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半微視的汎関数によるKohn-Sham型理論の核子・原子核散乱への拡張的応用

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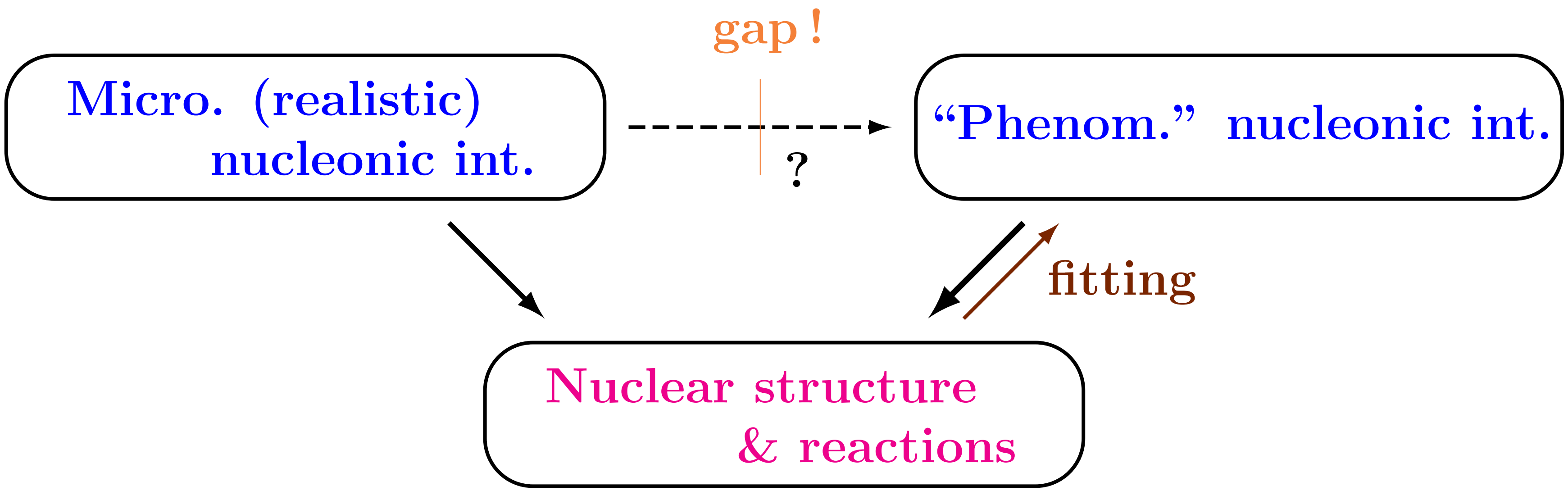
H.N. & K. Ishida, PRC 109, 044614; 110, 019901(E)
K. Ishida & H.N., PRC 111, 044610

Before starting ···

I. Introduction

★ **Nuclear properties & nucleonic interaction**



"gap" b/w micro. int. & phenom. int. (in KS / SCMF approach)

— needs to be filled !

- **micro. int.** → **precision for** E/A **: a few** $100\,\mathrm{keV} \approx 2\%$
  vs. **KS w/ phenom. int.**
  → **precision for** ^{208}**Pb : a few** MeV **out of** $1.6\,\mathrm{GeV} \approx 0.2\%$
- **phenom. int.** **(← fitting)**
  → **not extendable to unfitted quantities**

⇒ **KS approach w/ "semi-realistic int." (··· fine tuning)**

★ **“Minimal composition” ($\approx$ “essence”) of Kohn-Sham theory**

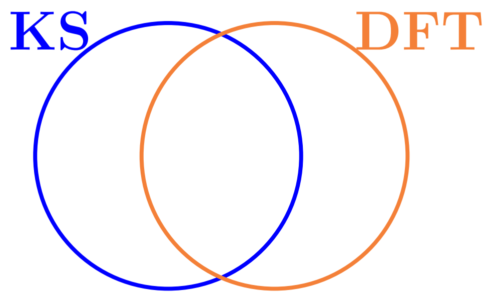
H.N., Phys. Scr. 98, 105007

$$E^{\mathrm{KS}}[\varrho] \quad \leftarrow \quad E\big[\boldsymbol{Q}[\varrho]\big] = \min_{\Psi \to \boldsymbol{Q}} E[\Psi]$$

$\boldsymbol{Q}$: **principal variables**

$\big(n(\boldsymbol{r}) \in \boldsymbol{Q}$ $\cdots$ **unnecessary !**$\big)$

ϱ : **1-b. DM**



★ **“v-representability” = differentiability** $\leftrightarrow$ **universality of** $E^{\mathrm{KS}}[\varrho]$

— **should not be disregarded !**

★ **KS orbitals $\cdots$ artifact ?** $\rightarrow$ **“quasiparticle” à la Landau**

(= dressed particle)

$$|\Psi^{\mathrm{exact}}\rangle = \mathcal{U}(0,-\infty)\,|\Phi^{\mathrm{KS}}\rangle \qquad \mathcal{U}(t,t_0): \ \textbf{time-evolution op.}$$

$$\big(H \ \rightarrow \ h^{\mathrm{KS}} + (H - h^{\mathrm{KS}})\,e^{\eta t}\big)$$

$\because$) **adiabatic theorem**

$$= \mathcal{U}(0,-\infty)\Big[\prod_{i=1}^{N} a_i^{\mathrm{KS}\,\dagger}|0\rangle\Big] = \prod_{i=1}^{N} \underbrace{\big[\mathcal{U}(0,-\infty)\,a_i^{\mathrm{KS}\,\dagger}\,\mathcal{U}^{-1}(0,-\infty)\big]}_{\text{“quasiparticle”}}|0\rangle$$

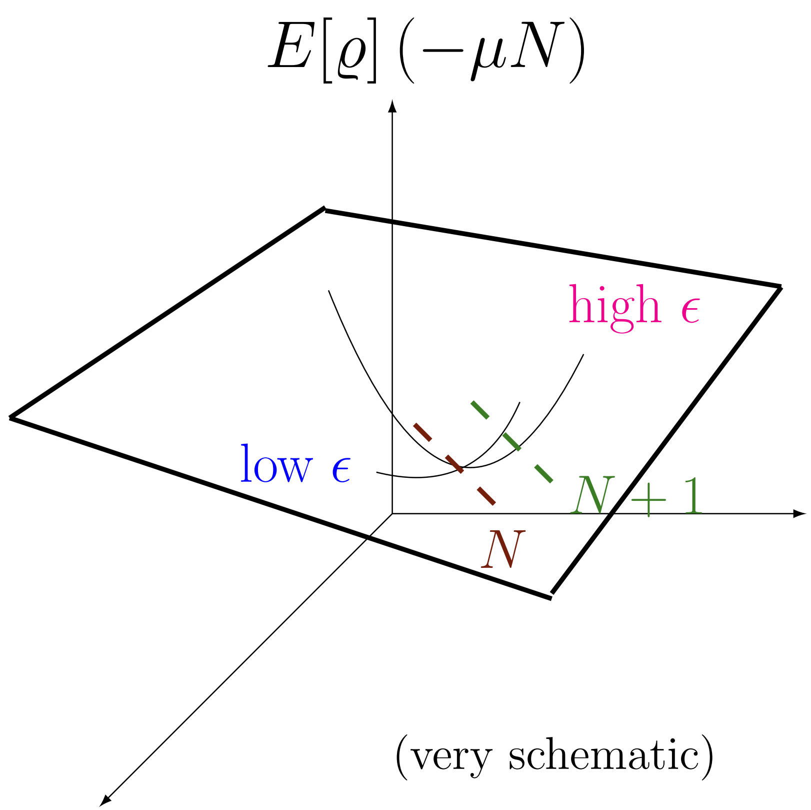
★ Extension of KS approach to s.p. states at $\epsilon > 0$

structure of $E^{\mathrm{KS}}[\varrho]$? **(— complicated manifold)**

$$h^{\mathrm{KS}} = \frac{\delta E^{\mathrm{KS}}}{\delta \varrho} = \frac{\boldsymbol{p}^2}{2M} + \underset{\uparrow}{U^{\mathrm{KS}}} \to \text{s.p. states}$$

containing many-body corr.

