Non-conformal Line Defects and ETH in AdS_3/CFT_2

Yuefeng Liu

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IAS, Kyushu University

New Advancements on Defects and Applications

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Defect in QFT

- Recently a surge of interest in defects in QFT:
  Higher form symmetry, Non-invertible symmetry
  D-brane/Orientifold in string theory (worldsheet)
  Boundary CFT, Crosscap CFT [Yuya Kusuki's talk]
  Island formula in double holography
  ...
- Defects: ubiquitous extended objects in QFT
  probe non-local phenomena inaccessible by local operators
- Viewed from symmetry (↘) and generality (↗):
  topological defect
  conformal defect
  non-conformal defect
- Quantities for non-conformal defects? [Yuya Kusuki's talk]
  defect partition function?
  defect correlation function?

Intuition from AdS/CFT

- General finite Temperature AdS/CFT set up
  consider codimension-one defects on boundary CFT
  make it ”simple” and heavy!

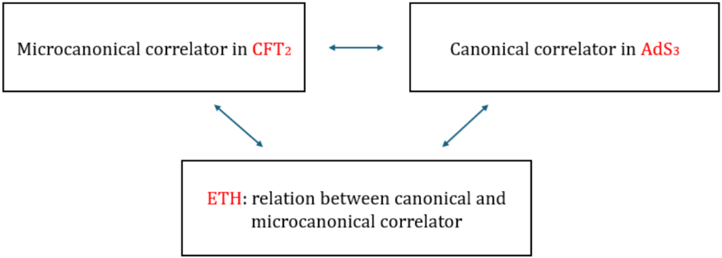
- Heavy: proportional to N or c
  bulk classical/semi-classical geometry dominant
  Simple: single trace operator, fundamental field
  single particle bulk state

- Bulk geometry: backreacted by a spherical domain wall
  codimension one in the bulk
  made of dust-like particles describable by a perfect fluid
  free falling guided by effective potential

- Questions:
  derive purely from field theory technique?
  how to prove its validity?

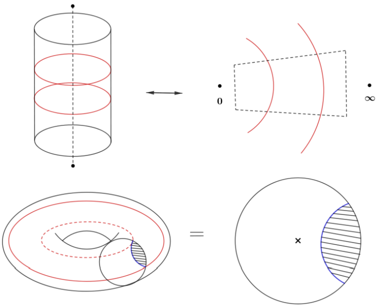
Non-conformal line defect in AdS_3/CFT_2

- Yes, we can!
- But only in holographic CFT_2 right now.
- Prove by matching between three pillars:
  ETH, AdS_3 and CFT_2



Why only in AdS_3/CFT_2 ?

- CFT_2: infinite symmetry enhancement
  states grouped into Virasoro representation
- Holographic CFT_2: large c, sparse low lying spectrum
  good enough, vacuum Virasoro block approximation
- AdS_3 gravity: no propagating local degree of freedom
  only topology change



CFT$_2$: Monodromy Method

- Want to compute
  defect partition functions:
  $\langle vac|D^\dagger(\tau_2)D(\tau_1)|vac\rangle$ and $\langle vac|D^\dagger(\tau_4)D(\tau_3)D^\dagger(\tau_2)D(\tau_1)|vac\rangle$
  defect correlator: $\langle vac|\mathcal{O}(-\infty)D^\dagger(\tau_2)D(\tau_1)\mathcal{O}(\infty)|vac\rangle$

- However, although we know how to compute four point function of local operators, how to compute correlator of line defects?

