3 dimensional AdS (traversable) wormhole via Janus or non-local $T\bar{T}$ deformation

YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

Taishi Kawamoto

Yukawa Institute for Theoretical Physics

PhD. student

京都大学基礎物理学研究所
重力量子情報研究センター

Based on work with N.Nakamura, R.Maeda and T.Takayanagi,
arXiv: **2502.03531.** published in JHEP
And ongoing work additionally with J. Harper

@New advancements on defects and their applications

,

INTRODUCTION: WORMHOLES

- ✓Wormholes are important objects in (quantum) gravity
- ✓They appear various aspects
  - ✓Breaks a global symmetry

    [Hawking, Lavrelashvifi, Rubakov , Tinyakov , Coleman , Strominger , Giddins]
  - ✓Related to the entanglement structure of gravity

    [Suskind , Maldacnea , Raamsdonk]
  - ✓ non-pertubative objects which rescue the information loss paradox

    [Saad , Shenker , Stanford , several replica wormholes papers]
  - ✓Give a protocol of quantum teleportation via AdS/CFT if they are traversable

    [Gao, Jafferis, Wall]

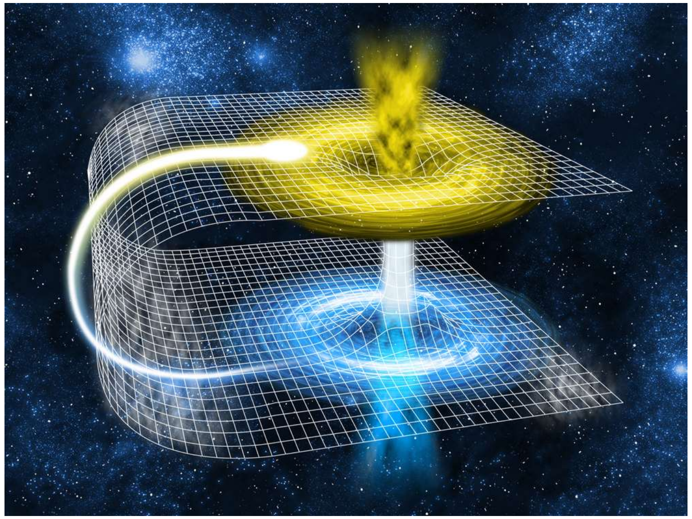


Figure from : What are wormholes? | Space

WORMHOLE IN ADS/CFT: PREVIOUS WORK

✓Previous work consider the coupling between two systems induces the traversable wormhole

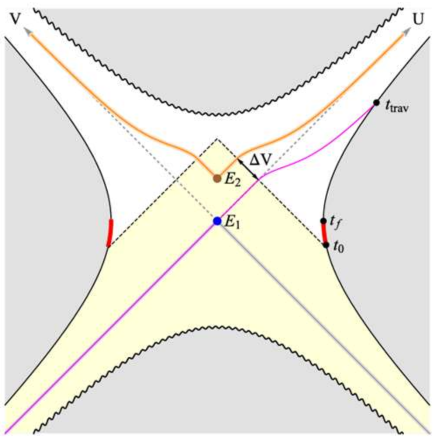
$$H_{int} = \int d^d x\, \mathcal{O}_L(x)\mathcal{O}_R(x)$$

✓[Gao,Jafferis,Wall] AdS3 and $\mathcal{O}$ is light single operator

⇒Mixed boundary condition induces the Casimir energy in the bulk

(Traversable wormhole is prohibited by averaged null energy condition)

✓[Maldacena,Qi] SYK model (AdS2)



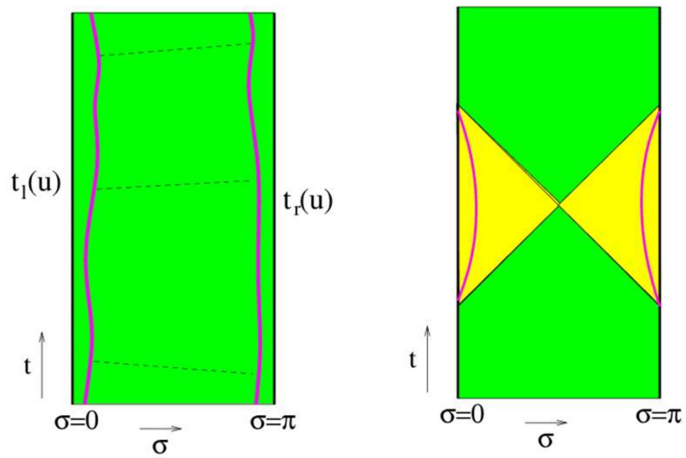


Figure from their papers

MOTIVATION FOR OUR WORK

✓Unsatisfactory point of previous work

1. GJW is gravitational perturbation $= O(G_N^0)$
2. MQ is low dimensional ⇒SYK/JT is special ?

✓Motivation

we would like to construct

explicit configurations of wormholes in 3d AdS

with huge backreaction and nice boundary interpretation

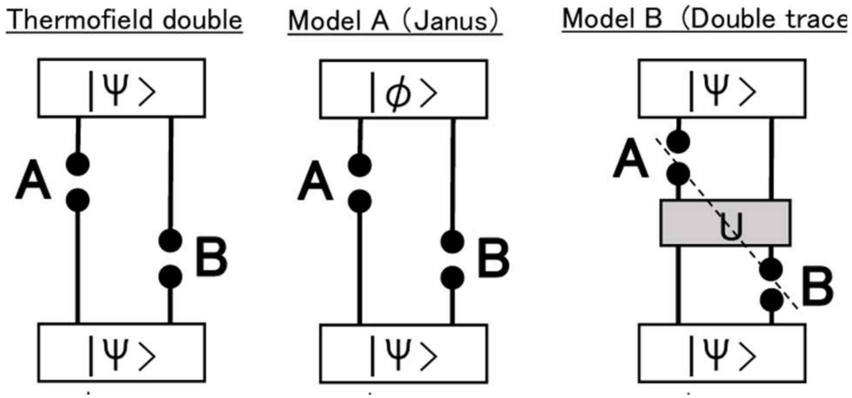
CONTENTS

FINDINGS: TWO TYPE OF WORMHOLE

✓We find there are two types of wormhole in AdS via concrete CFT deformations

1. Model A: via Janus deformation
2. Model B: Via non-local double trace or TT bar deformation:

✓With quantum circuit like understating there are completely different path integrals and distinguished by entanglement entropy (not in my talk)