- **minimization**
  - $\leftrightarrow$ **small-“curvature” submanifold**
  - $\leftrightarrow$ **low-ϵ s.p. states**
- **high-ϵ s.p. states (incl. $\epsilon > 0$)**
  - $\leftrightarrow$ **large-“curvature” submanifold**
- **v-representability (for g.s.)**
  - $\leftrightarrow$ **universality w.r.t. particle # ?**



(very schematic)

U^{KS} **at $\epsilon > 0$** $\leftrightarrow$ **real part of optical pot.** $\mathcal{V}^{\mathrm{SFP}}$

many-body corr. $\leftrightarrow$ “dynamical pol. effect”

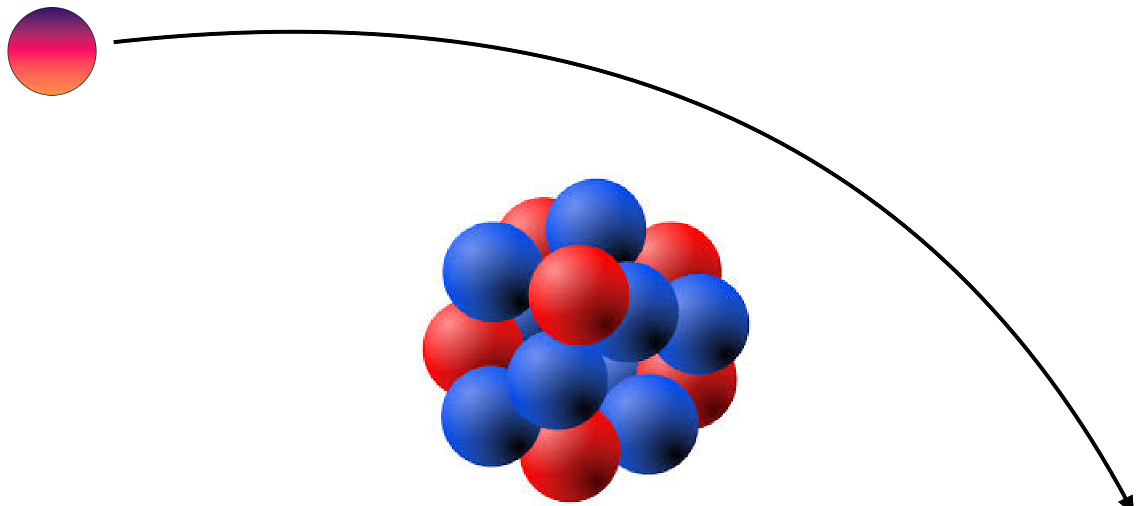
imag. part ? — beyond MF (KS)

cf. $\delta^2 E/\delta\varrho^2$ $\leftrightarrow$ **RPA or TDKS**

★ N-A scattering

$$|\Psi_A\rangle \otimes |\psi_N\rangle \quad \rightarrow \quad \left\{ \begin{array}{ll} |\Psi_A\rangle \otimes |\psi_N\rangle & \textbf{(elastic)} \\ \left. \begin{array}{ll} |\Psi_A^*\rangle \otimes |\psi_N\rangle & \textbf{(inelastic)} \\ |\Psi_{A_1}\rangle \otimes |\Psi_{A_2}\rangle & \\ \vdots & \end{array} \right\} \rightarrow \text{“}\textbf{absorption}\text{”} \end{array} \right.$$

channel branching — beyond KS



“KS approach” to elastic channel : $E^{\rm KS}[\varrho] \quad \rightarrow \quad \left\{ \begin{array}{l} |\Psi_A\rangle \approx |\Phi_A^{\rm KS}\rangle \\ (\mathcal{V}^{\rm SFP} \rightarrow)\ |\psi_N\rangle \end{array} \right.$

(dressed nucleon)

II. Self-consistent single-nucleon potential

★ **Self-consistent pot. from eff. int. (or EF)** $\Leftarrow$ **variational principle**

$$E[\varrho] = \sum_{\alpha\alpha'} \langle\alpha|\frac{\boldsymbol{p}^2}{2M}|\alpha'\rangle\, \varrho_{\alpha'\alpha} + \frac{1}{2}\sum_{\alpha\alpha'\beta\beta'} \langle\alpha\beta|\hat{v}_{\rm eff}|\alpha'\beta'\rangle\, \varrho_{\alpha'\alpha}\, \varrho_{\beta'\beta}$$

$$\rightarrow \quad h|\alpha^{(0)}\rangle = \epsilon_{\alpha^{(0)}}|\alpha^{(0)}\rangle \quad \textbf{with} \quad h = \sum_{\alpha\alpha'} |\alpha\rangle \left.\frac{\delta E}{\delta\varrho_{\alpha'\alpha}}\right|_{\varrho^{(0)}} \langle\alpha'| = \frac{\boldsymbol{p}^2}{2M} + U$$

input : eff. int. (or EF) $\cdots \quad \hat{v}_{\rm eff} = \sum_i C_i[n(\boldsymbol{r})]\cdot\hat{w}_i$

$\hat{w}_i$ **: two-body operator**

$$\rightarrow \quad U|\alpha\rangle = \sum_i \sum_{\beta\beta'} \big\langle *\beta\big|C_i[n^{(0)}(\boldsymbol{R}_{\alpha\beta})]\cdot\hat{w}_i\big|\alpha\beta'\big\rangle\, \varrho^{(0)}_{\beta\beta'}$$

$$+ \frac{1}{2}|\alpha\rangle \sum_i \sum_{\alpha''\alpha'\beta\beta'} C_i'[n^{(0)}(\boldsymbol{r}_\alpha)]\, \big\langle\alpha''\beta\big|\delta(\boldsymbol{r}_\alpha - \boldsymbol{R}_{\alpha''\beta})\cdot\hat{w}_i\big|\alpha'\beta'\big\rangle\, \varrho^{(0)}_{\alpha'\alpha''}\, \varrho^{(0)}_{\beta'\beta}$$

$\uparrow$

rearrangement term

$$\begin{cases} \min E_A[\varrho^{(0)}]\ \textbf{(iterative)} & \rightarrow\ |\Phi_A\rangle \\ U\ (=\mathcal{V}^{\rm SFP})\ \textbf{from}\ E_{A+1}[\varrho^{(0)}+\delta\varrho] & \rightarrow\ |\psi_N\rangle \end{cases} \qquad \big(\varrho^{(0)}\ \leftrightarrow\ |\Phi_A\rangle\big)$$

★ **Finite-range eff. int. ($\leftrightarrow$ non-local EF)**

— possibly important to cover high ϵ

- **SCMF code: `MFGSB` $\rightarrow$ accessible @ Chiba U. Repository**

  (`https://opac.ll.chiba-u.jp/da/curator/900123722/?lang=1`; **DOI: 10.20776/900123722**)

  - **GEM (multi-range Gaussian bases)**
  - **adapted to finite-range int. $\cdots$ Yukawa f.f. (& tensor force)**
  - **able to describe halo**

- **complete formulae for $U^{\mathrm{KS}}\,(=\mathcal{V}^{\mathrm{SFP}})$ $\rightarrow$ derived (up to $\hat{v}_{ij}^{(\mathrm{LS})}$ & $\hat{v}_{ij}^{(\mathrm{TN})}$)**

