

New Advancements on defects and their applications

Yukawa Institute of Theoretical Physics, 15/7/2025

Energy transport in 2D holographic interfaces

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with A. Banerjee, work in progress
with A. Banerjee, T. Kibe, A. Mukhopadhyay, work in progress

previous work 2006.11333, 2212.14058, 2404.02149

C. Bachas, S. Bagueira, S. Chapman, D. Ge, A. Mukhopadhyay, T.
Schwarzman

Outline

- CFT boundaries and interfaces
- Holographic defect model
- Energy transmission
- Nambu-Goto interfaces
- Interfaces in $T\bar{T}$ -deformed CFTs
- Conclusions

Motivation

Boundaries and interfaces exist in the real world in condensed matter systems, analyzed in CFT since '90 Affleck, Ludwig, Cardy ...

Non-local operators give additional structure in the theory beyond local OPE and correlators Seiberg, ...

Interfaces between CFTs can be related to RG flows

Brunner, Roggenkamp

Most explicit examples from free theory, topological, or perturbation theory

Holography gives access to a large number of solvable cases

Boundary 2d CFT

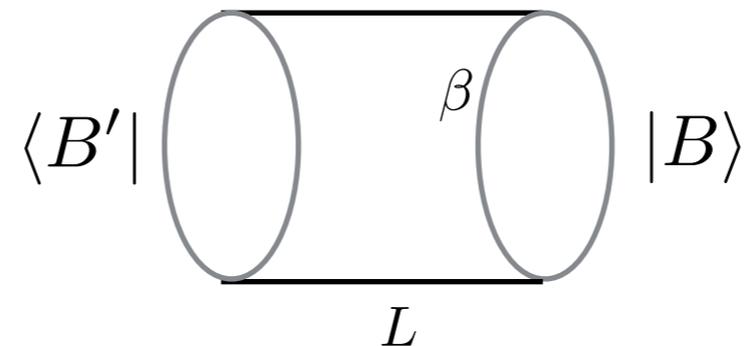
$$(T(z) - \bar{T}(\bar{z}))|B\rangle = 0 \quad \text{at } z = \bar{z}$$

$$\langle T(z)T(z) \rangle = \frac{c/2}{(z-w)^4}, \quad \langle T(z)\bar{T}(\bar{w}) \rangle = \frac{c/2}{(z-\bar{w})^4}$$

Conformal anomaly

$$\delta_\lambda \log Z = \frac{c}{48\pi} \left(\int_M \lambda R + 2 \int_{\partial M} \lambda k \right)$$

Boundary entropy



$$L/\beta \gg 1$$

$$S = -\beta^2 \frac{\partial Z}{\partial \beta} = \frac{\pi c}{3\beta L} + s_B + s_{B'}$$

$$s_B = \log \langle 0|B \rangle = \log g_B$$

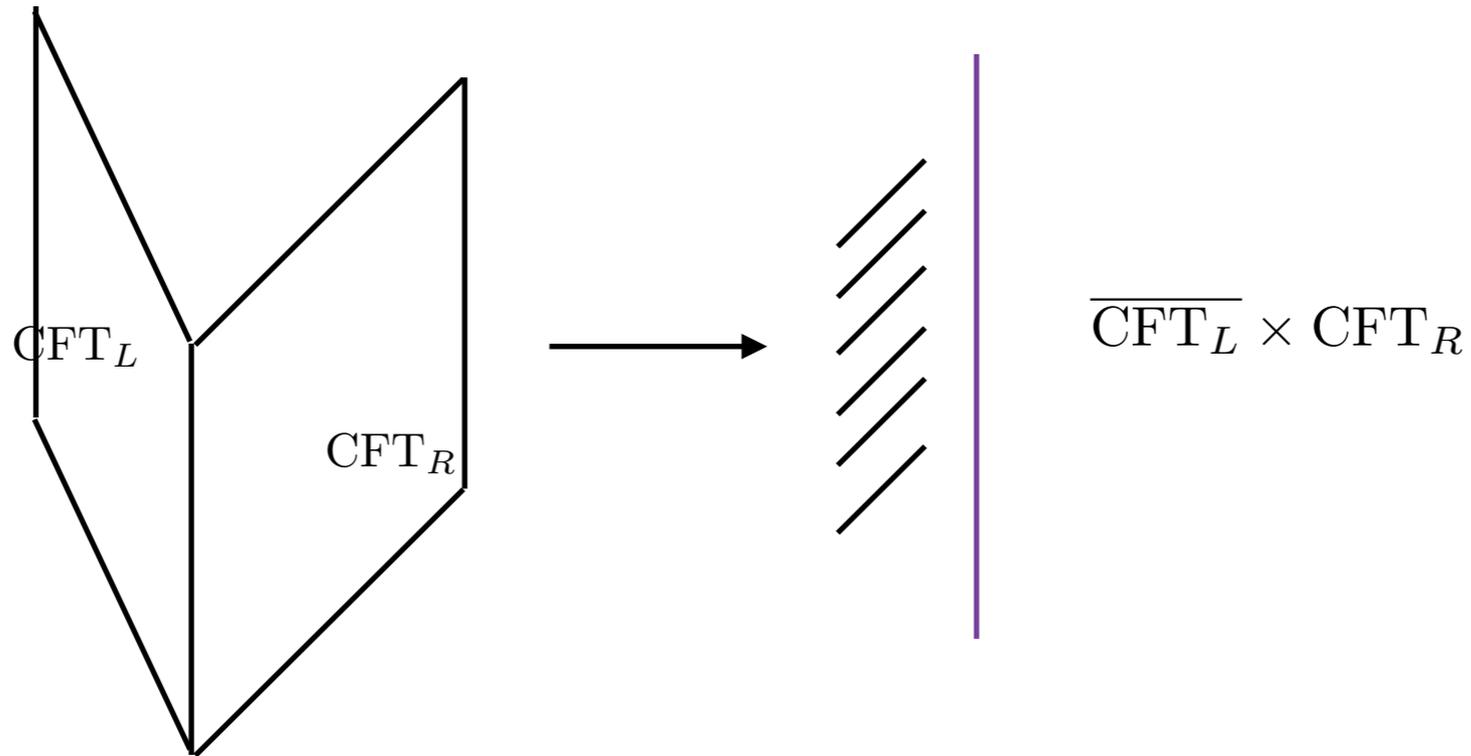
Affleck-Ludwig

The g-factor appears also in the entanglement entropy

Calabrese-Cardy

$$S_{EE} = \frac{c}{3} \log \frac{\ell}{\epsilon} + s_B + c'$$

Conformal interface (defect)



mapped to a BCFT by folding, but not generic situation

$$T_{tot} = T_L + T_R$$

$$T_{rel} = c_L T_R - c_R T_L \quad \text{spin-2 primary operator}$$

$$\langle T_{tot} T_{rel} \rangle = 0$$

Quella-Runkel-Watts 07

Billo-Goncalves-Lauria-Meineri 16

$$\langle T_L(z)T_R(w) \rangle = \frac{a/2}{(z-w)^4} \quad \langle T_L(z)\bar{T}_R(w) \rangle = 0$$

$$\langle T_L(z)\bar{T}_L(w) \rangle = \frac{(c_L - a)/2}{(z-\bar{w})^4} \quad \langle T_R(z)\bar{T}_R(w) \rangle = \frac{(c_R - a)/2}{(z-\bar{w})^4}$$

displacement operator = response to displacement of the defect

$$D(x) = \frac{\delta \log Z[\gamma_{\mu\nu}, X(x)]}{\delta X(x)} = T_L(x) + \bar{T}_L(x) - T_R(x) - \bar{T}_R(x)$$

$$\langle D(x)D(y) \rangle = \frac{2(c_L + c_R - 2a)}{(x-y)^4}$$

It is unclear whether a is related to an anomaly

No new coefficients in 3-pt correlators, infinitely many from 4-pt

Reflection and transmission coefficient

$$\mathcal{R} = \frac{\langle T_L \bar{T}_L + T_R \bar{T}_R \rangle}{\langle (T_L + \bar{T}_R)(\bar{T}_L + T_R) \rangle} = 1 - \frac{2a}{c_L + c_R} \quad \mathcal{T} = 1 - \mathcal{R}$$

Unitarity gives a bound $\left(\frac{c_L - c_R}{c_L + c_R} \right)^2 \leq \mathcal{R} \leq 1$

It follows from the positivity of the matrix of scalar products of $T_{tot}, T_{rel}, \bar{T}_{rel}$