- Review: Four point correlator in holographic CFT$_2$

$$\langle\mathcal{O}(0)\mathcal{O}(x)\mathcal{O}(1)\mathcal{O}(\infty)\rangle = \sum_p a_{12}^p a_{34}^p |\mathcal{F}(h_p, x)|^2 \approx e^{-\frac{c}{6}f(\frac{h_i}{c},\frac{h_p}{c},x)}|_{h_p=0}$$

- BPZ equation + Analytic structure:
  f is determined by the solution of a monodromy problem

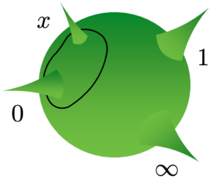
$$\chi''(z) + T(z)\,\chi(z) = 0, \quad \frac{\partial f(z)}{\partial z} = c_2(z)$$

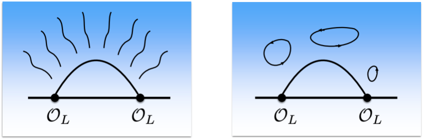
Monodromy Method: continued

- $c_2(z)$ is an "accessory" function in $T(z) = \sum_i \frac{6h_i/c}{(z-z_i)^2} - \frac{c_i(z)}{z-z_i}$
- Second order linear homogeneous PDEs, solutions $\psi_{1,2}$ take $\psi_{1,2}$ on a closed contour around one or more singularities they undergo monodromy

$$(\psi_1, \psi_2) \to (\psi_1, \psi_2).M \quad \text{Tr}M \propto \cos(\pi\sqrt{1 - 24h_p/c})$$

- Approximations in heavy-heavy-light-light $\langle HHLL \rangle$





$\epsilon_L \propto h_L/c$ $1/c$

Line defect: continuum limit

- How to realize line defects?
  N number of uniformly spaced same local operators
  vacuum block channel
  take continuum limit

- Infinite number of poles?
  Impossible to solve the equation

- From monodromy method and vacuum approximation
  non-local defect are indeed much harder
  but uniformly non-local line defect is easier!

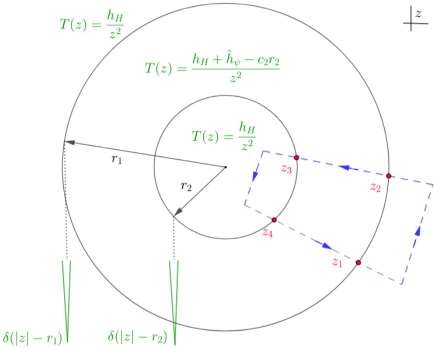
- Why easier?
  singularity structures simplify a lot due to uniform OPE channel between pairs of local operators on line defects

$$T(z) = T_H + \sum_{j=1}^{2} \sum_{i=1}^{n} \frac{h_\psi}{(z - z_j^{(i)})^2} - \frac{c_{\psi,i}}{z - z_j^{(i)}}$$

Continuum limit: continued

- Rotational symmetry is important, one c_i for one defect
- Integration well organized the pole structure

$$T(z,\bar{z}) = T_H + \frac{(\hat{h}_\psi - c_1 r_1)\Theta_{r_1} + (\hat{h}_\psi - c_2 r_2)\Theta_{r_2}}{z^2} + \frac{\hat{h}_\psi [r_1 \delta_{r_1} + r_2 \delta_{r_2}]}{z^2}$$



Line defect: solution

- Monodromy condition

$$M = J_2(z_1) J_2^{-1}(z_2) J_1(z_3) J_1^{-1}(z_4) = I_{2\times 2}$$

- Solution:

$$(\frac{r_2}{r_1})^{2\rho_2} = \frac{(\rho_2 - \rho_1 + \hat{h}_\psi)(\rho_1 + \rho_2 + \hat{h}_\psi)}{(\rho_2 + \rho_1 - \hat{h}_\psi)(\rho_2 - \rho_1 - \hat{h}_\psi)}$$

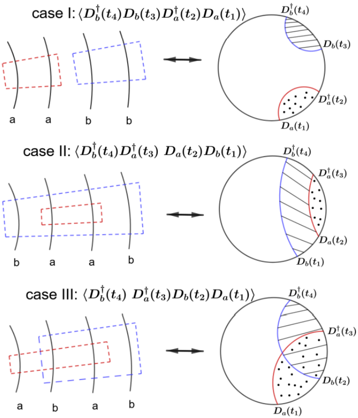
  and

$$\rho_1 = \frac{\sqrt{1 - 4h_H}}{2}, \quad \rho_2 = \frac{\sqrt{1 - 4(h_H + \hat{h}_\psi - c_2 r_2)}}{2}$$

