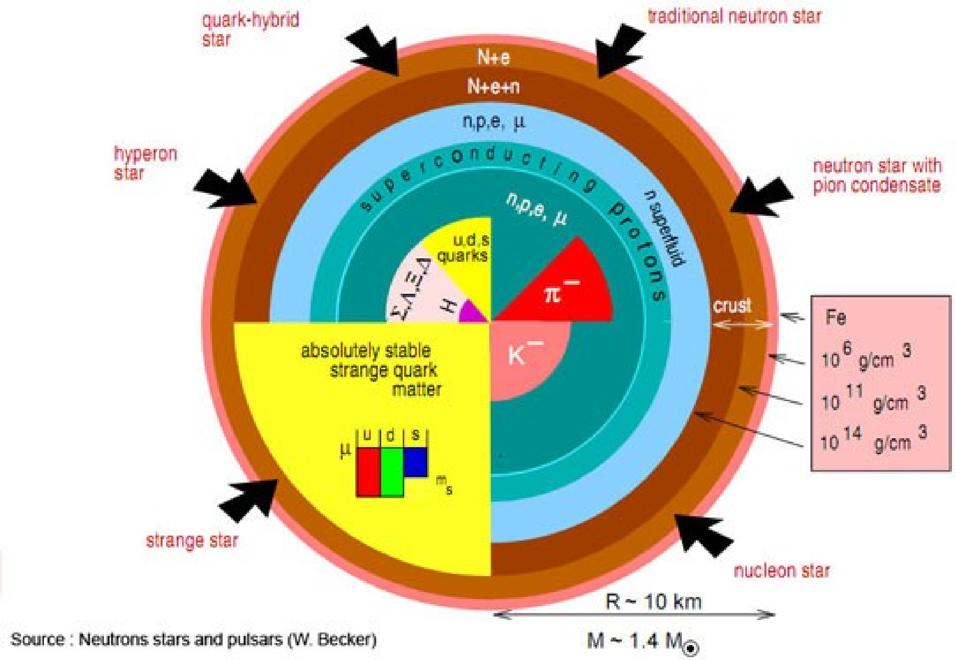
found in $SU(3)_F$ limit, ie. heavy u,d quark world

4. Is the H a compact six-quark object or a tight BB bound state?

➔ Both.
There is no distinct difference between two in QCD.
Note that baryon is made of three quarks in QCD.
Imagine a compact 6-quark object in $(0S)^6$ configuration.
This configuration can be re-written in a form of
$(0S)^3 \times (0S)^3 \times \mathrm{Exp}(-a\, r^2)$ with relative coordinate r.
This demonstrate that a compact six-quark object,
at the same time, has a BB type configuration.
In LQCD simulation at $SU(3)_F$ limits, we've established
existence of a $B=2$, $S=-2$, $I=0$ stable QCD eigenstate.