A topological interface has $\mathcal{R} = 0$, only possible for $c_L = c_R$

Fusion $\mathcal{I}_{12} = \mathcal{I}_1 \circ \mathcal{I}_2 \quad \mathcal{R}_{12} \neq \mathcal{R}_1 + \mathcal{R}_2$

However if \mathcal{I}_2 is topological $\mathcal{R}_{12} = \mathcal{R}_1$

Example: free massless scalar

Bachas-de Boer-Dijkgraaf-Ooguri 01

Matching condition at interface

$$\begin{pmatrix} \partial_- \phi_L \\ \partial_+ \phi_R \end{pmatrix} = S \begin{pmatrix} \partial_+ \phi_L \\ \partial_- \phi_R \end{pmatrix}$$

$$S = \begin{pmatrix} -\cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & \cos(2\theta) \end{pmatrix}, \quad S' = \begin{pmatrix} \cos(2\theta) & -\sin(2\theta) \\ \sin(2\theta) & \cos(2\theta) \end{pmatrix}$$

$$\mathcal{R} = \cos^2(2\theta)$$

$$g = \sqrt{\frac{k_1 k_2}{\sin(2\theta)}}$$

Bachas-Brunner 07

winding

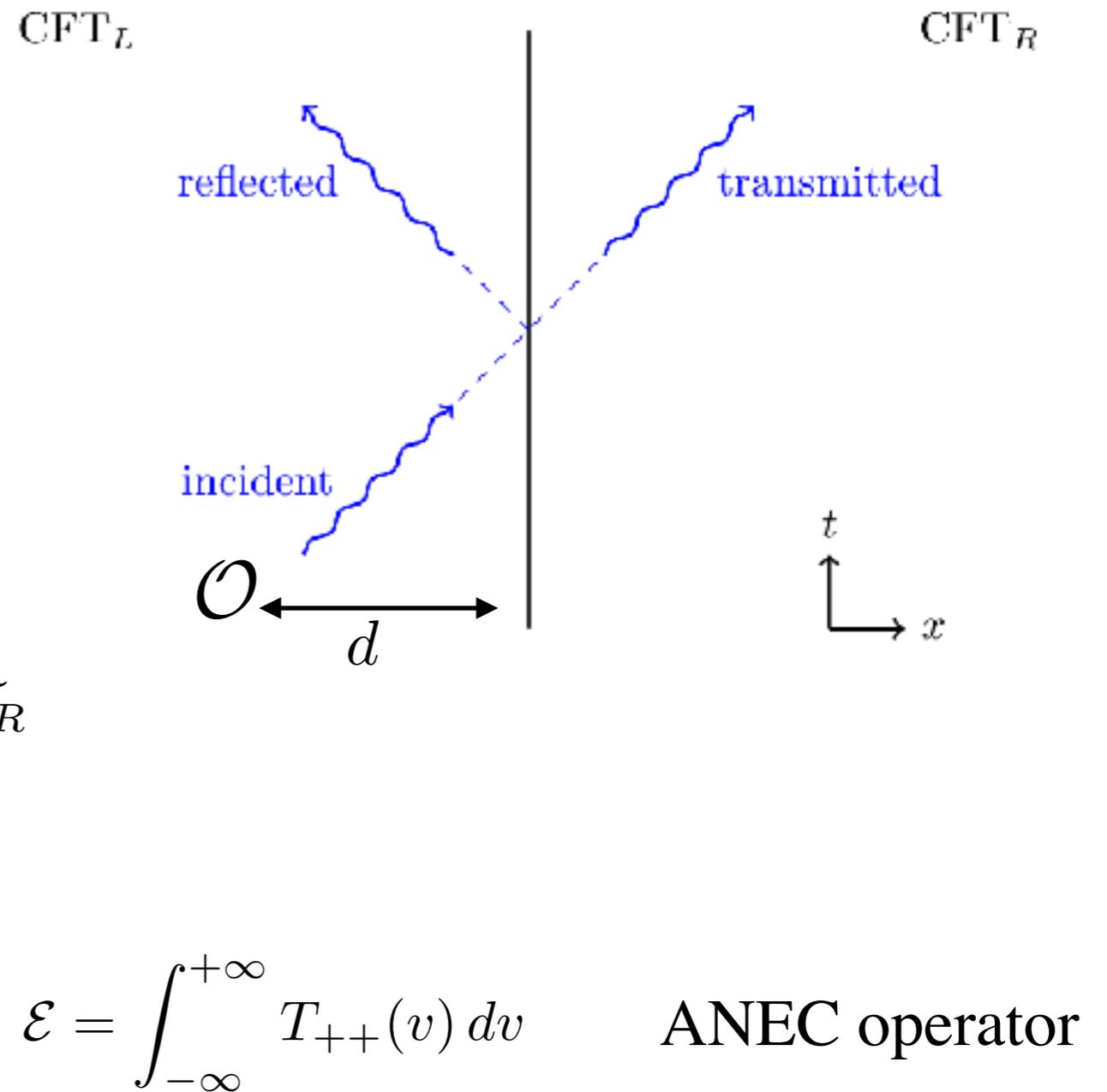
g and \mathcal{R} are generically independent

Lorentzian picture:
scattering of energy on the defect

Directional coefficient:

$$\mathcal{T} = \frac{c_L \mathcal{T}_L + c_R \mathcal{T}_R}{c_L + c_R} \quad c_L \mathcal{T}_L = c_R \mathcal{T}_R$$

$$\mathcal{T}_L = \lim_{d \rightarrow \infty} \frac{\langle \mathcal{O}, d | \mathcal{E}_R | \mathcal{O}, d \rangle}{\langle \mathcal{O}, d | \mathcal{E}_L | \mathcal{O}, d \rangle}$$



Universality: transmission of energy independent of the details of the incoming excitations (e.g. how the state is prepared)

The argument is easier for states created by holomorphic quasi-primary operators

$$\langle O_L^1(z_1)T_R(z)O_L^2(z_2)\rangle = \frac{c_{12}T_R}{(z_1 - z_2)^{h_1+h_2-2}(z_1 - z)^{h_1-h_2+2}(z_2 - z)^{h_2-h_1+2}}$$

One can use the OPE when $z_1 \rightarrow z_2$, and only operators with the same quantum numbers as T contribute

Assuming that no other spin-2 holomorphic currents exist, the correlator is fixed by $\langle T_L T_R \rangle \sim a$

$$\langle O_L^1(z_1)T_R(z)O_L^2(z_2)\rangle = \frac{a}{c_L} \langle O_L^1(z_1)T_L(z)O_L^2(z_2)\rangle \quad \rightarrow \quad \mathcal{T}_L = \frac{a}{c_L}$$

ANEC (positivity of the averaged null-energy in any state) implies

$$0 \leq \mathcal{T}_L \leq \min\left(1, \frac{c_R}{c_L}\right), \quad 0 \leq \mathcal{T}_R \leq \min\left(1, \frac{c_L}{c_R}\right)$$

$$0 < a < \min(c_L, c_R) \quad \frac{|c_L - c_R|}{c_L + c_R} \leq \mathcal{R} \leq 1$$

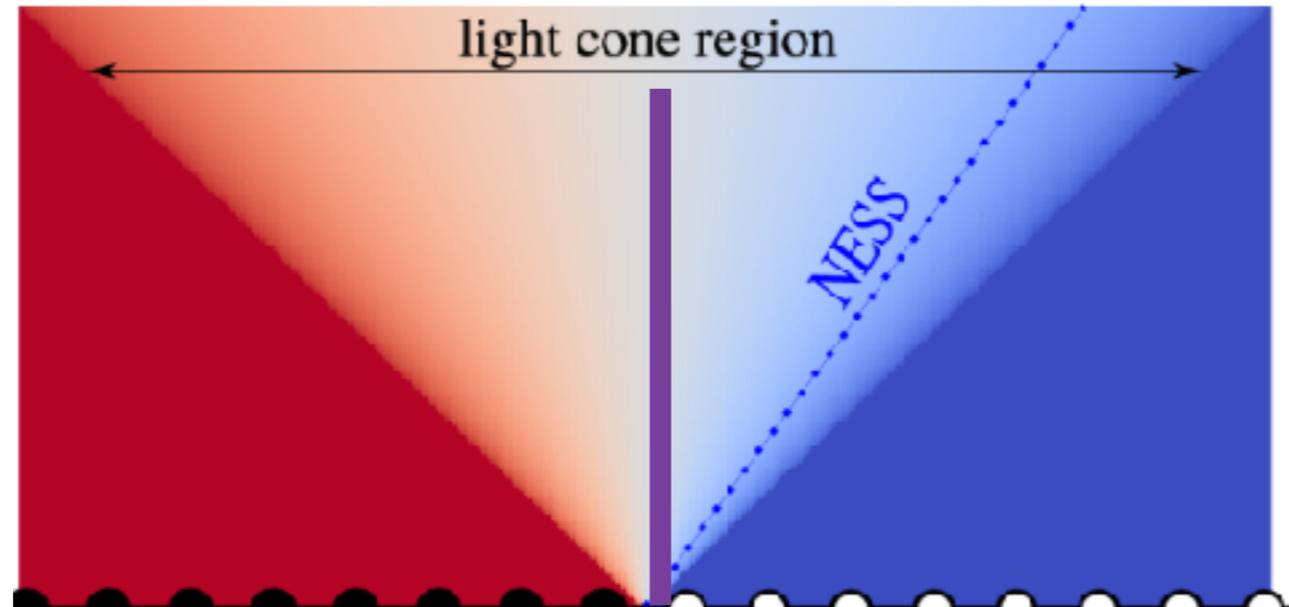
The lower bound is stronger than the unitarity bound