  H.N. & K. Ishida, PRC 109, 044614

- **new code applicable to non-local optical pot. $\rightarrow$ available**

  (*e.g.* `SIDES`)

exchange term of $\hat{v}_{ij}^{(\mathrm{LS})}$:

$$
\begin{aligned}
F^{(\mathrm{exc,LS})}_{\ell j,\tau_\alpha}\,\mathcal{R}_{\ell j}(r) = {}& \frac{1}{2}\sum_{\substack{\ell_\alpha j_\alpha \\ \nu_\alpha \nu_{\alpha'}(\in A)}} (2\ell_\alpha+1)\,(2j_\alpha+1)\,\varrho^{(\tau_\alpha \ell_\alpha j_\alpha)}_{\nu_\alpha \nu_{\alpha'}} \sum_\lambda (2\lambda+1)\,(\ell_\alpha\,0\,\lambda\,0\,|\,\ell\,0)^2 \begin{Bmatrix} \ell_\alpha & 1/2 & j_\alpha \\ \lambda & 0 & \lambda \\ \ell & 1/2 & j \end{Bmatrix}^2 \\
& \times \left\{ j(j+1) - \ell(\ell+1) - \frac{3}{4} + j_\alpha(j_\alpha+1) - \ell_\alpha(\ell_\alpha+1) - \frac{3}{4} \right\} \\
& \times \int r'^2 dr'\, g_\lambda(r,r')\, R^*_{\nu_{\alpha'}\ell_\alpha j_\alpha}(r')\, R_{\nu_\alpha l_\alpha j_\alpha}(r)\, \mathcal{R}_{\ell j}(r') \\
& + \sum_{\substack{\ell_\alpha j_\alpha \\ \nu_\alpha \nu_{\alpha'}(\in A)}} (2\ell_\alpha+1)\,(2j_\alpha+1)\,\varrho^{(\tau_\alpha \ell_\alpha j_\alpha)}_{\nu_\alpha \nu_{\alpha'}} \\
& \times \sum_{\lambda\lambda'\lambda''\lambda'''\kappa} (2\kappa+1)\,(\ell_\alpha\,0\,\lambda''\,0\,|\,\ell\,0)\,(\ell_\alpha\,0\,\lambda'''\,0\,|\,\ell\,0) \begin{Bmatrix} \ell_\alpha & 1/2 & j_\alpha \\ \kappa & 0 & \kappa \\ \ell & 1/2 & j \end{Bmatrix} \begin{Bmatrix} \ell_\alpha & 1/2 & j_\alpha \\ \lambda' & 1 & \kappa \\ \ell & 1/2 & j \end{Bmatrix} \\
& \times \int r'^2 dr'\, g_\lambda(r,r')\, R^*_{\nu_{\alpha'}\ell_\alpha j_\alpha}(r') \Bigg[\frac{\sqrt{3}}{2}\sqrt{\ell\,(\ell+1)\,(2\ell+1)} \Big(\sqrt{2\kappa+1}\,\delta_{\lambda\lambda'}\delta_{\lambda\lambda''}\delta_{\lambda\lambda'''}\, W(\ell\,1\,\ell_\alpha\,\lambda\,;\,\ell\,\kappa) \\
& - \frac{6r}{r'}\sqrt{(2\lambda'+1)\,(2\lambda''+1)}\,(\lambda\,0\,1\,0\,|\,\lambda''\,0)\,(\lambda\,0\,1\,0\,|\,\lambda'''\,0)\, W(\lambda\,1\,\kappa\,1\,;\,\lambda'\,1) \\
& \times \{ \sqrt{2\kappa+1}\,\delta_{\lambda'\lambda'''}\, W(\ell\,1\,\ell_\alpha\,\lambda''\,;\,\ell\,\kappa)\, W(\lambda\,1\,\kappa\,1\,;\,\lambda''\,1) \\
& + \sqrt{2\lambda'+1}\,\delta_{\kappa\lambda'''}\, W(\ell\,1\,\ell_\alpha\,\lambda''\,;\,\ell\,\lambda')\, W(\lambda\,1\,\lambda'\,1\,;\,\lambda''\,1)\} \Big)\, R_{\nu_\alpha \ell_\alpha j_\alpha}(r)\, \mathcal{R}_{\ell j}(r') \\
& - (-)^{\lambda'+\kappa}\frac{\sqrt{3}}{2}\sqrt{l_\alpha\,(l_\alpha+1)\,(2l_\alpha+1)} \Big(\sqrt{2\kappa+1}\,\delta_{\lambda\lambda'}\delta_{\lambda\lambda''}\delta_{\lambda\lambda'''}\, W(\ell_\alpha\,1\,\ell\,\lambda\,;\,\ell_\alpha\,\kappa) \\
& - \frac{6r'}{r}\sqrt{(2\lambda'+1)\,(2\lambda''+1)}\,(\lambda\,0\,1\,0\,|\,\lambda''\,0)\,(\lambda\,0\,1\,0\,|\,\lambda'''\,0)\, W(\lambda\,1\,\kappa\,1\,;\,\lambda'\,1)
\end{aligned}
$$

$$
\begin{aligned}
& \times \{ \sqrt{2\kappa+1}\,\delta_{\lambda'\lambda'''}\, W(\ell_\alpha\,1\,\ell\,\lambda''\,;\,\ell_\alpha\,\kappa) W(\lambda\,1\,\kappa\,1\,;\,\lambda''\,1) \\
& + \sqrt{2\lambda'+1}\,\delta_{\kappa\lambda'''}\, W(\ell_\alpha\,1\,\ell\,\lambda''\,;\,\ell_\alpha\,\lambda')\, W(\lambda\,1\,\lambda'\,1\,;\,\lambda''\,1)\} \Big)\, R_{\nu_\alpha \ell_\alpha j_\alpha}(r)\, \mathcal{R}_{\ell j}(r') \\
& - 3\sqrt{2\,(2\lambda'+1)}\,\delta_{\kappa\lambda''}\delta_{\lambda'\lambda'''}\,(\lambda\,0\,1\,0\,|\,\lambda''\,0)\,(\lambda\,0\,1\,0\,|\,\lambda'''\,0) W(\lambda\,1\,\kappa\,1\,;\,\lambda'\,1) \\
& \times \left\{ r\, R_{\nu_\alpha \ell_\alpha j_\alpha}(r)\, \frac{d\mathcal{R}_{\ell j}(r')}{dr'} - (-)^{\lambda'+\kappa}\, r'\, \frac{dR_{\nu_\alpha \ell_\alpha j_\alpha}(r)}{dr}\, \mathcal{R}_{\ell j}(r') \right\} \Bigg]. \qquad \text{(B14)}
\end{aligned}
$$

★ **Eff. int.** (or **EF**) to be used $\cdots$ $\hat{v}_{ij} = \hat{v}_{ij}^{(\mathrm{C})} + \hat{v}_{ij}^{(\mathrm{LS})} + \hat{v}_{ij}^{(\mathrm{TN})} + \hat{v}_{ij}^{(\mathrm{C}\rho)}$

M3Y int. $\cdots$ **Yukawa f.f.** $\rightarrow$ **fit to** G**-matrix (@** $n \approx n_0/3$**)**

'**M3Y-P**n' H.N., PRC 68, 014316

- **modifying M3Y-Paris** $\begin{cases} \text{replace short-range part of } \hat{v}^{(\mathrm{C})} \text{ by } \hat{v}^{(\mathrm{C}\rho)} \\ \text{enhance } \hat{v}^{(\mathrm{LS})} \quad (\leftrightarrow \ell s \text{ splitting}) \end{cases}$
- **keeping** $\hat{v}_{\mathrm{OPEP}}^{(\mathrm{C})}$ **(longest-range part)**
- **no change for** $\hat{v}^{(\mathrm{TN})}$ **from M3Y-Paris — realistic tensor force**

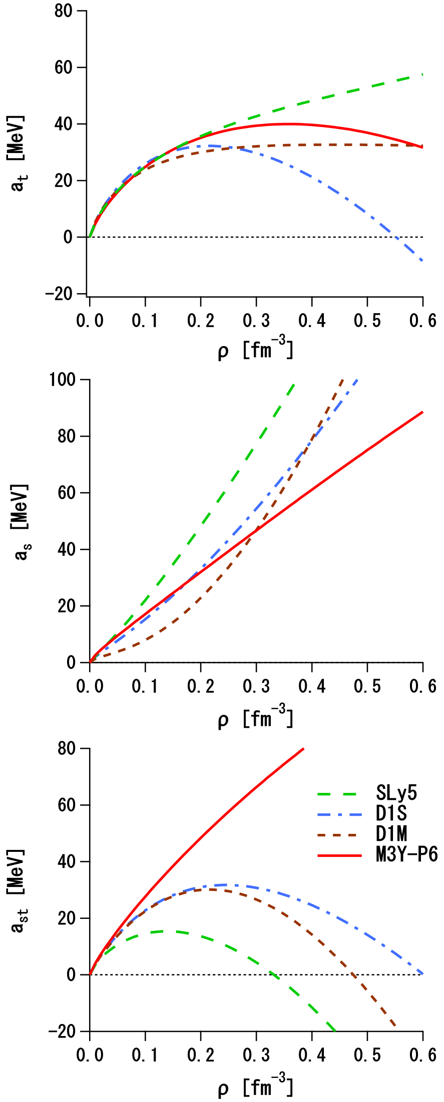
M3Y-P6: H.N., PRC 87, 014336; IJMPE 29, 1930008