Introduction



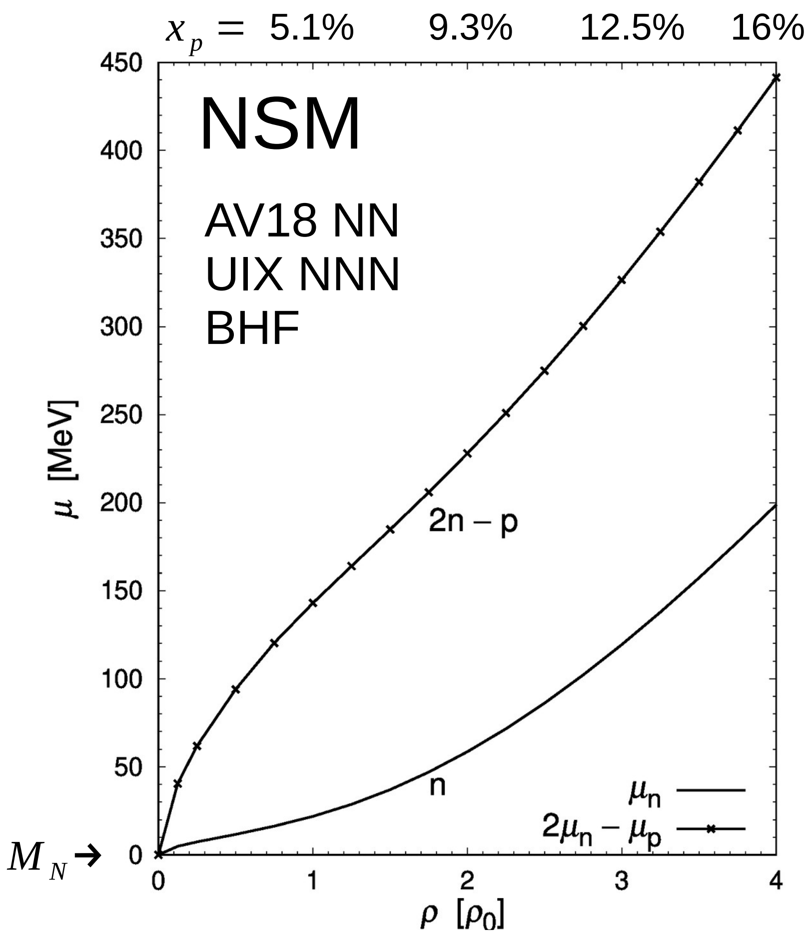


- Hyperon is a serious subject in physics of NS.
  - Does hyperon appear inside neutron star core?
  - How EoS of NS mater can be so stiff with hyperon?

cf. PSR J1614-2230 1.97±0.04 $M_{\odot}$

- This is a tough and very challenging problem. Because...

Chemical potentials in NSM



- Neutron Star Matter : ANM + e^-, μ^- @$Q=0$, β-eq.

- Parabola approx. for ANM

$$\mu_p(\rho;\beta) = \mu_N^{SNM}(\rho) + \beta^2 \frac{d E^{sym}(\rho)}{d\rho} - \beta(\beta+2)\, E^{sym}(\rho)$$

$$\mu_n(\rho;\beta) = \mu_N^{SNM}(\rho) + \beta^2 \frac{d E^{sym}(\rho)}{d\rho} - \beta(\beta-2)\, E^{sym}(\rho)$$

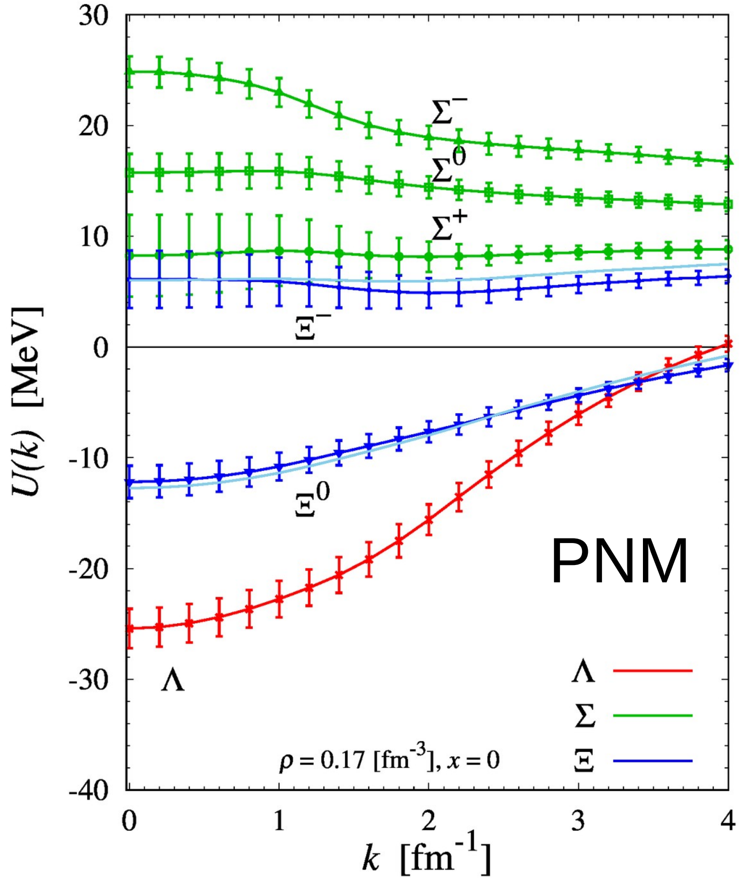
$$4E^{sym}(\rho) = \mu_n^{PNM}(\rho) - \mu_p^{PNM}(\rho), \quad \beta = 1 - 2x_p$$