It should follow from unitarity of an S-matrix in a larger space of in and out states but it is not known how to characterize it

Non-equilibrium steady states

Doyon, Bernard, Vichi

Bachas, Chen, Papadopoulos



$$\langle T_{xt}(x=0) \rangle_{NESS} = \frac{\pi}{12} c_1 \mathcal{T}_1 \left(\frac{1}{\beta_1^2} - \frac{1}{\beta_2^2} \right)$$

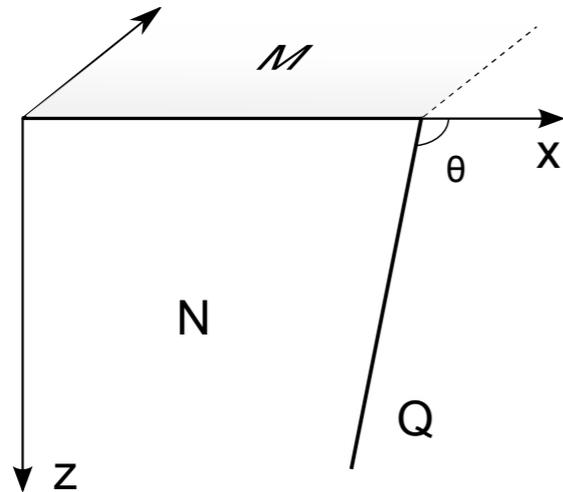
Without defect, the full large deviation function is known

$$F(\beta, \lambda) = \lim_{t \rightarrow \infty} \frac{1}{t} \log \langle e^{i\lambda t T_{xt}} \rangle = f(\beta_1, \lambda) + f(\beta_2, -\lambda)$$

$$f(\beta, \lambda) = \frac{\pi c}{12} \left(\frac{1}{\beta - i\lambda} - \frac{1}{\beta} \right)$$

Holographic BCFT

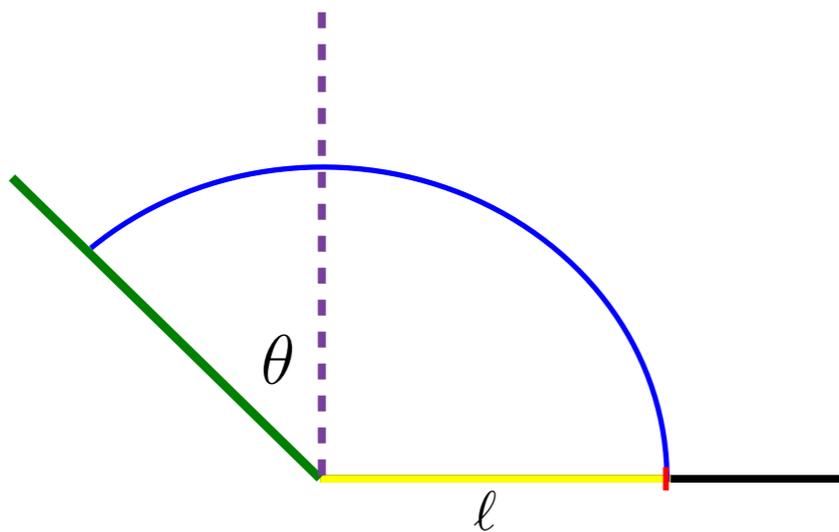
Takayanagi 11



Neumann b.c. for gravity on Q

$$Q : x = z \cot \theta$$

$$\sigma = \frac{\sin \theta}{8\pi G\ell} \quad \text{brane tension}$$

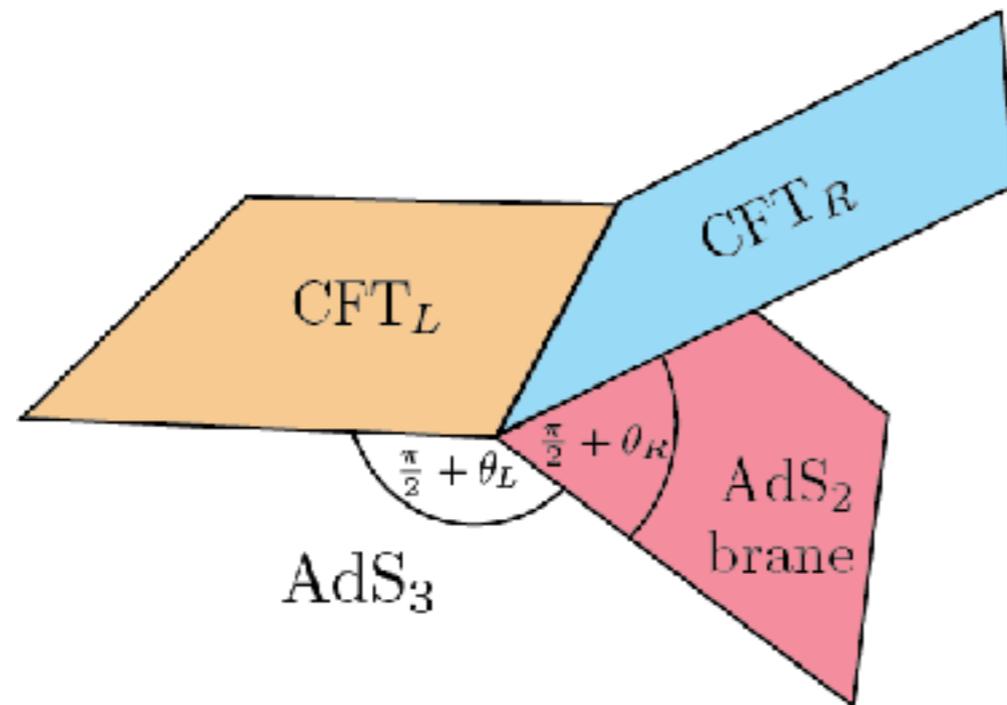


$$S_{EE} = \frac{c}{6} \log \frac{\ell}{\epsilon} + \frac{c}{6} \operatorname{arctanh}(8\pi G\ell\sigma)$$

g-factor determined by the brane tension

Holographic ICFT

Bachas 02
Azeyanagi-Karch-
Takayanagi-Thompson 07



$$S = (S_{EH} + S_{GH})_L + (S_{EH} + S_{GH})_R - \sigma \int_Q \sqrt{-\gamma}$$

Israel matching conditions

$$\gamma_{L,\alpha\beta} = \gamma_{R,\alpha\beta}$$

$$K_{L,\alpha\beta} - K_{R,\alpha\beta} = 8\pi G\sigma \gamma_{\alpha\beta}$$

$$\ell_Q = \frac{\ell_L}{\cos \theta_L} = \frac{\ell_R}{\cos \theta_R} = \frac{\tan \theta_L + \tan \theta_R}{8\pi G\sigma}$$

Stable solution for

$$\left| \frac{1}{\ell_L} - \frac{1}{\ell_R} \right| \leq 8\pi G\sigma \leq \left| \frac{1}{\ell_L} + \frac{1}{\ell_R} \right|$$

