Quantum Quenhes from Quantum Fields

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Based on:

- A. Chalabi, C.K., & C. Su, ArXiv:2503.22598, Phys. Lett. B 866 (2025) 139512
- C.K., & K. Zarembo, ArXiv:2412.17479, JHEP05 (2025) 207
- M. de Leeuw, C.K. & G. Linardopoulos , ArXiv:1802.01598, Phys.Lett. B 781 (2018) 238

New Advancements on Defects and their Applications
YITP, Kyoto University
July 16th, 2025

AdS/CFT

$\mathcal{N} = 4$ SYM in 4D $\longleftrightarrow$ IIB strings on $AdS_5 \times S^5$

- Conformal symmetry
- Supersymmetry
- Planar integrability

AdS/dCFT

$\mathcal{N} = 4$ SYM in 4D with co-dim d defect $\longleftrightarrow$ IIB strings on $AdS_5 \times S^5$ with probe brane

- Conformal symmetry partially broken
- Supersymmetry partially or completely broken

In the planar limit, $N \to \infty$:

Motivation

- Insights on the interplay between conformal symmetry, supersymmetry and integrability
- Tests of AdS/dCFT dictionary for set-ups with and without supersymmetry (so far all positive)
- Exact results for novel types of observables such as one-point functions, bulk-to-boundary correlators etc.
- Interesting connections to statistical physics: matrix product states and quantum quenhes.
- Possible cross-fertilization with the boundary conformal bootstrap program.

Plan of the talk

I. Defect set-ups

II. Connections to Quantum Quenches and MPS

III. Integrability Primer

IV. Closed expressions for one-point functions

V. Summary & Open problems

The defect set-ups

Nahm pole defects :
of co-dimension d
(All 1/2 BPS)

$$\langle \Phi \rangle \sim \frac{\Phi^{\rm cl}}{r} \longleftarrow \text{distance to defect}$$

- $d = 1$: Domain wall defect:

$$\langle \phi_i \rangle \neq 0, \qquad i = 1, 2, 3.$$

Karch & Randall ‘02

de Leeuw, CK. & Zarembo ‘15

- $d = 2$: Gukov-Witten surface defect:

$$\langle \phi_i \rangle \neq 0, \qquad i = 1, 2, \qquad \langle A_3 \rangle \neq 0.$$

Gukov & Witten ‘08

Chalabi, C.K & Su ‘2025

- $d = 3$ ’t Hooft line (monopole):