- A wide range of validity!
  any line defect separations
  any conformal dimensions of $\mathcal{O}$ and D (dominant?)
- Non-perturbative/exact in $\frac{h_\psi}{c}$ expansion
  perturbative in $1/c$ expansion

Four non-conformal line defects

- Unlike Euclidean correlator of local operators
  three different configurations



Prove it: AdS/CFT & ETH

- The above calculation use many approximations
  especially a ”mysterious” smooth limit
- We want to prove it using AdS/CFT!
  micro-canonical correlator (2d Plane, field side)
  vs. canonical correlator (2d Torus, gravity side)
- The bridge is the ETH (Eigenstate Thermalization Hypothesis)
  assume matrix elements of the line defect

$$D_{nm} \approx g_1(\bar{E})\delta_{nm} + e^{-S(\bar{E})/2} g_2(\bar{E}, \omega)^{1/2} R_{nm}$$

  in high energy/chaotic sector of the spectrum
- Validity range of ETH? need modifications
  line defects are heavy
  completely delocalize over spatial slice
  however simple!

Analytic form of defect correlator

- In the probe limit $\hat{h}_\psi \to 0$

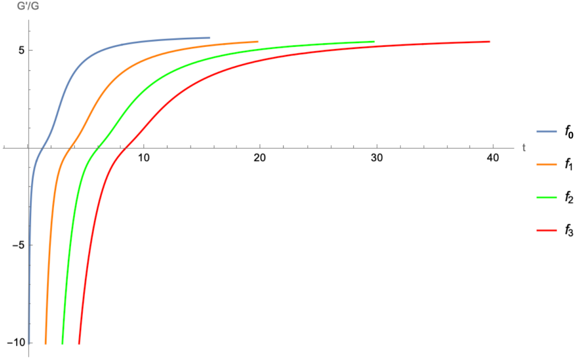
$$\log G_{e_H} = c_0(h_H, \hat{h}_\psi) - \frac{2c\hat{h}_\psi}{3} \log\sinh\frac{\sqrt{1-4h_H}\,\tau}{2} + \frac{c\hat{h}_\psi^2}{3}\coth\frac{\sqrt{1-4h_H}\,\tau}{2} \times \left(\tau\coth\frac{\sqrt{1-4h_H}\,\tau}{2} - \frac{2}{\sqrt{1-4h_H}}\right) + O(\hat{h}_\psi^3)$$

- To linearized order
  similar to the thermal correlator of local operators
  ETH is held by "heavy", but "simple" line defect $D(\tau)$
  in holographic CFT_2

- Perturbative solutions are singular
  at infinite number of points: $\tau_l = \frac{2l\pi}{\sqrt{4h_H - 1}}$, $l \in \mathbb{Z}$
  resolved by non-perturbatively sum
  over full probe corrections h_ψ / c

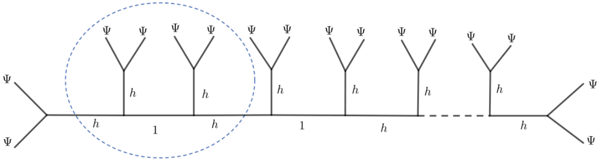
Analytic property: resummation

- Exact in $\frac{h_p}{c}$ expansion
  resummation automatically



Future directions

- Non-vacuum blocks: rotational symmetry
  (Island formula in CFT language?)



- Lorentzian behavior in late time regime t $\to \infty$
  two point correlator (forbidden singularity, information loss)
  four point correlator (chaotic property, OTOC)
- Under defect RG flow
  line defect $D(t)$ flow to a conformal defect?
  poss a factorization property ?
  factorizes into conformal boundary conditions

The End

Thank You!