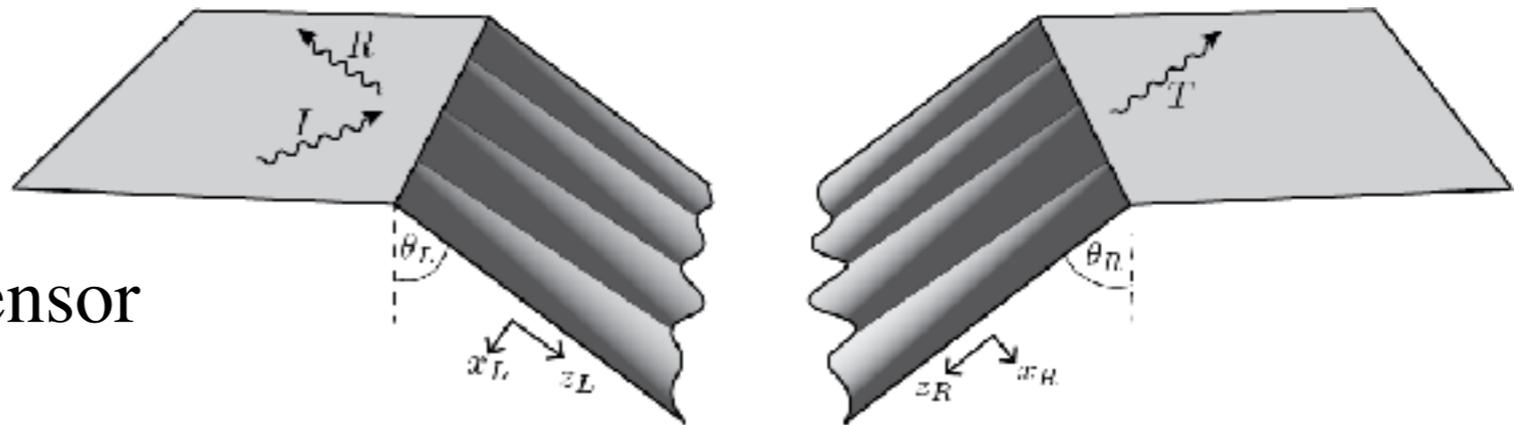
Minimal, bottom-up model used to compute EE, complexity etc

Holographic energy scattering

Away from the brane we have locally AdS3

$$ds^2 = \ell^2 \frac{dy^2}{y^2} + \left(\frac{\ell^2}{y^2} g_{ij}^0 + g_{ij}^2 + y^2 g_{ij}^4 \right) dx^i dx^j$$

$$g_{ij}^2 = 4G\ell \langle T_{ij} \rangle$$



No sources, only vev of e.m. tensor

$$\langle T_L \rangle = \epsilon \left[\mathbf{1} e^{i\omega(t_L - u_L)} d(t_L - u_L)^2 + \mathcal{R}_L e^{i\omega(t_L + u_L)} d(t_L + u_L)^2 \right] + c.c.$$

$$\langle T_R \rangle = \epsilon \mathcal{T}_L e^{i\omega(t_R - u_R)} d(t_R - u_R)^2 + c.c.$$

The nontrivial part is solving the matching conditions

$$\mathcal{T}_L = \frac{2 \cos \theta_R}{\cos \theta_R(1 + \sin \theta_L) + \cos \theta_L(1 + \sin \theta_R)} = \frac{2}{\ell_L} \left[\frac{1}{\ell_L} + \frac{1}{\ell_R} + 8\pi G\sigma \right]^{-1}$$

$$\left| \frac{c_L - c_R}{c_L + c_R} \right| \leq \mathcal{R} \leq \frac{c_L^2 + c_R^2}{(c_L + c_R)^2}$$

Lower bound = ANEC bound

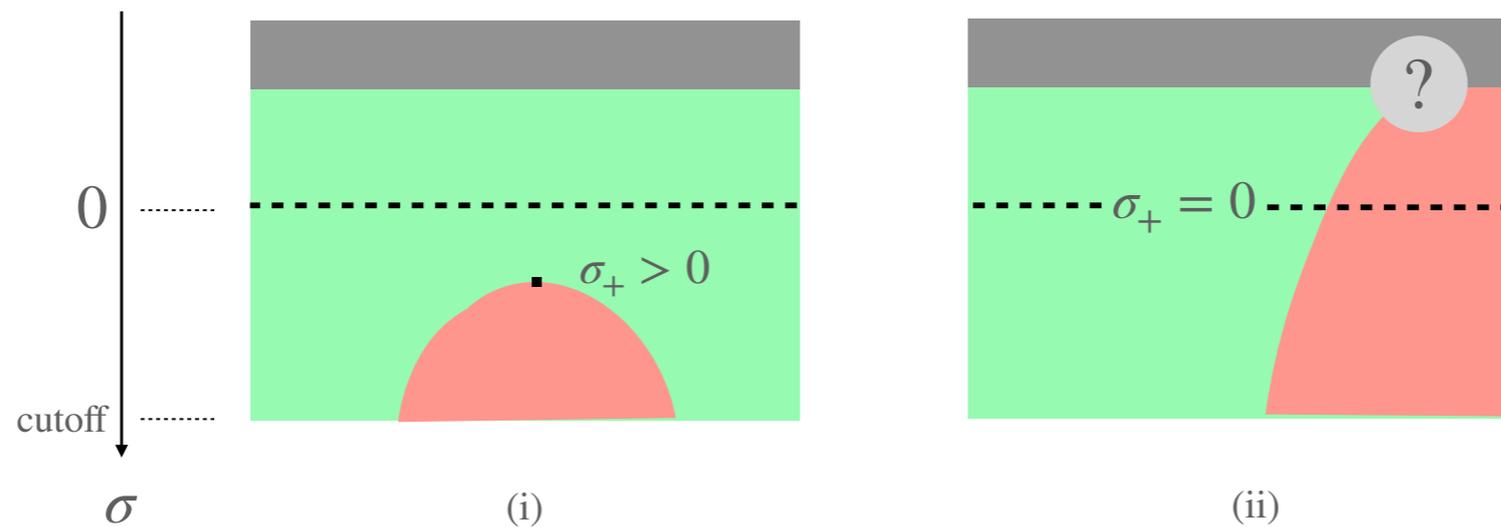
Upper bound: for $c_L = c_R$ $0 \leq \mathcal{R} \leq \frac{1}{2}$

Total reflection requires $\frac{c_L}{c_R} \rightarrow 0$ and infinite tension limit

Same result obtained from the holographic dual of the NESS setup

Bachas, Chen, Papadopoulos 21

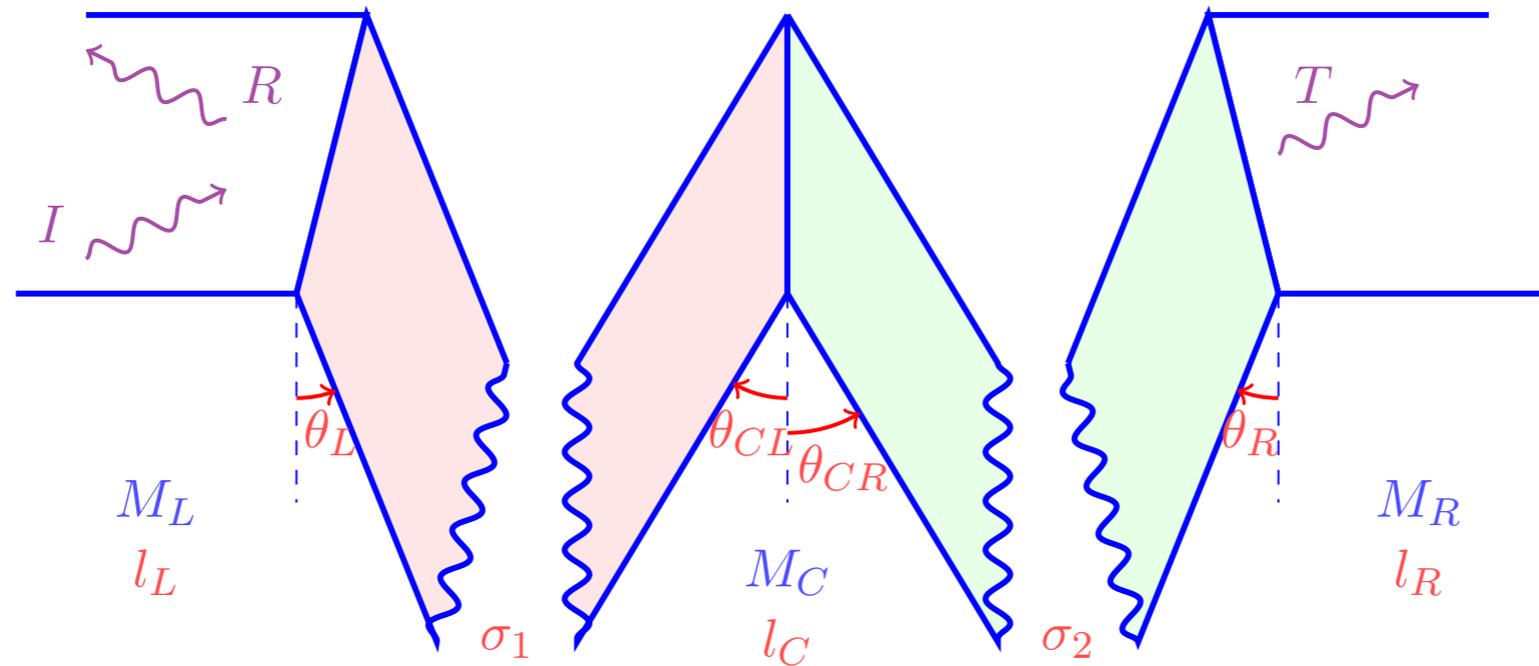
Take a brane joining two BTZ black holes