2024 Dirac Medal and Prize Award Ceremony

https://www.youtube.com/watch?v=ajIDjAUjOmY&t=42s

MODEL A: JANUS TYPE DEFORMATION

JANUS SOLUTION

✓We consider a bulk gravitational theories with dilaton

$$I_{\rm grav} = \frac{1}{16\pi G_{\rm N}} \int d^3x \sqrt{g} \left[R[g] - g^{ab}\partial_a\varphi\partial_b\varphi + 2 \right].$$

✓ This has clear Janus solutions with AdS2 slices [Bak-Gutperle-Hirano, 03, 07]

$$ds^2 = f(\mu)(d\mu^2 + ds^2_{\rm AdS_2}), \qquad \varphi = \varphi(\mu).$$

$$-\mu_0 \le \mu \le \mu_0,$$

$$\mu_0 = \sqrt{\frac{2}{1+\chi}} \int_0^1 \frac{ds}{\sqrt{(1-s^2)\left(1-\frac{1-\chi}{1+\chi}s^2\right)}} = \sqrt{\frac{2}{1+\chi}} K\left(\frac{1-\chi}{1+\chi}\right)$$

where γ or $\chi = \sqrt{1-2\gamma^2}$ is a parameter which characterizes the solutions

JANUS SOLUTION

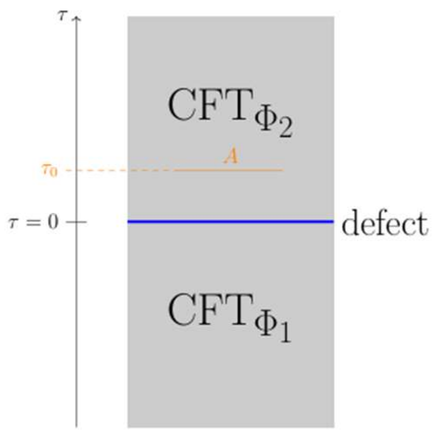
✓ Depending of the form of AdS2 metric, we have variety of situations

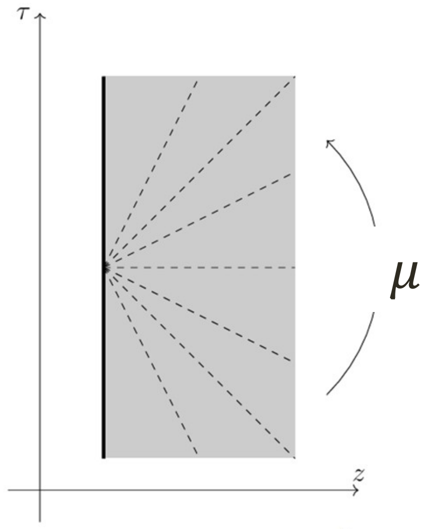
1. Poincare AdS2 ⇒Interface CFT on the plane

$$ds^2_{\mathrm{AdS}_2} = \frac{dx^2 + d\xi^2}{\xi^2}$$

$$\mathcal{L} = \mathcal{L}_{\mathrm{CFT}} + \gamma \mathcal{L}_{\mathrm{interface}}$$

Coupling strength of interface





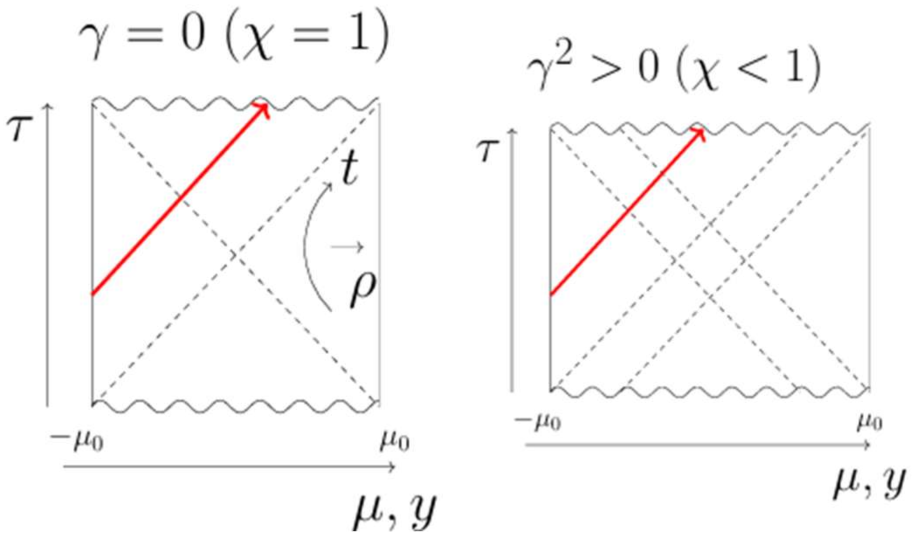
TIME DEPENDENT JANUS SOLUTION

Take AdS2 blackhole [Bak-Gutperle-Hirano; 07]

$$ds^2_{\mathrm{AdS}_2} = -d\tau^2 + r_0^2 \cos^2 \tau d\theta^2, \quad (-\frac{\pi}{2} \leq \tau \leq \frac{\pi}{2}).$$