$\cdots$ "**semi-realistic**" **int. — compromise of micro. & phenom. int.**
or "fine tuning" of micro. int.
($\cdots$ **attempt to fill the gap !**)

(**Gogny-D1S** & **Skyrme-SLy4** $\cdots$ **for comparison)**

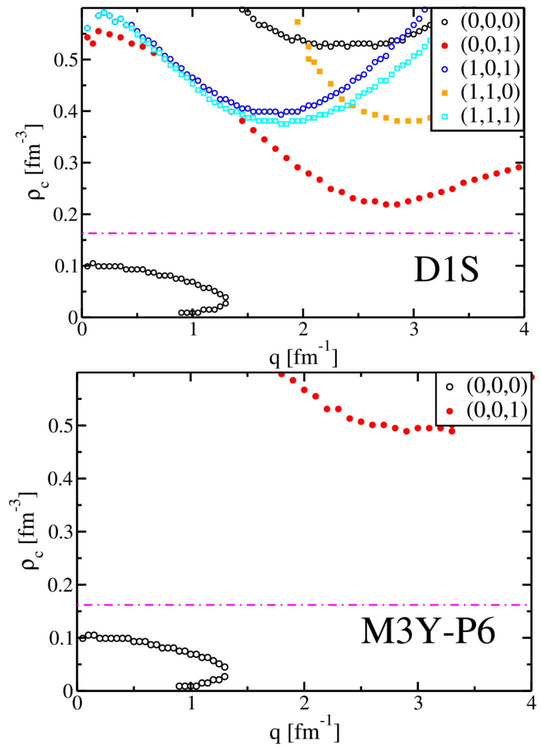
★ Unphysical instabilities (— in many SCMF int. even at MF level ?)

Curvatures of E/A against spin & isospin exc. :



H.N., PRC 87, 014336 ('13)

Instabilities in nuclear matter — *e.g.* response fn. (with $\hat{v}^{(\mathrm{TN})}$) :

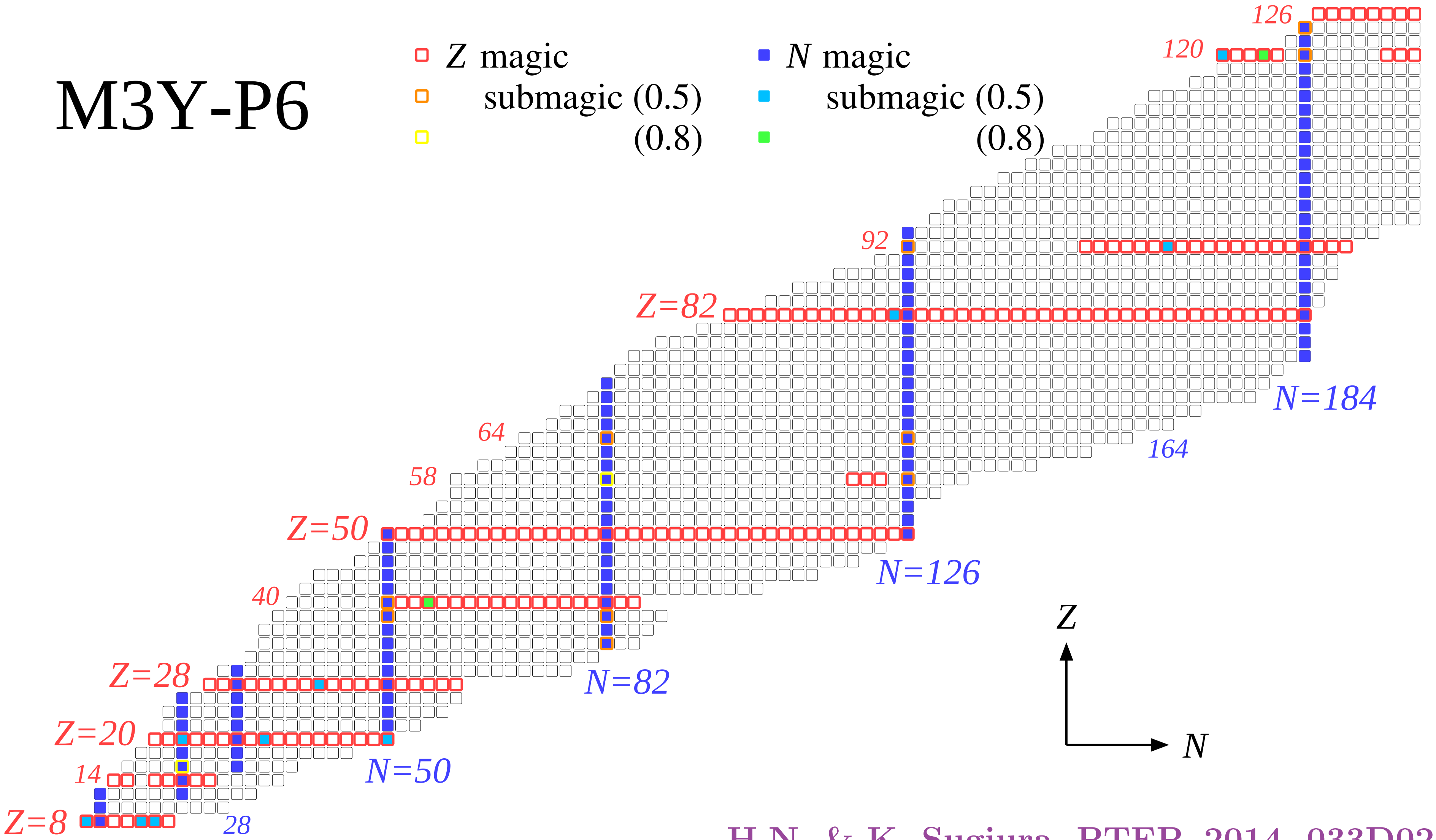


"A quite remarkable result is that the Nakada's interactions are essentially free from instabilities up to** $\approx 2\rho_0$**"

D. Davesne *et al.*, PPNP 120, 103870 ('21)

★ **S.p. pot. @ $\epsilon < 0$ — *e.g.* magic numbers**

← quenching of pair correlation



H.N. & K. Sugiura, PTEP. 2014, 033D02

⇒ magic # compatible with almost all available data !

($\cdots$ good s.p. pot. at $\epsilon < 0$)

★ **S.p. pot. @** $\epsilon > 0 \quad \cdots \quad A \to \infty$ **limit (homogeneous nuclear matter)**

$$\langle \boldsymbol{k}\sigma\tau|U|\boldsymbol{k}\sigma\tau\rangle \approx U_0(\epsilon_N;n) + \tau\,U_1(\epsilon_N;n)\,\eta_t\,; \qquad \eta_t := \sum_{\tau'} \tau' n_{\tau'}/n$$