Questions $\&$ Comments

Backup: ETH

- Relation between correlator in canonical ensemble and micro-canonical ensemble:

$$\partial_\tau \log \mathcal{Z}_{\beta^*} G_{\beta^*;2}(t) = \partial_\tau \log G_{E_H;2}(\tau)$$

- Gravity "like" canonical correlator, thus from $Z_\beta G_\beta(\tau)$ to $G_{M_m}(\tau)$, we have

$$G_{E_H}(\tau) = \int d\beta \, e^{-S(E_H)+\beta E_H} \mathcal{Z}_\beta G_\beta(\tau) \sim e^{-S(E_H)+\beta^* E_H} \mathcal{Z}_{\beta^*} G_{\beta^*}(\tau)$$

  where $\beta^* = \beta^*(E_H, \tau)$ are determined by condition

$$E_H + \partial_\beta \log \mathcal{Z}_\beta G_\beta(\tau)|_{\beta=\beta^*} = 0$$

- Physical meaning: up to multiplicative factor, canonical correlator equal to micro-canonical one at special mass and micro-canonical correlator equal to canonical one at special temperature.

Backup: Israel junction condition

- The Euclidean action of Einstein gravity with a domain wall in AdS_3 is given by

$$I = -\frac{1}{16\pi G}\int \sqrt{g}(R - 2\Lambda) + \int_{\mathcal{W}} \sqrt{h}\sigma$$

- Glue two Euclidean Schwarzshild-AdS regions $BTZ_\pm$ by Israel junction condition together with local geometries

$$ds_\pm^2 = f_\pm(r)dt_\pm^2 + \frac{dr^2}{f_\pm(r)} + r^2 d\phi^2 \; f_\pm(r) = r^2 - 8GM_\pm$$

- Israel junction condition:

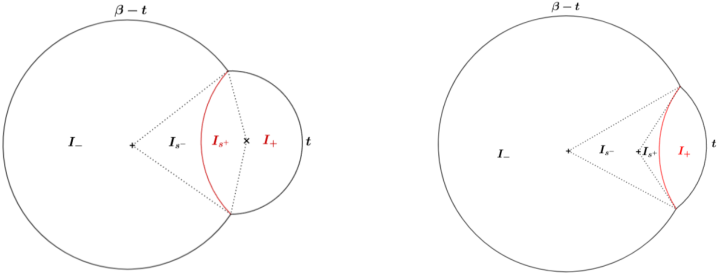
$$\Delta h_{ab} = 0, \quad \Delta K_{ab} - h_{ab}\Delta K = -8\pi G T^W_{ab}$$

where $(h^\pm_{ab}, K^\pm_{ab})$ are induced metrics and extrinsic curvatures on domain wall $\mathcal{W}$ and $\Delta h_{ab} = h^+_{ab} - h^-_{ab}$ and $\Delta K_{ab} = K^+_{ab} - K^-_{ab}$.

Backup: Non-trivial backreaction

- Non-trivial backreaction already at classical order.
- Thermal partition function $Z_\beta G_\beta(\tau)$ is

$$Z_\beta G_\beta(t) \approx e^{-\Delta I}, \quad \Delta I = I_- + I_+ + I_{s-} + I_{s+} + I_{ct}$$



Backup: Spinning line defect

- Junction condition: the volume form of the induced metric is continuous across the shell (not induced metric itself), and a constraint on the vielbeins e^a on both sides of the shell should be continuous

$$[\mathsf{Vol}] = 0, \quad \iota[\mathrm{e}^a \wedge [\mathrm{e}^b]] = 0.$$

where ι denoting the pull-back of differential forms to the worldvolume of the shell.

- gravitational action

$$I_{\mathsf{tot}} = I + I_{ct} + I_{\mathsf{spin}},$$

where

$$I_{\mathsf{spin}} = \frac{s}{4G} \int d\psi d\ell n_2 \cdot \nabla n_1.$$