- ✓ $\gamma = 0$→usual BTZ blakhole
- ✓$0 < \gamma < \gamma_c$→ Boundaries are getting

(The value of μ_0 is getting larger)

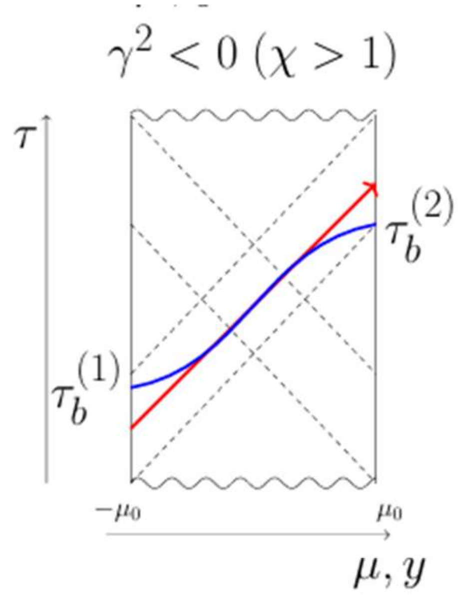


JANUS TRAVSERBABLE WORMHOLE

✓Now we consider $\gamma^2 < 0$ or $\chi > 1$.

✓ Then, we have traversable wormhole

[TK Maeda Nakamura Takayanagi]



Two ways of understanding

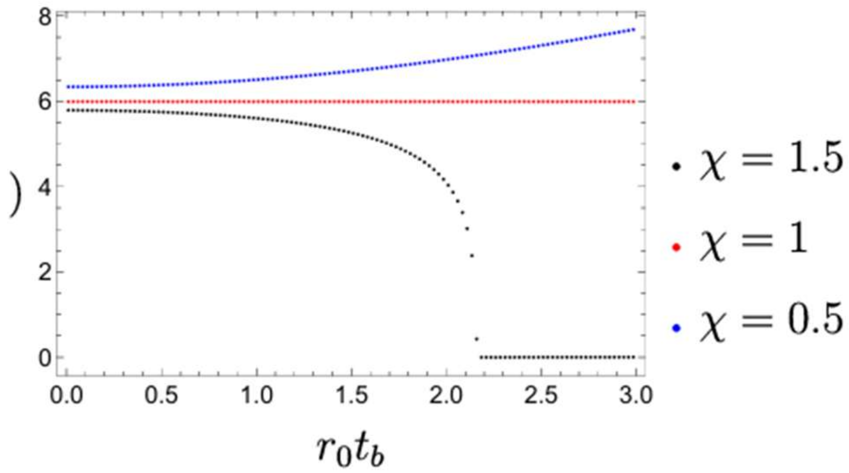
•Two boundaries are getting close

$=$ μ_0 is getting small

• Dilaton is pure imaginary

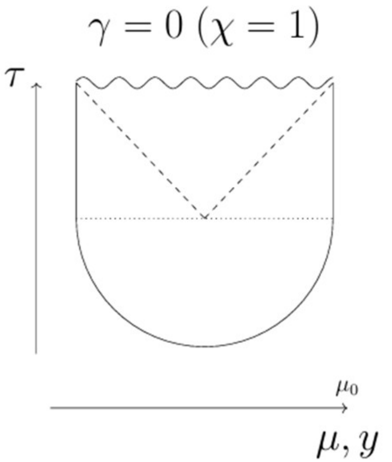
$\Rightarrow$ violation of energy condition

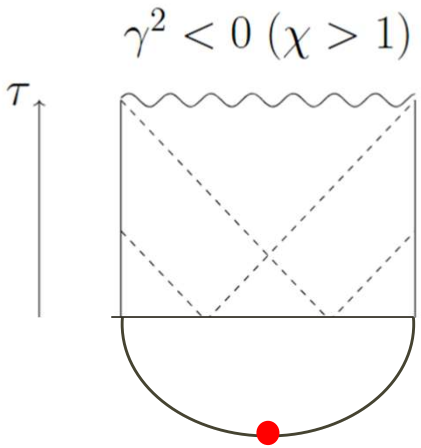
Geodesic Length



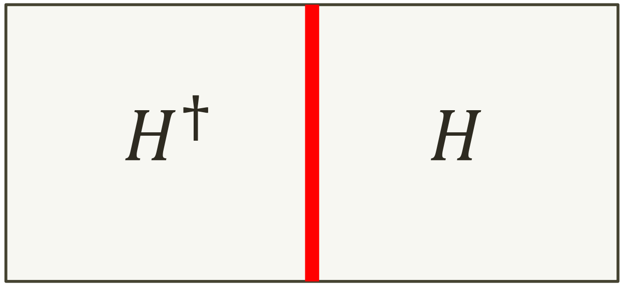
BOUNDARY PERSPECTIVE:

✓We think the path integral corresponds to the quantum quench





Non-Hermite interface



$$\mathrm{Tr}[e^{-\beta H}] = \langle TFD(\beta)|TFD(\beta)\rangle$$

$$\mathrm{Tr}\left[e^{-\frac{\beta H}{2}} e^{-\frac{\beta H^\dagger}{2}}\right] = \langle\, \overline{TFD}|TFD\rangle$$

⇒density matrix is non-Hermite

Similar to [Bak-Gutperle-Karch]

$VHV^{-1} = H^\dagger$ duality defect?