(non-locality $\to k \to \epsilon_N$**)**

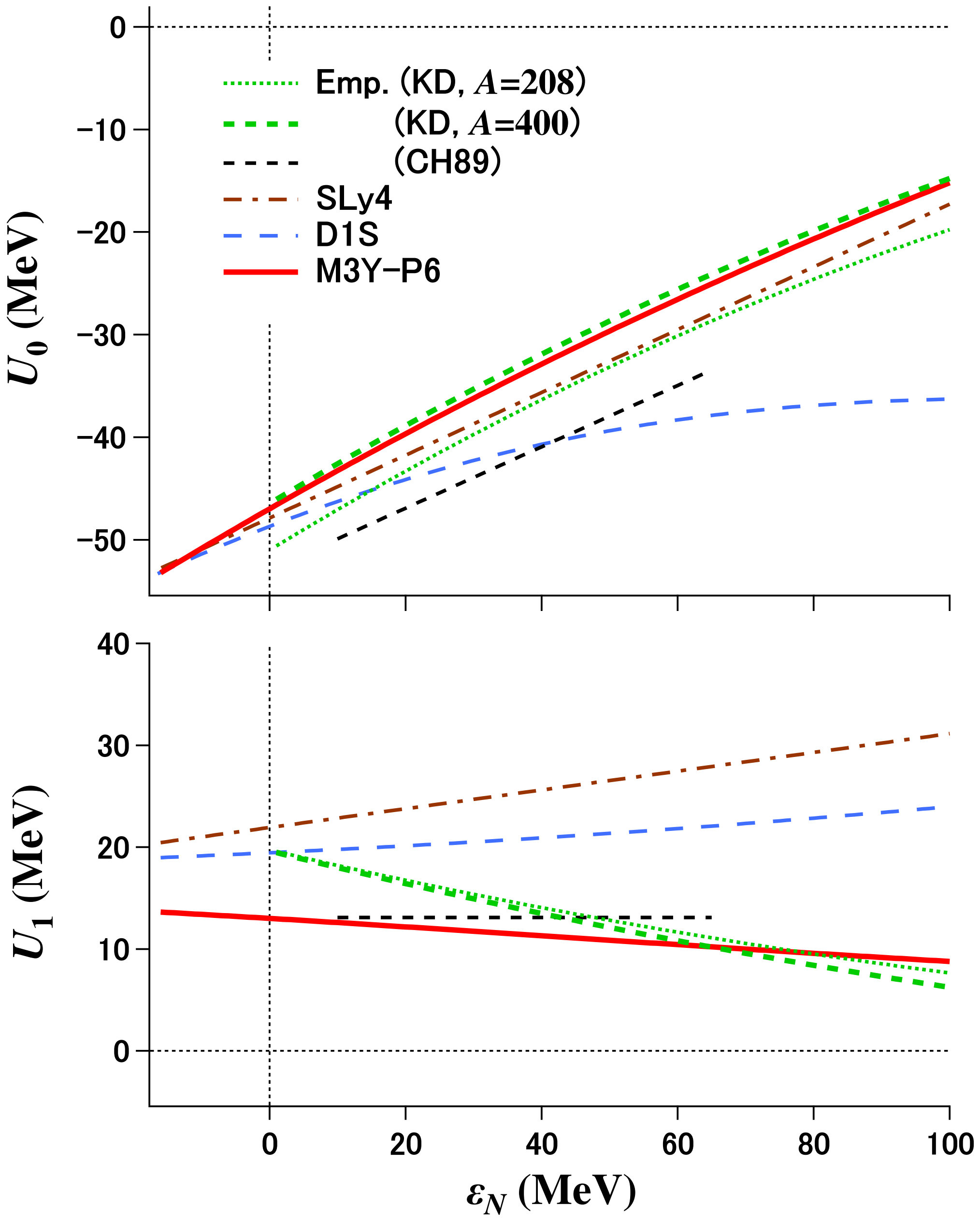
$$\updownarrow$$

$$\lim_{A\to\infty} \mathcal{V}^{\mathrm{emp}}(\boldsymbol{r}=0) \approx U_0^{\mathrm{emp}}(\epsilon_N;\rho_0) - \tau\,U_1^{\mathrm{emp}}(\epsilon_N;\rho_0)\,\frac{N-Z}{A}\,; \qquad \tau = \begin{cases} +1 & \textbf{for } p \\ -1 & \textbf{for } n \end{cases}$$

(local, A**- &** ϵ_N**-dep.)**

$\mathcal{V}^{\mathrm{emp}} \leftarrow$
- **Koning-Delaroche** $(0.001 \le \epsilon_N \le 200\,\mathrm{MeV})$ **NPA 713, 231**
- **'CH89'** $(10 \le \epsilon_N \le 65\,\mathrm{MeV})$ **Phys. Rep. 201, 57**

(KD $\cdots$ **care needed for** $A \to \infty$
$\because$ $(c_1 A)$ **term with small** $|c_1|$**)**



Emp. (KD, A=208)
(KD, A=400)
(CH89)
SLy4
D1S
M3Y-P6
U_0 (MeV)
U_1 (MeV)
ε_N (MeV)

★ **S.p. pot. @** $\epsilon > 0$ $\cdots$ **finite (doubly magic) nuclei**

$$|\Psi_{A+1}\rangle \approx |\Phi_A\rangle \otimes |\psi_N\rangle$$

$|\Phi_A\rangle \ \leftarrow$ **"SCMF" (or KS)** $(\epsilon < 0)$

$|\psi_N\rangle \ \leftarrow \mathcal{U} = \mathcal{V}^{\mathrm{SFP}} + i\mathcal{W}^{\mathrm{emp}}$ $(\epsilon > 0)$

$$\begin{cases} \mathcal{V}^{\mathrm{SFP}} = U \text{ — self-consistent pot.} \\ \qquad \cdots \text{ non-local, } \epsilon_N\text{-indep.} \\ \mathcal{W}^{\mathrm{emp}} \leftarrow \text{KD} \cdots \text{ local, } A\text{- \& } \epsilon_N\text{-dep. (empirical)} \end{cases}$$

$|\Phi_A\rangle$ & $\mathcal{V}^{\mathrm{SFP}}$

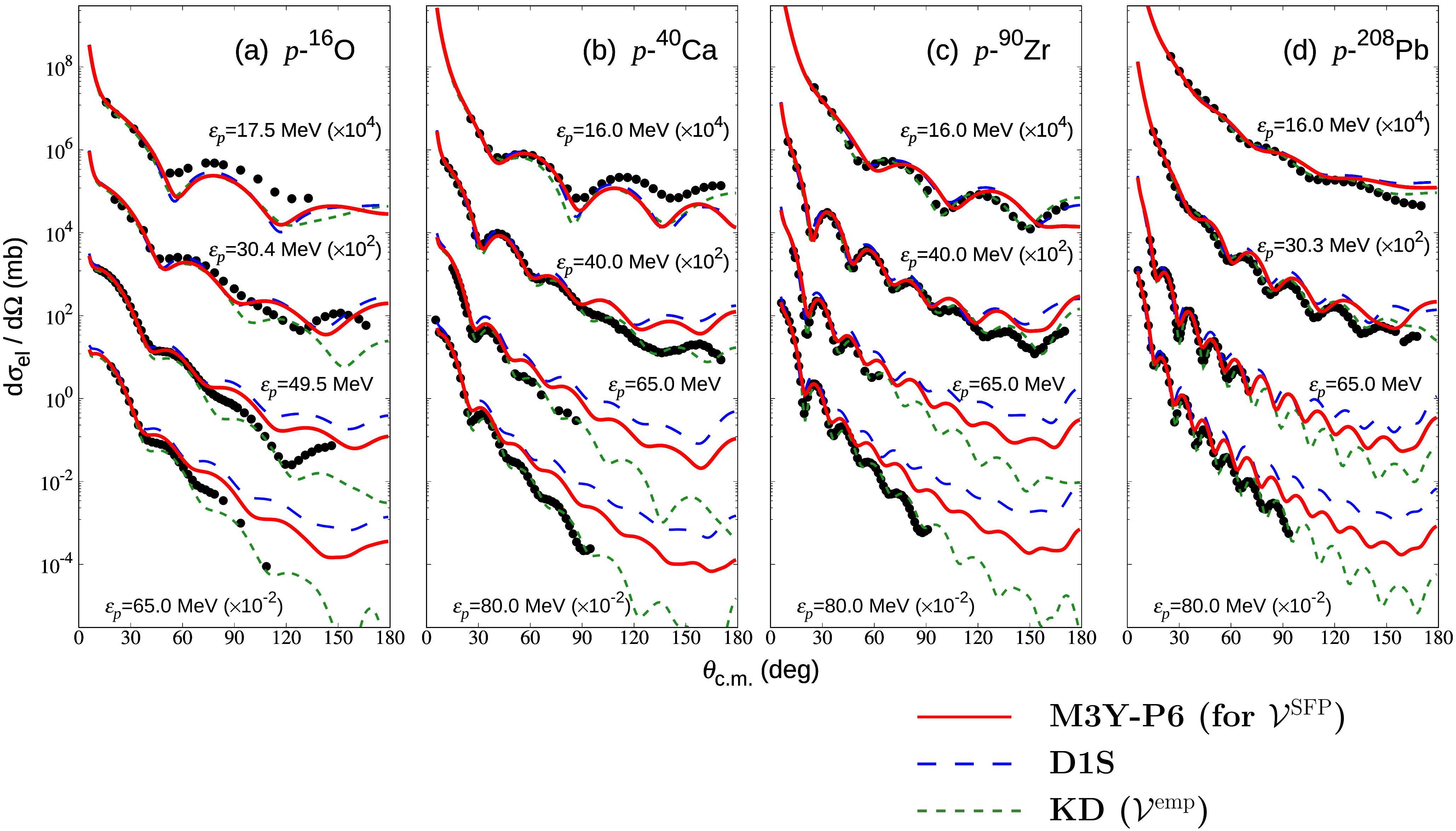
$\nwarrow$

same framework, same eff. int.