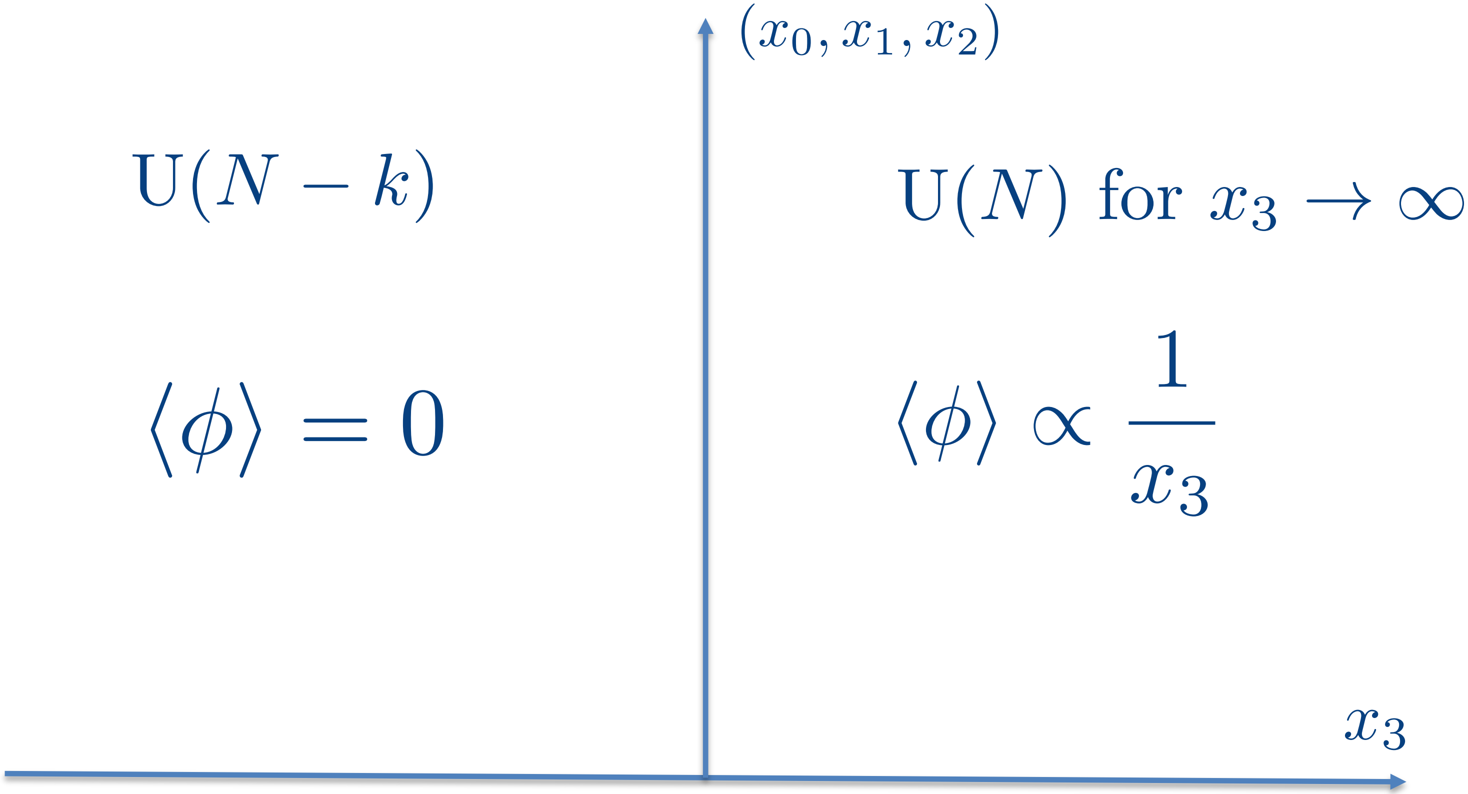
$$\langle \phi_i \rangle \neq 0, \qquad i = 1, \qquad \langle A_i \rangle \neq 0, \qquad i = 1, 2, 3.$$

Kapushtin ‘05

C.K & Zarembo ’23, ‘24

The domain wall set-up

$$\mathcal{N} = 4 \quad \text{SYM}$$



(x_0, x_1, x_2)

$\mathrm{U}(N-k)$

$\mathrm{U}(N)$ for $x_3 \to \infty$

$$\langle \phi \rangle = 0$$

$$\langle \phi \rangle \propto \frac{1}{x_3}$$

x_3

The Classical Fields

Assume only x_3 -dependence and $x_3 > 0$, $A_\mu^{\rm cl} = 0,$ $\Psi_A^{\rm cl} = 0$

Classical e.o.m.:

(x_3 is distance to defect)

$$\frac{d^2\phi_i^{\rm cl}}{dx_3^2} = [\phi_j^{\rm cl}, [\phi_j^{\rm cl}, \phi_i^{\rm cl}]] \, .$$

Solution:

$$\phi_i^{\rm cl} = \frac{1}{x_3} \begin{pmatrix} (t_i)_{k\times k} & 0 \\ 0 & 0 \end{pmatrix}, \; i = 1, 2, 3$$

Constable, Myers & Tafjord '99

$$\phi_4^{\rm cl} = \phi_5^{\rm cl} = \phi_6^{\rm cl} = 0$$

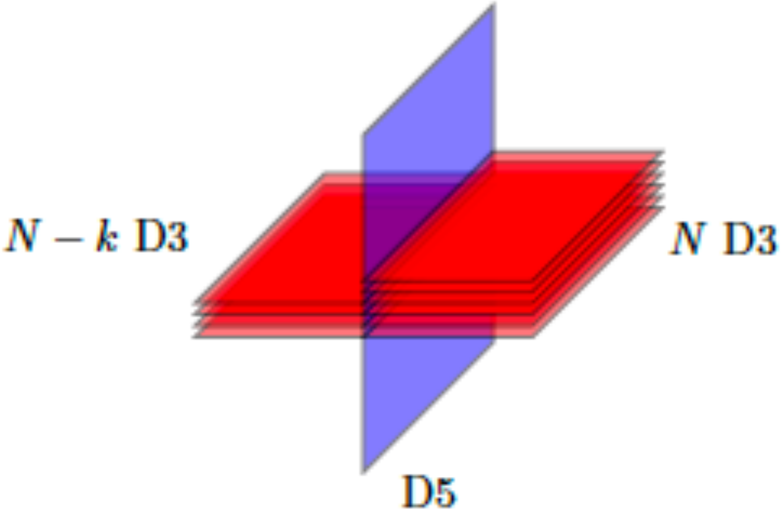
$$[t_i, t_j] = i\, \epsilon_{ijl}\, t_l$$

i.e. t_i, $i = 1, 2, 3$ constitute a k-dimensional irreducible representation of SU(2)