☚realized in pseudo Hermite case and realized as kind of modular conjugation in the bulk

[TK, Maeda, Nakamura, Takayanagi on-going and TK, Miyaji, Numasawa, Tasuki on-going about SYK]

MODEL B:NON-LOCAL TT BAR DEFORMATION

NON-LOCAL TT BAR DEFORMATION

✓Here we discuss the other deformation

⇒non-local TT bar deformation between two CFTs

$$\frac{\partial}{\partial\mu}S_{[\mu]} = \frac{1}{2}\int d^2x\sqrt{\gamma}\epsilon_{ac}\epsilon_{bd}(T_{[\mu]}{}^{(1)})^{ab}F(-\Box_\gamma)(T_{[\mu]}{}^{(2)})^{cd},$$

✓Why "TT bar" and "non-local" ?

1. TT bar deduces the backreaction.
2. But local TT bar is irrelevant deformation

⇒ Backreaction only happens around the boundary

⇒ Non-locality makes only IR mode can contributes!

NON-LOCAL TT BAR

✓Here we focus on single side deformation

$$S_{T\overline{T}} = -\frac{1}{2}\int d^2x\sqrt{\gamma_{[\mu]}}\epsilon_{ac}\epsilon_{bd}T^{ab}F(-\Box_\gamma)T^{cd}.$$

(Actually two side one is very difficult…. 😢)

✓ We can discuss the flow equation and gravity dual

✓To this end, we review the local TT bar deformation

TT BAR DEFORMATION (REVIEW)

✓TT bar deformation is an irrelevant deformation of 2D QFTs

[Zamolodchikov, Cavagli`a, Negro, Sz´ecs´enyi R. Tateo]

$$\frac{\partial}{\partial\mu}S_{[\mu]} = \int d^2x\sqrt{\gamma}\left[\left(T_{[\mu]}\right)_{ab}\left(T_{[\mu]}\right)^{ab} - \left(\left(T_{[\mu]}\right)^a_a\right)^2\right],$$

✓This defines the sequence of QFTs whose stress tensors are labeled by parameter μ.

✓Interestingly, the RG flow is completely solvable and deformed theories have nice gravity dual! [McGough, Mezei , H. Verlinde]

TT BAR FLOW EQUATION (USUAL CASE)

- ✓Formalism [Guica, Monten] : consider the flow of stress tensor and metric both
- ✓The resulting flow equation

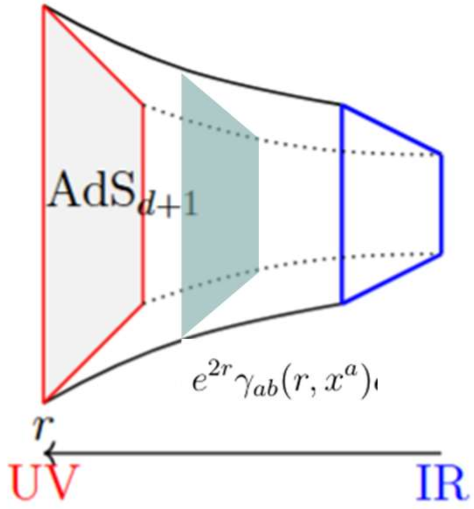
$$\partial_\mu \gamma_{[\mu]ab} = -2(\hat{T}_{[\mu]})_{ab}, \; \partial_\mu (\hat{T}_{[\mu]})_{ab} = -(\hat{T}_{[\mu]})_{ac}(\hat{T}_{[\mu]})^c_b, \; \partial_\mu \Big((\hat{T}_{[\mu]})_{ac}(\hat{T}_{[\mu]})^c_b \Big) = 0.$$

$$(\hat{T}_{[\mu]})_{ab} := (T_{[\mu]})_{ab} - (T_{[\mu]})^c_c (\gamma_{[\mu]})_{ab}$$

- ✓Surprisingly the flow solution is solved!

$$\partial^3_\mu \gamma_{[\mu]ab} = 0. \quad \Rightarrow \quad \gamma_{[\mu]ab} = \gamma_{[0]ab} - 2\mu(\hat{T}_{[0]})_{ab} + \mu^2 (\hat{T}_{[0]})_{ac}(\hat{T}_{[0]})^c_b.$$

GRAVITY DUAL OF TT DEFORMED THEORY



✓Fefferman-Graham gauge for asymptotically AdS

$$ds^2 = dr^2 + e^{2r}\gamma_{ab}(r, x^a)dx^a dx^b,$$
$$\gamma_{ab}(r, x^a) = g_{(0)ab}(x^a) + e^{-2r}g_{(2)ab}(x^a) + e^{-4r}g_{(4)ab}(x^a) + \cdots .$$

✓For asymptotically AdS3 case, Einstein eq. tells us expansion end in 4th order

[Skenderis]

$$g_{(4)} = \frac{1}{4}g_{(2)}(g_{(0)})^{-1}g_{(2)}, \qquad g_{(0)ab} = \gamma_{[0]ab},\ g_{(2)ab} = 8\pi G_{\mathrm{N}}\left(\hat{T}_{[0]}\right)_{ab}.$$

FINITE CUT OFF INTERPRETATION

✓ Comparison

Solution of TT bar flow eq

$$\gamma_{[\mu]ab} = \gamma_{[0]ab} - 2\mu(\hat{T}_{[0]})_{ab} + \mu^2(\hat{T}_{[0]})_{ac}(\hat{T}_{[0]})^c_b.$$

Solution of Einstein eq at finite cut off

$$\gamma_{[\mu]} = g_{(0)} + e^{-2r_c} \cdot g_{(2)} + e^{-4r_c} \cdot g_{(4)}.$$

$$g_{(4)} = \frac{1}{4} g_{(2)}(g_{(0)})^{-1} g_{(2)}, \quad g_{(0)ab} = \gamma_{[0]ab}, \; g_{(2)ab} = 8\pi G_N \left(\hat{T}_{[0]}\right)_{ab}.$$

⇨ **TT deformed theory is dual to gravity with finite cut-off surface at**

$$e^{-2r_c} = -\frac{\mu}{4\pi G_N}.$$

Also other quantities (thermodynamic quantities) are matched!