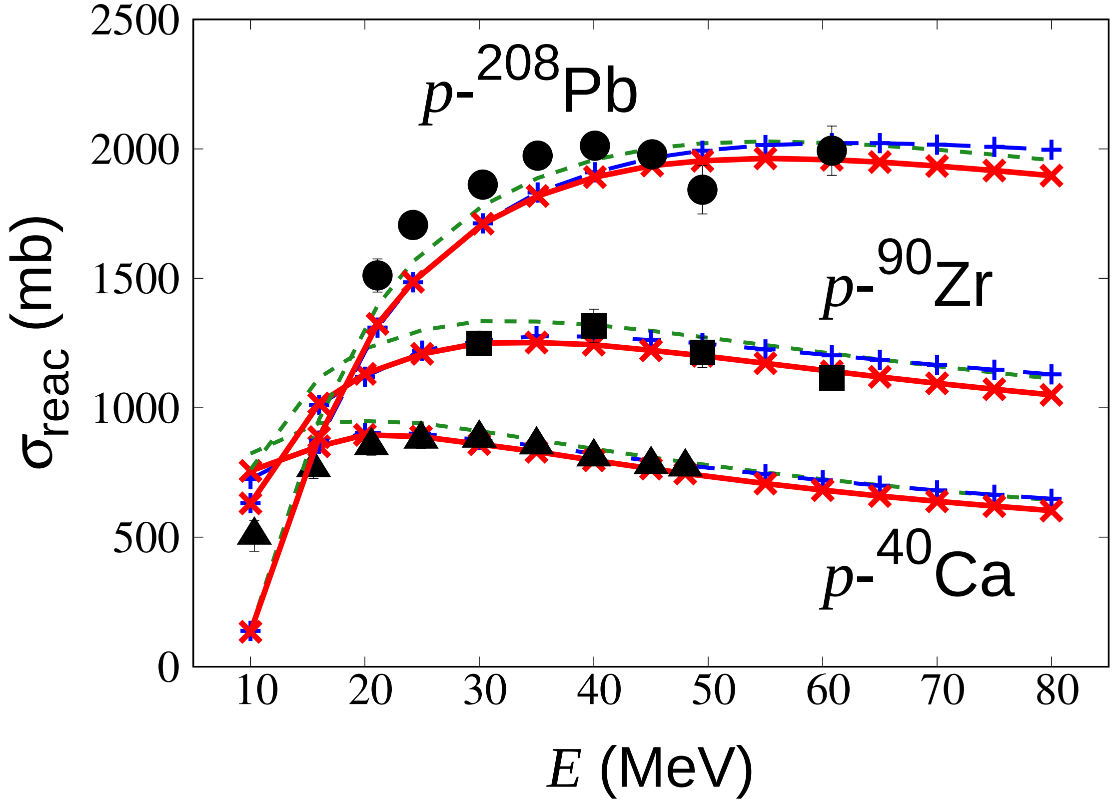
$\Rightarrow$ **observables** $\left(\dfrac{d\sigma_{\mathrm{el}}}{d\Omega}, \sigma_{\mathrm{reac}} \text{ or } \sigma_{\mathrm{tot}}, A_y, \textit{etc.}\right)$ **via** `SIDES` **code**

CPC 254, 107340

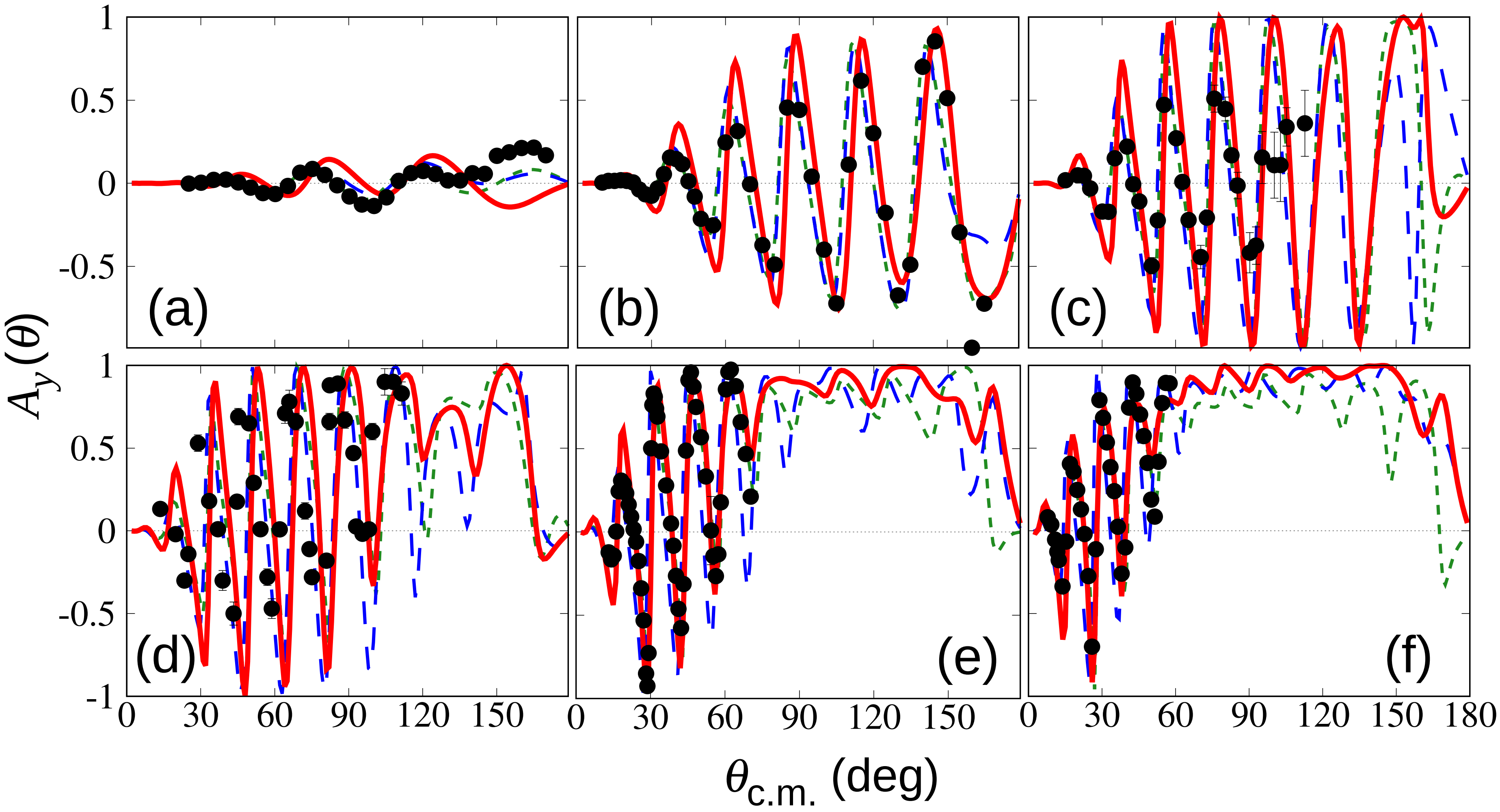
- **cross sections (up to angle-dep.)**



- **validity for $\mathcal{W}^{\rm emp}$ combined with $\mathcal{V}^{\rm SFP}$?**