Set-up ½ BPS (for appropriate choice b.c. for zero-modes, Gaiotto & Witten '08)

The domain wall set-up --- The string theory side

	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
D3	×	×	×	×						
D5	×	×	×		×	×	×			



Karch & Randall ’01,

Geometry of D5 brane: $AdS_4 \times S^2$

Background gauge field: k units of magnetic flux on S^2

One-point functions in the domain wall dCFT

$$\langle \mathcal{O}_{\Delta}^{\text{bulk}}(x)\rangle = \frac{C}{|x_3|^{\Delta}}$$

Cardy ´84
McAvity & Osborn '95

Normalization given by: $$\lim_{x_3\to\infty}\langle \mathcal{O}_{\Delta}^{\text{bulk}}(y+x)\mathcal{O}_{\Delta'}^{\text{bulk}}(z+x)\rangle = \frac{\delta_{\Delta\Delta'}}{|y-z|^{2\Delta}}$$

Due to vevs scalar operators can have non-zero 1-pt fcts at tree-level

$$\langle \mathcal{O}_{\Delta}(x)\rangle = (\text{Tr}(\phi_{i_1}\ldots\phi_{i_\Delta})+\ldots)\,|_{\phi_i\to\phi_i^{\text{cl}}=\frac{t_i}{x_3}}$$

Tree level and one-loop 1-pt functions of *conformal* scalar operators can be found in closed form using the tools of integrability

deLeeuw, C.K.
& Zarembo '15,

Buhl-Mortensen de Leeuw,
C.K & Zarembo '16

de Leeuw, C.K &
Mori, '17.

de Leeuw, C.K &
Linardopoulos, '18.

C.K, Müller,
Zarembo '20

An exact formula for any loop order can be found by integrability bootstrap arguments

Komatsu
& Wang '20

Gombor
& Bajnok '20

AdS/CFT Integrability

Conformal operators $\longleftrightarrow$ String states, (AdS/CFT)

$\updownarrow$ $\updownarrow$

Eigenstates of integrable super spin chain: $|\mathbf{u}\rangle$

Minahan.
Zarembo ‘02
Beisert, CK
Staudacher 03

Main examples: $\mathcal{N} = 4$ SYM (4D), ABJM theory (3D), ...
Large-N limit

Integrability in AdS/dCFT

Co-dimension d defect $\longleftrightarrow$ Probe brane

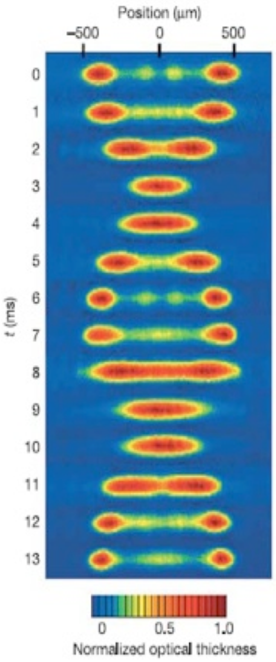
$\updownarrow$ $\updownarrow$

(Integrable) boundary state $|\Psi_0\rangle$ of spin chain

De Leeuw, C.K.
Zarembo ‘15

$\langle\Psi_0|\mathbf{u}\rangle$ is the one-point function of the operator described by $|\mathbf{u}\rangle$

Quantum Quenches and Overlaps



Set out quantum system in initial state $|\Psi_0\rangle$
which is not an eigenstate of its Hamiltonian $\mathcal{H}_0$

Study time development of local observable

$$\langle \mathcal{O}(t)\rangle = \langle \Psi_0|e^{i\mathcal{H}_0 t}\mathcal{O}e^{-i\mathcal{H}_0 t}|\Psi_0\rangle$$

$$= \sum_{\mathbf{u},\mathbf{v}} \langle \Psi_0|\mathbf{u}\rangle\langle \mathbf{u}|\mathcal{O}|\mathbf{v}\rangle\langle \mathbf{v}|\Psi_0\rangle e^{-i(E_{\mathbf{v}}-E_{\mathbf{u}})t},$$

$$\mathcal{H}_0|\mathbf{u}\rangle = E_{\mathbf{u}}|\mathbf{u}\rangle$$

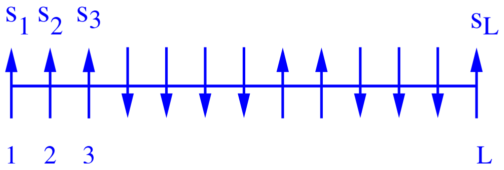
Assume $\mathcal{H}_0$ Hamiltonian of an integrable system

When and how can $\langle \Psi_0|\mathbf{u}\rangle$ be calculated in closed form?