FINITE CUT OFF FROM HOLOGRAPHIC RENORMALIZATION

✓We can see the finite cut-off interpretation on the level of gravitational action via holographic renormalization

✓To do systematic computation, the Hamilton Jacobi formalism is useful [Skenderis, Papadimitriou].

✓ADM decomposition $ds^2 = dr^2 + h_{ab}(r, x^a) dx^a dx^b$

✓ Dilatation operator $\delta_D := \int_{\Sigma_r} d^2x \, 2h_{ab} \frac{\delta}{\delta h_{ab}}.$ → equivalent to ∂_r

✓Hamilton's principal action

$$\mathcal{S}[\gamma] = \int_{\Sigma_r} d^2x \sqrt{h} \mathcal{L}.$$

$$\mathcal{S} = \sum_{k=0} \mathcal{S}_{2-2k}, \; \delta_D \mathcal{S}_{2k} = 2k \, \mathcal{S}_{2k}.$$

momentum $\Pi_{ab} = \frac{1}{\sqrt{h}} \frac{\delta \mathcal{S}}{\delta h^{ab}}$

Conformal anomaly [Skenderis]

✓Expansion

$$\mathcal{L} = \mathcal{L}_{(0)} + \tilde{\mathcal{L}}_{(2)} \log e^{-2r} + \mathcal{L}_{(2)} + \mathcal{L}_{(4)} + \cdots ,$$

$$\delta_D \mathcal{L}_{(n)} = -n\mathcal{L}_{(n)},\ n \neq 2,\ \delta_D \tilde{\mathcal{L}}_{(2)} = -2\tilde{\mathcal{L}}_{(2)}$$

$$\delta_D \mathcal{L}_{(2)} = -2\mathcal{L}_{(2)} - 2\tilde{\mathcal{L}}_{(2)}.$$

↔ $\mathcal{L}_{(n)} \sim e^{-nr},\ \sqrt{h} \sim e^{2r}.$

→ $$S = S_{\rm reg} + S_{\rm ct} = \int_{\Sigma_r} d^2x \sqrt{h} \mathcal{L}_{(2)}. \qquad S_{\rm ct} = -\int_{\Sigma_r} d^2x \sqrt{h} \Big(\mathcal{L}_{(0)} + \tilde{\mathcal{L}}_{(2)} \log e^{-2r} \Big).$$

Renormalized action

✓Momentum

$$\Pi^{ab} := \frac{1}{\sqrt{h}} \frac{\delta}{\delta h_{ab}} \int_{\Sigma_r} d^2x \sqrt{h} \mathcal{L} = {\Pi_{(0)}}^{ab} + \tilde{\Pi}^{ab}_{(2)} \log e^{-2r} + {\Pi_{(2)}}^{ab} + {\Pi_{(4)}}^{ab} + \cdots$$

✓$\mathcal{L}_{(2k)}$ $\Pi_{(2k)}$ are recursively determined by Hamilton constraint.

From def
$$\Pi^{ab} \delta_D h_{ab} = 2\Pi = \frac{1}{\sqrt{h}} \delta_D (\sqrt{h} \mathcal{L}) + (\text{total derivative}).$$

Hamilton constraint
$$0 = \mathcal{H} = 16\pi G_{\rm N} \big(\Pi^b_a \Pi^a_b - \Pi^2 \big) + \frac{1}{16\pi G_{\rm N}} R[h] + \frac{1}{8\pi G_{\rm N}}.$$

Start point of recursion (asymptotic Ads)
$${\Pi_{(0)}}^{ab} = -\frac{1}{16\pi G_{\rm N}} h^{ab}.$$

✓Result

$$\mathcal{L}_{(0)} = \Pi_{(0)} = -\frac{1}{8\pi G_{\rm N}}, \tilde{\Pi}_{(2)} = 0,$$
$$\tilde{\mathcal{L}}_{(2)} = -\Pi_{(2)}, \mathcal{L}_{(4)} = -\Pi_{(4)}, \cdots$$

$$\Pi_{(2)} = -\frac{1}{32\pi G_{\rm N}} R[h],\ \tilde{\mathcal{L}}_{(2)} = \frac{1}{32\pi G_{\rm N}} R[h].$$

✓Higher order?=irreverent deformation

$$\Pi_{(4)} = -8\pi G_{\rm N}\big({\Pi_{(2)}}^2 - {\Pi_{(2)}}^b_a {\Pi_{(2)}}^a_b\big),$$
$$\mathcal{L}_{(4)} = -\Pi_{(4)} = -8\pi G_{\rm N}\big({\Pi_{(2)}}^b_a {\Pi_{(2)}}^a_b - {\Pi_{(2)}}^2\big).$$

TT bar deformation with coupling

$$e^{-2r_c} = -\frac{\mu}{4\pi G_{\rm N}}.$$

✓Boundary action

$$S_{\rm CFT} := \lim_{r_c\to\infty} \int_{\Sigma_{r_c}} d^2x\sqrt{h}\mathcal{L}_{(2)},$$
$$S_{\rm finite} := \int_{\Sigma_{r_c}} d^2x\sqrt{h}\big(\mathcal{L}_{(2)} + \mathcal{L}_{(4)}\big).$$
$$= S_{\rm CFT}[g_{(0)}] - 2\pi G_N e^{-2r_c} \int d^2x\sqrt{g_{(0)}}\left(\langle T^{ab}\rangle^{\rm CFT}_{g_{(0)}} \langle T_{ab}\rangle^{\rm CFT}_{g_{(0)}} - \left(\langle T^a_a\rangle^{\rm CFT}_{g_{(0)}}\right)^2\right)$$