σ redshift factor on the brane

$$\sqrt{-\gamma} = \frac{A}{(\sigma - \sigma_+)(\sigma - \sigma_-)}$$

Regular solutions require $\sigma_+(M_1, M_2, J) = 0 \implies J(M_1, M_2)$

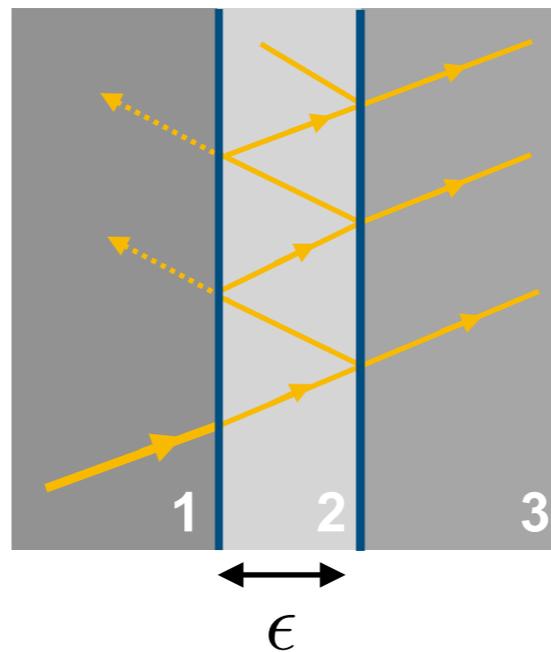


3 parameters: σ_1, σ_2, l_C

$$\mathcal{T}_L = \frac{2}{l_L} \left[\frac{1}{l_L} + \frac{1}{l_R} + 8\pi G(\sigma_1 + \sigma_2) \right] \quad \text{tension is additive}$$

$$\log g = \log g_1 + \log g_2 \quad \text{non-trivial fusion}$$

The additivity of the tension admits a nice CFT interpretation

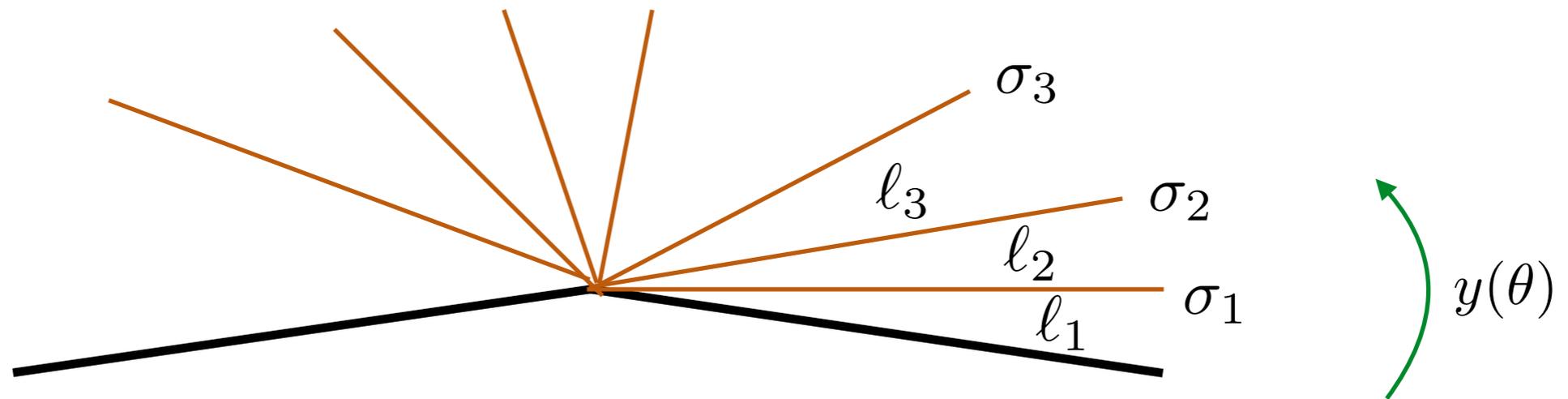


$$\mathcal{T}_{1 \rightarrow 3} = \mathcal{T}_{1 \rightarrow 2} \mathcal{T}_{2 \rightarrow 3} \left[1 + \mathcal{R}_{2 \rightarrow 3} \mathcal{R}_{2 \rightarrow 1} + \dots \right. \\ \left. + (\mathcal{R}_{2 \rightarrow 3} \mathcal{R}_{2 \rightarrow 1})^m + \dots \right]$$

The scattering is classical, no quantum interference

Not true in general - e.g. for the free scalar interface a putative effective tension would not be additive

Extension: multiple branes



An infinite number of branes can approximate a smooth defect geometry supported by a scalar field

$$ds^2 = dy^2 + a^2(y)\gamma_{ij}dx^i dx^j \quad \phi(y)$$

For slices with maximal symmetry (i.e. AdS) one has

$$T_{\mu\nu}^{\phi} = \partial_{\mu}\phi\partial_{\nu}\phi - \frac{1}{2}g_{\mu\nu}((\partial\phi)^2 + V(\phi)) = f(y)\Pi_{\mu\nu} + \Lambda_{eff}(y)g_{\mu\nu}$$

delta functions

 $\sigma_i\delta(y - y_i)$

step functions

 $-\frac{1}{\ell_i^2} \quad y_i < y < y_{i+1}$

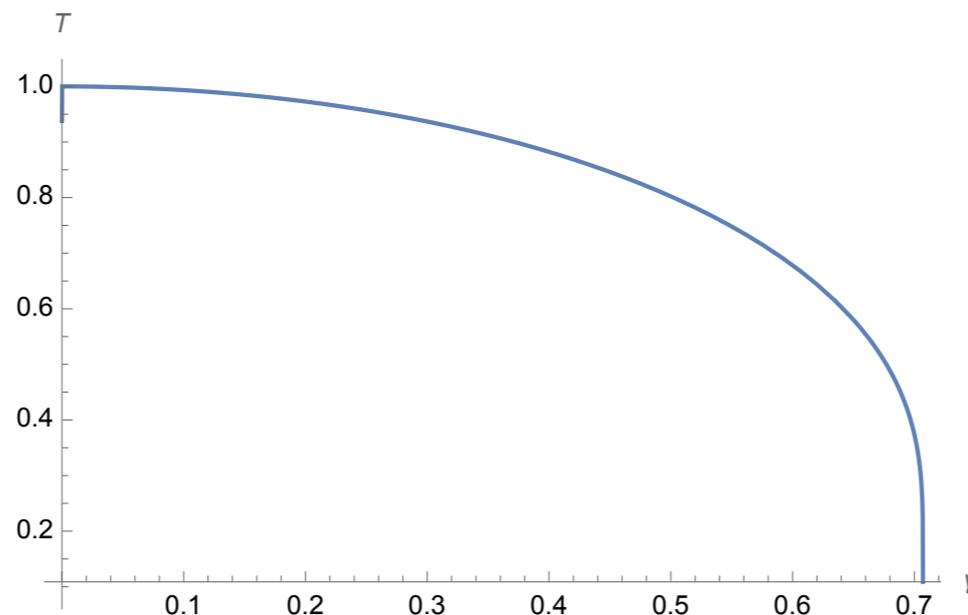
In the continuum limit $\Lambda = -\frac{1}{2}\phi'^2 + V, \quad d\sigma = \phi'^2 dy$

Effective tension $\sigma_{eff} = \int_{-\infty}^{+\infty} dy \phi'^2 = \int dy (T_{yy} - \frac{1}{n} \Pi^{\mu\nu} T_{\mu\nu})$