Of relevance for

- Time development after quantum quench
  (post-quench steady state, post-quench entanglement dynamics)
- Correlation functions in AdS/dCFT

Integrable Quenches



$s_{L+m} = s_m$ $\quad |\Psi\rangle = |s_1 s_2 s_3 \dots s_L\rangle$

Eigenstates: $H_0|\mathbf{u}\rangle = E_0|\mathbf{u}\rangle$

Integrable Quench: $\langle\Psi_0|\mathbf{u}\rangle$ computable in closed form

Identified types of relevance for AdS/dCFT:

Matrix product states: $|\Psi_0\rangle = |\mathrm{MPS}\rangle = \sum_{\{s_i\}} \mathrm{Tr}(t_{s_1} \dots t_{s_L})|s_1 \dots s_L\rangle$

De Leeuw, C.K., Zarembo '15

Ex: Heisenberg spin chain $\quad |\mathrm{MPS}\rangle = \mathrm{Tr} \prod_{l=1}^{L} (|\uparrow\rangle_l \otimes t_1 + |\downarrow\rangle_l \otimes t_2)^L$

Integrable Quenches

Identified types of relevance for AdS/dCFT:

Valence Bond States: $|\Psi_0\rangle = |\mathrm{VBS}\rangle = |K\rangle^{\otimes \frac{L}{2}}, \quad K = \sum_{s_1,s_2} K_{s_1,s_2}|s_1 s_2\rangle$

C.K., Müller, Zarembo ‘20

Of possible relevance for AdS/CFT:

Cross cap states: $|C\rangle = |c\rangle\rangle^{\otimes L/2}$, where $|c\rangle\rangle = |\uparrow\rangle_j |\uparrow\rangle_{\frac{L}{2}+j} + |\downarrow\rangle_j |\downarrow\rangle_{\frac{L}{2}+j}$

Caetano, Komatsu ‘21

One-point functions and $|\mathrm{MPS}\rangle$

General scalar conformal operator

$$\mathcal{O}_L(x) = \Psi^{i_1\ldots i_L}\mathrm{Tr}(\phi_{i_1}\ldots\phi_{i_L}) \qquad i_1,\ldots,i_L \in \{1,2,\ldots,6\}$$

$$\equiv \text{Eigenstate of integrable } SO(6) \text{ spin chain, } |\mathbf{u}\rangle$$

$$\mathrm{Tr}(\phi_{i_1}\phi_{i_2}\ldots\phi_{i_L}) \sim |s_{i_1}s_{i_2}\ldots s_{i_L}\rangle$$

Minahan & Zarembo

Due to the vevs scalar operators can have 1-pt fcts already at tree level

$$\langle\mathcal{O}_L(x)\rangle = \frac{1}{x_3^L}\,\Psi^{i_1\ldots i_L}\mathrm{Tr}(t_{i_1}^{(k)}\ldots t_{i_L}^{(k)}) \equiv \frac{C_k}{x_3^L}$$

Matrix product state (of bond dimension $k>1$) associated with defect

$$|\mathrm{MPS_k}\rangle = \sum_{i_1,\ldots,i_L}\mathrm{tr}\,(t_{i_1}^{(k)}\ldots t_{i_L}^{(k)})|s_{i_1}\ldots s_{i_L}\rangle,$$