- **spin-dep. ?**

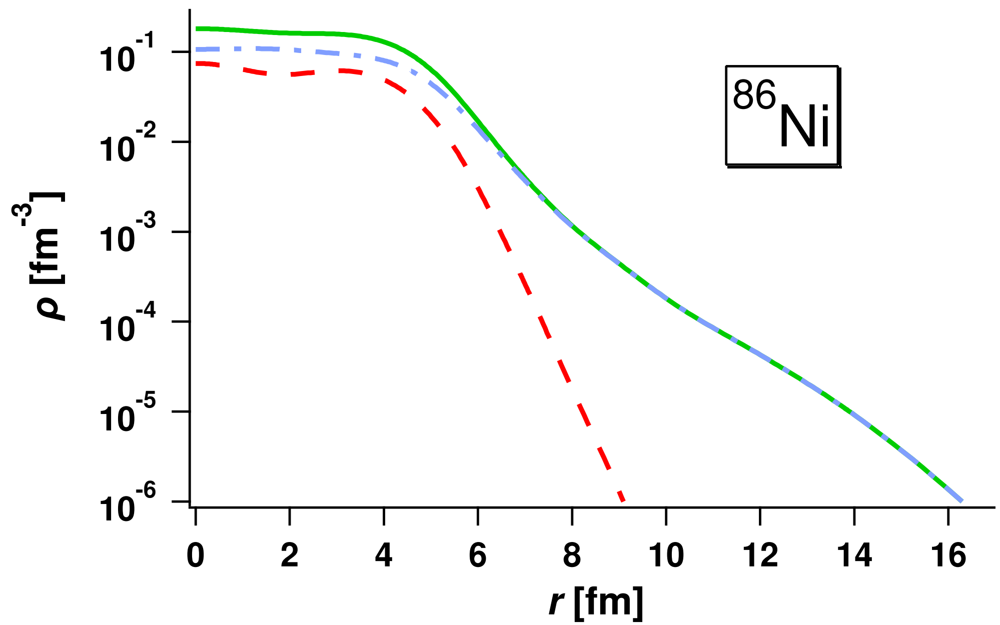


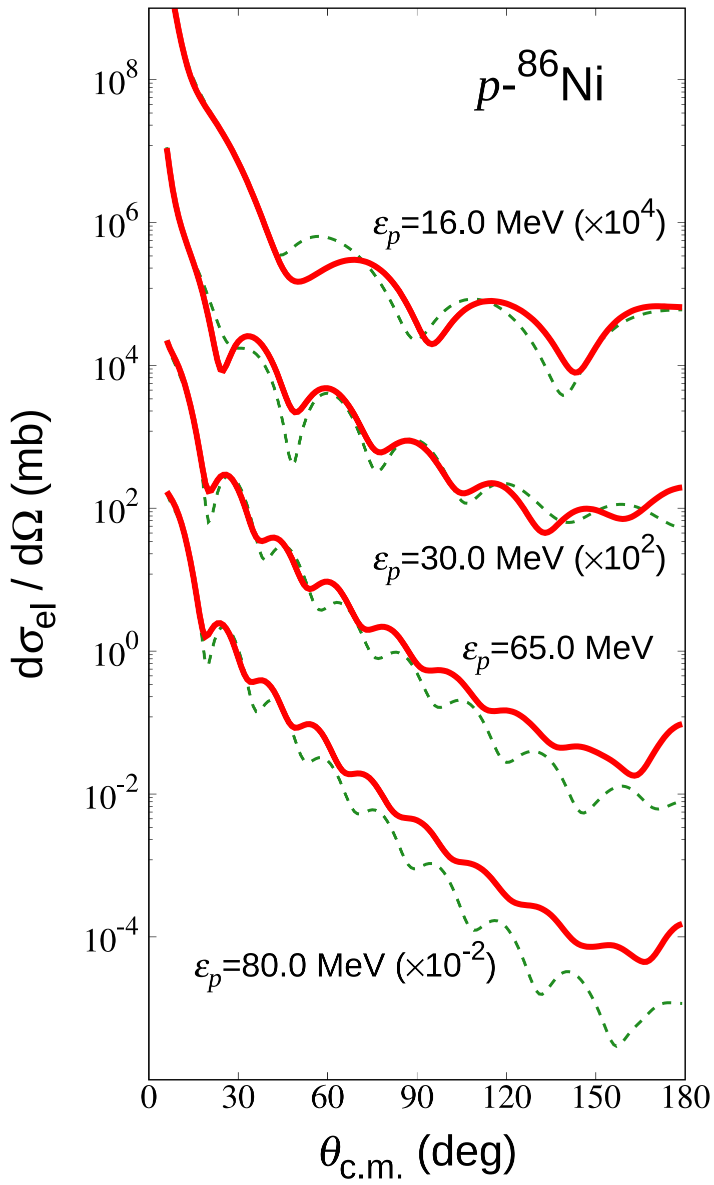
p-^{208}**Pb,** $\epsilon_p = 16 - 80\,\mathrm{MeV}$

$\Longrightarrow$ **good s.p. pot. in** $-20 \lesssim \epsilon \leq (65-80)\,\mathrm{MeV}$ **!**

- elastic scatt. off halo nuclei ? $\cdots$ $\mathcal{U}^{\rm emp}$ — questionable !

  $\rightarrow$ ^{86}Ni : (nearly) doubly magic with $n2s_{1/2}$ occ. PRC 81, 051302(R)





★ Validity / limitation of (Brieva-Rook) local approx.

NPA 291, 299 & 317; 297, 206

- **Brieva-Rook LA** $\cdots$ $\mathcal{V}^{\mathrm{SFP}} \leftarrow n_{p,n}^{(0)}(\boldsymbol{r})$ **only !**

  — **valid for "any"** A **&** ϵ_N **?**

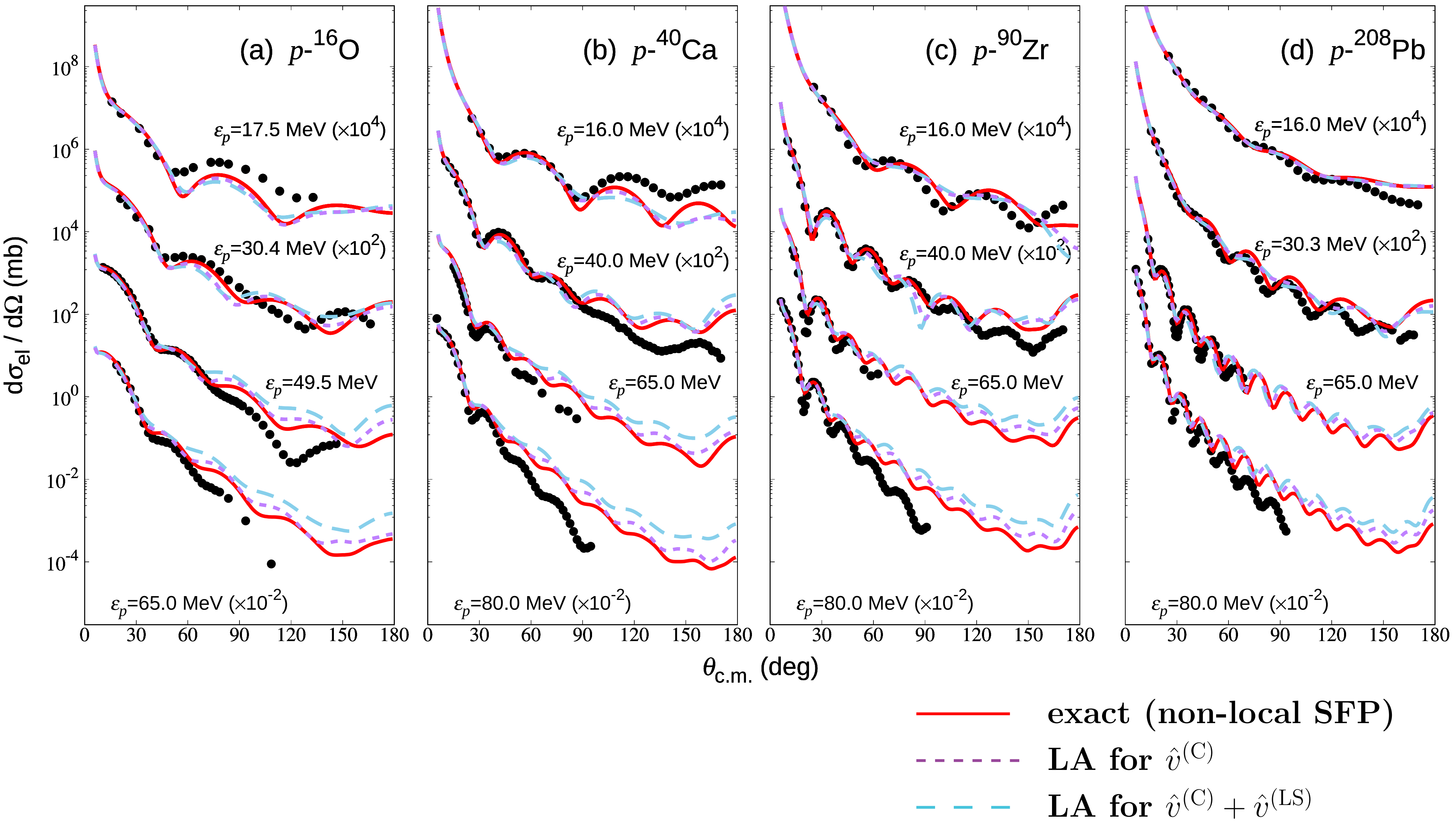
- **beyond** $n_{p,n}^{(0)}(\boldsymbol{r})$ **?** $\rightarrow$ $\varrho^{(0)}(\boldsymbol{r},\boldsymbol{r}')$ **(1-b. DM)**