NON-LOCAL TT BAR

✓We are doing same things for non-local TT bar

$$S_{T\overline{T}} = -\frac{1}{2}\int d^2x\sqrt{\gamma_{[\mu]}}\epsilon_{ac}\epsilon_{bd}T^{ab}F(-\Box_\gamma)T^{cd}.$$

✓Hubbard-Stratonovich technique [Cardy]

$$\begin{aligned} e^{-W_{[\mu+\Delta\mu]}} &= \int [dX]e^{-S_{[\mu]}[X;\gamma_{[\mu]}]}e^{\frac{\Delta\mu}{2}\int d^2x\sqrt{\gamma_{[\mu]}}\epsilon_{ab}\epsilon_{cd}T^{ab}F(-\Box_\gamma)T^{cd}} \\ &= \int [dX][dh]e^{-S_{[\mu]}[X;\gamma_{[\mu]}]}e^{-\frac{1}{8\Delta\mu}\int d^2x\sqrt{\gamma_{[\mu]}}\epsilon^{ab}\epsilon^{cd}h_{ab}F^{-1}(-\Box_\gamma)h_{cd}+\frac{1}{2}\int d^2x\sqrt{\gamma_{[\mu]}}T^{ab}h_{ab}}. \end{aligned}$$

✓Saddle point ⇒ flow equation for metric and effective action

$$\partial_\mu \gamma_{[\mu]ab} = -2F(-\Box_\gamma)\left(\hat{T}_{[\mu]}\right)_{ab},$$

$$\partial_\mu W_{[\mu]}[\gamma_{[\mu]}] = -\frac{1}{2}\int d^2x\sqrt{\gamma_{[\mu]}}\epsilon^{ac}\epsilon^{bd}(\hat{T}_{[\mu]})_{ab}F(-\Box_\gamma)(\hat{T}_{[\mu]})_{cd}.$$

✓Flow equation for stress tensor

$$\partial_\mu\left(\hat{T}_{ab}\right) = -\frac{1}{2}\left(\hat{T}_{ac}\hat{S}^c_b + \hat{T}_{bc}\hat{S}^c_a - 3\hat{T}\hat{S}_{ab} + 3\hat{T}\hat{S}_{ab}\right) - N_{ab}$$

$$\hat{S}_{ab} = F(-\Box_\gamma)\hat{T}_{ab} \quad U_{ab} = F'(-\Box_\gamma)(\hat{T}_{ab} - \gamma_{ab}\hat{T}).$$

$$\begin{aligned} N^{ab} := \Big[& (\nabla^a\hat{T}_{cd})(\nabla^b U^{cd}) + (\nabla^b\hat{T}_{cd})(\nabla^a U^{cd}) - \gamma^{ab}\Big((\Box\hat{T}_{cd})U^{cd} + (\nabla_e\hat{T}_{cd})(\nabla^e U^{cd})\Big) \\ & + \Box(\hat{T}^a{}_c U^{bc}) + \Box(\hat{T}^b{}_c U^{ac}) - 2\nabla_e(\hat{T}^a{}_c\nabla^e U^{bc}) - 2\nabla_e(\hat{T}^b{}_c\nabla^e U^{ac}) \\ & + \nabla_c\nabla^b(\hat{T}^a{}_d U^{cd}) + \nabla_c\nabla^a(\hat{T}^b{}_d U^{cd}) - 2\nabla_c(\hat{T}^a{}_d\nabla^b U^{cd}) - 2\nabla_c(\hat{T}^b{}_d\nabla^a U^{cd}) \\ & - \nabla^e\nabla^a(\hat{T}_{ec}U^{bc}) - \nabla^e\nabla^b(\hat{T}_{ec}U^{ac}) + 2\nabla^e(\hat{T}_{ec}\nabla^a U^{bc}) + 2\nabla^e(\hat{T}_{ec}\nabla^b U^{ac})\Big], \end{aligned}$$

✓ Super complex 😭

✓But at least we may observe $\partial^3_\mu \gamma_{[\mu]ab} \neq 0.$ (※Usual TT bar is magical!!)

→ **Not in quadratic in μ**

POSSIBILITY FOR GRAVITY DUALS

We consider three possibilities

A)a finite cut-off holography within Einstein gravity though the finite cut-off surface may not be $r\ =\ \mathrm{const.}$

B)No gravity duals… 🤐

C)a finite cut-off holography with $r\ =\ \mathrm{const.}$, the bulk gravity is no longer the Einstein gravity.

POSSIBILITY FOR GRAVITY DUALS

We consider possibility C)

we have a finite cut-off holography with $r = \mathrm{const.}$, the bulk gravity is no longer the Einstein gravity.

The Einstein gravity gives Fefferman-Graham expansion with finite terms.
But for more generic gravity, we do not need it.

→ Consider a bulk modified gravity

SUPPORTING OUR POSSIBILITY

- ✓Now we consider the following non-local gravity

$$H = \int_{\Sigma_r} d^d x (N\mathcal{H} + N_a \mathcal{H}^a),$$
$$\frac{1}{\sqrt{h}}\mathcal{H} = 16\pi G_{\rm N}\left(\Pi^a_b F(-e^{2r}\Box_h)\Pi^b_a - \Pi F(-e^{2r}\Box_h)\Pi\right) + \frac{1}{16\pi G_{\rm N}}(R[h]+2),$$
$$\mathcal{H}^a = -2D_b\left(\frac{1}{\sqrt{h}}\Pi^{ab}\right).$$