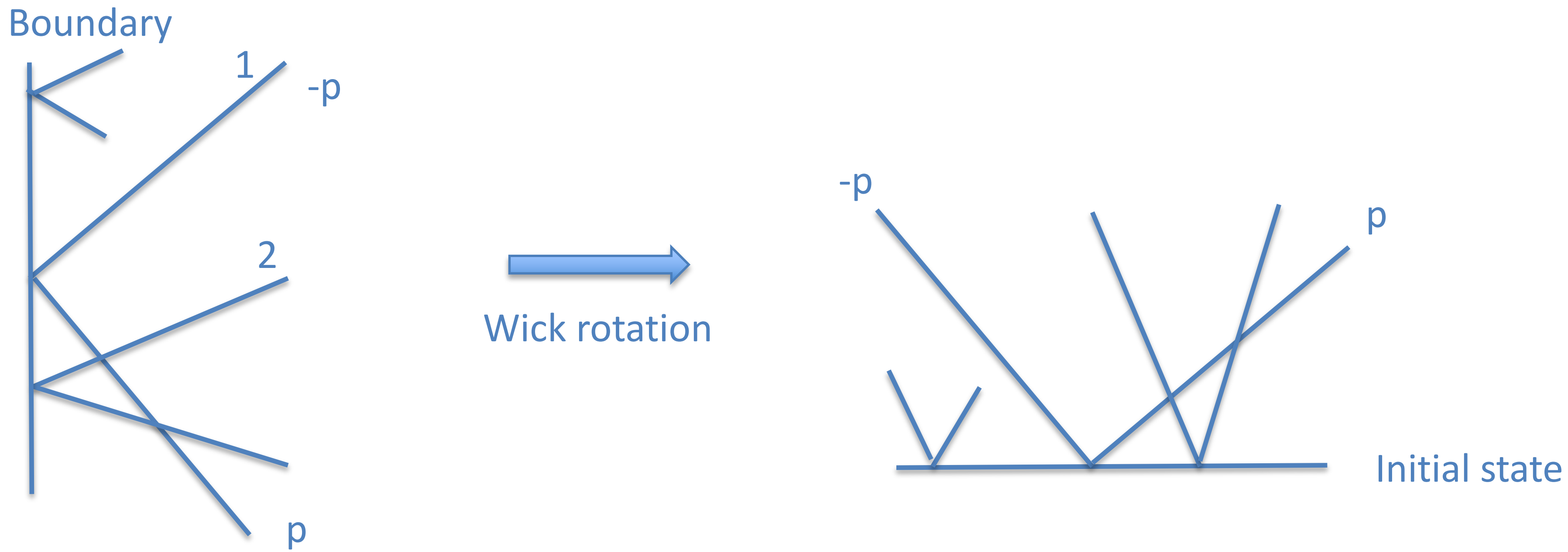
de Leeuw, C.K. & Zarembo '15

Object to calculate $\qquad C_k\,(\mathbf{u}) = \dfrac{\langle\mathrm{MPS}_k\,|\mathbf{u}\rangle}{\langle\mathbf{u}\,|\mathbf{u}\rangle^{\frac{1}{2}}}$

Integrable boundaries in integrable QFTs

Ghoshal,
Zamolodchikov '93

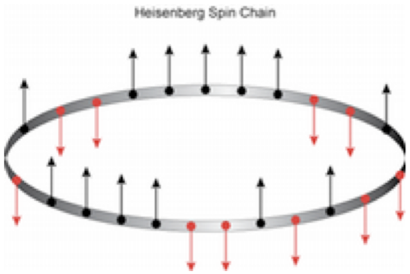
- No particle production or annihilation
- Pure reflection, possibly change of internal quantum numbers
- Yang-Baxter relations fulfilled (order of reflection does not matter)



Pure reflection
+BYBE for reflection matrix

Entangled (p,-p) pairs
+KYBE for initial state

Integrability test



Ex: Heisenberg spin chain

Vacuum State: All spins down $|0\rangle$

Excited states with M excitations: $|\{p_i\}_{i=1}^{M}\rangle$

L conserved charges, $\hat{Q}_n$, with eigenvalues Q_n

$$Q_n\left(\{p_i\}\right) = (-1)^n\, Q_n\left(\{-p_i\}\right)$$

Piroli, Pozsgay
Vernier '17

Bajnok, Gombor '20

Integrable initial state: $\widehat{Q}_{2m+1}|\Psi_0\rangle = 0, \quad \forall\, m$

$$\Pi\, T(u)\, \Pi\, |\Psi_0\rangle = T(u)|\Psi_0\rangle$$

Parity

Transfer matrix

T explicitly known – Easy to carry out concrete checks

Elements of the language of integrability

Eigenstates with M excitations described in terms of M momenta $p_1, \ldots, p_M$ or rapidities $u_i = \frac{1}{2}\coth(p_i/2)$

$$1 = \left(\frac{u_k - \frac{i}{2}}{u_k + \frac{i}{2}}\right)^L \prod_{j \neq k}^{K} \frac{u_k - u_j + i}{u_k - u_j - i} = e^{i\chi_k}, \qquad k = 1, \ldots M$$