  advantage of PW $\cdots$
  - $\varrho^{(0)}$ **within int. range : well examined !**
  - **influence of** $\hat{v}_{ij}^{(\mathrm{LS})}$ **(&** $\hat{v}_{ij}^{(\mathrm{TN})}$**) can be investigated**

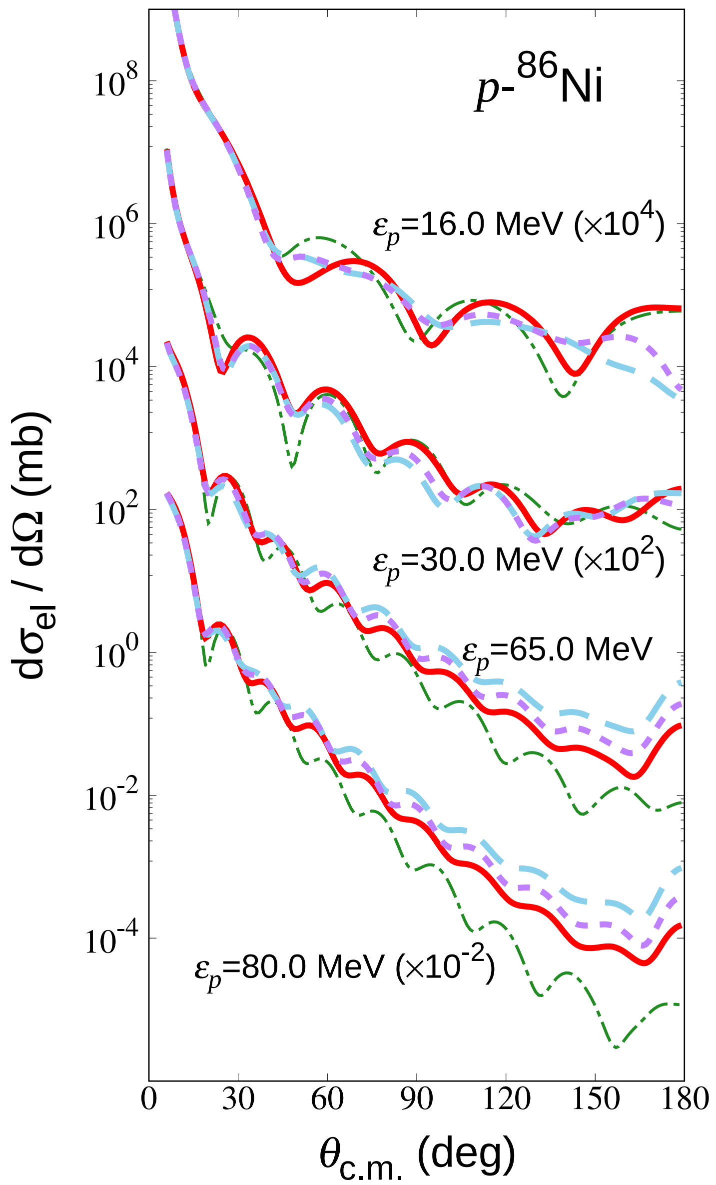
cf. H.F. Arellano *et al.*, PRC 42, 652

K. Minomo *et al.*, JPG 37, 085011

- **cross sections (up to angle-dep.)**



- **scatt. off halo nuclei ?**



exact (non-local SFP)

LA for $\hat{v}^{(\mathrm{C})}$

LA for $\hat{v}^{(\mathrm{C})} + \hat{v}^{(\mathrm{LS})}$

KD (empirical)

○ **notable influence of LA** @ $\varepsilon_p = 16\,\mathrm{MeV}$

$\leftrightarrow$ **halo**

○ **coupling to continuum**

— $\mathcal{V}^{\mathrm{SFP}}$ **good enough ?**

$\rightarrow$ **future exp. ?**

V. Summary & perspective

★ **Self-consistent single-nucleon pot.**

$\cdots$ **extention of KS approach w.r.t.** ϵ

$\rightarrow$ **target w.f. &** $\mathcal{V}^{\mathrm{SFP}}$ **(non-local,** ϵ_N**-indep.)**

$\searrow$

N**-**A **scatt. (**$A \rightarrow \infty$ **limit & finite nuclei)**

- **Wide energy range (**$\epsilon \lesssim 80\,\mathrm{MeV}$**) can be covered by a single energy-indep. int.!** (— **demonstrated by M3Y-P6)**

  $\cdots$ **power of KS approach w/ semi-realistic int.**

  $\hookrightarrow$ **filling the gap b/w micro. & phenom. int. ("fine tuning")**

- **Optical pot. @ unstable nuclei**

  $\cdots$ **beyond empirical pot.** ***e.g.*** **halo effects**

- **(Brieva-Rook) LA**
  - **insufficient @ large** q **(**$\epsilon_N \lesssim 80\,\mathrm{MeV}$**)**
  - **significant influence of LA for** $\hat{v}^{(\mathrm{LS})}$
  - **insufficient for halo nuclei (even at low** q**)**

★ Implication

- **Extendability with enhanced analyticity of $E^{\mathrm{KS}}[\varrho]$**

  ($\leftrightarrow$ v-representability)

  $\hookrightarrow$ **collection of well-examined s.p. pot. & levels up to high-ϵ**

- **Future extension to finite-T ?** **(optimistic view)**

  existence of $F_T^{\mathrm{KS}}[\varrho] = (E - TS)^{\mathrm{KS}}[\varrho]$ @ each T

  $\leftarrow \min_{\{\Psi\}\to \boldsymbol{Q}} F_T$ **: constrained search**

  dominance of s.p. d.o.f. @ higher-T $\rightarrow$ rationality of $F_T^{\mathrm{KS}}[\varrho]$

  — analyticity w.r.t. T ? $\cdots$ U^{KS} @ high-ϵ ?

  $\hookrightarrow$ **EoS @ finite-T consistent with exp. data**

  cf. detailed balance $\rightarrow$ no absorption @ equilibrium

  $\epsilon \lesssim 80\,\mathrm{MeV} \rightarrow T \lesssim 30\,\mathrm{MeV}$ $\Rightarrow$ **applicability to supernovae !(?)**