- ✓Assume non-locality factor $F(\cdot)$ has expansion $F(-e^{2r}\Box_h) = \sum_{k=0} f_k(-e^{2r}\Box_h)^k,$
- ✓Because of explicit r-coordinate dependence, dilatation operator is modified

$$\delta_D = \int d^2x\left(\partial_r + 2h_{ab}\frac{\delta}{\delta h_{ab}}\right)$$

✓The order of dilatation is given by

$$\delta_D(F(-e^{2r}\Box_h)X_{ab}) = -\lambda F(-e^{2r}\Box_h)X_{ab},$$
$$\delta_D(F(-e^{2r}\Box_h)X) = -(\lambda+2)F(-e^{2r}\Box_h)X.$$

✓Similar to the usual TT bar we do the expansion

$$\mathcal{L} = \mathcal{L}_{(0)} + \tilde{\mathcal{L}}_{(2)}\log e^{-2r} + \mathcal{L}_{(2)} + \mathcal{L}_{(4)} + \cdots,$$
$$\delta_D\mathcal{L}_{(n)} = -n\mathcal{L}_{(n)},\ n \neq 2,\ \delta_D\tilde{\mathcal{L}}_{(2)} = -2\tilde{\mathcal{L}}_{(2)}$$
$$\delta_D\mathcal{L}_{(2)} = -2\mathcal{L}_{(2)} - 2\tilde{\mathcal{L}}_{(2)}.$$
$$\Pi^{ab} := \frac{1}{\sqrt{h}}\frac{\delta}{\delta h_{ab}}\int_{\Sigma_r} d^2x\sqrt{h}\mathcal{L} = {\Pi_{(0)}}^{ab} + \tilde{\Pi}^{ab}_{(2)}\log e^{-2r} + {\Pi_{(2)}}^{ab} + {\Pi_{(4)}}^{ab} + \cdots$$

✓Solve the Hamilton constraint recursively with

$${\Pi_{(0)}}_{ab} = Ph_{ab},$$

Zero-th order

$$\Pi_{(0)ab} = Ph_{ab} \qquad P = -\frac{1}{\sqrt{f_0}}\frac{1}{16\pi G_{\rm N}},$$

Second order

$$\Pi_{(0)}{}_a^b F(-e^{2r}\Box_h)\Pi_{(2)}{}_b^a + \Pi_{(2)}{}_a^b F(-e^{2r}\Box_h)\Pi_{(0)}{}_b^a - \Pi_{(0)}F(-e^{2r}\Box_h)\Pi_{(2)} - \Pi_{(2)}F(-e^{2r}\Box_h)\Pi_{(0)}$$
$$= -\frac{1}{(16\pi G_{\rm N})^2}R[h].$$

$\Rightarrow$ $$\Pi_{(2)} = -\frac{1}{\sqrt{f_0}}\frac{1}{16\pi G_{\rm N}}Q(-e^{2r}\Box_h)R[h]. \quad Q(-e^{2r}\Box_h) = \sum_{k=0}^{\infty} q_k(-e^{2r}\Box_h)^k,\ q_0 = \frac{1}{2},\ q_1 = -\frac{f_1}{4f_0},\cdots.$$

$\Rightarrow$ $$\int d^2x\sqrt{h}\tilde{\mathcal{L}}_{(2)} = \frac{1}{32\pi G_{\rm N}}\frac{1}{\sqrt{f_0}}\int d^2x\sqrt{h}R[h], \qquad \tilde{\Pi}^{(2)}_{ab} = 0.$$

Fourth order

$$\Pi_{(4)} = -\frac{16\pi G_{\rm N}}{\sqrt{f_0}}Q(-e^{2r}\Box_h)\big(\Pi_{(2)}{}_a^b F(-e^{2r}\Box_h)\Pi_{(2)}{}_b^a - \Pi_{(2)}F(-e^{2r}\Box_h)\Pi_{(2)}\big).$$

NON-LOCAL GRAVITY INDUCE NON-LOCAL TTBAR

✓Finite cut-off renormalized action

$$S_{\text{finite}} = \int_{\Sigma_{r_c}} d^2x\sqrt{h}\left(\mathcal{L}_{(2)} + \mathcal{L}_{(4)}\right)$$

$$= S_{\text{CFT}}[\gamma] + \frac{2\pi G_{\text{N}}\, e^{-2r_c}}{\sqrt{f_0}} \int d^2x\sqrt{\gamma}\, \left\langle T^{ab}\right\rangle_{\gamma}^{\text{CFT}} F(-\Box_{\gamma}) \left\langle \hat{T}_{ab}\right\rangle_{\gamma}^{\text{CFT}} .$$

✓So far we do not mention the form of $F(x)$. We require

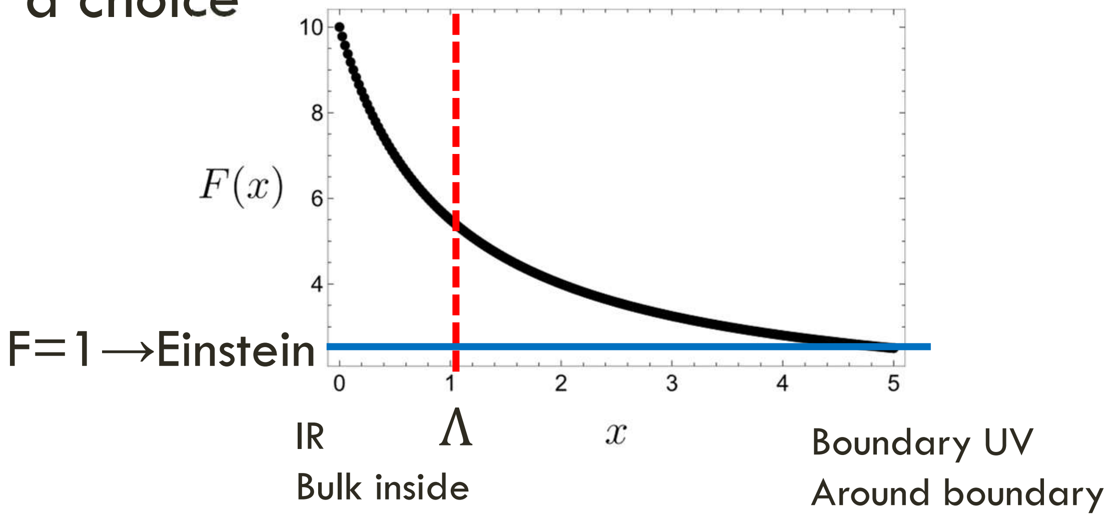
① Boundary UV degree of freedom is suppressed