Heisenberg
spin chain

Can be encoded in $\boxed{\text{Baxter polynomial}}$ $Q(u) = \prod_{i=1}^{M}(u - u_i)$

$$|\mathbf{u}\rangle = |\{u_i\}\rangle = \hat{B}(u_1)\ldots\hat{B}(u_M)|0\rangle$$

$$\langle\mathbf{u}|\mathbf{u}\rangle = \det G(\{u_i\}), \qquad \boxed{\text{Gaudin determinant}} \qquad G_{kj} = \frac{\partial\chi_k}{\partial u_j}$$

$$\hat{\mathbf{Q}}_{\mathbf{2n+1}}|\Psi_0\rangle = 0 \implies \boxed{\text{Pairing}}$$

$\langle\Psi_0|\mathbf{u}\rangle \neq 0$ iff roots are paired $\{u_i, -u_i\}_{i=1}^{K_u}$

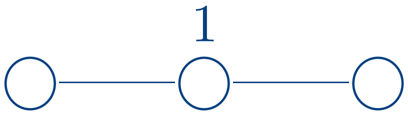
Overlaps with δ-states

$$SU(2): \quad |\Psi_0\rangle = (|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)^{\otimes L/2}, \quad C^2 = \frac{Q(0)}{Q(\frac{i}{2})} S \det G$$

Poszgay '18

$$SO(6): \quad |\Psi_0\rangle = (|XX\rangle + |YY\rangle + |ZZ\rangle + |\bar{X}\bar{X}\rangle + |\bar{Y}\bar{Y}\rangle + |\bar{Z}\bar{Z}\rangle)^{\otimes L/2} \; ,$$

$$C^2 = \frac{Q_1(0)Q_2(0)Q_3(0)}{Q_1\left(\frac{i}{2}\right) Q_2\left(\frac{i}{2}\right) Q_3\left(\frac{i}{2}\right)} S \det G$$

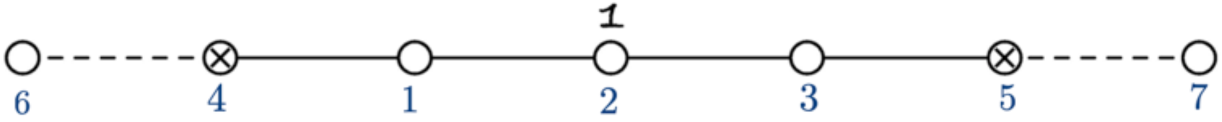


de Leeuw, Gombor, C.K.,
Linardopoulos, Pozsgay '19

Gombor '21

AdS/CFT

$PSU(2,2|4)$:



$$C^2 = \frac{Q_1(0)Q_2(0)Q_3(0)Q_4(0)Q_5(0)}{Q_1(\frac{i}{2})Q_2(\frac{i}{2})Q_3(\frac{i}{2})Q_6(0)Q_6(\frac{i}{2})Q_7(0)Q_7(\frac{i}{2})} S \det G$$

C.K.,
Zarembo
'22

From $|\delta\rangle$ to $|\mathrm{MPS}_k\rangle$ by dressing

Example for SU(2) chain

$$C_{|\mathrm{MPS}_k\rangle} = \sum_{a=-\frac{k-1}{2}}^{a=\frac{k-1}{2}} a^L \frac{Q\left(\frac{ik}{2}\right) Q\left(\frac{ik}{2}\right)}{Q\left((a-\frac{1}{2})i\right) Q\left((a-\frac{1}{2})\,i\right)} \sqrt{\frac{Q(0)Q\left(\frac{i}{2}\right)}{Q\left(\frac{ik}{2}\right) Q\left(\frac{ik}{2}\right)} S \det G}$$

Find a relation à la (systematic recursive strategy)

$$|\mathrm{MPS}_k\rangle = \hat{T}^{(k-1)}(\mathbf{u_{k-1}})|\delta\rangle + \alpha_1 \hat{T}^{(k-2)}(\mathbf{u_{k-2}})|\delta\rangle + \ldots$$