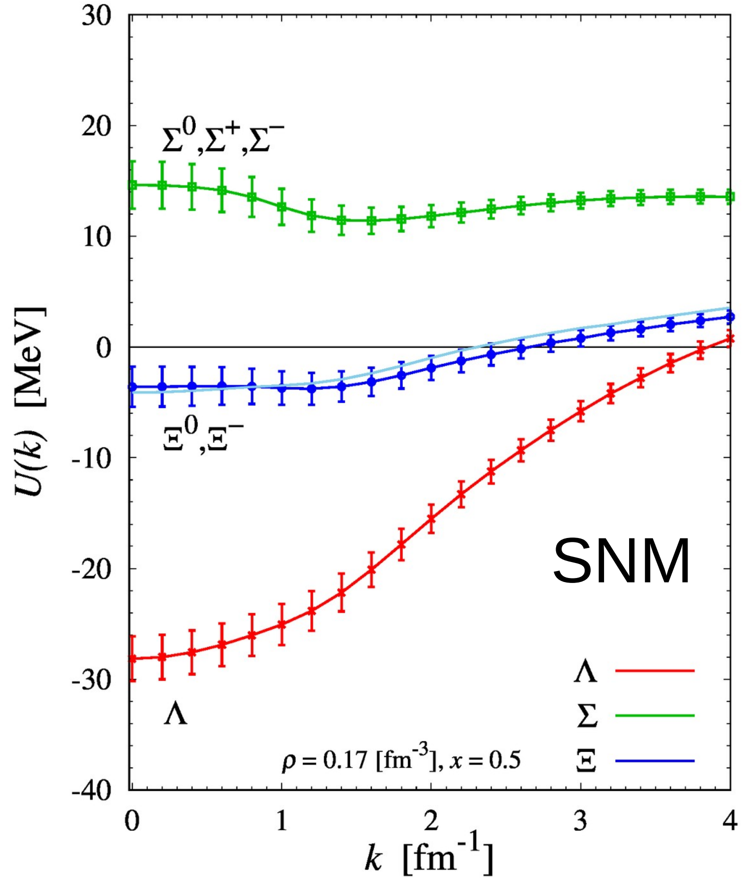
- Hyperon chemical in NSM $\quad \mu_Y(\rho) \simeq M_Y - M_N + U_Y^{ANM}(0;\rho)$

- Hyperons appear as $\quad n \to Y^0 \quad$ when $\quad \mu_n > \mu_{Y^0}$

$\quad nn \to pY^- \quad$ when $\quad 2\mu_n > \mu_p + \mu_{Y^-}$

Hyperon single-particle potentials





@ $\rho = 0.17\ [\mathrm{fm}^{-3}]$

Preliminary

- Skybule curves show $U_\Xi(k)$ w/ original $S=-2$ BB potentials inclding explicit $SU(3)_F$ breaking.
- We see that the flavor symmetric approximation used in the blue ones is reasonable for ΞN, YY forces.

Hyperon single-particle potentials

Preliminary

- Breakdown of $U_Y(0\,;\rho_0)$ in SNM including spin,iso-spin multiplicity

	I=1/2						total
Λ	$^1S_0$	$^3S_1$	$^3D_1$				
	−3.49	−24.84	0.18				−28.16

	I=1/2			I=3/2			total
Σ	$^1S_0$	$^3S_1$	$^3D_1$	$^1S_0$	$^3S_1$	$^3D_1$	
	7.43	−9.28	0.07	−4.97	21.80	−0.43	14.62

	I=0			I=1			total
Ξ	$^1S_0$	$^3S_1$	$^3D_1$	$^1S_0$	$^3S_1$	$^3D_1$	
	−3.15	−5.36	−0.30	7.12	−2.41	−0.08	−4.11

U_Ξ w/ original BB potentials inclding explicit flavor SU(3) breaking

Hyperon single-particle potentials

- Breakdown of $U_Y(0\,;\rho_0)$ in SNM inclding sipn multiplicity

	Yn			Yp			total
	$^1S_0$	$^3S_1$	$^3D_1$	$^1S_0$	$^3S_1$	$^3D_1$	
Λ	−1.75	−12.42	0.09	−1.75	−12.42	0.09	−28.16
Σ^0	1.23	6.26	−0.18	1.23	6.26	−0.18	14.62
Σ^+	6.19	−3.84	−0.04	−3.72	16.35	−0.32	14.62
Σ^-	−3.72	16.35	−0.10	6.19	−3.84	−0.04	14.62
Ξ^0	−1.45	−5.62	−0.04	6.05	−2.50	−0.02	−3.60
Ξ^-	6.05	−2.50	−0.02	−1.45	−5.62	−0.04	−3.60