Janus geometry solution of type IIB reduced on $AdS_3 \times S^3 \times M_4$

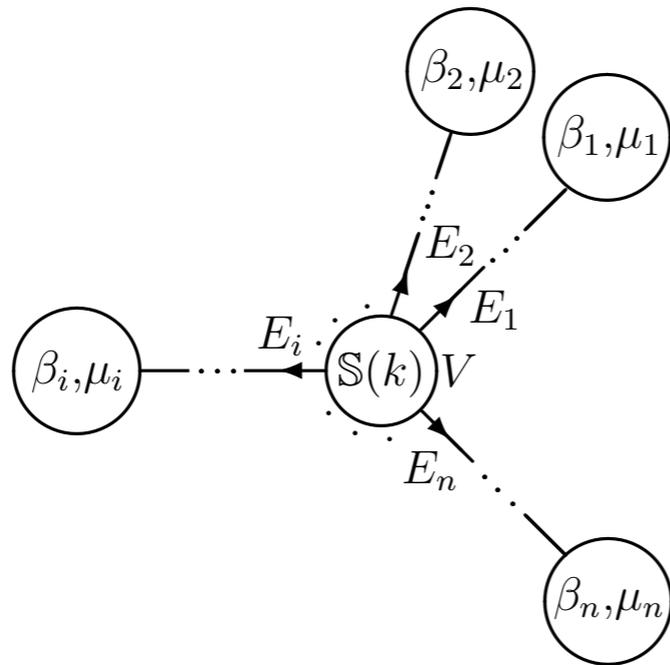
$$a^2(y) = \frac{1}{2} \left(1 + \sqrt{1 - 2\gamma^2} \cosh(2y) \right) \quad 0 < \gamma < \frac{1}{\sqrt{2}}$$

$$\phi(y) = \phi_0 + \frac{1}{\sqrt{2}} \log \left(\frac{1 + \sqrt{1 - 2\gamma^2} + \sqrt{2}\gamma \tanh y}{1 + \sqrt{1 - 2\gamma^2} - \sqrt{2}\gamma \tanh y} \right)$$



A free non-conformal case: Schroedinger junction

Mintchev 11 ...



$$\left(i\partial_t + \frac{1}{2m} \partial_x^2 \right) \psi(t, x, i) = 0$$

$$\lim_{x \rightarrow 0^+} \sum_{j=1}^n [\lambda(\mathbb{I} - \mathbb{U})_{ij} - i(\mathbb{I} + \mathbb{U})_{ij} \partial_x] \psi(t, x, j) = 0$$

$$S(k) = - \frac{[\lambda(\mathbb{I} - \mathbb{U}) - k(\mathbb{I} + \mathbb{U})]}{[\lambda(\mathbb{I} - \mathbb{U}) + k(\mathbb{I} + \mathbb{U})]}$$

In the state with finite temperatures and chemical potentials, the energy current is

$$\langle T_{xt}(i) \rangle_{\beta, \mu} = \frac{1}{m} \int_0^\infty \frac{dk}{2\pi} k \omega(k) \sum_{j=1}^n [\delta_{ij} - |S_{ij}(k)|^2] d_j(k)$$

Depending on the parameters, there can be bound states at the junction

Nambu-Goto from Israel matching conditions 2404.02149

Solving the junction conditions perturbatively in the string tension, at leading order the brane satisfies the Nambu-Goto equations

The NG modes can be uplifted to higher-order solutions but get sourced from reparametrization modes

This gives a class of generalizations of the static interface

On the BTZ black-hole background

$$ds^2 = \frac{dz^2}{\frac{z^2}{l^2} - 4\mu^2 l^2} - \left(\frac{z^2}{l^2} - 4\mu^2 l^2 \right) dt^2 + \frac{z^2}{l^2} dx^2$$

$$t_{L,R}(\tau, \sigma) = \tau \pm \tau_a(\tau, \sigma), \quad z_{L,R}(\tau, \sigma) = \sigma \pm \sigma_a(\tau, \sigma), \quad x_{L,R}(\tau, \sigma) = x_s(\tau, \sigma) \pm x_a(\tau, \sigma),$$

Perturbation around $x = 0$

$$\tau_a(\tau, \sigma) = \epsilon \left(\alpha_h + \frac{\sigma}{2\sqrt{\sigma^2 - 4\mu^2 l^4}} (-\beta_h e^{2\mu\tau} + \gamma_h e^{-2\mu\tau}) \right)$$

$$\sigma_a(\tau, \sigma) = \epsilon \left(\mu\sqrt{\sigma^2 - 4\mu^2 l^4} (\beta_h e^{2\mu\tau} + \gamma_h e^{-2\mu\tau}) \right)$$

$$x_{a,1}(\tau, \sigma) = \epsilon \left(-\frac{\lambda l^3}{2\sigma} \right)$$

$$x_{s,1}(\tau, \sigma) = \epsilon \left(\frac{8l^4 \mu^4 A_0 e^{-4\mu\tau}}{\sigma(4\mu^2 l^4 - \sigma^2)} \right)$$

with Dirichlet boundary conditions $x_s = x_a = 0$ at $\sigma \rightarrow \infty$

A_0 NG mode $\alpha_h, \beta_h, \gamma_h$ rigid parameters

The NG modes break the conformal invariance; in the CFT they can be understood as the effect of a one-sided conformal transformation to undo the time reparametrization at the interface

$$t_R = f(t_L) \quad x_R^\pm \rightarrow \tilde{x}_R^\pm = f^{-1}(x_R^\pm)$$

$$f^{-1}(x) = x + \epsilon (-2\alpha_h - \gamma_h e^{-2\mu x}) + \epsilon^2 \left(-\frac{\mu e^{-4\mu x} (A_0 \lambda + 2\gamma_h^2 l + 4\alpha_h \gamma_h l e^{2\mu x})}{2l} \right)$$

The Ward identities are modified

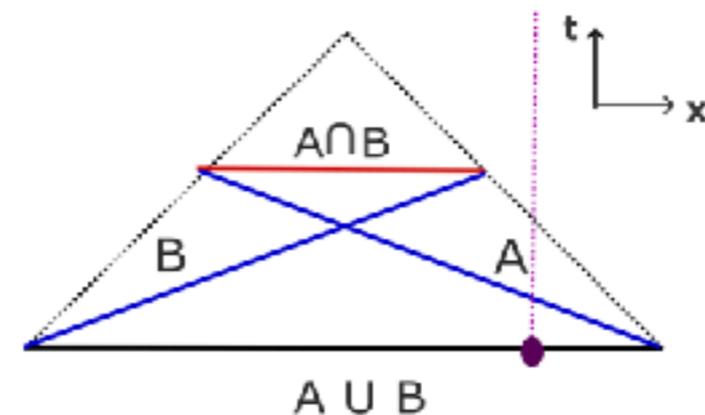
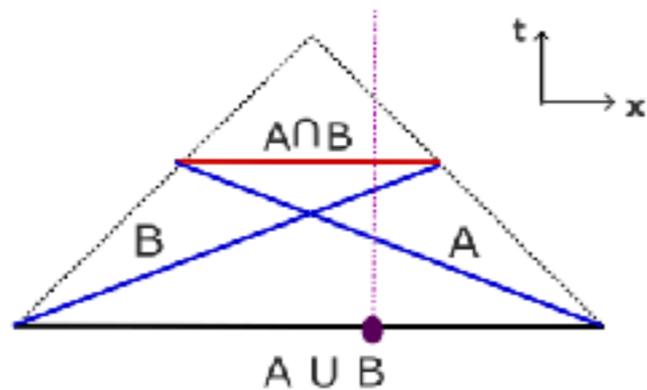
$$\partial_\mu T^{\mu t} = \delta(x) A^t \quad (= 0)$$

$$\partial_\mu T^{\mu x} = \delta(x) A^x \quad \left(= \epsilon^2 \frac{2A_0 c \lambda \mu^4 e^{-4\mu t}}{\pi l} \right)$$

The entanglement entropy across the interface depends non-trivially on all the parameters

We compute it in a double expansion in ϵ, δ where $x_{L,R} = \delta \frac{y_{L,R}}{\mu}$

Strong Sub-Additivity $S(A) + S(B) - S(A \cup B) - S(A \cap B) \geq 0$



We find that SSA is satisfied up to $O(\epsilon^2 \delta^2)$ without restricting the values of the parameters