Transfer matrix

de Leeuw, Gombor, C.K., Linardopoulos, Pozsgay '19

Gombor, C.K., Qian '24

Take the inner product with eigenstate $|\mathbf{u}\rangle$

More general integrability and pairing conditions

Gombor & Bajnok '20

Generalized integrability condition

Factorizable T-matrix: $T(u) = T_+(u)T_-(u)$

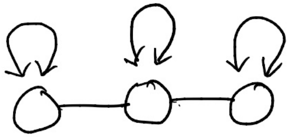
$$\langle\Psi_0|\Pi\, T_\pm(u)\,\Pi\; = \langle\Psi_0|T_\pm(u), \qquad \text{uncrossed}$$

$$\langle\Psi_0|\Pi\, T_\pm(u)\,\Pi\; = \langle\Psi_0|T_\mp(u), \qquad \text{crossed}$$

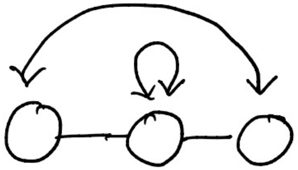
Generalized pairing condition

$$\{u_{aj}\} = \{-u_{\sigma(a)j}\}, \qquad a = 1, \ldots, \#\text{ of nodes}$$

$\sigma(a) = \mathrm{Id}$, chiral overlap



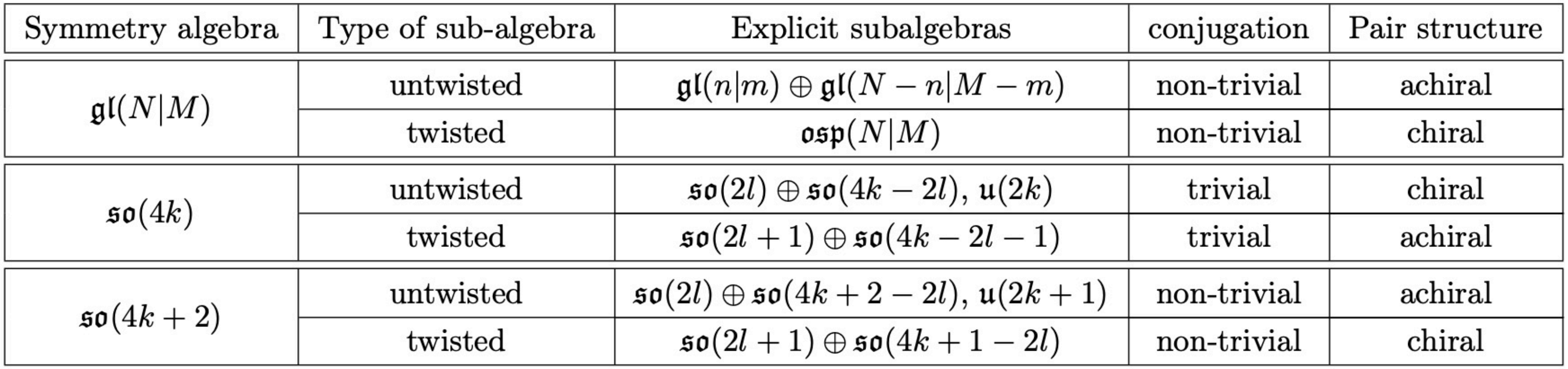
$\sigma(a) \neq \mathrm{Id}$, achiral overlap



General pairing condition

Gombor &
Bajnok '20

Gombor '24



Symmetry algebra	Type of sub-algebra	Explicit subalgebras	conjugation	Pair structure
$\mathfrak{gl}(N\|M)$	untwisted	$\mathfrak{gl}(n\|m) \oplus \mathfrak{gl}(N-n\|M-m)$	non-trivial	achiral
	twisted	$\mathfrak{osp}(N\|M)$	non-trivial	chiral
$\mathfrak{so}(4k)$	untwisted	$\mathfrak{so}(2l) \oplus \mathfrak{so}(4k-2l)$, $\mathfrak{u}(2k)$	trivial	chiral
	twisted	$\mathfrak{so}(2l+1) \oplus \mathfrak{so}(4k-2l-1)$	trivial	achiral
$\mathfrak{so}(4k+2)$	untwisted	$\mathfrak{so}(2l) \oplus \mathfrak{so}(4k+2-2l)$, $\mathfrak{u}(2k+1)$	non-trivial	achiral
	twisted	$\mathfrak{so}(2l+1) \oplus \mathfrak{so}(4k+1-2l)$	non-trivial	chiral

For the integrable $SO(6)$ spin chain of relevance for $\mathcal{N}=4$ SYM:

Domain wall: subalgebra $so(3) \oplus so(3)$ twisted, pair structure chiral

GW-Surface: subalgebra $so(2) \oplus so(4)$ untwisted, pair structure achiral

't Hooft line: subalgebra $so(5)$ twisted, pair structure chiral

Overlaps with $|\mathrm{MPS}\rangle$ directly

Gombor '24

Solve KT-relation

$$\sum_{k,\gamma} K^{\alpha,\gamma}_{i,k}(u)\langle\Psi^0_{\gamma,\beta}|T_{kj}(u) = \sum_{k,\gamma}\langle\Psi^0_{\alpha,\gamma}|\widehat{T}_{ik}(-u)K^{\gamma\beta}_{kj}(u)$$

Overlap can be extracted from $K(u)$
by recursive procedure

Universal form

$$\frac{\langle \mathrm{MPS}_{d_b}\,|\mathbf{u}\rangle}{\langle \mathbf{u}\,|\mathbf{u}\,\rangle^{\frac{1}{2}}} = \left\{\sum_{k=1}^{d_b}\beta_k\prod_{j=1}^{n_+}F_j(u_j^+)\right\}\sqrt{\frac{\det G_+}{\det G_-}}$$

(Square roots of) Baxter polynomials

Gukov Witten surface defects in $\mathcal{N} = 4$ SYM

	Ordinary	Rigid
Supersymmetry	1/2 BPS	1/2 BPS
D3-brane geometry	$AdS_3 \times S^1$ Cone-shaped in AdS	$AdS_3 \times S^1$ Cone collapsed
Parameters	Continuous	Discrete
Symmetries	$PSU(1,1\|2) \times PSU(1,1\|2) \times SO(2)$	$SU(1,1\|2) \times SU(1,1\|2)$
Background	Commutative	Non-commutative

Gukov Witten surface defects in $\mathcal{N} = 4$ SYM

Ordinary case

$$\Phi^{\rm cl} = \frac{1}{\sqrt{2}z}{\rm diag}((\beta_1 + {\rm i}\gamma_1)\mathbb{1}_{N_1}, \dots, (\beta_M + {\rm i}\gamma_M)\mathbb{1}_{N_M})\,, \quad z = re^{i\psi}$$

$$A^{\rm cl} = {\rm diag}(\alpha_1\mathbb{1}_{N_1}, \alpha_2\mathbb{1}_{N_2}, \dots, \alpha_M\mathbb{1}_{N_M})d\psi, \qquad \alpha_i \text{ real and periodic}$$

Transverse coordinates: r, ψ

Rigid case

$$A^{\rm cl} = \frac{t_3}{\log\frac{r}{r_0}}d\psi\,, \qquad \Phi^{\rm cl} = \frac{t_1 + {\rm i}t_2}{\sqrt{2}z\log\frac{r}{r_0}}\,,$$

t_1, t_2, t_3 k-dimensional irreducible representation of $SU(2)$

Integrability properties of Gukov-Witten defects

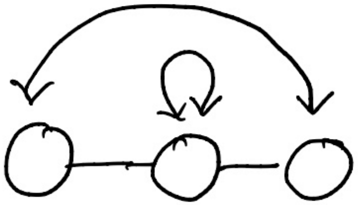
	Ordinary	Rigid
$SU(2)$-sector	Trivial	Integrable $\forall\, k \in \boldsymbol{N}, k \geq 2$
$SO(6)$ sector	Integrable $\forall\, \beta_i, \gamma_i$	Integrable for $k = 2$
SL(2)-sector	Non-integrable	Integrable for $k = 2$
Higher loops	—	Possibly for $k = 2$

Chalabi, C.K &
Su ‘2025

Partial results
A. Holguin ‘25

One-point functions for rigid Gukov-Witten defects

$$\langle \mathcal{O}_L(x) \rangle \propto \frac{\langle \mathrm{MPS} \,|\mathbf{u}\rangle}{\langle \mathbf{u}\,|\mathbf{u}\,\rangle^{\frac{1}{2}}} = \sqrt{\frac{Q_2(i/2)}{Q_2(0)} \frac{\det G_+}{\det G_-}}$$



Any operator built from scalars $\phi_1, \ldots, \phi_6$

Future Directions

- Carry out higher loop integrability bootstrap of Gukov-Witten surface defects.
- Complete the all order integrability bootstrap of the ´t Hooft line
- Develop boundary conformal bootstrap for Nahm pole defects

Thank you