② Remove finite cut-off effect ($r_c \to \infty$)

③ Around the boundary, bulk is Einstein gravity

✓These requests are satisfied by a choice

$$F(x) = \frac{x/\Lambda^2 + e^{4r_c}\overline{\mu}^2}{x/\Lambda^2 + 1}$$



✓Finally we see the non-local TT bar deformation

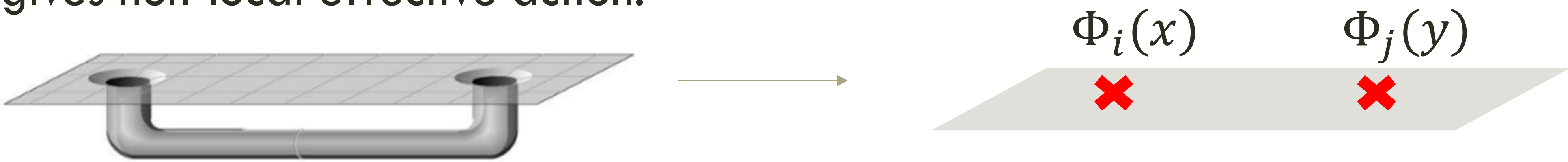
$$S_{\text{finite}} = S_{\text{CFT}}[g_{(0)}] - \frac{2\pi G_{\text{N}}\, e^{-2r_c}}{\sqrt{f_0}} \int d^2x \sqrt{g_{(0)}}\, \langle T^{ab} \rangle^{\text{CFT}}_{g_{(0)}} F(-\Box_{g_{(0)}}) \left\langle \hat{T}_{ab} \right\rangle^{\text{CFT}}_{g_{(0)}} .$$

which lives on asymptotic infinity ($r_c = \infty$)

⇒Non-local TT bar deformed holographic CFT= Non-local gravity.

WORMHOLE IN THE BULK

- ✓We may think that the origin of the non-locality is wormhole.
- ✓If we focus on macroscopic mode whose scale is larger than the wormhole, wormhole can be effectively regarded as local operators[Coleman].
- ✓And the summation over the wormhole configuration in the path integrals gives non-local effective action.



$$I_{\text{eff}} = \frac{1}{2} \int d^{d+1}x \sqrt{g(x)} \int d^{d+1}y \sqrt{g(y)} \sum_{i,j} \Delta_{ij} \Phi_i(x) G(x,y) \Phi_j(y)$$

- ✓Our case wormhole which connects two points in $r = const.$ may give a non-local action with size $l_{\text{WH}} \sim k_{\text{bulk}}^{-1} \sim e^r \Lambda$

SUMMARY AND DISCUSSION

✓In this talk, I discussed two concrete AdS3 examples of (traversable) wormhole in AdS/CFT

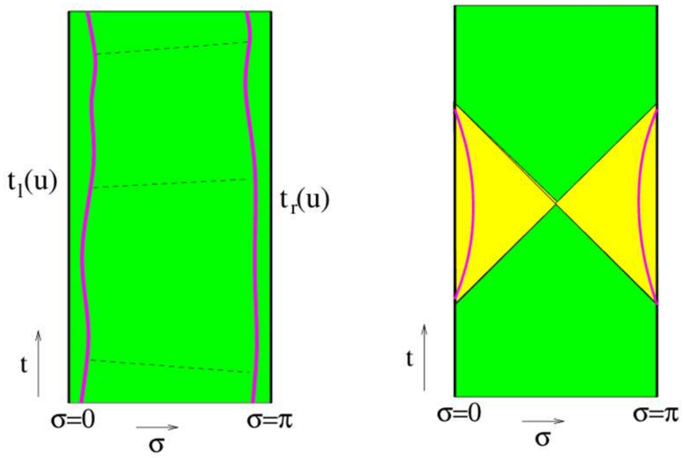
1. Model A: Wormhole via Janus deformation ⇒no-interaction and non-hermite density matrices
2. Model B: via non-local TT bar deformation

⇒non-local bulk gravity ☚microscopic wormholes

✓Actually these two situations are distinguished by pseudo entropy which I do not discuss

THANK YOU FOR LISTENING !!!!

POSSIBILITY FOR GRAVITY DUALS



We consider three possibilities

A)a finite cut-off holography within Einstein gravity though the finite cut-off surface may not be $r = \text{const.}$

Criticism for A)

This possibility is similar to the Maldacena Qi. Suppose we take undeformed metric as $\gamma_{[0]} = \delta_{ab}$ and we consider inhomogeneous cut off surface $r = r(x^a)$. Then, the induced metric does not keep the Poincare symmetry. This means that this possibility denoted for higher dimension (SYK is 1D).

POSSIBILITY FOR GRAVITY DUALS

We consider three possibilities

A)a finite cut-off holography within Einstein gravity though the finite cut-off surface may not be $r = \mathrm{const.}$

B)No gravity duals…🤐

Criticism for B)
We would like to expect that the perturbation from holographic CFTs have some gravity duals.
(Question to the audience) Do you know some examples?