Away from conformality: $T\bar{T}$ deformation

Zamolodchikov '04

Smirnov, Zamolodchikov '16

Irrelevant perturbation of the Lagrangian

$$\delta_\sigma \mathcal{L} = \frac{\delta\sigma}{2} \det T_{\mu\nu}^{(\sigma)} = -\frac{\delta\sigma}{2\pi^2} (T\bar{T} - \Theta^2) \quad \Theta = T_\mu^\mu$$

Energy levels satisfy the Burgers equation

$$\partial_\sigma E_n(R, \sigma) = E_n \partial_R E_n + \frac{1}{R} P_n^2(R)$$

CFT in finite volume

$$E_n(\sigma) = -\frac{R}{\sigma} \left(1 - \sqrt{1 + 2\sigma \frac{E_n(0)}{R} + \frac{\sigma^2}{R^2} P^2} \right)$$

$\sigma < 0$ complex spectrum $\sigma > 0$ Hagedorn growth

Integrability-preserving deformation of the S-matrix by CDD factor

$$S_{ij}^{kl} \rightarrow S_{ij}^{kl} e^{i\sigma\delta_{ij}} \quad \delta_{ij} = \epsilon_i p_j - \epsilon_j p_i = \begin{cases} m^2 \sinh(\theta_i - \theta_j), & m > 0 \\ \Lambda^2 e^{\theta_i - \theta_j}, & m = 0 \end{cases}$$

Dynamical coordinate transformation

Conti, Negro, Tateo '18
Cardy '19

deformed CFT = CFT in deformed geometry

$$h_{\alpha\beta} = \partial_{(\alpha}\xi_{\beta)}$$

$$\xi_0 = -2\pi \left(\int_x^\infty - \int_{-\infty}^x \right) T_{10}(t, x') dx' = P_{<} - P_{>}$$

$$\xi_1 = 2\pi \left(\int_x^\infty - \int_{-\infty}^x \right) T_{00}(t, x') dx' = E_{>} - E_{<}$$

Particles experience an energy-momentum-dependent time delay

The geometry is **dynamical**, i.e. state-dependent

Evolution equation for the correlators

Cardy '19

$$\partial_\sigma \Phi^\sigma(x) = \epsilon^{ab} \epsilon^{ij} \int_x dx'_j T_{ai}^\sigma(x') \partial_b \Phi^\sigma(x)$$

From the leading OPE singularity

$$\Phi^\sigma(k) \sim e^{-\sigma \log(\Lambda/\mu) k^2} \Phi^0(k)$$

It can be interpreted as a Brownian motion of the position x with diffusivity $\sim \sigma \log \Lambda$

Particles behave as if they had a finite size

Cardy-Doyon '20

Holographically, **cutoff at finite radius**
or better **mixed boundary conditions**

McGough, Mezei, Verlinde '16

Guica, Monten '19

$$ds^2 = \ell^2 \frac{d\rho^2}{4\rho^2} + \frac{1}{\rho} g^{(0)}(x) + g^{(2)}(x) + \rho g^{(4)}(x) \quad \text{asymptotically-AdS}_3 \text{ in FG coords}$$

$$\text{Undeformed :} \quad \gamma_0 = g^{(0)}, \quad T_0 = \frac{1}{8\pi G\ell} g^{(2)}$$

$$\text{Deformed :} \quad \gamma_\mu = g^{(0)} + \mu g^{(2)} + \mu^2 g^{(4)} \quad (\text{induced metric at } \rho = \mu)$$

$$\hat{T}_\mu = \frac{1}{8\pi G\ell} (g^{(2)} + 2\mu g^{(4)}) \quad \hat{T}_{\alpha\beta} = -\epsilon_{\alpha\gamma}\epsilon_{\beta\delta} T^{\gamma\delta} = T_{\alpha\beta} - \gamma_{\alpha\beta} T_\gamma^\gamma$$

$$\mu = -\frac{c}{12\pi\ell^2} \sigma \quad \text{semiclassical regime: } c \rightarrow \infty \text{ with } \sigma c \text{ fixed}$$

Black-hole thermodynamics



Flow equation

$$\begin{aligned}\gamma_{\alpha\beta}^{[\mu]} &= \gamma_{\alpha\beta}^{[0]} - 2\mu\hat{T}_{\alpha\beta}^{[0]} + \mu^2\hat{T}_{\alpha\rho}^{[0]}\gamma^{[0]\rho\sigma}\hat{T}_{\sigma\beta}^{[0]}, \\ \hat{T}_{\alpha\beta}^{[\mu]} &= \hat{T}_{\alpha\beta}^{[0]} - \mu\hat{T}_{\alpha\rho}^{[0]}\gamma^{[0]\rho\sigma}\hat{T}_{\sigma\beta}^{[0]}.\end{aligned}$$

$\partial_\mu(\sqrt{\gamma_\mu}R_\mu) = 0$ the flow preserves flatness of the boundary metric

The Hamiltonian constraint for *pure gravity* imposes the trace relation

$$\text{tr}T^{[\mu]} = \mu \det T^{[\mu]}$$

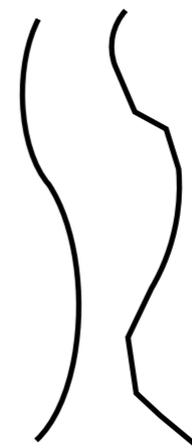
There are flat coordinates such that $\gamma_{\alpha\beta}^{[0]}dx^\alpha dx^\beta = dudv$

$$\gamma_{\alpha\beta}^{[\mu]}dX^\alpha dX^\beta = dUdV$$

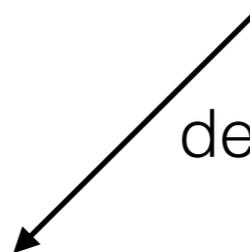
Problem: the change of coordinate generically does not preserve the position of the interface

(\tilde{u}, \tilde{v}) (u, v)

conformal map



deformation map

 (U, V)

$$\tilde{u} = u - 2\mu\alpha(u), \tilde{v} = v - 2\mu\bar{\alpha}(v)$$

$$U = u - 2\mu\bar{h}(v), \tilde{V} = v - 2\mu h(u)$$

$$T_{uu} = h'(u), T_{vv} = \bar{h}'(v)$$

The coordinate change is done independently on the two sides

4 functions $\alpha_{L,R}, \bar{\alpha}_{L,R}$

$$\text{At the interface} \quad X_L = X_R = 0$$

$$T_L = T_R$$

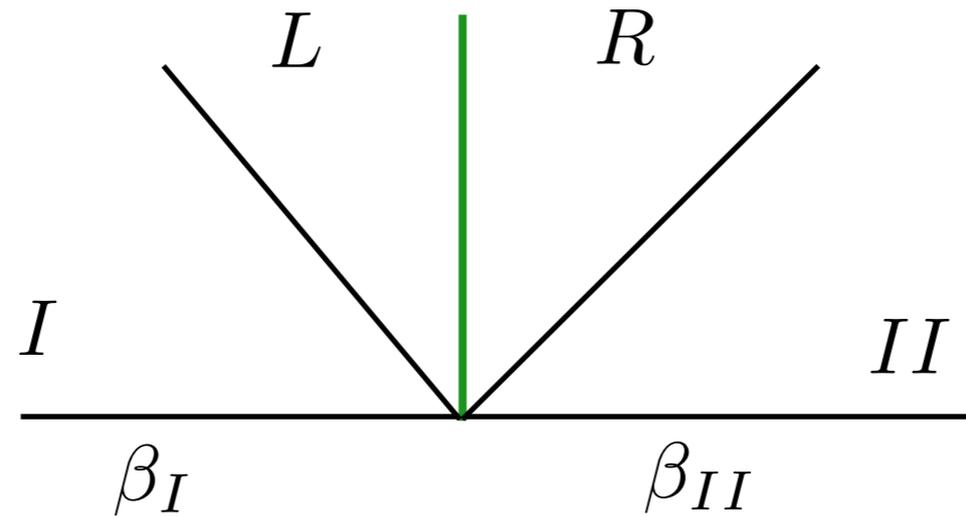
One function is unfixed: the results will not depend on it

We start with a solution of the interface conditions in the CFT - the sequence of maps defines the interface of the deformed theory

$$T_{\tilde{u}\tilde{u}}(\tilde{u}) = \frac{h'(u)}{(1 - 2\mu\alpha'(u))^2} \quad T_{\tilde{v}\tilde{v}}(\tilde{v}) = \frac{\bar{h}'(v)}{(1 - 2\mu\bar{\alpha}'(v))^2}$$

$$T_{UU} = \frac{h'}{1 - 4\mu^2 h' \bar{h}'} \quad T_{VV} = \frac{\bar{h}'}{1 - 4\mu^2 h' \bar{h}'} \quad T_{UV} = -\frac{2\mu h' \bar{h}'}{1 - 4\mu^2 h' \bar{h}'}$$

1 - Non-Equilibrium Steady State



$$T_{\tilde{u}\tilde{u}}^i = \frac{M^i + J}{2} \quad T_{\tilde{v}\tilde{v}}^i = \frac{M^i - J}{2} \quad J = \frac{\mathcal{T}_0}{\mathcal{R}_0} \frac{M_L - M_R}{2}$$

matching across the shock-waves determines $M_L, M_R(\beta_I, \beta_{II})$

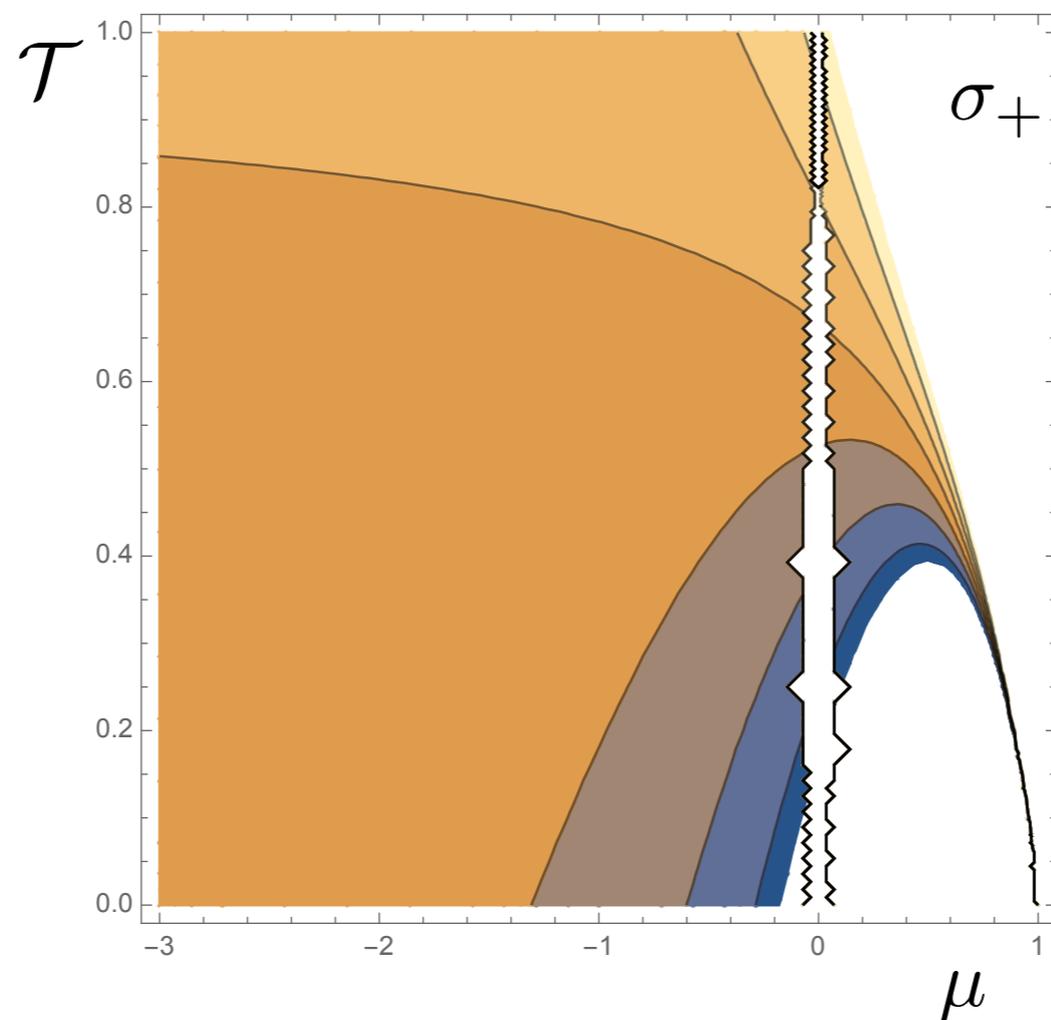
$$T_{VV}^L = \mathcal{R} T_{UU}^L + \mathcal{T} T_{VV}^R$$

$$T_{UU}^R = \mathcal{T} T_{UU}^L + \mathcal{R} T_{VV}^R$$

$$T_{UU}^R - T_{VV}^R = T_{UU}^L - T_{VV}^L \implies \mathcal{R} + \mathcal{T} = 1$$

The transport coefficients are *state-dependent*, i.e. depend on β_L, β_R

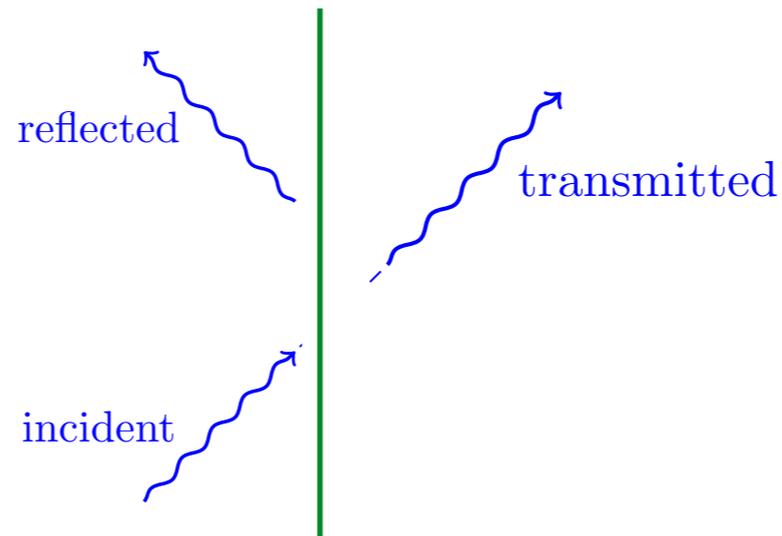
They can be found in an expansion in μ or numerically, and agree with the holographic result



$\mu = 1$ Hagedorn bound

Unitarity bound $0 \leq \mathcal{T} \leq 1$ always respected

2 - Scattering of plane waves



$$T_{\tilde{u}\tilde{u}}^L = \epsilon(A_1 e^{i\omega\tilde{u}} + A_2 e^{i\nu\tilde{u}})$$

$$T_{\tilde{u}\tilde{u}}^R = \epsilon\mathcal{T}_0(A_1 e^{i\omega\tilde{u}} + A_2 e^{i\nu\tilde{u}})$$

$$T_{\tilde{v}\tilde{v}}^L = \epsilon\mathcal{R}_0(A_1 e^{i\omega\tilde{u}} + A_2 e^{i\nu\tilde{u}})$$

In the deformed theory, different frequencies interact

We solve the problem in an expansion in the amplitude of the waves

The interface matching conditions have to be generalized

$$T_{VV}^L = \mathcal{R}_0 T_{UU}^L + \mathcal{T}_0 T_{VV}^R + M(T_{UU}^L, T_{VV}^R)$$

$$T_{UU}^R = \mathcal{T}_0 T_{UU}^L + \mathcal{R}_0 T_{VV}^R - M(T_{UU}^L, T_{VV}^R)$$

$$M(x, y) = \mathcal{R}_0 \mathcal{T}_0 [4\mu(x^2 - y^2) + 4\mu^2(x - y) ((5\mathcal{T}_0 - 2)(x^2 + y^2) + (6\mathcal{T}_0 - 2)xy)] \\ + \mathcal{O}(\epsilon^4)$$

Satisfies basic consistency checks

Non-linear but **local**

Universal function : coefficients are frequency-independent

Summary

We derived holographically the energy reflection and transmission coefficients of a conformal defect

Result consistent with unitarity but also with stricter ANEC bound

The tension of the brane appears to be related to energy transmission more than to entropy, although our model cannot really distinguish (only one parameter)

Multiple-brane setup can vastly enlarge the class of holographic defects and allow to study some top-down models

The $T\bar{T}$ deformation and NG interfaces offers a window into a class of solvable non-CFT defects

Open questions

Coefficients for charge transport

Universality of transmission coefficient in gravity

Scattering interpretation of higher-point coefficients

Generalization to higher-dimensional CFTs

Non-conformal interfaces, behavior of transport under RG flow

Quantization of the non-linear matching conditions

More general ANEC bounds on transport functions

Thank you!